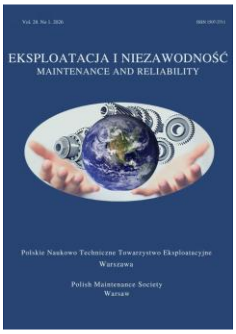

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Reliability allocation of mechanical transmission system considering motion accuracy stability and G-RCF model

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Highlights

- A reliability allocation method for mechanical transmission system is proposed.
- A G-RCF model for meta-action unit is established.
- A mathematical model for reliability optimization allocation is established.

Abstract

The reliability of mechanical transmission systems and the stability of motion accuracy have a significant impact on the performance of CNC equipment. Existing studies have rarely established accurate reliability optimization allocation models. Therefore, a reliability allocation method that incorporates motion accuracy stability and a generalized reliability-cost function (G-RCF) is proposed. Firstly, the mechanical transmission system is decomposed by using the meta-action theory to obtain the meta-action units (MAUs). The motion accuracy stability, structural complexity, and comprehensive maintenance cost factors of MAUs are analyzed, and the traditional reliability-cost function is modified to obtain the generalized reliability-cost function. Then, taking the minimum generalized cost as target, a mathematical model for optimizing the allocation of system reliability is established. Using the intelligent algorithm to obtained the reliability design values of each MAU. Finally, the effectiveness of the proposed method is demonstrated through an engineering example.

Keywords
mechanical transmission system, reliability allocation, motion accuracy stability, meta-action, generalized reliability-cost function (G-RCF)

1. Introduction

The mechanical transmission system is a critical component of high-end equipment such as CNC machine tools, and its reliability significantly impacts the comprehensive performance of CNC equipment. Therefore, it is of great importance to conduct accurate fault diagnosis and reliability assessment [1-3]. Therefore, to ensure the reliability level of mechanical transmission systems, the reliability design must be implemented. Reliability design primarily includes reliability allocation and reliability prediction [4-6]. Reliability allocation involves rationally allocating the system's reliability design requirements to each subsystem, thereby guaranteeing the reliability level of the entire machine system [7].

Regarding the reliability allocation of mechanical systems, scholars have conducted in-depth research. For instance, Yang et al. [8] utilized the FMECA method to modify the criticality factors of subsystems and combined objective data to obtain the reliability allocation factors, achieving reliability allocation for CNC lathes. Yu et al. [9] systematically analyzed the

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influencing factors of reliability costs and the severity of failures in mechanical systems, and proposing a novel fuzzy reliability allocation method based on multi-criteria decision method (MCDM). Cheng et al. [10] improved the traditional FOO method by using fuzzy allocation method to achieve the flexible allocation of system reliability. Du et al. [11] employed a fuzzy evaluation method to determine the relative importance of components and integrated their failure impacts to realize reliability allocation for remanufactured CNC lathes. Cao et al. [12] comprehensively considered common cause fault factors in analyzing the severity of component failures, which improved the rationality of system reliability allocation. Bai et al. [13] quantified epistemic uncertainty by using the Dempster-Shafer (D-S) evidence theory and proposed a reliability allocation method for multi-state systems, demonstrating higher efficiency and accuracy compared to traditional approaches. Cheng et al. [14] introduced a new reliability allocation method for CNC machine tools by integrating trapezoidal intuitionistic fuzzy numbers with the TOPSIS method. Comparing with the traditional Analytic Hierarchy Process (AHP), the reliability allocation results of this method are more reasonable. Du et al. [15] achieved reliability allocation for CNC gear hobbing machines by deriving remanufacturing coefficients for subsystems through a comprehensive remanufacturing evaluation method. Cheng et al. [16] proposed a novel reliability allocation method for machine tools by considering multiple influencing factors and combining subjective and objective weights, effectively addressing uncertainties and ambiguities in the allocation process. However, existing reliability allocation methods generally assume independence between influencing factors and subsystems, leading to reduced accuracy in allocation results. Therefore, Gu et al. [17] developed a reliability allocation method that accounts for correlations among influencing factors and subsystem failures, significantly reducing the complexity of reliability design for CNC machine tools.

The above research indicates that current reliability analysis methods for mechanical systems still follow those used for electronic products, employing a structural decomposition approach of "whole machine-component-part." However, the mechanical systems fundamentally differ from electronic products in their functional formation processes. Mechanical systems achieve specified functions and performance through the mutual motion of components. Ignoring these characteristics during reliability allocation for mechanical systems can compromise the accuracy of the allocation. To address this issue, Li and Yu et al. [18-20] systematically analyzed the unique features of mechanical systems to propose the meta-action theory, and applied it to the reliability analysis of CNC machine tools. Subsequently, numerous scholars have conducted the related research. Chen et al. [21] introduced a multi-criteria decision-making reliability allocation method by integrating DEMATEL, ULOWA, and PROMETHEE II methods based on the meta-action theory. In applying the meta-action theory to reliability allocation, it is assumed that adjacent meta-actions are independent. However, there are complex interactions between meta-actions in mechanical systems. In order to improve the accuracy of reliability allocation, Li et al. [22] analyzed the impact of motion stability in meta-action units on overall machine performance and proposed a reliability allocation method for mechanical transmission systems that considers performance stability. Chen [23] analyzed the failure correlation between meta-actions by using the Copula theory and proposed a new reliability method. Traditional reliability allocation method heavily relies on the expert knowledge, leading to insufficient objectivity in decision-making. To overcome this problem, Zhang et al. [24] proposed a hybrid reliability allocation method combining meta-action theory, MCDM, and MOO methods. Li et al. [25] proposed a reliability allocation method that combines qualitative and quantitative data analysis from the perspective of the entire product lifecycle, taking into account factors such as part recycling and performance stability. This method improves the objectivity and rationality of reliability allocation results.

The Meta-action unit(MAU) is the smallest motion unit for the mechanical transmission system, which can be designed and analyzed independently. Taking the MAU as the analysis object for reliability design and analysis conforms to the design and manufacturing laws of the mechanical transmission system and can better ensure the reliability of it. Therefore, the meta-action theory has obvious advantages in the research of reliability allocation of the mechanical transmission system, and providing a more reasonable and accurate method for the reliability design of it. However, current reliability allocation processes based on meta-action theory predominantly employ cost-unconstrained methods, meaning that comprehensive cost constraints are not imposed during reliability allocation. Although the influence of reliability cost factors is considered in the allocation process, there has been no pursuit of reliability allocation with the goal of minimizing costs. Product cost is a primary concern for enterprises. By establishing the reliability cost function model and developing the cost-constrained reliability allocation method, optimal reliability allocation under minimum cost conditions can be achieved for systems [26]

Through the above analysis, extensive research achievements have been obtained in reliability allocation for mechanical transmission systems. However, existing studies still exhibit significant shortcomings:

(1) In traditional reliability-cost function models, only reliability improvement costs are considered, while the impacts of maintainability and subsystem structural complexity on comprehensive costs are neglected.

