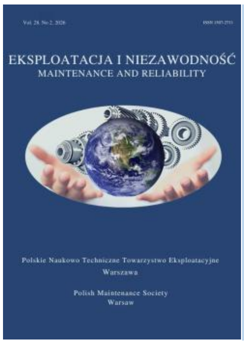

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Analysis of failure data for estimating availability and reliability indicators using statistical and stochastic methods

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Highlights

- A novel 7-state semi-Markov model of vehicle operation was developed.
- Reliability models of technical objects were developed based of failure data.
- Reliability and availability indicators were computed and validated.
- Statistical and stochastic methods were integrated and compared.

Abstract

This paper presents a novel implementation of statistical and stochastic methods for estimating and evaluating reliability and availability indicators in technical systems. Using empirical failure data from a real-world military transport system, we introduce an innovative 7-state model that provides a detailed representation of operational phase of the systems. The research integrates Markov and semi-Markov processes to accurately model state transitions, particularly addressing scenarios where traditional Markov models are insufficient due to non-exponential state distributions. Our findings demonstrate that both statistical and stochastic methods yield closely aligned reliability and availability indicators, validating the robustness of the proposed methodologies. This research not only advances the accuracy of reliability assessments but also identifies actionable improvements to enhance operational readiness. They provide a comprehensive framework for analyzing and improving the operational efficiency of technical systems, with broader applications across various engineering fields.

Keywords

semi-Markov process; reliability modeling; maintenance; availability indicators; vehicle

1. Introduction

Ever-growing requirements for the reliability and safety of technical objects and systems generate the need to continuously improve theoretical models to describe them. In addition to the accuracy of the theoretical model's representation of reality, the economic aspect related to the high research costs in operational conditions should also be considered. Due to the above, modeling reliability is an extremely difficult issue, but due to its importance, it constitutes an area of research in modern scientific publications [1, 22].

Description of the functioning of objects using deterministic models may entail the possibility of omitting the impact of random factors affecting their operation. Probabilistic models are free from this drawback and consider all phenomena affecting objects' functioning. Probability distributions constitute a basis for building probabilistic models. Knowing the form of the distribution describing the failure rate of a technical object, it is possible to predict its operating time with a certain probability. Knowing the course of the probability of

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a failure over time, one can take preventive actions before the risk of failure reaches its limit value, thus avoiding financial losses and ensuring an appropriate level of safety [47].

The basis for proper and effective management in technical object operation systems is the correct analysis and assessment of the processes taking place there. Statistical methods are among the basic methods for examining the reliability and availability of objects and systems. They enable the characterization of the operation process without the use of complicated mathematical methods and expensive computer programs.

Using stochastic processes, the exploitation process model can be presented by describing subsequent changes in the states of objects over a period of time. After identifying all possible states of objects (systems), they are aggregated using the method of successive approximations in accordance with the adopted modeling objective [26]. In this paper, as a result of the identification of the actual transport system and the multi-state facility operation process implemented in it, significant operational states were determined. Then, possible transitions between the distinguished operational states were determined, creating a graph of interstate transitions.

The reliability and technical availability of vehicles are one of the main determinants of the effectiveness of modern, advanced transport systems. The specificity of vehicles operating in military transport systems implies the need to restore technical suitability after random failures. The occurrence of failure to the means of transport during transport processes translates into disruptions to the functioning of the entire system [10, 32]. In many technical systems, due to the lack of quick and elastic supplies of necessary materials and components, the time of unsuitability of means of transport that suffered a mechanical failure is a significant factor that reduces the availability indicator values [39]. Reliability models are useful for forecasting and evaluating the maintenance and repair needs of vehicles used by, among others, army [49], police [7, 29], health service [24], transport companies [5, 44] or service patrols [15]. Therefore, availability and reliability indicators must be carefully analyzed and monitored, especially during crises and wars.

This study aims to implement and compare the suitability of two methods for assessing the availability and reliability of technical objects. The proposed approach is an extension and complements existing statistical methods and stochastic modeling in terms of their applications to the field of engineering and technical sciences [4, 7, 16]. An original algorithm for analyzing the reliability of technical objects was constructed. The model includes a reliability testing methodology based on failure analysis, which reduces the costs related to conducting expensive experimental tests or time-consuming simulations. The results of the analyses allow for the assessment of the reliability and availability of vehicles, which consuming simulations. The results of the analyses allow for the assessment of the reliability and availability of vehicles, which reflects the technical aspect of the effectiveness of the means of transport. The proposed method makes it possible to identify possible components that can improve the availability indicators. The research was carried out using MATLAB and STATISTICA software.

The main contributions of the paper are as follows:

- To create a novel algorithm for applying statistical and stochastic methods;
- Developing of a new 7-state model will capture the complex operational phases of military transport systems, enhancing the accuracy of reliability and availability indicators.
- Identification of critical factors that influence reliability and availability measures. Proposing actionable improvements to it.
- Establishment of a comprehensive methodology for assessing and improving the operational efficiency of technical systems.
- Applying the methodology to real-world data from a military transport system demonstrates its utility and robustness.
- The presented methodology can be extended to other technical systems in various engineering fields for reliability assessment and process optimization.

The paper has been divided into five sections. Section 2 presents the current state of knowledge regarding the use of statistical and stochastic methods in reliability and availability modeling. Then, based on the adopted assumptions, a description of statistical and stochastic methods was made along with the methodology of their application (Section 3).

Based on operational data empirically obtained from a real military transport system, Section 4 presents the practical application of the proposed approach to analyzing the operational process and reliability of trucks. Based on empirical data and statistical methods, the values of reliability indicators and availability measures of the transport system were calculated. Next, the possibility of using Markov and semi-Markov processes for a given transport system was investigated. A model of the operation process was developed using a semi-Markov process. Then, the research results obtained using statistical and stochastic methods were compared. Section 5 summarizes The entire publication, including the conclusions and future works.

Notifications	
LDA	Life Data Analysis
MSCF	Mean Security Capacity to Failure
MTBF	Mean Time Between Failures
MTTD	Mean Time To Diagnose
MTFF	Mean Time to Failure
MTTS	Mean Time To Supply
MTTR	Mean Time To Repair
RAM	Reliability, Availability, Maintainability
SMP	Semi-Markov Process
MSE	Mean Squared Error
AIC	Akaike Information Criterion
PCC	Pearson Correlation Coefficient
R(·)	Reliability function
F(·)	Cumulative distribution function
k_{gt}	Availability coefficient for statistical method
k_{gw}	Internal (technical) availability coefficient for statistical method
k_m	Temporary indicator of technical Unavailability for statistical method
$\boldsymbol{Q_{ij}(t)}$	Matrix of the renewal kernel
$\boldsymbol{\Lambda}$	Transition intensity matrix
λ_{ij}	The intensity of the state changes
N	Matrix of the number of interstate transitions
p_{ij}	Probability of transition from state *i* to state *j*
P	Stochastic matrix
π	Ergodic probability vector
p_j	Ergodic probability of being in state *j* element of the vector π.
K_{gt}	availability coefficient for stochastical method
K_{gw}	Internal (technical) availability coefficient for stochastical method

2. Literature review

This section contains reviews of the literature devoted to modeling the reliability and availability of technical objects using statistical and stochastic methods. The main research goals of the authors were the following analyses: reliability [8, 28, 36, 58], availability [20, 28, 57, 66], determination of optimal repair intervals [37, 41, 45, 59] and reduction of operating costs of the tested systems [18, 19, 31, 46].

Statistical methods are one of the basic ways of creating reliability models. Based on the operational data of transport companies in [5], the technical availability indicators of selected vehicles were determined and compared. It was proved that the availability indicator does not depend directly on the object’s age and mileage. Czarnowska and Migawa [34] determined the availability indicators of transport means and, based on them, created the basis for building a mathematical model of the operation process of the objects under study. In [38], using a linear econometric model, the author presented an attempt to forecast the reliability of machines, taking into account their seasonality. He demonstrated that the range of data and its randomness limit the use of econometric modeling. Using the Kaplan-Meier estimator, Selech and Andrzejczak [47], examined the reliability of the driver cabin lock in a rail vehicle. The method of examining the temperature profile in aircraft commutators presented by Wawrzyński et al. [55] allowed for the estimation of changes in the values of their diagnostic parameters and their operating time. Based on the availability and reliability metrics, Kokieva et al. [27] discussed the methodology for calculating, analyzing and increasing the reliability of objects and systems. When characterizing statistical methods in reliability testing, it should be emphasized that understanding various types of failures by the users and statistical analysis of data on MTBF and MTTF are the basis for applying an appropriate policy to optimize availability and reliability. Żyluk et al. [66] proved that the logistic needs of operational support of a military aircraft can be defined based on the MTTF indicator. The presented method takes into account accidental failure to components that could not be predicted at the spare parts scheduling stage, identifies failure patterns and allows for logistic planning of the supply process of failure-prone parts, thus increasing the system's operational safety. In [52] the MTBF indicator was used to quantitatively describe technical objects' reliability level. Bai et al. [3] used MTTF and MSCF indicators to calculate availability in the platooning system.

