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Reliability assessment for small-sample accelerated degradation testing with Weibull pseudo-failure lifetimes

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Abstract

To overcome the prolonged test durations and large sample sizes inherent in conventional accelerated degradation testing (ADT), this paper develops a novel reliability assessment framework based on Weibull pseudo-failure lifetimes. Uniquely, the proposed method enables high-confidence reliability assessment using data from only two accelerated stress levels. First, a mathematical relationship between the use-level reliable life and the Weibull distributional parameters at accelerated levels is established. Subsequently, a joint confidence distribution for these parameters is constructed, facilitating an explicit formula for the exact one-sided lower confidence limit of the reliable life. Monte Carlo simulations and case studies demonstrate that the proposed framework significantly reduces the sample size and duration, while enhancing theoretical rigor and assessment precision compared to traditional multi-level asymptotic methods.

Keywords: *accelerated degradation testing, small-sample reliability, Weibull distribution, pseudo-failure lifetime, exact confidence limit*

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Highlights

- Developed a small-sample ADT framework using Weibull pseudo-failure lifetimes.
- Reduced testing to two stress levels, significantly cutting sample size and costs.
- Derived an explicit formula for the exact confidence limit of reliable life.

1. Introduction

In the field of aerospace, ensuring the high reliability and long service life of critical components is essential for both system safety and mission success. However, traditional reliability testing becomes impractical, as obtaining sufficient degradation or failure information within short development cycles is increasingly difficult [1,2]. To circumvent these limitations, accelerated degradation testing (ADT) has been widely adopted as an efficient engineering solution [3–5]. By subjecting products to elevated stress levels to accelerate performance degradation, ADT utilizes degradation data and acceleration models to extrapolate reliability metrics under normal operating conditions.

Among various ADT analytical frameworks, the pseudo-failure lifetime method based on degradation trajectories is widely utilized due to its physical intuitiveness and implementation simplicity. This approach characterizes the reliability of individual units by defining their lifespan as the first passage time when a parameterized trajectory crosses a predefined failure threshold. Depending on the depth of mechanistic understanding, these degradation trajectory models are generally divided into physics-based and empirical models [6]. When failure mechanisms are explicit, physics-based approaches are preferred. For example, theories such as fracture dynamics [7,8], stress relaxation [9], and generalized reaction-diffusion [10] have been successfully applied to model the degradation in composite materials, electromagnetic relays, and MOSFETs, respectively. Conversely, for complex systems where establishing precise physical relationships is challenging, empirical function models provide a flexible and robust alternative. Various functional forms have been explored in the literature, including three-parameter power functions for rubber aging [11], double-exponential models for organic light-emitting diodes brightness decay [12], and optimal elementary function selection for Hall current sensors [13].

Although the pseudo-failure lifetime method effectively shortens test duration, traditional ADT still requires testing across multiple stress levels, frequently leading to large sample sizes, long testing duration, and high costs [14–16]. Furthermore, when extrapolating results to the normal operating conditions, it is difficult to simultaneously satisfy high confidence requirements and ensure prediction accuracy for the one-sided lower confidence limits (LCLs) of reliable life [17,18]. Currently, maximum likelihood estimation (MLE) and best linear unbiased estimation (BLUE) are the predominant inference methods in ADT [19]. However, the MLE method exhibits severe errors and instability under small sample sizes. Moreover, its estimation of LCLs relies on the large-sample asymptotic normality assumption, which often leads to coverage probabilities (CPs) that fail to meet the nominal confidence levels in small-sample scenarios [20,21]. This renders the method non-conservative for high-reliability applications. On the other hand, the BLUE method typically requires testing at multiple stress levels to ensure precision, imposing a heavy test burden. Furthermore, estimating LCLs through BLUE requires complex distributional assumptions for the estimators, which introduces additional errors [22]. Therefore, BLUE is often limited to point estimation [19]. Given these limitations, achieving high-confidence reliability assessment under small-sample conditions remains a critical challenge in ADT research.

To address small-sample constraints, prior studies have established reliability assessment frameworks for ADT assuming exponential or lognormal distributions [23–25]. While the Weibull distribution offers broader applicability for modeling the life of electromechanical products, the development of exact inference methods for it in small-sample ADT scenarios remains inadequately addressed. To bridge this gap, this paper develops a novel reliability assessment framework for small-sample ADT under Weibull assumptions. This method enables an exact evaluation of the one-sided LCL for reliable life using data from only two accelerated stress levels, thereby significantly reducing sample size and test duration. The proposed method first establishes a mathematical relationship between the reliable life at normal operating conditions and the Weibull parameters under accelerated stress levels. Subsequently, a joint confidence distribution for these parameters is constructed using the pseudo-failure lifetimes. Finally, explicit calculation formulas for the exact LCLs are derived, providing a robust theoretical foundation for reliability-based maintenance decisions.

2. Small-sample ADT scheme and data pre-processing

2.1. Two-level ADT design

Let S_0 denote the normal operating stress level. To address the constraints of small-sample testing, this study employs a two-level ADT scheme with accelerated stress levels S_1 and S_2, where $S_0 < S_1 < S_2$. A total sample of size n is randomly allocated into two groups of sizes n_1 and n_2, such that $n_1 + n_2 = n$ and $n_i \geq 2$ for $i = 1,2$. These groups are subjected to accelerated stress levels S_1 and S_2, respectively. The degradation measurement of the j-th unit under stress level S_i at inspection time t_{ijk} is denoted by x_{ijk}. The resulting ADT dataset is defined as:

$$D = \{(t_{ijk}, x_{ijk}) | i = 1,2; j = 1,2,\cdots,n_i; k = 1,2,\cdots,m_{ij}\} \quad (1)$$

where m_{ij} is the number of measurements for the j-th unit under S_i.