(2) The motion accuracy stability of mechanical transmission systems is a critical performance indicator. Traditional methods in reliability analysis fail to account for the impact of this factor on comprehensive costs.

In the actual working process of the mechanical transmission system, maintenance and repair are important means to ensure its reliability. During the maintenance and repair process, operations such as disassembly and assembly of the system are required. The maintainability of the system and the complexity of assembly determine the cost of maintenance and repair. Considering maintainability and structural complexity can more accurately reflect the comprehensive reliability cost of the mechanical transmission system.

The stability of the motion accuracy of the mechanical transmission system determines the performance stability and reliability of the equipment system. Therefore, during the reliability allocation of the mechanical transmission system, the influence of the stability of the unit's motion accuracy needs to be considered. If the stability of the unit's motion accuracy has a significant impact on the system performance stability, a higher reliability should be allocated. However, allocating a higher reliability leads to a higher comprehensive cost. Therefore, it is necessary to comprehensively analyze the impact of the stability of motion accuracy and reliability on the comprehensive cost.

To address the above issues, a reliability allocation method for mechanical transmission systems is proposed in this paper, that incorporates motion accuracy stability and a generalized reliability-cost function（G-RCF）. First, based on the meta-action theory, the mechanical transmission system is decomposed structurally into MAUs. By comprehensively analyzing the maintainability and structural complexity of these MAUs, maintenance correction coefficients and complexity correction coefficients are derived respectively. Next, the motion transmission process of the mechanical transmission system is rigorously examined to establish a motion accuracy stability model, through which motion accuracy stability correction coefficients for each MAU are systematically calculated. Subsequently, expanding upon the traditional reliability-cost function (RCF) model, the maintenance correction coefficients, complexity correction coefficients, and motion accuracy stability correction coefficients are integrated to formulate a generalized reliability-cost function(G-RCF) model specifically tailored for MAUs. Finally, targeting the minimization of generalized costs, a mathematical model for reliability optimization allocation of the mechanical transmission system is established, achieving rational allocation of the system reliability. The research flowchart is illustrated in Figure 1.

The remainder of this paper is organized as follows. the meta-action theory and the traditional RCF model are introduced in Section2. In Section 3, the motion accuracy stability coefficients, maintenance cost coefficients, and comprehensive complexity coefficients of the MAUs are calculated, and a G-RCF model for the MAUs is established based on these parameters. A reliability optimization allocation model for the mechanical transmission system is proposed in Section 4. An application case is analyzed in Section 5. Finally, the conclusion and future works are presented in Section 6.

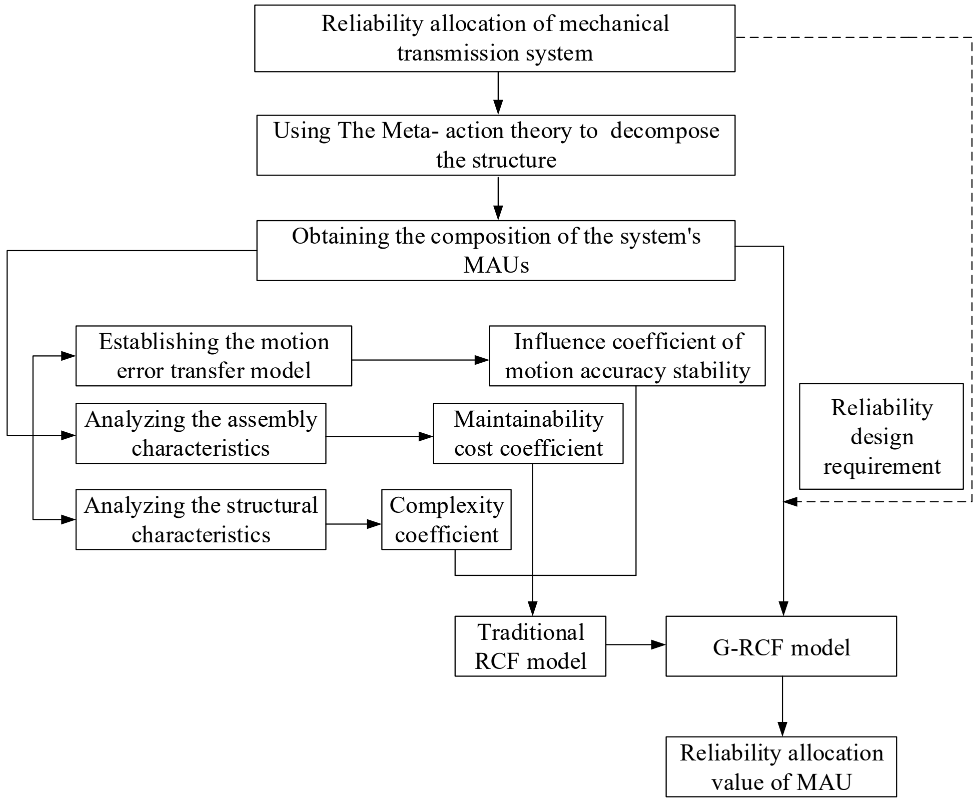

Fig. 1. The research flowchart.

2. Preliminaries

2.1. Meta-action theory

Li and Yu et al. proposed a meta-action theory that is more applicable to mechanical products by systematically analyzing the characteristics and functional formation process of mechanical systems and comparing them with traditional electronic products [18-20]. In decomposing mechanical system structures, this theory defines the most fundamental motion as the minimum unit, i.e. Meta-action Unit (MAU). And this theory believes that the functionality and performance of MAUs form the foundation of system functionality and performance. This theory has been widely applied in various fields of mechanical systems including precision [27], reliability [28-31] and maintainability [32].

The MAU requires the coordination of multiple parts to complete the specified motion, as shown in Figure 2 (a), which is a typical MAU structural model.

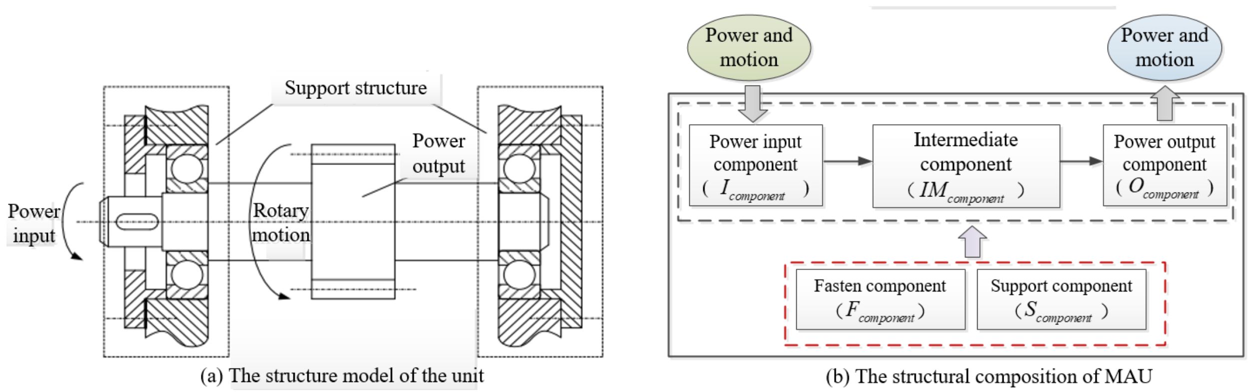

Fig. 2. The structural of the typical MAU.