Test results obtained using statistical methods constitute the basis for further reliability research. Koohsari et al. [28] and Nurcahyo et al. [37], based on MTBF and MTTR, conducted a RAM analysis and proved that appropriate maintenance planning significantly influences machines' readiness. Similar research was carried out in [45], where Saini et al., additionally using a genetic algorithm and particle swarm optimization, indicated possible directions for improving the load haul dump machines' functional parameters. Using statistical methods, extended by using the Laplace transform and the R programming language, enabled Szkutnik-Rogoż et al. [51] to create a universal method of operation process modeling.

After analyzing the current studies, it should be stated that Markov or semi-Markov models of the exploitation process are widely used in science and technology and are an area of interest for many authors. Depending on the modelling goal, Markov and semi-Markov models may have a different number of states in the phase space. In [53], based on a 2-state Markov model, the author created a control rule to simulate and optimize energy saving in the manufacturing system line. In [19], Markov processes were used to determine fueling patterns for hydrogen-powered vehicles and drivers' behaviors in the South African transport sector. Raven in [30] used a 4-state Markov model to present a model of the operation process with the expectation of use. He proved that the estimated indicators and measures essential for the vehicle operation, i.e. repair defectiveness, repair intensity, usage intensity and failure intensity, may be useful in the operation management process. In [20], Itkin analyzed farm availability indicators based on the 5-state Markov model. According to his estimations, the power of the tested wind farm is covers only 54% of the electricity demand and its expansion is necessary. In [48], a 6-state Markov model was used for modelling the repair policy. Estimating parameters for semi-Markov models is more difficult, which makes them less popular. However, due to less restrictive requirements regarding the form of distribution of the studied variables (any variables), they constitute a universal tool for modelling operational processes. Grabski [16] presented a general model of technical objects and proved the theory regarding the Markov recovery equations for the conditional reliability function with a semi-Markov failure rate process. Borucka et al., [7] using semi-Markov processes, characterized the process of operating police cars. It developed a model based on three states (usage, parking and repair). By examining the intensity of use and the time of failure-free operation of the vehicles, she estimated the level of readiness and showed that the analyzed transport system had a satisfactory stationary availability factor.

Table 1. The comparative summary of the literature review.

Methods and approaches	Indicators	Model	Case study	Purpose of research	Result and conclusions	Papers
Statistical	Availability	-	Vehicles in transport company	Determining readiness indicators for transport vehicles.	The basis for constructing mathematical models to evaluate availability.	[34]
	Reliability	-	Agricultural machines	Analyze reliability and readiness indicators.	Identified operational parameters to enhance equipment utilization.	[27]
	MTFF	Normal, Lognormal, Exponential, Weibull, Logistic, Loglogistic Gumbel	Rail vehicles components	Select criteria for fitting time-to-failure distributions.	Developed methods for robust data-model fit for predictive maintenance.	[47]
Statistical and probabilistic methods		Normal, Lognormal. Exponential, Weibull	Helicopters	Predict spare part needs using reliability indexes.	Highlighted logistical improvements based on MTTF predictions.	[66]
Statistical and genetic algorithm	MTBF, MTTR	Normal, Exponential Weibull, Lognormal	Load haul dump machines	Optimize failure and repair parameters.	Enabled planning for advanced maintenance strategies.	[45]
Markov process	-	Exponential 2-state model	Manufacturing line system	Increase efficiency and energy savings	Develop control rule to simulate and optimize energy saving	[53]
	-	Exponential 3-state model	Hydrogen fuel vehicles	Understanding stochastic refueling behavior	Develop refueling trend algorithms and behavioral patterns.	[19]
	-	Exponential 4 -state model	General technical objects	Exploring component sequencing in redundancy strategies	Provided formulas for reliability in mixed redundancy strategies.	[18]
	Availability	Exponential 5-state model	Wind farm	Estimating availability indicators of real case study	Quantified electricity demand met by wind power.	[20]

Methods and approaches	Indicators	Model	Case study	Purpose of research	Result and conclusions	Papers
	-	Weibull 5-state model	City buses	Develop a universal method for comparing maintenance scheduling policies across technical systems.	Introduced a structured framework that optimizes maintenance schedules, reducing downtime and improving system readiness.	[35]
	Reliability	7-state model	MMC	Short-term reliability assessment and maintenance decision support	Model supports maintenance planning and enhances operational reliability.	[60]
	-	Weibull Gamma Lognormal 8-state model	Police cars	Diagnostics and readiness evaluation	Introduced a predictive readiness model for maintenance systems.	[29]
Statistical and semi-Markov process	MTTF, MTBF, MTTR, MTTD, MTTS, availability, reliability	Normal, Exponential Weibull, Lognormal Gamma 7-state model	Military trucks	Implementation and comparison of statistical and stochastic methods	The use of stochastic processes makes it possible to identify operating conditions that affect the improvement of the values of availability and reliability indicators.	This paper

Based on empirical data in [40], a 3-state semi-Markov model was developed for vehicles used in a military transport system, which is less accurate than the original 9-state model. However, it is still a reliable representation of the operation process under study, and at the same time, it significantly simplifies calculations related to the analysis of reliability. The model of operational reliability of road machinery developed in [14] enables estimation of its operational reliability. Sanchez-Herguedas et al. [46] based on empirical data on o-rings belonging to the refrigerated exhaust system of a marine diesel engine, developed a 3-state semi-Markov model to optimize the preventive maintenance interval. In [35] authors considered a 5-state model of preventive repairs and component replacements depending on the age of city buses. The authors proved that taking into account the criterion of profit in time and the readiness factor, optimal times for preventive replacements of components can be determined. Zhang et al. [60] developed a 7-state semi-Markov model for reliability evaluation of modular multilevel converter (MMC) systems in flexible DC power transmission networks. In their approach, the state space was divided into three main operational states (locked, full voltage, zero voltage), three corresponding alarm states, and a complete failure state. This structure allows for detailed modeling of both normal operation and degraded or alarm conditions of the converter, utilizing real-time operational data and non-exponential sojourn time distributions. The focus is on enabling condition-based maintenance and supporting decision-making in power engineering. Kozłowski et al. [29], examining an 8-state semi-Markov model, presented a method for detecting hidden factors and their impact on the system reliability. In the field of IT, Ivanchenko et al. [21] and Mengistu et al. [33] develop multi-state semi-Markov models for availability assessment, accounting for a very large number of states (up to 19) and complex scenarios of failures, including deliberate malicious impacts.

The literature review showed that the analysis of appropriate indicators for monitoring the technical condition of objects is the basic criterion for managing the operational processes of technical objects. The scientific literature includes many studies on modeling the availability and reliability of technical object operation systems. It is worth emphasizing that although the analyzed models are based on similar mathematical foundations, despite their shared advancement in the assessment of reliability and availability, they differ in the level of detail of the represented states, the application context, and the type of input data used. Compared to these studies, the proposed 7-state semi-Markov model aims to capture the full operational lifecycle of a military transport system, with states reflecting not only basic functioning, downtime, and repair, but also logistical phases such as waiting for parts. This structure balances the need for sufficient detail with practical tractability for parameter estimation and result interpretation. It is less complex than the high-dimensional models used in IT infrastructure studies, but offers a more nuanced description of operational reality than traditional 2- or 3-state models commonly used in transport or manufacturing systems.

Based on the literature review, a methodological gap was identified in the form of a lack of validation of stochastic modeling based on the results of statistical methods. Additionally, it should be noted that the presented analysis fills the subject gap in the lack of implementation of semi-Markov models to reflect the truck operation process. Table 1 summarizes data regarding the research methods used to analyze the survival of technical objects, i.e. statistical and stochastic.

3. Methodology

Implementing of operational research involves monitoring operational incidents (both planned and accidental) generated by various data sources (operational areas), systematically archiving them on primary storage media, and subsequently transferring, verifying, and processing them in a centralized data repository. The data collection, validation and mathematical modeling process was conducted in four stages (Fig. 1).

