

Remaining useful life prediction via a bivariate Wiener process incorporating complex correlation structures

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Abstract

Remaining useful life (RUL) prediction is critical to enhance the reliability of the device. But such prediction is difficult owing to the multiple health indicators (MHI) in the degradation process. However, the complex correlation between MHI is generally ignored in current models, which leads to inaccurate issues in RUL prediction. Bivariate degradation serves as the basis for the multivariate degradation modeling in RUL prediction. Therefore, to solve the above issues, a novel RUL prediction method via a bivariate Wiener process incorporating complex correlation structures is proposed in this paper, which considers the intricate nonlinear correlation between MHI, and a RUL prediction model based on the bivariate Copula function (BCF) is constructed. After that, the Akaike Information Criterion is utilized to select the optimal BCF for analyzing the correlation structure of the two health indicators. Finally, considering the complex correlation in the degradation process, XJTU-SY datasets are used as an example for the effectiveness and predictability of the proposed method.

Keywords: bivariate Wiener process, multiple health indicators, copula function, multivariate degradation modeling, complex correlation

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Highlights

- A bivariate Wiener process RUL prediction method with the complex correlation is proposed.
- The RUL prediction model based on the bivariate Copula function is proposed.
- The bearing test data is provided to illustrate the applicability of the proposed method.

1. Introduction

The maintenance of equipment is crucial during operation, and the remaining useful life (RUL) prediction of prognostics and health management (PHM) is an indispensable means to ensure the reliability, and safety of the equipment. In practice, the prediction accuracy of the RUL has a significant impact on the safety, reliability and maintenance cost of the equipment [1,2]. Due to the interference of various random factors, the degradation process of equipment has uncertainty and variability. In view of this, it is necessary to seek a reasonable RUL prediction model to realize accurate prediction. Currently,

the degradation statistics data-driven method has become an important research direction in the field of reliability and safety. The Wiener process is widely used in stochastic process modeling owing to its good mathematical properties and flexibility [3–5].

In current studies, the Wiener process model exhibits significant advantages in degradation modeling, the derivation of the probability density function (PDF) of RUL, and the estimation of the model parameters [6–8]. Multiple health indicators (HI) usually exist in the degradation process of the equipment, and changes in the indicators will affect the degradation process of the equipment. The RUL prediction method based on the multivariate Wiener process has received extensive research in the field of RUL prediction. The basic idea is simultaneously to use the Wiener process to model and analyze MHI, and establish a suitable degradation model for RUL based on the complex correlation between MHI. Some studies assume that the HI of the equipment are independent of each other, follow a known multidimensional distribution, and

are linearly correlated [9–11]. For example, Jin and Matthews [12] used the bivariate Wiener process to describe the degradation process and studied the design of the bivariate degradation experiments under the non-equilibrium measurement conditions. Similarly, Liu et al. [13] developed the covariance matrix to describe the correlation between the two health indicators to achieve the RUL prediction of lithium-ion batteries, and Bayesian updating was utilized to estimate the model parameters.

Considering the comprehensive impact of the complex operating environments, operating modes, and loads, the degradation processes characterized by the various HI are usually interact and influence each other beyond simple linear correlations. The correlation of the health indicators, including the nonlinear correlation and directional degradation, has been explored recently. It is challenging to directly measure the correlations of MHI because of the indicator correlations change over time. To address the above issue, the known Copula function is introduced to describe the correlation between the health indicators [14–16]. For example, Wang et al. [17] employed the Wiener process to describe the degradation process of the equipment and used the Copula function to depict the correlation between the two health indicators. In the degradation modeling process, the influence of impact factors in the degradation process was considered, and the reliability function was proposed to evaluate the reliability of the equipment. Wang et al. [18] directly used the Frank Copula function to describe the correlation between the two health indicators and constructed the degradation model based on the bivariate Wiener model for the degradation process of the equipment. Furthermore, considering the influence of time scale changes, Wang et al. [19] proposed a RUL prediction model based on the bivariate Wiener process, and the Copula function was used to describe the correlation in MHI. Wang et al. [20] extended the previous study and developed a RUL method, which considered the influence of measurement errors in the degradation modeling process and derived the PDF of RUL under the bivariate degradation. For the correlations between the more complex multiple failure modes, Song et al. [21] utilized the Copula function to analyze and established the lifetime distribution model of the bearing based on Copula theory. Wu et al. [22] proposed a novel multi-indicator state-

space model and RUL prediction method based on the multiple indicators, the three correlation relationships between the degradation indicators and features are considered in RUL prediction.

Even though the degradation models considering complex correlation have provided some good results, some factors limiting the practicality need to be explored further. However, existing studies usually assume that MHI are independent of each other or follow a multidimensional distribution, which rarely concentrates on the complex correlation between MHI. Additionally, the current studies mostly suppose that the correlations follow the known function under the circumstances of the complex correlation between the various performance indicators, which leads to unreasonable descriptions of the complex correlation between MHI, and thus affects the accuracy of the RUL prediction. The prediction model is unable to accurately achieve the important HI of the data if not consider the complex correlation between the HI, which causes the prediction result to deviate from the actual situation and leads to a decrease in prediction accuracy. For example, since an increment of the bearing speed can cause an increase in temperature, the correlation between the speed and temperature is not considered, or the single-type data is only utilized for inference, the actual lifetime of the bearing can be easily underestimated, which results in the premature failure of the bearing. Therefore, to promote the accuracy of the RUL prediction and reliability evaluation, it is necessary to focus on studying the RUL prediction based on the Wiener process considering the impact of MHI.