The structural composition of the MAU is shown in Fig. 2(b), including the power input component ($I_{component}$), intermediate component ($IM_{component}$), support component ($S_{component}$), fasten component ($F_{component}$), and power output component ($O_{component}$). Among these, the $I_{component}$ receives motion and power, then transmits them through the $IM_{component}$ to the

$O_{component}$. The $F_{component}$ and $S_{component}$ collectively ensure the transmission of power and motion within the unit。

For mechanical transmission systems in high-end equipment, the power and motion are typically provided by power sources such as servo motors. The power and motion generated by the power source are transmitted through MAUs to the executive components of the system. Consequently, the composition of the mechanical transmission system can be derived from the meta-action theory, as illustrated in Fig. 3.

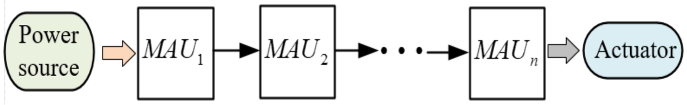

Fig. 3. The composition of mechanical transmission system.

2.2. Reliability-cost function (RCF)

The RCF is the basis for reliability allocation, where a rational and accurate model of this function enables the establishment of precise mapping relationships between system reliability and cost under various influencing factors. Through analysis of existing RCF models, the exponential model is considered effective for accurately characterizing the relationship between reliability and cost in system design. Specifically, the RCF for the *i*-th system can be expressed as [33-34]:

$$C_i(R_i) = \beta(i) \cdot exp\left(\alpha(i) \cdot \frac{R_i - R_{i,min}}{R_{i,max} - R_i}\right) \tag{1}$$

Where, $C_i$represents the comprehensive cost of the i-th system; $\alpha_i$denotes the comprehensive complexity coefficient of the i-th system, $0 < \alpha_i < 1$. And the more complex the unit structure, the higher the production, manufacturing and assembly costs will be, resulting in a higher overall cost; $\beta(i)$ indicates the initial cost of the *i*-th system; R_i stands for the reliability design value of the i-th system, where $R_{i,\ min}$ and $R_{i,\ max}$ respectively represent the existing design reliability and the maximum achievable reliability under current conditions for the *i*-th system.

3. Generalized reliability-cost function (G-RCF) of MAU

3.1. Impact analysis on motion accuracy stability of MAU

The stability of motion accuracy in mechanical transmission systems determines the comprehensive performance level of integrated machinery systems. Structural decomposition analysis reveals that motion and power transmission within such systems are achieved through constituent MAUs. Consequently, the system's motion accuracy stability is governed by the stability of motion performance in individual MAUs.

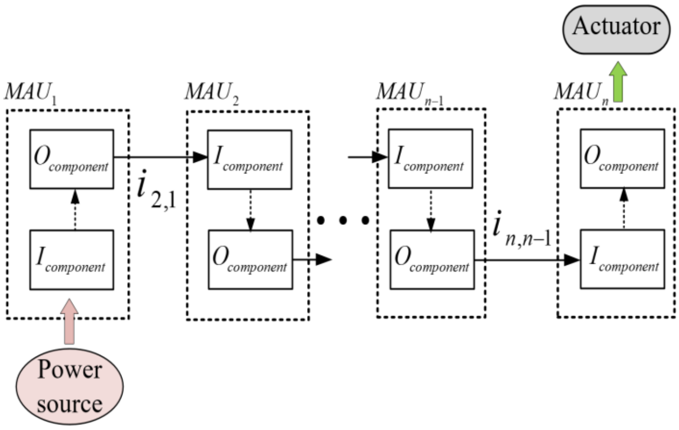

Fig. 4. The motion transmission diagram of mechanical transmission system.

Figure 4 illustrates the transmission process of motion and power between MAUs within the mechanical transmission system. The system comprises *n* MAUs, where the transmission ratio between $O_{component}$ of a preceding MAU and the $I_{component}$ of the subsequent MAU is denoted as $i_{n,n-1}$. Based on the interconnection relationships between adjacent MAUs, the integrated motion error model of the system is derived [25]:

$$\theta_{MC} = \frac{\theta_1}{\prod_{k=2}^{n-1} i_{k,k-1}} + \cdots + \frac{\theta_m}{\prod_{k=m}^{n-1} i_{k+1,k}} + \cdots + \frac{\theta_{n-1}}{i_{n-1,n}} + \theta_n \tag{2}$$

Where, θ_{MC} denotes the motion error of the execution component in the mechanical transmission system, and θ_m represents the motion error of the $O_{component}$ in the *m*-th MAU.

As derived from Eq. (2), the motion error of the execution component in a mechanical transmission system is influenced by the motion errors of all MAUs within the system, with each unit contributing a distinct influence coefficient. Specifically, the influence coefficient of the motion error from the *m*-th MAU on the system's execution component motion error is expressed as:

$$Se(m) = \frac{\partial(\theta_{MC})}{\partial(\theta_m)} \tag{3}$$

Where, $Se(m)$ denotes the influence coefficient of the motion error for the *m*-th MAU. Consequently, the influence coefficient of the *m*-th MAU on the motion accuracy stability of the system is derived as:

$$F(m) = 0.6^{\left(log_{10}\left(\frac{1}{(Se(m))}\right)\right)} \tag{4}$$

Through the aforementioned analysis and calculations, the influence of motion errors in the $O_{component}$ of each MAU on the system's execution component has been obtained. However, during motion transmission between MAUs, power is typically transferred through gear pairs, where the upstream unit's $O_{component}$ acts as the driving gear and the downstream unit's $I_{component}$ serves as the driven gear. Consequently, for the MAUs located in the central region of the system, motion errors in both their $I_{component}$ and $O_{component}$ affect the system's motion accuracy. When calculating the motion accuracy stability coefficient of such units, it is necessary to comprehensively consider the combined effects of the $I_{component}$ and $O_{component}$. For a mechanical transmission system containing n MAUs, the composite influence degree of the *i*-th MAU on the system's motion accuracy stability is formulated as:

$$\begin{cases} \zeta(i) = F(i), i = 1 or i = n \\ \zeta(i) = (F(i) + F(m+1)), 2 \leq i \leq n-1 \end{cases} \tag{5}$$

The composite influence coefficient of motion accuracy stability for all MAUs in the mechanical transmission system is formulated as:

$$\xi(i) = \frac{\zeta(i)}{max[\zeta(i)]} \tag{6}$$

Where, $\xi(i)$ represents the composite influence coefficient of motion accuracy stability for the *i*-th MAU, and $0 < \xi(i) \leq 1$.