2.2. Pseudo-failure lifetime extraction

Assume that the performance degradation path of the product over time t is described by the trajectory model:

$$x = f(t; \boldsymbol{\theta}) \quad (2)$$

where x is the critical degradation performance, $f(\cdot)$ is a monotonic function determined by the underlying failure mechanism or data trends, $\boldsymbol{\theta}$ is the unknown parameter vector. Typical functional forms include the power law model for lithium battery capacity fade [26] and stress relaxation in springs [27]:

$$x = \theta_1 + \theta_2 t^{\theta_3} \quad (3)$$

and the exponential model for light-emitting diodes brightness decay [28] and rubber aging [29]:

$$x = \theta_1\, exp\left(\theta_2 t^{\theta_3}\right) \quad (4)$$

For the j-th unit under stress level S_i, the parameter vector $\widehat{\boldsymbol{\theta}}_{ij}$ can be estimated via non-linear least squares fitting. Given a failure threshold x_{th}, the pseudo-failure lifetime $\hat{t}_{ij}$ can be obtained via the inverse function:

$$\hat{t}_{ij} = f^{-1}\left(x_{\mathrm{th}}; \hat{\boldsymbol{\theta}}_{ij}\right) \tag{5}$$

Consequently, a set of pseudo-failure lifetimes is obtained. For statistical analysis, these values are arranged as order statistics: $t_{i1} \le t_{i2} \le \cdots \le t_{in_i}$.

3. Statistical models

It is assumed that the product lifetime t at any constant stress level S_i follows a Weibull distribution, with the reliability function expressed as:

$$R(t; \alpha, \beta_i) = exp\left[-\left(\frac{t}{\beta_i}\right)^{\alpha}\right] \tag{6}$$

where $\alpha > 0$ is the constant shape parameter, and $\beta_i > 0$ is the stress-dependent scale parameter.

3.1. Acceleration model

The relationship between the reliable life and the stress level is typically described by a log-linear acceleration model [30,31]:

$$ln\, t_{Ri} = a + b\varphi(S_i) \tag{7}$$

where $t_{Ri} = R^{-1}(R; \alpha, \beta_i) = \beta_i(-\,ln\, R)^{\frac{1}{\alpha}}$ represents the quantile of the Weibull distribution corresponding to reliability R, a and b are model parameters to be determined, and $\varphi(S_i)$ is a known function of the stress level S_i. Depending on the type of stress, $\varphi(S_i)$ can be expressed by:

$$\varphi(S_i) = \begin{cases} \frac{1}{S_i}, & \text{Arrhenius model} \\ ln\, S_i\,, & \text{Inverse power law model} \\ S_i, & \text{Exponential model} \end{cases} \tag{8}$$

For example, the Arrhenius model is applicable when thermal stress is used [32], while the inverse power law model is suitable when voltage, humidity, or mechanical load is applied as the accelerated stress [33,34].

3.2. Expression of the use-level reliable life

Let t_{R1} and t_{R2} denote the reliable lives at reliability R under stress levels S_1 and S_2 , respectively. By applying the acceleration model in Eq. (7) to these two levels, the model parameters a and b can be obtained as:

$$\begin{cases} a = \frac{\varphi(S_2)\, ln\, t_{R1} - \varphi(S_1)\, ln\, t_{R2}}{\varphi(S_2) - \varphi(S_1)} \\ b = \frac{ln\, t_{R2} - ln\, t_{R1}}{\varphi(S_2) - \varphi(S_1)} \end{cases} \tag{9}$$

Subsequently, by introducing a stress weighting factor ω:

$$\omega = \frac{\varphi(S_2) - \varphi(S_0)}{\varphi(S_2) - \varphi(S_1)} > 1 \tag{10}$$

the logarithm of the reliable life at the normal operating stress level can be expressed as a linear weighted combination of the logarithms of the reliable lives under the two accelerated stress levels:

$$ln\, t_{R0} = a + b\varphi(S_0) = \omega\, ln\, t_{R1} + (1-\omega)\, ln\, t_{R2} \tag{11}$$

Finally, by substituting the quantile function $ln\, t_{Ri} = ln\, \beta_i + \alpha^{-1}\, lnln\left(\frac{1}{R}\right)$, the explicit expression is derived as:

$$ln\, t_{R0} = \omega\, ln\, \beta_1 + (1-\omega)\, ln\, \beta_2 + \alpha^{-1}\, lnln\left(\frac{1}{R}\right) \tag{12}$$

Eq. (12) directly links the reliable life t_{R0} under normal operating conditions to the distribution parameters $(\alpha, \beta_1, \beta_2)$ at accelerated stress levels. This formulation provides the theoretical foundation for the subsequent derivation of the exact one-sided LCL of t_{R0}.

4. Reliability assessment for small-sample ADT

Focusing on small-sample scenarios, this section proposes a novel reliability assessment framework for ADT under the Weibull distribution. Explicit formulas are derived to evaluate the one-sided LCLs of both the reliable life and the reliability function under normal operating stress level.

4.1. Exact LCL for reliable life

Theorem 1. Let $\{t_{i1} \le t_{i2} \le \cdots \le t_{in_i}\}$ be a set of pseudo-failure lifetimes obtained under accelerated stress level S_i for $i = 1,2$. Given a nominal confidence level γ and target reliability R, the exact one-sided LCL of the reliable life t_{R0} under the normal operating stress level S_0 is given by $t_{RL0,\gamma}$:

$$t_{RL0,\gamma} = \Psi^{-1}(\gamma) \tag{13}$$

$$\Psi\left(t_{RL0,\gamma}\right) = \int_0^1 \int_0^1 F\left[\frac{2T_{1,n_1}(\alpha(z))}{\beta\left(y,z,t_{RL0,\gamma}\right)^{\alpha(z)}}; 2n_1\right] dydz \tag{14}$$

which satisfies the exact coverage requirement:

$$P\left(t_{RL0,\gamma} \le t_{R0}\right) = \gamma \tag{15}$$

where $\Psi^{-1}(\gamma)$ is the inverse function of $\Psi\left(t_{RL0,\gamma}\right)$, $F(\cdot, \nu)$ is the cumulative distribution function (CDF) of the χ^2 distribution with ν degrees of freedom. The integrand components $\alpha(z)$ and $\beta\left(y, z, t_{RL0,\gamma}\right)$ are defined as:

$$\alpha(z) = W^{-1}[\chi_z^2(2n-4)] \tag{16}$$

$$\beta\left(y, z, t_{RL0,\gamma}\right) = exp\left\{\frac{1}{\omega\alpha(z)}\left[\alpha(z)\, ln\, t_{RL0,\gamma} + (1-\omega)\, ln\left(\frac{\chi_{1-y}^2(2n_2)}{2T_{2,n_2}(\alpha(z))}\right) - lnln\left(\frac{1}{R}\right)\right]\right\} \tag{17}$$

Here, z and y are probability indices associated with the confidence distribution of α and the conditional confidence distribution of β_2 given α , respectively. Specifically, $\alpha(z)$ represents the value of α corresponding to cumulative probability level z under its confidence distribution. Meanwhile, $\beta\left(y, z, t_{RL0,\gamma}\right)$ represents the corresponding value of β_1

associated with $t_{RL0,\gamma}$.