To address the above discussions and challenges, the RUL prediction via a bivariate Wiener process incorporating complex correlation structures is proposed in the paper. The main contributions of the work are summarized as follows:

- (1) A novel bivariate degradation model is developed by combining the Wiener process model and the copula theory. This framework captures the synergistic degradation mechanisms of multiple performance metrics, and the complex correlation structures are addressed. Compared to existing degradation models, the bivariate Wiener process degradation model is constructed, which establishes the foundation for the study of multivariate degradation models. Unlike existing studies, which often assume that MHI are independent of each

other or follow a multidimensional distribution, the nonlinear correlation of the health indicators is considered in the proposed model.

(2) Aiming at the unreasonable description of the complex related structure of MHI leading to lower prediction accuracy, the copula theory is introduced in the RUL prediction model. Different from the existing studies that assume the complex correlation structure of MHI follows a known function, three commonly used Copula functions are used to describe the complex correlation structures. Thus, the RUL prediction model based on the bivariate Copula function (BCF) is proposed. On this basis, the Akaike Information Criterion (AIC) is utilized for the choice of the appropriate BCF, and the RUL prediction method is developed. In addition, the adaptive estimation of the model parameters is achieved.

(3) The proposed model is demonstrated through the bearing degradation case study. The results demonstrate that the Frank copula outperforms the other two copula functions in characterizing the complex correlation structures of bearing health indicators. This work provides a methodological framework for degradation modeling of complex systems. Furthermore, it offers new insights into reliability prediction for nonlinear degradation processes.

The rest of this paper consists of the following parts. Section 2 describes the bivariate degradation modeling and provides the RUL prediction method incorporating the complex correlation structures. The model parameter estimation is discussed in Section 3. In Section 4, a case study is presented to verify the effectiveness of the application of the proposed method. The whole thesis and future work are summarized in Section 5.

2. Degradation modeling based on the bivariate Wiener process and RUL prediction

2.1. Degradation modeling on the bivariate Wiener process

Existing studies usually construct the degradation model by using the fused indicators based on the Wiener process. However, the physical meaning of the health indicator is missing, and it is difficult to intuitively reflect the relationships between the health indicators. To overcome the shortcomings of the above-mentioned methods and preserve the characteristics of the individual health indicator, MHI are modeled and

analyzed simultaneously in the thesis. The multivariate Wiener process is introduced to better describe the degradation model.

It is assumed that the device contains n health indicators, the degradation process is $X(t) = \{X^{(1)}(t), X^{(2)}(t), \dots, X^{(n)}(t)\}$, where $\{X(t), t \geq 0\}$ denote the n -dimensional Wiener process. Let $X(t) = X^{(1)}(t), X^{(2)}(t), \dots, X^{(m)}(t)$, the degradation process $\{X^{(m)}(t), t \geq 0\}$ of the m th health indicator is described as follows:

$$X^{(m)}(t) = x_0^{(m)} + \lambda^{(m)}t + \sigma^{(m)}B(t) \quad (1)$$

where $X^{(m)}$ is the degradation of the m th health indicator. $x_0^{(m)}$ is the initial degradation state of the m th health indicator. $\lambda^{(m)}$ represents the drift coefficient of the m th health indicator, which is the degradation rate. $\sigma^{(m)}$ is the diffusion coefficient of the m th health indicator. $B(t)$ is a standard Wiener process. It is worth noting that if $m=1,2$, the degradation process $\{X^{(1)}(t), X^{(2)}(t)\}$ is the bivariate Wiener process. In addition, it is worth mentioning that the observation time corresponding to the two health indicators is identical for the same device. The device is considered to be in failure if the degradation of any one of the health indicators reaches the failure threshold.

The bivariate Wiener process has the following properties: (1) $\{X^{(1)}(0), X^{(2)}(0)\} = \{0,0\}$, where $\{X^{(1)}(t), X^{(2)}(t)\}$ is right continuous at $t = 0$. (2) The increments of different intervals $X^{(m)}(t_1), X^{(m)}(t_2) - X^{(m)}(t_1), \dots, X^{(m)}(t_n) - X^{(m)}(t_{n-1})$ are independent. The increment at any time $X^{(m)}(t_i) - X^{(m)}(t_k)$ obeys the normal distribution, with the mean $\lambda^{(m)}(t_i - t_k)$ and the variance $\rho(\sigma^{(m)})^2(t_i - t_k)$, where ρ is the correlation coefficient between the two degradation processes.

2.2. Correlation analysis of the bivariate copula function

From the above analysis, it can be concluded that complex correlation needs to be considered when conducting the RUL prediction based on the MHI. The complex correlation structure can be effectively described by the Copula function, which is capable of linking the joint distribution of the multivariate random variables to the respective univariate marginal distributions [23].

The joint distribution function can be derived by Sklar's theorem [24], provided that the marginal probability density functions of each random variable are given. As stipulated in Sklar's theorem, if the specific form of a BCF is available, the bivariate joint distribution function can be constructed flexibly.

Three common types of the BCFs are summarized in Table 1 [25].

Table 1. Common forms of three BCFs.

