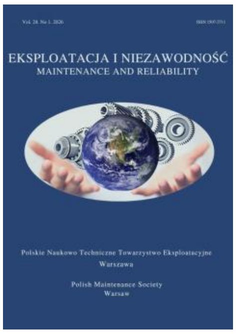

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Reliability Estimation of Retraction Mechanism Kinematic Accuracy under Small Sample

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Highlights

- The model is based on time-domain fuzzy Bayesian network.
- It combines the fuzzy fault tree and Bayesian network reliability analysis methods.
- The method provides robust probabilistic assessments.

Abstract

This study addresses high fault uncertainty, time-varying dynamics and non-reversible reasoning in natural gas station regulators by integrating fuzzy fault tree analysis with Bayesian networks. The approach combines component-level reliability models to tackle complex structural uncertainties and dynamic failure scenarios more accurately than traditional methods like binary-state event relationships or T-S fuzzy gates. By leveraging explicit causal links through fuzzy logic while enabling probabilistic predictions using Bayesian inference over time-varying dependencies, the method provides robust probabilistic assessments. The subsequent calculation of posterior probability and sensitivity identifies weak links influencing the system's reliability. Experimental validation demonstrates its capability to identify critical weak points affecting system performance, affirming applicability and potential impact on design optimization and maintenance strategies for real-world installations.

Keywords
natural gas station. pressure regulating system, fuzzy fault tree, bayesian network, reliability analysis

1. Introduction

Natural gas pipeline expansions lead to increased commissioning of transmission stations, which rely on critical systems like pressure regulation for efficient operation. However, the complexity of these systems can cause unforeseen issues. Effective maintenance, especially for components like pressure regulators, is essential to ensure both employee safety and operational productivity. Malfunctions in this system could jeopardize personnel safety and station performance. Therefore, maintaining the reliability of pressure regulation systems through regular inspections and preventive measures is critical for safe and stable gas transmission operations.

The reliability analysis of the system can provide the basis for the safety evaluation of the system and the formulation of the corresponding maintenance strategy [1; 2]. The common methods for system reliability analysis include fault tree analysis (FTA)[3], Monte Carlo simulation analysis process [4; 5; 6], failure mode consequences and hazard analysis (FMECA) [7; 8], Markov process analysis [9; 10] and Bayesian network analysis [11; 12; 13; 14]. Bayesian network analysis has been perfected and developed by many experts and scholars into a theoretical method for analyzing uncertainty expression systems. Yakowitz S J elaborated on the Bayesian network and applied it in

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engineering [15]. Researchers Minn and Zhang both pay attention to fault diagnosis and analyze information about uncertain system failures [16; 17]. In reference [18], a Bayesian approach is proposed to allow the corrosion model parameters to be updated according to the evolution 2of environmental conditions. This method makes full use of the advantages of Bayesian networks and solves the uncertainty problem in the process of network fault diagnosis. However, the fault mechanism of the actual natural gas station system is much more complex. The previous fault tree model has many deficiencies: (1) it needs to understand the logical relationship and probability between events accurately. (2) Each event can only be described by two states. (3) Traditional BN requires node states to be discretized (such as "normal/fault"), while faults in natural gas pressure systems often exhibit continuous or polymorphic characteristics (such as gradual pressure drop, partial valve blockage, sensor drift, etc.). (4) Traditional BN requires an accurate conditional probability table (CPT), but in practical systems, the fault logic is often fuzzy (such as "slightly higher pressure may cause slight valve jamming").

This paper introduces a novel approach by combining the T-S fuzzy gate fault tree with Bayesian networks, achieved through the conversion of the fuzzy gate rules of the former into the conditional probability table of the latter. Its advantages are: (1) Fine modeling of polymorphic faults; (2) Collaborative expression of fuzzy logic and probability; (3) Time domain dynamic adaptability; (4) Handling of complex coupling relationships. suitable for scenarios involving polymorphic continuous fault diagnosis, risk assessment in uncertain environments, and preventive maintenance decision-making. Given the inherent complexity of the system, the multitude of potential errors, and the diverse range of data sources, this article harnesses the efficient parallel bidirectional inference capability of Bayesian networks to its full extent. Through the integration of reliability models for each component and their hierarchical traversal in the time domain, the proposed methodology enables uncertain fault diagnosis for industrial systems. The implementation of this method results in the construction of a Bayesian network, and its effectiveness is demonstrated through the diagnosis of faults in the regulation system's regulator skid. This component, being one of the most critical and frequently-faulting elements in a natural gas station, highlights the practical significance of the developed approach.

2. Method

2.1. Reliability Modeling

The reliability analysis model employed in this study assesses the reliability and lifespan of each component across various time periods. Serving as the cornerstone of this paper, this model provides the essential data support for the subsequent exploration of the fuzzy Bayesian network in the temporal dimension throughout the manuscript.

2.1.1. Reliability Modeling Process

The primary objective of reliability modeling is to utilize historical failure data of equipment components, coupled with statistical modeling techniques, to extract pertinent lifespan information [19]. This enables the representation of failure patterns, providing theoretical underpinnings for equipment maintenance. Consequently, it facilitates the formulation of more rational and scientifically grounded maintenance strategies. The process of reliability modeling is depicted in Figure 1, and the specific modeling steps are as follows:

1. Fault Data Preprocessing: Utilizing the historical fault records of related equipment at the gas transmission station, determine the time intervals between failures to establish separate fault data points. Afterward, arrange the component failure data by type of equipment and location in ascending order, remove any outliers, and obtain standardized reliability analysis data.

2. Fitting Distribution Models: It is possible to identify one or more applicable distributions for a given set of failure data. To conduct a fitting analysis, it is crucial to comprehend the appropriateness of common distribution models and to choose an appropriate distribution function. The best distribution function can be identified through validation methods.

3. Parameter Estimation: In order to conduct reliability analysis accurately, it is crucial to determine the parameters of the distribution function selected in step 2. This further step of parameter estimation directly impacts the calculation of final reliability indicators.

4. Calculating Reliability Indices: Reliability indices are primarily used to characterize the failure patterns in equipment fault data. They can uncover potential issues, allowing for the

optimization of existing maintenance procedures by directly influencing the maintenance content through reliability indices.