In summary, the comprehensive impact coefficient of the stability of the motion accuracy of the MAU reflects the degree of influence of the motion error of the MAU on the motion accuracy of the system execution component. Therefore, in the process of reliability design and analysis, for the MAUs with a large comprehensive impact coefficient on the stability of motion accuracy, higher reliability indicators should be allocated. This can reduce the impact of MAUs on the stability of system motion accuracy and improve the stability of mechanical transmission system motion accuracy.

3.2. Structural Complexity Analysis of MAU

The various components inside the meta action unit ensure the functionality and performance of the meta action through complex assembly and mutual motion relationships. Within the unit, the contact surfaces between components are categorized into fixed interfaces and moving interfaces, with the motion characteristics primarily realized through the moving interfaces. As indicated by the structural configuration of MAUs, the components responsible for transmitting motion and power include $I_{component}$, $IM_{component}$, and $O_{component}$. During power transmission, the greater the number of moving interfaces, the more intricate the motion transmission path becomes. Therefore, the structural complexity of the i-th MAU can be quantitatively analyzed by integrating the number of moving interfaces with the quantities of the $I_{component}$, $IM_{component}$, and $O_{component}$. The computational model is as follows:

$$Co(i) = \frac{n_{djs}(i)}{n_{in}(i) + n_{out}(i) + n_{mid}(i)} \tag{7}$$

Where, $n_{djs}(i)$ denotes the number of moving interfaces between components in the i-th MAU, $n_{in}(i)$ represents the quantity of $I_{component}$ in the i-th MAU, $n_{out}(i)$ indicates the count of $O_{component}$ in the i-th MAU, $n_{mid}(i)$corresponds to the number of $IM_{component}$ in the i-th MAU.

If the mechanical transmission system contains *n* MAUs, the comprehensive structural complexity coefficient of the *i*-th MAU is:

$$\alpha(i) = \frac{Co(i)}{min[Co(i)]} \tag{8}$$

Where, $\alpha(i)$denotes the structural complexity coefficient of the *i*-th MAU, and$\alpha(i) \geq 1$.

3.3. Comprehensive Maintenance Cost Analysis of MAU

In the full life cycle of the mechanical transmission system, the comprehensive cost factors include design cost and maintenance cost. As the minimal functional unit, the MAU can be independently designed and maintained. Being a typical mechanical unit, the maintenance cost of the MAU primarily refers to the disassembly and assembly cost, which encompass economic costs, time expenditure, labor resources, and equipment usage during the unit's disassembly and assembly processes. Therefore, the maintenance cost coefficient of the MAU is quantitatively calculated using the comprehensive assembly complexity.

Table.1. The APA complexity of component.

Attribute	Description	Average complexity
Size/ Length （L）	L >15mm	0.75
	5mm< L≤15mm	0.8
	L≤5mm	1.0
Thickness/ Diameter （D）	D>10mm	0.3
	5mm< D≤10mm	0.5
	D≤5mm	1.0
Symmetry （$\gamma+\lambda$）	$\gamma+\lambda$ <360°	0.5
	360°≤$\gamma+\lambda$ <540°	0.7
	540°≤$\gamma+\lambda$ <720°	0.9
	$\gamma+\lambda$ =720°	1.0
Mass （K）	K<2.5kg	0.3
	2.5kg≤K<10kg	0.6
	K≥10kg	1.0
Difficulty of grasping	No tools required	0.6
	Using universal tool	0.8
	Using professional tool	1.0
Auxiliary motion	Without auxiliary	0.5
	One hand	0.7
	Both hands	1.0

Table.2. The AOA complexity of component.

Attribute	Description	Average complexity
Difficulty in positioning	Easy	0. 5
	Fairly difficult	0.75
	Difficulty	1.0
Assembly resistance	Smaller	0.3
	Larger	0.5
	Great	1.0
Line of sight	No occlusion	0.35
	Partial occlusion	0.73
	Completely occlusion	1.0
Tightening method	Bend	0.35
	Rivet bond	0.5
	Screw thread	0.85
	Plastic deformation	1.0
Detection method	Visualization	0.3
	Simple tools	0.65
	Complex equipment	1.0
Job requirement	No requirement	0.3
	Job rotation	0.7
	Specialized position	1.0
Auxiliary equipment	No tools required	0.3
	Simple tools	0.56
	Professional tools	1.0

Success rate of one assembly (P)	P >95%	0.3
	80%< P≤95%	0.57
	65%< P≤80%	0.85
	P≤65%	1.0

The comprehensive assembly complexity of mechanical systems includes the complexity of the assembly pre-processing attributes (APA) and the complexity of the assembly operation attributes (AOA) of components [33,36]. The APA complexity and AOA complexity of parts are primarily derived from accumulated empirical data on part assembly processes. Scholars have summarized and analyzed this accumulated empirical data to determine the average complexity levels of APA and AOA for components, as shown in Table 1 and Table 2.

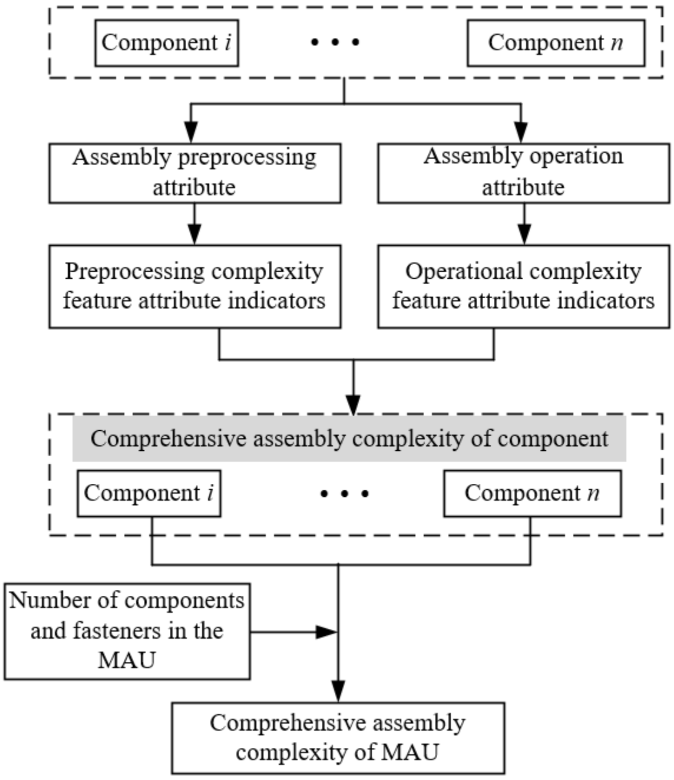

Fig. 5. Calculation of the comprehensive assembly complexity of MAU

The calculation process for the comprehensive assembly complexity of MAU is illustrated in Figure 5. Based on the data from Table 1 and Table 2, the APA and AOA complexity values of each feature for the i-th part (including $I_{component}$, $O_{component}$, $IM_{component}$, and $S_{component}$) in the unit are obtained. Subsequently, the average APA and AOA complexity values for the i-th part are calculated as follows:

$$\begin{cases} A_P^i = \frac{1}{p} \cdot \sum_{j=1}^{p} A_{P,j}^i \\ A_O^i = \frac{1}{q} \cdot \sum_{j=1}^{p} A_{O,j}^i \end{cases} \tag{9}$$

Where, $A_{P,j}^i$ and $A_{O,j}^i$ denote the APA complexity and AOA complexity of the j-th feature of the part, respectively.

