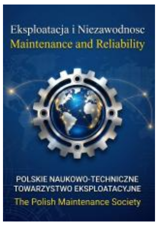

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A hierarchical Bayesian reliability assessment method for vehicle operational tests under zero-failure data

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Abstract

No-failure data are common in vehicle Operational Test (OT) because of limited test mileage and high reliability, which poses challenges for reliability assessment. This study proposes a hierarchical Bayesian method for OT reliability assessment by incorporating Developmental Test (DT) information. Under the exponential lifetime assumption, DT failure data and the engineering knowledge that OT conditions are more severe are used to construct a Shifted-scaled Beta prior for the OT failure rate. A multi-chain Metropolis-within-Gibbs MCMC algorithm with engineering constraints is developed to infer the posterior distribution and estimate the one-sided credible lower bound of the Mean miles between failures (MTBF) in OT. The case study and sensitivity analysis indicate that the proposed method can provide reasonable and interpretable reliability estimates under limited-mileage zero-failure OT conditions.

Keywords: *operational test, reliability assessment, zero-failure data, hierarchical Bayesian method, shifted-scaled Beta distribution*

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Highlights

- Proposes a hierarchical Bayesian method for vehicle reliability with zero-failure data.
- Constructs a Shifted-scaled Beta prior reflecting harsher operational test environments.
- Integrates developmental test information into operational test reliability evaluation.
- Improves reliability inference by controlled borrowing of cross-stage information.
- Obtains interpretable MTBF lower bounds under limited-mileage zero-failure OT conditions.

1. Introduction

In modern warfare, vehicles are no longer just transport tools. They are a key link for force maneuver and deployment, sensing and command, and logistics support. Vehicle reliability under real combat conditions directly affects combat effectiveness. The OT phase bridges vehicle development and operational use. Its environment is close to real combat, and operating conditions are complex and variable. OT results provide critical evidence for judging reliability in real operations [1]. Therefore, strict reliability assessment in OT is essential for ensuring vehicles can adapt to battlefield conditions and accomplish missions. It directly determines whether vehicles are dependable and durable in combat.

In actual vehicle OT, high costs, long durations, and complex environments result in test mileage far below that of DT [2]. With advances in modern manufacturing, vehicles also exhibit high reliability. The combination of limited effective test mileage and high vehicle reliability often leads to no-failure data during OT. Specifically, vehicles may experience no failures throughout the entire OT. While this no-failure phenomenon partly reflects good reliability, the lack of detailed failure information limits statistical insight, making quantitative reliability assessment very difficult. Improper handling can easily misjudge vehicle reliability in operational use, affecting equipment approval decisions and constraining military combat capability. Therefore, obtaining robust reliability estimates under no-failure conditions in OT remains a major challenge in current equipment reliability assessment.

Accurate reliability evaluation under zero-failure conditions in the OT phase is crucial for deciding whether a vehicle can enter mass production and operational deployment. First, the limited statistical information in zero-failure OT data must be fully exploited. Reliability metrics should be computed with rigorous methods and judged against current reliability standards. Second, the limited zero-failure OT mileage alone cannot fully represent reliability under real combat conditions. The evaluation should incorporate the characteristics of zero-failure data. It should also use multi-source prior information, such as the DT results and expert knowledge, to supplement and constrain the inference. This improves robustness and credibility. Therefore, a scientific and robust conclusion requires making the best use of limited data, computing reliability metrics properly, and building a robust assessment model under data scarcity by integrating multi-source information. This ensures the final results are robust and credible.

For scientific computation and standardized evaluation of reliability metrics, many methods have been proposed. William et al. [3] derived a lower confidence bound for MTBF under an exponential model with zero failures (here, MTBF denotes the Mean miles between failures). They used a chi-square quantile with degrees of freedom $2(r+1)$ and discussed the conservatism of this method. Sundberg et al. [4] systematically compared seven ways to construct confidence limits. They proved that the estimate is most reliable when the chi-square quantile with degrees of freedom $2(r+1)$ is used, and this approach later became the most authoritative core method in domestic and international standards. Chen et al. [5] were among the first to develop confidence-limit methods for zero-failure data. They provided optimal lower confidence bounds for the exponential, Weibull, and lognormal cases, which simplifies computation while improving efficiency. Wang et al. [6] proposed a modified likelihood method for zero-failure test data. By adjusting the conventional likelihood, it enables stable estimation of reliability metrics such as failure rate and the Mean miles between failures. These studies laid the foundation for reliability-metric computation under zero failures. However, most of them are built on a single distribution assumption and single-phase test data. The results are highly sensitive to test mileage. The methods also focus mainly on statistical index construction and make limited use of multi-source information. This restricts robustness and credibility under data scarcity and does not fit practical vehicle reliability assessment in the OT phase.

For zero-failure reliability assessment models based on multi-source information and prior knowledge, extensive Bayesian work has been reported. Martz et al. [7] were among the first to apply Bayesian methods to zero-failure reliability assessment. By introducing prior information, they obtained more reasonable conclusions and laid the foundation for later improvements and applications. Miller et al. [8] combined the conservative "Rule of 3" upper-bound formula with an input distribution and Bayesian prior updating. They derived a reasonable upper confidence limit for software failure rate under zero failures, which addressed the common zero-failure issue in software testing. Han [9] first proposed a multi-level Bayesian method for zero-failure data. It greatly reduced prior sensitivity while delivering robust reliability estimates, and it enabled the later development of E-Bayes[10] and M-Bayes [11]. Yoon et al. [12] systematically analyzed the advantages and limitations of Bayesian zero-failure reliability tests in sample-size design. Their study provided important theoretical support for Bayesian reliability modeling, multi-source information fusion, and prior-sensitivity analysis under zero failures. Overall, these studies established an important basis for zero-failure modeling with multi-source priors. However, current research lacks a unified way to fuse information from different sources. It does not clarify how to integrate DT information in an OT setting. In addition, prior hyperparameter ranges are often chosen empirically, with limited justification of their rationality and robustness. Some methods focus mainly on describing parameter posteriors, with a weak mapping to reliability metrics. Therefore, an assessment method is needed that can effectively fuse prior information from DT and the characteristics of zero-failure data, determine robust hyperparameter ranges in a principled way, and accurately map parameter posteriors to specific reliability metrics.

To address the difficulty of scientifically assessing vehicle reliability under no-failure conditions in OT, this study develops a hierarchical Bayesian reliability assessment framework that incorporates DT information. Assuming an exponential lifetime distribution, a flexible Beta distribution family is used to construct the prior model. Failure information from DT is then integrated, and engineering knowledge of the harsher OT environment is applied to build a Shifted-scaled Beta distribution as the prior for the OT failure rate. By establishing the relationship between DT and OT failure rates and combining hyperparameter values, a hierarchical Bayesian model is formed. A multi-chain Metropolis-within-Gibbs MCMC algorithm is designed for statistical inference of all parameters. The one-sided credible lower bound of the MTBF in OT is then obtained. Case studies based on actual vehicle test data are conducted, along with sensitivity analysis and convergence diagnostics, to verify the robustness and engineering applicability of the proposed method. The results show that the method can provide robust reliability estimates under no-failure OT conditions, offering a new solution for vehicle reliability assessment in such scenarios.

The main contributions of this study are summarized as follows.

- A reliability assessment framework for zero-failure vehicles in OT is established. OT is characterized by limited test mileage and frequent zero-failure observations. Under the assumption of an exponential lifetime distribution, a reliability assessment framework based on hierarchical Bayesian statistics is developed. The framework enables probabilistic modeling of extremely limited OT mileage and zero-failure data. It provides a methodological foundation for vehicle reliability evaluation in the OT stage.
- An improved Bayesian prior construction method that integrates DT information is proposed. Beta distributions replace traditional uniform priors, and failure information from DT is incorporated to build a Shifted-scaled Beta prior. This effectively fuses DT and OT information at the prior level while maintaining the physical meaning and valid range of the failure rate parameters and accounting for no-failure data characteristics.
- An interpretable cross-stage information-borrowing mechanism is established. Beta distribution parameters control the transfer of DT information to OT. The model reasonably sets hyperparameter ranges based on mission profile similarity, vehicle technical state consistency, environmental stress differences, and assessment risk preference. This ensures proper use of cross-stage information and avoids biases from simply merging data or ignoring DT information.
- Robust estimation and rigorous validation of reliability metrics under no-failure OT conditions are achieved. Using the established model, the one-sided credible upper bound of the OT failure rate and the one-sided credible lower bound of MTBF are obtained via MCMC. Convergence diagnostics and parameter sensitivity analysis with OT data confirm the rationality of hyperparameter choices and demonstrate the method's robustness and engineering applicability.

The remainder of this paper is organized as follows. Section 2 reviews related work. Section 3 presents the proposed framework in detail. Section 4 reports the numerical results and analysis. Section 5 concludes the paper and outlines future work.

2. Related works

2.1. Reliability assessment methods for zero-failure data

The core challenge of reliability assessment under zero failures is to make interpretable inferences on reliability metrics when failure information is missing and statistical information is extremely limited. The U.S. military handbook MIL-HDBK-781A [13] and the Chinese national standard GB/T 12678-2021[14] provide a one-sided lower confidence bound formula for MTBF under frequentist inference and an exponential lifetime assumption. The formula is widely used in military, aerospace, and automotive reliability practice because it is simple and engineering-friendly. Xia et al. [15] proposed a grey bootstrap method based on information-poor theory for zero-failure reliability analysis. It generates many simulated samples from a small zero-failure dataset and estimates the lifetime distribution via an empirical failure-probability function to support reliability inference. Li et al. [16] studied the zero-failure case for high-quality long-life products. They proposed a framework that uses a balanced distribution-curve method for reliability point estimation and the bootstrap for confidence intervals. By decoupling point and interval estimation, they improved the accuracy of point estimates under zero failures. Li et al. [17] further addressed zero-failure reliability assessment for Weibull lifetime models. They developed a set of confidence-limit methods, including a one-sided confidence-limit method and an optimal confidence-limit method, which greatly improves engineering efficiency in handling zero failures. Under a Weibull lifetime model, Zhang et al. [18] constructed and validated an unbiased estimator of the scale parameter. They also provided an effective confidence interval for the scale parameter and verified its coverage, offering a new perspective on zero-failure reliability inference. These methods provide useful insights. However, standard and confidence-limit methods still depend heavily on test mileage. Most studies also focus on the lifetime distribution itself and lack a systematic characterization of the mapping from lifetime models to reliability metrics.

2.2. Bayesian methods for zero-failure reliability assessment: applications and improvements

In zero-failure reliability assessment, missing failure events and extremely limited statistics make reliability parameter estimation unstable. Bayesian statistics can mitigate this issue because it can fuse multi-source information. Liang et al. [19] studied small-sample, zero-failure torpedo loading test data. They proposed an information-fusion Bayesian reliability model. Under small-sample zero failures, it avoids overly aggressive conclusions and shows good robustness. Liu et al. [20] addressed heavily censored data in high-reliability equipment. They proposed an improved E-Bayes method that outputs both point estimates and lower confidence bounds for reliability metrics. This overcomes the limitation of classical E-Bayes, which provides only point estimates, and it became a basis for later E-Bayes extensions. Zheng et al. [21] proposed a two-sided modified Bayesian reliability method. It applies two corrections, including posterior adjustment and physical constraints on failure rate. Under zero failures, it yields higher estimation accuracy and more reasonable interval widths than classic methods. Ou et al. [22] proposed an improved Bayesian method based on the folded normal distribution. By introducing a folded-normal prior that captures the distributional behavior of failure rate, it delivers more reasonable and accurate estimates than existing Bayesian methods and offers a new modeling idea for zero-failure data. These studies explored zero-failure reliability assessment under the Bayesian framework from multiple angles. However, most methods still focus on modeling the failure-rate behavior itself. They give insufficient attention to failure-rate prior information. As a result, they cannot fully exploit the latent value of DT data in an OT setting.