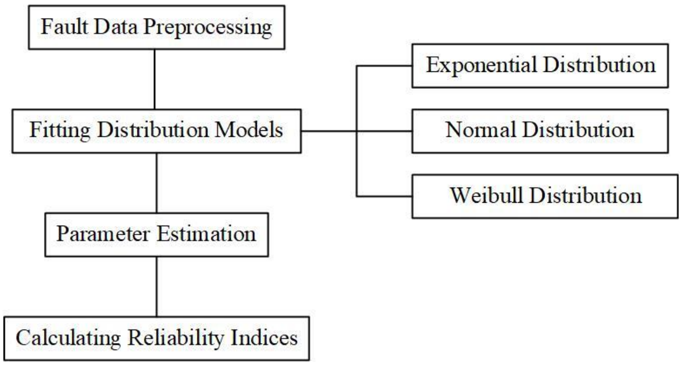


Figure 1. The process of reliability modeling.

2.1.2. Reliability Metrics

Lifespan analysis is based on the theoretical techniques of reliability analysis [20], which defines reliability as the capacity of mechanical equipment to fulfill designated duties within defined time frames and under specific conditions. Failure occurs when this specified capability is lost, often characterized as a fault for repairable equipment. Reliability metrics are utilized to quantify the level of equipment reliability. Several common reliability indicators include the following:

1. Reliability

Reliability is the probability that a mechanical device can perform its specified function within a defined time period and under specified conditions. It is a probabilistic quantification of reliability over time. Let T be a random variable representing the lifespan of the mechanical equipment. The probability that the product accomplishes its specified function within a fixed time t is given by:

$$R(t) = P(T > t) = \int_{r}^{\infty} f(t)dt \quad (1)$$

2. Probability Distribution Function

The probability distribution function of failure data, also known as the unreliability function, represents the probability that a mechanical device cannot perform its specified function within a defined time period and under specified conditions [21]. Its relationship with the reliability function can be expressed as follows:

$$F(t) = 1 - R(t) \quad (2)$$

3. The failure rate function When the sample size is relatively large, the approximate median rank is often used to estimate the empirical distribution function of failure data.

$$F_n(t_i) = \frac{i-0.3}{n+0.4}(i = 1,2,\cdots,n) \quad (3)$$

In the equation: i represents the index of failure data, and n represents the total number of failure data points.

2.1.3. Reliability Distribution Model

The equipment lifespan distribution refers to the distribution pattern of equipment failure data. It involves using statistical analysis methods to fit the equipment's failure data into a specific distribution, estimating relevant parameters, and ultimately analyzing various related indicators [22]. Below is a brief introduction to three commonly encountered lifespan distribution models in engineering applications:

1. Exponential Distribution [23]: The exponential distribution is often utilized to model the lifespan distribution of components, such as electronic devices, software, and related parts in complicated systems with numerous components. These components generally display a constant failure rate, signifying that their failures are mainly random events and are not heavily dependent on time. The reliability function for the exponential distribution is determined by:

$$R(t) = e^{-\lambda t} \quad (4)$$

2. Normal Distribution [24]: According to survey results, in non-lifespan situations, over 80% of equipment failure issues follow a normal distribution pattern. In mechanical products and structural engineering, the normal distribution is a common distribution model used to study the strength and stress distribution of mechanical structures. Additionally, for components prone to wear-related failures, such as gears and bearings, their failure distribution typically conforms to a normal distribution. The reliability function for the normal distribution is given by:

$$R(t) = \frac{1}{\sqrt{2\pi\sigma}} \int_{t}^{\infty} e^{-\frac{t-\mu^2}{2\sigma^2}} dt \quad (5)$$

3. Weibull Distribution [25]: The Weibull distribution is a frequently observed distribution pattern in mechanical equipment failure problems, similar to the normal distribution. It is utilized to represent the initial, random, and wear-out failure periods of equipment and has a close relationship with the bathtub curve. The Weibull distribution's reliability function is provided as follows:

$$R(t) = e^{-\left(\frac{t-t_0}{\eta}\right)^{\beta}} \quad (6)$$

β represents the shape parameter, η represents the scale parameter, andt_0 represents the location parameter. Whent_0 =

0, the three-parameter Weibull distribution becomes a two-parameter Weibull distribution.

2.1.4. Distribution Parameter Estimation

Two common statistical methods used for parameter estimation of distribution models are graphical methods and analytical methods. Graphical methods include probability plot methods and hazard rate plot methods, among others. Analytical methods include moment estimation, least squares method, and maximum likelihood estimation, among others. Analytical methods are less influenced by the size of the data sample compared to graphical methods and are therefore often used as the primary parameter estimation methods. In this paper, the least squares method is used as an estimation method for the parameters.

The least squares method is commonly used to estimate the distribution parameters A and B of a linear model $y = wx + b$. It aims to minimize the sum of squared differences between the estimated values and the observed values, making it an optimization goal of the least squares method. Therefore, the objective function can be represented as the formula:

$$F = argmin \sum_{i=1}^{n} L_i^2 = argmin \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2 \tag{7}$$

To take the derivatives of the objective function F with respect to w and b:

$$\begin{cases} \frac{\partial F}{\partial w} = -2\sum_{i=1}^{n} x_i\left(Y_i - \hat{Y}_i\right) = 0 \\ \frac{\partial F}{\partial b} = -2\sum_{i=1}^{n} x_i\left(Y_i - \hat{Y}_i\right) = 0 \end{cases} \tag{8}$$

Solving the above equations yields:

$$\begin{cases} w = \frac{\sum_{i=1}^{n} x_i Y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} Y_i}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)} \\ b = \frac{1}{n}\sum_{i=1}^{n}\left(Y - wx_i\right) \end{cases} \tag{9}$$

Using the above method to solve the parameters of the Weibull distribution and applying linear transformation, we obtain:

$$ln\left(-ln\left(1 - F(t)\right)\right) = \beta(ln\, t - ln\, \eta) \tag{10}$$

Let $y = ln\left[ln\left(\frac{1}{1-F(t)}\right)\right]$, $x = ln(t)$, The transformed linear equation is then:

$$y = \beta x - \beta\, ln\, \eta \tag{11}$$

Where $w = \beta$and $b = -\beta\, ln\, \eta$.

After obtaining the parameters w and b through the above calculation process, you can then reverse-calculate the two parameters of the Weibull distribution. Additionally, by introducing the Mean Square Error (MSE) and theR^2coefficient of determination, you can assess the goodness of fit of the linear model, further evaluating the accuracy of the distribution parameters. The formulas for calculating the Mean Square Error and the R^2coefficient are as follows:

$$MSE = \frac{1}{n}\sum_{i}^{n}\left(y_i - \hat{y}_i\right)^2 \tag{12}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{n}\left(y_i - \overline{y_i}\right)^2} \tag{13}$$

Where y_i represents the actual values, i.e., empirical distribution values; $\hat{y}_i$ represents the fitted predicted values; $\overline{y_i}$represents the mean of the actual values. A smaller MSE indicates a better model performance for that parameter, and an R^2 value closer to 1 indicates a better fit.