The comprehensive assembly complexity of the i-th component in the MAU is calculated as follows:

$$A(i) = A_P^i + A_O^i \tag{10}$$

If the number of $I_{component}$, $O_{component}$, $IM_{component}$, and $S_{component}$ in the MAU is m, the assembly complexity index of the unit is derived based on the comprehensive assembly complexity of each part, as follows:

$$IA_{MAU} = \frac{1}{m} \sum_{i=1}^{m} A(i) \tag{11}$$

Based on the obtained assembly complexity index of the unit, the total number of unit components, and the total number of fasteners, the comprehensive assembly complexity of the unit is calculated as follows:

$$C(MAU) = N_p \cdot IA_{MAU} \cdot log\,2\left(N_f\right) \tag{12}$$

Finally, based on the comprehensive assembly complexity of each unit in the mechanical transmission system, the comprehensive maintenance cost coefficient of each unit is calculated as follows:

$$\lambda(i) = \frac{log_{10}(C(i))}{min[log_{10}(C(i))]} \tag{13}$$

Where, $\lambda(i)$ denote the comprehensive maintenance cost coefficient of the i-th MAU, and $\lambda(i) \geq 1$. $C(i)$represent the comprehensive assembly complexity of the i-th MAU.

3.4. Generalized reliability-cost function (G-RCF) model

In the RCF model shown in Eq. (1), $\beta(i)$denotes the initial cost of the i-th MAU. However, determining the initial cost for each MAU is challenging. Therefore, the revised RCF of the MAU is obtained by simplifying the conventional RCF model:

$$G_i(R_i) = \frac{C_i(R_i)}{\beta(i)} = exp\left(\alpha(i) \cdot \frac{R_i - R_{i,min}}{R_{i,max} - R_i}\right) \tag{14}$$

Where, $G_i(R_i)$ denotes the cost coefficient of the i-th MAU when the reliability is R_i, and $G_i(R_i) \geq 1$. The revised RCF model eliminates the influence of initial costs and simplifies the computational process.

To further account for the influence of the comprehensive maintenance cost and motion accuracy stability of MAUs on their reliability, the G-RCF is refined by incorporating the comprehensive maintenance cost coefficient and the motion accuracy stability coefficient, thereby establishing the G-RCF model of MAU:

$$\tilde{G}_i(R_i) = \frac{1}{\xi(i)} \cdot \lambda(i) \cdot exp\left(\alpha(i) \cdot \frac{R_i - R_{i,min}}{R_{i,max} - R_i}\right) \quad (15)$$

Where, $\tilde{G}_i(R_i)$denotes the generalized cost coefficient of the *i*-th MAU when the reliability is R_i; $\xi(i)$ denotes the motion accuracy stability influence coefficient of the *i*-th MAU in the mechanical transmission system, and $\lambda(i)$ represents the comprehensive maintenance cost coefficient of the *i*-th MAU.

In the process of studying the G-RCF in this article, we assume that after repairing and maintaining the MAU, the function and performance of the unit will be restored to normal level.

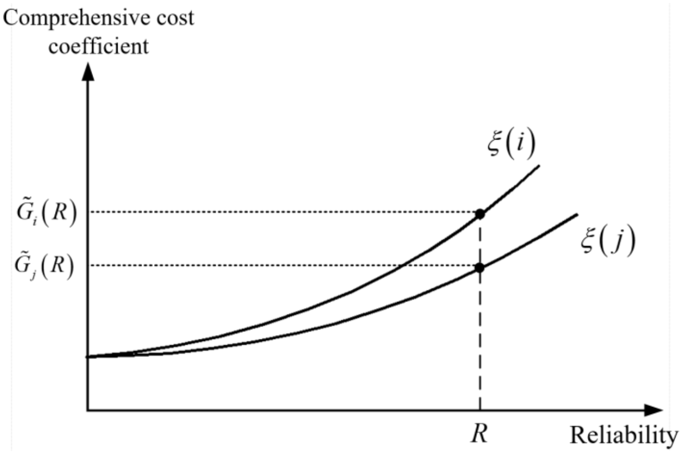

Fig. 6. The relationship between $\xi(i)$,$\tilde{G}_i(R)$ and *R*.

To further analyze the impact of the precision stability influence coefficient of MAUs on their reliability and generalized comprehensive cost, the relationship among MAU's reliability, comprehensive cost coefficient, and motion precision stability influence coefficient are calculated using the Eq. (15). The results are illustrated in Figure 6.

In the figure, the comprehensive motion precision stability influence coefficients of the *i*-th and *j*-th MAU are denoted as $\xi(i)$ and $\xi(j)$, respectively, and $\xi(i) < \xi(j)$. This indicates that the *j*-th MAU has a relatively large impact on the motion accuracy stability of the mechanical transmission system. Therefore, a relatively high reliability should be allocated. However, the reliability optimization allocation model presented in Eq. (14) prioritizes minimizing the comprehensive cost as the basis for reliability distribution. To address this, by taking the reciprocals of the comprehensive motion precision stability influence coefficients, the G-RCF model for MAUs is obtained, as shown in Eq. (15). By incorporating $\xi(i)$ and $\xi(j)$, it is demonstrated that when the system reliability is R, the comprehensive cost coefficient of the *i*-th unit exceeds that of *j*-th unit. This approach ensures that the *j*-th MAU is allocated a higher reliability level, effectively translating the influence of motion precision stability into total cost coefficient relationships.

4. Reliability optimization allocation of mechanical transmission system

If the reliability design value of the mechanical transmission system is $[R_S^*]$, and the system comprises *n* MAUs. Then, based on the G-RCF model for MAUs is shown in Eq. (15), the mathematical model for reliability optimization allocation of the system can be derived as follows:

$$\min[\tilde{G}_S] = \sum\nolimits_{i=1}^{n}[\tilde{G}_i(R_i)]$$
$$= \sum_{i=1}^{n} \frac{1}{\xi(i)} \cdot \lambda(i) \cdot exp\left(\alpha(i) \cdot \frac{R_i - R_{i,min}}{R_{i,max} - R_i}\right)$$
$$\text{s.t.}\begin{cases} R_S = \prod\nolimits_{i=1}^{n} R_i \geq [R_S^*] \\ R_{i,min} \leq R_i \leq R_{i,max}, i = 1,2,3,\dots n \end{cases} \quad (16)$$

Where, $[\tilde{G}_S]$ denotes the comprehensive cost coefficient of the system, and R_S represents the calculated reliability value of the system.