In these equations, $W(\alpha) = 2\sum_{i=1}^{2}\sum_{j=1}^{n_i-1} ln\left[\frac{T_{i,n_i}(\alpha)}{T_{i,j}(\alpha)}\right]$ is a monotonically increasing function regarding α, and $W^{-1}(\cdot)$ denotes its inverse. $T_{i,j}(\alpha) = \sum_{k=1}^{j} t_{ik}^{\alpha} + (n_i - j)t_{ij}^{\alpha}$. $\chi_z^2(\nu)$ and $\chi_{1-y}^2(\nu)$ are the z -th and $(1-y)$-th lower quantiles of the χ^2 distribution with ν degrees of freedom, respectively.

Since the $t_{RL0,\gamma}$ in Eq. (13) exhibits strict monotonicity with respect to γ, the value of $t_{RL0,\gamma}$ can be efficiently determined through numerical iteration. Specifically, an initial value for the one-sided LCL, denoted as t_{RL0}^*, is first assigned. Then, its corresponding confidence level γ^* can be calculated as:

$$\gamma^* = \int_0^1\int_0^1 F\left[\frac{2T_{1,n_1}(\alpha(z))}{\beta(y,z,t_{RL0}^*)^{\alpha(z)}}; 2n_1\right] dydz \tag{18}$$

The numerical computation formula is given by:

$$\gamma^* = \frac{1}{M^2}\sum_{j=1}^{M}\sum_{k=1}^{M} F\left[\frac{2T_{1,n_1}(\alpha_j)}{\left(\beta_{jk}(t_{RL0}^*)\right)^{\alpha_j}}; 2n_1\right] \tag{19}$$

where the components are defined as follows:

$$\alpha_j = W^{-1}\left[\chi_{1-\gamma_j}^2(2n-4)\right] \tag{20}$$

$$\beta_{jk}(t_{RL0}^*) = exp\left\{\frac{1}{\omega\alpha_j}\left[\alpha_j\, ln\, t_{RL0}^* + (1-\omega)\, ln\left(\frac{\chi_{\gamma_k}^2(2n_2)}{2T_{2,n_2}(\alpha_j)}\right) - lnln\left(\frac{1}{R}\right)\right]\right\} \tag{21}$$

$$\gamma_l = 1 - \frac{(l-0.5)}{M}, \quad l = 1,2,\cdots,M \tag{22}$$

In the numerical implementation, α_j in Eq. (20) denotes the value of α at the j -th integration node, while $\beta_{jk}(t_{RL0}^*)$ in Eq. (21) denotes the corresponding value of β_1 at the (j,k)-th grid point for the current estimate t_{RL0}^*. Here, j and k are indices of the numerical integration. M represents the number of integration intervals. Simulation experiments indicate that when $M \geq 1000$, the numerical error of Eq. (19) is negligible.

The solution is obtained through an iterative process by adjusting the initial value t_{RL0}^* until the calculated γ^* converges to the nominal confidence level γ within a specified tolerance. Upon convergence, the final t_{RL0}^* is determined as the exact one-sided LCL, denoted as $t_{RL0,\gamma}$.

Proof. Let t_{RL0}^* be an initial value for the one-sided LCL of the reliable life under S_0. The associated coverage probability γ^* is defined as:

$$\gamma^* = P(t_{RL0}^* \leq t_{R0}) \tag{23}$$

By conditioning on the shape parameter α and scale parameter β_2, this probability can be expanded using the law of total probability:

$$\begin{aligned}\gamma^* &= \int_0^{+\infty}\int_0^{+\infty} P\left(t_{RL0}^* \leq t_{R0}\middle|\alpha,\beta_2\right)h\left(\beta_2\middle|\alpha\right)g\left(\alpha\right)d\beta_2 d\alpha \\ &= \int_0^1\int_0^1 P\left(t_{RL0}^* \leq t_{R0}\middle|\alpha,\beta_2\right)dH\left(\beta_2\middle|\alpha\right)dG\left(\alpha\right) \\ &= \int_0^1\int_0^1 \gamma_0\left(\alpha,\beta_2,t_{RL0}^*\right)dH\left(\beta_2\middle|\alpha\right)dG\left(\alpha\right)\end{aligned} \tag{24}$$

where $h(\beta_2|\alpha)$ and $H(\beta_2|\alpha) = 1 - F\left[\frac{2T_{2,n_2}(\alpha)}{\beta_2^{\alpha}}; 2n_2\right]$ are the conditional probability density function (PDF) and CDF of the confidence distribution for β_2 given α, derived from $\frac{2T_{2,n_2}(\alpha)}{\beta_2^{\alpha}} \sim \chi^2(2n_2)$. Likewise, $g(\alpha)$ and $G(\alpha) = F[W(\alpha); 2n-4]$ denote the PDF and CDF of the confidence distribution for α, constructed from the pivotal quantity $W(\alpha) \sim \chi^2(2n-4)$ [35]. The term $\gamma_0(\alpha,\beta_2,t_{RL0}^*)$ represents the conditional probability $P(t_{RL0}^* \leq t_{R0}|\alpha,\beta_2)$. Based on the relationship $\frac{2T_{1,n_1}(\alpha)}{\beta_1^{\alpha}} \sim \chi^2(2n_1)$ and Eq. (12), this probability satisfies:

$$ln\, t_{RL0}^* = \frac{\omega}{\alpha} ln\left[\frac{2T_{1,n_1}(\alpha)}{\chi_{\gamma_0(\alpha,\beta_2,t_{RL0}^*)}^2(2n_1)}\right] + (1-\omega)\, ln\, \beta_2 + \frac{1}{\alpha} lnln\left(\frac{1}{R}\right) \tag{25}$$