The same approach of linear transformation can be applied to the other two distributions, and the results are as shown in Table 1:

Table 1. Linearization of common distribution.

Distribution type	Y	X	A	B
Exponential Distribution	$Y = ln\left[\frac{1}{1-F(t)}\right]$	$X = t$	$A = \lambda$	$B = 0$
Normal Distribution	$Y = z$	$X = t$	$A = \frac{1}{\sigma}$	$B = -\frac{\mu}{\sigma}$
Weibull Distribution	$Y = ln\left\{ln\left[\frac{1}{1-F(t)}\right]\right\}$	$X = ln(t)$	$A = \beta$	$B = -\beta\, ln\, \eta$

2.1.5. Goodness-of-Fit Test

When using the above method to calculate the assumed distribution parameters for mechanical equipment failure data, it's essential to further test the four types of lifespan distribution models, analyze the differences between the lifespan distribution model and the actual distribution, and then choose the lifespan distribution type that provides the best fit. When conducting goodness-of-fit tests, the steps for hypothesis testing generally include:

1. First, establish the null hypothesis$H_0$and the alternative hypothesisH_1.

H_0: The sample data follows the given distribution model.

H_0: The sample data does not follow the given distribution model.

2. Choose an appropriate test statistic.

3. Determine an appropriate significance levelα. Reject the null hypothesis within the significance level, otherwise, accept it. In general, $\alpha$is often set to 0.05.

4. Based on the distribution of the test statistic, calculate the p-value. The p-value measures the probability of observing the sample's differences due to sampling error and helps decide whether to accept or reject the null hypothesis. If p ¡ 0.05, then the null hypothesis$H_0$is rejected in favor ofH_0.

This article uses the K-S test [55] to test the goodness of fit of the distribution, because the amount of fault data obtained from historical data is relatively small, and the K-S test is more suitable for small sample data. The basic method is to arrange n experimental data (t, t,..., t_n) in ascending order, and then calculate the probability distribution value F (ti) corresponding to each data based on the pre assumed probability distribution F (t). Compare it with the cumulative frequency function value (t) of the random sample, and take the maximum difference D, that is:

$$D = 1 \le i \le n\boldsymbol{max} \mid Fn(ti) - F0(ti) \mid \qquad (14)$$

Among them, D is called the test statistic, which compares D with the critical value D (which can be obtained by looking up a table). If D<Dn, the null hypothesis H is accepted, otherwise it is rejected.

2.1.6. Hypothesis Rationality Analysis

The following main assumptions were made when conducting reliability modeling in this study:

1. Assumption of Independence of Fault Events: Assuming that the faults of each component of the equipment occur independently.

2. Distribution selection assumption: Assuming that the equipment failure time follows a Weibull distribution, normal distribution, or exponential distribution.

To verify the impact of these assumptions on the model results, we conducted a sensitivity analysis:

1. Assumption of independent fault events: The object of this study is the monitoring regulator valve. Due to the fact that the monitoring pressure regulating valve is composed of multiple components, such as the commander diaphragm, commander sealing ring, solenoid valve, commander filter element, etc., each component has its specific function. If a component experiences a serious malfunction, the pressure regulating valve will immediately stop working and undergo testing. Therefore, after the failure occurs, the pressure regulating valve will not be able to continue to use, and the failure will be found and handled in a timely manner. Based on this, this study believes that failures between various components occur independently. This assumption holds true. In practical applications, the failures of different components will not affect each other, and there is no need for complex analysis of their interrelationships.

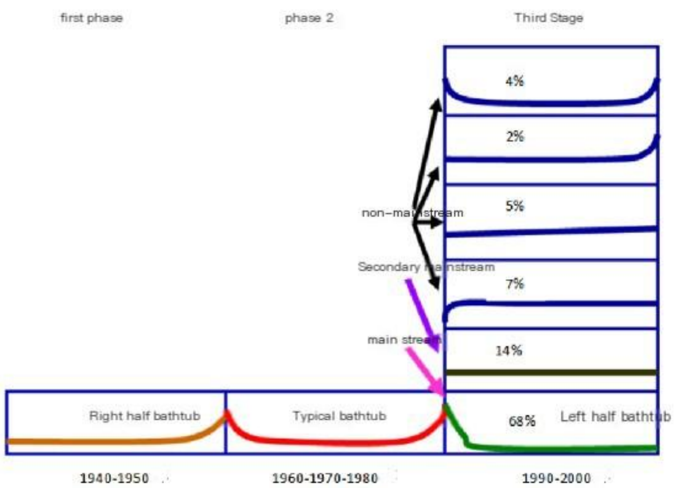


Figure 2. Bathtub curve chart.

2. Distribution selection assumption: This study assumes that the failure time of equipment follows a Weibull distribution, normal distribution, or exponential distribution. Previous studies have shown that normal distribution, Weibull distribution, and exponential distribution can cover 90% of common failure modes. The typical life distribution types are shown in the figure on the right. Among the six distribution types, 68% of equipment failures belong to the left bathtub curve, 14% of equipment have a constant failure rate, and 7% of equipment failures first increase and then stabilize. Therefore, the exponential life distribution curve, normal life distribution curve, logarithmic normal distribution, and Weibull life distribution curve can describe more than 90% of the life distribution situation. Based on this, using these four life distributions for modeling is scientific.

Sensitivity analysis shows that the assumptions of fault event independence and distribution selection have a relevant impact on the reliability analysis results, and these assumptions are reasonable for the structural characteristics and usage of monitoring and regulating valves.

2.2. fuzzy Bayesian network

To address the complexities of industrial production environments and to calculate the impact of different system components on system reliability more accurately, this study makes full use of the parallel bidirectional reasoning capabilities of Bayesian networks, combined with the advantages of fuzzy fault trees [26]. By calculating reliability and failure rate models for each component and employing Bayesian network [27] hierarchical traversal in both temporal and parameter domains, it assesses the reliability and sensitivity of mixed uncertainty. This method takes into account the polymorphic and fuzziness of failure states in complex system reliability calculations, the uncertainty in logical relationships between event failures, and the variation of component failure rates over time. It allows for reverse inference using posterior probabilities, constructs Bayesian fuzzy network models using T-S fuzzy fault trees [28], and combines the temporal and parameter domains, effectively leveraging their strengths while overcoming their limitations.