Eq. (16) represents a typical single-objective optimization mathematical model, which can be well solved using mature swarm intelligence algorithms such as multi-island genetic algorithm method and particle swarm optimization method.

5. Case study

5.1. Reliability allocation of the NC rotary workbench system

The NC rotary workbench system is one of the critical functional components in high-end CNC equipment such as CNC grinding machine and CNC machining center. Its reliability significantly impacts the functionality of the entire CNC equipment system. Reasonable design of the reliability of the NC rotary workbench system can effectively ensure the functionality of the entire system. Taking a specific model of the NC rotary workbench system as an example, its designed reliability is R_S=0.85, and the mean time between failures (MTBF) is 1500*h*, the positioning precision is 30". And the 3D model of the system is shown in Figure. 7.

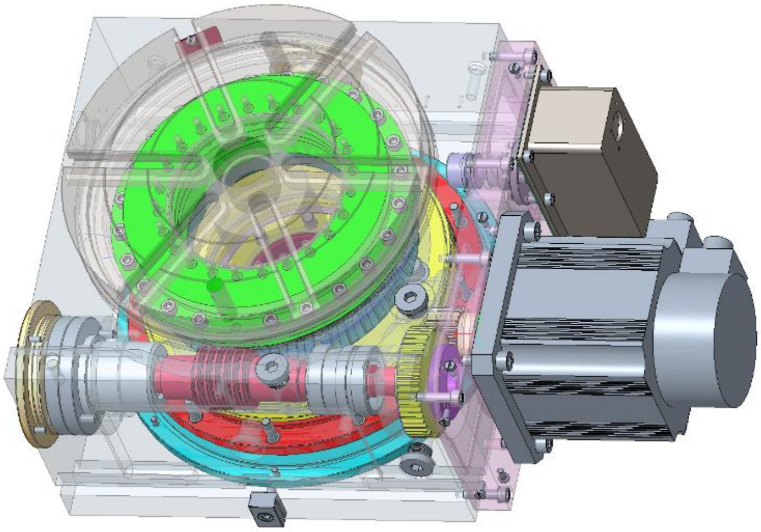

Fig. 7. The 3D model of the system.

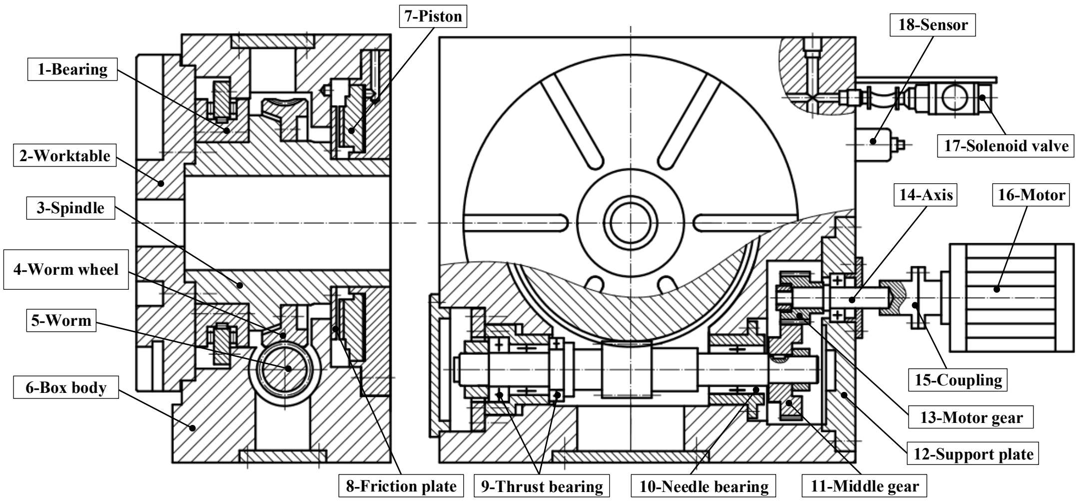

Fig. 8. Structure composition of NC rotary workbench system.

The mechanical transmission system primarily consists of the box body, motor gear, input gear, worm, worm wheel, worktable, and other components, as illustrated in Figure 8[25].

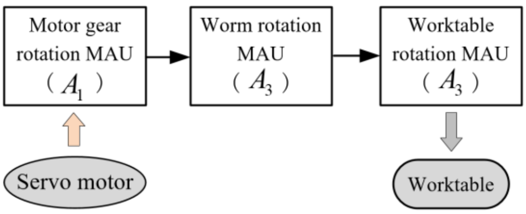

Fig. 9. Structural decomposition of NC rotary workbench system.

By applying the meta-action theory and based on the structural composition and functional components of the NC rotary workbench system, the system is decomposed into three MAUs, i.e the motor gear rotation MAU (A_1), the worm rotation MAU (A_2), and the worktable rotation MAU (A_3), as illustrated in Figure 9.

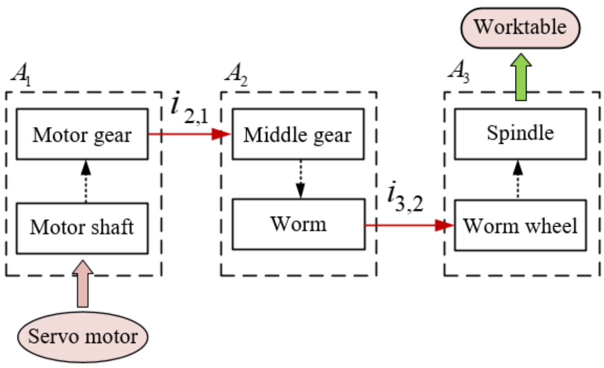

Fig. 10. The motion transmission diagram of NC rotary workbench system.

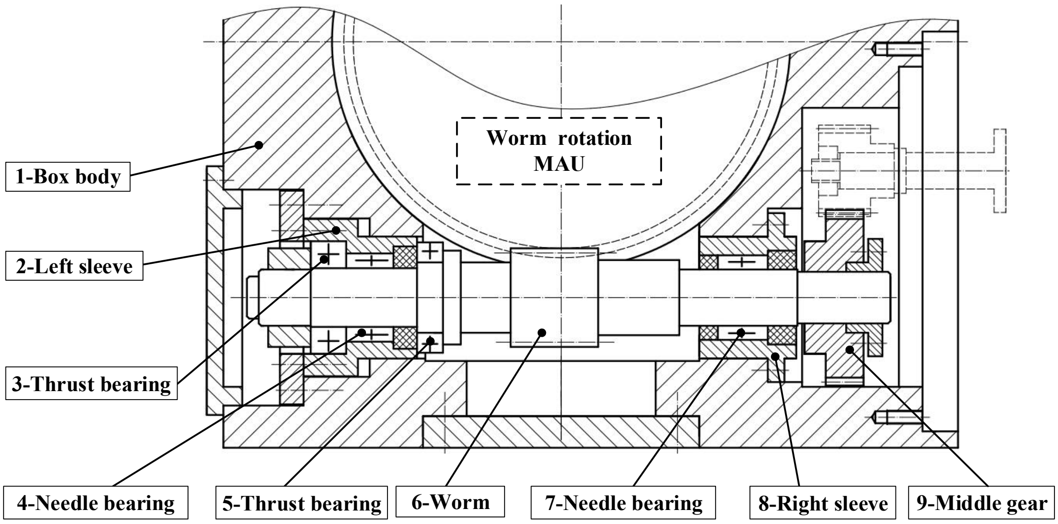

Fig. 11. The structure of worm rotation MAU.