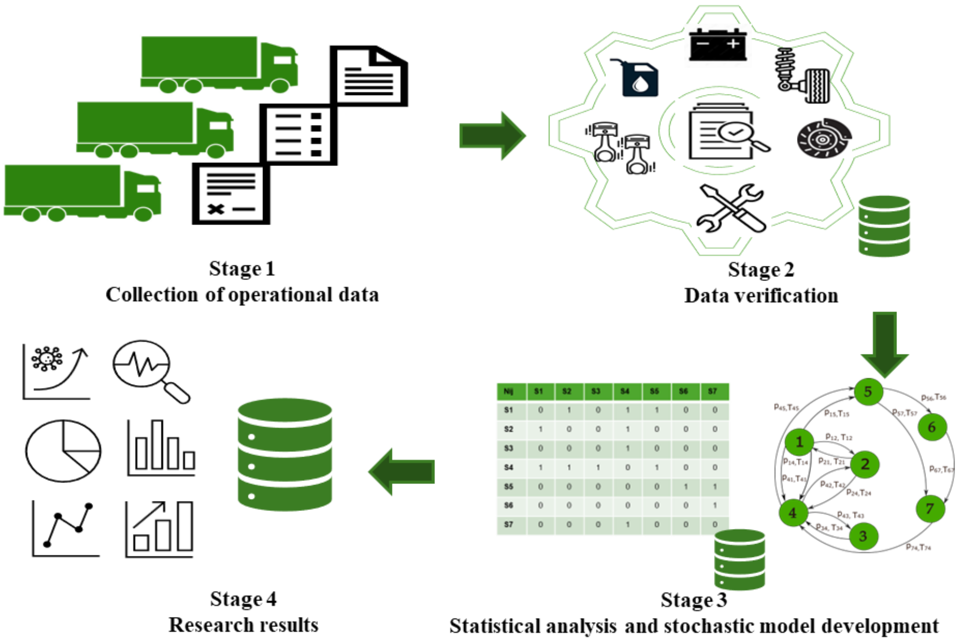


Fig. 1. Data collection, validation and mathematical modeling process.

The initial step in developing the mathematical model involved collecting empirical data from a real-world operational system. The study sample consisted of military vehicles utilized by the Polish Armed Forces between 2019 and 2021, as detailed in Section 4. Data was extracted from traditional paper-based documentation, including departure orders, technical service sheets, operation plans, and technical condition reports. The collected information included start and end times of operations, distance traveled, fuel consumption, scope and duration of maintenance and repair activities, and a list of spare parts and materials used. This phase established a comprehensive dataset capturing key operational parameters and maintenance records, forming the foundation for subsequent analysis.

The second step focused on the verification and validation of the collected data. The initial research sample comprised 70 military vehicles; however, due to incomplete operational records that impeded the accurate reconstruction of detailed phase trajectories, 20 vehicles were excluded from the analysis. To construct the preliminary operational model, the method of successive approximations was applied:

- Identification of all recorded operational states,
- Systematic aggregation of states aligned with the modeling objectives,
- Elimination of states that did not influence the reliability and availability analysis.

Initially, 32 distinct states were identified, each corresponding to a relevant phase of vehicle operation as defined by applicable technical regulations and the authors' expert knowledge. Each state was subsequently evaluated based on the following criteria: frequency of occurrence, average dwell time, importance to the structure of state transitions (including high transition intensity and probability), and functional similarity to other states.

States that occurred infrequently or for short durations, as well as those exhibiting similar functional roles or stochastic behavior, were aggregated to simplify the model without compromising its accuracy. Additionally, states characterized by zero or negligible probability of interstate transitions were excluded from the final model. This systematic refinement process enabled the development of a robust operational model that accurately reflects the real-world behavior of the system, while eliminating irrelevant or redundant data. We believe this detailed methodological clarification enhances the model's transparency, facilitates reproducibility, and supports its potential generalizability to other vehicle systems or operational contexts.

The result of the research is an original 7-state model of the operation process of Iveco Stralis vehicles, as shown in Fig. 2.

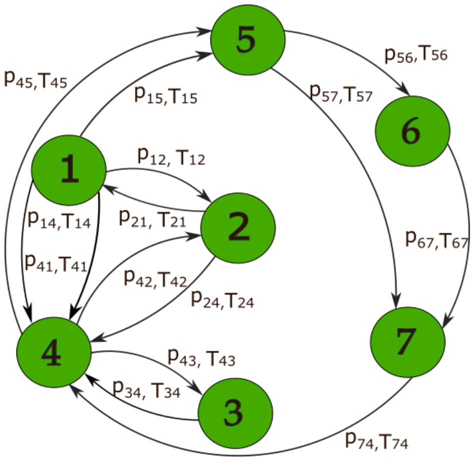


Fig. 2. Process states in the operating system of military trucks.

The graph is an interpretation of the analyzed exploitation process, in which the vertices are operational states and the arcs are possible transitions between states. Each of the technical objects (means of transport) in operation may at any given time t be in only one of the highlighted states $S_i \in S$, forming a finite set **S** of operational states of the technical object.

Operation is understood as the transition of the objects between identified operating states, i.e.:

- ***S*₁ - task execution** - carrying out transport of cargo from central warehouses to local warehouse,
- ***S*₂ - refueling** - refilling operating fluids,
- ***S*₃ - standby/parking in the garage** - time after completion of other tasks. The vehicle is reliable and parked in a garage,
- ***S*₄ - service** - includes scheduled (planned) technical maintenance and inspections. If a technical failure requiring withdrawal from service is detected during this state, the vehicle transitions to S_5. Minor issues that do not require withdrawal remain classified under S_4."
- ***S*₅ - failure** - covers any unplanned interruption in operation that directly affects reliability or safety. This state includes the period of diagnosis and verification of the failure, starting from the initial detection (whether during operation or scheduled service) until the need for repair is confirmed. Both failures occurring during operation and those detected during scheduled maintenance are included. Failure results in a transition to an unfit state for a period of renewal.
- ***S*₆ - awaiting repair** - period when the vehicle is out of service and waiting for resources needed to initiate repair (e.g. spare parts, tools, or personnel). The vehicle remains non-operational until the necessary resources become available and repair can begin.
- ***S*₇ - repair** - operations aimed at restoring readiness of the technical objects or their resources by removing any malfunctions (damage).

At stage 3, using the verified dataset, the phase space of the analyzed operational process was defined. Preliminary databases were compiled using MS Excel and then translated into source databases optimized for further engineering and computational studies. For each vehicle, the balance of operating conditions was verified, detailing the duration of each operating condition over the three-year study period. This verification provided an accurate record of system condition transitions and operational schedules.

The final stage involved constructing an operational database containing detailed records of state transitions. The operational database was formatted for compatibility with engineering analysis tools, enabling advanced studies in reliability engineering and availability modeling.

The compiled database serves as a critical input for reliability and availability analysis, facilitating the application of advanced engineering software to model and simulate the behavior of technical operating systems.

The structured data collection, verification, and processing approach yielded a comprehensive operational dataset, capturing the dynamics of military vehicle operations over an extended period. The resulting database provides a reliable foundation for conducting advanced studies on the reliability and availability of technical systems, supporting further development of mathematical models, predictive maintenance strategies, and operational optimization tools. This process ensures the validity and applicability of the dataset for future engineering research.

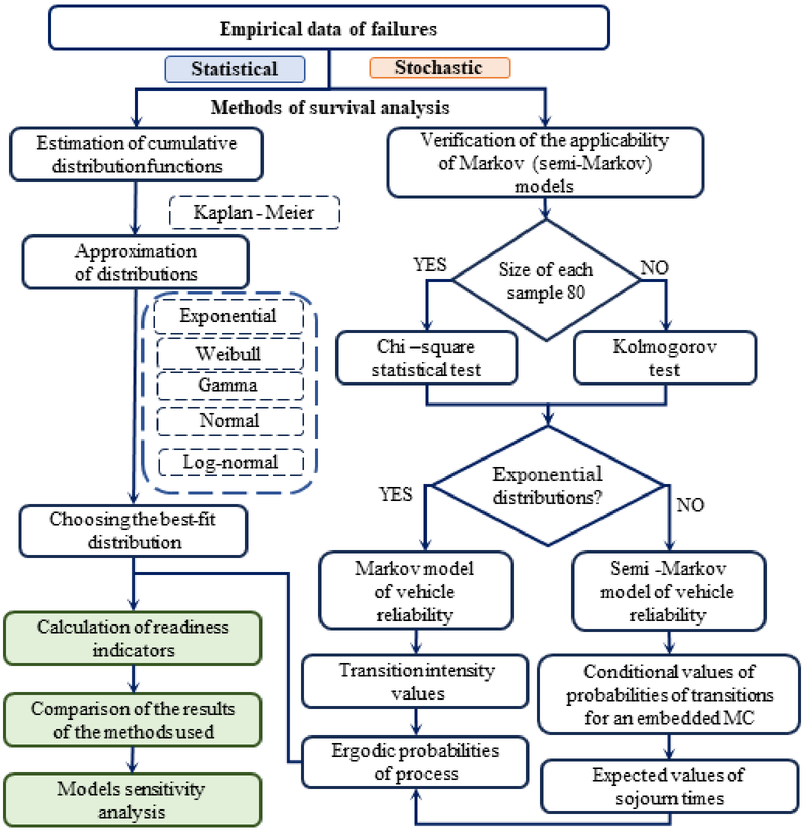


Fig. 3. Flowchart of the reliability and availability modeling.

As mentioned in the introduction and shown in Fig. 3, based on empirical data on the operation process of technical objects, two methods used to analyze the reliability and availability of technical objects will be compared.