2.2.1. T-S fuzzy fault tree

The T-S fuzzy model replaces the logic gate in the traditional fault tree [29]. The normal and fault states are commonly used in traditional two-state systems to describe the failure states of the basic components of the system, while in practical applications, the system and components often show a variety of fault states and fault degrees. For example, the occurrence of some basic events cannot lead to the direct occurrence of the top event system failure, and the system may only be in the stage of minor failure. In this paper, fuzzy number (0,0.5, 1) is used to describe the three fault states of the system (no fault, slight fault, fault). Figure 2 shows the T-S fuzzy fault tree model.

$x_1, x_2, \cdots, x_n (n = 1,2,\cdots.n)$ represents the bottom event, $y_m (m = 1,2,\cdots.m)$, intermediate event variable, the system output is the top even variable is $G_1, G_2, \cdots, G_j (j = 1,2,\cdots.j)$ represents T-S fuzzy gate. Fuzzy numbers $x_n^i (i = 0,0.5,1)$, $y_m^i (i = 0,0.5,1)$, and $T_q (q = 0,0.5,1)$ are used respectively to represent the fault state of the corresponding event. The fuzzy rules of the local T-S fuzzy gate fault tree composed of y_1, T-S gates G_1, and x_1, x_2 and x_3 can be represented in Table 2.

2.2.2. Bayesian network

BN (Bayesian Network) is a probabilistic causal network expressed in the form of graph theory, which is widely used in probabilistic reasoning with uncertain pins. Bayesian networks can be bidirectional reasoning [30; 31]. Using conditional probability boxes and prior probabilities, forward inference can be achieved by calculating the probability of any node in the network. Give each root node a prior probability, and give each child node a conditional probability table. The basis of Bayesian networks is shown in Figure

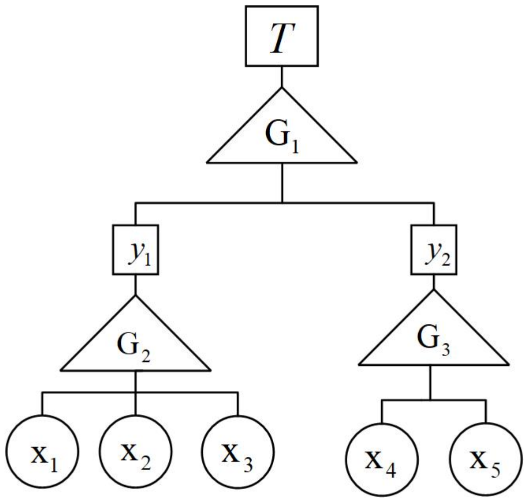


Figure 3. T-S fuzzy fault tree.

Table 2. Fuzzy gate rule.

rule	x_1	x_2	$\cdots$	x_3	y_i		
					0	0.5	1
1	x_1^i	x_2^i	$\cdots$	x_n^i	$P^{G_1}(y_1^0)$	$P^{G_1}(y_1^{0.5})$	$P^{G_1}(y_1^1)$

3. Where x_1, x_2, x_3, x_4, x_5 represents the bottom event (root node), y_1, y_2 represents the middle event (middle node), and T represents the top event (leaf node).

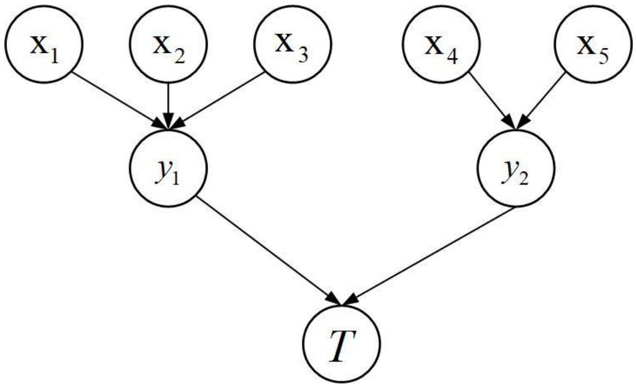


Figure 4. Bayesian Network.

2.2.3. Mapping of T-S Fuzzy fault Tree to Fuzzy Bayesian networks

The T-S fuzzy fault tree analysis method is capable of expressing the polymorphism of event fault states and addressing the uncertainty in the logical relationship between fuzziness and fault events during the reliability analysis of a system. However, its modeling and analytical capabilities are limited, resulting in inefficiency and the inability to perform reverse reasoning. On the other hand, Bayesian networks are well-suited for the reliability analysis of complex multistate systems and for expressing the unclear mechanisms of faults. While Bayesian networks enable backward reasoning using posterior probabilities, establishing the model can be challenging. Therefore, we propose using the T-S fuzzy fault tree to construct a Bayesian fuzzy network model. This approach leverages the advantages of both methods and overcomes their respective limitations in a coherent manner.

T-S fuzzy fault tree mapping to BN is mainly divided into two steps:

1) The nodes of the T-S fuzzy fault tree correspond to each node of BN.

2) Determine the conditional probability table.

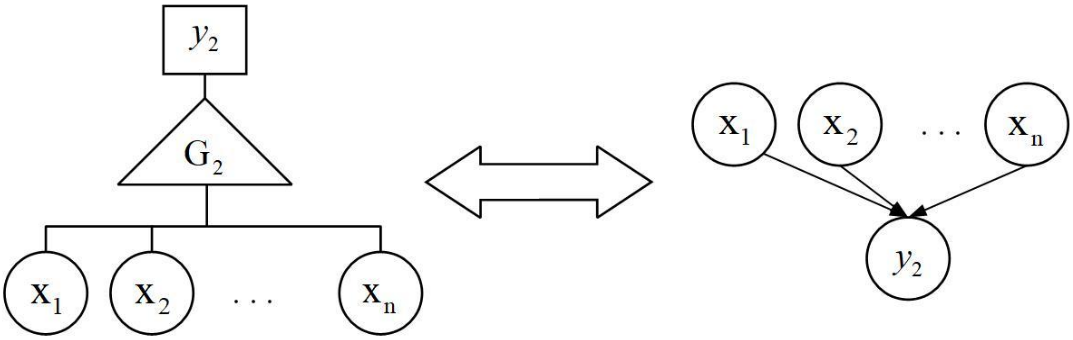


Figure 5. T-S fuzzy fault tree mapping to BN directed acyclic graph.

