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## Risk assessment of a production system based on a technological description model

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### Highlights

- Method for assessing the risk of exceeding production time in manufacturing process.
- Risk calculation considers technological route structure and statistical operation traits.
- Factors of equipment breakdown and adjustment states are included in risk calculation.
- Method estimates processing time and losses due to unproduced parts.
- A production line with seven sequential operations is analyzed for risk assessment.

### Abstract

The paper presents a method for assessing the risk of exceeding the agreed production time for a batch of products using the statistical modeling of a production system. The approach considers the structure of the technological route and statistical characteristics of operations. It accounts for both the probability of a given state, such as equipment failure or adjustment, and the distribution of time spent in that state. A production line with seven sequential operations is analyzed. Risk is defined as the probability that production time will exceed the planned order completion time. The method estimates total processing time and potential losses due to unproduced parts. The results show that batch processing time follows a distribution close to the normal law. This provides a basis for optimizing the exploitation and reliability of manufacturing systems, ensuring their efficiency and reducing downtime.

### Keywords

production risk, reliability, production system model, risk assessment, random process realization, transport delay

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### 1. Introduction

Analyzing and assessing risks in manufacturing processes enable engineers and managers to make informed decisions focused on minimizing adverse effects and improving the performance of the production system [1-3]. Therefore, to protect companies from increased risk, legal requirements are increasingly being introduced, mandating the disclosure of the adopted risk management strategy. Widely implemented standardization standards, such as ISO 9001 or ISO 31010, also require the risk of activity to be estimated in their regulations [4, 5]. In this regard, both large and small enterprises in various industries are paying increasing attention to production risk

management and their comprehensive assessment.

Production risk is most commonly defined as the product of the probability of occurrence and the magnitude of losses caused by risk factors [6]. Another widely accepted definition describes risk as the difference between the defined and achieved objectives of a production system, resulting from the influence of disruptive factors [7, 8]. In other words, risk in a production system is "the probability of a negative deviation (loss) from the planned objective occurring during the manufacturing process" [8, 9]. This means that the assessment of production risk at the operational level (manufacturing

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system) is associated with calculating the probability of producing a batch of products within the timeframe agreed upon in the contract. Consequently, the focus shifts toward evaluating the likelihood of failing to complete a production order and quantifying potential losses resulting from various technological and non-technological risk factors.

The literature offers a wide range of methods for risk analysis and assessment. These methods are compiled and organized in the IEC 31010:2019 "Risk management – Risk assessment techniques" standard, which is part of a series of risk management standards supporting ISO 31000 6. A review of risk assessment methods and techniques reveals their immense diversity. The differences pertain to various aspects, including the nature of the methods (quantitative or qualitative), the management level at which they can be applied.

A deeper analysis of these methods, along with the fact that the FMEA method is the most commonly used in risk analysis and assessment, highlights that achieving a common framework for assessing technological risks remains a significant challenge 10. This is primarily explained by the fact that even a simple version of a production line in the form of sequential technological operations is a complex stochastic dynamic system with the presence of a transport delay. As a rule, technological operations are performed in parallel and are interconnected. In addition, parallel processing of different parts takes place at different technological operations. Technological trajectories of movement of individual parts along a technological route are interconnected. Stopping the processing of parts at one technological operation due to the presence of one or another production risk factor does not mean the stopping of technological processing at related technological operations, but leads to the formation of inter-operational backlogs before the technological operation during equipment downtime [11, 12].

This study addresses a pressing industrial problem: order lead time estimation in batch manufacturing. Unpredictable delays caused by stochastic disruptions such as equipment failures and material shortages pose significant risks to production schedules. Despite the wide coverage of these disruptions in the lean and FMEA literature, the issue of quantifying order lead times remains underdeveloped and requires further research.

The main purpose of the paper is to assess the risks and losses associated with a simple and commonly used production system containing a linear production line, represented by a sequence of technological operations in accordance with the technological route at the enterprise. The paper presents a methodology for assessing production risks leading to quantitative losses in the production system. To simplify the presentation of the material, a one-dimensional technological space is used, with the total time the part spends in the technological process chosen as the coordinate axis. The selection of the coordinate space and its metrics enables the calculation of various macroparameters of the production system, which determine both the technological indicators of production and the financial indicators of the enterprise. To illustrate the methodology for assessing production system risk, the paper uses an example of a production line for manufacturing wooden single-leaf windows in a small carpentry enterprise [13]. To construct the method for assessing production risks, a technological description of the production system is employed [14, 15]. The paper demonstrates a method for assessing the risk associated with the timely production of a batch of wooden single-leaf windows within a specified time interval.