2.3. Bayesian zero-failure test design and sensitivity analysis

Beyond metric inference, Bayesian statistics is also crucial for designing reliable tests and building sound assessment models under zero failures. Roos et al. [23] proposed an RLMC-based local sensitivity analysis framework for Bayesian hierarchical models. It enables joint diagnosis of prior sensitivity and model identifiability. The framework can quickly locate highly sensitive parameters and guide prior refinement. Nott et al. [24] addressed inconsistencies between priors and data. They proposed a prior-expansion approach for prior–data conflict checking. It can handle conflict detection across multiple prior levels in hierarchical models and is useful for diagnosing complex Bayesian models. Sara et al. [25] performed simulation studies to analyze sensitivity for three default priors. These include an uninformative but unsuitable prior, a vague but suitable prior, and an empirical Bayes prior. They explained how different prior choices affect posterior inference. Li et al. [26] proposed a Bayesian reliability demonstration test-planning framework based on generalized statistics. Case studies show that it can plan demonstration tests effectively under small samples and support reasonable reliability decisions. These studies provide useful references for zero-failure reliability assessment. However, most focus on local parameter sensitivity. They lack a comprehensive robustness analysis of assessment results. As a result, they cannot fully demonstrate the robustness and scientific validity of reliability conclusions under zero-failure OT conditions.

3. Methodology

3.1. Problem statement and data structure representation

3.1.1. Modeling vehicle reliability assessment in OT under zero failures

This study focuses on vehicle reliability assessment during the OT phase. Vehicle reliability testing is conducted in two stages: DT under standard conditions, and OT under near-operational conditions. Reliability assessment in DT mainly evaluates whether vehicles meet reliability requirements under controlled conditions, while OT assessment emphasizes comprehensive evaluation in near-operational scenarios. DT is conducted on standard roads, producing abundant driving data. OT, however, is limited by cost, duration, and logistics, often providing only hundreds of kilometers of test data. With modern vehicles generally exhibiting high reliability, OT tests frequently yield no-failure data. Since vehicles undergoing OT have passed DT and condition verification, reliability indicators meet requirements under specified conditions, creating continuity in technology and data between the two stages. Therefore, failure information from DT can provide prior information for OT reliability assessment. This study addresses how to effectively leverage no-failure data patterns and DT failure information to construct a reasonable hierarchical Bayesian prior model, enabling valid inference of the posterior distribution and related reliability metrics under zero-failure conditions.

Assuming vehicle lifetimes follow an exponential distribution, let the failure rate in the DT phase be λ_{DT} and in the OT phase be λ_{OT}. The vehicle reliability metric is the Mean miles between failures (*MTBF*), with $MTBF_{DT}$ and $MTBF_{OT}$ for DT and OT, respectively. Under the exponential lifetime assumption, MTBF is the reciprocal of the failure rate, i.e., $MTBF = 1/\lambda$. To fully integrate λ_{DT}-related information with no-failure characteristics of λ_{OT}, the posterior distribution of λ_{OT} is solved, enabling reasonable estimation of the one-sided upper bound λ_{OTU} and the one-sided lower bound $MTBF_{OTL}$. To achieve this, a hierarchical Bayesian modeling approach is introduced. A conditional prior based on the Shifted-scaled Beta distribution is constructed for λ_{OT}, realizing effective transfer of DT information to OT reliability assessment.

This study adopts the exponential distribution assumption as an engineering approximation for OT scenarios with limited mileage and no-failure data, rather than assuming a constant failure rate over the vehicle's entire life cycle. The Weibull distribution is more suitable when mileage and failure samples are sufficient. Using a Weibull model under limited OT mileage and no failures introduces an additional shape parameter, making posterior inference highly dependent on the prior and reducing the robustness of reliability estimates. In practical equipment testing, vehicles entering OT have completed DT and condition verification, with stable technical states. If no significant wear or degradation is observed during limited OT mileage, the failure process can be considered a homogeneous Poisson process dominated by random failures. Additionally, the calculation formula for the one-sided confidence lower bound of the vehicle MTBF in GB/T12678-2021 [14] is based on the exponential lifetime assumption. In practice, engineering evaluation of vehicle reliability metrics also typically uses the exponential assumption as a baseline approximation. Therefore, the exponential distribution assumption provides acceptable engineering accuracy for this study's specific limited-mileage, no-failure OT scenario, and is adopted as the vehicle lifetime model. For long-term service equipment or datasets with sufficient failure samples, the Weibull model is more appropriate, which can be explored in future research.

3.1.2. Vehicle reliability test data structure and basic statistical model

Vehicle reliability test data can be represented as follows.

$$\mathcal{D}_m = \{(S_m, N_m): m = DT, OT\} \tag{1}$$

Here, m denotes the test phase, including DT and OT, S_m is the total test mileage in phase m, and N_m is the total number of failures observed in phase m.

When vehicle lifetime follows an exponential distribution, the failure rate is constant. The failure probability density function is:

$$f(S,\lambda) = \lambda e^{-\lambda S}$$
$$f(S,\gamma) = \frac{1}{\phi} e^{-\frac{S}{\phi}} \tag{2}$$

Here, ϕ denotes the Mean miles between failures, i.e., the vehicle MTBF.

Under the exponential lifetime assumption, the failure process is equivalent to a Poisson process with mileage as the scale. That is, we have:

$$N_m \mid \lambda_m, S_m \sim Poisson(S_m \lambda_m) \tag{3}$$

3.2. Construction of the zero-failure hierarchical prior model

3.2.1. Analysis of the Bayesian hierarchical model construction approach

Relying solely on limited OT mileage makes robust reliability assessment difficult, and posterior inference of the failure rate is highly sensitive to prior assumptions and test mileage.

Therefore, this study employs a hierarchical Bayesian model, incorporating abundant DT information into the prior construction of the OT failure rate, enhancing both the robustness and information utilization of vehicle reliability assessment under no-failure OT conditions.

In vehicle reliability theory, the failure rate λ is a dimensional rate parameter with a theoretical range of $\lambda \in (0,\infty)$. Currently, the highly flexible Beta distribution is often used for reliability assessment with no-failure data. However, the standard Beta distribution defined on $(0,1)$ is typically used for probability parameters or normalized dimensionless variables and cannot directly serve as a prior for the vehicle failure rate. Therefore, this study does not assume that the OT failure rate λ_{OT} directly follows a Beta distribution. Instead, based on engineering experience and actual no-failure OT observations, λ_{OT} is set within a finite, engineering-feasible interval, then normalized to the $(0,1)$ range, and the Beta prior is constructed on the normalized variable.

From the vehicle reliability testing process, OT is conducted based on prior DT. Although the two stages have different environments and objectives, failure data from DT provides important information for OT reliability assessment. DT is performed in controlled standardized environments such as test grounds and dedicated tracks, while OT targets realistic and complex battlefield conditions with higher uncertainty and harsher environmental stresses. Under these constraints, the OT failure rate λ_{OT} is necessarily greater than the DT failure rate λ_{DT}, i.e., $\lambda_{OT} > \lambda_{DT}$. An engineering upper bound λ_U is introduced and set such that $\lambda_{DT} < \lambda_{OT} < \lambda_U$. This upper bound reflects the maximum λ_{OT} based on engineering experience, providing a valid support range for constructing normalized variables and eliminating extreme failure rates that clearly do not match the vehicle's actual reliability level.

Under the above interval constraints, we define the normalized variable λ:

$$\lambda = \frac{\lambda_{OT}-\lambda_{DT}}{\lambda_U-\lambda_{DT}}, \quad 0 < \lambda < 1 \tag{4}$$

The OT failure rate λ_{OT} is then given by:

$$\lambda_{OT} = \lambda_{DT} + (\lambda_U - \lambda_{DT})\lambda \tag{5}$$

From Equation (5), as λ decreases, λ_{OT} decreases and approaches λ_{DT}. When $\lambda = 0$, $\lambda_{OT} = \lambda_{DT}$. As λ increases, λ_{OT} approaches λ_U. When $\lambda = 1$, λ_{OT} equals its upper bound λ_U. In this study, the dimensionless variable λ is assumed to follow a Beta distribution. Through the linear transformation in Equation (5), the standard Beta distribution is mapped to the interval $(\lambda_{DT}, \lambda_U)$, and λ_{OT} is constructed as a Shifted-scaled Beta prior. This prior ensures flexibility in shape adjustment while maintaining the dimensional consistency and valid range of the failure rate. The probability density function of the standard Beta distribution is:

$$\pi(\lambda \mid \alpha,\beta) = \frac{1}{B(\alpha,\beta)}\lambda^{\alpha-1}(1-\lambda)^{\beta-1} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\lambda^{\alpha-1}(1-\lambda)^{\beta-1} \tag{6}$$

where α and β denote the hyperparameters of the Beta distribution. Different values of α and β determine different shapes of the Beta distribution. Figures 1 and 2 illustrate the probability density curves and cumulative distribution functions of the Beta distribution under different values of α and β, respectively.

Figures 1 and 2 illustrate the probability density and cumulative distribution functions of the Beta distribution for different α and β values. The Beta distribution's advantage lies in expressing prior knowledge of vehicle failure rates through its hyperparameters. Since no failures occur during OT, lower OT failure rates should have higher prior probability, and the prior distribution of λ_{OT} should be decreasing. From Equation 5, when the normalized variable λ is decreasing, λ_{OT} is also decreasing. Observing Figures 1 and 2, the ranges of the Beta hyperparameters that produce a decreasing function are $\alpha \in (0,1]$ and $\beta > 1$. Figure 1 shows that when $\alpha < 1$, the Beta probability density tends to infinity near 0, causing strong shrinkage toward λ_{DT}. Under no-failure and limited OT mileage conditions, this can bias the prior toward λ_{DT}, overestimating $MTBF_{OT}$. Sensitivity analysis later shows that α significantly affects the $MTBF_{OT}$ lower bound. Therefore, for model simplification and to improve posterior robustness, this study sets $\alpha = 1$, preventing excessive prior concentration near λ_{DT} and reducing the risk of overestimating OT reliability under no-failure and limited mileage. With α fixed, β still controls the concentration of λ_{OT} toward λ_{DT}. Based on prior studies, larger β produces thinner tails in the Beta distribution, reducing Bayesian estimation robustness [10] . Thus, β is set within a reasonable range $\beta \in [1, \beta_{max}]$ and treated as a hyperparameter in the hierarchical Bayesian model rather than a fixed constant. The specific range of β_max is discussed in the context of vehicle reliability assessment cases.

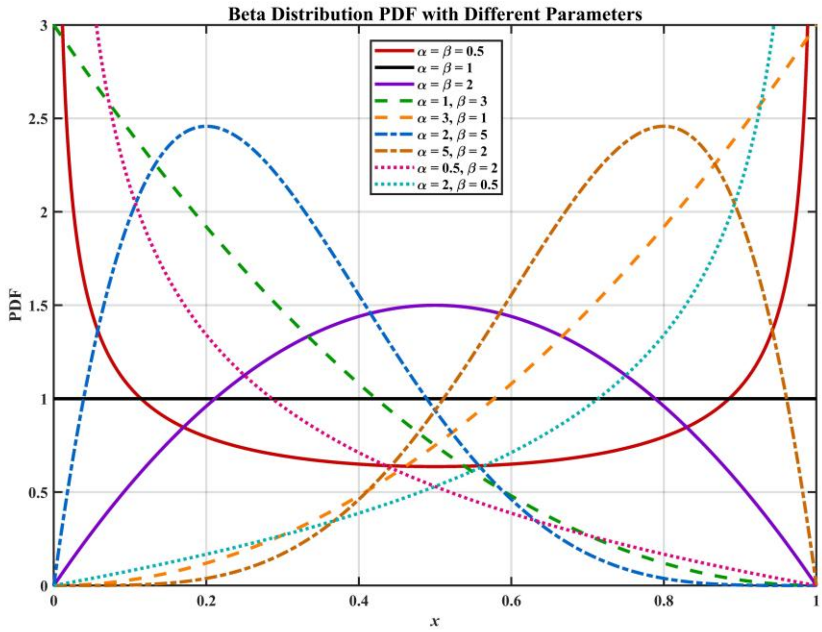


Figure 1. Probability density curves of the Beta distribution under different hyperparameter values.