Reliability is the most important property of objects or systems and characterizes their ability to perform their functions during normal operation. In a probabilistic approach, the reliability function is defined as the probability that no failure will occur during the execution of the task (use), i.e. in the service life range (0, *t*), according to the relation [23, 63, 64]:

$$R(t) = P(T \geq t) \text{ for } t \geq 0, \quad (1)$$

the statistical expression of the reliability function has the following form:

$$R(t) = \frac{N(0)-n(t)}{N(0)}. \quad (2)$$

In the context of vehicles and transport systems, reliability refers to the ability of vehicles, infrastructure and supporting systems to operate without failure and as expected, while ensuring safety and operational efficiency. The indicators will be discussed later in the article.

The study of reliability characteristics using statistical methods begins with estimating the reliability function. Then, using analytical models, an approximation of this function is performed. The next step is calculating availability indicators.

Reliability analysis using stochastic methods begins with checking the applicability of the Markov or semi-Markov model, i.e. verifying the exponential distribution of time characteristics. For this purpose, the Kolmogorov-Smirnov test (for a sample of less than 80) or the $\chi 2$ test is performed. In the case of exponential distribution, the Markov model is used, and the intensity values of interstate transitions are calculated based on it. Using the semi-Markov model, the value of the conditional transition probabilities for the inserted Markov chain and the expected value of the time between subsequent transitions are determined. The values of the ergodic probabilities of the inserted Markov chain are then calculated. The result of the research is an identified subset of technical availability states and the determination of the availability value of a technical object. Validation of both methods is carried out by comparing the obtained results of reliability and availability indicators. After positive validation, it is assumed that the obtained results will be analyzed.

3.1. Statistical methods

Multiple research methods are used to analyze the reliability and availability of technical objects and systems, the use of which most often involves in-depth mathematical knowledge and dedicated specialized software [2, 52]. Statistical methods enable a relatively easy description of phenomena. Data on failures is the basis for developing reliability models, which is the basis for determining reliability measures and statistics.

A. Reliability estimation

As shown in Fig. 3, the estimation of the reliability function value will be based on empirical data on technical object failures. Operational data in the analyzed, real transport system are censored. Therefore, the Kaplan-Meier estimator was used to estimate the survival function and determine the empirical distribution function [25, 67], according to the formulas:

$$\hat{R}(x) = \prod_{i:x_l \leq \leq x}(1 - \frac{d_i}{n_i}), \tag{3}$$

$$\hat{F}(x) = 1 - \prod_{i:x_l \leq \leq x}(1 - \frac{d_i}{n_i}), \tag{4}$$

where d_i is the number of objects that failed when the x_i value was reached, while n_i is the number of all technical objects that worked correctly until the value x_i was reached.

B. Lifetime distribution (Reliability distributions)

The results of many years of research on reliability show that distinctive distributions of reliability characteristics can be assigned to specific technical objects and typical types of failure, which are defined models of their reliability [23, 42, 67]. Knowledge about the distribution types is necessary for managing and forecasting the operation of machines and devices. Knowing the form of the distribution describing the failure rate of a technical object, it is possible to forecast its operating time with a certain probability. Knowing the course of the probability of failure over time, one can take preventive actions before the risk of failure reaches its limit value, thus avoiding financial losses and ensuring an appropriate level of safety.

In the literature on technical objects, various families of probability distributions are used as time-to-failure models. The most commonly used distributions in Life Data Analysis (LDA) are the exponential, Weibull, gamma, normal and lognormal distributions [23, 67].

A number of measures and indicators are used in the literature to determine the accuracy of model fit to empirical or estimated values. The authors of this paper used the following indicators to assess the fit of the developed models to the estimated values: correlation coefficient R, Mean Squared Error (MSE), Pearson Correlation Coefficient (PCC), and Akaike Information Criterion (AIC).

C. Availability indicators

The described objects are most often characterized by binary random variables, i.e. variables taking two values - zero and one, and working and failure in operation systems [51]. The tools used to properly assess the operation process of technical objects are reliability and availability indicators. When using them, it is possible to quantitatively assess and compare the reliability of objects. The technical objects can be characterized as a function of time [h, mth] or, as in the case of vehicles, mileage [km] [66].

The basic measure describing the reliability of repairable objects is the Mean Time Between Failures (MTBF). It is expressed as the quotient of the total operation time of all objects to their total number of failures according to the formula:

$$MTBF = \frac{\sum_{i=1}^{N} T_i}{n}, \tag{5}$$

where N – the number of objects, T_i – the time of correct operation of the i-th object, n – the total number of failures.

For irreparable objects, Mean Time To Failure (MTTF) indicators are used, defined according to the following formula:

$$MTTF = \frac{\sum_{i=1}^{N} T_W}{n}, \tag{6}$$

where T_W – the operation time of the i-th object.

Mean Time To Repair (MTTR) expresses the average time to repair a system or object after a failure. The lower the MTTR indicator, the faster the repair process and the higher the system's availability to perform tasks. It is expressed with the following formula:

$$MTTR = \frac{\sum_{i=1}^{N} T_R}{n}, \tag{7}$$

where T_R – i-th overhaul time.

Mean Time To Diagnose (MTTD) represents the average time to diagnose a system or object after a failure:

$$MTTD = \frac{\sum_{i=1}^{N} T_D}{n}, \tag{8}$$

where T_D – i-th diagnosis time.

Mean Time To Supply (MTTS) that represents the average waiting time for spare parts:

$$MTTS = \frac{\sum_{i=1}^{N} T_S}{n}, \tag{9}$$

where T_S – the spare parts waiting time.

The relations between the characteristics mentioned above are given in Fig. 4.

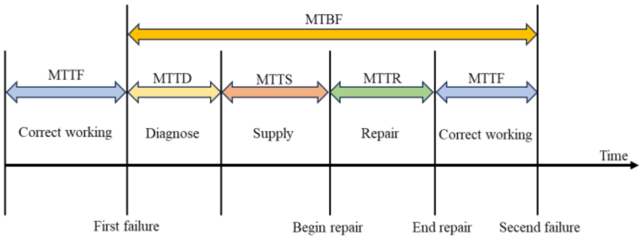


Fig. 4. A schematic diagram of MTTF, MTTD, MTTS, MTTR, and MTBF.

The characteristics that represent the ability of an object or system to work correctly or take action at a random moment t are measures of availability.

The availability factor determines the probability that, at time t, the object is in a state of availability. It is determined as the quotient of the total time spent in states of availability to the total time spent in states of availability and unavailability to perform tasks by the relation:

$$k_{gt} = \frac{\sum_{i=1}^{N} T_G(t)}{\sum_{i=1}^{N} T_G(t) + \sum_{i=1}^{N} T_N(t)}, \tag{10}$$

where: $\sum_{i=1}^{N} T_G(t)$– the sum of times spent in the availability state, $\sum_{i=1}^{N} T_N(t)$– the sum of times in the unavailability state.

Due to the fact that the time spent in an unavailability state is generally not the time of effective repair, but, as shown in Fig. 4, it includes the time for diagnosis and the logistic delay resulting from the inability to carry out the repair for organizational or technical reasons or the lack of spare parts, the following technical (internal) unavailability factor is used:

$$k_{gw} = \frac{\sum_{i=1}^{N} T_G(t)}{\sum_{i=1}^{N} T_G(t) + \sum_{i=1}^{N} T_R(t)}, \tag{11}$$

where: $\sum_{i=1}^{N} T_R(t)$ – the sum of times effective repair.

A coefficient that practically characterizes the system's unavailability to undertake the task at the moment t is a temporary indicator of technical unavailability k_m. It does not characterize the technical condition of the fleet of cars in the system, nor their suitability to complete the task within the time interval $(t,\ t+\Delta t)$. The coefficient is a basic measure of unavailability in real operation systems (e.g. car fleet) calculated according to the formula:

$$k_m = \frac{N_e - n_t}{N_e} = \frac{N(t)}{N_e}, \tag{12}$$

where: N_e – is evidential (regular) state of the car fleet, N(t) – is number of cars technically suitable for operation at the moment t, n_t – is a number of cars technically unsuitable for operation (located in $\{S_5, S_6, S_7\}$ at the moment t).

3.2. Stochastic methods

Stochastic processes are a set of mathematical models that characterize random events observed over time. The stochastic approach recognizes the irregularity and randomness of events, probabilities, and average values. Stochastic processes are widely used in various fields of science, including statistics [9], economics [61], engineering [6], medicine [54], construction [11], mechanics [29], communications and IT [43] or social sciences [13]. Stochastic methods are constantly improved and developed but, at the same time, poorly standardized. One of the most frequently used methods in describing the operation processes of technical objects are Markov and semi-Markov processes. They constitute a group of analytical methods based on the analysis of random processes focused on determining the probability of conditional interstate transition.

As indicated in [12, 39], it is considered a methodological error to assume that the studied process is a Markov process without verifying its properties first. It has been proven in the source literature that this may result in incorrect analysis results and final conclusions. In [50] it was shown that the differences in the values of the calculated limiting probabilities for the Markov process differ significantly from the values of the semi-Markov process (even beyond 530%). Moreover, a difference in the calculated value of the technical availability coefficient (almost half the size) was observed in the case of the discussed processes. Similar conclusions were obtained in [39], where the semi-Markov model compared to the Markov model reached a mean absolute percentage error (MAPE) of 351.92%. In accordance with the adopted research methodology presented in Fig. 3, the applicability of Markov and semi-Markov models can be determined using Kolmogorov-Smirnov tests.

