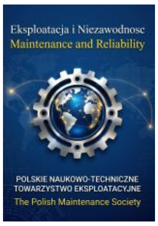

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Dynamic pricing strategy for maintenance services based on equipment degradation and game theory

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Abstract

In equipment maintenance, traditional methods often fail to characterize the equipment status, complicating service valuation. This study proposes a dynamic pricing strategy that integrates Gamma degradation modeling with Stackelberg game theory. Focusing on a single piece of equipment, the Gamma process is used to model its stochastic degradation, while preventive maintenance, replacement, and corrective maintenance strategies are combined to form a state-dependent dynamic maintenance rule. Using the Stackelberg game framework, key indicators like expected maintenance frequency and cost are embedded into the pricing model, creating the maintenance-decision pricing mechanism. Monte Carlo simulation solves the game equilibrium for the optimal inspection interval and price. Results show that an 11-day inspection interval achieves system equilibrium, with an optimal price of CNY 1.261 million. This strategy significantly improves the pricing scheme's adaptability to the actual equipment conditions and enhances its industrial practicality.

Keywords: *maintenance, pricing, equipment degradation, game theory, inspection interval*

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Highlights

- Integrates equipment degradation with game theory for dynamic maintenance pricing.
- Derives expected maintenance costs and downtime via Gamma process degradation model.
- Establishes a Stackelberg game to balance provider profit and customer utility.
- Uses Monte Carlo simulation to optimize inspection intervals and service prices.

1. Introduction

In the field of industrial equipment maintenance services, as enterprises' requirements for the equipment operational stability and production efficiency continue to increase, third-party specialized equipment maintenance services have gradually become an important choice for a wide range of enterprises to optimize resource allocation, enhance maintenance efficiency, and focus on core businesses. However, industrial equipment inevitably experiences performance degradation under long-term, high-load operating conditions. This degradation process exhibits significant stochastic and cumulative characteristics, which directly determine the maintenance frequency, cost structure, and risk of unplanned downtime during the maintenance process. These factors collectively form the foundation of the value of maintenance services, thereby profoundly influencing service pricing decisions. Therefore, constructing a pricing strategy that can accurately quantify and dynamically respond to the equipment degradation state has become an urgent problem to be solved in the field of industrial equipment maintenance management.

At present, extensive studies have been conducted on stochastic process modeling of equipment degradation in the fields of fault prediction and maintenance strategy optimization [1–4]. Common modeling methods include the Wiener process, IG process, and Gamma process [5–7]. Among these, the Gamma process can accurately characterize the non-negativity, randomness, and temporal correlation of degradation increments, and has thus attracted extensive attention in maintenance optimization. For example, Finkelstein et al. [8] modeled equipment degradation using the Gamma process and significantly reduced the long-run cost rate by setting a failure threshold. Yao et al. [9] investigated system maintenance strategies based on a non-homogeneous Gamma degradation process, and obtained the optimal maintenance actions to minimize the long-run average cost in each decision epoch by examining three non-periodic inspection policies. Therefore, the Gamma process is adopted for degradation modeling in this study. Nevertheless, most existing studies focus on degradation modeling and maintenance decision-making per se, and their integration with maintenance service pricing decisions remains inadequate. Few studies have embedded equipment degradation state as an endogenous variable into pricing models, thus failing to accurately portray the complete mechanism by which degradation dynamically drives service price formation by affecting maintenance costs and downtime risks. This problem makes it impossible to accurately quantify the true value of maintenance services, leaves pricing decisions without a fair value benchmark, and consequently amplifies the inherent goal conflicts in service transactions.

Currently, the core contradiction in maintenance service pricing lies in the inherent divergence of objectives between the service provider and the customer. The service provider focuses on covering its maintenance costs and maximizing profits, and thus tends to set a relatively high service price. In contrast, the customer urgently needs to control maintenance expenditures while maximizing equipment reliability and minimizing losses from production interruptions. Existing research on maintenance service pricing models has mostly focused on methods such as static optimization or contract design, neglecting the dynamic coupling relationship between the equipment degradation process and service strategies [10–13]. Although some studies have employed degradation processes or introduced warranty contracts to address product performance deterioration, most lack the dynamic responsiveness to the actual operating condition of equipment and thus fail to establish an effective balancing mechanism between the conflicting interests of both parties. For instance, Zhu et al. [14] optimized condition based maintenance for gamma deteriorating systems under performance based contracting. However, their single party optimization framework fails to capture the bilateral strategic interaction between the service provider and the customer, which is essential for degradation driven pricing. Consequently, these approaches cannot accurately reflect service costs that vary with equipment degradation, nor can they meet the dynamic demands of enterprises for production stability.

Game theory, as an effective tool for resolving multi-agent decision-making conflicts, provides a theoretical framework for reconciling the aforementioned bilateral interest contradictions. Existing studies have validated the prominent advantages of game theory in portraying multi-party strategic interactions and realizing coordinated pricing [15–18]. Among them, the Stackelberg game has inherent merits in depicting the maintenance service scenario where the service provider makes pricing decisions first and customers respond afterward, since it can effectively characterize the leader-follower sequential decision-making interaction. For example, the Stackelberg game has successfully achieved the optimization of strategies and payoffs in research on demand response pricing [19,20] and degradation-driven contract design [21]. This offers a direct paradigm for this study to address dynamic game issues in maintenance services; hence, the Stackelberg game is selected to construct the maintenance service pricing model in this study. Although this framework has been widely adopted, in the specific field of equipment maintenance services, existing research mostly focuses on market-level competition or macro-system optimization, and fails to integrate the evolution process of physical states into the game model. As a result, pricing decisions lack a scientific basis for reflecting the genuine value of maintenance services.

In summary, equipment degradation directly affects downtime and service costs. However, traditional maintenance service pricing methods, for example cost-plus pricing or fixed-period contract models, have not effectively quantified this dependency, nor have they sufficiently embedded the degradation state into the dynamic mechanism of maintenance service pricing. These conventional approaches fail to capture the time-varying nature of failure risk and maintenance costs, often leading to either overpricing that deters customers or underpricing that erodes the service provider's profit. Moreover, traditional methods do not incorporate the customer's utility with respect to downtime, and thus lack a mechanism to balance the interests of both parties. This failure limits the practicality of existing models in real-world industrial scenarios. Therefore, this study introduces an equipment degradation model into the pricing of maintenance services, proposing a dynamic pricing game model for maintenance services based on equipment degradation. The main contributions are as follows:

(1) An equipment degradation model based on the Gamma process is established. By integrating three strategies—preventive maintenance, preventive replacement, and corrective maintenance—a state-dependent dynamic maintenance strategy is constructed. Key indicators, such as the expected maintenance frequency and maintenance cost for each maintenance strategy, are derived.

(2) Based on the Stackelberg game, indicators such as expected maintenance frequency and maintenance cost are used as core inputs to construct a bi-level optimization pricing game model. This model aims to achieve the dual objectives of maximizing the service provider's profit and the enterprise's utility, thereby formulating a maintenance-decision pricing mechanism.