The scientific novelty of the presented solution lies in the development of an assessment of production risk based on the relationship between the technological description of the production process (micro level of description) and the flow description of the production process (macro level of description). The set of technological trajectories of individual products determine the macro parameters of the production system, such as inter-operational backlogs, the flow of products processed in the technological operation, which in turn affect the process of manufacturing the product, changing its technological trajectory.

Through this study, the authors aim to identify effective risk management strategies tailored to the specific requirements of the manufacturing sector. The paper is organized as follows. The section 2 characterizes the production risk. The section 3 describes a model of a production line consisting of the M-sequential technological operations. The section 4 presents a model of a single process operation based on statistical characteristics of the process operation execution time. In the

section 5, the technological process of manufacturing a batch of 60 wooden single-leaf windows is considered as a numerical example to test the effectiveness of the proposed method. Finally, the conclusion is given in the last section.

## 2. Production risk

Manufacturing a product in accordance with a given production technology is a purposeful process of sequentially changing the state of a part as a result of the transfer of technological resources to it. The transfer of technological resources occurs during the execution of technological operations. The standards of technological resources required to perform each technological operation are determined by the technological process of manufacturing the product. The generally accepted approach is to rate the required technological resources for each technological operation. A typical example is the minimization of costs related to materials and production processing time. The resource norm is usually considered to be a value close to the average value of consumed resources, calculated for a sufficiently large batch of products. The state of a part during processing can be represented by a point in a multi-coordinate space, each axis of which characterizes the amount of a certain type of resource transferred to the product. The process of changing the state of a part can be represented by a sequence of points in the specified multi-coordinate space, which form the technological trajectory of manufacturing the product [14]. The process of transferring technological resources is a random process and depends on many production factors and non-production factors [14-16]. As a result of the influence of these factors, the technological trajectory in multi-coordinate space deviates from the product manufacturing trajectory determined by the technological process. Deviations in technological trajectories determine production risk.

The specifics of production require a different approach to risk compared to fields like finance, where higher risk often correlates with the potential for greater returns. In production systems, risk is better defined as the probability of a negative event (risk factor) causing losses, such as unproduced items or uncompleted technological operations [1, 8, 17]. This study defines production system risk as the impact of disruptive factors (risk factors) on the achievement of production goals outlined in production plans. Risk analysis and assessment often

focus on a single aspect or functional area, with production companies typically prioritizing: a) performance risk (the likelihood that production lines, machines, or personnel fail to meet expected efficiency) [16, 18, 19]; b) scheduling risk (the possibility of missing production deadlines) [20-22]; c) cost risk (the probability of exceeding planned production or project costs) [20, 23].

In production management literature, losses are interpreted in two contexts. The first relates to the efficiency of machine utilization, measured by OEE (Overall Equipment Effectiveness), which captures losses from machine downtime, performance inefficiencies, and quality defects. The second, broader interpretation aligns with this study's risk definition, viewing losses as unmet system goals, which can vary in dimension (e.g., quantitative, financial, efficiency, or quality-related) and organizational level (e.g., strategic, project, process, or product) [24]. At the operational level, production goals are typically set in terms of the quantity and quality of goods specified in production plans.

The concept of production losses also appears frequently in Lean Manufacturing (LM) literature, where losses are equated with waste (Japanese: muda; English: waste), representing resource-consuming activities that do not add value for internal or external customers. Eliminating such losses enhances competitiveness and efficiency. Losses can originate at any stage of the production system, from raw material procurement to final product delivery. Given the complexity and integration of production systems, various frameworks are used to categorize and analyze loss causes, such as [25, 26]: a) 4M (Machine, Material, Method, Man); b) 5M (4M + Management); c) 5M+E (5M + Environment).