D. Continuous-time Markov process

A quantitative characteristic of the Markov process is the transition intensity matrix **Λ**, according to formula (13):

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1(k-1)} & \lambda_{1k} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2(k-1)} & \lambda_{2k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{(k-1)} & \lambda_{(k-1)2} & \cdots & \lambda_{(k-1)(k-1)} & \lambda_{(k-1)k} \\ \lambda_{k1} & \lambda_{k2} & \cdots & \lambda_{k(k-1)} & \lambda_{kk} \end{bmatrix}. \tag{13}$$

Matrix **Λ** components on the main diagonal of this matrix are negative or equal to zero, while the remaining components are non-negative. Moreover, the sum of the components of each row is zero, and their values are calculated as the intensity of the state changes according to the formula (14):

$$\lambda_{ij} = \frac{1}{T_{ij}}, \tag{14}$$

where T_{ij} – average time spent in state S_i before state S_j.

The transition intensity matrix **Λ** cannot be a direct basis for assessing the availability of a technical object[65]. For this reason, the ergodic probabilities of the Markov process for the entire set of operating states are calculated by solving the matrix equation (15) [40]:

$$[\mathbf{p_j}]^T \cdot \mathbf{\Lambda} = \mathbf{0}, \tag{15}$$

along with the condition of the system normalization (16):

$$\sum_{j=1}^{n} \pi_j = 1. \tag{16}$$

E. Semi-Markov process

Semi-Markov processes are a generalization of Markov processes for which the times of presence in individual states can have any distribution of time characteristics. This feature means semi-Markov processes have a wider range of applications than Markov processes.

The semi-Markov process is constructed using a two-dimensional Markov chain, the so-called Markov renewal process. It is defined by the matrix of the renewal kernel **Q**(t) according to the equation [7, 17, 34]:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{12}(t) & \cdots & Q_{1(k-1)}(t) & Q_{1k}(t) \\ Q_{21}(t) & 0 & \cdots & Q_{2(k-1)}(t) & Q_{2k}(t) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ Q_{(k-1)}(t) & Q_{(k-1)2}(t) & \cdots & 0 & Q_{(k-1)k}(t) \\ Q_{k1}(t) & Q_{k2}(t) & \cdots & Q_{k(k-1)}(t) & 0 \end{bmatrix}. \tag{17}$$

$\boldsymbol{Q_{ij}}(t)$ matrix elements are the conditional transition probabilities from state S_i to state S_j and depend on the state duration distribution function S_i before moving transition to state S_j, according to the equation:

$$Q_{ij}(t) = p_{ij} F_{ij}(t), \tag{18}$$

where p_{ij} denotes the probability of transition from state S_i to state S_j, and $F_{ij}(t)$ is the cumulative distribution function of the time of presence in state S_i before transition to state S_j.

The first stage of the study is the creation of a Markov chain. Based on the obtained empirical data on interstate transitions, a matrix of the number of interstate transitions was created according to the formula:

$$\mathbf{N} = \begin{bmatrix} 0 & n_{12} & \cdots & n_{1(k-1)} & n_{1k} \\ n_{21} & 0 & \cdots & n_{2(k-1)} & n_{2k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ n_{(k-1)} & n_{(k-1)2} & \cdots & 0 & n_{(k-1)k} \\ n_{k1} & n_{k2} & \cdots & n_{k(k-1)} & 0 \end{bmatrix}. \tag{19}$$

It constitutes the basis for estimating the probability of transitions p_{ij} of stochastic matrix **P** [29, 62]. The values of these estimators are the state transition probabilities from state S_i to state S_j. They are calculated based on empirical data according to the relation:

$$p_{ij} = \frac{n_{ij}}{\sum_{j=1}^{k} n_{ij}}, \tag{20}$$

where: n_{ij} - number of transitions from state S_i to state S_j.

Matrix of conditional probabilities of interstate transitions **P** has the following form:

$$\mathbf{P} = \begin{bmatrix} 0 & p_{12} & \cdots & p_{1(k-1)} & p_{1k} \\ p_{21} & 0 & \cdots & p_{2(k-1)} & p_{2k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{(k-1)} & p_{(k-1)2} & \cdots & 0 & p_{(k-1)k} \\ p_{k1} & p_{k2} & \cdots & p_{k(k-1)} & 0 \end{bmatrix}, \tag{21}$$

assuming the fulfillment of the stochastic matrix condition [29, 35]:

$$\sum_{j=1}^{k} p_{ij} = 1. \tag{22}$$

The next step is to calculate the ergodic probability of the embedded Markov chain by solving the equation [40]:

$$(\mathbf{P^T} - \mathbf{I}) \cdot \mathbf{\Pi} = \mathbf{0}, \tag{23}$$

along with the condition of normalization:

$$\sum_{j=1}^{k} \pi_{ij} = 1. \tag{24}$$

Then, if the inserted Markov chain exhibits ergodicity and there are expected values $E(T_i)$ of times of presence in states, ergodic values of probabilities p_j are determined for the semi-Markov process according to the relation [12]:

$$p_j = \frac{\pi_j E(T_j)}{\sum_{i=1}^{k} \pi_j E(T_j)}, \tag{25}$$

$$E(T_j) = \sum_{i=1}^{k} p_{ij} E(T_{ij}), \tag{26}$$

where π_i is the ergodic probability of the inserted Markov chain for state S_i, and $E(T_{ij})$ is the expected value of the direct state transition time from state S_i to state S_j.

4. Results and discussions

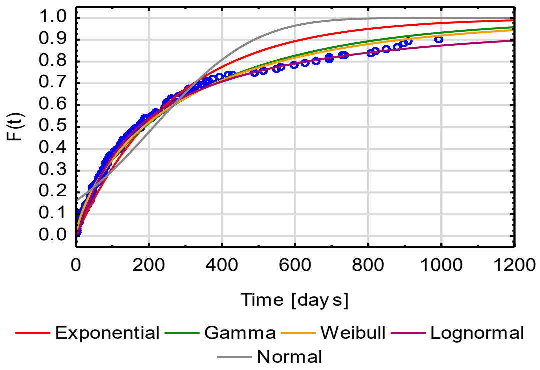
This section presents a computational example based on the transport system established in the Polish Armed Forces. The failure data was collected using a test sample of 50 Iveco Stralis tractor units operated for three years from 01/01/2019 to 31/12/2021). They have been operated by the Polish Armed Forces since 2003. The vehicle's empty weight is 10,500 kg, and the payload is 13,500 kg. The tractor unit is designed to transport bulk, palletized or containerized loads over long distances and is adapted to work with semi-trailers with a total weight of up to 19,000 kg. The sample can be considered relatively homogeneous with varying scales of operation intensity since only one vehicle model was tested.

All selected vehicles are of the same model and were operated within the same organizational structure, which minimizes technical variability and enhances the representativeness of the sample. The vehicles performed comparable transport tasks (mainly long-distance transportation of bulk, palletized, or containerized cargo) under similar operational conditions. The vehicles were not limited to a specific geographic location but operated within a national military transport network in Poland. No special roles or missions were assigned to the vehicles in the sample beyond standard military logistics. There were no known operational restrictions or preferential allocation that would systematically bias the reliability or availability results. All vehicles were operated according to a standard military logistics schedule. The operating system did not employ a strict 8-, 12-, or 24-hour shift pattern. Instead, vehicle utilization was determined by mission requirements and the operational plan, with vehicles being dispatched as needed. The analysis is therefore based on actual recorded operation and downtime periods (start and end times of tasks, maintenance, refueling, etc.), not on theoretical maximum utilization or imposed shift cycles. This approach reflects real-world operating conditions and is consistent with military logistics practice, where operation is demand-driven rather than fixed-shift-based.

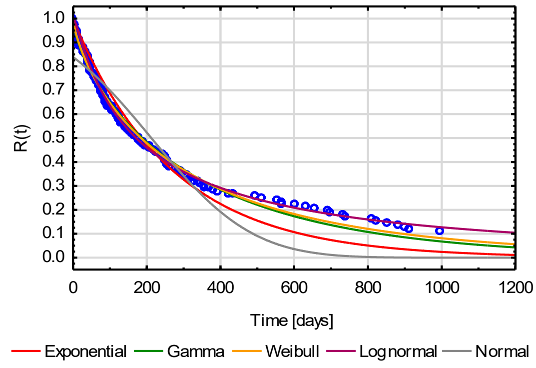
4.1. Results of the statistical approach

Based on operational data, a statistical analysis of vehicles used in the Polish Armed Forces was performed.