Therefore, $\gamma_0(\alpha,\beta_2,t_{RL0}^*)$ can be solved by:

$$\gamma_0(\alpha,\beta_2,t_{RL0}^*) = F\left[\frac{2T_{1,n_1}(\alpha)}{\beta(\alpha,\beta_2,t_{RL0}^*)^{\alpha}}; 2n_1\right] \tag{26}$$

where:

$$\beta(\alpha,\beta_2,t_{RL0}^*) = exp\left\{\frac{1}{\omega\alpha}\left[\alpha\, ln\, t_{RL0}^* - (1-\omega)\alpha\, ln\, \beta_2 - lnln\left(\frac{1}{R}\right)\right]\right\} \tag{27}$$

Substituting Eq. (26) into Eq. (24), it can be obtained as:

$$\gamma^* = \int_0^1\int_0^1 F\left[\frac{2T_{1,n_1}(\alpha)}{\beta(\alpha,\beta_2,t_{RL0}^*)^{\alpha}}; 2n_1\right] dH(\beta_2|\alpha)\, dG(\alpha) \tag{28}$$

To facilitate calculation, let $z = G(\alpha)$ and $y = H(\beta_2|\alpha)$. Then, α and β_2 can be expressed as functions of z and y:

$$\alpha(z) = G^{-1}(z) = W^{-1}[\chi_z^2(2n-4)] \tag{29}$$

$$\beta_2(y,z) = H^{-1}(y|\alpha) = \left[\frac{2T_{2,n_2}(\alpha(z))}{\chi_{1-y}^2(2n_2)}\right]^{\frac{1}{\alpha(z)}} \tag{30}$$

Finally, substituting these transformed variables into Eq. (28) yields the exact calculation formula for the actual confidence level γ^*:

$$\gamma^* = \int_0^1\int_0^1 F\left[\frac{2T_{1,n_1}(\alpha(z))}{\beta(y,z,t_{RL0}^*)^{\alpha(z)}}; 2n_1\right] dydz \tag{31}$$

where:

$$\beta(y,z,t_{RL0}^*) = exp\left\{\frac{1}{\omega\alpha(z)}\left[\alpha(z)\, ln\, t_{RL0}^* + (1-\omega)\, ln\left(\frac{\chi_{1-y}^2(2n_2)}{2T_{2,n_2}(\alpha(z))}\right) - lnln\left(\frac{1}{R}\right)\right]\right\} \tag{32}$$

Consequently, as demonstrated in Eqs. (31) and (32), γ^* is a strictly monotonically decreasing continuous function of t_{RL0}^* with a range of (0,1). There exists a unique t_{RL0}^* such that the confidence level γ^* equals the nominal confidence level $\gamma \in$

(0,1). The value of t_{RL0}^* at this convergence point corresponds to the exact one-sided LCL of the reliable life, $t_{RL0,\gamma}$.

By discretizing the integration domain into an $M \times M$ grid, the confidence level γ^* can be approximated via numerical quadrature, as detailed in Eqs. (19)–(22). The detailed numerical calculation procedure is presented in Algorithm 1.

Algorithm 1: Numerical solution procedure for one-sided LCL $t_{RL0,\gamma}$

Input:
Pseudo-failure lifetimes $\{t_{i1} \le t_{i2} \le \cdots \le t_{in_i}\}$ for $i = 1,2$; Reliability R; Confidence level γ; Stress weighting factor ω; Grid size M; Convergence tolerance ε.

Output:
The one-sided LCL of reliable life under S_0: $t_{RL0,\gamma}$.

1. **Initialize** search boundaries $[t_{min}, t_{max}]$.
2. **Generate** grid nodes $\gamma_l = 1 - \frac{(l-0.5)}{M}$ for $l = 1,2,\cdots,M$.
3. **Pre-compute** α_j values using Eq. (20) for $j = 1,2,\cdots,M$.
4. **While** $(t_{max} - t_{min} > \varepsilon)$ **do**
5. Update initial LCL: $t_{mid} \leftarrow (t_{max} + t_{min})/2$.
6. Update $\beta_{jk}(t_{\text{mid}})$ values using Eq. (21) for all j, k.
7. Calculate current confidence γ^* using Eq. (19):
$$\gamma^* \leftarrow \frac{1}{M^2}\sum_{j=1}^{M}\sum_{k=1}^{M} F\left[\frac{2T_{1,n_1}(\alpha_j)}{\left(\beta_{jk}(t_{mid})\right)^{\alpha_j}}; 2n_1\right]$$
8. **If** $\gamma^* > \gamma$ **then** $t_{min} \leftarrow t_{mid}$ **else** $t_{max} \leftarrow t_{mid}$
9. **End While**
10. **Return** $t_{RL0,\gamma} = t_{\text{mid}}$.

4.2. Exact LCL for reliability

By using the equivalence relationship between the confidence limits for reliable life and reliability, the assessment method for the reliable life LCL presented in Theorem 1 can be directly extended to evaluate the LCL of the reliability.

Corollary 1. For a given confidence level γ and a specified mission time t under the normal operating stress level S_0, the one-sided LCL of the reliability $R_0(t)$, denoted as $R_{\text{L0},\gamma}(t)$, can be determined by solving:

$$R_{\text{L0},\gamma}(t) = \Psi^{-1}(\gamma) \tag{33}$$

where $\Psi(\cdot)$ is defined by:

$$\Psi\left(R_{\text{L0},\gamma}(t)\right) = \int_0^1\int_0^1 F\left[\frac{2T_{1,n_1}(\alpha(z))}{\beta\left(y,z,R_{\text{L0},\gamma}(t)\right)^{\alpha(z)}}; 2n_1\right] dydz \tag{34}$$

In this formula, the term $\beta\left(y,z,R_{\text{L0},\gamma}(t)\right)$ is obtained by substituting $t_{RL0,\gamma}$ with the mission time t, and the reliability R with the unknown variable $R_{\text{L0},\gamma}(t)$ in Eq. (17), resulting in:

$$\beta\left(y,z,R_{\text{L0},\gamma}(t)\right) = exp\left\{\frac{1}{\omega\alpha(z)}\left[\alpha(z)\ln t + (1-\omega)\ln\left(\frac{\chi^2_{1-y}(2n_2)}{2T_{2,n_2}(\alpha(z))}\right) - \ln\ln\left(\frac{1}{R_{\text{L0},\gamma}(t)}\right)\right]\right\} \tag{35}$$

The resulting LCL satisfies the exact confidence requirements:

$$P\left(R_{\text{L0},\gamma}(t) \le R_0(t)\right) = \gamma \tag{36}$$

5. Numerical simulation and comparative analysis

To evaluate the performance of the proposed method in small-sample scenarios, extensive Monte Carlo simulations were conducted, comparing it against the traditional MLE and BLUE methods in ADT.