It is a characteristic of directed acyclic graphs in Bayesian networks that non-descendant nodes are independent of each other. Therefore, when the top event y_2 fails, the posterior failure probability of the bottom event x_2 is:

$$P(y_2 = y_2^i) = \sum_{x_1, x_2, \cdots, x_n} P(x_1, x_2, \cdots, x_n, y_2 = y_2^i) = P(y_2 = y_2^i | x_1 = x_1^i, \cdots, x_n = x_n^i) \quad (15)$$

2.2.4. Probability importance of the root node

The significance of a root node's contribution to the failure of a leaf node is crucial in system reliability analysis and fault diagnosis. The data obtained from BN importance analysis are computed based on the mean value of the evidence interval.

Definition 1: When the root node's fault state is $x_i^{k_i}$, the probability importance of the leaf node's fault state is:

$$I_{T_q}^{P_r}(x_i = x_i^{k_i}) = P(T = T_q \backslash x_i = x_i^{k_i}) - P(T = T_q \backslash x_i = 0) \quad (16)$$

In the equation 15, $P(T = T_q \backslash x_i = x_i^{k_i})$ represents the occurrence probability of leaf node T failure state being T_q when the fault function state of root node x_i is $x_i^{k_i}$, and $I_{T_q}^{P_r}(x_i = x_i^{k_i})$ represents the occurrence probability of leaf node T failure state being when the fault state of root node x_i is $x_i^{k_i}$.

Definition 2: The probability importance degree of root node xi to the failure state of leaf node T_q is:

$$I_{T_q}^{Pr}(x_i) = \frac{1}{h_i-1}\sum_{h_j=4}^{h_j} I_{T_q}^{Pr}\left(x_i = x_i^{k_i}\right)$$

(17)

Where, h_i is the number of failure states of root nodex_i.

2.2.5. Sensitivity

Sensitivity reflects the responsiveness of leaf nodes to the extent of failure. Analyzing sensitivity allows us to discern the impact of distribution parameters of root nodes on the reliability relationship of leaf nodes. This analysis can result in significant variations in leaf node reliability even when the degree of root node failure changes minimally. Identifying high-risk events that contribute to system failure becomes possible through this approach, thereby offering a theoretical foundation for establishing risk control measures. Definition 1: The sensitivity of root node xi to the failure state of leaf node$T_q$is:

$$S_{T_q}^{(I)}\left(x_i = x_i^{k_i}\right) = \frac{I_{T_q}^{Pr}\left(x_i=x_i^{k_i}\right)}{P(T=T_q \backslash x_i=0)} \quad (18)$$

Where: $I_{T_q}^{Pr}\left(x_i = x_i^{k_i}\right)$ is the probability importance of the root node.

According to Equation 17, the sensitivity of root node x to the failure state of leaf node T is:

$$S_{T_q}^{(I)}(x_i) = \frac{1}{k_i-1}\sum_{a_i=1}^{k_i} S_{T_q}^{(I)}\left(x_i = x_i^{k_i}\right) \quad (19)$$

Where, $h_i$is the number of failure states of root nodeh_i.

3. Experiment

3.1. Reliability Assessment Case Study

The failure data for this analysis is derived from a valve level sensor for monitoring regulator failure data provided by a gas transmission site. Since valve failure cycles are typically long and single-device failures are rare, This paper chose to analyze typical cases by monitoring the same manufacturer and size regulator. This approach was taken to minimize potential result inaccuracies arising from data variations.

3.1.1. Data preprocessing

A total of 51 sets of fault interval values of Supervisory Regulators seats were collected from the gas transmission stations, and according to the preprocessing method, the fault interval values were sorted from smallest to largest. According to the pre-processing method, the observed values of failure interval time are sorted from smallest to largest, and the specific data are shown in Table 3:

Table 3. fault interval values of Supervisory Regulators' seats.

serial number	commissioning time	downtime	failure interval
1	2010/12/6	2018/4/15	20
2	2017/4/16	2017/9/23	160
3	2017/6/13	2017/11/25	165
⋮	⋮	⋮	⋮
49	2003/10/6	2015/11/10	4418
50	2003/10/16	2017/10/11	5109
51	2003/10/1	2017/10/12	5125

Table 4. The failure data, the empirical probability distribution observations (F).

serial number	F
1	0.013619
2	0.033074
3	0.052529
⋮	⋮
49	0.947471
50	0.966926
51	0.986384

After obtaining the failure data, the empirical probability distribution observations (F) for failure interval times were calculated using the previously introduced median rank formula, as shown in Table 4. Additionally, an empirical probability distribution graph was generated, as depicted in Figure 5.

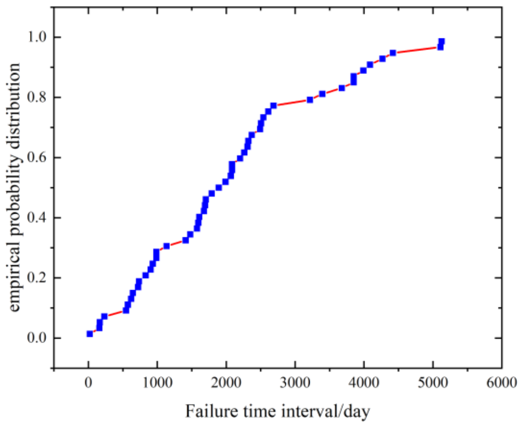
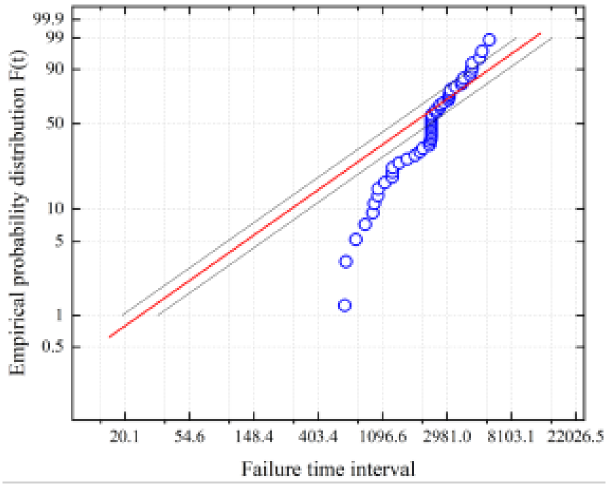


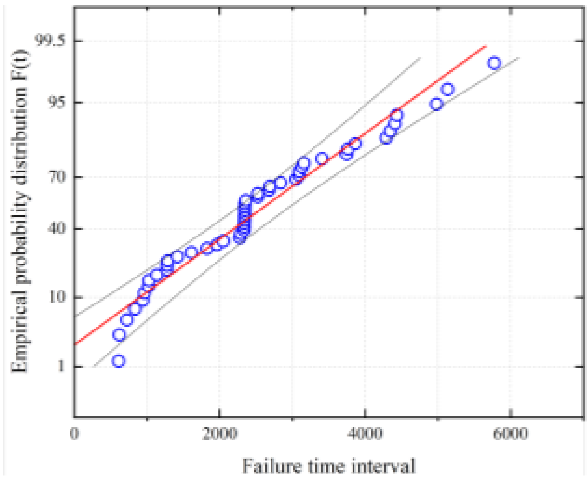
Figure 6. The failure data, the empirical probability distribution observations (F).