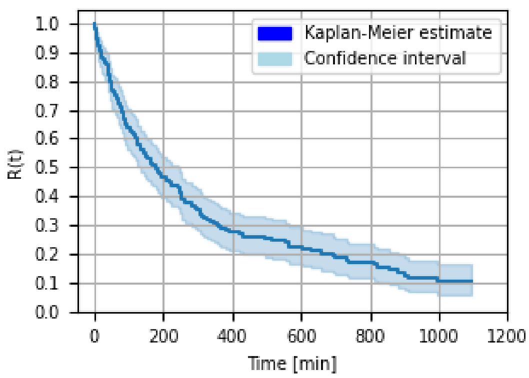
The Kaplan-Meier estimator was used to estimate the values of the unreliability and reliability functions. Based on empirical data on the times between failures of military vehicles according to relations (3) and (4), the reliability and failure functions were estimated. The result's graphical interpretation is presented in Fig. 5.



(a)



(b)



(c)

Fig. 5. Estimation and approximation of functions: (a) - Cumulative distribution function approximation, (b) - Reliability function approximation, (c) - Kaplan-Meier estimation.

Assessment of the accuracy of matching the developed models to empirical values is an essential element of reliability analyses. AIC index is the primary and decisive criterion for selecting the best-fitting distribution. MSE is reported as a secondary criterion, providing a direct measure of the overall deviation between the empirical Kaplan-Meier curve and the fitted distribution. PCC and R are presented as auxiliary measures. They indicate the strength of the linear relationship between the empirical and fitted values. While a high value of R or PCC confirms the quality of fit, these coefficients are not used as the decisive criteria for selecting the best model.

In this study, the reliability function models developed in this paper showed a high degree of fit to the estimated values. Table 2 lists the values of correlation coefficient R, MSE, PCC, and AIC for each distribution. For gamma, Weibull and lognormal models, the R coefficient reached a value of above 0.99. The Pearson Correlation Coefficient for the normal model is an outlier from the others. Of the 5 proposed distributions, we consider the one with the lowest AIC index value to be the best suited. Unless the MSE values for the fitted models are too close to each other, the smallest AIC will be at the smallest MSE.

Before fitting theoretical distributions, the empirical distributions of sojourn times for key transitions were examined. Representative histograms are provided in Appendix A to illustrate the underlying variability in the data and justify the selection of candidate distributions.

Reliability indicators are the measures that provide information about the quality and functioning of the operation process. These factors characterize the intensity of use, identify the time of failure detection, determine the time required for repairs, assess the availability of components or parts that must be repair or replacement, and define the organization and equipment of the service base. Based on the relations (5)-(9) and the total duration of individual states S_i the values of reliability indicators for Iveco Stralis vehicles were calculated, the summary of the values is presented in Table 3.

MTBF value was 268.11 [days], which proves the high level of vehicle reliability. Moreover, it can be assumed that the strategies used and the operation system implemented allow for the efficient removal of malfunctions. The high level of training of the fault diagnosis personnel and their efficiency is characterized by MTTD at the level of only 29 [minutes], which is 0.02 [days]. Moreover, the MTTR value of 18.80 [days] may indicate a small number of repair personnel, poor training and improper equipment at the repair stations. However, the system could be improved by reducing the waiting time for spare parts, which is likely due to logistical delays caused by complex purchasing procedures.

Table 3. The value of reliability indicators.

Reliability indicator	**Time [min]**	**Time [days]**
MTBF	366081	268.11
MTTD	29	0.02
MTTS	16996	11.80
MTTR	27071	18.80

Other measures used to assess the ability to work properly are availability factors. One is the momentary technical availability factor, which determines how many vehicles are ready to perform a task at a random moment t. According to the data given in Fig. 6, the system's temporary technical availability remains above 82% throughout the tested operation process. Moreover, the analyzed transport system has a high level of redundancy, which increases the safety, durability and reliability of the transport objects and systems.

Moreover, in accordance with the relation (10)-(11), the values of availability coefficient k_{gt} = 0.934 and internal (technical) availability coefficient k_{gw} = 0.989 were calculated. Taking into account the level of availability and reliability indicators accepted by the Polish Armed Forces, the value 0.95 k_{gt} is slightly below the requirements. The value of coefficient k_{gt} is lower than k_{gw}, since the internal availability factor k_{gw} does not include, among others, downtimes caused mainly by waiting for spare parts. Therefore, maintenance downtimes can be limited to increase its value to acceptable levels. Fig. 7 shows the impact of shortening MTTS on the value of indicator k_{gt}.

Table 2. Accuracy of fitting models to data.

Model	***a***	***b***	***AIC***	***R***	***MSE***	**PCC**
Exponential	0.0037	-	-145.5865	0.9877	0.0025	0.9941
Gamma	0.6800	0.0021	-361.9451	0.9964	0.0006	0.9957
Weibull	298.8060	0.7619	-420.4997	0.9973	0.0004	0.9971

Model	*a*	*b*	*AIC*	*R*	*MSE*	PCC
Lognormal	**5.1448**	**1.5461**	**-527.1318**	**0.9980**	**0.0002**	**0.9987**
Normal	212.8367	215.0102	13.3573	0.9535	0.0073	0.9538

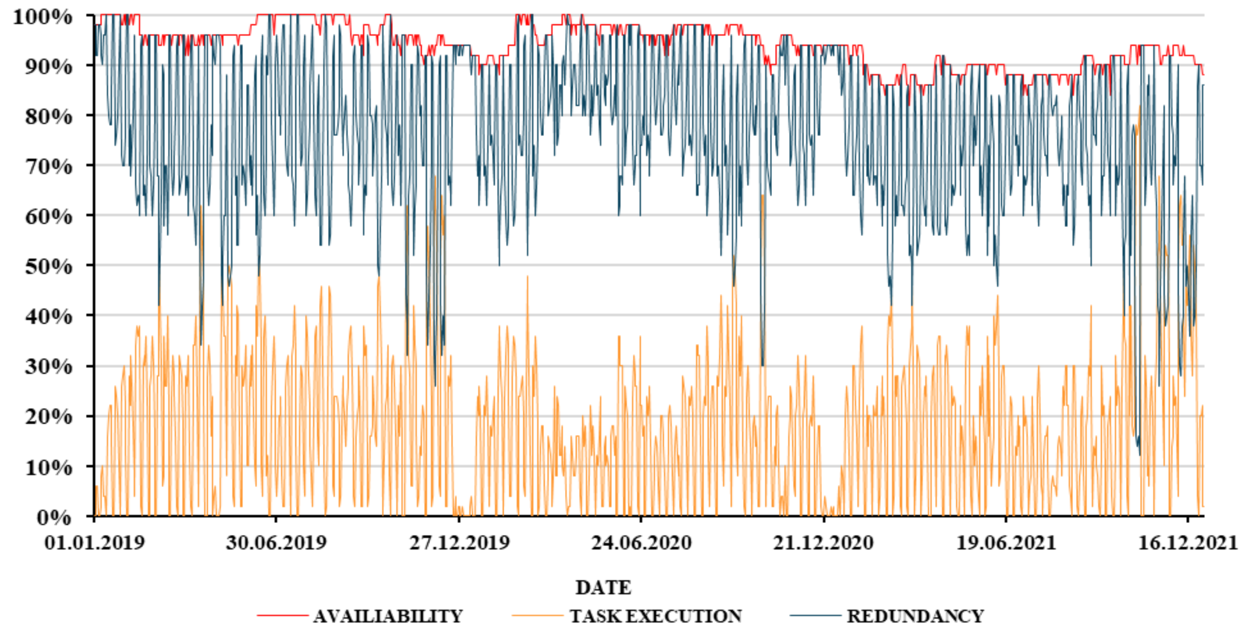


Fig. 6. Momentary values of the technical availability indicator.

It should be noted that reducing the waiting time for spare parts significantly affects the availability index, which increases and approaches the value of internal (technical) availability. It can be observed that reducing the maintenance downtime by 40% results in its improvement by 0.017 and reaching the value of 0.951, which meets the requirements for the Polish Armed Forces transport systems.

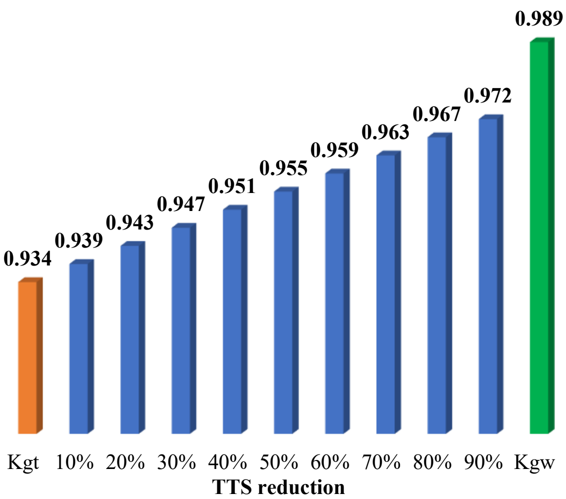


Fig. 7. Availability indicators values obtained after TTS reduction.