5.1. Validation of coverage probabilities

The Weibull characteristic life β_i is assumed to follow the Arrhenius model: $\ln\beta_i = -10 + \frac{6000}{S_i}$. The shape parameter α is set to 0.5, 1.0, and 3.0, respectively. The normal operating stress is defined at $S_0 = 293$ K, with accelerated stress levels configured at $S_1 = 343$ K and $S_2 = 373$ K. To simulate small-sample scenarios, sample sizes are restricted to $n_1 = n_2 \in \{3,5,10\}$. A Monte Carlo simulation consisting of 10^6 independent replications was executed. Given that BLUE is predominantly limited to point estimation because the construction of LCLs requires pivotal quantities whose distributions are not readily available, the comparative study focuses on evaluating the proposed method against the conventional MLE method. In each iteration, both methods are employed to estimate the one-sided LCL of the reliable life at a reliability level of 0.95 under S_0.

Table 1 details the CPs for both methods under different shape parameters and nominal confidence levels. The results demonstrate that the proposed method exhibits superior performance in small-sample scenarios. Across all considered configurations, the CPs of the proposed method align consistently with the nominal confidence levels, even in the extreme small sample case. This confirms that the proposed approach serves as an exact assessment method in the frequentist sense.

In contrast, the MLE method shows a clear tendency towards overestimation. Its CPs consistently fall below the nominal levels, indicating that the estimated LCLs are biased upwards, thereby severely underestimating the actual failure risk. This deviation is particularly critical in the scenario with $\alpha = 1$ and $n_1 = n_2 = 3$, where the actual coverage drops to 69.6% (against a 90% nominal target) and 75.6% (against a 95% nominal target). Such severe under-coverage implies a dangerous underestimation of failure risk, rendering the MLE method unsuitable for safety-critical applications in aerospace where high-confidence assessment is essential.

Table 1. Comparison of CPs for the one-sided LCLs of reliable life.

α	(n_1, n_2)	Proposed method		MLE method	
		$\gamma = 0.90$	$\gamma = 0.95$	$\gamma = 0.90$	$\gamma = 0.95$
0.5	(3,3)	0.900	0.950	0.699	0.758
	(5,5)	0.900	0.949	0.771	0.832
	(10,10)	0.901	0.951	0.832	0.887
1.0	(3,3)	0.900	0.951	0.696	0.756
	(5,5)	0.900	0.950	0.766	0.829
	(10,10)	0.899	0.949	0.820	0.881
3.0	(3,3)	0.900	0.950	0.698	0.760
	(5,5)	0.900	0.950	0.760	0.824
	(10,10)	0.899	0.950	0.816	0.878

5.2. Comparative analysis

As demonstrated in Section 5.1, the MLE method is unsuitable for small-sample applications because it fails to meet confidence requirements. Furthermore, since deriving LCLs for the two-parameter Weibull distribution using the BLUE method necessitates complex distributional approximations, this section evaluates the proposed method against the BLUE method using the exponential distribution which serves as a special case of the Weibull distribution with shape parameter $\alpha = 1$.

In this comparative study, the product lifetime is assumed to follow an exponential distribution, where the relationship between mean life β_i and stress level S_i is governed by the inverse power law: $ln\, \beta_i = 21 - 4.3\, ln\, S_i$. Pseudo-failure lifetimes across five accelerated stress levels were simulated, with a sample size of 3 per level as detailed in Table 2. For the normal operating stress level of $S_0 = 10$ V, the 90% one-sided LCL of the mean life was evaluated using both methods and compared against the true value of $\beta_0 = 6.61 \times 10^4$ h.

Table 2. Simulated pseudo-failure times for the exponential distribution case.

Stress level (V)	Pseudo-failure lifetime (h)		
20	3424	3737	4919
25	307	693	3577
30	48	365	721
35	78	370	494
40	36	54	91

As summarized in Table 3, the proposed method utilizes data from only two stress levels (total sample size $n = 6$), whereas the BLUE method employs the full dataset from five levels (total sample size $n = 15$). Despite this difference in data usage, the LCL estimated by the proposed method (5.79×10^4h) remains highly consistent with the BLUE result (5.84×10^4h). For this simulated dataset, both estimated LCLs fall below the true mean life $\beta_0 = 6.61 \times 10^4$h . This comparison demonstrates that the proposed methodology achieves precision comparable to the traditional BLUE method while realizing a 60% reduction in sample size, thereby offering significant effectiveness.

Table 3. Comparison of reliability assessment results for the exponential distribution case.

$\beta_0 (10^4$ h$)$	$\beta_{L0,\gamma} (10^4$ h$)$	
	Proposed method (2 levels, 6 units)	BLUE method (5 levels, 15 units)
6.61	5.79	5.84

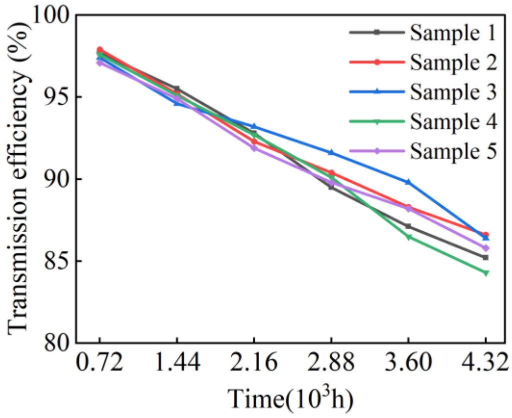
In summary, the proposed method demonstrates superior performance in small sample scenarios, where its CP consistently aligns closely with the nominal confidence level. In contrast, the MLE method fails to meet these confidence requirements, rendering it unsuitable for practical engineering applications. Furthermore, the BLUE method necessitates complex assumptions when applied to the Weibull distribution. Consequently, BLUE is typically confined to the point estimation of reliable life and is inadequate for evaluating LCLs. Most notably, the proposed method offers a substantial reduction in experimental cost, saving approximately 60% of test specimens compared to the BLUE method.