(1) Calculating the comprehensive influence coefficient of motion accuracy stability

The power transmission process of the NC rotary workbench system is shown in Figure 10. Establishing the worktable rotation error according to Eq. (4):

$$\theta_{RTS} = \frac{\theta_{A1}}{i_{2,1} \cdot i_{3,2}} + \frac{\theta_{A2}}{i_{3,2}} + \theta_{A3} \tag{17}$$

Where, θ_{RTS} denotes the worktable rotation error; $i_{2,1}$ represents the transmission ratio of the gear pair, and $i_{2,1} = 2$; $i_{3.2}$represents the transmission ratio of the worm gear pair , and $i_{3.2} = 80$; θ_{A1}, θ_{A2} andθ_{A3} represent the motion errors of the motor gear MAU, worm rotation MAU, and worktable rotation MAU, respectively.

The motion accuracy stability influence coefficients of the three MAUs are obtained respectively using the calculation methods of Eq. (3-6)

$$\begin{cases} \xi(A_1) = 0.32 \\ \xi(A_2) = 0.38 \\ \xi(A_3) = 1.0 \end{cases} \tag{18}$$

(2) Calculating the structure complexity coefficient of the unit

The structural complexity coefficients of each MAU are calculated using the methodology outlined in Section 2.2. Taking the A_2 as an example, its structural composition is shown in Figure 11. Here, the $I_{component}$ and $O_{component}$ are designated as 12-input gear and 8-worm, respectively, and the $IM_{component}$ including 3-left thrust bearing, 5-left shaft sleeve, 6-left needle roller bearing, 7-right thrust bearing, and 9-right shaft sleeve. By applying Eq. (7), the structural complexity of this unit is calculated as $Co(A_2) = 0.63$. Using the same method, the complexities of the other units are determined as $Co(A_1) = 0.33$ and $Co(A_3) = 0.75$. Subsequently, the structural complexity coefficients for all units are derived through Eq. (8)

$$\begin{cases} \alpha(A_1) = 1.0 \\ \alpha(A_2) = 1.91 \\ \alpha(A_3) = 2.27 \end{cases} \tag{19}$$

(3) Calculating the maintenance cost coefficient of the unit

Taking the A_2 as an example, and the calculation method in Section 2.3 is used to calculate the comprehensive maintenance cost coefficient of the unit.

Tab.3. The APA complexity of component inA_2.

Name	APA complexity								
	Quantity	Size	Thickness	Symmetry	Weight	Grasping difficulty	Auxiliary action	p	A_P
Worm end cap	1	0.75	0.3	0.7	0.3	0.6	0.7	6	0.56
Sealing ring	1	0.75	1.0	0.7	0.3	1.0	0.5	6	0.71
Adjusting pad	1	0.75	1.0	0.7	0.3	0.6	0.7	6	0.68
Worm shaft cover	1	0.75	0.5	0.5	0.3	0.6	0.7	6	0.56

Thrust bearing	2	0.75	0.3	0.7	0.3	0.6	0.5	6	0.53

Name	APA complexity								
	Quantity	Size	Thickness	Symmetry	Weight	Grasping difficulty	Auxiliary action	p	A_P
Left nut	1	0.75	1.0	0.7	0.3	0.6	0.7	6	0.68
Left shaft sleeve	1	0.75	0.5	0.7	0.3	0.8	0.7	6	0.63
Bearing retaining ring	1	0.75	0.5	0.7	0.3	0.6	0.7	6	0.59
Needle bearing	2	0.75	1.0	0.7	0.3	0.8	1.0	6	0.76
Worm	1	0.75	0.3	0.7	0.6	0.8	1.0	6	0.69
Snap ring	2	0.75	1.0	0.7	0.3	1.0	1.0	6	0.79
Right shaft sleeve	1	0.75	0.5	0.7	0.3	0.8	1.0	6	0.68
Rotating shaft oil seal	1	0.75	0.5	0.7	0.3	1.0	1.0	6	0.71
Middle gear	1	0.75	0.3	0.7	0.3	0.6	0.7	6	0.56
key	1	0.75	1.0	0.7	0.3	0.8	1.0	6	0.76
Right nut	1	0.75	0.5	0.7	0.3	0.6	0.7	6	0.59

Tab.4. The AOA complexity of component in A_2.

Name	AOA complexity										
	Quantity	Difficulty in positioning	Assembly resistance	Line of sight	Tightening method	Detection method	Job requirement	Auxiliary equipment	Success rate of one assembly	q	A_O
Worm end cap	1	0.75	0.3	0.35	0	0.3	0.3	0.3	0.3	7	0.37
Sealing ring	1	0.5	0.3	0.35	0	0	0.3	0.3	0.3	6	0.35
Adjusting pad	1	0.75	0.3	0.35	0	1.0	0.3	0.3	0.85	7	0.55
Worm shaft cover	1	1.0	0.3	0.35	0	0.3	0.3	0.3	0.57	7	0.45
Thrust bearing	2	0.5	1.0	0.73	0	1.0	0.56	0.7	0.85	7	0.76
Left nut	1	0.5	0.5	0.35	0.85	0.65	0.3	0.3	0.3	8	0.47
Left shaft sleeve	1	1.0	1.0	1.0	0	1.0	1.0	0.7	0.85	7	0.95
Bearing retaining ring	1	0.75	1.0	1.0	0	0.65	0.56	0.7	0.57	7	0.75
Needle bearing	2	1.0	1.0	0.73	0	1.0	1.0	1.0	1.0	7	0.96
Worm	1	1.0	1.0	1.0	0	1.0	1.0	1.0	1.0	7	1.0
Snap ring	2	0.75	0.3	0.73	1.0	0.3	0.3	0.3	0.57	8	0.53
Right shaft sleeve	1	1.0	1.0	1.0	0	1.0	1.0	0.7	0.85	7	0.95
Rotating shaft oil seal	1	0.5	0.5.	0.73	1.0	0.3	0.3	0.3	0.3	8	0.43
Middle gear	1	1.0	1.0	0.73	0	1.0	0.7	1.0	0.85	7	0.91
key	1	0.3	0.5	0.73	0	0.3	0.3	0.3	0.57	7	0.46
Right nut	1	1.0	1.0	0.73	0	0.65	0.3	0.3	0.57	7	0.65