Copula function	Prior estimation
Frank	$-\frac{1}{\theta} \ln(1 + (e^{-\theta u^{(1)}} - 1)(e^{-\theta u^{(2)}} - 1)/(e^{-\theta} - 1))$
Clayton	$\max\left\{-[(u^{(1)})^{-\theta} + (u^{(2)})^{-\theta} - 1]^{\frac{-1}{\theta}}, 0\right\}$
Gumbel	$\exp\left\{-[(-\ln u^{(1)})^{\theta} + (-\ln u^{(2)})^{\theta}]^{\frac{1}{\theta}}\right\}$

By computing the partial derivatives of $C(u^{(1)}, u^{(2)}; \theta)$ about $u^{(1)}$ and $u^{(2)}$, the probability density function (PDF) of

the bivariate Frank Copula function, Clayton Copula function, and Gumbel Copula function can be obtained as:

$$c(u^{(1)}, u^{(2)}; \theta) = \frac{\theta(1 - e^{-\theta})e^{-\theta(u^{(1)} + u^{(2)})}}{[(e^{-\theta} - 1) + (e^{-\theta u^{(1)}} - 1)(e^{-\theta u^{(2)}} - 1)]^2} \quad (2)$$

$$c(u^{(1)}, u^{(2)}; \theta) = (1 + \theta)(u^{(1)}u^{(2)})^{-\theta-1}[(u^{(1)})^{-\theta} + (u^{(2)})^{-\theta} - 1]^{\frac{-1}{\theta}-2} \quad (3)$$

$$c(u^{(1)}, u^{(2)}; \theta) = \frac{C(u^{(1)}, u^{(2)}; \theta)(\ln u^{(1)} \ln u^{(2)})^{\theta-1}}{u^{(1)}u^{(2)}[(-\ln u^{(1)})^{\theta} + (-\ln u^{(2)})^{\theta}]^{\frac{1}{\theta}-2}} \times \left\{ [(-\ln u^{(1)})^{\theta} + (-\ln u^{(2)})^{\theta}]^{\frac{1}{\theta}} + \theta - 1 \right\} \quad (4)$$

It can be concluded that bivariate random vectors with tail asymptotic symmetry can be described by the bivariate Frank Copula. The asymmetric bivariate random vectors with lower tail dependence and upper tail asymptotics can be described by the bivariate Clayton Copula, and asymmetric tail bivariate random vectors with upper tail dependence and lower tail asymptotics can be described by the bivariate Gumbel Copula. Therefore, most of the complex correlation structures can be characterized by the above three common BCFs, and the selection of the three BCFs to describe the correlation structure of the two health indicators is reasonable.

2.3. Bivariate Copula function selection based on AIC

Based on the above analysis, the BCF enables the construction of a joint distribution from the marginal distributions, regardless of the specific forms of those marginals. From the PDF of BCFs, it can be observed that the complex correlations described by different BCFs are not identical, even the parameter θ is same. Due to the significant impact of BCF selection on the analysis results of prediction model, it is critical to choose a appropriate model to represent the potential degradation process. To better characterize the complex correlation structure of the two health indicators and improve RUL prediction accuracy, the Akaike

Information Criterion (AIC) [26] is employed to select the optimal BCF model.

In general, a higher likelihood value indicates a better fitting performance. The number of unknown parameters corresponds to the independent variables of the model, a model with more parameters typically offers greater flexibility and thus higher fitting accuracy. However, an excessive number of parameters may lead to overfitting and increase uncertainty, which can degrade the predictive performance. Therefore, a comprehensive criterion is needed. AIC is commonly employed for this purpose and can be expressed as

$$AIC = -2 \log(A) + 2m \quad (5)$$

where A is the likelihood function corresponding to the model, m is the number of model parameters.

The AIC balances fitting accuracy against model complexity, with lower value indicating superior performance. Consequently, the BCF with the smallest AIC is considered optimal. The selection criterion ensures that the selected Copula function not only adequately describes the complex correlation structure of the two health indicators but also avoids over-parameterization.

2.4. RUL prediction model based on the Bivariate copula function

Accurate RUL prediction requires the integration of complex correlation structures of the health indicators into the modeling framework. In this paper, the BCF is employed to capture the complex correlation structure of the two health indicators, and the RUL prediction model based on the BCF is constructed.

It is assumed that the failure thresholds $\omega^{(1)}$ and $\omega^{(2)}$ corresponding to the bivariate degradation process $\{X^{(1)}(t), X^{(2)}(t)\}$, respectively. $x_k^{(m)} = X^{(m)}(t_k)$ is denoted as

$$L_k = \inf\{l_k: X^{(1)}(t_k + l_k) \geq \omega^{(1)} \text{ or } X^{(2)}(t_k + l_k) \geq \omega^{(2)} | X^{(1)}(t_k) < \omega^{(1)}, X^{(2)}(t_k) < \omega^{(2)}\} \quad (7)$$

It is assumed that the posteriori estimation of the drift coefficient λ for the degradation process based on the univariate Wiener process $\{X(t), t > 0\}$ obeys the Gaussian distribution $\lambda \sim N(\hat{\lambda}_{k|k}, P_{k|k})$. According to the theorem in [27] and its corresponding distribution function, considering the condition

$$f_L^{(m)}(l_k | X_{1:k}^{(m)}) = \frac{\omega^{(m)} - x_k^{(m)}}{\sqrt{2\pi l_k^3 (P_{k|k}^{(m)} l_k + \sigma^{2(m)})}} \exp \left[-\frac{(\omega^{(m)} - x_k^{(m)} - \hat{\lambda}_{k|k}^{(m)} l_k)^2}{2 l_k (P_{k|k}^{(m)} l_k + \sigma^{2(m)})} \right] \quad (8)$$