According to systems theory, a system is an organized, purposeful set of elements with specific properties and relationships [27]. Every system is a whole conventionally distinguished from its environment, has a defined structure (organization), is designed to fulfill specific tasks and goals, influences neighboring systems and its environment, evolves over time, and is subject to disruptive factors originating from its surroundings. The state of a production system at an arbitrary point in time can be represented in a multi-coordinate technological space as a set of states of a large number of parts. The averaging of the states of a large number of parts is

determined by the macroparameters of the state of the production system. Quantitative and quantitative assessment of macro-parameters of the state associated with specific risk factors makes it possible to develop risk management systems for a manufacturing enterprise 28.

### 3. Production line model

Consider a linear production line consisting of  $M$  sequential technological operations. The state of the system is determined through the states of the total number  $N$  of parts (products) in the production process. As a result of performing a technological operation, a part (product) transitions from one state to another state determined by the technological process. During the transition of a product from operation to operation, there is a gradual transformation of technological resources (raw materials, materials, worker labor) into a finished product as a result of the targeted influence of technological equipment 14. For a part that has completed the processing process at the  $m$ -th technological operation ( $m = 1..M$ ) at time  $t$ , the average time  $t_{\psi}$  for technological processing of the part is:

$$t_{\psi} = \sum_{k=1}^m \eta_{mean\ k}, \quad t_{d\psi} = \sum_{k=1}^M \eta_{mean\ k}, \quad (1)$$

$$\eta_{mean\ k} = M[\eta_k],$$

where  $\eta_{mean\ k}$  is the average execution time of the  $k$ -th technological operation;  $t_{d\psi}$  is the average total time of technological processing of a part in all  $M$  operations of the technological process. The time  $\eta_{mean\ k}$  does not include downtime of the part while waiting for the start of processing at the  $m$ -th technological operation. To describe the state of the part in the production process, let's introduce dimensionless parameters:

$$\tau = \frac{t}{t_{d\psi}}, \quad \tau_{\psi} = \frac{t_{\psi}}{t_{d\psi}}, \quad \tau_{\psi} \in [0,1], \quad \vartheta_{mean\ k} = \frac{\eta_{mean\ k}}{t_{d\psi}}, \quad \vartheta_{mean\ k} = M[\vartheta_k]. \quad (2)$$

Dimensionless time  $\tau_{\psi}$  determines the completion rate of the part processing process. The completion rate of the processing after processing on the last operation is equal to 1. For a part incoming for processing for the first technological operation, the completion rate of the processing is 0. To describe the state of a part during the production process, let's introduce the technological space  $(\tau, \tau_{\psi}, \vartheta)$  14. Each  $n$ -th part located in the technological process corresponds to a point in the technological space  $D_n(\tau_{p\ n}, \tau_{\psi\ p\ n}, \vartheta_{p\ n})$ . This point

characterizes the state of the  $n$ -th part at time  $\tau = \tau_{p\ n}$ :

1) part has an average processing time  $\tau_{\psi} = \tau_{\psi\ p\ n}$  (technological process completion rate  $\tau_{\psi\ p\ n}$ ). The part is located on the  $m$ -th technological operation, the number of which is determined by the inequality

$$\sum_{k=1}^{m-1} \vartheta_{mean\ k} < \tau_{\psi\ p\ n} \leq \sum_{k=1}^m \vartheta_{mean\ k}. \quad (3)$$

2) the part is being processed. The processing time is  $\vartheta = \vartheta_{p\ n}$ , in general, different from the average processing time for a technological operation  $\vartheta_{mean\ m}$ . During the processing, the value of the completion rate changes according to the following law

$$\tau_{\psi\ p\ n} = \sum_{k=1}^{m-1} \vartheta_{mean\ k} + \frac{\tau - \tau_{n\ m}}{\vartheta_{p\ n}} \vartheta_{mean\ m}. \quad (4)$$

Complete processing of the part will occur within an interval of time  $\vartheta_{p\ n} = (\tau - \tau_{n\ m})$ . Then the process completion rate after performing a technological operation  $\tau_{\psi\ p\ n}$  is calculated based on the expression (4):

$$\tau_{\psi\ p\ n} = \sum_{k=1}^{m-1} \vartheta_{mean\ k} + \frac{\vartheta_{p\ n}}{\vartheta_{p\ n}} \vartheta_{mean\ m} = \sum_{k=1}^m \vartheta_{mean\ k} \quad (5)$$

The technological process completion coefficient  $\tau_{\psi\ p\ n}$  when performing  $m$ -th technological operation changes linearly from the initial value  $\sum_{k=1}^{m-1} \vartheta_{mean\ k}$ , when the part entered the technological operation to the value  $\sum_{k=1}^m \vartheta_{mean\ k}$ , when the part completed processing in the technological operation.