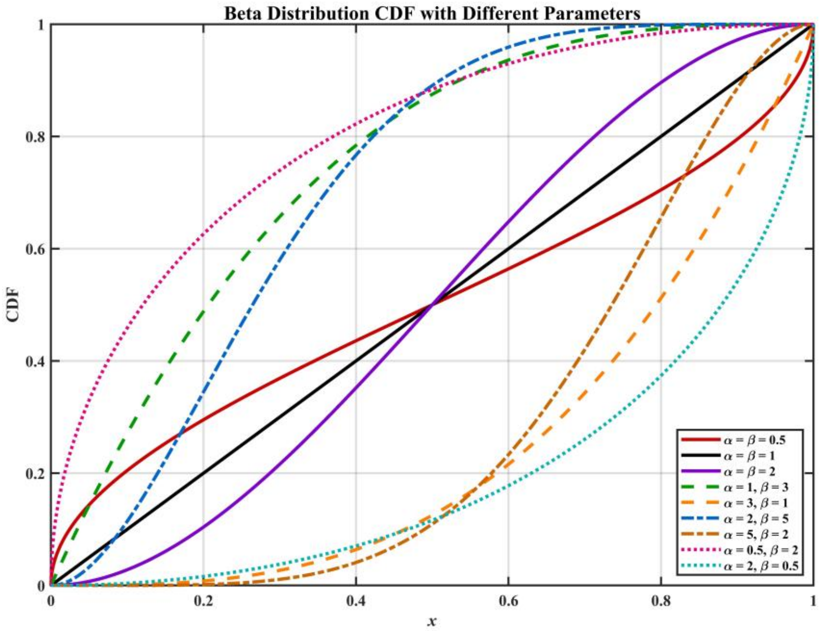


Figure 2. Cumulative distribution functions of the Beta distribution under different hyperparameter values.

Based on the above analysis, we construct a Bayesian hierarchical model for the unknown parameters $\{\lambda_{DT}, \lambda_{OT}, \beta\}$. A hard constraint $\lambda_{DT} < \lambda_{OT} < \lambda_U$ is introduced in the prior of λ_{OT} to reflect the harsher OT conditions and provide prior information for λ_{OT}. It should be noted that the prior distribution of λ_{OT} is not a standard Beta distribution, but a Shifted-scaled Beta prior constructed via the normalized variable λ. This design maintains the physical meaning of the failure rate while unifying DT information, OT no-failure characteristics, and conservative engineering constraints.

3.2.2. Prior construction for the DT failure rate λ_{DT}

Vehicle lifetime is assumed to follow an exponential distribution, and DT is conducted under controlled conditions, so failure samples are sufficient. In the hierarchical prior model, λ_{DT} mainly provides a baseline constraint for λ_{OT}. We therefore adopt a conjugate-prior approach and assign a Gamma prior to λ_{DT}. That is,

$$\lambda_{DT} \sim Gamma(a_0, b_0) \tag{7}$$

Its probability density function is:

$$\pi(\lambda_{DT}) = \frac{b_0^{a_0}}{\Gamma(a_0)} \lambda_{DT}^{a_0-1} \exp(-b_0 \lambda_{DT}) \tag{8}$$

Here, the two Gamma hyperparameters a_0 and b_0 are both set to 0.001 to approximate a noninformative prior. This lets the posterior of λ_{DT} be dominated by the abundant DT data and prevents the prior from unduly influencing λ_{DT}.

3.2.3. Prior construction for the OT failure rate $\boldsymbol{\lambda_{OT}}$

Based on the analysis in Section 3.2.1, given $\lambda_{OT} \in (\lambda_{DT}, \lambda_U)$, to more accurately characterize the prior distribution of λ_{OT}, this study incorporates the DT failure rate into the prior specification of λ_{OT} and defines the normalized variable $\lambda \mid \alpha, \beta \sim Beta(\alpha, \beta)$. The prior distribution of λ_{OT} is then constructed as a Shifted-scaled Beta distribution. That is:

$$\lambda_{OT} \sim Beta(\lambda_{DT}, \lambda_U, \alpha, \beta) \tag{9}$$

Here, values of λ_{OT} are within the range $(\lambda_{DT}, \lambda_U)$, α and β are the hyperparameters of the Beta distribution with $\alpha = 1$ and $\beta > 1$.

The prior probability density function of λ_{OT} can then be derived from the probability density function of the Beta distribution as follows:

$$\pi(\lambda_{OT} \mid \lambda_{DT}, \lambda_U, \alpha, \beta) = \frac{(\lambda_{OT}-\lambda_{DT})^{\alpha-1}(\lambda_U-\lambda_{OT})^{\beta-1}}{B(\alpha,\beta)(\lambda_U-\lambda_{DT})^{\alpha+\beta-1}} = \frac{\beta(\lambda_U-\lambda_{OT})^{\beta-1}}{(\lambda_U-\lambda_{DT})^{\beta}}, \lambda_{OT} \in (\lambda_{DT}, \lambda_U) \tag{10}$$

Equation (10) represents the prior probability density function of λ_{OT}. This prior is constructed based on the Beta distribution. On one hand, it retains the flexibility of the Beta family in modeling vehicle failure rates under no-failure conditions. When $\alpha = 1$ and $\beta > 1$, the prior density decreases with increasing λ_{OT}, reflecting the higher likelihood of lower failure rates under no-failure data. On the other hand, it embeds DT information into the prior of λ_{OT}, and as β increases, λ_{OT} tends to concentrate near λ_{DT}. This prior specification unifies the characteristics of no-failure OT data with cross-stage reliability information.

3.2.4. Prior construction for the hyperparameter $\boldsymbol{\beta}$

From the analysis in Section 3.2.1, as β increases, the distribution of λ_{OT} becomes more concentrated near λ_{DT}. To avoid imposing overly strong subjective assumptions on β that could reduce Bayesian robustness, this study treats β as a hyperparameter to be estimated in the hierarchical Bayesian model and assigns it a uniform prior. According to Section 3.2.1, $\beta \in [1, \beta_{max}]$, that is:

$$\beta \sim Uniform(1, \beta_{max}) \tag{11}$$

When $\beta = 1$ and $\alpha = 1$, the prior distribution of λ_{OT} becomes uniform over $(\lambda_{DT}, \lambda_U)$, indicating that the model does not favor lower failure rates but distributes them evenly within the feasible range. Therefore, the minimum value of $\beta = 1$ corresponds to the most conservative estimate. Since the normalized variable λ follows a Beta distribution, when the Beta hyperparameter $\alpha = 1$, its prior mean is as follows:

$$E(\lambda) = E\left(\frac{\lambda_{OT}-\lambda_{DT}}{\lambda_U-\lambda_{DT}}\right) = \frac{1}{1+\beta} \tag{12}$$

From Equation (12), as β increases, the prior mean of λ_{OT} decreases, indicating that its distribution concentrates more around λ_{DT}. This reflects a stronger transfer of DT information to OT reliability assessment and β_{max} can be interpreted as the maximum strength allowed for such information transfer in the hierarchical Bayesian model. When β_{max} is large, the model allows λ_{OT} to concentrate more strongly toward λ_{DT}. When β_{max} is small, its allowable range is narrower, indicating weaker information borrowing, suitable for scenarios with large differences between DT and OT profiles, significant environmental stress differences, and high operational risk. Since the similarity between DT and OT varies across vehicle types, profiles, and mission scenarios, β_{max} is not fixed to a single value. Instead, it is selected based on vehicle type, test stage, OT operational environment, and risk preference.

3.3. Posterior distribution derivation

Under the exponential lifetime assumption, the vehicle failure process can be modeled as a Poisson process. In the OT stage, when the vehicle travels S_{OT} kilometers and no failures are observed, the corresponding likelihood function is given by:

$$\mathcal{L}(0 \mid \lambda_{OT}) = exp(-S_{OT}\lambda_{OT}) \tag{13}$$

In DT, if the vehicle runs S_{DT} kilometers and observes N_{DT} failures, the likelihood function is:

$$\mathcal{L}(N_{DT} \mid \lambda_{DT}) = \frac{(S_{DT}\lambda_{DT})^{N_{DT}}}{N_{DT}!} exp(-S_{DT}\lambda_{DT}) \tag{14}$$

In summary, the priors for all parameters and the likelihoods for DT and OT are now specified. The set of parameters to be estimated is θ as follows:

$$\theta = \{\lambda_{DT}, \lambda_{OT}, \beta\} \tag{15}$$

We then derive the joint posterior distribution of all parameters using Bayesian Formula:

$$\pi(\theta \mid \mathcal{D}) \propto \pi(\lambda_{OT} \mid \lambda_{DT}, \lambda_U, \alpha, \beta)\pi(\lambda_{DT})\pi(\beta)\mathcal{L}(N_{DT} \mid \lambda_{DT})\mathcal{L}(0 \mid \lambda_{OT}) \quad (16)$$

Equation (16) gives the form of the joint posterior distribution, and the parameters must satisfy the following constraints:

$$0 < \lambda_{DT} < \lambda_{OT}, \lambda_{DT} < \lambda_{OT} < \lambda_U, 1 \le \beta \le \beta_{max} \quad (17)$$

To facilitate subsequent MCMC inference, we take the natural logarithm on both sides of (16). The joint log-posterior distribution is:

$$log\, \pi\, (\theta \mid \mathcal{D}) \propto (a_0 + N_{DT} - 1)log(\lambda_{DT}) - (b_0 + S_{DT})\lambda_{DT} - S_{OT}\lambda_{OT} + log(\beta) + (\beta - 1)log(\lambda_U - \lambda_{OT}) - \beta log(\lambda_U - \lambda_{DT}) - log(\beta_{max} - 1) \quad (18)$$

where $a_0 = b_0 = 0.001$ and $\beta_{min} = 1$ .The log-posterior formulation avoids numerical underflow caused by repeated multiplication of small probability terms. It also improves numerical stability in subsequent MCMC iterations and facilitates the construction of Metropolis–Hastings acceptance ratios for each parameter. The joint log-posterior distribution forms the basis of the subsequent MCMC algorithm.

