

Maintenance tasks scheduling in a limited space with limited resources

Xu Luo^{1,2} , Jinlong Zhao^{1,3,*}, Wenjin Zhu⁴ , Shigang Zhang^{1,2}, Bin Wu⁴

¹ College of Intelligence Science and Technology, National University of Defense Technology, China

² Key Laboratory of Equipment State Sensing and Smart Support, National University of Defense Technology, China

³ China North Vehicle Research Institute, China

⁴ School of Mechanical Engineering, Northwestern Polytechnical University, China

* Corresponding Author: zhaojinlong817@sohu.com

Abstract

Maintenance tasks scheduling is one of the essential factors in maintenance engineering, which is the bridge connecting maintenance decision optimization and the implementation of maintenance activities. The success of a maintenance policy depends on the spare parts logistics, the technicians, the tool kit and reasonable task scheduling under the proper environment. This paper focuses on the maintenance task scheduling problem in confined spaces under resource-constrained scenarios (e.g., maintenance task planning during level maintenance or annual inspections in ship engine rooms or offshore drilling platforms), where spatial limitations and multi-resource availability jointly impact maintenance efficiency. Considering the constraints of different priority of maintenance tasks, unequal required area for maintenance activities and corresponding maintenance time with different resource limits, it is important to organize and schedule the maintenance activities in a way which can minimize the total maintenance duration. In this paper the main objective is to obtain the optimal order and resource allocation level of each maintenance task within minimal completion time under a given irregular limited area. The problem is modelled as a mixed integer quadratic programming problem, and numerical case studies with 12 maintenance tasks are used to illustrate the optimal solutions under different scenarios and to verify the effectiveness of the proposed model. The study provides a very prototype model and based on which some dynamic scheduling strategy can be proposed through modifying the task priority, maintenance resource level and parameters of limited space.

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Highlights

- Maintenance tasks scheduling is developed in an engine room with irregular layout.
- The priority is used to simplify the logical relationship among maintenance tasks.
- The maintenance resource and time of each maintenance task are not the same.
- Each maintenance task needs a required work area which cannot be overlapping.
- The scheduling problem is modelled by a mixed integer quadratic programming problem.

1. Introduction

Maintenance is a very important way that keeps the desired system reliability and significantly reduces system security risk for any mechanical equipment in various fields, such as aviation industry, naval vessels, nuclear power plants, etc. Current

mainstream and existing reliability modelling concerning maintenance models can be classified as follows:

(1) Degradation modelling and remaining useful life (RUL) estimation by statistical methods and by stochastic process: In [1] the remaining useful life prognostic is utilized to make maintenance decision to minimize production fluctuations in a factory which consists of multiple complex systems using redundant components. In [2] a multi-state system subject to competing and hidden failures by degradation and random shocks is studied where the impact of imperfect inspection is considered.

(2) Integration maintenance decision with multiple constraints: In [3] a tri-objective optimization problem is modelled based on the maintenance of a multi-unit nuclear power plant sites, where the unavailability, risk and cost are minimized. In [4] the framework of predictive Maintenance

(PdM) solutions for industry 5.0 is discussed with the related domains including the human, task, organization and environment. In [5] a multi-stage hybrid assembly flow-shop scheduling is incorporated with considerations for job release times and service tasks to implement preventive maintenance. In [6] the limited maintenance resources is considered to jointly optimize the fleet-level selective maintenance and repairpersons assignment. In [7] a resource-constrained project scheduling problem with state-dependent maintenance activity motivated by aircraft maintenance projects is studied.

(3) Upstream such as product design, spare parts and logistics: The design and sequential preventive maintenance schedule are integrated with minimal repair of a system and a two-stage stochastic programming model is deduced where the numbers of components to be used in the subsystems are optimized in the first stage by [8]. In [9] a m-out-of-n standby system with imperfect element reactivation is considered where the mission success probability and expected mission cost are evaluated.

(4) Integrated maintenance with production and other indicators: In [10] the maintenance tasks and production are scheduling simultaneously to minimize the makespan and the total maintenance cost. In [11] the information collected from physical assets for predictive maintenance is planned to be utilized by a digital twin way integrating the prediction and production by dynamic simulation. In [12] the selective maintenance for part of the system is considered with uncertain operating conditions for the next mission. With the framework of Markov decision process, the mission-critical systems whose deterioration can be controlled by adjusting the performance levels is studied by [13]. To schedule maintenance and production activities for a job-shop production system, a rolling-horizon approach is proposed through dynamic opportunistic maintenance grouping and modelled by a mixed-integer nonlinear mathematical model in [14].

The above literatures about maintenance modelling and optimization are all concerned with several factors: 1) Uncertainty of RUL and failures result in stochastic maintenance decision; 2) Maintenance decision should be made by considering constraints in practice, including maintenance time window, limited resources and so on. Thus it is necessary to establish a model which could bridge the gap between

maintenance decision and maintenance task scheduling.

1.1. Maintenance scheduling in industrial practice

Most existing literatures focus on reliability modelling and maintenance optimization. However, the mission success of maintenance activities was rarely being studied due to it crosses several disciplines. Taking a broad definition, maintenance is a comprehensive set of behaviors composed of decision-making, specific maintenance activities, spare parts inventory, tools and maintenance personnel and all the other relevant activities. The successful implementation of the maintenance strategy depends on many factors. For example, an early study [15] is about the scheduling of workers in a constricted area motivated by an aircraft maintenance scheduling problem. Later [16] considers the maintenance tasks with limited resources. In [17] workers-constrained shutdown maintenance scheduling with flexible skills is investigated. For the detailed and refined maintenance activity scheduling and optimization model can be referred to [18], which is a flexible job-shop scheduling problem considering machine capacity, time lags, holding times, and sequence-dependent setup times.

Maintenance task scheduling is the bridge of maintenance planning and maintenance management to industrial practice. Recently some researchers concentrate on building the maintenance scheduling models including the diverse-skilled technician, routing of the crew, the maintenance resource and the maintenance environment in [19]. In [20] the authors dealt with the resource assignment problem for maintenance activities scheduling by generating an optimal sequence of tasks. In [21] a multi-criteria methodology to insert new tasks into a planned maintenance tasks schedule is proposed with the aim of finding the best resource and time to perform each maintenance task. Aircraft maintenance management is modelled as task network-based project scheduling in [22], where the resource constraints, coordination of multiple projects and dynamic re-scheduling are considered. In [23] a two-stage iterative heuristic algorithm is proposed to optimize the maintenance service network by optimizing the workforce capacity in repair shops with a single location repair shop through cross-training.

1.2. Maintenance scheduling in limited areas

Apart from the typical representatives highlighted above,

maintenance tasks scheduling with limited resources and limited areas distinguishes itself. Restricted areas are defined as places where access is limited, or where certain environmental conditions or safety requirements need to be considered. Examples of limited areas include nuclear power plants, oil refineries, engine room, and chemical plants. In the engine room of a vessel, large equipment, such as steam turbines, booster boilers, propulsion and other electromechanical equipment are located in. The facility layout problem of the engine room or cabins has been studied by [24,25] usually with the objectives of overall material handling costs, number of workshops, utilization ratio of workshop and the safety factors. The optimum layout considering efficiency and quality of maintainability design of a ship cabin is proposed by [26], where the maintenance operating space and such practical factors for routine operation and maintenance are considered. Further in [27] the worker skills and ergonomic constraints are considered.