4.2. Results of the stochastic approach

Stochastic modeling of the operation process allows for capturing the inherent randomness of state transitions and the variability of sojourn times in different operational phases of the vehicles.

To enhance understanding of the processes occurring in the analyzed operating system and enable a more accurate interpretation of the distribution characteristics, Appendix A presents raw empirical data in the form of residence time histograms for selected transitions between states in a 7-state operating model. Each histogram shows the observed distribution of transition times *Tij* between specific operational states of military vehicles. Analysis of these histograms clearly indicates that the empirical distributions of sojourn times frequently deviate from the classical exponential distribution.

With regard to these observations, the next step in the analysis was to formally verify whether the Markov model could be applied to the analyzed data or whether a more flexible semi-Markov model was necessary.

As mentioned in Section 3, testing the reliability using stochastic methods will begin with checking the applicability of the Markov or semi-Markov model for the test sample. Both proposed methods for verifying the exponential distribution are based on the empirical distribution function and belong to the group of nonparametric tests. χ^2 test is one of the most popular ones utilized by the authors of [29, 56], however, due to the limitation of a minimum size of the sample, it cannot always be used [29]. Due to the conditions of applicability of the proposed nonparametric tests and the sample size of less than 80 (T_{45}, T_{56}, T_{67}), verification of the empirical distribution with the theoretical distribution will be performed using the Kolmogorov-Smirnov test. For hypothesis H_0 it was assumed that the distribution of times for individual interstate transitions has an exponential distribution. Alternative hypothesis H_1 contradicts this assumption. The critical values (K) of the Kolmogorov-Smirnov test for α = 0.05 can be estimated according to the formula:

$$D(0.05; y) = \frac{1.358}{\sqrt{y}+0.12+\frac{0.11}{\sqrt{y}}}. \tag{27}$$

When $U_n \in K$, we reject hypothesis H_0 and adopt H_1.

The analysis results are presented in Table 4.

Table 4. Results of Kolmogorov-Smirnov test.

T_{ij}	Sample size y	U_n statistic	Critical range	Hypothesis
T_{12}	5063	0.1356	[0.0190, 1]	H_1
T_{14}	7356	0.2706	[0.1420, 1]	H_1
$\boldsymbol{T_{15}}$	**95**	**0.1167**	**[0.1375, 1]**	$\mathbf{H_0}$
T_{21}	2962	0.4241	[0.0249, 1]	H_1
T_{24}	3436	0.4115	[0.0231, 1]	H_1
T_{34}	12044	0.4166	[0.0124, 1]	H_1
T_{41}	9569	0.6184	[0.0139, 1]	H_1
T_{42}	1591	0.4243	[0.0339, 1]	H_1
T_{43}	11809	0.4958	[0.0125, 1]	H_1
T_{45}	78	0.2803	[0.1515, 1]	H_1
T_{56}	46	0.7994	[0.1963, 1]	H_1
T_{57}	112	0.6126	[0.1268, 1]	H_1
$\boldsymbol{T_{67}}$	**37**	**0.2029**	**[0.2183, 1]**	$\mathbf{H_0}$
T_{74}	148	0.5997	[0.1105, 1]	H_1

Only two time characteristics, T_{15} and T_{67}, reached the values of the Kolmogorov-Smirnov test statistics, which are not included in the critical value. Therefore, only two time characteristics have an exponential distribution, while the remaining 12 do not meet this condition. This means that there is a justified need to use the semi-Markov model to describe the operation process of goods and passenger vehicles. The obtained results are the basis for developing semi-Markov models of the operation process of military vehicles.

Furthermore, the statistical robustness of the analysis is supported by the size of the dataset. The total number of interstate transitions for the sample was 54,565, with a statistical average of 1,059 transitions per vehicle. Therefore, the presented test sample is statistically reliable.

$$\mathbf{N} = \begin{bmatrix} 0 & 5063 & 0 & 7356 & 95 & 0 & 0 \\ 2962 & 0 & 0 & 3436 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12044 & 0 & 0 & 0 \\ 9569 & 1591 & 11809 & 0 & 78 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 46 & 112 \\ 0 & 0 & 0 & 0 & 0 & 0 & 37 \\ 0 & 0 & 0 & 148 & 0 & 0 & 0 \end{bmatrix}. \tag{28}$$

Based on the matrix (28), the values of interstate transition probabilities of the inserted Markov chain were estimated and presented using the matrix (29):

$$\mathbf{P} = \begin{bmatrix} 0 & 0.404587 & 0 & 0.587822 & 0.007591 & 0 & 0 \\ 0.462957 & 0 & 0 & 0.537043 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0.041520 & 0.069033 & 0.512388 & 0 & 0.003384 & 0 & 0 \\ 0.028701 & 0 & 0 & 0.235650 & 0 & 0.138973 & 0.338369 \\ 0 & 0 & 0 & 0 & 0.554217 & 0 & 0.445783 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{29}$$

After substituting the numerical data into equation (23), the following system of equations was obtained:

$$\begin{cases} -\pi_1+0.404587\pi_2+0.587822\pi_4+0.007591\pi_5=0 \\ 0.462957\pi_1-\pi_2+0.537043\pi_4=0 \\ -\pi_3+\pi_4=0 \\ 0.415195\pi_1+0.069033\pi_2+0.512388\pi_3-\pi_4+0.003384\pi_5=0 \\ 0.287009\pi_1+0.235650\pi_4-\pi_5+0.138973\pi_6+0.338369\pi_7=0 \\ 0.554217\pi_5-\pi_6+0.445783\pi_7=0 \\ \pi_5-\pi_7=0 \end{cases} \tag{30}$$

By solving the Chapman-Kolmogorov system of equations (30), the ergodic probabilities for the Markov chain were estimated - their values are given in Fig. 8 and Table 5. The highest observation probabilities were recorded for operational states π_4 = 0.4195, π_1 = 0.2330, π_3 = 0.2149 and π_2 = 0.1232. This is a desirable phenomenon and proves the high availability of the objects. The lowest probability of entries was observed for undesirable states (unsuitability $\{S_5, S_6, S_7\}$) π_7 = 0.0024, π_5

= 0.0061, π_6 = 0.0008.

It should be noted that the calculated limit probabilities refer only to the frequency of observations and do not consider the duration of individual states. Based on formula (25), the values of the ergodic probabilities of the semi-Markov model were calculated.

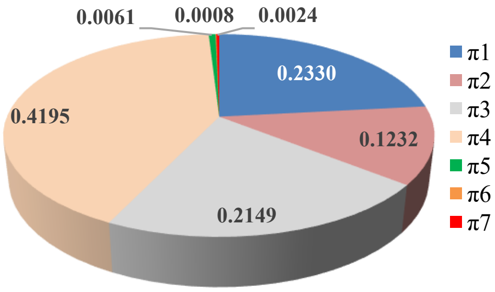


Fig. 8. Ergodic probabilities $\boldsymbol{\pi_j}$ for the homogeneous Markov chain.

The obtained values in individual operating states are given in Table 5. State S_3 reaches the highest values, i.e. standby/parking in the garage (over 84%), while state S_1 reaches 7%. The remaining limit values are satisfactorily low, which confirms the relatively low failure rate of the fleet vehicles.

Table 5. Ergodic probabilities of embedded Markov Chain and semi-Markov process.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7
π_j	0.2330	0.1232	0.2149	0.4195	0.0061	0.0008	0.0024
p_j	0.0773	0.0007	0.8298	0.0099	0.0293	0.0432	0.0097
p_j [%]	7.73	0.07	84.98	0.99	2.93	4.32	0.97
$E(T_j)$ [min]	486.8	37.4	18.6	9.5	1375.2	4270.6	1418.7
$E(T_j)$ [day]	0.34	0.03	0.01	0.01	0.96	2.97	0.99

Based on the ergodic probability values of the semi-Markov process, p_j availability indicators K_{gt} and K_{gw} were calculated. For the analyzed exploitation process, states S_1, S_2, S_3, and S_4 were considered states of suitability, and states S_5, S_6, and S_7 were considered states of unsuitability. Availability indicator K_{gt} for the analyzed 7-state semi-Markov model was calculated as the sum of the conditional probabilities of the appropriate reliability states:

$$K_{gt} = P(A|\Omega) = \frac{P(A\cap\Omega)}{P(\Omega)} = \frac{p_1+p_2+p_3+p_4}{\sum_{j=1}^{7} p_j}, \tag{31}$$

where A = {S_1, S_2, S_3, S_4} is the set of suitability states, and Ω is the phase space of the model and A ∈ Ω.

Based on Eq. (31), the indicator value of K_{gt} = 0.9296 was obtained. Calculations of the internal availability indicator K_{gw} was made using the relation:

$$K_{gw} = P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{p_1+p_2+p_3+p_4}{p_1+p_2+p_3+p_4+p_7}, \tag{32}$$

where B is the set of operational states, excluding failure states S_5 and waiting for repair S_6.

The indicator value of K_{gw} = 0.9880 was obtained.