### 4. Technological operation model

The execution time of the  $m$ -th technological operation is a random variable  $\vartheta_m$  with a distribution density  $f_m(\vartheta_m)$  that depends on a large number of production factors, the statistical characteristics of which are presented in Table 1 13. In the simplest case, if a factor is characterized by the probability of occurrence and average duration, then the distribution density  $f_m(\vartheta_m)$  of the random variable  $\vartheta_m$  can be written as

$$f_m(\vartheta_m) = \sum_{k=0}^K r_{k,m} \delta(\vartheta_m - \vartheta_{mean\ k,m}), \quad 1 = \int_0^{\infty} \delta(\vartheta_m - \vartheta_{mean\ k,m}) d\vartheta_m \quad (6)$$

where  $\delta(\vartheta)$  is the Dirac function. To integrate the distribution density  $f_m(\vartheta_m)$  over the range of changes of a random variable  $\vartheta_m$ , it is obtained:

$$1 = \int_0^{\infty} f_m(\vartheta_m) d\vartheta_m = \int_0^{\infty} \sum_{k=0}^K r_{k,m} \delta(\vartheta_m - \vartheta_{mean\ k,m}) d\vartheta_m = \sum_{k=0}^K \int_0^{\infty} r_{k,m} \delta(\vartheta_m - \vartheta_{mean\ k,m}) d\vartheta_m = \sum_{k=0}^K r_{k,m} = r_{0,m} + r_{1,m} + r_{2,m} + r_{3,m} + r_{4,m} + r_{5,m} + r_{6,m}. \quad (7)$$

Thus, as a result of processing the part at the  $m$ -th technological operation, one of the states occurs: a) the part is directly processed. The state probability is  $r_0$ . The average processing time is  $\vartheta_{mean\ 0,m}$ ; b) additional adjustments and settings. The state probability is  $r_1$ . The average time for setting up or reconfiguring technological parameters of an operation is  $\vartheta_{mean\ 1,m}$ ; c) breakdowns. The state probability is  $r_2$ . The average time required to perform repair work to restore equipment functionality is  $\vartheta_{mean\ 2,m}$ ; d) delays or shortages in the delivery of materials. The state probability  $r_3$ . The average

time required to provide raw materials for a technological operation is  $\vartheta_{mean\ 3,m}$ ; e) poor quality of materials. The state probability is  $r_4$ . The average time to change technological parameters of an operation as a result of using low-quality materials is  $\vartheta_{mean\ 4,m}$ ; f) differences in technological and real times. The state probability is  $r_5$ . The average time to resolve nonconformities is  $\vartheta_{mean\ 5,m}$ ; g) absence. The state probability is  $r_6$ . The average downtime for a process operation due to the absence of a worker with the required qualifications is  $\vartheta_{mean\ 6,m}$ .

Table 1. Statistical characteristics of factors characterizing a technological operation 13.

| Name factor (the operation state)                | proba-bility | Statistical characteristics of event duration |                        |
|--|--------------|---|------------------------|
|  |              | average time                                  | standard deviation     |
| characteristics workstation                      | $r_0$        | $\vartheta_{mean\ 0,m}$                       | $\vartheta_{std\ 0,m}$ |
| additional adjustments and settings              | $r_1$        | $\vartheta_{mean\ 1,m}$                       | $\vartheta_{std\ 1,m}$ |
| breakdowns                                       | $r_2$        | $\vartheta_{mean\ 2,m}$                       | $\vartheta_{std\ 2,m}$ |
| delays or shortages in the delivery of materials | $r_3$        | $\vartheta_{mean\ 3,m}$                       | $\vartheta_{std\ 3,m}$ |
| poor quality of materials                        | $r_4$        | $\vartheta_{mean\ 4,m}$                       | $\vartheta_{std\ 4,m}$ |
| differences in technological and real times      | $r_5$        | $\vartheta_{mean\ 5,m}$                       | $\vartheta_{std\ 5,m}$ |
| absence  | $r_6$        | $\vartheta_{mean\ 6,m}$                       | $\vartheta_{std\ 6,m}$ |