and

$$F_L^{(m)}(l_k | X_{1:k}^{(m)}) = 1 - \Phi \left(\frac{\omega^{(m)} - x_k^{(m)} - \hat{\lambda}_{k|k}^{(m)} l_k}{\sqrt{l_k (P_{k|k}^{(m)} l_k + \sigma^{2(m)})}} \right) + \exp \left[\frac{2 \hat{\lambda}_{k|k}^{(m)} (\omega^{(m)} - x_k^{(m)})}{\sigma^{2(m)}} + \frac{2 P_{k|k}^{(m)} (\omega^{(m)} - x_k^{(m)})^2}{\sigma^{4(m)}} \right] \times \Phi \left(\frac{2 P_{k|k}^{(m)} (\omega^{(m)} - x_k^{(m)}) l_k + \sigma^{2(m)} (\hat{\lambda}_{k|k}^{(m)} l_k + \omega^{(m)} - x_k^{(m)})}{\sigma^{2(m)} \sqrt{l_k (P_{k|k}^{(m)} l_k + \sigma^{2(m)})}} \right) \quad (9)$$

where $\omega^{(m)}$ is the failure threshold of the m th health indicator, $m=1, 2$. $\hat{\lambda}_{k|k}^{(m)}$ and $P_{k|k}^{(m)}$ are the expectation and variance of the posterior estimation $\lambda^{(m)}$ of the m th health indicator at time t_k , respectively. Φ represents the standard normal distribution.

According to the correlation analysis of the BCF, when the

$$f_L^{union}(l_k) = c(F_L^{(1)}(l_k | X_{1:k}^{(1)}), F_L^{(2)}(l_k | X_{1:k}^{(2)}); \theta) \prod_{m=1}^2 f_L^{(m)}(l_k | X_{1:k}^{(m)}) \quad (10)$$

where $F_L^{(1)}(l_k | X_{1:k}^{(1)})$ and $F_L^{(2)}(l_k | X_{1:k}^{(2)})$ are the edge distribution functions of RUL for the two health indicators at time t_k , respectively, $c(F_L^{(1)}(l_k | X_{1:k}^{(1)}), F_L^{(2)}(l_k | X_{1:k}^{(2)}); \theta)$ is the PDF of BCF.

$$F_T(t) = P(T < t) = F_T^{(1)}(t) + F_T^{(2)}(t) - C(F_T^{(1)}(t), F_T^{(2)}(t); \theta) \quad (11)$$

the monitoring data of the two degradation processes at time t_k . In the concept of first hitting time, the failure time, denoted as T , is defined as the time at which the degradation value of any individual health indicator first reaches or surpasses its predetermined failure threshold. Its expression is:

$$T = \inf\{t: X^{(1)}(t) \geq \omega^{(1)} \text{ or } X^{(2)}(t) \geq \omega^{(2)}\} \quad (6)$$

According to the failure threshold ω_1, ω_2 , and the monitoring data $X_{1:k}^{(m)} = \{x_1^{(m)}, x_2^{(m)}, \dots, x_k^{(m)}\}$, the RUL of the device at time t_k is defined as:

of the bivariate Wiener degradation process, given the monitoring data up to time t_k and the posteriori estimation $\lambda^{(m)}$, the edge PDF and distribution function of RUL for the m th health indicator at time t_k are respectively expressed as follows:

edge PDF and distribution function of RUL for the two health indicators are obtained, a specific copula function can be used to describe the complex correlation structure based on the Sklar theorem. In view of this, the joint PDF of RUL for the two health indicators at time t_k can be expressed as:

It is known that the degradation path of the device at time t is $X(t) = \{X^{(1)}(t), X^{(2)}(t)\}$, if the device is still in the normal operation state at time t . According to the definition of the failure time T , the lifetime distribution function is described as:

where $F_T^{(1)}(t)$ and $F_T^{(2)}(t)$ represent the lifetime edge distribution functions of the two independent variables $x^{(1)}$ and $x^{(2)}$, respectively.

Proceeding to the next step, the distribution function of the

$$F_L(l_k) = F_L^{(1)}(l_k|X_{1:k}^{(1)}) + F_L^{(2)}(l_k|X_{1:k}^{(2)}) - C(F_L^{(1)}(l_k|X_{1:k}^{(1)}), F_L^{(2)}(l_k|X_{1:k}^{(2)}); \theta) \quad (12)$$

and

$$f_L(l_k) = f_L^{(1)}(l_k|X_{1:k}^{(1)}) + f_L^{(2)}(l_k|X_{1:k}^{(2)}) - f_L^{union}(l_k|X_{1:k}^{(m)}) = f_L^{(1)}(l_k|X_{1:k}^{(1)}) + f_L^{(2)}(l_k|X_{1:k}^{(2)}) - c(F_L^{(1)}(l_k|X_{1:k}^{(1)}), F_L^{(2)}(l_k|X_{1:k}^{(2)}); \theta) \prod_{m=1}^2 f_L^{(m)}(l_k|X_{1:k}^{(m)}) \quad (13)$$

The PDF of RUL governs the probability of failure at any given time. Therefore, the RUL prediction value is often defined as the time point with the highest PDF value. Compared to existing studies, the proposed approach integrates complex correlation structure of the health indicators into the prediction model by deriving the RUL PDF based on the BCF, which also reflects the impact of the complex correlation on the RUL prediction results.