Using the data in Table 1 and Table 2, the APA complexity and AOA complexity of characteristic components in the A_2 are analyzed and presented in Table 3 and Table 4. Following the calculation methods outlined in Eq. (9-12), the comprehensive assembly complexity of A_2 is determined as $C(A_2) = 205.86$. Using the same methodology, the $C(A_1) = 71.26$ and $C(A_3) = 802.85$ are obtained, respectively. Subsequently, the comprehensive maintenance cost coefficients for each MAU are calculated through Eq. (13):

$$\begin{cases} \lambda(A_1) = 1.0 \\ \lambda(A_2) = 1.25 \\ \lambda(A_3) = 1.57 \end{cases} \tag{20}$$

(4) Establishing the reliability optimization allocation mathematical model

Based on the reliability optimization mathematical model shown in Eq. (16), and incorporating the comprehensive influence coefficient of precision stability, structural complexity coefficient, and comprehensive maintenance cost coefficient of each MAU, the reliability optimization mathematical model for the NC rotary table system is derived as follows:

$$\min[\tilde{G}_S] = \sum_{i=1}^{3} [\tilde{G}_i(R_i)]$$

$$= \sum_{i=1}^{3} \frac{1}{\xi(A_i)} \cdot \lambda(A_i) \cdot exp\left(\alpha(A_i) \cdot \frac{R_i - R_{i,min}}{R_{i,max} - R_i}\right)$$

$$\text{s.t.} \begin{cases} R_S = \prod_{i=1}^{3} R_i \geq [R_S^*] \\ 0.6 \leq R_i \leq 0.99, i = 1,2,3 \end{cases} \tag{21}$$

To enhance analysis and computational efficiency, the minimum reliability of each MAU in the NC rotary table system is 0.6, and the maximum reliability is 0.9924.

Using a multi-island genetic algorithm to solve the optimization mathematical model defined in Eq. (21), the reliability allocation values and comprehensive generalized cost coefficients for each MAU is obtained, as shown in Table 5

Tab.5. Reliability allocation results.

	A_1	A_2	A_3	$[\tilde{G}_S]$
R	0.9483	0.9465	0.9478	4.22×10^4

5.2. Comparative analysis of results

To demonstrate the rationality of the reliability allocation results, the mechanical system reliability is allocated using the methods from references [15] and [19], with the outcomes presented in Table 6. A comparative analysis of reliability for three MAUs is shown in Figure 11.

Tab.6. Comparative analysis of results.

	Reference [18]	Reference [22]	Proposed method
A_1	0.9608	0.9338	0.9483
A_2	0.9387	0.9425	0.9465
A_3	0.9425	0.9658	0.9478
$[\tilde{G}_S]$	7.36×10^5	1.31×10^7	4.22×10^4

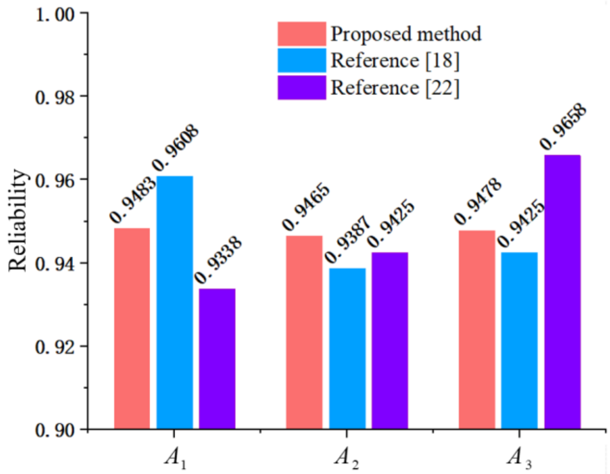

Fig. 12. Comparative analysis of different methods.

The results in Table 6 demonstrate that the generalized cost obtained by the proposed method is minimized compared to the reliability allocation results derived from the methods in references [18] and [22]. Reference [18] allocated higher reliability to unit A_1 while assigning lower reliability to A_2 and A_3 due to its emphasis on comprehensive cost factors during analysis. In contrast, reference [22] incorporated both comprehensive cost factors and motion accuracy stability considerations. Although A_2 and A_3 have higher structural complexity and require higher costs to improve their reliability, considering the impact of motion accuracy stability, A_2 and A_3 still allocate higher reliability, resulting in higher generalized costs. Both references [18] and [22] employed MCDM for reliability allocation in mechanical transmission systems. However, their analytical processes relied heavily on expert decision-making information with significant uncertainty impacts, and focused solely on comprehensive cost factor evaluations rather than optimizing for generalized cost minimization.

The comparative reliability allocation results in Figure 12 reveal that the proposed method allocates higher reliability to unit A_1 compared to reference [22], while A_3 receives significantly lower reliability than in reference [22], yet higher than in reference [18]. Notably, the reliability allocations for A_2 across all three methods are relatively close. This discrepancy arises because A_3 has a substantial impact on the motion accuracy stability of the mechanical transmission system, and improving its reliability entails significantly higher costs. Conversely, A_1 exhibits minimal influence on motion accuracy stability, but its simple structure and lower maintenance costs allow reliability enhancement at reduced expenses

6. Conclusion

This paper proposes A reliability allocation method for mechanical transmission systems is proposed in this article, that considers motion accuracy stability and a G-RCF. The developed reliability optimization allocation model quantitatively incorporates motion accuracy stability and maintenance cost factors, generating a reliability allocation scheme that minimizes the combined design, manufacturing, and maintenance costs of the system while satisfying both system reliability constraints and motion stability requirements.

(1) The meta-action theory is introduced to structurally decompose the mechanical transmission system and obtaining the MAUs. Taking the MAUs as analysis objects, the factors including their motion accuracy stability, structural complexity, and comprehensive maintenance costs are analyzed. Then, the unit motion accuracy stability influence coefficient, unit structural complexity coefficient, and unit comprehensive maintenance cost coefficient are obtained.

(2) The traditional RCF model is modified using these coefficients (motion accuracy stability influence coefficient, structural complexity coefficient, and comprehensive maintenance cost coefficient) to establish a G-RCF model for MAUs. Subsequently, with the objective of minimizing generalized costs, a reliability optimization allocation mathematical model for the mechanical transmission system is developed. Intelligent algorithms are employed to solve this model, and obtaining the reliability design value of each MAU. Finally, a reasonable allocation of reliability for the system is achieved.

The calculation results of the method described in this article are not affected by the expert decision-making process, and the reliability allocation results are more reasonable. In future research, the reliability allocation model of the dual-drive system will be studied. At the same time, combined with experimental and operation and maintenance data, a more accurate and applicable reliability allocation model will be established to further improve the performance of the mechanical transmission system and numerical control equipment.

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