Among the competing objectives integrated with the interest of multiple players including manufacturers, customers and operators, efficiency of maintainability is one of the hidden problems easy to be neglected. However, maintenance is the last but not the least. During a long-lasting mission, the engineers and maintenance staffs carry out periodic maintenance and condition-based maintenance for safety and efficient operation. The time window for all types of inspection and maintenance activities on board of all the large equipment are limited, which put forward the requirement of the efficiency of maintenance. On one hand, the size of maintenance staffs and maintenance resource (such as tools, spare parts) are limited during the maintenance mission. Thus, when facing several maintenance tasks in sequence, the maintenance staffs and resource may not be available all the time. On the other hand, from the perspective of mechanism structure design and space layout, the engine room is limited by the height and length and the size of passages between machines are limited. Maintenance activities require specific and sufficient work area as safe operating space due to physical and psychological factors from the aspect of operating tools and technicians. Thus, some maintenance activities cannot be implemented due to the conflicts of staffs, resources, space and time. The main challenges of maintenance scheduling in limited areas include:

- Limited access: limited areas often have limited access, which can make it difficult to implement maintenance activities as by [26].
- Safety requirements: limited areas often have specific safety requirements that need to be met during maintenance, which can add complexity to the scheduling process.
- Environmental conditions: limited areas may be subject to specific environmental conditions, such as high temperatures or corrosive environments, that need to be taken into account when scheduling maintenance as by [4].
- Time windows: in some limited areas, maintenance may need to be performed during specific time windows to minimize disruptions or potential hazards, which can also make scheduling more difficult.
- To address these challenges, researchers have proposed various models and algorithms for maintenance scheduling optimization. Some of the approaches include:
- Mathematical models: researchers have proposed mathematical models for maintenance scheduling optimization, such as linear programming and mixed integer programming [28,29,30].
- Optimization techniques: researchers have proposed the use of optimization techniques such as genetic algorithms, simulated annealing, and particle swarm optimization to solve maintenance scheduling problems in limited areas or with limited resources [31,32].
- Simulation: researchers have also proposed the use of simulation to evaluate the performance of different maintenance schedules and identify the most effective strategies [19].

Nevertheless, to the best knowledge of the authors, the study of scheduling of maintenance tasks based on a limited area with irregular layout considering space conflict is rare. For the most of existing literatures, the space conflict due to the limited area is simplified as a factor which decides the sequential relationship or structure between nodes of task network. The influence of space conflict in the limited area has never been fully investigated. In order to complete the maintenance tasks in time within limited resources and limited area, meanwhile keep

the safety and reliability, this study develops an integrated model to balance the resource allocation, order of tasks and space conflict. The contribution of the paper is to develop a very prototype model which put forward the effects of space conflicts when scheduling tasks and based on which some dynamic scheduling strategy considering randomness can be proposed through modifying the task priority, maintenance resource level and location parameters.

The remainder of the paper is organized as follows. Section 2 presents the model background and the elementary assumptions. Section 3 provides a detailed description of the mathematical model considered in the study. Section 4 provides a comprehensive numerical experiments and optimal solution verification. The conclusions and directions of future work are presented in Section 5.

2. Maintenance in a limited space: background and assumptions

2.1. Elementary of maintenance model

Due to the scale and the operating mechanism of limited area, such as in a ship engine room, maintenance on board usually consists of periodic or routine maintenance, preventive/condition-based maintenance, corrective/emergency maintenance and recently the so-called predictive maintenance. In this study, we present a simplified and general model whose assumptions are abstracted based on the practical experience for equipment maintenance and repair.

2.1.1. Maintenance task

This study focus on the implementation success of the scheduled maintenance tasks. Due to the limited maintenance staff and limited space in machine room onboard, sometimes different maintenance activities have to wait in queue until the necessary staff, resource and space are in position. Without considering the specific and useful purpose of each maintenance activities corresponding to each machine, we label each specific maintenance activities as maintenance task $\{M_1, M_2, \dots, M_n\}$ and simplify it by the following four properties: priority of maintenance task, maintenance resource, maintenance space and maintenance time duration.

In the numerical examples, each maintenance task represents an equipment-specific maintenance job in a confined engine-room-like workspace rather than a purely abstract job.

Representative assets include pumps, valves, filters, heat exchangers, auxiliary piping modules and similar service equipment commonly arranged in close proximity in ship engine rooms or offshore utility compartments. The maintenance activities considered in this prototype model correspond to typical onboard interventions such as inspection, disassembly, cleaning, replacement of wear parts, alignment, tightening and functional testing. The detailed engineering content of each task is abstracted into four scheduling attributes, namely task priority, maintenance resource demand, maintenance space requirement and maintenance duration.

2.1.2. Priority of maintenance task

The priority of a maintenance task concerns to the failure consequence of the involved component, the cost of corrective maintenance and the other inherent properties related to the component. The priority of a maintenance task can change with time:

- Scenario 1): for condition-based/predictive maintenance of a critical component, the maintenance is scheduled in advance with uncertainty. During the waiting time, with more incoming information, the confidence of the real state of the component will be updated and the classification of the priority will be modified.
- Scenario 2): for some critical components with redundancy, one of them is scheduled for maintenance with a given classification of priority. As the component is in the waiting queue, the health state of the redundancy degrade drastically, then it is necessary to upgrade the priority of the maintenance task in waiting queue.

The priority of maintenance task M_i is denoted as ρ_i . In this prototype model the priority of maintenance task keeps constant until its maintenance activity completed. We stipulate that tasks with higher priority must be completed before tasks with lower priority. The tasks with the same priority level will be scheduled due to the other constraints.

2.1.3. Maintenance resource and maintenance time duration

The implementation of maintenance task needs staff with toolkit and spare parts, which can be referred to maintenance resources. The time required to perform maintenance task is partly dependent on the resource that it is allocated. Usually the

maintenance resource is limited onboard and the ship in the distant ocean is inconvenient to be supplemented or rescued. The maintenance resource is one of the prime constraints which affect the scheduling and completion of maintenance task. In this study we do not differentiate the diverse categories of maintenance resource and assume that the total amount of maintenance resource is denoted by R . The maintenance resource is assumed to be reusable and thus the total maintenance resource occupied during the maintenance mission at any time should be less than the threshold R . When a maintenance task is completed, the occupied resource will be released and be available for the other maintenance tasks immediately. The time duration of each maintenance task is dependent on the level of maintenance resource allocated on the maintenance task.

2.1.4. Maintenance space

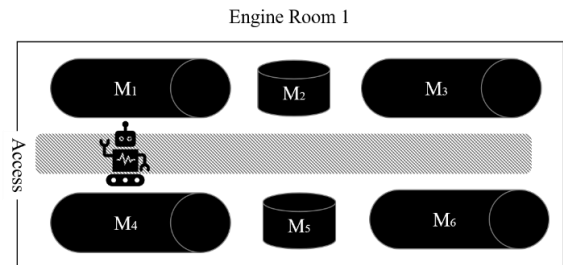
Due to the limited space and the layout in the engine room, the effective space for maintenance operation is limited. Figure 1 gives two examples of simplified two-dimensional layouts of two engine rooms with 6 the maintenance tasks labelled as M_i , $i = 1, 2, \dots, 6$. The rectangle texture area shows the accessible area for the maintenance crew with toolkit and spare parts. When M_1 or M_4 is being performed, the pathway to the accessible area of M_2, M_3, M_5 and M_6 is blocked down. The similar problem has been studied widely in the flowshop scheduling [33,34] and automated guided vehicles (AGVs) scheduling field [35] and truck scheduling problem at dock [36]. But our problem is different in the following ways:

The operating conditions motivating the model are those of confined technical compartments in which several maintenance crews work under limited access, narrow passages and safety-clearance requirements. Typical environmental conditions include elevated temperature, humidity, vibration, oil contamination and, for offshore platforms, potentially corrosive marine exposure. These factors do not change the deterministic optimization structure directly in the present paper, but they justify why non-overlapping work envelopes and coordinated resource allocation are essential practical constraints.

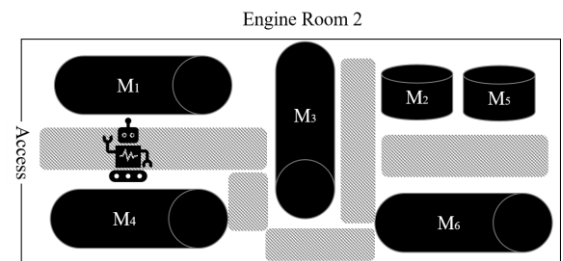
- For the classic AGV scheduling problem, the discharge time at each discharging place is assumed to be equal. But in this study, due to the complexity of different

maintenance tasks, the time of each maintenance task is unequal.

- The layout of the port is usually regular and symmetrical, but the layout of the engine room in this study is irregular.
- The maintenance task needs a required work area during maintenance activity.



(a) Regular layout



(b) Irregular layout

Figure 1. Regular and irregular layouts of two engine rooms.