4.3. Limitations

Despite the valuable insights provided by the proposed methodology and results, several limitations should be considered when interpreting the findings of this study:

- **Data selection:** The analysis was limited to 50 military vehicles of the same model, used under similar conditions, which reduces technical variability but limits generalizability to other vehicle types or operational contexts.
- **Data sources:** Operational data were based on paper records, increasing the risk of incompleteness and errors; 20 vehicles were excluded due to missing data.
- **Model selection:** The suitability of Markov or semi-Markov models was statistically verified. Only 2 of 14 transitions met the exponential distribution assumption, necessitating the use of the semi-Markov model.
- **Deterministic assumptions:** The process was treated as stochastic, but some deterministic factors (e.g., periodic maintenance) exist. A higher share of such elements could limit the applicability of these models.

These limitations should be considered when interpreting and generalizing the findings. While the methodology is universal and can be applied to other technical systems, the numerical results obtained in this study may not be directly generalizable to different vehicle types or operational conditions. Further research using broader datasets and alternative data sources is recommended.

5. Comparison of the results

The comparison of the test results obtained using statistical and stochastic methods is given in Table 6 and Fig. 9.

The values of the availability indicators k_{gt} and K_{gt}, at the level of 0.92, do not meet the acceptable level availability indicators in the Polish Armed Forces, which have been set at the level of ≥ 0.95. The probability value of the ergodic state S_6 has the greatest impact on reducing the availability rate. This condition corresponds to a situation where the vehicle has broken down, is inoperable and requires repair. Still, due to the lack of technical or organizational abilities or the lack of spare parts, it is not in a state of repair (S_7). The main reasons for this situation are logistical delays related to the limited availability of spare parts and the lack of qualified technical personnel at a given time. As far as the properties of the device are concerned, an acceptable level of availability can be achieved by reducing the time the object stays in state S_6. This can be done by maintaining higher levels of spare parts and simplifying purchasing procedures. By improving the operation of the logistics system, it is possible to achieve availability levels k_{gt} and K_{gt} at a level close to the values of the indicators k_{gw} and K_{gw}. Technical availability indicators (internal) k_{gw} and K_{gw} strictly illustrate the device's properties, and for the examined case they reached values above 0.98.

Comparing the results obtained using two methods, differences in the values of indicators of staying in operational states were obtained in the range of -10.2% to 117.0%. The percentage differences between the models reached very similar (almost identical) values for the suitability states S_1, S_2, S_3, and S_4, where they differed by less than 3%. The greater differences in results were obtained for states of unsuitability S_6 (5.1%) and S_7 (10.2%). The relative differences in the probability values of remaining in State S_5 reached as much as 117.0%. Moreover, states S_5, S_6 , and S_7 being a set of unsuitability states, have reached positive differences. This high value is primarily due to the highly skewed (left-skewed) distribution of the data, with a significant predominance of short sojourn times in State S_5, alongside a few much larger values. In such cases, relative error indicators become highly sensitive to small differences between the results obtained from the two approaches, especially when the absolute values are small. The differences obtained in interpretation using statistical and stochastic methods are because availability coefficients k_{gt} and k_{gw} reflect the results for the test sample. The value of the availability indicators K_{gt}, i K_{gw} is theoretical and represents the probability at infinity to which the availability factor is predicted to tend.

Table 6. The comparison of the test results obtained by statistical and stochastic methods.

Statistical			**Stochastic**			**Differences**	
Measure	Indication	Value	Measure	Indication	Value	Value	[%]
Availability indicators	k_{gt}	0.9340	Availability indicators	K_{gt}	0.9296	-0.0044	-0.5%
	k_{gw}	0.9890		K_{gw}	0.9880	-0.0006	-0.1%
Percentage of the duration of each operating condition S_i for the test sample	S_1 [%]	7.59	Ergodic probabilities of the SMP	p_1 [%]	7.46	-0.1300	-1.7%
	S_2 [%]	0.07		p_2 [%]	0.07	0.0000	0.0%
	S_3 [%]	84.82		p_3 [%]	84.44	-0.3800	-0.4%
	S_4 [%]	0.97		p_4 [%]	0.98	0.0100	1.0%
	S_5 [%]	1.35		p_5 [%]	1.33	-0.0200	-1.5%
	S_6 [%]	4.11		p_6 [%]	4.58	0.4700	11.4%
	S_7 [%]	1.08		p_7 [%]	1.13	0.0500	4.6%

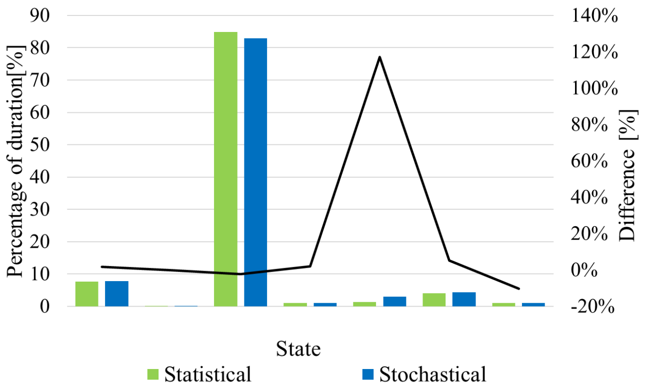


Fig. 9. Percentage differences of state time durations using statistical and stochastic methods.

6. Conclusions and future works

The reliability and availability of technical objects and systems are extremely important and often analyzed in the source literature. This study presents a novel approach to evaluating and optimizing the reliability and availability of technical systems through the implementation and comparison of statistical methods and semi-Markov processes. By applying these methods to empirical failure data from a military transport system, this research provides a significant advancement in understanding and addressing the complexities of operational processes in technical systems.

Based on the statistical analysis of technical object failures, reliability and availability indicators were estimated, which enable monitoring of the effectiveness of the transport system. The tested trucks have high reliability, which can be identified by a relatively long time of failure-free operation, i.e. MTBF = 268.11 [days]. The reduction in system reliability indicators is mainly affected by the MTTS indicator at the level of 11.8 [days]. The remaining 2 indicators, i.e. MTTD and MTTR have a significantly smaller impact on vehicle reliability.

The model creation algorithm using the Markov and semi-Markov processes proposed in this paper was implemented into the truck operation process. Based on the empirical operation process analysis, a 7-state phase space was identified. Based on the verification of the exponential distributions of time characteristics using the Kolmogorov-Smirnov test, the need to use the semi-Markov process was confirmed. The value of the matrix of conditional probabilities of interstate transitions of the inserted Markov chain was estimated based on real data. Then, by solving the matrix equation, the values of ergodic probabilities were calculated to determine accessibility indicators.

To sum up, statistical methods only allow for assessing system availability without indicating possible directions for its improvement. However, it has been shown that reducing of state time S_6 by 40% improves the k_{gt} indicator value by 0.017 to reach the value of 0.951.

In turn, using the semi-Markov processes enables accurate mapping of the actual course of state changes in the identified phase space. The semi-Markov model is useful in the context of process analysis in terms of identifying the causes of reduced availability rates. In accordance with the developed and implemented 7-state model in the analysis of truck operation processes, it has been shown that there is a significant effect on the expected time of staying in state S_6 (waiting for spare parts), which is a key factor in the significant drop in the value of indicator K_{gt} compared to the value of indicator K_{gw}.

It should be emphasized that the obtained reliability indicators values for both methods indicate a properly planned and implemented operation process from the standpoint of technical availability of equipment operated in the transport system. Convergent results obtained using statistical and stochastic methods prove the correctness of the proposed methodology and calculations.

This study contributes to the field of reliability and availability modeling by providing a validated, universal framework that combines statistical and stochastic methods. The proposed semi-Markov model not only estimates performance indicators but also identifies key areas for process improvement, bridging the gap between theory and application. This research lays the groundwork for optimizing system performance and readiness in diverse technical domains by addressing critical factors such as logistical delays. A major contribution of this research is the introduction of a 7-state semi-Markov model, which provides a detailed representation of operational states, including critical phases such as downtime due to logistical delays. This model overcomes the limitations of traditional Markov models, which assume exponential state distributions, by accommodating non-exponential behaviors. The methodology introduced in this study is not limited to the military transport system. Its adaptability makes it applicable to other high-stakes technical systems, such as healthcare, aerospace, and public transportation, where reliability and availability are critical.

The obtained results constitute the basis for further research on the reliability characteristics of the operation process. The next step may be to apply the proposed approach to RAM analysis. Moreover, the authors see the possibility of implementing and consolidating the information on failures with operational management software. The methodology proposed in this paper is a universal tool that can be used to analyze and assess any technical object's operation. Furthermore, future work will focus on validating the proposed methodology using additional datasets from different vehicle types or operational contexts, in order to further assess and improve the generalizability of the results. The flexibility of the proposed methodology makes it suitable for other complex systems, including urban infrastructure, energy networks, and industrial manufacturing processes. On this basis, it is possible to develop an effective repair and maintenance strategy, the implementation of which would positively impact reducing repair time and increasing the system's efficiency.

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