(3) The Monte Carlo method is employed to simulate stochastic degradation paths under different inspection intervals, thereby obtaining the optimal solution for both the inspection interval and pricing.

The research indicates that this method can effectively balance the dual objectives of cost control for the service provider and downtime minimization for the customer, enhancing the adaptability of the pricing strategy to the actual equipment status and its practicality for industrial scenarios.

2. Decision-making pricing mechanism for equipment maintenance services

In the field of equipment maintenance services, the decision-making process between a service provider and an enterprise is essentially a dynamic interactive process of multi-objective optimization, as shown in Fig. 1. The core objective of the service provider is to cover maintenance costs and maximize profits by optimizing the inspection interval and pricing scheme. The core demand of the customer is to minimize equipment downtime, ensure production continuity, and control maintenance expenditures to maximize comprehensive utility. The pricing game for equipment maintenance services proposed in this study is a dynamic game process. In the game interaction, the maintenance service provider can simulate customer responses under different strategies by iteratively optimizing the inspection interval and pricing, thereby offering dynamic pricing schemes to achieve profit maximization. The customer, in turn, can dynamically adjust their decision on whether to accept the service by responding to the service provider's pricing scheme. This behavior adheres to the rational economic agent hypothesis, which entails selecting the optimal decision scheme while satisfying the customer's minimum utility condition.

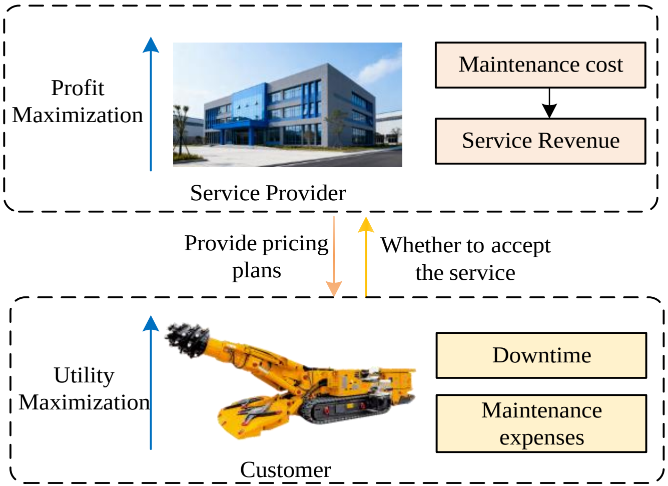

Figure 1. Dynamic interaction between the two parties in maintenance services.

However, prior to the game, if the equipment's degradation state is unknown, it is difficult to accurately estimate maintenance costs or downtime, leading to a maintenance strategy that is ultimately unscientific. Therefore, the equipment degradation process is embedded into the game model, and a dynamic maintenance pricing game model is established. This model comprises three modules: the equipment degradation process, maintenance strategy rule determination, and the pricing game model. The overall logic is presented as follows: First, taking a single piece of equipment as the study object, a Gamma process is employed to model the stochastic degradation increments of its critical components, generating the probability distribution of the equipment state under different inspection intervals. Second, based on the equipment state and combined with predefined threshold parameters for preventive maintenance and corrective maintenance, a maintenance strategy is dynamically selected. This step calculates key performance indicators for the chosen strategy, such as the expected number of maintenance actions, total maintenance cost, and total downtime. Finally, the service provider, acting as the leader, first formulates a combined strategy for inspection intervals and pricing. The customer, as the follower, then calculates a comprehensive utility based on the price and expected downtime. This process iterates through multiple rounds of gameplay until an equilibrium is reached between the objectives of both parties. The entire workflow couples stochastic degradation paths with state-dependent decision-making via Monte Carlo simulation, ensuring the pricing strategy's capability for dynamic response to the actual operational state of the equipment.

3. State-dependent dynamic maintenance strategy based on equipment degradation

The equipment degradation process serves as the foundation for formulating maintenance strategies. By quantifying the evolution law of natural equipment degradation, the expected number of each type of maintenance action can be derived, which in turn yields key parameters such as maintenance cost and downtime. These parameters provide input data for the subsequent game-theoretic analysis between the two parties.

The relevant parameters and their meanings are listed in Table 1.

Table 1. Model parameters.

Parameters	Meaning
X_t	Equipment state
N_i	Number of inspections
N_p	Number of preventive maintenance actions
N_r	Number of preventive replacement actions
N_f	Number of corrective maintenance actions
w_p	Preventive maintenance threshold
w_f	Corrective maintenance threshold
T	Inspection interval
L	Length of the maintenance period
η	Shape parameter
λ	Rate parameter
φ	Number of inspections performed
θ	Maintenance effect factor
ϑ	Residual degradation coefficient

3.1. Equipment degradation model based on a gamma process

The Gamma process is suitable for modeling damage that accumulates gradually over time through a series of small increments, such as wear, fatigue, and corrosion. Consequently, it has been widely adopted in the field of equipment degradation modeling [22, 23].

To characterize the performance deterioration of equipment between adjacent inspection points, this study introduces the independent increment, ΔX=X_(t+1)-X_t, representing the stochastic change in equipment degradation from time t to t+1. This increment possesses the properties of independence and stationarity, aligning with the fundamental assumptions of the Gamma process. To quantify the distribution characteristics of this random increment, it is necessary to establish its probability density function. This function describes the likelihood of the degradation amount residing within a specific interval during a unit inspection period, thereby providing the probabilistic foundation for subsequent calculations of the expected number of maintenance actions. Let the independent increment ΔX=X_(t+1)-X_t follow a Gamma distribution with a shape parameter η and a rate parameter λ. Its probability density function is given in Eq. (1). Serving as the fundamental input for modeling the degradation process, this function reflects the inherent physical law governing equipment performance deterioration.

$$f(x) = G(x|\eta, \lambda) = \frac{\lambda^{\eta} x^{\eta-1} e^{-\lambda x}}{\Gamma(\eta)} \ (x \geq 0) \tag{1}$$

In Eq. (1), η is the dimensionless shape parameter, which characterizes the aging behavior of the degradation process. The rate parameter λ represents the average rate of degradation increment per unit time. Since the basic unit of time in this study is the year, the unit of λ is $1/\mathrm{year}$, ensuring the dimensional consistency of the probability density function. Over any time interval, the degradation increment ΔX follows a gamma distribution $Gamma(\eta \cdot \Delta t, \lambda)$, and its expected degradation is $E[\Delta X] = (\eta \cdot \Delta t)/\lambda$, with the dimension being "dimensionless (relative degradation)" , which is consistent with the definition of probability.