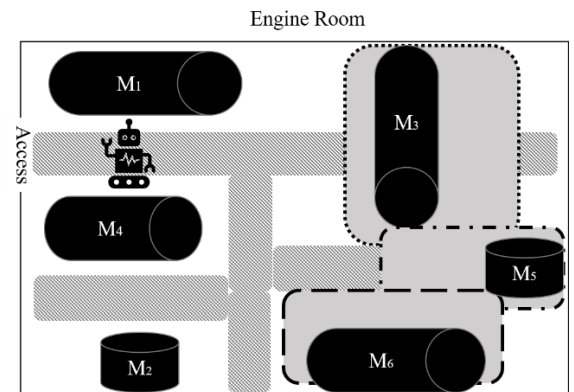


Figure 2. Engine room with a maintenance task in progress.

Besides, due to the size of the spare parts and the layout of the equipment, each maintenance task requires a certain work area to carry out. Thus, in some cases even the pathway is available, the residual space is not enough for a maintenance task. Figure 2 gives an example where a maintenance task (M_3) is in progress and the shadow surrounding M_3 with dotted line is the space that M_3 needs to take. And even though the pathway to M_5 and M_6 are accessible, the work area of these maintenance tasks marked by dash dot and dash respectively are partly overlapping with M_3 . Thus these maintenance task cannot be

implemented in the same time and the sequence of maintenance tasks should be optimized.

3. Mathematic model of maintenance in a limited space

3.1. Assumptions and definitions

3.1.1. Maintenance space

The time required to perform maintenance tasks is denoted by t_i of task M_i with amount of maintenance resource γ_i allocated to M_i . In order to simplify the model, the machines with different area occupancies in the engine room is labelled as maintenance tasks M_i , $i = 1, 2, \dots, n$ with the meshed layout. For example, a layout with size 6×8 units with n rectangular shadow area is shown by Fig 3. And the rules of numbering of the maintenance tasks is based on a coordinate system and starts from the two-dimensional origin (0,0) and ends at (8,6) as shown by Fig 3. Each maintenance task is identified according to its coordinates of the four corner points of the rectangle shadow and labelled by ascending order according to the abscissa and ordinate. For example, the machine located in top left corner is labelled as M_6 with coordinate $\{(0,5),(0,6),(4,5),(4,6)\}$ and the machine located in the bottom right-hand corner is labelled as M_3 with coordinate $\{(6,1),(6,2),(8,1),(8,2)\}$ in Fig 3. Notice that in this study, the situation of diagonally connected of two machines(maintenance tasks) doesn't included, indicating that the maintenance tasks are disjointed among each other.

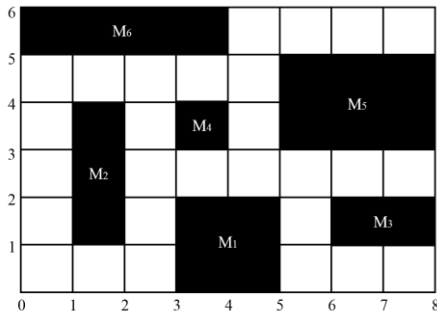


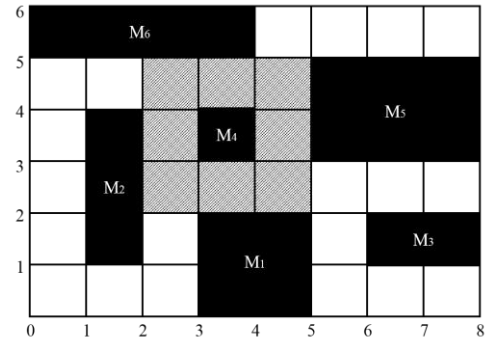
Figure 3. Engine room with 6 labelled maintenance tasks.

The area of the maintenance task M_i is defined based on the coordinate and precisely by $S_i = (x_i^r - x_i^l)(y_i^u - y_i^d)$, where x_i^l and x_i^r are two coordinates of the M_i along the horizontal direction, y_i^d and y_i^u are two coordinates of the M_i along the vertical direction respectively.

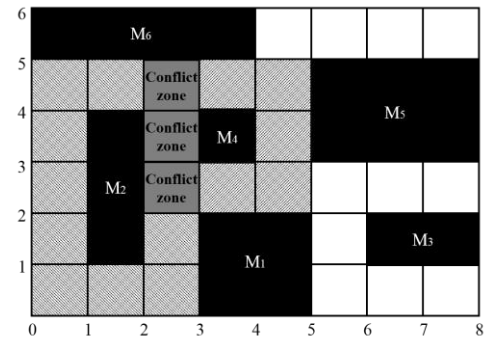
3.1.2. Required space of maintenance tasks

In this study, the time spending on the path to each machine is negligible compared to the time duration of maintenance. As

mentioned in Section 2.1.4, the implementation of each maintenance task needs some specific work area which cannot overlap among different maintenance tasks. Based on the Section 3.1.1 for each M_i the required work area is defined based on $A(l_i, r_i, d_i, u_i)$, where l_i and r_i denote the number of rows of mesh located in the left and right of M_i and similarly u_i and d_i denote the number of rows of mesh located upper and down of M_i . And the required work area is the surrounded area defined by the intersection of the four coordinates of the four corners minus the area of M_i . Figure 4 gives an example of M_4 with $A(l_4, r_4, d_4, u_4) = (1, 1, 1, 1)$, where the texture area surrounding the M_4 is the required work area of M_4 . If $A(l_2, r_2, d_2, u_2) = (1, 1, 1, 1)$ for M_2 given $A(l_4, r_4, d_4, u_4) = (1, 1, 1, 1)$, then the work areas of M_2 and M_4 are overlapping, which leads to the conflict between M_2 and M_4 if they are scheduled to be maintained in the same time.



(a) Work area of a maintenance task



(b) Conflict of work areas of two tasks.

Figure 4. Layout of maintenance tasks with required work area.

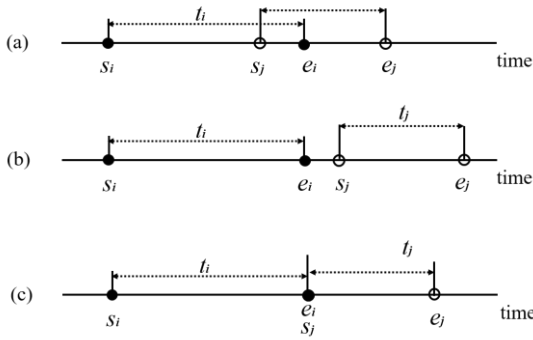
3.1.3. Priority constraint

For clarity, in Eqs. (1)-(4), s_i and s_j denote the starting times of tasks M_i and M_j , respectively; t_i and t_j denote their maintenance durations; and $e_i = s_i + t_i$ and $e_j = s_j + t_j$ denote their completion times. The corresponding indicator in Eq. (1) takes value 1 when the two execution intervals overlap in time and 0 otherwise. In

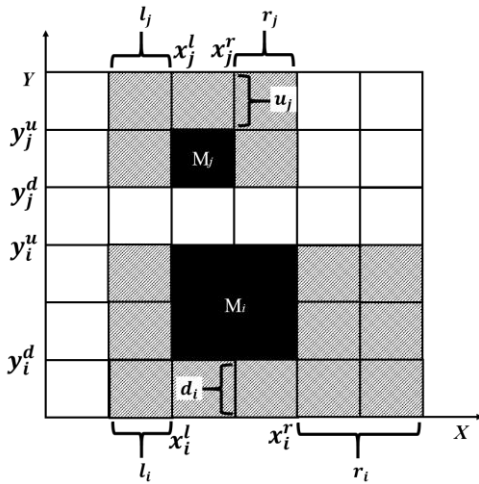
the space-related expressions, the geometric description of each task is determined by its rectangular location in the meshed layout together with the required work-area extension $A(l_i, r_i, d_i, u_i)$, where l_i , r_i , d_i and u_i denote the left, right, lower and upper buffer widths required for safe maintenance operations.

The priority of maintenance task M_i denoted by ρ_i , is fixed and is given at the beginning for the present model. The priority of maintenance task is represented by the rule that the finish time of maintenance tasks with high priority should be no later than the starting time of the maintenance task with low priority. Let the starting time of maintenance task M_i be s_i and the maintenance duration time be t_i . Then $e_i = s_i + t_i$ is the time when M_i is finished. The indicator function that indicates the conflict of start time of maintenance tasks M_i and M_j is denoted by $I_{T_{ij}}$ and is defined as follows:

$$I_{T_{ij}} = \begin{cases} 1 & (s_i - s_j)(s_j - e_i) > 0 \\ 0 & \text{others} \end{cases} \quad (1)$$



(a) 3 scenarios of time line of two maintenance tasks



(b) Conflict of work areas of two maintenance tasks

Figure 5. Schema of potential conflict of maintenance time and work area between two maintenance tasks.