3. Model parameter estimation

The estimation of the parameters in Equation (13) is presented. As demonstrated above, accurate RUL prediction required to estimate the model parameters $\theta^{(m)} = [a_0^{(m)}, p_0^{(m)}, Q^{(m)}, \sigma^{2(m)}]^T$ and θ , where θ denotes the correlation coefficient between the two health indicators. To achieve this objective, the expectation maximization (EM) algorithm and maximum likelihood estimation (MLE) are introduced for the parameter estimation. The EM algorithm is first utilized to derive optimal estimations of the primary degradation parameters. Following this, the MLE method is specifically applied to estimate the correlation

$$\ell(\theta^{(m)}|\hat{\theta}_k^{(m)(i)}) = \mathbb{E}_{\lambda^{(m)}|X_{1:k}^{(m)}, \hat{\theta}_k^{(m)(i)}} \{ \ln f(X_{1:k}^{(m)}, \lambda^{(m)}|\theta^{(m)}) + \ln f(\lambda^{(m)}|\theta^{(m)}) \} \quad (15)$$

where the drift coefficient vector is $\lambda^{(m)} = \{\lambda_1^{(m)}, \lambda_2^{(m)}, \dots, \lambda_k^{(m)}\}$, $\hat{\theta}_k^{(m)(i)}$ represents the estimation result of the i th iteration based on the monitoring data $X_{1:k}^{(m)}$, $\hat{\theta}_k^{(m)(i)} = [a_{0,k}^{(m)(i)}, p_{0,k}^{(m)(i)}, Q_k^{(m)(i)}, \sigma_k^{2(m)(i)}]^T$.

$$\hat{\theta}_k^{(m)(i+1)} = \arg\max_{\theta} \{ \mathbb{E}_{\lambda^{(m)}|X_{1:k}^{(m)}, \hat{\theta}_k^{(m)(i)}} [\ln f(X_{1:k}^{(m)}, \lambda^{(m)}, \theta^{(m)}) + \ln f(\lambda^{(m)}|\theta^{(m)})] \} \quad (16)$$

Equation (15) and Equation (16) are iterated circularly until the set convergence criteria are satisfied based on the EM

RUL is calculated by the relationship between the lifetime and RUL. If the posterior estimation $\lambda^{(m)}$ and the monitoring data $X_{1:k}^{(m)}$ up to time t_k are known, the distribution function and PDF of RUL based on the BCF are respectively represented by:

parameter θ , which is used to adjust the prediction model and obtain accurate prediction results.

3.1. Adaptive parameter estimation

In order to incorporate the accumulated monitoring data up to the current time into the degradation modeling and RUL prediction framework, a linear state space model is introduced to characterize the degradation process of the device. The model is formulated as follows:

$$\begin{cases} \lambda_k^{(m)} = \lambda_{k-1}^{(m)} + w_k^{(m)} \\ x_k^{(m)} = x_{k-1}^{(m)} + \lambda_{k-1}^{(m)}(t_k - t_{k-1}) + v_k^{(m)} \end{cases} \quad (14)$$

where $w_k^{(m)}$ and $v_k^{(m)}$ are random error terms, $w_k^{(m)} \sim N(0, Q^{(m)})$, $v_k^{(m)} \sim N(0, \sigma^{2(m)}(t_k - t_{k-1}))$. The drift coefficient $\lambda^{(m)}$ is a hidden variable.

The EM algorithm is introduced to estimate the model parameter $\theta^{(m)}$. Firstly, the joint log likelihood function is calculated. Considering the bivariate Wiener process degradation model and the monitoring data, the joint log likelihood function of $\theta^{(m)}$ can be given by:

Subsequently, the estimates of the EM algorithm are derived based on the joint log likelihood function, which is expressed as:

algorithm, and then the iteration is terminated. A series of optimal estimation results for the model parameters $\hat{\theta}_k^{(m)(i+1)}$

are obtained, which are formulated by:

$$\begin{aligned}
(a_0^{(m)})_k^{(i+1)} &= \hat{\lambda}_{0|k}^{(m)} \\
(p_0^{(m)})_k^{(i+1)} &= p_{0|k}^{(m)} \\
(Q^{(m)})_k^{(i+1)} &= \frac{1}{k} \sum_{j=1}^k [\mathbb{E}_{\lambda^{(m)}|X_{1:k}^{(m)}, \theta_k^{(m)(i)}}(\lambda_j^{2(m)}) - 2\mathbb{E}_{\lambda^{(m)}|X_{1:k}^{(m)}, \theta_k^{(m)(i)}}(\lambda_j^{(m)}\lambda_{j-1}^{(m)}) + \mathbb{E}_{\lambda^{(m)}|X_{1:k}^{(m)}, \theta_k^{(m)(i)}}(\lambda_{j-1}^{2(m)})] \\
(\sigma^{(m)})_k^{(i+1)} &= \frac{1}{k} \sum_{j=1}^k \frac{[(X_j^{(m)} - X_{j-1}^{(m)})^2 + (t_j - t_{j-1})^2 \mathbb{E}_{\lambda^{(m)}|X_{1:k}^{(m)}, \theta_k^{(m)(i)}}(\lambda_{j-1}^{2(m)})]}{t_j - t_{j-1}} - \frac{1}{k} \sum_{j=1}^k \frac{2\hat{\lambda}_{j-1|k}^{(m)}(X_j^{(m)} - X_{j-1}^{(m)})(t_j - t_{j-1})}{t_j - t_{j-1}}
\end{aligned} \tag{17}$$

where $\mathbb{E}_{\lambda^{(m)}|X_{1:k}^{(m)}, \theta_k^{(m)(i)}}(\lambda_j^{2(m)})$ and $\mathbb{E}_{\lambda^{(m)}|X_{1:k}^{(m)}, \theta_k^{(m)(i)}}(\lambda_j^{(m)}\lambda_{j-1}^{(m)})$ can be calculated by the properties of the variance covariance and Rauch-Tung-Striebel smoothing algorithm [28].