3.2. State-dependent dynamic maintenance strategy

The equipment degradation state serves as the core input that directly dictates the maintenance strategy. Drawing on the previous research [10,24], this study categorizes maintenance strategies into preventive maintenance, preventive replacement, and corrective maintenance. Assuming the maintenance period is L and the service provider conducts regular inspections every T days, the total number of inspections over the maintenance period is:

$$N_i = L/T (T \in Z^+) \tag{2}$$

Assuming inspection points at $t_m = 1T, 2T, \dots, kT$ (where $k \in \{1,2,3,\dots\}$). At any inspection point, the equipment is inspected, and the state of its critical component is X_{tm}. The corresponding maintenance strategy is adopted based on the interval to which X_{tm} belongs, resulting in a post-maintenance equipment state denoted as X'_{tm}. If the equipment degradation state $X_{tm} < w_p$, no maintenance is performed, and the state retains the characteristics of the previous cycle.

If the equipment degradation state $X_{tm} \in [w_p, w_f)$ and $k \neq \varphi$, then preventive maintenance (PM) is performed on the equipment within the maintenance period. The expected number of such maintenance actions is given by Eq. (3). In this case, the equipment state is reduced to $\theta X_t (0 < \theta < 1)$. Furthermore, a residual degradation coefficient ϑ is introduced, meaning the post-maintenance state satisfies $X'_{tm} = max(\theta X_t, \vartheta w_p)$. This reflects the engineering reality that the equipment retains a baseline level of wear even after maintenance, while also reducing the risk of unexpected failures in subsequent inspection intervals.

$$E(N_p) = \sum_{k=1}^{N_i} \int_{w_p}^{w_f} \Omega_{kT}(x) dx \tag{3}$$

In the equation, $\Omega_{kT}(x)$ denotes the probability density function that the equipment degradation state $x \in [w_p, w_f)$ at the k-th inspection. This function is recursively derived based on the initial state, the increment distribution $f(x)$, and the maintenance rules.

When the equipment degradation state $X_{tm} \in [w_p, w_f)$ and the number of inspections reaches φ, a preventive replacement (PR) is executed on the equipment within the maintenance period. The expected number of such maintenance actions is given by Eq. (4). After replacement, the equipment returns to its initial state, i.e., $X'_{tm} = 0$.

$$E(N_r) = \sum_{k=1}^{N_i/\varphi} \int_{w_p}^{w_f} \Omega_{k\varphi T}(x) dx \tag{4}$$

Failures can occur at any time during the maintenance period. Once an equipment failure occurs, corrective maintenance (CM) is performed. The expected number of such maintenance actions is given by Eq. (5). After repair, the equipment is likewise restored to its initial state, i.e., $X'_{tm} = 0$.

$$E(N_f) = \sum_{m=1}^{L} \int_{w_f}^{\infty} \Omega_m(x) dx \tag{5}$$

To facilitate understanding, the process of inspecting and maintaining equipment during the maintenance period is illustrated using Fig. 2 as an example. Specifically, at the inspection point t_{k+1}, the equipment has been inspected φ times, thus triggering a PR action, after which the equipment returns to its initial state.

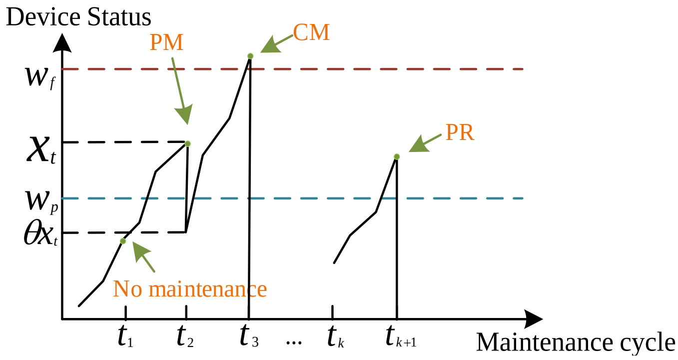

Figure 2. Equipment inspection and maintenance operations.

To solve Eq. (3), (4), and (5), it is necessary to derive the probability density function of equipment degradation for each time period. Since the equipment is assumed to be new at the initial time, when $m = 1$, $\Omega_1(X_t) = f(X_t)$. When $m \geq 2$, $\Omega_m(X_t)$ is jointly determined by the probability density function $\Omega_{m-1}(X_t)$ at time $m-1$ and the probability density function of the increment X_{t-1} from time $m-1$ to m. Assuming at inspection point t_{m-1}, the equipment degradation state before maintenance is b and after maintenance is b'; at inspection point t_m, the equipment state before maintenance is a and after maintenance is a'. Then, $X_{t-1} = a - b'$, as shown in Fig. 3.

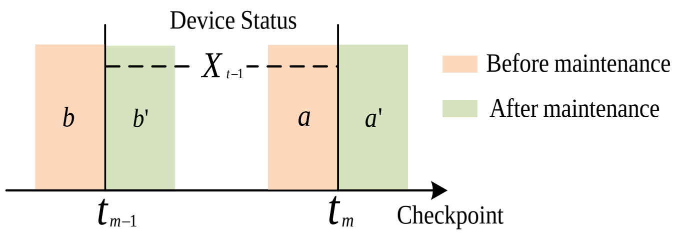

Figure 3. Relationship between equipment state and inspection points.

When the equipment state detected at time t_{m-1} requires PM, $b' = \theta b$. Consequently, the probability density function of the degradation state a at time t_m is given by:

$$\Omega_{kT}(a)=\int_{w_p}^{w_f}\Omega_{m-1}(b)f(a-\theta b)db \tag{6}$$

When the number of inspections reaches φ and the equipment state reaches the PM threshold, $b'=0$. Consequently, the probability density function of the degradation state a at time t_m is given by:

$$\Omega_{k\varphi T}(a)=\int_{w_p}^{w_f}\Omega_{m-1}(b)dbf(a) \tag{7}$$

When the equipment state detected at time t_{m-1} requires CM, $b'=0$. Consequently, the probability density function of the degradation state a at time t_m is given by:

$$\Omega_m(a)=\int_{w_f}^{\infty}\Omega_{m-1}(b)dbf(a) \tag{8}$$

4. Maintenance service pricing model based on Stackelberg game

4.1. Problem description

On the basis of the defined dynamic equipment maintenance strategy, this study constructs a Stackelberg game model with the service provider as the leader and the customer as the follower. Taking a single piece of equipment as the study object, the model combines the three strategies of preventive maintenance, preventive replacement, and corrective maintenance. Specifically, throughout the maintenance period L, the service provider conducts regular inspections of the equipment every T days and selects the corresponding maintenance action based on the equipment degradation state. From a game-theoretic perspective, the service provider aims to maximize profit by reducing maintenance costs and increasing service revenue, while the customer seeks to maximize its own utility by balancing total downtime and pricing. Accordingly, the Stackelberg game proceeds in two stages: the service provider, as the leader, first formulates a dynamic strategy comprising the inspection interval and the optimal price; the customer, as the follower, then decides whether to accept the service based on this strategy.

The relevant parameters and their meanings are listed in Table 2.

Prior to establishing the pricing game model for equipment maintenance services, it is assumed that the interaction between both parties satisfies the following conditions:

(1) Both participants, the service provider and the customer, are rational, meaning they reasonably seek to optimize their respective decision-making objectives.