6. Engineering case study

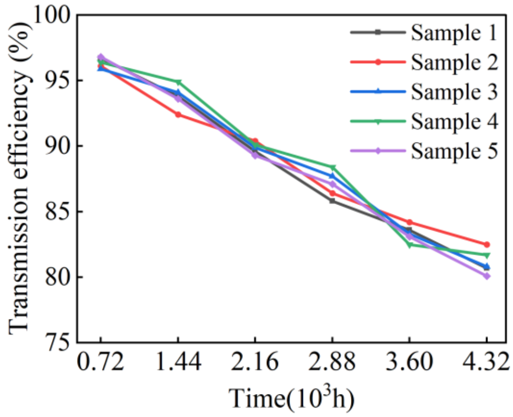
This section presents a case study on harmonic reducers to demonstrate the proposed method. As critical precision transmission elements in aerospace and robotics, harmonic reducers are characterized by high precision, efficiency, and longevity. Ensuring their reliability is essential for system safety and mission success. However, constrained by prohibitive manufacturing costs and limited development cycles, it is often impractical to conduct long-term testing with large sample sizes.

Consequently, ADT emerges as the dominant means for assessing their reliability.

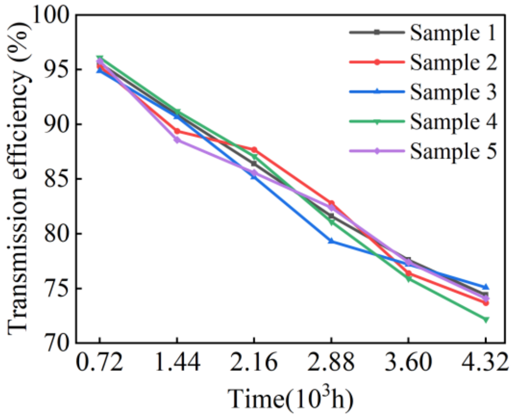
Figure 1 shows the measured degradation paths of a harmonic reducer [36]. Torque is selected as the accelerated stress. Four accelerated torque stress levels of 10 N·m, 20 N·m, 30 N·m, and 40 N·m were chosen, with five specimens allocated to each level. Transmission efficiency was recorded monthly throughout the test for the four accelerated stress levels.



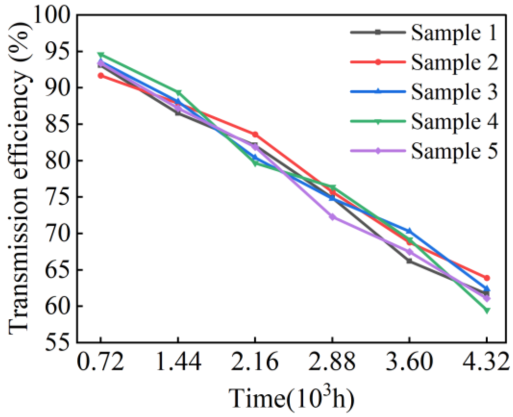
(a) Degradation data under 10 N·m



(b) Degradation data under 20 N·m



(c) Degradation data under 30 N·m



(d) Degradation data under 40 N·m

Figure 1. Degradation paths of harmonic reducers under accelerated stress levels.

Table 4. Pseudo-failure lifetimes of harmonic reducers.

Stress level (N·m)	Pseudo-failure lifetime (h)				
10	7935	8156	8338	8704	9609
20	5744	5911	6289	6856	7067
30	4058	4286	4497	4836	5179
40	2838	3033	3362	3526	3754

As observed from the degradation trajectories in Figure 1, the transmission efficiency exhibits an approximately linear decreasing trend over time at all accelerated stress levels. Moreover, the applied torque is assumed to accelerate the degradation rate without changing the dominant failure mechanism, as the degradation paths under different torque levels exhibit similar monotonic trends. Therefore, a linear degradation model, $x = \theta_1 + \theta_2 t$, is adopted to describe the performance evolution of each specimen. With the failure threshold x_{th} set as a 30% loss in transmission efficiency, the pseudo-failure lifetime is determined as $\hat{t}_{ij} = \frac{(x_{\text{th}} - \hat{\theta}_{1ij})}{\hat{\theta}_{2ij}}$. The resulting pseudo-failure lifetimes are listed in Table 4.

Life data from the highest stress level $S_2 = 40$ N · m and the lowest stress level $S_1 = 10$ N · m are selected as inputs. The objective is to evaluate the one-sided LCL of the reliable life under the normal operating stress level of $S_0 = 5$ N · m, with a confidence level of $\gamma = 0.90$ and a reliability of $R = 0.95$. The detailed procedure is described as follows:

Step 1: Formulation of the use-level reliable life relationship. Based on the inverse power law model, establish the mathematical relationship between the use-level reliable life t_{R0} and the distribution parameters $(\alpha, \beta_1, \beta_2)$ at accelerated stress levels:

$$ln\, t_{R0} = \omega\, ln\, \beta_1 + (1-\omega)\, ln\, \beta_2 + \alpha^{-1}\, lnln\left(\frac{1}{R}\right) \quad (37)$$

where ω is the stress weighting factor:

$$\omega = \frac{ln\, S_2 - ln\, S_0}{ln\, S_2 - ln\, S_1} = 1.5 \quad (38)$$

Step 2: Initial LCL estimate and confidence level calculation. An initial estimate for the LCL is assigned as $t^*_{RL0} = 6{,}000$ h. Based on Eq. (19), the corresponding exact confidence level γ^* is calculated as:

$$\gamma^* = 0.9986 > \gamma \quad (39)$$

Step 3: Iterative calibration and exact LCL determination. Iteratively adjust the initial estimate until the calculated γ^* converges to the nominal confidence level γ. Consequently, the exact one-sided LCL for the reliable life t_{R0} is obtained as:

$$t_{RL0,\gamma} = 9{,}135 \text{ h} \quad (40)$$

This LCL indicates that with 90% confidence, the true lifetime at which 95% of the harmonic reducer population survives is at least 9,135 h. From a maintenance perspective, the estimated LCL is more informative than a point estimate when conservative decision-making is required. Specifically, the obtained value of 9,135 h can be treated as a reliability-guaranteed maintenance threshold under the target reliability level of 0.95 and the confidence level of 0.90. In practice, it can provide a quantitative basis for arranging preventive replacement or inspection actions before the accumulated degradation risk becomes unacceptable.