3.4. MCMC inference algorithm and reliability metric computation

3.4.1. MCMC algorithm design

In the hierarchical Bayesian model constructed in this study, the prior of λ_{OT} is a Shifted-scaled Beta prior defined on $(\lambda_{DT}, \lambda_U)$, and β is treated as a hierarchical hyperparameter in posterior inference. Therefore, the joint posterior has no standard conjugate form, lacks an analytical solution, and cannot be directly sampled using Gibbs. Consequently, this study employs a component-wise multi-chain Metropolis-within-Gibbs MCMC algorithm for posterior sampling of the parameters. In each iteration, λ_{DT}, λ_{OT}, and β are updated sequentially, with each component updated using Metropolis-Hastings sampling. At the *t*-th iteration, the parameter state is $\theta^{(t)} = \{\lambda_{DT}^{(t)}, \lambda_{OT}^{(t)}, \beta^{(t)}\}$, and the algorithm updates the parameters in the following sequence:

$$\lambda_{DT} \to \lambda_{OT} \to \beta \quad (19)$$

First, update the Developmental Test failure rate λ_{DT}, ensuring that the candidate value λ_{DT}^* satisfies the following constraints:

$$0 < \lambda_{DT}^* < \lambda_{OT}^{(t-1)} \quad (20)$$

Then, update the Operational Test failure rate λ_{OT}, ensuring that the candidate value λ_{OT}^* satisfies the following constraints:

$$\lambda_{DT}^{(t)} < \lambda_{OT}^* < \lambda_U \quad (21)$$

Finally, update the hyperparameter β, ensuring that the candidate valueβ^*satisfies the following constraints:

$$1 \le \beta^* \le \beta_{max} \quad (22)$$

If any of the three candidate parameter values violate the constraints, the candidate is immediately rejected. When the constraints are satisfied, each candidate is accepted according to its own acceptance probability. For the DT failure rate λ_{DT} and OT failure rate λ_{OT}, standard normal random walks are used as proposal distributions. Their acceptance probability calculations and sampling procedures follow the standard Metropolis–Hastings random walk. The proposal distributions for λ_{DT} and λ_{OT} are given by:

$$\lambda_{DT}^* = \lambda_{DT}^{(t-1)} + \sigma_{DT}\varepsilon_{DT}, \quad \varepsilon_{DT} \sim N(0,1) \quad (23)$$

$$\lambda_{OT}^* = \lambda_{OT}^{(t-1)} + \sigma_{OT}\varepsilon_{OT}, \quad \varepsilon_{OT} \sim N(0,1) \quad (24)$$

where σ_{DT} and σ_{OT} are the proposal step sizes, and the acceptance probabilities for λ_{DT} and λ_{OT} are A_{DT} and A_{OT}, respectively.

$$A_{DT} = min\{1, exp(\Delta_{DT})\}, \ \Delta_{DT} = log\, \pi\left(\lambda_{DT}^*, \lambda_{OT}^{(t-1)}, \beta^{(t-1)} \mid \mathcal{D}\right) - log\, \pi\left(\lambda_{DT}^{(t-1)}, \lambda_{OT}^{(t-1)}, \beta^{(t-1)} \mid \mathcal{D}\right) \quad (25)$$

$$A_{OT} = min\{1, exp(\Delta_{OT})\}, \ \Delta_{OT} = log\, \pi\left(\lambda_{DT}^{(t)}, \lambda_{OT}^*, \beta^{(t-1)} \mid \mathcal{D}\right) - log\, \pi\left(\lambda_{DT}^{(t)}, \lambda_{OT}^{(t-1)}, \beta^{(t-1)} \mid \mathcal{D}\right) \quad (26)$$

When updating the hyperparameter β, due to its range $1 \le \beta \le \beta_{max}$ to ensure that candidate values remain positive and improve sampling stability, a random walk in the log space is used as the proposal distribution, defined by the following transformation:

$$u = log(\beta - \beta_{min} + 1) = log(\beta), \ \beta_{min} = 1 \quad (27)$$

Then, its proposal distribution is defined as follows:

$$u^* = u^{(t-1)} + \sigma_\beta \varepsilon_\beta, \quad \varepsilon_\beta \sim N(0,1) \quad (28)$$

Here, σ_β is the proposal step size, $\beta^* = exp(u^*)$, and the acceptance probability of β is A_β:

$$A_\beta = min\{1, exp(\Delta_\beta)\}, \ \Delta_\beta = log\, \pi\left(\lambda_{DT}^{(t)}, \lambda_{OT}^{(t)}, \beta^* \mid \mathcal{D}\right) - log\, \pi\left(\lambda_{DT}^{(t)}, \lambda_{OT}^{(t)}, \beta^{(t-1)} \mid \mathcal{D}\right) + (u^* - u^{(t-1)}) \quad (29)$$

After T iterations, discard the first T'samples as burn-in. Then, thin the remaining samples at interval L to reduce autocorrelation. Combine the chains after sampling to form the posterior sample set as follows:

$$\{\lambda_{DT}^{(k)}, \lambda_{OT}^{(k)}, \beta^{(k)}\}_{k=1}^{K} \tag{30}$$

Here, K is the total number of posterior samples after merging.

The pseudocode for the entire multi-chain Metropolis-within-Gibbs MCMC algorithm is as follows:

Algorithm Multi-chain Metropolis-within-Gibbs MCMC sampler

Input: Test data$\mathcal{D}_m = \{(S_m, N_m): m = DT, OT\}$; hyperparameter s$a_0$,$b_0$,$\alpha$,$\beta_{min}$, $\beta_{\max}$ λ_U; MCMC settings$C$,T,T',L; proposal scales σ_{DT},σ_{OT},σ_β. credibility level$1-\gamma$ (set$\gamma = 0.05$in this work).

Output: Posterior samples $\{\lambda_{DT}^{(k)}, \lambda_{OT}^{(k)}, \beta^{(k)}\}_{k=1}^{K}$; reliability metrics: one-side$1-\gamma$upper credible limit of Failure rate $\lambda_{1-\gamma}^{U}$and one-side$1-\gamma$lower credible limit of$MTBFMTBF_{1-\gamma}^{L}$.

for chain$c = 1, \dots, C$**do**
  Initialize $\lambda_{DT}^{(0)}, \lambda_{OT}^{(0)}, \beta^{(0)}$ such that$0 < \lambda_{DT} < \lambda_{OT} < \lambda_U$ and βmax_{min}; evaluate$\ell^{(0)}$.
  for $t = 1, \dots, T$ to $T-1$ **do**
    //Updateλ_{DT}
    $\lambda_{DT}^{*} \leftarrow \lambda_{DT}^{(t-1)} + \sigma_{DT} \cdot N(0,1)$
    if$0 < \lambda_{DT}^{*} < \lambda_{OT}^{(t-1)}andv \sim U(0,1) < exp(\Delta\ell)$ **then** accept; **else** reject.
    //Updateλ_{OT}
    $\lambda_{OT}^{*} \leftarrow \lambda_{OT}^{(t-1)} + \sigma_{OT} \cdot N(0,1)$
    if$\lambda_{DT}^{(t)} < \lambda_{OT}^{*} < \lambda_Uandv \sim U(0,1) < exp(\Delta\ell)$ **then** accept; **else** reject.
    //Updateβ**via log-space random walk**
    $u^{*} \leftarrow log(\beta^{(t-1)} - \beta\beta_{min}; \beta^{*} \leftarrow exp(u^{*}) + \beta_{min}$
    if$\beta *_{max_{min}}$and$v \sim U(0,1) < exp(\Delta\ell + u^{*} - u^{(t-1)})$ **then** accept; **else** reject.
    if$t \le T'$**then** adaptively tuneσ_{DT},σ_{OT},$\sigma_\beta$based on acceptance rates.
  end for
end for
Discard the firstT'burn-in iterations, thin by interval$L$, and merge$C$chains, yielding$K = C(T - T')/L$posterior samples.
//Compute reliability metrics at credibility level$1-\gamma = 95\%$
$\lambda_{1-\gamma}^{U} \leftarrow Q_{1-\gamma}(\{\lambda^{(k)}\})$ **//95% one-sided upper credible limit of failure rate**
$MTBF_{1-\gamma}^{L} \leftarrow Q_{\gamma}(\{1/\lambda^{(k)}\})$ **// 95% one-sided lower credible limit of**$MTBF$
Return$\{\lambda_{DT}^{(k)}, \lambda_{OT}^{(k)}, \beta^{(k)}\}, \lambda_{1-\gamma}^{U}, MTBF_{1-\gamma}^{L}$.

Note: $\Delta\ell$denotes the log-posterior difference, i.e., ℓ(proposal) $-$ ℓ(current)

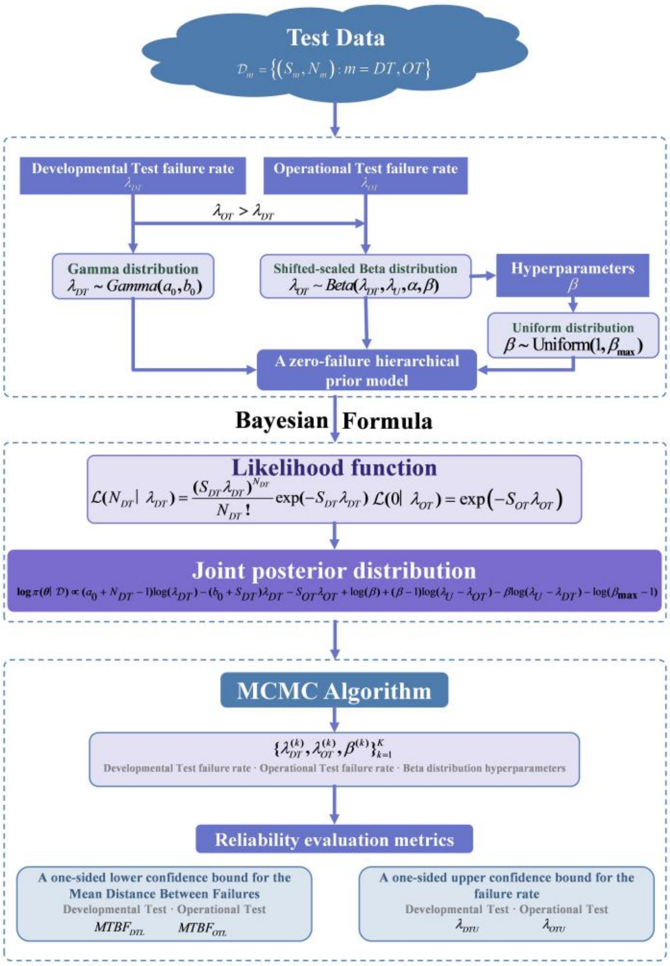


Figure 3. Flowchart of the hierarchical Bayesian reliability assessment method.

3.4.2. Reliability metric calculation

After obtaining the posterior sample set $\{\lambda_{DT}^{(k)}, \lambda_{OT}^{(k)}, \beta^{(k)}\}_{k=1}^{K}$, the vehicle reliability metrics can be computed, yielding the point estimates and interval estimates for each metric.

At a 95% credible level, the posterior samples are sorted in ascending order, and the 95th percentile of λ_{DT} and λ_{OT} is taken, yielding the 95% one-sided upper limits λ_{DTU} and λ_{OTU} for the failure rates in the two test phases.

Under the assumption of an exponential lifetime distribution, the failure rate can be mapped to the Mean miles between failures for the two test phases as follows

$$\begin{cases} MTBF_{DT} = \frac{1}{\lambda_{DT}} \\ MTBF_{OT} = \frac{1}{\lambda_{OT}} \end{cases} \tag{31}$$

Among them, $MTBF_{DT}$ and $MTBF_{OT}$ correspond to the Mean miles between failures for vehicles in the Developmental Test and Operational Test phases, respectively. Then, the 5th percentiles of $MTBF_{DT}$ and $MTBF_{OT}$ are taken as the one-sided lower confidence limits of the mean miles between failures for the two phases, denoted as $MTBF_{DTL}$ and $MTBF_{OTL}$, which serve as the required metric values in this study.

The overall procedure of the proposed method is illustrated in Figure 3.

4. Case study

4.1. Test background and data description

This study uses real test data from a certain type of vehicle for the case study. Reliability tests were conducted for this vehicle in both the developmental test (DT) stage and the operational test (OT) stage. The specific test data are as follows:

Table 1. Reliability test data of a certain type of vehicle.

Test Phase	Total Test Mileage	Number of Test Vehicles	Total Number of Failures
Developmental Test	34837	2	12
Operational Test	815	13	0

Table 1 presents the data differences for a specific vehicle across the DT and OT phases. The DT is conducted in a controlled, standardized environment with manageable and repeatable conditions. Consequently, the test mileage per vehicle is relatively long, allowing substantial accumulation of failure data and reflecting the vehicle's baseline reliability. In contrast, the OT evaluates the vehicle's reliability under near-real combat conditions. It is typically performed alongside combat scenarios, requiring a realistic battlefield environment with high uncertainty and harsh conditions. Due to cost constraints, only limited reliability driving tests are feasible, often resulting in no-failure or minimal-failure data. Under such conditions, reliability conclusions are highly sensitive to test mileage and the statistical methods used, making it difficult to obtain reasonable assessments based solely on limited OT data. However, since the OT follows the DT and both phases involve the same vehicle type, there is continuity in failure mechanisms and technical status. Adequate failure information from the DT can serve as prior knowledge for OT reliability assessment. Incorporating DT failure data into the OT evaluation compensates for limited OT data, enhancing information utilization and improving the validity of reliability conclusions under small-sample no-failure conditions. The reliability test data in Table 1 for the selected vehicle precisely exhibit the characteristics of “limited OT mileage with no failures and sufficient DT mileage and failure samples,” making them suitable for the hierarchical Bayesian reliability assessment method proposed in this study for no-failure OT data.