(2) The interaction and transaction between the service provider and the customer are fair and equitable.

(3) The downtime for a single inspection is negligible.

(4) After performing preventive replacement or corrective maintenance on the equipment, the equipment can be restored to its initial state, i.e., $X'_{tm}=0$.

Table 2. Parameters of the equipment maintenance service pricing game model.

Parameters	Meaning
s	Customer
g	Service provider
V	Total profit of the service provider
U	Customer utility
P	Service price
C_c	Total maintenance cost
C_i	Total inspection cost
C_p	Total preventive maintenance cost
C_r	Total preventive replacement cost
C_f	Total corrective maintenance cost
D_t	Total downtime
D_P	Downtime per preventive maintenance action
D_r	Downtime per preventive replacement action
D_f	Downtime per corrective maintenance action
ω_1	Downtime weight factor
ω_2	Price weight factor

4.2. Pricing game model based on bilevel optimization

Bi-level optimization provides a systematic mathematical approach for solving the Stackelberg game equilibrium problem. The upper-level problem describes the profit maximization objective of the service provider, while the lower-level problem depicts the customer's decision-making behavior of pursuing its own utility maximization based on the given pricing strategy.

4.2.1. Upper-level model

(1) Objective Function of the Service Provider

In this study, the maintenance service provider aims to maximize the maintenance profit for a single piece of equipment, which is directly related to the total maintenance cost.

$$\begin{aligned}E(C_c)&=C_i+C_p+C_r+C_f\\&=C'_i*E(N_i)+C'_p*E(N_p)+C'_r*E(N_r)+C'_f*E(N_f)\end{aligned} \tag{9}$$

where C'_i, C'_p, C'_r, and C'_f denote the unit cost of inspection, preventive maintenance, preventive replacement, and corrective maintenance, respectively.

Therefore, the service provider's objective function is formulated as follows:

$$V_{max} = P - E(C_c) = P - [C'_i * E(N_i) + C'_p * E(N_p) + C'_r * E(N_r) + C'_f * E(N_f)] \quad (10)$$

(2) Constraints

To ensure that the Stackelberg game model reflects the key economic principles of actual transactions, reasonable constraints must be imposed on the service provider's pricing decisions. The following constraints are constructed:

$$E(C_c) \times (1 + R) \leq P \leq P_{max} \quad (11)$$

Eq. (11) defines a key constraint for the service provider's pricing decision, requiring that the price simultaneously meets the service provider's minimum profit margin R and does not exceed the customer's stipulated maximum allowable price.

$$\begin{cases} P > 0, \\ C_c > 0, \\ V > 0, \\ T \in Z^+ \end{cases} \quad (12)$$

Eq. (12) specifies that the price, total maintenance cost, service provider's profit, and inspection interval must always be positive.

4.2.2. Lower-level model

(1) Objective Function of the Customer

In the lower-level model, the decision objective of the customer is to maximize its comprehensive utility function. This utility function comprehensively reflects the customer's trade-off between equipment downtime and service price.

First, equipment downtime is the most critical factor influencing customer utility. The total downtime $D_t(T)$ is composed of the cumulative downtime caused by all maintenance activities, and serves as a core indicator for measuring production interruptions induced by the service. Its expression is as follows:

$$E(D_t) = E(N_P) \times D_P + E(N_r) \times D_r + E(N_f) \times D_f \quad (13)$$

Second, the service price is another critical factor influencing the customer's decision. To comprehensively characterize the customer's preference in trading off price against downtime, this study employs a weighted normalization method to construct the comprehensive utility function. That is, the customer utility for a single piece of equipment over the entire service period is given by:

$$U = \omega_1 \times (1 - E(D_t)/D_{max}) + \omega_2 \times (1 - P/P_{max}) \quad (14)$$

Where D_{max} denotes the upper limit of total downtime per equipment acceptable to the customer over the entire service period; P_{max} denotes the maximum price acceptable to the customer; ω_1 and ω_2 (with $\omega_1 + \omega_2 = 1$) are weighting factors that characterize the customer's relative preference for controlling downtime and service price, respectively.

(2) Constraints

The constraints of the lower-level model aim to capture the baseline requirements for the customer to accept the service. Specifically, this study considers the following three types of constraints:

$$A = \begin{cases} 1, U \geq U_0 \\ 0, otherwise \end{cases} \quad (15)$$

Eq. (15) represents the customer's decision function: the customer will accept the service if and only if the utility derived from the service provider's pricing strategy is no less than its reservation utility, U_0.

$$E(D_t) \leq D_{max} \quad (16)$$

Eq. (16) states that the total downtime per equipment over the service period must not exceed the customer's maximum tolerable downtime threshold, D_{max}.

$$P^* \leq P_{max} \quad (17)$$

Eq. (17) states that the final optimal price, P^*, determined for the service period must not exceed the customer's maximum acceptable price, P_{max}.

4.3. Solution of the pricing game model based on Monte Carlo Simulation

4.3.1. Existence and uniqueness of solutions

The core of the model proposed in this study is to solve for the optimal pricing P and inspection interval T so as to maximize the objective functions of both the service provider and the customer. The proof for the existence and uniqueness of the equilibrium solution is given as follows.

(1) Proof of Solution Existence

The feasible set, defined jointly by the service provider's pricing constraints and the customer's participation constraints, ensures the boundedness and closure of the decision space.

$$\Gamma = \begin{Bmatrix} (P,T)|\omega_1 \times (1 - E(D_t)/D_{max}) + \omega_2 \times (1 - P/P_{max}) \geq U_0 \\ E(D_t) \leq D_{max} \\ P \geq E(C_c) \times (1 + R) \\ T \in \{1,2,\ldots L\} \end{Bmatrix} \quad (18)$$

where $E(D_t)$ and $E(C_c)$ are the expected total downtime and expected total maintenance cost, respectively, for a given inspection interval T, and R denotes the minimum profit margin.

First, when the service provider adopts basic pricing strategies such as cost-plus pricing, there always exists an inspection interval T that can make the customer's utility no less than its minimum acceptable level U_0; therefore, the feasible set is non-empty. Meanwhile, since the inspection interval T is confined to a finite set of integers, and the corresponding price P is constrained within a closed interval, the feasible set is compact. Second, the service provider's profit function $V(P,T)$ is continuous over the feasible set. Although T is a discrete variable, the expected cost and expected downtime, which are based on the Gamma process, are continuous functions of T; hence, the profit function is continuous.

Given the conditions above, according to the Weierstrass Extreme Value Theorem, the continuous function $V(P,T)$ must attain a maximum on the non-empty compact set Γ, i.e., a Stackelberg equilibrium solution (T^*, P^*) exists.