$$\begin{aligned} \vartheta_{mean\ m} &= \int_0^{\infty} \vartheta_m f_m(\vartheta_m) d\vartheta_m = \int_0^{\infty} \vartheta_m \sum_{k=0}^K r_{k,m} \delta(\vartheta_m - \vartheta_{mean\ k,m}) d\vartheta_m = \sum_{k=0}^K \vartheta_{mean\ k,m} r_{k,m} = \\ &= \vartheta_{mean\ 0,m} r_{0,m} + \vartheta_{mean\ 1,m} r_{1,m} + \vartheta_{mean\ 2,m} r_{2,m} + \vartheta_{mean\ 3,m} r_{3,m} + \vartheta_{mean\ 4,m} r_{4,m} + \\ &\quad \vartheta_{mean\ 5,m} r_{5,m} + \vartheta_{mean\ 6,m} r_{6,m}. \end{aligned} \quad (8)$$

Let's determine the average time for performing a technological operation. Multiply the distribution density  $f_m(\vartheta_m)$  by  $\vartheta_m$  and integrate over the range of changes with a random variable  $\vartheta_m$ :

When constructing a technological process, the condition for the equality of the values of the average processing time of a part at each technological operation is often chosen as the conditions for synchronizing the production line:

$$\vartheta_{mean\ 0,m} \cong \vartheta_{mean\ 0,n}, \quad m \neq n. \quad (9)$$

The presence of factors  $(r_1, \dots, r_6)$  causes desynchronization of the production line, the productivity of the technological equipment of which satisfies condition (9). Taking into account the factors  $(r_1, \dots, r_6)$  when determining the average execution time of a technological operation (8), the synchronization condition (9) takes the form:

$$\vartheta_{mean\ m} \cong \vartheta_{mean\ n}, \quad m \neq n. \quad (10)$$

The last condition requires the presence of a control system for the flow parameters of the production line, such as the productivity of technological equipment and the amount of interoperational backlogs before each technological operation [12, 15]. Let's consider the movement of a batch of  $N$  parts along a production line consisting of  $M$  sequential technological operations. Let's assume that the production line does not contain parts from other batches in processing. Then the processing time of a batch of parts at the  $m$ -th technological operation is a random variable

$$V_m = \sum_{n=1}^N (\vartheta_{n,m} + \theta_{n,m}). \quad (11)$$

where  $\vartheta_{n,m}$  is the processing time of the  $n$ -th part at the  $m$ -th technological operation. This time is determined by the combination of all factors influencing the duration of processing

of a part at the  $m$ -th technological operation;  $\theta_{n,m}$  is downtime of the  $n$ -th part after performing the  $m$ -th technological operation, waiting for the start of its technological processing. The amount of downtime depends on the state of the parts received for processing at the  $(m+1)$ -th technological operation. The downtime of the  $n$ -th part at the  $m$ -th technological operation can be represented by the expression

$$\theta_{n,m} = \max\left(\left[\sum_{k=1}^{m+1}(\vartheta_{(n-1),k} + \theta_{(n-1),k}) - \vartheta_{n,m} - \sum_{k=1}^{m-1}(\vartheta_{n,k} + \theta_{n,k})\right], 0\right), 0 < m < M. \quad (12)$$

Downtime of the  $n$ -th part after the first technological operation

$$\theta_{n,m=1} = \max\left(\left[\sum_{k=1}^{m+1}(\vartheta_{(n-1),k} + \theta_{(n-1),k}) - \vartheta_{n,m}\right], 0\right). \quad (13)$$

Downtime of the  $n$ -th part before the start of processing the first technological operation (conventionally can be taken as the downtime after performing the zero operation, namely the period of time from the start of the arrival of a batch of parts for processing until the start of processing of the  $n$ -th part in the first technological operation):

$$\begin{aligned} \theta_{n,m=0} &= \max\left(\left[\sum_{k=1}^{m+1}(\vartheta_{(n-1),k} + \theta_{(n-1),k})\right], 0\right) \\ &= \vartheta_{(n-1),1} + \theta_{(n-1),1} \\ &= \vartheta_{(n-1),1} + \vartheta_{(n-2),1} + \theta_{(n-2),1} \\ &= \sum_{i=1}^{n-1} \vartheta_{i,1} \end{aligned} \quad (14)$$