4.2. Analysis of hierarchical Bayesian reliability evaluation results

In terms of the experimental setup, this study implements the multi-chain Metropolis-within-Gibbs MCMC algorithm using MATLAB R2024b, running four chains in total. Each chain performs 30,000 iterations, with the first 5,000 discarded as burn-in, and a sampling interval set to 5. Based on Table 1 data, the OT failure rate λ_{OT} is assigned an engineering upper bound λ_U of 0.01, corresponding to a minimum vehicle $MTBF_{OT}$ of 100 km under the exponential lifetime assumption. The upper bound excludes failure rate values that clearly conflict with OT reliability and provides a support range for the Shifted-scaled Beta prior.

From Section 3.2, β_{max} represents the maximum strength of information transfer from DT to OT allowed by the hierarchical Bayesian model. If β_{max} is too large, the model may overly assume that λ_{OT} is close to λ_{DT}, leading to excessive reliance on DT information and weakening the influence of OT data. This can result in overly optimistic OT reliability estimates when OT mileage is limited and no failures occur. To reasonably represent the relationship between DT and OT, while facilitating subsequent case validation, this study restricts β_{max} to integer values between 2 and 9. The corresponding reliability estimates for different β_{max} values at a 95% credible level are shown in Table 3.

Table 2. Reliability evaluation results under different values of $\boldsymbol{\beta_{max}}$.

β_{max}	λ_{DTU}	λ_{OTU}	$MTBF_{DTL}$	$MTBF_{OTL}$
2	0.0005126	0.003707	1950.85	269.70
3	0.0005149	0.003434	1942.11	291.21
4	0.0005178	0.003213	1931.10	311.21
5	0.0005137	0.003057	1946.69	327.13
6	0.0005121	0.002911	1952.70	343.54
7	0.0005130	0.002767	1949.50	361.43
8	0.0005149	0.002702	1942.11	370.13
9	0.0005116	0.002584	1954.75	387.07

Table 2 presents vehicle reliability assessment results for different values of β_{max}. It shows that when β_{max} ranges from 2 to 9, the 95% one-sided credible lower limit of OT phase mean miles between failures $MTBF_{OTL}$ increases monotonically from 269.70 km to 387.07 km, while the 95% one-sided credible upper limit of OT phase failure rate λ_{OTU} decreases monotonically from 0.003707 to 0.002584. This is consistent with Section 3.2.4, indicating that as β_{max} increases, the model allows stronger DT information to migrate to OT, causing the posterior distribution of λ_{OT} to contract toward λ_{DT} and compress the upper tail of the failure rate, resulting in a higher $MTBF_{OTL}$ and lower λ_{OTU}. Meanwhile, for different values of β_{max}, the 95% one-sided credible lower limit of DT phase mean miles between failures $MTBF_{DTL}$ remains stable between 1931.10 km and 1954.75 km, and the 95% one-sided credible upper limit of DT phase failure rate λ_{DTU} remains stable between 0.0005116 and 0.0005178. This indicates that DT phase failure rates are primarily determined by sufficient DT mileage and observed failures, and are not affected by the OT prior structure or β_{max}.

This trend indicates that the hierarchical Bayesian model can control the strength of information borrowed from DT to OT through β_{max}. A smaller β_{max} suggests the model perceives large differences between OT and DT in task profiles, road conditions, and operational environments, allowing only weak information transfer. A larger β_{max} implies the model considers the vehicle's technical state and usage conditions consistent across the two test phases, so more DT information can be borrowed. An increase in β_{max} does not indicate pursuit of a better reliability estimate, but reflects the OT reliability assessment under different information borrowing assumptions. In practice, DT data cannot be completely ignored or simply equated to OT data; the appropriate β_{max} should be selected based on the differences between the two test phases.

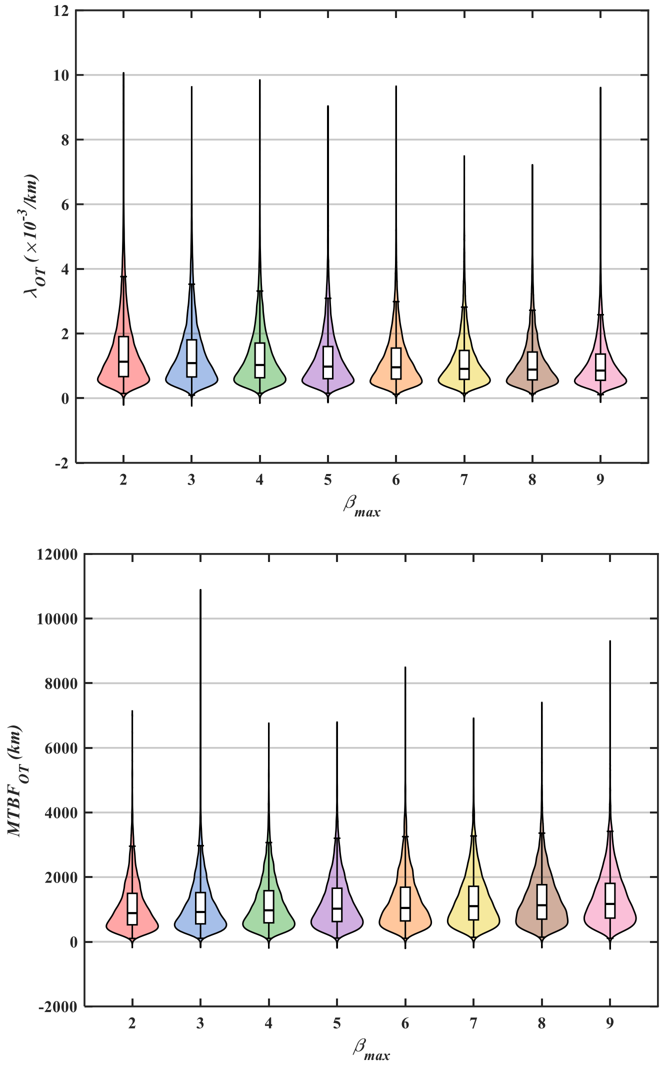


Figure 4. Posterior distributions of the reliability metrics λ_{OT} and $MTBF_{OT}$ for different β_{max} values.

Figure 4 presents the posterior distributions of λ_{OT} and $MTBF_{OT}$ under different values of β_{max} using violin plots. The left subplot shows the posterior distribution of the OT failure rate λ_{OT}. The right subplot shows the posterior distribution of the mean miles between failures in the OT stage $MTBF_{OT}$. Both subplots are visualized using violin plots. The width of each violin represents the magnitude of the posterior probability density. The embedded boxplot indicates the median and the interquartile range of the reliability parameters. The box corresponds to the interquartile range (IQR), the central line represents the median, and the upper and lower whiskers correspond to $Q_1 - 1.5IQR$ and $Q_3 + 1.5IQR$, respectively. The extended ends of the violin reflect the tail range of the posterior distribution.

Figure 4 shows that as β_{max} increases, the transfer intensity of DT information to OT strengthens. The posterior distribution of λ_{OT} gradually contracts toward the low-failure-rate region, while the posterior distribution of $MTBF_{OT}$ shifts toward higher values. This reflects the influence of DT information borrowing strength on the posterior inference in OT reliability evaluation. At different β_{max} values, the posterior distribution of λ_{OT} exhibits typical right-skewed characteristics, with most probability concentrated in the low-failure-rate region and a long right tail. The narrow left tails in the violin plots occur because small-probability higher-failure-rate events are still allowed under no-fault OT conditions. The extended right tails indicate that, in limited OT mileage without failures, the possibility of extremely low failure rates cannot be excluded, so the minimum λ_{OT} maps to a very high $MTBF_{OT}$. This pattern aligns with the features of no-fault OT data: on one hand, OT experienced no failures, giving low failure rates high posterior probability; on the other hand, the OT mileage of 815 km is short, so small-probability higher failure rates cannot be fully ruled out, maintaining some right-tail uncertainty. The long-tail structure in the violin plots demonstrates that, due to weak constraints from no-fault data, OT reliability evaluation should focus on risk indicators such as the upper quantile of λ_{OT} and the lower quantile of MTBF. This study adopts the 95% one-sided upper limits of λ_{OT} and $MTBF_{OT}$ as reliability evaluation metrics, providing a direct reflection of reliability risk boundaries of concern in equipment testing and validation.

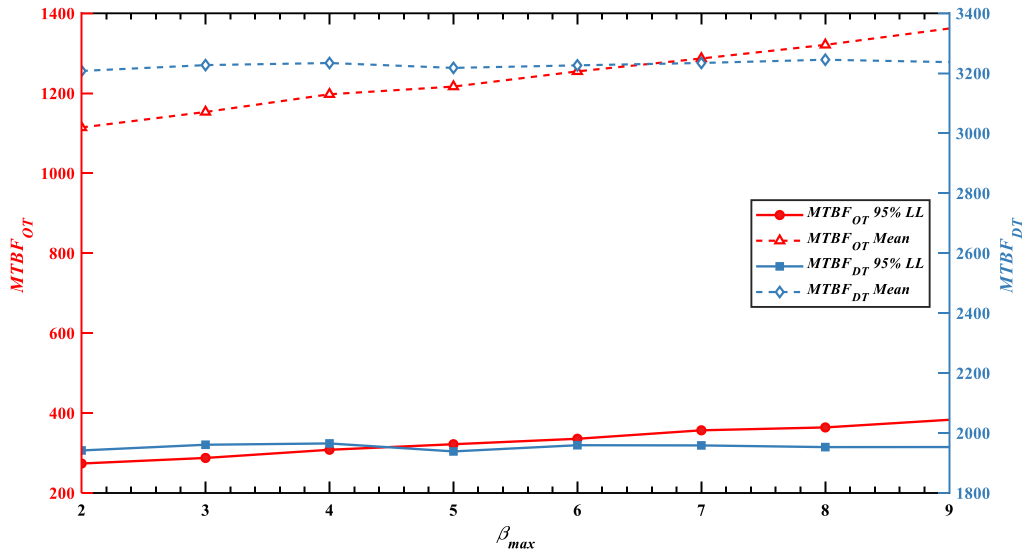


Figure 5. Posterior statistics of reliability indicators under different values of β_{max}.

Figure 5 shows the trends of posterior statistics of MTBF for the DT and OT phases under different β_{max} values. The horizontal axis represents different β_{max} values, the left vertical axis represents $MTBF_{OT}$, and the right vertical axis represents $MTBF_{DT}$. The solid lines indicate the 95% one-sided credible lower limits of MTBF for both phases, while the dashed lines represent the posterior means of MTBF for both phases.

From Figure 5, as β_{max} increases from 2 to 9, the mean of $MTBF_{OT}$ and its 95% one-sided credible lower limit both show a monotonic increasing trend. This aligns with the results in Table 2, indicating that β_{max} significantly affects reliability estimates in the OT phase. The reason is that β_{max} controls the maximum strength of DT information borrowed into the OT phase. When β_{max} increases, the hierarchical Bayesian model allows stronger information borrowing, causing the posterior distribution of λ_{OT} to shrink toward λ_{DT}, resulting in a monotonic increase of $MTBF_{OT}$ mean and credible lower limit $MTBF_{OTL}$. In contrast, for the DT phase, as β_{max} increases, the mean and credible lower limit of $MTBF_{DT}$ remain largely stable, with the mean around 3200 km and the credible lower limit around 1950 km. This indicates that the posterior inference of DT failure rate is primarily determined by its own sufficient test mileage and failure counts, and is almost unaffected by β_{max}.