As shown by Fig 5(a), between two maintenance tasks M_i

and M_j , three scenarios are exclusive: (a)conflicted with $I_{T_{ij}} = 1$, (b)disjointed with $I_{T_{ij}} = 0$ and (c)successive with $I_{T_{ij}} = 0$.

3.1.4. Space conflict

Recall the definition of x_i^l , x_i^r , y_i^u , y_i^d and $A(l_i, r_i, d_i, u_i)$ by Fig 5(b). Based on the analysis of Eq 1, the indicator function indicating the conflict of maintenance work area of maintenance tasks M_i and M_j is denoted by $I_{S_{ij}}$ and is defined as follows: Accordingly, Eqs. (2) and (3) describe whether the required work areas of M_i and M_j overlap in the horizontal and vertical directions, respectively. The related indicators equal 1 when overlap exists in the considered direction and 0 otherwise; Eq. (4) then combines the two directional conditions to determine whether a spatial conflict is present.

$$I_{S_{ij}}(x) = \begin{cases} 1 & (x_i^l - l_i - x_j^l + l_j)(x_j^l - l_j - x_i^r - r) > 0 \\ 0 & \text{others} \end{cases} \quad (2)$$

where $i, j \in N, i \neq j$. $(x_i^l - l_i - x_j^l + l_j)(x_j^l - l_j - x_i^r - r) < 0$ means that M_i and M_j are conflicted with each other in horizontal direction and hence $I_{S_{ij}}(x) = 1$ given $x_i^l - l_i < x_j^l - l_j$. For the scenario of $x_i^l - l_i > x_j^l - l_j$, the similar function can be deduced.

$$I_{S_{ij}}(y) = \begin{cases} 1 & (y_i^d - d_i - y_j^d + d_j)(y_j^d - d_j - y_i^u - u) > 0 \\ 0 & \text{others} \end{cases} \quad (3)$$

where $i, j \in N, i \neq j$. $(y_i^d - d_i - y_j^d + d_j)(y_j^d - d_j - y_i^u - u)$ means that M_i and M_j are conflicted with each other in vertical direction and hence $I_{S_{ij}}(y) = 1$ given $y_i^d - d_i < y_j^d - d_j$. For the scenario of $y_i^d - d_i > y_j^d - d_j$, the similar function can be deduced.

$$I_{S_{ij}} = I_{S_{ij}}(x) * I_{S_{ij}}(y), i, j \in N, i \neq j \quad (4)$$

If and only if both $I_{S_{ij}}(x) = 1$ and $I_{S_{ij}}(y) = 1$, i.e., the conflict occurs both with the direction of horizon and vertical, M_i and M_j are conflicted with each other.

3.2. Model 1: unlimited maintenance resource

The MIQP formulation was chosen because the prototype model contains pairwise non-overlap logic linking task timing and workspace conflict. In the present formulation, these pairwise relations can be expressed directly and compactly. A linearized MILP reformulation is possible, but it would require additional auxiliary binary variables and big-M constraints for many task pairs. This would enlarge the formulation and may also weaken the linear relaxation if the big-M values are conservative. Since the main purpose of this study is to establish a clear exact prototype model for limited-space maintenance scheduling, the

MIQP form was retained in order to keep the conflict structure explicit and the formulation relatively compact.

The operation of each maintenance task is characterized by two variables: the starting time s_i and the maintenance duration time t_i , and is denoted by (s_i, t_i) . The maintenance decision is to optimize the s_i and t_i in order to minimize the total maintenance time duration T_{end} .

$$\min \max_{i \in N} (s_i + t_i) \quad (LP1)$$

$$s.t. \sum_{i=1}^n O_{ij} \leq 1, \text{ for } i, j = 1, 2, \dots, n \quad (LP1-1)$$

$$\sum_{j=1}^n O_{ij} \leq 1, \text{ for } i, j = 1, 2, \dots, n \quad (LP1-2)$$

$$\sum_{i=1}^n \sum_{j=1}^n O_{ij} = 1, \text{ for } i, j = 1, 2, \dots, n \quad (LP1-3)$$

$$\sum_{j=1}^n O_{ij} (\rho_i - \rho_j) \geq 0, \text{ for } i, j = 1, 2, \dots, n \quad (LP1-4)$$

$$\sum_{j=1}^n O_{ij} (s_i - s_j) \leq 0, \text{ for } i, j = 1, 2, \dots, n \quad (LP1-5)$$

$$I_{Tij} \cdot I_{Sij} = 0, \text{ for } i, j = 1, 2, \dots, n \quad (LP1-6)$$

Constraints of the integer programming problem (LP1-1, LP1-2 and LP1-3) guarantee that the order of maintenance tasks $M_i, i \in N$ should satisfy the following condition, i.e., for each $M_i, i \in N$ there exists only one preceding maintenance task and only one succeed maintenance task except the first (which has no preceding task) and the last (which has no succeed task) according to the order. Thus an auxiliary variable is introduced and is denoted by $O = (O_{ij})_{n \times n}$, $O_{ij} \in \{0, 1\}$, where O is a matrix with size $n \times n$. Hence n maintenance tasks generate a sequence of tasks with length $n-1$. According to the following example,

$$O = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (5)$$

the above matrix O (Eq 5) presents the sequence as $1 \rightarrow 2 \rightarrow 3 \dots \rightarrow n \rightarrow n-1$ and decides the maintenance order be as: the maintenance starts from M_1 then followed by M_2 and eventually ends when M_n is completed. Notice that matrix O is a designed auxiliary decision variables which guarantee the procedures of optimization to be proceeded. The optimal decision variables are (s_i, t_i) , i.e., the optimal starting time and maintenance time duration for each maintenance task M_i .

Constraints (LP1-4) guarantee the property of priority of maintenance task. Constraints (LP1-5) make sure that the starting time of maintenance task with higher priority will be earlier than the one with lower priority.

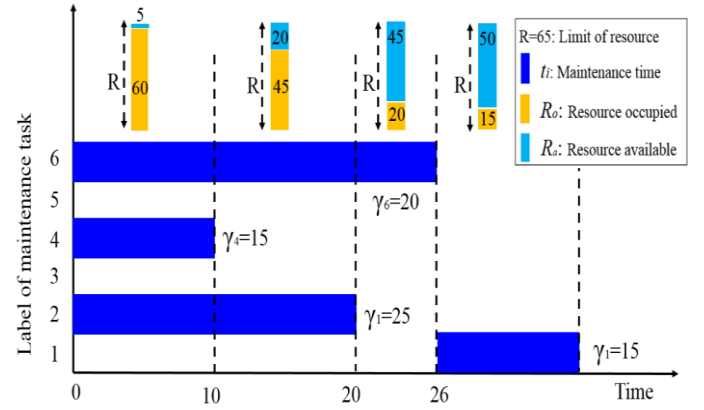


Figure 6. Constraint of resource with sliding window.

Constraints (LP1-6) make sure that the work area and the time are feasible. Constraints (LP1-6) can be satisfied if and only if $I_{Tij} = 0$ and $I_{Sij} = 0$.

3.3. Model 2: limited maintenance resource

In this subsection, the maintenance resource is reusable and limited with a constant limit R , which means that the cumulative maintenance resource that the undergoing maintenance tasks occupied should be less than R from the beginning to the end of the total maintenance mission. And further, each maintenance task can be completed with three levels from high, medium to low with different maintenance time duration corresponding to three levels of resource allocation: (t_i^k, γ_i^k) , where $0 < \gamma_i^k \leq R, i \in N, k = 1, 2, 3$ corresponding to high, medium and low level respectively. Once the resource of amount of γ_i^k is distributed to M_i , the resource will keep as constant as γ_i^k from time moment s_i to moment $s_i + t_i^k$. For simplification, the restrict of maintenance resource is considered by discrete time unit and is followed the following optimization problem as:

$$LP1 \quad (LP2) \quad \sum_{i=1}^n \sum_{j=1}^n O_{ij} (\gamma_i^k + \gamma_j^h) \leq R, \text{ for } i, j = 1, 2, \dots, n; k, h \in \{1, 2, 3\} \quad (LP2-1)$$

where LP1 and variables O_{ij} are defined in Section 3.2.