(2) Proof of Solution Uniqueness

At equilibrium, to maximize profit, the service provider sets the price P at the maximum level acceptable to the customer, i.e., $P^* = P_{max}(T^*)$. Consequently, the service provider's decision can be simplified to choosing only the inspection interval T, and its profit function becomes:

$$\pi(T) = P_{max}(T) - E(C_c)$$
$$= P_{max} \times [1 - \frac{1}{\omega_2}(U_0 - \omega_1(1 - E(D_t)/D_{max}))] - E(C_c) \quad (19)$$

The core of proving uniqueness lies in demonstrating that the service provider's profit function $\pi(T)$ is strictly unimodal over the strategy space. First, the expected total maintenance cost, $E(C_c)$, exhibits a typical U-shaped curve. An overly short inspection interval leads to frequent checks and high costs, while an overly long interval results in insufficient maintenance, causing a sharp increase in failure repair costs. Therefore, there exists an "economically optimal inspection interval" that minimizes the cost. Second, the maximum price acceptable to the customer, $P_{max}(T)$, is a monotonically decreasing function of T. This is because a longer inspection interval leads to increased expected downtime and reduced customer utility, consequently lowering the maximum price the customer is willing to pay. Finally, the profit function $\pi(T)$ is the difference between a monotonically decreasing function and a U-shaped function. This composite function first increases and then decreases within its domain, possessing a unique maximum point T^*.

For the discrete inspection interval strategy space, the aforementioned unimodal property guarantees that at most one maximum point exists that satisfies the local optimality condition $\pi(T^*)$ is greater than both $\pi(T^* - 1)$ and $\pi(T^* + 1)$, thereby ensuring the uniqueness of the integer optimal solution T^*.

In summary, the Stackelberg game model constructed in this study satisfies the mathematical conditions for the existence and uniqueness of an equilibrium solution.

4.3.2. Monte Carlo Simulation Solution procedure design

This study designs a systematic solution procedure. The overall procedure is illustrated in Fig. 4.

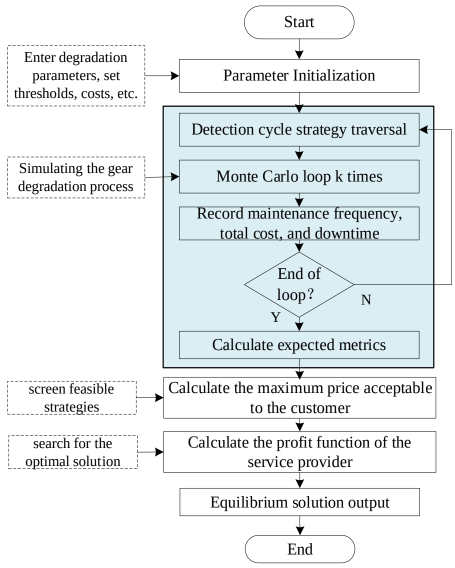

Figure 4. Solution procedure for the Stackelberg game based on the Monte Carlo Method.

The solution procedure for the Stackelberg game equilibrium based on the Monte Carlo simulation is as follows:

Step 1: Parameter Initialization. Input the equipment degradation parameters, cost and downtime parameters, utility function parameters, and the number of Monte Carlo simulations K. Determine the search range for the inspection interval.

Step 2: Inspection Interval Strategy Enumeration. For each candidate inspection interval T, perform K independent Monte Carlo simulation runs to simulate the equipment degradation process and the corresponding maintenance strategies. Record the counts of maintenance actions, cumulative costs, and downtime, and then calculate the expected metrics.

Step 3: Customer Response Decision. For each inspection interval T, based on the obtained expected downtime and the utility function formula, calculate the maximum acceptable price $P_{max}(T)$ at which the customer's utility is exactly equal to its reservation utility U_0.

Step 4: Service Provider Profit Optimization. The service provider, as the leader, aims to maximize profit. For each T, the service provider sets the price to the customer's maximum acceptable price, $P = P_{max}(T)$, and consequently calculates the profit function. It then searches within the feasible strategy set for the inspection interval that maximizes profit, $T^* = argmax\pi(T)$. This search is performed by enumerating all candidate T values and comparing their $\pi(T)$ values.

Step 5: Equilibrium Solution Output. Output the optimal inspection interval T^* and the corresponding optimal price $P^* = P_{max}(T^*)$. Calculate the equilibrium state metrics, including the service provider's maximum profit V_{max} and the customer's maximum utility U_{max}.

5. Numerical experiments

In this section, numerical examples are presented to demonstrate the effectiveness of the proposed model. A sensitivity analysis is conducted on key factors to examine the impact of input uncertainty on the results, thereby deriving practical insights.

A case study is conducted focusing on a MG300/730-AWD type shearer, with a maintenance service enterprise and a coal mining enterprise as service provider and customer, respectively. The structure of this shearer is shown in Fig. 5. The gear in the cutting unit, as a main load-bearing component of the shearer, can impact coal mining safety and production efficiency upon failure. Therefore, the cutting unit gear is selected as the critical component for investigating equipment maintenance strategies and pricing [25,26].

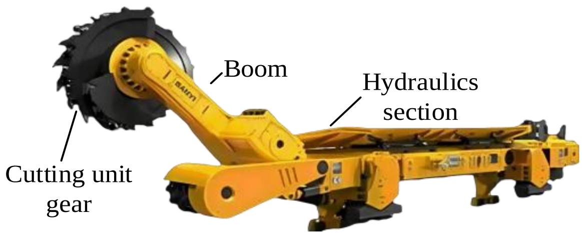

Figure 5. Structure of the shearer.

The game model parameters, including maintenance costs and downtime, are determined by integrating the actual requirements of both the service provider and the customer, historical maintenance records, and the maintenance accounting records of the cooperating coal mine enterprise. The specific values are presented in Table 3.

Table 3. Parameter settings for the case study.

Parameter Symbol	Parameter Description	Value	Unit
L	maintenance period	3	years
U_0	Reservation utility	0.75	-
C'_i	Single inspection cost	9,000	CNY
C'_p	Single PM cost	13,000	CNY
C'_r	Single PR cost	40,000	CNY
C'_f	Single CM cost	16,000	CNY
D_P	Single PM downtime	3	hours
D_r	Single PR downtime	15	hours
D_f	Single CM downtime	18	hours
D_{max}	Maximum acceptable downtime	1,000	hours
P_{max}	Maximum acceptable price for customer	1,400,000	CNY
ω_1	Downtime weight factor	0.8	-
ω_2	Price weight factor	0.2	-

For equipment degradation and associated model parameters, the shape parameter $\eta = 2$ and rate parameter $\lambda = 1.5$ (unit: 1/year) of the Gamma process are determined based on prior relevant literature [27] and calibrated using actual operational degradation data of the shearer; the maintenance effect factor $\theta = 0.5$ and residual degradation coefficient $\vartheta = 0.3$ are established based on mining equipment maintenance benchmarks combined with expert experience; the preventive maintenance threshold $w_p = 0.35$, corrective maintenance threshold $w_f = 0.55$, and preventive replacement threshold $\varphi = 4$ inspections are determined based on historical failure statistics of the cooperating enterprise combined with expert maintenance experience.