There is no downtime after completing the last technological operation:

$$\theta_{n,M} = 0. \quad (15)$$

The statistical characteristics of a random variable  $y = \theta_{n,m}$  are determined through the known distribution density  $f(x)$  of the random variable  $x$ :

$$\begin{aligned} x &= \sum_{k=1}^m (\vartheta_{(n-1),k} + \theta_{(n-1),k}) \\ &\quad - \sum_{k=1}^{m-1} (\vartheta_{(n-1),k} + \theta_{n,k}) \end{aligned} \quad (16)$$

in the following way

$$\begin{aligned} m_y &= \int_{-\infty}^{\infty} \max(x, 0) f(x) dx = \int_0^{\infty} x f(x) dx, \\ \sigma_y^2 &= \int_{-\infty}^{\infty} \max(x, 0)^2 f(x) dx - m_y^2 = \\ &\quad \int_0^{\infty} x^2 f(x) dx - m_y^2 \end{aligned} \quad (17)$$

Analytical determination of statistical characteristics is associated with significant difficulties. In this regard, to calculate statistical characteristics, let's use the implementation of a random process characterizing the processing of a batch of

parts on the production line. Let's consider the movement of two arbitrary  $(n-1)$ -th and  $n$ -th parts along a production line. The  $n$ -th part arrives at the production line after a time interval

$$\tau_{n,1} - \tau_{(n-1),1} = \theta_{n,1}, \quad (18)$$

after the start of processing the  $(n-1)$ -th part. During technological operations,  $n$ -th part completes processing at the last  $M$ -th operation after a time interval

$$\tau_{n,M} - \tau_{(n-1),M} = \sum_{m=1}^M \theta_{n,m}. \quad (19)$$

The total processing time of a batch of parts is determined through the time interval required to process the first part

$$\tau_{1,M} = \sum_{m=1}^M \vartheta_{1,m} + \theta_{1,m}, \quad \theta_{1,m} = 0 \quad (20)$$

and the total time interval required for processing  $(N-1)$  at the last technological operation in accordance with expression (19):

$$\tau_{N,M} - \tau_{2,M} = \sum_{n=2}^N (\tau_{n,M} - \tau_{(n-1),M}) = \sum_{n=2}^N \sum_{m=1}^M \theta_{n,m}. \quad (21)$$

Summarizing the results obtained, we obtain the total processing time of a batch  $N$  parts on a production line consisting of  $M$  technological operations:

$$\begin{aligned} \tau_{batch} &= \tau_{N,M} - \tau_{1,1} = \sum_{m=1}^M \vartheta_{1,m} + \\ &\quad \sum_{n=2}^N \sum_{m=1}^M \theta_{n,m} = \sum_{m=1}^M \vartheta_{1,m} + \\ &\quad \sum_{n=2}^N \sum_{m=1}^M \theta_{n,m}, \end{aligned} \quad (22)$$

The processing time of a batch of  $N$  parts  $\tau_{batch}$  is a random variable, which is expressed through the sum of  $N \cdot M$  independent random variables  $\vartheta_{1,m}, \theta_{n,m}$ . Let us determine the statistical characteristics of a random variable  $\tau_{batch}$ :

$$\begin{aligned} m_{\tau_{batch}} &= M[\tau_{batch}] = M\left[\sum_{m=1}^M \vartheta_{1,m} + \sum_{n=2}^N \sum_{m=1}^M \theta_{n,m}\right] = \sum_{m=1}^M M[\vartheta_{1,m}] + \\ &\quad \sum_{n=2}^N \sum_{m=1}^M M[\theta_{n,m}] = \end{aligned} \quad (23)$$

$$\begin{aligned} &= \sum_{m=1}^M \vartheta_{mean\ m} + (N-1) \sum_{m=1}^M \theta_{mean\ m}, \\ \sigma_{\tau_{batch}}^2 &= D[\tau_{batch}] = D\left[\sum_{m=1}^M \vartheta_{1,m} + \sum_{n=2}^N \sum_{m=1}^M \theta_{n,m}\right] = \sum_{m=1}^M D[\vartheta_{1,m}] + \\ &\quad \sum_{n=2}^N \sum_{m=1}^M D[\theta_{n,m}] = \\ &= \sum_{m=1}^M \vartheta_{std\ m}^2 + (N-1) \sum_{m=1}^M \theta_{std\ m}^2. \end{aligned} \quad (24)$$