Constraints (LP2-1) restrict the maintenance resource under the limit during all time duration of the maintenance mission. Figure 6 provides an illustration of the constraint of (LP2-1), which indicates that the maintenance resource occupied by M_i will be released when M_i is completed. Thus the resource constraint is in a way of sliding window.

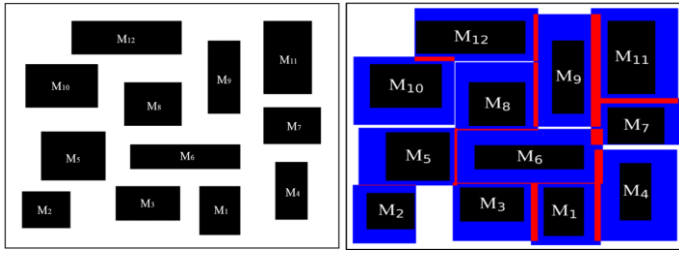


Figure 7. Case study: a two-dimensional layout with 12 maintenance tasks.

4. Solutions and experimental results of maintenance in a limited space

4.1. Parameters setting

A case of a two-dimensional layout with 12 maintenance tasks is considered as shown in Fig 7. By meshing densely, the 12 maintenance tasks are characterized as the following parameters in Table 1 where x_i^l and y_i^d denote the coordinates of the left lower corner of M_i , $x_i^r - x_i^l$ and $y_i^u - y_i^d$ denote the length and the width of the location of M_i . l_i , r_i , d_i and u_i denote the required work area surrounding the maintenance task M_i . ρ_i denotes the

priority of M_i and the higher value of priority means the sooner the starting time should be arranged. The figure in the left of Fig 7 is the original layout of the machine room and the figure in the right presents the required area for maintenance of M_i , where M_i is colored by black, the required area for maintenance operation is colored by blue and the conflict of required area is highlighted by red.

In the case study, the 12 tasks can be interpreted as representative maintenance jobs distributed over a confined engine-room-like layout. They correspond to equipment items or subsystems such as pumps, valve groups, filters, auxiliary drives or pipe-connected service units. The associated maintenance actions are routine or preventive interventions, for example inspection, opening/access preparation, part replacement, cleaning, adjustment and final testing. The operating context is a limited-access maintenance period, such as annual inspection or level maintenance, in which several jobs compete simultaneously for manpower, tools and safe workspace.

Table 1. Parameters setting of 12 maintenance tasks.

Coordinates \ Tasks	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}
x_i^l	542	54	312	751	107	352	718	336	565	64	718	191
y_i^d	36	55	75	78	185	216	284	333	366	382	419	528
$x_i^r - x_i^l$	111	132	176	87	176	301	156	156	88	198	131	301
$y_i^u - y_i^d$	131	98	92	155	131	65	99	118	198	118	198	89
l_i	35	38	20	69	75	55	46	37	51	45	46	4
r_i	46	4	38	71	20	52	46	35	46	35	63	35
d_i	25	38	52	55	13	39	2	12	35	46	25	20
u_i	13	22	13	36	12	44	25	54	71	20	34	34
t_i	2	3	2	1	4	1	3	2	1	3	2	4
Priority ρ_i \ Tasks	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}
Case1	1	1	1	1	1	1	1	1	1	1	1	1
Case2	1	2	3	4	5	6	7	8	9	10	11	12
Case3	3	3	3	3	2	2	2	2	1	1	1	1
Case4	2	1	3	2	2	2	3	1	1	1	1	3

4.2. Experimental verification of the optimality of solutions

The solutions of the mixed integer quadratic programming problem 4.2.3 and LP2 are based on Python and Gurobi Optimizer. First, we will design the following experiments to verify the optimality of the solutions. For the 12-task instance considered in this study, the MIQP model was implemented in Python in PyCharm 2022.1 (Professional Edition) and solved

with Gurobi 10.0.2.

4.2.1. Baseline experiment with the same level of priority

As shown by Table 1, for the column labelled as Case 1, the priority of each M_i takes the same value $\rho_i = 1$ for , which means that each M_i has the same chance to be selected as the first batch of task to be maintained. Similarly, Case 2 in Table 1 presents a fixed sequential arrangement of 12 maintenance tasks with 12 different values of priority. We set Case 1 and Case 2 as the

baseline experiments and find the optimal solutions of 4.2.3 and conduct some contrast experiments in the next two subsections.

4.2.2. Optimal solution of maintenance tasks scheduling with partly fixed order

In order to verify the optimality of the proposed 4.2.3, we can artificially design some potential suboptimal solutions by fixing the order of several M_i s as the following procedures:

- Let $P = [\rho_1, \dots, \rho_{12}] = [1, 1, \dots, 1]$ denote the priority vector of 12 M_i s of the baseline experiment according to Case 1 in Table 1.
- Assign higher priority to each of the 12 maintenance tasks: Let $P_i^o = [1, 1, \dots, 2, \dots, 1]$ denote the vector of the priority where the i th item is $\rho_i = 2, i = 1, 2, \dots, 12$. Then we can obtain 12 sets of cases where each P_i^o is corresponding to the scenario of assigning the first batch of task as $\{M_i\}$, and obtain 12 sets of optimal solutions

and the minimal time duration of the maintenance mission.

- Assign higher priority to two of the 12 maintenance tasks: Let $P_2^o = \underbrace{[1, 2, 3, 1, \dots, 1]}_{i_1, i_2}$ denote the priority vector where the i_1 th and i_2 th items are 2 and 3, $i_1 \neq i_2 = 1, 2, \dots, 12$. Then we can obtain 12×11 sets of cases and obtain 12×11 sets of optimal solutions and the minimal time duration of the maintenance mission.
- Assign higher priority to j of the 12 maintenance tasks: Let $P_j^o = \underbrace{[1, 2, 3, 4, \dots, j, j+1, 1, \dots, 1]}_{i_1, \dots, i_j}$ denote the priority vector where the i_1 th, $\dots$, i_j th items take values from 1 to $j+1$. Then we can obtain $12! \setminus j!$ sets of cases where each $P_j^o = [1, 2, \dots, j, j+1, 1]$ is corresponding to the situation of assigning the first batch of task from $M_{i_1}, \dots, M_{i_j}$, and obtain $12! \setminus j!$ sets of optimal solution.

Table 2. Comparison of optimal solutions of cases with assigned priority of random maintenance tasks.

T_{end}	P	P_1^o	P_2^o	P_3^o	P_4^o	P_5^o	P_6^o	P_7^o	P_8^o	P_9^o	P_{10}^o	P_{11}^o	P_{12}^o
min	7	8	10	11	13	15	18	19	19	22	22	25	28
max	7	11	14	15	18	21	22	24	26	26	27	28	28
mean	7	9.3	11.9	13.3	15.8	18.3	19.8	21.1	22.2	23.8	24.5	26.5	28

The results of the previous procedure are summarized by Table 2. As the numbers of maintenance tasks with different priority increase, the optimal maintenance time duration increases. For $P = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$ all the maintenance tasks are equally to be scheduled and yields to the minimal and optimal maintenance time duration among the $P_i^o, i = 1, 2, \dots, 12$ as Case 1 in Table 1. P_{12}^o means that the order of the 12 maintenance tasks are all fixed randomly and thus both the minimal and maximal of the optimal solution are the largest. Eventually, the Case 2 of Table 1 is one of the cases of P_{12}^o from $12!$ cases. The results are reasonable, because theoretically assigning higher priority to some M_i s will reduce the feasible solution space and thus the suboptimal solutions are obtained.

4.2.3. Optimal solution of maintenance tasks scheduling with fixed priority

In order to further verify the optimality of the proposed problem 4.2.3, we can assign several M_i s the same and higher priority as the following procedures:

- Let $P = [1, 1, \dots, 1]$ denote the vector of the priority of 12 M_i s of the baseline experiment according to Table 1.