5.1. Analysis of results

Based on the above data, numerical simulations were performed using Python. The average time for the equipment to degrade from its initial state to the preventive maintenance threshold is approximately 96 days, which defines the upper bound of the search range to prevent excessively long inspection intervals from causing uncontrolled failure risk. Simultaneously, an overly short inspection interval would lead to a sharp increase in inspection costs, and in actual underground mining conditions, frequent inspections would significantly disrupt production schedules. Therefore, to systematically analyze the impact of the inspection interval T on the game equilibrium, the strategy space was set from 7 to 96 days with a step size of 1 day. This method requires substantial computational resources. Preliminary experiments indicated that the defined range of T values is sufficient to capture the variation patterns between the inspection interval and the benefits for all parties, while keeping the computation time manageable.

The solution process relies on the Monte Carlo simulation method. To scientifically determine its optimal repetition count K, this study employs a systematic convergence analysis approach. By performing a sequential scan over the K value range [100, 4000] (step size 100) combined with multiple repeated experiments, key statistical metrics such as the Relative Standard Error (RSE) and the 95% confidence interval width were evaluated. The results show that when $K = 3000$, the RSE drops significantly to 0.33%, and the confidence interval width converges to 0.2470, indicating clear convergence. Considering statistical reliability, computational efficiency, and result stability comprehensively, $K = 3000$ achieves the best balance among all metrics and is therefore determined as the optimal Monte Carlo simulation count.

Through systematic calculation of the game equilibrium under different inspection intervals, the expected maintenance count, downtime, and cost metrics were obtained, retaining three decimal places. Selected calculation results are presented in Table 4.

Table 4. Game equilibrium results under different inspection intervals T.

T	V (CNY)	U	$E(C_c)$(CNY)	P (CNY)	$E(D_t)$ (h)	PM times	PR times	CM times
7	-207,820	0.750	1,524,890	1,317,080	77.307	2.818	1.013	2.981
8	-25,220	0.750	1,343,730	1,318,510	77.052	2.756	0.988	2.998
9	110,640	0.750	1,208,400	1,319,050	76.956	2.768	0.960	3.014
10	215,500	0.750	1,101,240	1,316,740	77.367	2.700	0.995	3.019
11	252,200	0.759	1,008,820	1,261,020	76.233	2.791	0.888	3.030
12	234,480	0.771	937,940	1,172,430	76.986	2.751	0.921	3.051
13	218,860	0.782	875,430	1,094,290	77.139	2.673	0.954	3.045
...	...	...	...	...	...	...	...	...
30	109,580	0.860	438,310	547,800	77.223	2.344	0.849	3.192
31	106,770	0.862	427,070	533,840	76.674	2.378	0.763	3.226
32	104,660	0.863	418,620	523,270	76.632	2.391	0.781	3.208
...	...	...	...	...	...	...	...	...
93	48,540	0.905	194,160	242,710	75.870	1.596	0.432	3.589
94	48,580	0.905	194,330	242,920	75.933	1.620	0.429	3.591
95	48,770	0.904	195,090	243,860	76.131	1.646	0.443	3.586
96	48,770	0.904	195,060	243,830	76.080	1.678	0.432	3.587

As can be seen from Table 4, when the inspection interval is short (e.g., $T = 7$ and 8 days), frequent inspections indeed capture the degradation state more promptly. However, the service provider's profit becomes negative, while the customer's utility remains at the level of its reservation utility. The fundamental reason for this phenomenon is that the shortened inspection interval leads to a significant increase in inspection frequency, causing inspection costs to constitute an excessively large portion of the total maintenance cost. As the leader, the service provider must prioritize ensuring that the customer's utility is no less than its reservation level when setting the price P. Consequently, it cannot raise the price sufficiently to cover the high inspection costs, resulting in negative profit. As T gradually increases, inspection costs decrease. However, the monitoring blind spot due to stochastic degradation enlarges. By adjusting the pricing while still satisfying the customer's utility constraint, the service provider's profit turns from negative to positive and gradually optimizes. This reveals that, under the balance of bilateral interests, excessively frequent inspections can actually harm the service provider's revenue due to soaring costs. When $T = 11$ days, the system reaches a game equilibrium state. The degradation process, by influencing the expected counts of the three maintenance activities (PM, PR, and CM), collectively determines the structure of the total maintenance cost. At this equilibrium point, the service provider can set a price that both covers the comprehensive maintenance cost driven by degradation and is acceptable to the customer. At this point, the service provider's maximum profit is CNY 252,200, the customer's utility increases to 0.759, the total cost is CNY 1,008,800, the price is CNY 1,261,000, and the total downtime is controlled at 76.23 hours.

Due to the large volume of data, to clearly show the key variation trends, this study selects data for inspection intervals T ranging from 7 to 30 days for analysis. Among these, the variation patterns of the cost structure and profit with the inspection interval T are shown in Fig. 6, and the variation of customer utility with the inspection interval is shown in Fig. 7. Fig. 6 clearly illustrates the variation of the service provider's profit and the total maintenance cost with the inspection interval T. When $T = 11$ days, the system reaches the game equilibrium, and the service provider's profit achieves its maximum value of CNY 252,200, while the corresponding total maintenance cost is CNY 1,008,800. The total cost shows a trend of first declining rapidly and then leveling off as T increases. This cost curve profoundly reflects the combined effect of the stochastic equipment degradation process and the state-dependent maintenance strategy. While a shorter inspection interval allows for more timely detection of degradation, it causes inspection costs to surge. Conversely, an overly long interval increases failure risk due to insufficient monitoring, thereby raising corrective maintenance costs. The peak of the profit curve precisely represents the optimal outcome of balancing these two opposing effects.

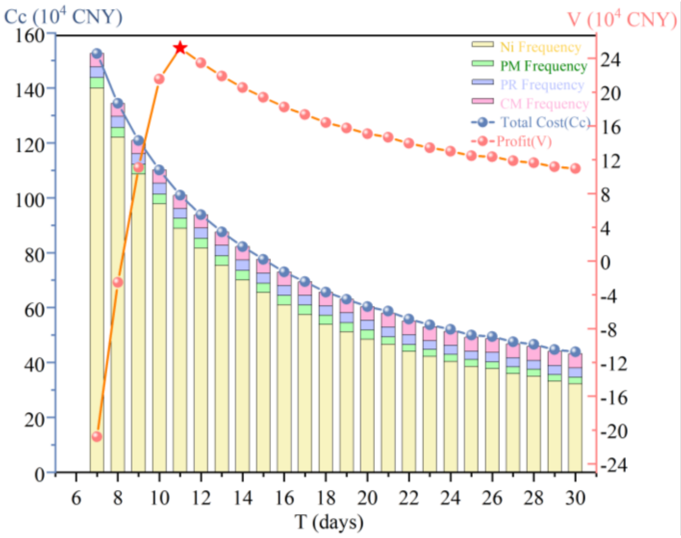

Figure 6. Variation of cost structure and profit with inspection interval.