3.2. Parameter estimation of the bivariate copula function

The MLE method is introduced to estimate the parameter θ based on the above model parameter estimation results, where θ is the parameter of the BCF, which reflects the complex correlation structure of the two health indicators in the copula function.

Firstly, the PDF of the BCF corresponding to the edge distribution function $F_L^{(1)}(l_k|X_{1:k}^{(1)})$ and $F_L^{(2)}(l_k|X_{1:k}^{(2)})$ of RULat time t_k is defined, and the log likelihood function is calculated by:

$$L(\theta) = \sum_{k=1}^n \ln c(F_L^{(1)}(l_k|X_{1:k}^{(1)}), F_L^{(2)}(l_k|X_{1:k}^{(2)}); \theta) \tag{18}$$

The maximum likelihood estimation of θ can be computed by:

$$\hat{\theta} = \operatorname{argmax}_{\theta} \{\partial L(\theta) / \partial \theta\} \tag{19}$$

4. Case study

In this section, two bearing samples are used to evaluate the applicability and effectiveness of the proposed method. The case study aims to evaluate the RUL prediction performance of the proposed method. Si method [27] and Zhang method [29] are taken as the benchmark of the single health indicator for RUL prediction based on the Wiener process. However, the method does not consider the complex dependence structure of the health indicators in the degradation process. Therefore, the above methods are selected for the performance comparison with the proposed method.

4.1. Data description

The Bearing1-3 of the XJTU-SY [30] bearing datasets is

selected for the validation of the proposed model. Time domain features are extracted from the vertical vibration signals, and the absolute mean and peak values are taken as the two performance indicators to characterize the degradation state of the bearing. The absolute mean and peak of Bearing1-3 are shown in Figure 1.

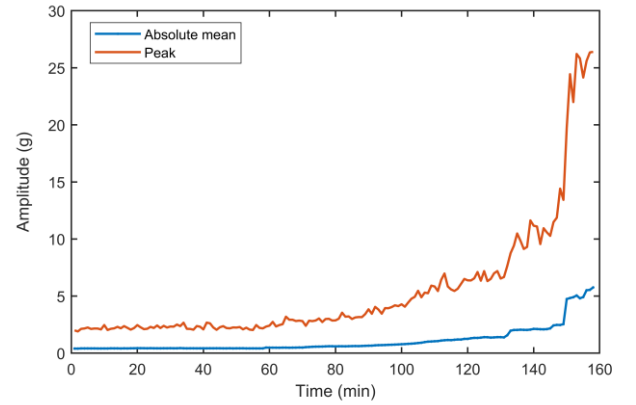


Figure 1. The absolute mean and peak of Bearing 1-3.

Considering the relevant requirements and assumptions of the bivariate Wiener process degradation modeling, the failure thresholds are set based on the degradation process of the entire life cycle of the bearing. The information of Bearing 1-3 are given in Table 2.

Table 2. The information of Bearing 1-3.

Performance indicator	Actual lifetime (min)	Time interval (min)	Failure threshold ω
Absolute mean	158	1	5
Peak value			24

The Copula function is used to describe the complex dependence of the health indicators. However, the Copula function is not the best choice if there is a strong linear correlation between the health indicators, because the linear correlation can be described through simple linear regression or other linear models. Therefore, it is necessary to first perform nonlinear correlation tests before conducting the copula

modeling. The Kendall correlation coefficient is applied to inspect the correlation between the peak and absolute mean. The Kendall correlation coefficient test results are shown in Table 3. The results indicate that the Copula function can be employed to describe the complex correlation structure of the two health indicators.

Table 3. Kendall correlation coefficient test results.

Result	Correlation coefficient	P-value
Test result	0.8555	0

4.2. Parameter estimation and the bivariate Copula function selection

The initial model parameters of each health indicator are set in this section. The initial model parameters of the two health indicators are given in Table 4.

Table 4. The initial model parameters of the two health indicators.

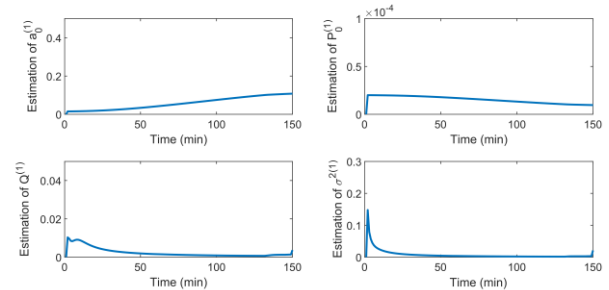
Health indicator	Initial model parameter
Absolute mean	$[0.015, 2 \times 10^{-5}, 0.01, 0.1]$
Peak	$[0.05, 1 \times 10^{-3}, 1 \times 10^{-4}, 0.1]$

The EM algorithm is introduced to update and estimate the model parameters based on the initial parameters (in Table 4). The model parameter update estimation results of the absolute mean and peak can be obtained as the running time increases. The model parameters of the above two health indicators that need to estimate are represented as $\theta^{(1)} = [a_0^{(1)}, p_0^{(1)}, Q^{(1)}, \sigma^{2(1)}]^T$ and $\theta^{(2)} = [a_0^{(2)}, p_0^{(2)}, Q^{(2)}, \sigma^{2(2)}]^T$, respectively. The parameter estimation results of the two health indicators are displayed in Figure 2.