- Assign the priority of a maintenance task: Let $P_i = [1, 1, \dots, 2, \dots, 1]$ denote the priority vector where the i th item is $\rho_i = 2, i = 1, 2, \dots, 12$. Then we can obtain 12 sets of cases where each P_i is corresponding to the situation of assigning the first batch of task as M_i , and obtain 12 sets of optimal solutions and the minimal time duration of the maintenance mission.
- Assign the priority of two maintenance tasks: Let $P_2 = \underbrace{[1, 2, 2, 1, \dots, 1]}_{i_1, i_2}$ denote the priority vector where the i_1 th and i_2 th items are 2, $i_1, i_2 = 1, 2, \dots, 12$. Then we can obtain 12×11 sets of cases and obtain $(12 \times 11)/2$ sets of optimal solutions and the minimal time duration of the maintenance mission.
- Assign the priority of j maintenance tasks: Let $P_j = \underbrace{[1, 2, 2, 2, \dots, 1]}_{i_1, \dots, i_j}$ denote the priority vector where the i_1 th, $\dots$, i_j th items take the same value as 2. Then we can obtain C_{12}^j sets of cases where each $P_j = [1, 2, \dots, 2, 1]$ is corresponding to the situation of assigning the first batch of task from $M_{i_1}, \dots, M_{i_j}$, and obtain $C_{12}^j = \frac{12!}{(12-j)!j!}$ sets of

optimal solutions.

Table 3. Comparison of optimal solutions of cases with partly assigned priority.

T_{end}	P	P_1^o	P_2^o	P_3^o	P_4^o	P_5^o	P_6^o	P_7^o	P_8^o	P_9^o	P_{10}^o	P_{11}^o	P_{12}^o
min	7	8	8	9	7	10	10	10	9	9	8	7	7
max	7	11		14	14	14	14	14	14	14	14	11	7
mean	7	9.3	10.3	11.0	11.5	12.6	12.8	12.3	11.4	10.8	10.2	9.1	7

It is obvious that the optimal solutions of the above procedure in Table 3 are better than the results of Table 2. Because for the operation of scheduling with fixed priority, some M_i s are assigned by the same and higher priority. Due to the other constraints of problem, the M_i s with the same priority will be scheduled and optimized further. Hence the feasible solution space is between the baseline case(optimal solution among all cases) and the worst case in Section 4.2.3.

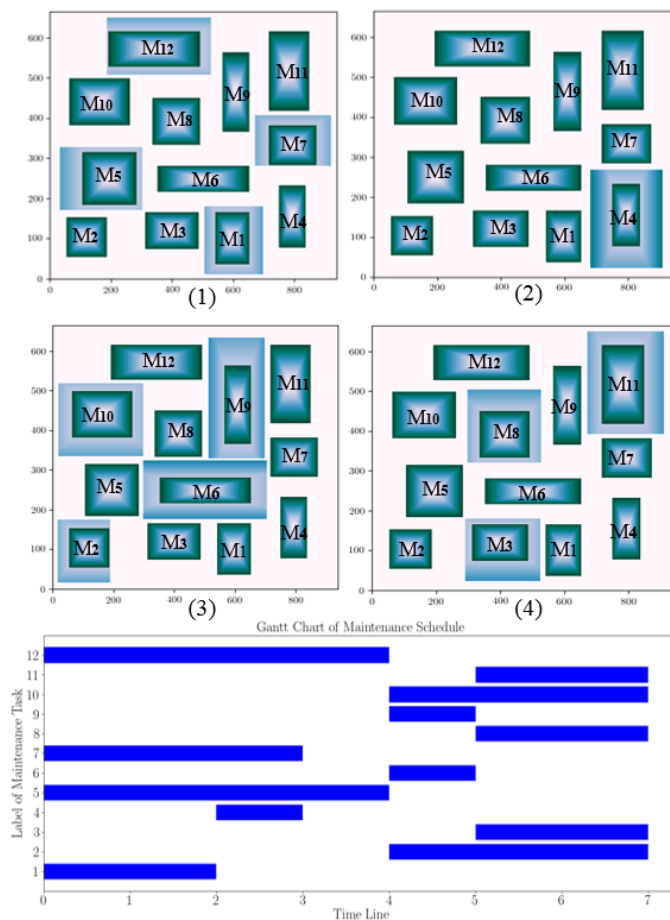


Figure 8. Track and Gantt chart of the optimal scheduling of Case 1 in Table 1.

4.3. Case study-model 1: unlimited maintenance resource

In this subsection, we focus on Case 1, 3 and 4 in Table 1. The track of the optimal of scheduling of maintenance tasks of Case 1 is shown by Figure 8, where the shadow surrounding each M_i

means M_i being under maintenance by taking the required work area. The Gantt chart by Figure 8 presents the scheduling and the procedure of the completion of maintenance tasks. The constraint of priority doesn't work and thus the maintenance time duration and the required work area of M_i dominate the optimality of Case 1 with the same priority of all M_i .

Case 3 is a regular ordering with three equally divided parts and the results are shown by Figure 9. The aim of Case 3 is to provide an intelligible example for our model and solution. It can be seen clearly that M_5, M_6, M_7 and M_8 are of the same priority taking value 2, yet the starting time of M_6 and M_8 are delayed due to the conflict of required work area. Case 4 shown by Figure 10 presents a random setting of priority and the optimal $T_{end} = 12$ which is between Case 1 and Case 3.

4.4. Case study-model 2: limited maintenance resource

Considering the limited maintenance resource through sliding window modeled by the mixed integer quadratic programming problem LP2 and illustrated by Figure 6, the extra parameters of the maintenance resource allocation with depending maintenance duration are listed in Table 4. t_i^1, t_i^2 and t_i^3 are corresponding to three levels of maintenance time duration for each M_i with three levels of resource γ_i^1, γ_i^2 and γ_i^3 corresponding to high, medium and low level respectively. Due to the constraint of resource, the computational complexity increases greatly and the computation time increases exponentially. Thus, in this section, we focus on the effect of the limitation of resource with the optimal mission completion time: (R, T_{end}) given the priority of all M_i in the same level.

Due to the results shown by Table 5, we can notice that as the limit of maintenance resource R increases, the optimal mission time T_{end} decreases and eventually when $R = 150$, the optimal solution is obtained with almost every γ_i^k and t_i^k take the high level. When the maintenance resource is very limited as under 80, the convergence of the optimal solution is quick. As a result, when $R > 150$, the increase of resource will not lead to the optimal solution be further optimized.

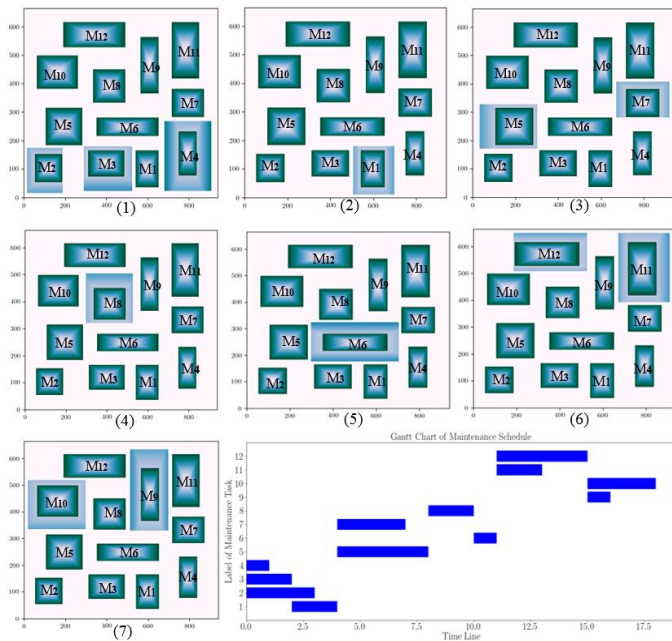


Figure 9. Track and Gantt chart of the optimal scheduling of Case 3 in Table 1.

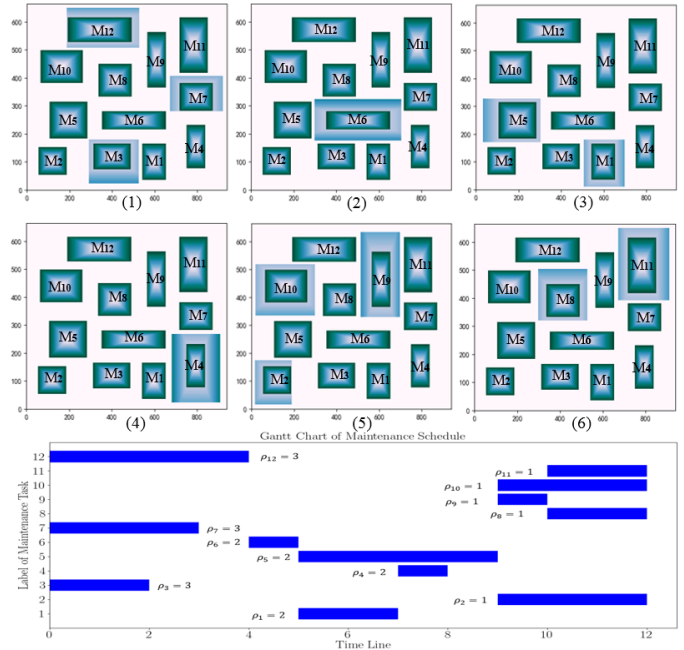


Figure 10. Track and Gantt chart of the optimal scheduling of Case 4 in Table 1.