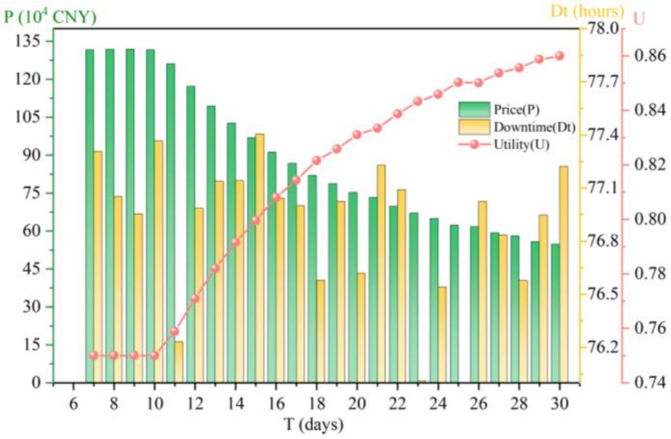

Figure 7. Variation of customer utility with inspection interval.

Fig. 7 shows the pattern of how customer utility varies with the inspection interval. On the one hand, because the degradation increments simulated by the Gamma process are stochastic, yet their expected values stabilize through Monte Carlo simulation, and because changes in the inspection interval have a compensating effect on the adjustment of preventive maintenance and corrective repair frequencies, the total downtime exhibits irregular variations with relatively small fluctuations. On the other hand, as the price gradually decreases, the customer utility shows a steady increasing trend, reflecting the direct influence of the price factor in the utility function. The phenomena described above collectively verify that the model proposed in this study accurately captures the stochasticity of equipment degradation and the dynamics of the bilateral game, demonstrating its scientific rigor and practical utility in balancing service provider profit and customer utility.

To empirically validate the superiority of the proposed model over traditional pricing methods, a benchmark comparison model is constructed. Specifically, this model adheres to the following rules: During the maintenance period, the inspection interval T is empirically set to 30 days, and preventive maintenance (PM) is performed every T days. Corrective maintenance (CM) is only conducted when the equipment degradation level X_t exceeds the corrective maintenance threshold w_f at any time, and the equipment state is reset to 0 upon completion of maintenance. Based on this maintenance strategy, the expected total maintenance cost $E(C_c)$ and expected total downtime $E(D_t)$ are obtained through Monte Carlo simulation. Subsequently, the service price is determined using the cost-plus pricing method, i.e., $P_{cp} = E(C_c) \times (1 + r)$, where the fixed profit margin r is set to 25%. Finally, the service provider's profit V and customer utility U are calculated. The comprehensive performance comparison between the proposed model and this traditional method is presented in Table 5.

Table 5. Performance comparison between the proposed model and Traditional Pricing methods.

	P(CNY)	$E(C_c)$ (CNY)	V(CNY)	$E(D_t)$ (hours)	U
The Proposed Model	1,261,000	1,008,800	252,200	76.23	0.759
Traditional Pricing Methods	637,100	509,600	127,400	154.84	0.785

The comparison results clearly reveal the core advantages of the proposed model over the traditional pricing method. Although the traditional method achieves a slightly higher customer utility through its low-price strategy, this comes at the cost of an expected total downtime as high as 154.84 hours. This reveals that its high utility essentially stems from a severe compromise on reliability risk, which is unsustainable in practical production. In contrast, the proposed model, through state awareness and joint optimization, significantly reduces downtime while creating higher service value, achieving approximately twice the profit of the traditional method.

The above analysis demonstrates that the maintenance service pricing decision model based on equipment degradation and Stackelberg game theory proposed in this study has significant advantages over traditional maintenance strategy pricing methods. It can effectively coordinate the bi-objective optimization of the service provider and the customer, achieving profit maximization while satisfying customer utility. Meanwhile, through Monte Carlo simulation, it effectively handles the stochasticity of equipment degradation, provides a data foundation for the formulation of state-dependent dynamic maintenance strategies and the solution of the game equilibrium, and realizes accurate quantification and dynamic optimization of maintenance strategies.

5.2. Sensitivity analysis

In this section, sensitivity analysis is conducted using the controlled variable method to evaluate the impact of several key parameters on the optimization results.

5.2.1. Sensitivity analysis of preventive maintenance parameters

As shown in Fig. 8, when the preventive maintenance threshold w_p is around 0.45, the service provider's profit reaches its peak of CNY 269,000. This phenomenon stems from the dual regulatory effect of w_p on the maintenance cost structure. When w_p is low, the equipment enters the preventive maintenance (PM) interval prematurely, resulting in an excessively high PM frequency. While this controls failure risk, the accumulated PM costs erode profits. Appropriately increasing w_p reduces unnecessary PM activities, significantly lowering the total maintenance cost and thereby driving profits upward. However, when w_p is too high, there is a lack of effective intervention before the equipment reaches the corrective maintenance threshold w_f, leading to an increased failure rate and a rise in the number of expensive corrective maintenance (CM). This causes the total cost to rebound and profits to decline.

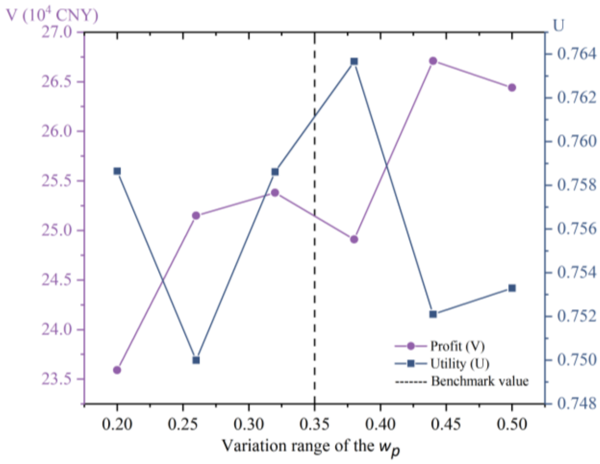

Figure 8. Impact of w_p variation on profit and utility.

Simultaneously, when w_p is set too low, the equipment triggers preventive maintenance prematurely, leading to an increased maintenance frequency. This, in turn, raises the total downtime, thereby reducing customer utility. As w_p increases to a moderate range, the frequency of preventive maintenance is appropriately controlled, total downtime decreases, and customer utility recovers accordingly. However, when w_p exceeds this range, delayed preventive intervention leads to a significant increase in equipment failure risk. The prolonged downtime caused by corrective repairs severely drags down customer utility, causing it to decline again. In practical management, $w_p \approx 0.41$ can be regarded as a potential trade-off point, where both the service provider's profit and the customer's utility are maintained at relatively high levels, achieving a relative balance of interests for both parties.