The combined results from Figures 4 and 5 show that changes in β_{max} mainly affect the OT reliability assessment, while the DT results remain essentially stable. This indicates that the hierarchical Bayesian model developed in this study can supplement inferences for OT no-failure data without altering the statistical conclusions from DT data itself, through an interpretable information-borrowing mechanism. The sensitivity of OT reliability results does not imply model instability; rather, it reflects a reasonable response to the extent DT information can be transferred under limited OT mileage and no failures. In practical equipment testing, it provides quantitative support for reliability judgments under different risk preferences. Smaller β_{max} values emphasize cautious assessment of differences between OT and DT environments, whereas larger β_{max} values indicate a certain similarity between the two test phases.

4.3. α Sensitivity analysis

To further examine the effect of the Beta distribution hyperparameter α on the reliability assessment results and to validate the appropriateness of fixing $\alpha = 1$, a sensitivity analysis was conducted for α in this study.

In the sensitivity analysis, α was set to $\alpha = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$, while β_{max} was varied from 2 to 9, forming 80 hyperparameter combinations. For each pair (α, β_{max}), the hierarchical Bayesian model was independently run, and the reliability metrics $MTBF_{OTL}$, $MTBF_{DTL}$, λ_{OTU}, and λ_{DTU} were computed for the two test phases. The sensitivity analysis results are shown in Figure 6.

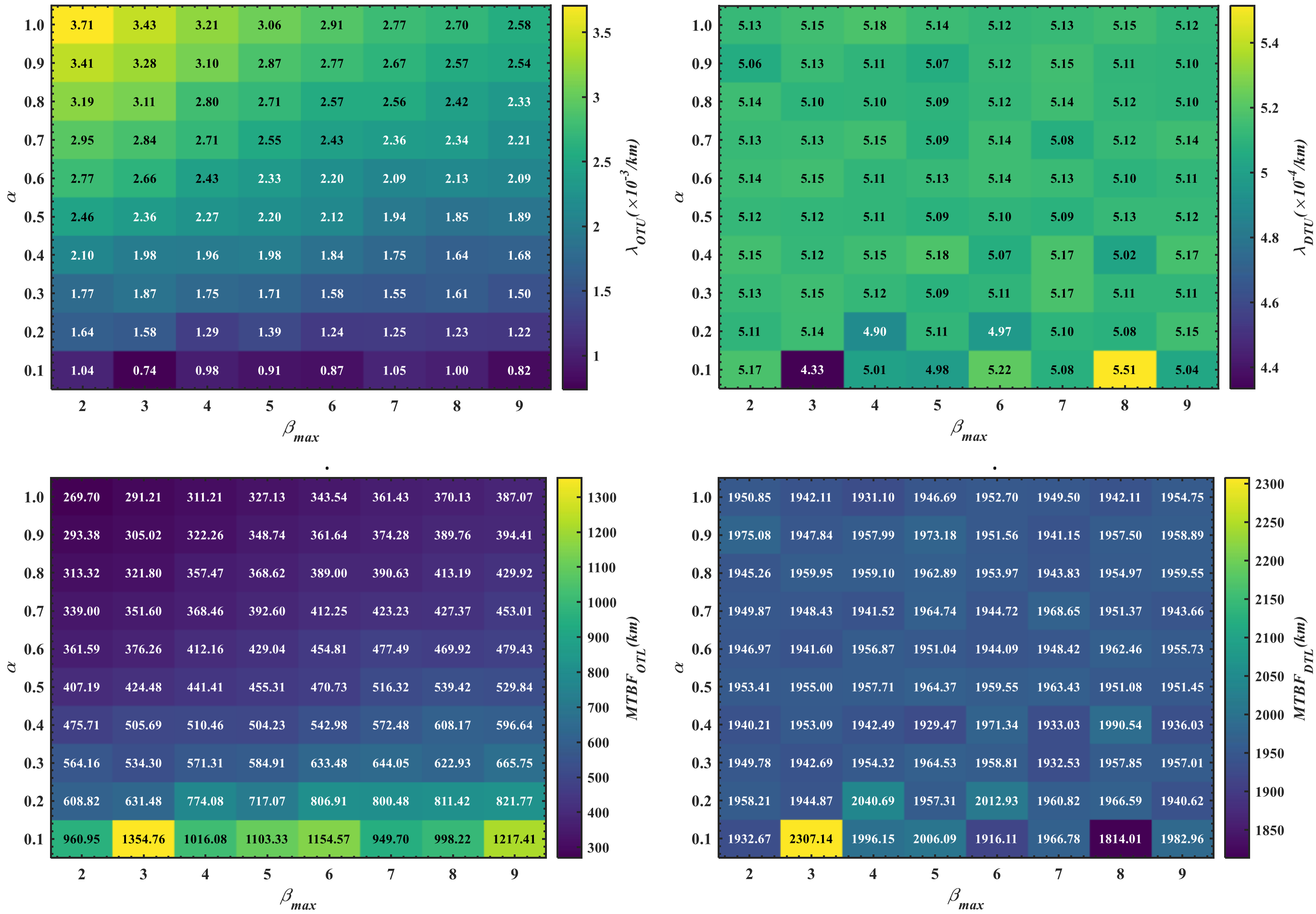


Figure 6. α sensitivity heatmap.

Figure 6 presents heatmaps showing the effect of different α values on vehicle reliability assessment results under fixed β_{max} settings. The horizontal axis represents β_{max}, and the vertical axis represents α. The two subplots on the left correspond to the OT reliability metrics λ_{OTU} and $MTBF_{OTL}$, while the two subplots on the right correspond to the DT reliability metrics λ_{DTU} and $MTBF_{DTL}$.

The two left subplots show that the OT reliability metrics are highly sensitive to the value of α. With β_{max} fixed, λ_{OTU} increases as α increases, whereas $MTBF_{OTL}$ decreases. For example, when $\beta_{max} = 5$, as α gradually increases, λ_{OTU} rises from 0.00091 to 0.00306, and the corresponding $MTBF_{OTL}$ decreases from 1103.33 to 327.13. Combined with the Beta probability density curves, this result indicates that α has an important effect on the upper tail of the posterior distribution of the OT failure rate. In contrast, the two right subplots show that as α varies, λ_{DTU} and $MTBF_{DTL}$ remain essentially stable. This indicates that the DT failure rate is mainly determined by its own sufficient test data, and changes in the hyperparameters of the hierarchical Bayesian model do not significantly alter the DT reliability conclusions.

In summary, the choice of α has an important impact on OT reliability assessment results. Combined with the Beta distribution shape analysis, when $\alpha < 1$, its probability density tends to infinity near 0, strongly pulling λ_{OT} toward λ_{DT}. This causes excessive borrowing of DT information, leading to unstable posterior estimates and overly optimistic OT reliability conclusions. The slight fluctuations in some results in the sensitivity heatmap also reflect posterior instability caused by $\alpha < 1$. Therefore, this study sets $\alpha = 1$ and uses different β_{max} values to flexibly control the borrowing strength of DT information. This avoids excessive prior concentration of the OT failure rate in the low-failure-rate region and provides more robust and reasonable reliability estimates under limited-mileage and no-failure OT conditions.

4.4. MCMC sampling performance analysis

The convergence and efficiency of MCMC sampling are essential for ensuring the reliability of inference results. To verify that the hierarchical Bayesian model proposed in this study produces stable results under different values of β_{max}, the sampling performance of the MCMC algorithm is comprehensively evaluated from three aspects: multi-chain convergence diagnostics, within-chain autocorrelation, and effective sample size.

Table 3. R-hat statistics of the three parameters for different β_{max} values.

β_{max}	2	3	4	5	6	7	8	9
λ_{DT}	1.00005	1.00023	1.00010	1.00015	1.00012	1.00011	1.00014	1.00014
λ_{OT}	1.00020	1.00023	1.00003	1.00031	1.00012	1.00000	1.00023	1.00015
β	1.00002	1.00001	1.00042	0.99999	1.00005	1.00005	1.00007	1.00048

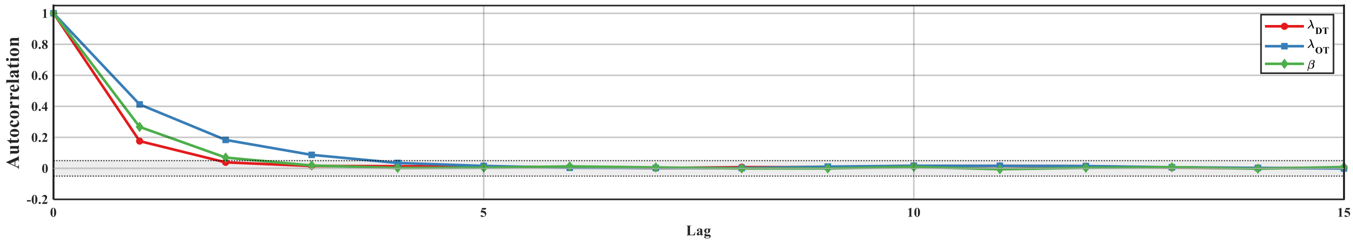
Table 3 gives the Gelman–Rubin statistics of λ_{DT}, λ_{OT}, and β under different β_{max} values. The R-hat statistic is used to determine whether multiple MCMC chains have converged to the same stationary distribution. Convergence is considered achieved when R-hat is less than 1.01 [27]. Table 3 shows that, for all β_{max} values, the R-hat statistics of the three estimated parameters are very close to 1 and far below 1.01. This indicates that the four MCMC chains used in this study are well mixed and have converged to the same posterior distribution. Therefore, the R-hat diagnostic results confirm that the proposed multi-chain Metropolis-within-Gibbs MCMC algorithm has good convergence within the specified β_{max} range. The resulting multi-chain posterior samples can be used to calculate vehicle reliability metrics.

To measure within-chain sample dependence, this study further calculated the autocorrelation function (ACF) of the posterior samples of the three parameters under different β_{max} values. The ACF measures the correlation of Markov chain samples at different lag orders. A faster ACF decay indicates weaker within-chain dependence, more effective independent samples, and higher mixing efficiency. The ACF results of the three parameters under different β_{max} values are shown in Figure 7.

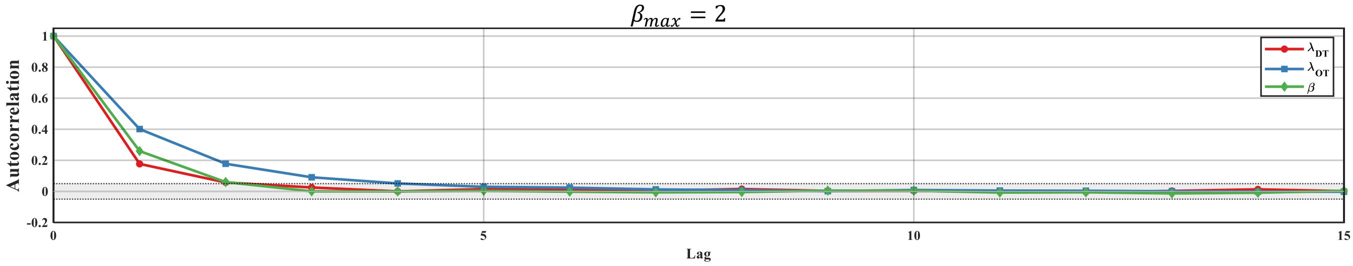
Figure 7 compares the ACF results of the three parameters in the hierarchical Bayesian model under different β_{max} values. The red solid line represents λ_{DT}, the blue solid line represents λ_{OT}, the green solid line represents β, and the gray shaded region indicates $|ACF| < 0.05$.