Table 4. Parameters setting of maintenance resource allocation with depending maintenance duration.

Level	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}
γ_i^1	25	35	30	10	50	10	35	25	10	35	30	50
γ_i^2	15	25	15	10	30	10	25	15	10	25	15	30
γ_i^3	10	15	10	5	15	5	15	10	10	15	10	15
t_i^1	1	2	1	1	2	1	2	1	1	2	1	2
t_i^2	2	3	2	1	4	1	3	2	1	3	2	4
t_i^3	3	4	3	2	6	2	4	3	1	4	3	6

From a scalability perspective, the present exact formulation is best interpreted as a prototype and benchmark model rather than a large-scale real-time scheduler. For an instance with n maintenance tasks, the core sequencing and workspace-conflict logic is generated at the task-pair level, so the number of unordered task pairs is $n(n-1)/2$. This gives 66 potential pairs for $n = 12$, 190 for $n = 20$, and 1225 for $n = 50$, even before introducing the additional coupling created by resource allocation and priority relations. In that sense, the difficulty does not arise only from the number of tasks themselves, but from the simultaneous interaction among starting times, pairwise ordering relations, workspace non-overlap, and, in Model 2, resource allocation over the planning horizon. More precisely, if H denotes the discretized mission horizon in Model 2, the sliding-window resource feasibility conditions introduce an additional $O(nH)$ layer of temporal coupling, because resource usage must remain feasible across time units while task durations themselves depend on the selected resource levels.

Priority constraints further couple tasks across groups and may also grow quadratically in the worst case, depending on how many cross-priority precedence relations are induced by the given priority structure. This helps explain why the exact MIQP can still solve the 12-task prototype case reported here, yet may become increasingly difficult when the number of tasks, the density of spatial interference, and the effective mission horizon grow simultaneously. Therefore, for larger industrial scenarios, the most promising direction is not simply to solve the same exact model at a larger scale, but to exploit the model structure: one may first build a conflict graph from the layout so that only genuinely interfering task pairs are kept in the core optimization model; one may also decompose sequencing and resource-allocation decisions, or split the problem by spatial zones and time windows within a rolling-horizon framework. In addition, heuristic and metaheuristic approaches can use the present MIQP as an exact benchmark or as a local improvement module, thereby preserving the engineering meaning of the space-

conflict constraints while improving scalability.

Table 5. Comparison of optimal solution with different limitation of maintenance resource.

	R = 60												T_{end}
Starting time	6	4	2	0	7	1	2	6	8	2	5	0	
Ending time	7	6	5	1	9	2	4	7	9	6	8	2	9
Resource level	1	1	3	1	1	1	1	1	1	3	3	1	
	R = 70												T_{end}
Starting time	3	6	6	7	4	2	0	7	2	0	2	2	
Ending time	6	8	7	8	6	3	2	8	6	2	5	4	8
Resource level	3	1	1	1	1	1	1	1	1	1	3	1	
	R = 80												T_{end}
Starting time	0	3	4	3	0	2	3	3	0	1	2	5	
Ending time	2	7	5	5	2	3	7	4	1	5	3	7	7
Resource level	2	3	1	3	1	1	3	1	1	3	1	1	
	R = 85												T_{end}
Starting time	3	0	6	1	3	2	0	0	3	1	2	5	
Ending time	6	3	7	2	5	3	2	1	4	5	3	7	7
Resource level	3	2	1	2	1	2	1	1	1	3	1	1	
	R = 90												T_{end}
Starting time	1	0	0	4	4	2	4	3	0	0	1	2	
Ending time	2	4	1	6	6	3	6	4	1	2	3	4	6
Resource level	1	3	1	3	1	1	1	1	1	1	2	1	
	R = 100												T_{end}
Starting time	5	0	1	3	3	0	2	5	1	0	0	2	
Ending time	6	3	3	5	5	1	6	6	2	2	1	6	6
Resource level	1	2	2	3	1	1	3	1	1	1	1	1	
	R = 110												T_{end}
Starting time	1	2	2	0	0	4	2	2	0	0	4	3	
Ending time	2	5	3	1	2	5	4	3	1	2	5	5	5
Resource level	1	1	1	2	1	1	1	1	1	1	1	1	
	R = 140												T_{end}
Starting time	3	3	4	0	0	2	0	4	2	0	3	3	
Ending time	4	5	5	2	2	3	2	5	3	2	4	5	5
Resource level	1	1	1	3	1	1	1	1	1	1	1	1	
	R = 150												T_{end}
Starting time	0	2	2	1	0	3	1	2	3	2	0	0	
Ending time	1	4	3	2	2	4	3	3	4	4	1	2	4
Resource level	1	1	1	1	1	2	1	1	1	1	1	1	

5. Conclusion and future work

This paper deals with the maintenance optimization problem in a limited space, where the maintenance tasks are carried out in a limited space, where the implementation of maintenance task should be subject to the pre-described priority level and the rule of no overlapping of the required work area. The maintenance resource is considered by the way of sliding window which is practical for the limited maintenance crew and special equipments. The model is formulated by a mixed integer quadratic programming problem and solved by Python and Gurobi solver by optimizing the starting time and the resource allocation. The numerical experiments are exploited and the optimal solutions are compared with different settings of numbers of maintenance tasks (up to 12 in an irregular layout of engine room), priority and limits of maintenance resource. The optimal upper limit of resource is given from the

perspective of minimizing the total maintenance time. In the future, the varying required work area for each maintenance task will be considered. Besides, the access of the engine room will be considered which will increase the part of path planning of the maintenance crew.

The present model is two-dimensional because it is intended as a first exact representation of workspace conflict in limited areas. Nevertheless, a direct engineering extension to three dimensions is meaningful. In a 3D formulation, each maintenance task would no longer be represented only by a planar work area, but by a work envelope with length, width and height, and conflict would be checked simultaneously in all three directions. This matters in engine rooms because two tasks may appear non-overlapping in the floor plan and still be incompatible in practice: for example, one crew may require scaffolding or lifting clearance above a machine while another crew works on nearby piping at a different elevation, so the two

activities interfere through vertical occupation rather than planar overlap. A 3D extension would therefore improve realism, but it would also require additional geometric

description of equipment, access corridors and safety clearance envelopes. For this reason, it is considered a natural next step after the 2D prototype developed in this paper.