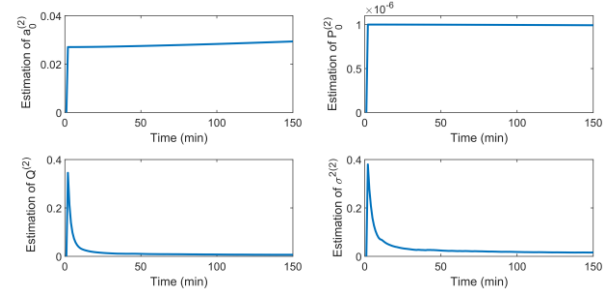
As shown in Figure 2, the model parameters gradually converge with the accumulation of monitoring data. The model parameters can be updated at each monitoring time after obtaining the new monitoring data, and the adaptive estimation of the model parameters are achieved. The estimated values of the parameter θ corresponding to the three commonly used BCFs are calculated based on the above parameter estimation results. Then, combining the parameter estimation results and the estimated values of θ , the RUL edge PDF of the two health indicators are computed by Equation (13). Finally, the optimal BCF is selected through AIC values. The AIC values of the three kinds of Copula functions are shown in Table 5.

From Table 5, it can be observed that the AIC value corresponding to the bivariate Frank Copula function is the

minimum. According to the judgment rule, the bivariate Frank Copula function is selected to analyze the complex correlation structure of the absolute mean and peak. Based on the above results, the MLE method is used to estimate the parameter θ of the bivariate Frank Copula function, and the estimation result is $\hat{\theta} = -2.0585$.



(a) Parameter estimation of absolute mean



(b) Parameter estimation of peak

Figure 2. Parameter estimation of the two health indicators.

Table 5. The AIC values of three kinds of Copula functions.

BCF	Frank Copula	Gumbel Copula	Clayton Copula
AIC	-117.5652	-54.0393	inf

4.3. Comparative result and analysis

To verify the effectiveness of the proposed method and highlight the importance of considering the complex correlation structure of MHI in RUL prediction, Si method [27] and Zhang method [29] are used to further compare and analyze with the proposed method. It is worth mentioning that the logarithmic linearization is applied in the monitoring data in the proposed method. Zhang method is an innovation and improvement of the Si method, which considers the nonlinear and staged characteristics of bearing degradation. It can better demonstrate the general applicability of the proposed method. Furthermore, a deeper comparison with existing studies is provided through the bivariate Gumbel Copula-based RUL prediction method.

The PDF of RUL is computed according to Equation (13)

and the above-mentioned estimation results of the model parameters. The proposed method is compared with Si method based on the absolute mean and peak value. For a clearer and more intuitive presentation, the comparison results of the PDF at ten monitoring time are given in Figure 3.

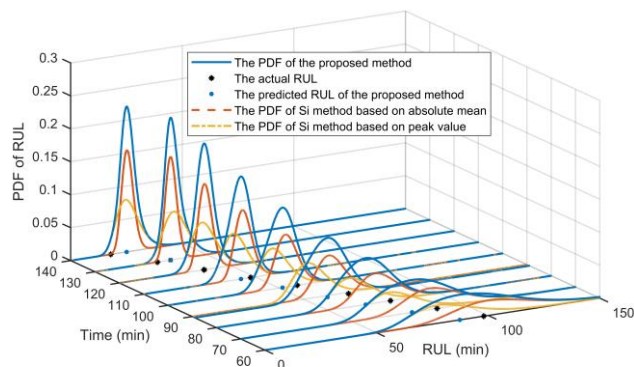


Figure 3. The PDF of RUL prediction.

Figure 3 shows that all of the actual RUL fall within the range of the PDF curve, which indicates that all three methods are effective. The PDF curve of RUL becomes sharper as the degradation data accumulates, which indicates that the prediction uncertainty of RUL is gradually decreasing. It is worth noting that the PDF curve of the proposed method has the characteristics of being the narrowest and sharpest, which further proves that the proposed method is conducive to reduce the prediction uncertainty of RUL.

Furthermore, the RUL estimation is calculated based on the PDF of RUL. The time corresponding to the maximum value of the PDF is taken as the prediction result of the proposed method. The RUL prediction results of three methods are displayed in Figure 4. As shown in Figure 4, the RUL predicted by the proposed method is mostly lower than the actual RUL and closest to the actual RUL, while the RUL of Si method based on absolute mean mostly lies above the true RUL. In maintenance decision-making, RUL prediction results are categorized into underestimation and overestimation, with the overestimation leading to overestimation of equipment reliability. Although the RUL of Si method based on peak value are mostly lower than the actual RUL, which significantly deviate from the actual RUL. From the reliability perspective, the maintenance strategy informed by the proposed method is conservative, which is generally recommended in maintenance decisions. The strategy helps to avoid overestimating the equipment reliability, thereby reducing associated safety risks and preventing failures.

The relative errors (RE) is used to further demonstrate the effectiveness of the proposed method. The RE of the three methods at each monitoring time are calculated based on the RUL prediction results. The RE of RUL estimation is shown in Figure 5.

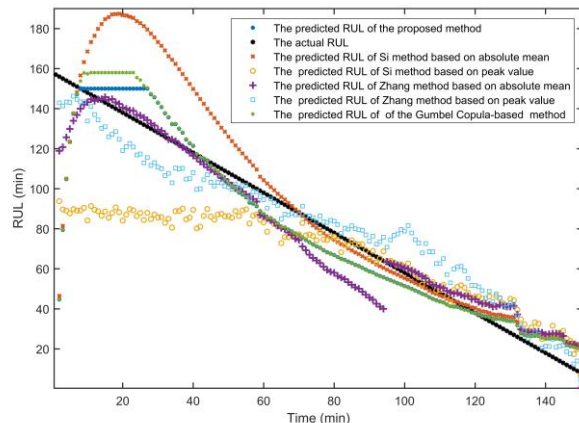


Figure 4. The prediction results of RUL.