Figure 7 shows that, under different β_{max} values, the ACF decay rates of the parameters differ only slightly. The ACFs of all three parameters decrease monotonically from 1 and decay to near 0 after lag order 5. This indicates strong independence between adjacent samples, good chain mixing, and a sufficient effective sample size. However, slight differences exist among the three parameters. λ_{OT} decays the slowest, indicating stronger sample correlation for λ_{OT}. This is because the OT phase has no failures and limited test mileage, making posterior inference more dependent on hierarchical prior constraints and more prone to slightly stronger correlation during sampling. Overall, the slowest decay of λ_{OT} is not due to sampling failure, but is a natural result of information scarcity under no-failure conditions. This is consistent with the statistical characteristics of OT reliability assessment.

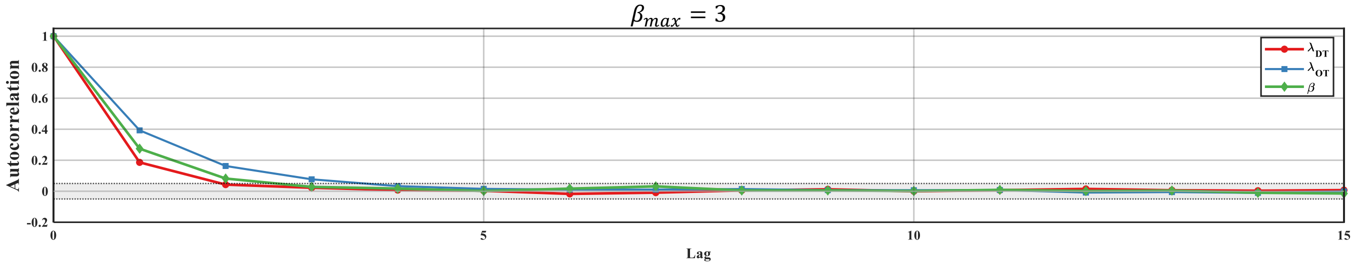
Furthermore, the research team evaluated the MCMC sampling quality from the perspective of effective sample size (ESS), with the results shown in Table 4.



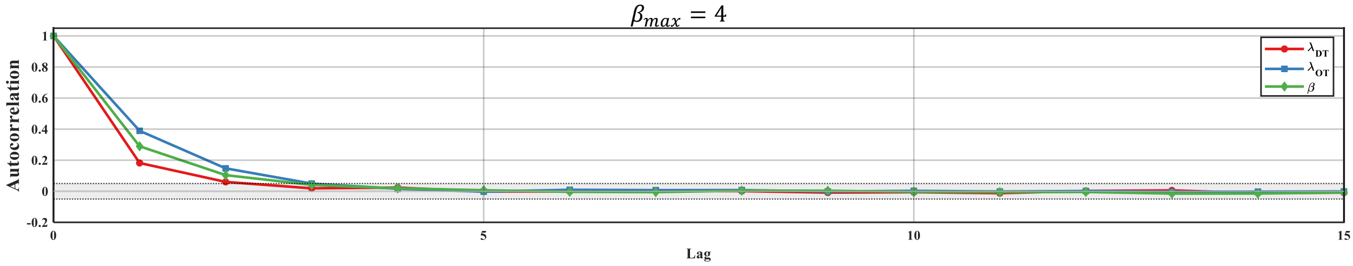
$\beta_{max} = 2$



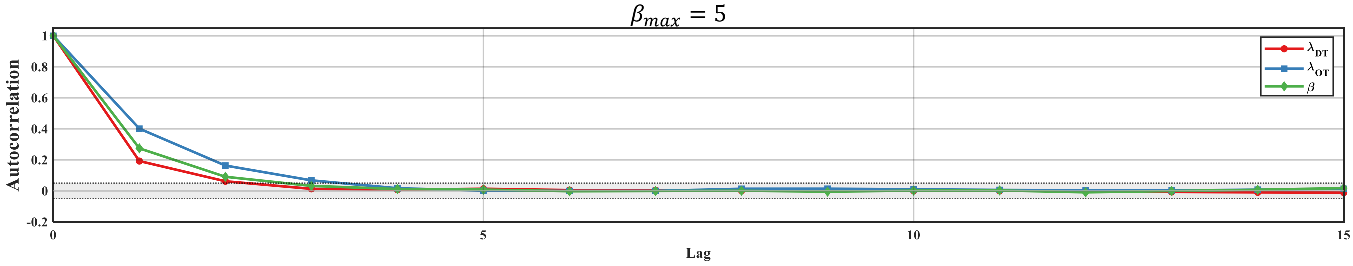
$\beta_{max} = 3$



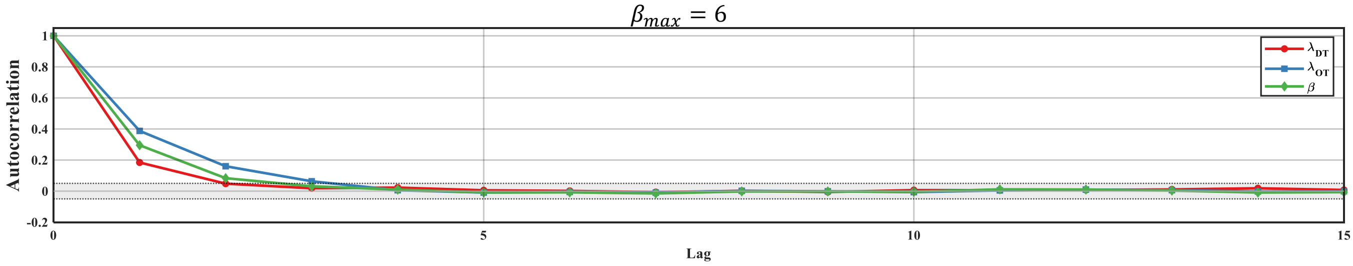
$\beta_{max} = 4$



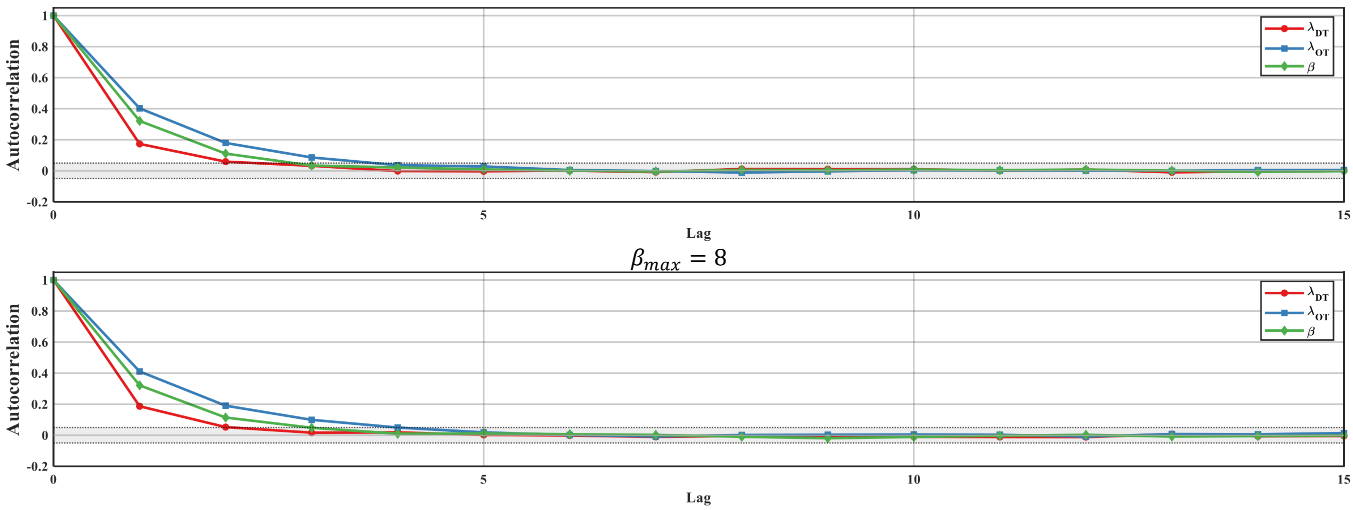
$\beta_{max} = 5$



$\beta_{max} = 6$



$\beta_{max} = 7$



$\beta_{max} = 8$

$\beta_{max} = 9$

Figure 7. ACF plots of the three parameters under different $\boldsymbol{\beta_{max}}$ values.

Table 4. ESS of the three parameters under different $\boldsymbol{\beta_{max}}$ values.

β_{max}	2	3	4	5	6	7	8	9
λ_{DT}ESS	13997	13165	13719	13134	13042	13654	13076	13231
λ_{OT}ESS	8212	7997	8581	9205	8706	8949	8303	7997
βESS	11688	12198	11307	10698	11152	10990	10351	10165

Table 4 shows the ESS values of the three parameters under different β_{max} values. As shown in Table 4, even in the most unfavorable case, the minimum ESS of the three parameters is still as high as 7997, far above the ESS threshold required for general posterior inference accuracy [27]. This means that, even under OT conditions with no failures and very limited mileage, the sampling settings used in this study can still provide many effective independent samples. Thus, the estimates of posterior means, quantiles, and credible intervals for the related parameters have high numerical stability. Among them, λ_{DT} has the highest overall ESS, indicating the best posterior sampling efficiency for the DT failure rate. This is because DT provides sufficient information, and its posterior inference is mainly determined by test data, leading to a more stable sampling process. In contrast, λ_{OT} has the lowest overall ESS. This is due to limited OT mileage and no failures, which make posterior inference rely more on the hierarchical prior and engineering constraints, resulting in relatively stronger correlation among Markov chain samples.

In summary, Table 3, Figure 7, and Table 4 jointly show that the proposed hierarchical Bayesian model has good multi-chain convergence, low within-chain correlation, and high ESS under different β_{max} settings. The proposed multi-chain Metropolis-within-Gibbs sampler achieves good convergence, high mixing efficiency, and many independent samples. The posterior inference results obtained from this MCMC sampling therefore have a sound and reliable numerical basis, providing strong numerical support for subsequent vehicle reliability metric evaluation.

4.5. Comparison of evaluation methods

To further verify the effectiveness of the proposed method, this section compares it with three representative reliability assessment methods. The first is the classical MTBF one-sided confidence lower limit method commonly used in current national and military standards. The second is the classical Bayesian one-sided credible lower limit method commonly used under no-failure conditions without incorporating DT information. The third is a two-layer Bayesian method that incorporates DT information and engineering constraints but uses a uniform prior. All four methods are used to calculate the one-sided lower limit of $MTBF_{OT}$ at the 95% confidence/credible level.

4.5.1. Classical MTBF one-sided confidence lower limit method in the national standard

First, the classical MTBF one-sided confidence lower limit method used in current national standards is considered [13,14]. The calculation formula is as follows:

$$MTBF_L = \frac{2S}{\chi^2[2(r+1),\gamma]} \quad (32)$$

Here, S denotes the total vehicle mileage, $\chi^2[2(r+1),\gamma]$ denotes the chi-square distribution value with $2(r+1)$ degrees of freedom at confidence level $1-\gamma$, and r denotes the total number of failures.

Substituting the OT no-failure data into Equation (32) gives $MTBF_{OTL} = 272.05$ km using the classical national-standard method. This method is simple to calculate and highly standardized, making it suitable for rapid preliminary reliability judgments in equipment test evaluation. However, it evaluates OT reliability using only limited OT mileage and zero-failure data. It ignores the relationship between DT and OT and does not use the sufficient DT information. As a result, it wastes useful information and cannot reliably reflect the true OT reliability level of the vehicle based on limited test samples.