References

1. He J, Khebbache S, Anjos MF, Hadji M. Optimization of maintenance for complex manufacturing systems using stochastic remaining useful life prognostics. *Computers & Industrial Engineering* 2023; 182: 109348. <https://www.doi.org/10.1016/j.cie.2023.109348>.
2. Zhang Y, Ouyang L, Meng X, Zhu X. Condition-based maintenance considering imperfect inspection for a multi-state system subject to competing and hidden failures. *Computers & Industrial Engineering* 2024; 188: 109856. <https://www.doi.org/10.1016/j.cie.2023.109856>.
3. Zhang S, Du M, Tong J, Li F. Multi-objective optimization of maintenance program in multi-unit nuclear power plant sites. *Reliability Engineering & System Safety* 2019; 188: 532-548. <https://www.doi.org/10.1016/j.ress.2019.03.034>.
4. van OB, van de CP, Basten R, Demerouti E. Predictive maintenance for industry 5.0: behavioural inquiries from a work system perspective. *International Journal of Production Research* 2023; 61(22): 7846-7865. <https://doi.org/10.1080/00207543.2022.2154403>.
5. Alexander DA, Pandey A, Kumar SK. Mathematical models for multi-stage hybrid assembly flow-shop scheduling with preventive maintenance and release times. *Computers & Industrial Engineering* 2023; 186: 109716. <https://doi.org/10.1016/j.cie.2023.109716>.
6. Chen Z, Chen Z, Zhou D, Pan E. Joint optimization of fleet-level sequential selective maintenance and repairpersons assignment for multi-state manufacturing systems. *Computers & Industrial Engineering* 2023; 182: 109411. <https://doi.org/10.1016/j.cie.2023.109411>.
7. Chen G, He W, Tian Y, Ma K. Resource-constrained project scheduling with multiple states: Bi-objective optimization model and case study of aircraft maintenance. *Computers & Industrial Engineering* 2024; 191: 110169. <https://doi.org/10.1016/j.cie.2024.110169>.
8. Zhu X, Bei X, Chatwattanasiri N, Coit WD. Optimal system design and sequential preventive maintenance under uncertain aperiodic-changing stresses. *IEEE Transactions on Reliability* 2018; 67(3): 907-919. <https://doi.org/10.1109/TR.2018.2798298>.
9. Levitin G, Xing L, Dai Y. Optimal operation and maintenance scheduling in m-out-of-n standby systems with reusable elements. *Reliability Engineering & System Safety* 2021; 211: 107582. <https://doi.org/10.1016/j.ress.2021.107582>.
10. Ali MB, Sassi M, Gossa M, Harrath Y. Simultaneous scheduling of production and maintenance tasks in the job shop. *International Journal of Production Research* 2011; 49(13): 3891-3918. <https://doi.org/10.1080/00207543.2010.492405>.
11. Arena M, Pasquale VD, Iannone R, Miranda S, Riemma S. A maintenance driven scheduling cockpit for integrated production and maintenance operation schedule. *Advances in Manufacturing* 2022; 10: 205-219. <https://doi.org/10.1007/s40436-021-00380-z>.
12. Ghorbani M, Noureldath M, Gendreau M. A two-stage stochastic programming model for selective maintenance optimization. *Reliability Engineering & System Safety* 2022; 223: 108480. <https://doi.org/10.1016/j.ress.2022.108480>.
13. Zhao X, He Z, Wu Y, Qiu Q. Joint optimization of condition-based performance control and maintenance policies for mission-critical systems. *Reliability Engineering & System Safety* 2022; 226: 108655. <https://doi.org/10.1016/j.ress.2022.108655>.
14. Hadi AS, Elham MJ, Reza TM, Mehdi RB. Dynamic opportunistic maintenance grouping in a lot streaming based job-shop scheduling problem. *Computers & Industrial Engineering* 2023; 183: 109424. <https://doi.org/10.1016/j.cie.2023.109424>.
15. Brimberg J, Hurley W, Wright R. Scheduling workers in a constricted area. *Naval Research Logistics* 1996; 43(1): 143-149. [https://doi.org/10.1002/\(SICI\)1520-6750\(199602\)43:1<143::AID-NAV9>3.0.CO;2-B](https://doi.org/10.1002/(SICI)1520-6750(199602)43:1<143::AID-NAV9>3.0.CO;2-B).
16. Ecker KH. Scheduling of resource tasks. *European Journal of Operational Research* 1999; 115: 314-327. [https://doi.org/10.1016/S0377-2217\(98\)00226-4](https://doi.org/10.1016/S0377-2217(98)00226-4).
17. Mehmet E, Rami A, Mahmoud A, Abdul RA. Workers-constrained shutdown maintenance scheduling with skills flexibility: Models and solution algorithms. *Computers & Industrial Engineering* 2022; 172: 108575. <https://doi.org/10.1016/j.cie.2022.108575>.
18. Boyer V, Vallikavungal J, Rodríguez XC, Salazar-Aguilar MA. The generalized flexible job shop scheduling problem. *Computers & Industrial Engineering* 2021; 160: 107542. <https://doi.org/10.1016/j.cie.2021.107542>.
19. Akl AM, Sawah SE, Chakraborty RK, Turan HH. A Joint Optimization of Strategic Workforce Planning and Preventive Maintenance Scheduling: A Simulation-Optimization Approach. *Reliability Engineering & System Safety* 2022; 219: 108175. <https://doi.org/10.1016/j.ress.2021.108175>.
20. Ait-Kadi D, Menye JB, Kane H. Resources assignment model in maintenance activities scheduling. *International Journal of Production*

Research 2011; 49(22): 6677-6689. <https://doi.org/10.1080/00207543.2010.518722>.

21. Marmier F, Varnier C, Zerhouni N. Proactive, dynamic and multi-criteria scheduling of maintenance activities. *International Journal of Production Research* 2009; 47(8): 2185-2201. <https://doi.org/10.1080/00207540802311106>.
22. Hao Q, Shen W, Xue Y, Wang S. Task network-based project dynamic scheduling and schedule coordination. *Advanced Engineering Informatics* 2010; 24: 417-427. <https://doi.org/10.1016/j.aei.2010.07.001>.
23. Turan HH, Kosanoglu F, Atmis M. A multi-skilled workforce optimisation in maintenance logistics networks by multithread simulated annealing algorithms. *International Journal of Production Research* 2021; 59(90): 2624-2646. <https://doi.org/10.1080/00207543.2020.1735665>.
24. Allahyari ZM, Azab A. Mathematical modeling and multi-start search simulated annealing for unequal-area facility layout problem. *Expert Systems with Applications* 2018; 91: 46-62. <https://doi.org/10.1016/j.eswa.2017.07.049>.
25. Guan C, Zhang Z, Liu S, Gong J. Multi-objective particle swarm optimization for multi-workshop facility layout problem. *Journal of Manufacturing Systems* 2019; 53: 32-48. <https://doi.org/10.1016/j.jmsy.2019.09.004>.
26. Luo X, Yang Y, Ge Z, Wen X, Guan F. Maintainability-based facility layout optimum design of ship cabin. *International Journal of Production Research* 2015; 53(3): 677-694. <https://doi.org/10.1080/00207543.2014.919416>.
27. Rinaldi M, Fera M, Bottani E, Grosse EH. Workforce scheduling incorporating worker skills and ergonomic constraints. *Computers & Industrial Engineering* 2022; 168: 108107. <https://doi.org/10.1016/j.cie.2022.108107>.
28. Rebai M, Kacem I, Adjallah KH. Earliness -tardiness minimization on a single machine to schedule preventive maintenance tasks: metaheuristic and exact methods. *Journal of Intelligent Manufacturing* 2012; 23: 1207-1224. <https://doi.org/10.1007/s10845-010-0425-0>.
29. Allahama H, Dalalaha D. MILP of multitask scheduling of geographically distributed maintenance tasks. *International Journal of Industrial Engineering Computations* 2022; 13: 119-134. <https://doi.org/10.5267/j.ijiec.2021.7.001>.
30. Golpîra H, Tirkolaei EB. Stable maintenance tasks scheduling: A bi-objective robust optimization model. *Computers & Industrial Engineering* 2019; 137: 106007. <https://doi.org/10.1016/j.cie.2019.106007>.
31. Zhang Y, Li C, Su X, Cui R, Wan B. A baseline-reactive scheduling method for carrier-based aircraft maintenance tasks. *Complex & Intelligent Systems* 2023; 9: 367-397. <https://doi.org/10.1007/s40747-022-00784-9>.
32. Li C, Zhang Y, Su X, Wang X. An Improved Optimization Algorithm for Aeronautical Maintenance and Repair Task Scheduling Problem. *Mathematics* 2022; 10: 3777. <https://doi.org/10.3390/math10203777>.
33. Cheng ETC, Gupta JND, Wang G. A review of flowshop scheduling research with setup times. *Production and Operations Management* 2000; 9: 262-282. <https://doi.org/10.1111/j.1937-5956.2000.tb00137.x>.
34. Liu D, Xu Z, Li F. A three-stage decomposition algorithm for decentralized multi-project scheduling under uncertainty. *Computers & Industrial Engineering* 2021; 160: 107553. <https://doi.org/10.1016/j.cie.2021.107553>.
35. Ji S, Luan D, Chen Z, Guo D. Integrated scheduling in automated container terminals considering AGV conflict-free routing. *Transportation Letters* 2021; 13(7): 501-513. <https://doi.org/10.1080/19427867.2020.1733199>.
36. Monemi RN, Gelareh S. Dock assignment and truck scheduling problem; consideration of multiple scenarios with resource allocation constraints. *Computers & Operations Research* 2023; 151: 106074. <https://doi.org/10.1016/j.cor.2022.106074>.