To further validate the superiority of the proposed method, a comparative analysis is conducted against the results obtained using the BLUE method reported in reference [36]. Whereas reference [36] utilized data from all 20 specimens across four stress levels to obtain a characteristic life estimate of 14,918 h, the proposed method achieves a comparable estimate of 14,107 h using only 10 specimens across two levels. This represents a relative deviation of approximately 5.4%, demonstrating that assessment accuracy comparable to the multi-level BLUE method is maintained despite a 50% reduction in sample size.

Beyond point estimation, the proposed framework addresses the inherent theoretical limitations of the traditional BLUE method, which is typically restricted to point estimation for Weibull distributions because the derivation of LCLs necessitates complex distributional assumptions for the estimators. Conversely, the proposed method not only achieves high-precision point estimation but also directly evaluates the exact one-sided LCL for the reliable life, thereby satisfying high-confidence requirements.

Table 5 presents a comprehensive comparison of the assessment results. It can be observed that while reducing the sample size by 50%, the proposed method yields a point estimate very close to that of the BLUE method. Moreover, it is capable of providing a 90% lower confidence limit for the reliable life that the traditional method cannot exactly derive.

Table 5. Comparison of reliability assessment results for harmonic reducer.

Method	Sample size	$\hat{\beta}_0$(h)	$t_{RL0,\gamma}$(h)
Proposed method	2 levels, 10 units	14,107	9,135
BLUE method	4 levels, 20 units	14,918	Unable to estimate

7. Conclusion

To address the challenges of reliability evaluation for high-reliability, long-lifespan products with small sample sizes, this paper proposes a reliability assessment method for ADT based on the Weibull pseudo-failure lifetimes. The main conclusions are as follows:

(1) The proposed method enables high-confidence reliability assessment using data from only two accelerated stress levels. This capability significantly reduces the experimental burden associated with multiple stress levels and large sample sizes in conventional ADT, thereby substantially improving engineering feasibility for long-life products.

(2) Under small-sample constraints, the MLE method suffers from pronounced instability and significant bias. Furthermore, its reliance on the asymptotic normality assumption for LCL estimation leads to CPs that fall below nominal confidence levels, rendering it non-conservative and potentially risky for safety-critical applications.

(3) Unlike the BLUE method which is primarily restricted to point estimation for Weibull distributions due to complex distributional requirements, the proposed exact

inference framework achieves exact evaluation of the one-sided LCL. Under the same sample-size conditions, the developed method yields superior assessment accuracy; conversely, it achieves comparable precision with a significantly smaller sample size than traditional approaches.

(4) Extensive Monte Carlo simulations demonstrate that the CPs of the proposed method align closely with nominal confidence levels. In contrast, the CPs of the MLE method are significantly lower, further validating the superiority of the proposed method in small-sample scenarios.

References

1. Meeker WQ, Escobar LA, Lu CJ. Accelerated degradation tests: modeling and analysis. *Technometrics* 1998; 40(2): 89-99. https://doi.org/10.1080/00401706.1998.10485191.
2. Ye ZS, Xie M. Stochastic modelling and analysis of degradation for highly reliable products. *Applied Stochastic Models in Business and Industry* 2015; 31(1): 16-32. https://doi.org/10.1002/asmb.2063.
3. Park JI, Bae SJ. Direct prediction methods on lifetime distribution of organic light-emitting diodes from accelerated degradation tests. *IEEE Transactions on Reliability* 2010; 59(1): 74-90. https://doi.org/10.1109/TR.2010.2040761.
4. Limon S, Yadav OP, Liao H. A literature review on planning and analysis of accelerated testing for reliability assessment. *Quality and Reliability Engineering International* 2017; 33(8): 2361-2383. https://doi.org/10.1002/qre.2195.
5. Tian R, Zhang F, Du H, Wang P. Exponential dispersion accelerated degradation modelling and reliability assessment considering initial value and processes heterogeneity. *Eksploatacja i Niezawodność – Maintenance and Reliability* 2024; 26(4): 191432. https://doi.org/10.17531/ein/191432.
6. Gorjian N, Ma L, Mittinty M, Yarlagadda P, Sun Y. A review on degradation models in reliability analysis. In: Kiritsis D, Emmanouilidis C, Koronios A, Mathew J (eds). Engineering Asset Lifecycle Management. London: Springer; 2010: 369-384. https://doi.org/10.1007/978-0-85729-320-6_42.
7. Bhuiyan FH, Fertig RS. Predicting matrix and delamination fatigue in fiber-reinforced polymer composites using kinetic theory of fracture. *International Journal of Fatigue* 2018; 117: 327-339. https://doi.org/10.1016/j.ijfatigue.2018.08.007.
8. Bhuiyan FH, Fertig RS. A PFA methodology to investigate UD composites in fatigue comprising a KTF-based model for matrix damage and stochastic fiber failure prediction. *Composite Structures* 2022; 279: 114724. https://doi.org/10.1016/j.compstruct.2021.114724.
9. Li Q, Lin Y, Wang S, Wang S, Zhu X. Storage life prediction method of the aerospace electromagnetic relays based on physics of failure and data-driven fusion. *IEEE Access* 2022; 10: 103303-103314. https://doi.org/10.1109/ACCESS.2022.3209205.
10. Kufluoglu H, Alam MA. A generalized reaction-diffusion model with explicit H-H_2 dynamics for negative-bias temperature-instability (NBTI) degradation. *IEEE Transactions on Electron Devices* 2007; 54(5): 1101-1107. https://doi.org/10.1109/TED.2007.893809.
11. Liu Q, Shi W, Chen Z, Li K, Liu H, Li S. Rubber accelerated ageing life prediction by Peck model considering initial hardness influence. *Polymer Testing* 2019; 80: 106132. https://doi.org/10.1016/j.polymertesting.2019.106132.
12. Bae SJ, Yuan T, Kim SJ. Bayesian degradation modeling for reliability prediction of organic light-emitting diodes. *Journal of Computational Science* 2016; 17(Part 1): 117-125. https://doi.org/10.1016/j.jocs.2016.08.006.
13. Chen T, Liu Z, Ju L, Lu Y, Wei S. Improved Bayes-based reliability prediction of small-sample Hall current sensors. *Machines* 2024; 12(9): 618. https://doi.org/10.3390/machines12090618.
14. Li XY, Wu JP, Liu L, Wen ML, Kang R. Modeling accelerated degradation data based on the uncertain process. *IEEE Transactions on Fuzzy Systems* 2019; 27(8): 1532-1542. https://doi.org/10.1109/TFUZZ.2018.2883016.
15. Zhang H, Yuan H, Li P. Estimation method for extremely small sample accelerated degradation test data. In: 2015 First International Conference on Reliability Systems Engineering (ICRSE); Beijing, China. IEEE; 2015: 1-5. https://doi.org/10.1109/ICRSE.2015.7366417.
16. Tseng ST, Balakrishnan N, Tsai CC. Optimal step-stress accelerated degradation test plan for gamma degradation processes. *IEEE Transactions on Reliability* 2009; 58(4): 611-618. https://doi.org/10.1109/TR.2009.2033734.
17. Ling MH, Tsui KL, Balakrishnan N. Accelerated degradation analysis for the quality of a system based on the gamma process. *IEEE Transactions on Reliability* 2015; 64(1): 463-472. https://doi.org/10.1109/TR.2014.2337071.
18. Liu L, Li XY, Zio E, Kang R, Jiang TM. Model uncertainty in accelerated degradation testing analysis. *IEEE Transactions on Reliability*