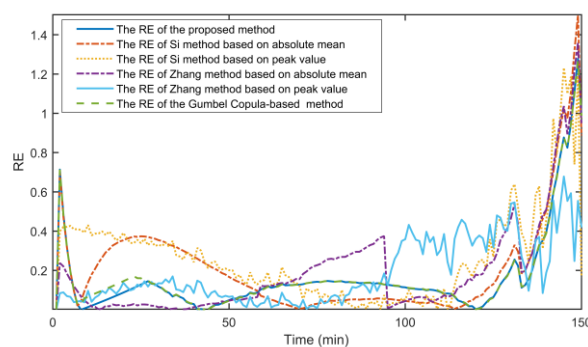


Figure 5. The RE of RUL estimation.

From Figure 5, it can be observed that the RE of the proposed method is significantly lower than the other methods. Specifically, the average RE of Si method based on absolute mean and the peak value are 0.2132 and 0.2665, respectively. The average RE of Zhang method based on absolute mean and the peak value are 0.1839 and 0.1914, respectively. The average RE of Gumbel Copula-based method is 0.1654. In contrast, the proposed method achieves an average RE of only 0.1594. Compared with the former two methods, the prediction accuracy of the proposed method is improved by 5.38% and 10.71%, respectively. The results indicate that the RUL prediction accuracy can be effectively improved by fully considering the complex correlation structures of performance indicators in the degradation modeling process. RE is categorized into four intervals to further investigate its distribution. The proportions of RE falling within each interval

are quantified, and the corresponding percentages are summarized in Table 6.

Table 6. Percentage of different RE intervals.

Method	Percentage of different RE intervals			
	(0, 0.15]	(0.15, 0.3]	(0.3, 0.5]	(0.5, +∞)
The proposed method	83.33%	6.00%	4.00%	6.67%
Si method based on absolute mean	53.33%	20.67%	18.00%	8.00%
Si method based on peak value	37.33%	23.33%	30.00%	9.34%
Zhang method based on absolute mean	58.67%	11.33%	10.00%	8.00%
Zhang method based on peak	62.27%	11.23%	20.67%	5.33%
The Gumbel Copula-based method	80.67%	8.66%	4.00%	6.67%

Notably, as given Table 6, the proposed method achieved a proportion of 83.33% within the RE range of (0, 0.15], while the range of the Gumbel Copula-based method is 80.67%. It is obvious that the prediction accuracy of the proposed method is better than the Gumbel Copula-based method. Accurate warning is essential for equipment reliability in the early degradation stage of bearing. In addition, it can be seen that the proposed method is superior to the other methods. Specifically, within this range, the Si method based on absolute mean and peak value yields proportions of only 53.33% and 37.33%, respectively. Thus, the proposed method exceeds these benchmarks by 30% and 46%, respectively. These findings clearly indicate that the proposed method delivers a lower average RE compared to the above RUL prediction methods, which rely on a single health indicator or assume the complex correlation structures of MHI follows a known function. This strongly validates the superiority of the proposed method in improving RUL prediction accuracy and further confirms the effectiveness of the proposed model.

5. Conclusion

To address the challenges of the complex correlation structures of MHI in RUL prediction modelling, this study proposed a RUL prediction via a bivariate Wiener process incorporating complex correlation structures. The proposed framework integrated the Wiener process, MHI degradation modeling, and the Copula theory to enhance the prediction accuracy under the complex correlation structures. Firstly, a bivariate degradation model was constructed that combined the Wiener process and the Copula theory. Two specific health indicators with complex correlation structures were separately subjected to bivariate degradation modeling. The bivariate Wiener process degradation model has the advantage of preserving the inherent

characteristics of the performance indicators. Then, the Copula theory was introduced to solve the issue of the lower prediction accuracy, which was caused by the unreasonable description of the complex correlation structure in RUL prediction. Specifically, the bivariate Frank Copula function, Gumbel Copula function, and Clayton Copula function were utilized to describe the complex correlation structures of the health indicators. The corresponding edge PDF and edge distribution function of RUL were derived. To characterize the dependency of two health indicators and improve prediction accuracy, the AIC was utilized for the choice of the optimal Copula function. In addition, the model parameters were estimated through the EM algorithm and MLE algorithm.

In the case study, the bearing degradation data were used to verify the accuracy and effectiveness of the proposed method, with particular attention to the complex correlation structures of the health indicators. The proposed method achieves an average RE of only 0.1594, whose prediction accuracy is better than other methods. Notably, the proposed method achieved a proportion of 83.33% within the RE range of (0, 0.15], which was 2.66% higher than the Gumbel Copula-based method. The results demonstrated that the Frank copula outperforms the other two copula functions in characterizing the complex correlation structures of bearing health indicators. Experimental results further validated that the RUL prediction model considering the complex correlation structures can not only effectively capture the individual degradation features of the original performance indicators but also accurately represent the complex correlation structures of the health indicators, thereby significantly improving the prediction accuracy.

However, despite the encouraging prediction results, there are still some issues that need further study. In terms of the indicator dimensionality, the paper is limited to a bivariate

degradation model. The study can extend the model to multivariate degradation scenarios. The description of the correlation between the various performance indicators in the

context of multivariate degradation states will be another direction worthy of study in the future.

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