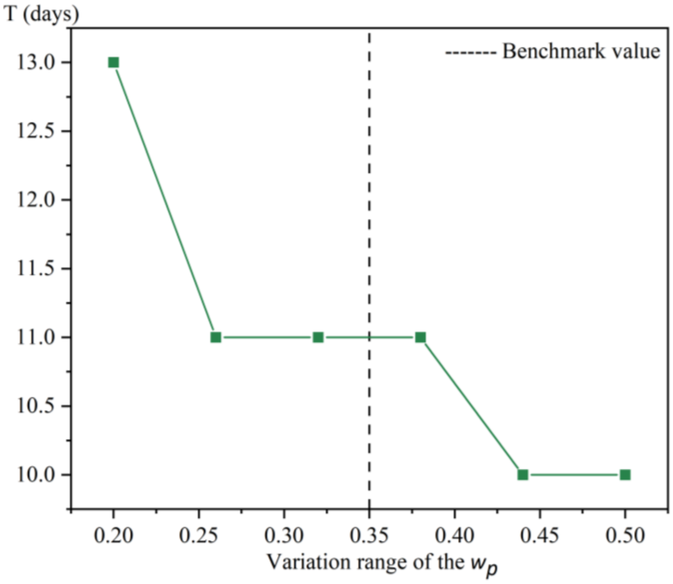

Figure 9. Impact of w_p variation on the optimal inspection interval.

Meanwhile, Fig. 9 reveals a negative correlation between w_p and the optimal inspection interval T^*. As w_p increases from 0.20 to 0.50, T^* decreases in a stepwise manner from approximately 13 days to about 10 days. The underlying mechanism is as follows: a higher w_p means allowing the equipment to operate at a higher degradation state, which increases the risk of sudden failure occurring between two inspections. To control this risk, more frequent inspections can be adopted to promptly detect and intervene in degradation states that might cross the threshold w_f. Conversely, a lower w_p sets a stricter safety margin, as the equipment is maintained in advance before reaching the corrective maintenance threshold w_f. Therefore, a longer inspection interval can be permitted without significantly increasing the failure risk.

5.2.2. Sensitivity analysis of various maintenance cost parameters

Maintenance costs are a core factor influencing service pricing and profit. As shown in Fig. 10, inspection cost has the highest sensitivity, a 50% change in it causes profit to drop significantly to approximately CNY 200,000. This is because inspection is the most frequently occurring activity, and variations in its cost directly and significantly impact the total cost through a multiplier effect. Corrective repair cost has the next highest sensitivity level. Its high unit cost, coupled with the longest associated downtime, means that an increase in this cost forces the model to favor more frequent PM or PR to prevent failures. However, the resulting increase in preventive costs cannot fully offset the impact of the rising corrective maintenance costs. Preventive maintenance and preventive replacement costs have lower sensitivity. The curves depicting their impact on profit are close to each other and exhibit a gentle slope.

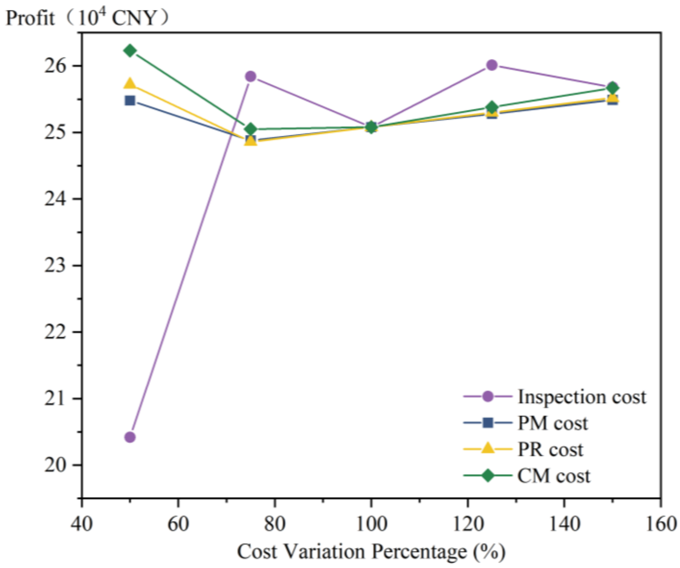

Figure 10. Impact of variations in different maintenance cost parameters on profit.

5.3. Managerial implications and insights

Synthesizing the above analysis results, this study proposes the following managerial implications for industrial equipment maintenance services. First, the analysis reveals a deeply coupled relationship between the preventive maintenance threshold and the inspection interval, which together form the core levers for risk control and cost management. Managers should collaboratively optimize both to adapt to the degradation characteristics and operational environment of specific equipment, thereby achieving optimal global economic benefit. Second, the high sensitivity of profit to inspection cost and corrective maintenance cost provides clear priority guidance for resource allocation and technical upgrades. Enterprises should prioritize investments in improving inspection efficiency to reduce inspection costs, and focus on reducing failure rates through measures such as enhancing preventive maintenance quality and improving the reliability of key components. This controls corrective maintenance costs at the source, and these investments will yield the highest cost-benefit returns. Finally, sensitivity analysis provides a quantitative basis for designing differentiated customer service and pricing strategies. Service providers can construct a series of service packages, ranging from "high-coverage, low-risk" to "economical, controlled-risk," based on different combinations of preventive maintenance thresholds and optimal inspection intervals. This achieves precise alignment between customer value perception and willingness to pay, thereby maximizing their own revenue while meeting diverse market needs, which significantly enhances its practical value and guiding significance in complex industrial operation and maintenance scenarios.

6. Conclusion and future work

This study proposes an optimization method for operation and maintenance service pricing decisions that integrates equipment degradation randomness and the bilateral service game. By accurately modeling the cumulative degradation characteristics of key equipment components using a Gamma process and constructing a dynamic pricing mechanism based on a Stackelberg game, it enables collaborative optimization of interests between the service provider and the client. The main conclusions are as follows:

(1) By transforming continuous degradation levels into expected maintenance frequencies via the Gamma process, this method accurately calculates corresponding operation and maintenance costs as well as downtime. This allows service pricing to dynamically respond to the equipment's operational state, providing a quantifiable theoretical foundation for addressing the challenge of insufficient pricing rationale in maintenance services.

(2) A state-dependent dynamic maintenance strategy was designed. By setting different thresholds and automatically triggering differentiated maintenance actions based on detected states, it achieves synergistic optimization of maintenance costs and production assurance in a stochastic degradation environment.

(3) The use of Monte Carlo simulation to handle stochastic degradation paths enhances the model's practicality and flexibility in complex industrial scenarios, providing quantifiable decision support for maintenance service pricing.

The proposed dynamic pricing mechanism achieves the synergistic optimization of maintenance cost control and production continuity assurance, improves resource allocation efficiency and the fairness of bilateral benefit distribution, and effectively addresses the lack of flexibility and state adaptability in traditional fixed-period pricing models.

However, this study has certain limitations. First, while the proposed modeling framework is theoretically applicable to all general industrial equipment with monotonic stochastic degradation characteristics, its empirical validation is currently limited to a single case of the MG300/730-AWD shearer. Further cross-validation across diverse equipment types is required to establish its broad generalizability. Meanwhile, the analysis reveals that both the preventive maintenance threshold and the inspection interval exert profound and coupled impacts on system performance. Future research should treat them jointly as endogenous decision variables and construct a joint dynamic optimization framework to capture deeper-level strategic interactions.

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