3.1.2. Parameter Estimation for the Lifespan Distribution Model

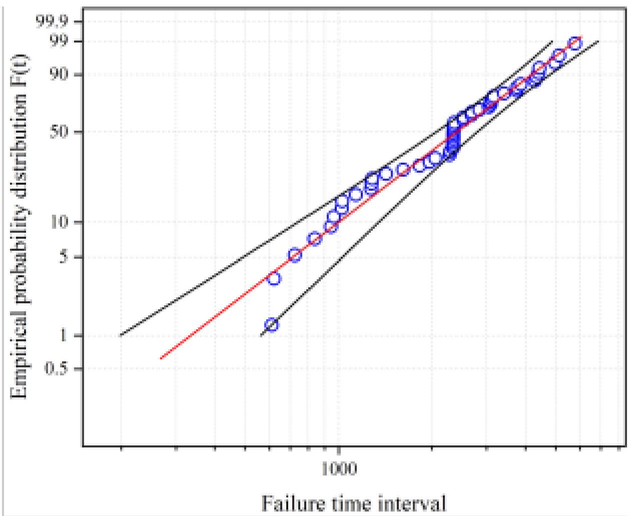
To further assess the distribution function type of the electric ball valve failure interval data, fits were performed with the three commonly used distributions introduced in section 2.1.5. The goodness of fit of these three assumed distributions was observed using probability plots. As seen in Figures 6, the fits for the normal distribution and Weibull distribution appear to be relatively good. Still, it is impossible to directly evaluate the goodness of fit visually. On the other hand, the fit for the exponential distribution appears to be relatively poorand the possibility of this model distribution can be ruled out.



a)



b)



c)

Figure 7. Three types of fitting.

Table 5. Fitting distribution parameters.

Lifespan Distribution Model	Distribution	Parameters
Normal Distribution	μ	σ
	2041.70588	1316.89019
Weibull Distribution	β	η
	1.49883	2244.24509

Regarding the linearly transformed failure interval data, parameter estimation was performed using the least squares method as introduced in Section 2.1.5 for fitting the transformed actuator filter cartridge failure interval data to the theoretical distribution. The calculation results are presented in Table 5.

3.1.3. Selection of Lifespan Distribution Model

To further validate the goodness of fit of the failure data to the four distributions, the Kolmogorov-Smirnov (K-S) test method was employed. The specific results are presented in Table 6:

Table 6. KS calibration results.

Distribution Type	Static	P-value	Determination
Normal Distribution	0.1532	0.2657	Obey
Weibull Distribution	0.1366	0.3968	Obey

From the hypothesis test results in Table 6, it can be observed that the failure data of the valve level sensor do not follow the exponential distribution. When comparing the normal distribution and Weibull distribution, the Weibull distribution has the largest p-value, indicating a higher probability that the failure data conforms to the Weibull distribution. Additionally, the value of the hypothesis statistic for the Weibull distribution is smaller than that for the other two distributions. Therefore, based on the above analysis, it can be concluded that the failure data of the valve level sensor follows a Weibull distribution.

3.1.4. Constructing Reliability Metrics Based on the optimized parameters of the Weibull distribution, the following can be calculated:

Reliability

$$R(t) = exp\left[-\left(\frac{t}{2244.24509}\right)^{1.49883}\right] \quad (20)$$

Probability Distribution Function

$$F(t) = 1 - R(t) = 1 - exp\left[-\left(\frac{t}{2244.24509}\right)^{1.49883}\right] \quad (21)$$

3.2. A case study of pressure regulating pry is carried out

3.2.1. Construct a fuzzy fault tree

When one or more components in the pressure-regulating pry system fail, the possibility of failure of the pressure-regulating pry is uncertain because of the different degrees of failure of each component: no fault, minor fault, and serious fault (complete fault). The failure of the pressure-regulating pry system is selected as the top event, and its T-S fuzzy fault tree is shown in Figure 7. The top event T is the output of T-S gate G1, and the middle event y is the output of G respectively. Table 7 shows the components corresponding to each event in the T-S fuzzy fault tree of the pressure regulating pry.

Assume that the common fault degree of the top event T is (0, 0.5, 1), where 0 indicates no fault, that is, the pressure regulator can work normally, the outlet pressure is stable, and the system can complete the specified working condition. 0.5 indicates a slight fault state, that is, the working state of the pressure regulating pry is not stable, but it can still complete most of the work tasks; 1 indicates a complete fault. The system cannot work properly and needs timely maintenance.

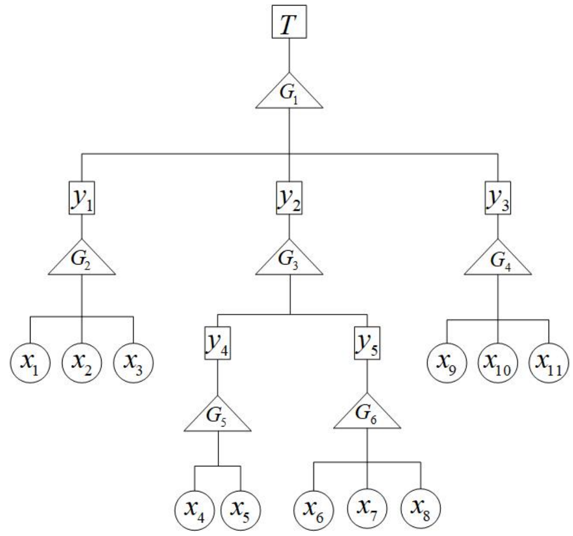


Figure 8. Fuzzy fault tree of pressure regulating pry system

Taking the intermediate event node y as an example, each row in Table

Table 7. Pry the parts corresponding to each event in the T-S fuzzy fault tree.