For a deterministic synchronized production line, for which the processing of parts at each technological operation occurs with the same productivity, the processing time of  $N$  parts  $m_{\tau_{batch}}$  determined by the following expressions:

$$\begin{aligned} \vartheta_{mean\ m} &= \vartheta_{mean}, \quad \theta_{mean\ 1} = \vartheta_{mean}, \\ \theta_{mean\ m} &= 0, \quad m > 1, \end{aligned} \quad (25)$$

$$\begin{aligned} m_{\tau_{batch}} &= \sum_{m=1}^M \vartheta_{mean} + (N-1) \sum_{m=1}^M \vartheta_{mean} = \\ &= M \vartheta_{mean} + (N-1) \vartheta_{mean}. \end{aligned} \quad (26)$$

For a deterministic desynchronized production line, the processing time of  $N$  parts  $m_{\tau_{batch}}$  is calculated as follows:

$$\sum_{m=1}^M \theta_{mean\ m} = \max(\vartheta_{mean}), \quad (27)$$

$$m_{\tau_{batch}} = \sum_{m=1}^M \vartheta_{mean\ m} + (N - 1) \max(\vartheta_{mean}) \quad (28)$$

The expressions (23), (24) can be used to assess the risk of producing a batch of products on schedule. Macroparameters of the processing process of a batch of parts are represented by the statistical characteristics of the processing of  $n$ -th part at  $m$ -th technological operation. By adding axes to the coordinate technological space, various macroparameters of risk assessment, both industrial and financial in nature, can be constructed 25.

## 5. Analysis of results

Let's consider a technological process of the enterprise X for producing single-leaf windows. Enterprise X is a small manufacturing plant located in Poland, near Wrocław. The facility produces single- and double-leaf windows in both standard sizes and custom dimensions. The plant is equipped with modern machinery for the production and processing of window joinery, along with extensive infrastructure, including a powder coating shop, a painting facility. The quality of the products is ensured at every stage of production. The model assumes that no overtime occurs. The technological route for

producing single-leaf windows consists 7 sequential technological operations, Table 2.

The technological process (list of technological operations and workplaces) in the production of single-leaf windows was selected for analysis 13. The average processing time of a part in minutes at the  $m$ -th technological operation is represented by the value  $\eta_{mean\ 0,m}$ , ( $\eta_{mean\ k,m}$ ,  $k = 0$ ). The standard deviation for the execution time of a technological operation is  $\eta_{std\ 0,m} = 0.2\eta_{mean\ 0,m}$ . When performing a technological operation, the technological equipment may be in one of the  $k$ -th state (Table 1) with probability  $r_k$  (Table 3). The probability values  $r_k$  for the  $m$ -technological operation are presented in Table 3, and correspond to the values given in the research paper [14]. The duration of stay in the  $k$ -th state характеризуется characterized by a distribution function with mathematical expectation  $\eta_{mean\ k,m}$  (Table 2) and standard deviation: a)  $\eta_{std\ 0,m} = 0.1\eta_{mean\ 0,m}$  for  $k > 0$ ; b)  $\eta_{std\ 0,m} = 0.1\eta_{mean\ 0,m}$  for  $k = 0$ . If the technological operation is in the state  $k > 0$ , then the time required to process the part consists of the time  $\eta_{k,m}$ , associated with solving the problem of exiting the  $k$ -th risk state and directly processing the part in the technological operation  $\eta_{0,m}$  and is equal  $\eta_m = (\eta_{k,m} + \eta_{0,m})$ .

Table 2. List of technological operations and workstations in the production of single-leaf windows: mean processing time, min.

| m     | Name of the technological operation                                       | mean processing time (min), $\eta_{mean\ k,m}$ |     |     |     |      |     |     |
|-------|---|--|-----|-----|-----|------|-----|-----|
|       |   | k=0  | k=1 | k=2 | k=3 | k=4  | k=5 | k=6 |
| E1    | Cutting (work place: saw)   | 70   | 250 | 210 | 420 | 630  | 70  | 420 |
| E2    | Straightening<br>(work place: planing machine)                            | 60   | 120 | 120 | 360 | 540  | 60  | 360 |
| E3    | Planing (work place: planing machine – thicknesser)                       | 70   | 120 | 140 | 420 | 630  | 70  | 420 |
| E4    | Profiling + cutting out (work place: bottom spindle milling machine)      | 163  | 340 | 640 | 700 | 1000 | 160 | 700 |
| E5    | Grinding (work place: wide belt grinder)                                  | 70   | 200 | 100 | 720 | 1000 | 70  | 720 |
| E6    | Assembling, drilling, removing finishing grinding (work place: worktable) | 140  | 360 | 700 | 680 | 950  | 140 | 680 |
| E7    | Impregnation and painting (work place: industrial painting workshop)      | 120  | 130 | 500 | 600 | 900  | 120 | 600 |
| Total |   | 693  |     |     |     |      |     |     |