2017; 66(3): 603-615. https://doi.org/10.1109/TR.2017.2696341.

19. Li S, Chen Z, Liu Q, Shi W, Li K. Modeling and analysis of performance degradation data for reliability assessment: a review. *IEEE Access* 2020; 8: 74648-74678. https://doi.org/10.1109/ACCESS.2020.2987332.
20. Tsai TR, Sung WY, Lio YL, Chang SI, Lu JC. Optimal two-variable accelerated degradation test plan for gamma degradation processes. *IEEE Transactions on Reliability* 2016; 65(1): 459-468. https://doi.org/10.1109/TR.2015.2435774.
21. Hong L, Ye ZS, Sari JK. Interval estimation for Wiener processes based on accelerated degradation test data. *IISE Transactions* 2018; 50(12): 1043-1057. https://doi.org/10.1080/24725854.2018.1468121.
22. Wang K, Zhang Z. Evaluation of a power module accelerated degradation test data based on Weibull pseudo-life method. In: 2020 IEEE 3rd International Conference of Safe Production and Informatization (IICSPI); Chongqing, China. IEEE; 2020: 215-219. https://doi.org/10.1109/IICSPI51290.2020.9332423.
23. Balakrishnan N, Kundu D, Ng KT, Kannan N. Point and interval estimation for a simple step-stress model with Type-II censoring. *Journal of Quality Technology* 2007; 39(1): 35-47. https://doi.org/10.1080/00224065.2007.11917671.
24. Zheng D, Fang X. Exact confidence limits for the acceleration factor under constant-stress partially accelerated life tests with Type-I censoring. *IEEE Transactions on Reliability* 2018; 67(1): 92-104. https://doi.org/10.1109/TR.2017.2747762.
25. Guo J, Fu H. Reliability assessment for small-sample accelerated life tests with normal distribution. *Machines* 2025; 13(9): 850. https://doi.org/10.3390/machines13090850.
26. Saxena S, Xing Y, Kwon D, Pecht M. Accelerated degradation model for C-rate loading of lithium-ion batteries. *International Journal of Electrical Power & Energy Systems* 2019; 107: 438-445. https://doi.org/10.1016/j.ijepes.2018.12.016.
27. Zhang TT, Gao ZG, Xu ZF, Gao CJ, Yan ZH, Qin ZS, Liu LL. Stress relaxation behavior and life prediction of 1Cr18Ni9 spring based on PSO-TCN-Attention. *Advances in Manufacturing* 2026; 14(3): 774-788. https://doi.org/10.1007/s40436-024-00538-5.
28. Huang J, Golubović DS, Koh S, Yang D, Li X, Fan X, Zhang GQ. Lumen degradation modeling of white-light LEDs in step stress accelerated degradation test. *Reliability Engineering & System Safety* 2016; 154: 152-159. https://doi.org/10.1016/j.ress.2016.06.002.
29. Guo X, Yuan X, Hou G, Zhang Z, Liu G. Natural aging life prediction of rubber products using artificial bee colony algorithm to identify acceleration factor. *Polymers* 2022; 14(17): 3439. https://doi.org/10.3390/polym14173439.
30. Bagdonavicius V, Nikulin M. Accelerated Life Models: Modeling and Statistical Analysis. Boca Raton, FL: Chapman & Hall/CRC; 2001. https://doi.org/10.1201/9781420035872.
31. Escobar LA, Meeker WQ. A review of accelerated test models. *Statistical Science* 2006; 21(4): 552-577. https://doi.org/10.1214/088342306000000321.
32. Barriga GDC, Ho LL, Cancho VG. Planning accelerated life tests under exponentiated-Weibull-Arrhenius model. *International Journal of Quality & Reliability Management* 2008; 25(6): 636-653. https://doi.org/10.1108/02656710810881926.
33. Bai DS, Chung SW. An accelerated life test model with the inverse power law. *Reliability Engineering & System Safety* 1989; 24(3): 223-230. https://doi.org/10.1016/0951-8320(89)90041-0.
34. Kececioglu DB, Jacks JA. The Arrhenius, Eyring, inverse power law and combination models in accelerated life testing. *Reliability Engineering* 1984; 8(1): 1-9. https://doi.org/10.1016/0143-8174(84)90032-5.
35. Wang BX, Yu K, Sheng Z. New inference for constant-stress accelerated life tests with Weibull distribution and progressively Type-II censoring. *IEEE Transactions on Reliability* 2014; 63(3): 807-815. https://doi.org/10.1109/TR.2014.2313804.
36. Sun X. Failure Mechanism and Life Prediction Technology of Harmonic Reducer. Anhui Normal University; 2018. https://doi.org/10.7666/d.D01559460.