Node code	Name of parts
x_1	Main diaphragm
x_2	Valve position indicator
x_3	Cutting spring
x_4	Main diaphragm
x_5	Valve level sensor
x_6	Conductor diaphragm
x_7	Conductor seal ring
x_8	Conductor solenoid valve
x_9	Main diaphragm
x_{10}	Valve seat
x_{11}	Flow sleeve
y_1	Safety stop valve
y_2	Monitor the pressure-regulating valve
y_3	Operating pressure-regulating valve
y_4	Main valve
y_5	Conductor
T	Pressure-regulating lever

3 represents a fuzzy rule. For example, the first and second lines represent the following rules:

Rule 1: If the fault status of x_1, x_2, x_3 is 0, the probability of$y_1$being 0 is 1, and the probability of 0.5 or 1 being 0 is 0.

Rule 2: If the fault state of x_1, x_2 is 0 and the fault state of x_3 is 0.5, then the probability of$y_1$being 0 is 0.2, that of 0.5 is 0.5, and that of 1 is 0.3.

3.2.2. The fuzzy fault tree is mapped to the Bayesian network

Each node in BN corresponds to each event in the T-S fuzzy fault tree of the pressure regulating pry respectively. The T-S fuzzy fault tree of the pressure regulating pry is mapped to the fuzzy BN model and the T-S fuzzy fault tree is mapped to the fuzzy Bayesian network:

In Figure 7 Fuzzy fault tree of pressure regulating pry system, $x_1, x_2, \cdots, x_{11}$is the root node (basic event), T is the leaf node (top event), and$y_1, y_2, y_3, y_4, y_5$is the intermediate node (intermediate event).

According to the historical big data and mechanical characteristics of the system, the appropriate distribution function is selected for fitting the bottom event, the optimal distribution function is determined by the checking method, and the parameters of the distribution function are obtained by the parameter estimation method. The distribution function and parameters are shown in Table 9.

Table 8. T-S fuzzy gate 2.

Rule	x_1	x_2	x_3	y_1			Rule	x_1	x_2	x_3	y_1		
				0	0.5	1					0	0.5	1
1	0	0	0	1	0	0	15	0.5	0.5	1	0	0.2	0.8
2	0	0	0.5	0.2	0.5	0.3	16	0.5	1	0	0.2	0.2	0.6
3	0	0	1	0	0.4	0.6	17	0.5	1	0.5	0	0.4	0.6
4	0	0.5	0	0.6	0.3	0.1	18	0.5	1	1	0	0.1	0.9
5	0	0.5	0.5	0.2	0.3	0.5	19	1	0	0	0.3	0.3	0.4
6	0	0.5	1	0	0.3	0.7	20	1	0	0.5	0.2	0.2	0.6
7	0	1	0	0.4	0.4	0.2	21	1	0	1	0	0.2	0.8
8	0	1	0.5	0.2	0.2	0.6	22	1	0.5	0	0.2	0.2	0.6
9	0	1	1	0	0.2	0.8	23	1	0.5	0.5	0.1	0.2	0.7
10	0.5	0	0	0.4	0.4	0.2	24	1	0.5	1	0	0	1
11	0.5	0	0.5	0.2	0.3	0.5	25	1	1	0	0.2	0.2	0.6
12	0.5	0	1	0	0.2	0.8	26	1	1	0.5	0.1	0.2	0.7
13	0.5	0.5	0	0.2	0.3	0.5	27	1	1	1	0	0	1
14	0.5	0.5	0.5	0.1	0.2	0.7							

In order to facilitate the introduction of the method and simplify the calculation, the following assumptions are made for the system reliability modeling:

(1) Failure events with a low probability or components that do not cause system failure will be ignored.

(2) For the bottom event x, the probability curve of fault degree 0 is $R(t)$, while the probability curve of fault degrees 0.5 and 1 is the same, both of which are $0.5\big(1 - r(t)\big)$.

(3) Set the time in days to 3000 days.

3.2.3. Posterior probability

Using Bayesian networks and evidence theory, the prior probability of each basic event involved in regulating pressure can be analyzed, enabling the identification of weak nodes that might decrease system reliability. Additionally, the a posteriori probability of the Bayesian network can be utilized in backward reasoning for fault diagnosis of the pressure-regulating system. This approach helps save maintenance time and reduces worker workload.

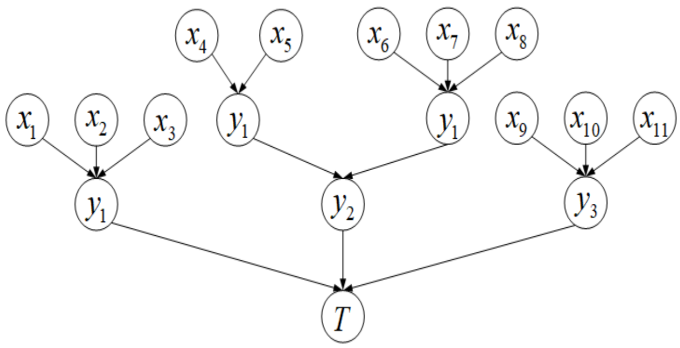


Figure 9. fuzzy Bayesian networks.

Table 9. Failure probability distribution of parts.

Part	Failure probability distribution	Parameter
x_1	Normal	$\mu = 3298, \sigma = 1313$
x_2	Normal	$\mu = 2274, \sigma = 1646$
x_3	Normal	$\mu = 5940, \sigma = 1289$
x_4	Weibull	$\beta = 1.0377, \mu = 1958$
x_5	Weibull	$\beta = 1.49883, \mu = 2244$
x_6	Weibull	$\beta = 0.9668, \mu = 1492$
x_7	Normal	$\mu = 2202, \sigma = 1415$
x_8	Weibull	$\beta = 1.1298, \mu = 1865$
x_9	Weibull	$\beta = 1.0787, \mu = 2266$
x_{10}	Normal	$\mu = 2744, \sigma = 1822$
x_{11}	Normal	$\mu = 6980, \sigma = 1206$

Figure 7 shows the reliability change curve of $x_1, x_2, \cdots, x_{11}$ when the regulator pry fails within 0-3000 days.

According to the analysis in Figure 9, when T (pressure regulator pry) fails, the reliability of x_6 (conductor diaphragm) declines the fastest within 0-1000 days. In 1000-2000 days, the x_7 (conductor seal ring) showed the fastest decline in reliability; There is little decrease in reliability of the flow sleeve of the regulator operating for 0-2000 days. Therefore, It can be inferred that in the event of pressure regulator failure, the conductor diaphragm and conductor seal ring have the highest likelihood of encountering failure.

3.2.4. Sensitivity

The sensitivity measures how leaf nodes respond to failure and analysis can determine how distribution parameters of root nodes affect leaf node reliability. Small changes in root node failure can result in significant differences in leaf node reliability, so high-risk events leading to system failure can be identified. This provides a theoretical foundation for risk control.