4.5.2. Classical Bayesian one-sided credible lower limit method

Next, the classical Bayesian one-sided credible lower limit method is further discussed [28]. This method only uses OT data and does not incorporate DT failure information. It assigns a uniform prior to the OT failure rate within a specified range. Without the constraint from DT information, λ_{OT} is assumed to follow a uniform distribution over $(0,\lambda_U)$. The value of λ_U is kept consistent with that in the proposed method, yielding:

$$\lambda_{OT} \sim U(0,\lambda_U), \ \lambda_U = 0.01 \quad (33)$$

The posterior distribution of λ_{OT} is then derived according to Bayesian Formula.

$$\pi(\lambda_{OT} \mid \mathcal{D}) = \frac{S_{OT}exp(-S_{OT}\lambda_{OT})}{1-exp(-S_{OT}\lambda_U)}, \quad 0 < \lambda_{OT} < \lambda_U \quad (34)$$

Given a 95% credible level, the one-sided credible upper limit λ_{OTU} of λ_{OT} satisfies:

$$\int_0^{\lambda_{OTU}} \pi(\lambda_{OT} \mid data) d\lambda_{OT} = 0.95 \quad (35)$$

According to Equation (35), λ_{OTU} can be obtained. Then, using the monotonic mapping $MTBF = 1/\lambda$ under the exponential lifetime assumption, the one-sided credible upper limit of the OT failure rate can be transformed into the one-sided credible lower limit of the vehicle MTBF in OT:

$$MTBF_{OTL} = \frac{1}{\lambda_{OTU}} \quad (36)$$

Substituting the OT no-failure data into this calculation gives $MTBF_{OTL} = 272.55$ km using the classical Bayesian one-sided credible lower limit method. This result is very close to that obtained by the national-standard classical MTBF one-sided confidence lower limit method. This indicates that, without incorporating DT information, even a Bayesian method is still mainly driven by OT mileage and no-failure data. The advantage of this method is its clear calculation process and its ability to provide the full posterior distribution of λ_{OT}. However, it still ignores DT information and cannot express the engineering knowledge that the OT failure rate is higher than the DT failure rate. Therefore, a reliability assessment method that can incorporate DT information and express the constraint $\lambda_{OT} > \lambda_{DT}$ is needed.

4.5.3. Two-layer Bayesian method with uniform prior

To further analyze the role of the Shifted-scaled Beta prior structure in the proposed method, a two-layer Bayesian method with a uniform prior is introduced as a comparative method. This method accounts for the engineering fact that the OT failure rate is higher and sets λ_{OT} to follow a uniform distribution over (λ_{DT},λ_U), yielding:

$$\lambda_{OT} \sim U(\lambda_{DT},\lambda_U), \ \lambda_U = 0.01 \quad (37)$$

The related settings for λ_{DT} are kept consistent with the proposed method, and $MTBF_{OTL}$ is calculated as 246.48 km using the MCMC algorithm. The $MTBF_{OTL}$ obtained by this method is slightly lower than those obtained by the national-standard method and the classical Bayesian one-sided credible lower limit method. This is because the method considers the engineering constraint $\lambda_{OT} > \lambda_{DT}$ and excludes the possibility that λ_{OT} lies between 0 and λ_{DT} in the prior, leading to a more conservative result. However, it assumes that λ_{OT} is uniformly distributed over (λ_{DT},λ_U). At the prior level, this means that all values in the feasible interval are equally likely, so λ_{OT} still has a relatively high probability in the high-failure-rate region. This approach only uses DT information as a lower-bound constraint and does not fully capture the reliability correlation between the two test phases. The assumption that high and low failure rates are equally likely also fails to reflect the support of OT zero-failure data for the low-failure-rate region. Under limited OT mileage, this may lead to overly conservative OT reliability estimates, which is unfavorable for equipment finalization decisions. In addition, this method lacks a mechanism for adjusting the strength of information borrowing, making it difficult to flexibly adjust the borrowing level according to test-phase similarity, mission profile differences, and evaluation risk preference. Therefore, the two-layer uniform Bayesian method can only serve as a comparison method for the proposed method. It shows that only introducing DT information and the engineering constraint $\lambda_{OT} > \lambda_{DT}$ is not sufficient to obtain reasonable reliability estimates. A hierarchical prior model that reflects zero-failure characteristics and information-borrowing strength is further needed.

4.5.4. Comprehensive comparative analysis of assessment methods

The proposed method further improves the prior construction of the OT failure rate based on the two-layer uniform-prior Bayesian method. It no longer assumes that λ_{OT} is uniformly distributed over $(\lambda_{DT}, \lambda_U)$. Instead, it fully accounts for the characteristics of no-failure data and constructs the prior of λ_{OT} as a Shifted-scaled Beta distribution over $(\lambda_{DT}, \lambda_U)$. This assigns higher prior probability to smaller λ_{OT} values while incorporating the engineering constraint $\lambda_{OT} > \lambda_{DT}$. This setting reflects both the engineering knowledge that lower failure rates are more likely under no-failure data and the objective fact that the OT failure rate is higher. The borrowing strength of DT information in OT can also be adjusted by reasonably selecting different β_{max} values, which makes full use of DT information and avoids information waste. As shown in Table 2, when $\beta_{max} = 2$, the proposed method gives $MTBF_{OTL} = 269.70$km, which is very close to the results of the national-standard method and the classical Bayesian one-sided credible lower limit method. This reflects a conservative estimate under weak information borrowing. In this case, the hierarchical Bayesian model allows only limited transfer of DT information to OT, making it suitable for scenarios where the two test phases differ greatly and the evaluation risk preference is conservative. When the test profiles are more similar, the environmental stress difference is smaller, and the technical state of the prototype is more consistent across the two phases, a larger β_{max} can be selected. In this case, the resulting $MTBF_{OTL}$ is higher than those from the national-standard method and the classical Bayesian one-sided credible lower limit method, indicating that the proposed method can produce a more reasonable assessment of the true OT reliability level by appropriately borrowing DT information when the two test phases have sufficient similarity.

β_{max} is a key factor in the hierarchical Bayesian model developed in this study. It represents the maximum strength allowed by the model for transferring DT information to OT. Vehicle reliability is affected by many factors, including road conditions, load state, mission profile, driving level, and environmental stress. Therefore, the information transferability between the two test phases should not be fixed for different vehicle types, but should be determined by comprehensively considering multiple factors.

Table 5. Selection Criteria for β_{max}.

Influencing Factor	Basis for Judgment
Mission Profile	Operating speed range, load condition, continuous operating time, vehicle usage mode
Vehicle Technical State	Major design configuration changes, Critical assembly replacement, Test vehicle batch consistency, Maintenance support strategy consistency, Failure mechanism consistency
Environmental Stress	Road conditions, climatic environment, vibration and shock, electromagnetic environment
Assessment Risk Preference	High safety-risk equipment, Mission-critical equipment, Difficulty of follow-on maintenance support, Severity of operational failure consequences

Table 5 provides the engineering basis for selecting β_{max}. It should be noted that the range of 2 to 9 discussed in this study is not the final allowable range of β_{max}. This setting is used only for case validation. In actual vehicle reliability assessment, the equipment contractor and user organization should jointly determine an appropriate β_{max} value by considering the factors in Table 5, expert engineering experience, and experience with similar equipment.

In summary, the proposed method does not simply pursue more optimistic or conservative reliability estimates. Instead, it provides a more reasonable assessment of vehicle reliability in OT by considering multi-source information, no-failure data characteristics, and differences between test phases. It avoids the information shortage caused by relying only on short-mileage no-failure OT data. It also avoids directly combining DT and OT data while ignoring phase differences. The method reflects the engineering knowledge that the OT failure rate is higher, while reasonably using the support of no-failure data for the low-failure-rate region. It can also adjust the degree of information borrowing according to the mission profile, vehicle technical state, environmental stress, and assessment risk preference. Therefore, it provides a more reasonable and interpretable quantitative basis for reliability assessment under no-failure OT data.

4.6. Discussion and limitations analysis

Based on the above case results, the proposed improved hierarchical Bayesian reliability assessment method incorporating DT information shows clear advantages for vehicle OT with no-failure data. Compared with the classical national-standard method and the classical Bayesian one-sided credible lower-limit method, this study no longer relies excessively on OT mileage as a single information source. Instead, DT information is introduced into OT reliability assessment through a hierarchical prior structure, and the constraint $\lambda_{OT} > \lambda_{DT}$ reflects the engineering knowledge that the OT environment is harsher. This improves the use of multi-stage test information. Compared with the two-level uniform-prior Bayesian method, the proposed method uses the Shifted-scaled Beta prior to reflect the higher likelihood of low failure rates under no-failure conditions. It avoids assigning excessive probability to high-failure-rate regions and thus provides posterior inference results that better match the characteristics of no-failure data.

The advantage of this study is not to obtain fixed higher or lower vehicle OT reliability estimates. Instead, it provides a multi-stage test information fusion framework that can be adjusted according to test-stage similarity and assessment risk preference. A smaller β_{max} is suitable for conservative assessment scenarios with large mission-profile differences. A larger β_{max} is suitable for scenarios where DT and OT have higher similarity. By reasonably determining β_{max} based on the vehicle mission profile, technical state, environmental stress, and assessment risk preference, more valuable vehicle OT reliability assessment results can be obtained.

Meanwhile, the MCMC diagnostics and sensitivity analysis show that, under the β_{max} settings used in this study, the multi-chain Metropolis-within-Gibbs algorithm provides stable numerical support for posterior inference. Fixing $\alpha = 1$ avoids unstable estimation caused by excessive concentration of the OT failure-rate prior near λ_{DT}. The information-borrowing strength can still be adjusted by controlling different β_{max} values. This demonstrates that the proposed method has good robustness under OT conditions with no failures and limited mileage.

However, several limitations of the proposed method should be noted. First, it is based on the constant failure-rate assumption of the exponential distribution. It is more suitable for OT scenarios where random failures dominate, test mileage is short, and no clear wear-out failure features are observed. Future work should consider wear-out effects in vehicle components and extend the failure-rate model to the Weibull distribution. Second, the method incorporates DT information into the posterior inference of the OT failure rate through engineering constraints and information-borrowing strength. This mechanism depends strongly on the accuracy of DT information. If DT data contain errors caused by random factors, the OT assessment results may be affected. Future work may develop a more reasonable Bayesian uncertainty propagation mechanism to quantify the differences between the two test phases and obtain more scientific reliability estimates under OT conditions. Third, this study provides selection principles for β_{max} based on mission-profile similarity, vehicle technical-state consistency, and other factors, and analyzes the assessment results under different β_{max} values. However, in actual vehicle OT reliability assessment, β_{max} is still mainly selected based on expert judgment rather than a fully data-driven method. Future work may develop a quantitative β_{max} selection method based on indicators such as mission-profile similarity and environmental stress ratio, thereby reducing subjectivity and improving the objectivity of vehicle reliability assessment under limited-mileage and no-failure OT conditions.

5. Conclusion and future work

To address the common no-failure problem in vehicle OT reliability assessment, this study proposes an improved hierarchical Bayesian reliability assessment method that incorporates DT information. Hierarchical priors are constructed separately for λ_{OT}, λ_{DT}, and hyperparameter β, with DT information introduced and a more reasonable Shifted-scaled Beta prior established for λ_{OT}. A multi-chain Metropolis-within-Gibbs MCMC algorithm is then designed to infer the joint posterior of model parameters and calculate the one-sided credible lower bound of MTBF for the OT phase. Results show that the proposed method effectively integrates DT information and OT no-failure data within a unified probabilistic framework, providing a reasonable assessment of vehicle OT reliability while avoiding information waste. Convergence diagnostics and sensitivity analysis indicate that the method offers robust posterior inference and demonstrates good engineering applicability under reasonable hyperparameter settings. Comparative evaluation shows that the method avoids the insufficient use of OT data alone while accounting for no-failure characteristics, and prevents environmental discrepancies from directly merging DT and OT data, thus providing a more reasonable, robust, and interpretable quantitative basis for reliability assessment under no-failure OT data. Future research may explore broader applicability using Weibull lifetime models and cross-phase failure information transfer mechanisms, and develop fully data-driven hierarchical Bayesian models to further enhance the scientific rigor and applicability of vehicle reliability assessment in complex OT environments.

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