According to the analysis of Figure 10, Figure 11 and Figure 12, x_6 (conductor diaphragm) and x_7 (conductor seal ring) have the highest sensitivity when the regulator pry is slightly faulty, but the lowest sensitivity x_2 (valve position indicator) is very close when the regulator pry is seriously faulty.

The x_5 (valve level sensor) and x_{11} (flow sleeve) are the opposite. Although x_{11} (flow sleeve) has a high sensitivity, it is not a high-risk event, because the probability of flow sleeve failure is small, that is, a small probability event.

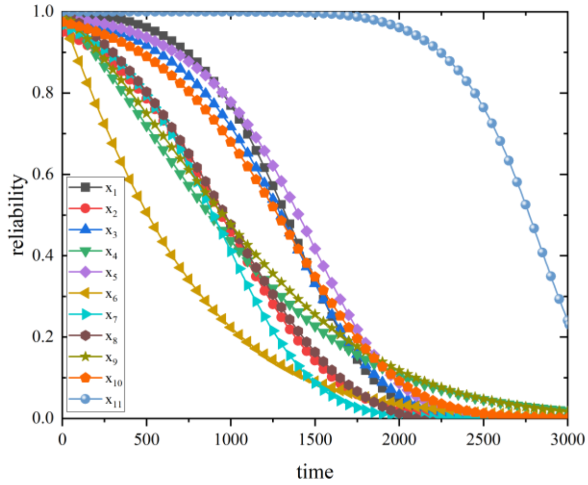


Figure 10. Reliability of $x_1, x_2, \cdots, x_{11}$ in case of failure of pressure regulating pry.

Based on sensitivity analysis, several system optimization plans can be proposed: (1) Priority allocation; (2) Dynamic adjustment of parameter threshold; (3) Redundant design enhancement; (4) Human computer interaction optimization. Regarding the maintenance schedule, it is possible to: (1) Dynamically sort the maintenance priority; (2) Optimization of preventive maintenance cycle; (3) Seasonal maintenance resource allocation; (4) Intelligent management of spare parts inventory.

4. Conclusion

This study tackles the challenge of diagnosing pressure lever faults by employing the fuzzy Bayesian network method alongside component reliability models. The method presented here successfully navigates the intricacies involved in the T-S fuzzy fault tree approach's inference and resolves issues related to determining the structure and conditional probability tables of Bayesian networks. Notably, it seamlessly integrates temporal domains, capitalizing on the individual strengths of each method. To validate the effectiveness of this approach, the regulation system's regulator skid in the processing system of the natural gas station is subjected to analysis. The method adeptly identifies weak areas within the system that periodically impact its reliability, offering valuable insights for optimizing maintenance procedures. This application demonstrates the robustness of the proposed methodology in addressing pressure lever faults and underscores its potential for enhancing the reliability of natural gas station systems. In the future, fuzzy C-means (FCM) clustering of polymorphic fault data can be used to generate fuzzy state partitioning.

Build a DBN structure with nodes containing fuzzy states (such as "pressure=high/medium/low") and fault modes. Dynamically adjust membership function parameters (such as mean/variance) based on real-time sensor data. Use approximate reasoning algorithms (such as Loop Belief Propagation) to accelerate calculations.

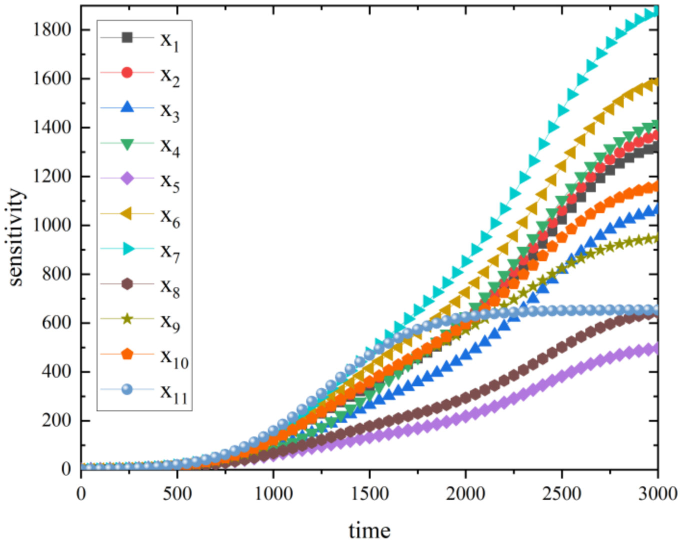


Figure 11. Sensitivity of each bottom event at $T = 0.5$

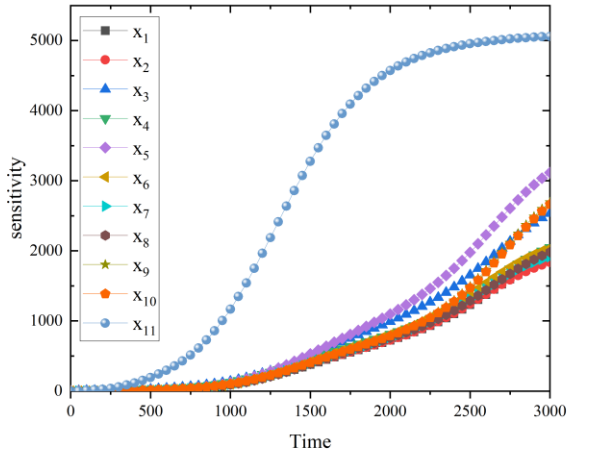


Figure 12. Sensitivity of each base event when $T = 1$

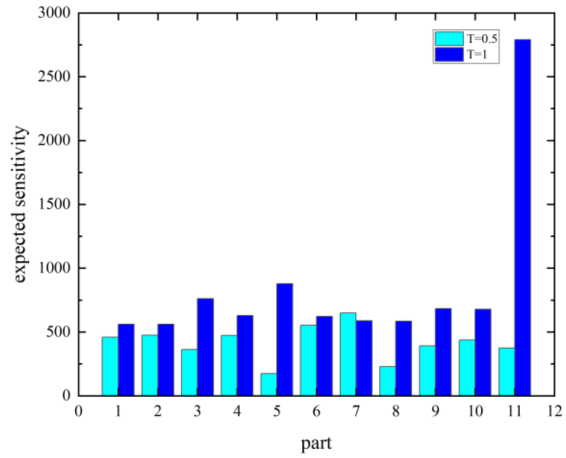


Figure 13. The expected sensitivity of each base event at $T = 0.5$ and $T = 1$.

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