Table 3. State probability,  $r_k$ .

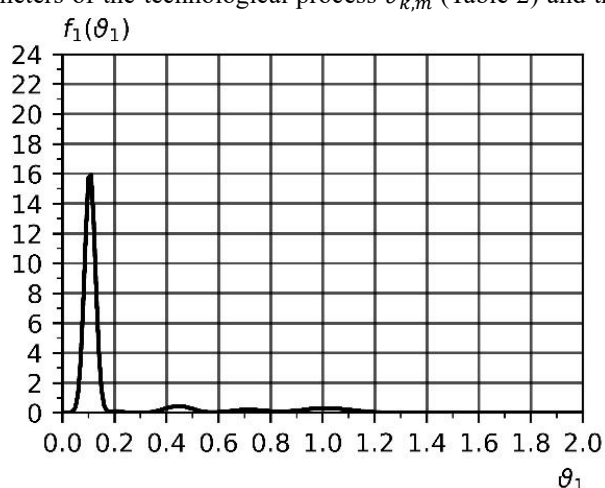
| m  | Name of the technological operation               | state probability, $r_k$ |              |       |       |              |       |              |
|----|---|--------------------------|--------------|-------|-------|--------------|-------|--------------|
|    |   | k=0                      | k=1          | k=2   | k=3   | k=4          | k=5   | k=6          |
| E1 | Cutting   | 0.834                    | 0.036        | 0.020 | 0.024 | <b>0.072</b> | 0.006 | 0.008        |
| E2 | Straightening                                     | 0.963                    | 0.002        | 0.002 |       |              | 0.009 | 0.024        |
| E3 | Planning  | 0.964                    | 0.002        | 0.004 |       |              | 0.006 | 0.024        |
| E4 | Profiling + cutting out                           | 0.896                    | <b>0.048</b> | 0.025 |       |              | 0.015 | 0.016        |
| E5 | Grinding  | 0.903                    | 0.025        | 0.042 |       |              | 0.006 | 0.024        |
| E6 | Assembling, drilling, removing finishing grinding | 0.956                    | 0.012        | 0.010 |       |              | 0.006 | 0.016        |
| E7 | Impregnation and painting                         | 0.880                    | 0.003        | 0.009 | 0.012 | 0.036        | 0.012 | <b>0.048</b> |

Using dimensionless notation (2), we present the parameters of technological operations  $\eta_{mean\ k,m}$  (Table 2) in the dimensionless form:  $\vartheta_{mean\ k,m} = \vartheta_{mean\ k,m} / t_{d\psi}$ ,  $t_{d\psi} = 693$ ,  $\vartheta_{std\ 0,m} = 0.1\vartheta_{mean\ 0,m}$ ,  $\vartheta_{std\ k,m} = 0.2\vartheta_{mean\ k,m}$ ,  $k > 0$  (Table 4). For the deterministic approximation used to describe the production process, the total production time in accordance with formula (28) is:

Table 4. Average dimensionless processing time.

| m  | Name of the technological operation               | average processing time (min), $\vartheta_{mean\ k,m}$ |       |       |       |       |       |       |
|----|---|--|-------|-------|-------|-------|-------|-------|
|    |   | k=0  | k=1   | k=2   | k=3   | k=4   | k=5   | k=6   |
| E1 | Cutting   | 0.101  | 0.361 | 0.303 | 0.606 | 0.909 | 0.101 | 0.606 |
| E2 | Straightening                                     | 0.087  | 0.173 | 0.173 | 0.519 | 0.779 | 0.086 | 0.519 |
| E3 | Planing   | 0.101  | 0.173 | 0.202 | 0.606 | 0.909 | 0.101 | 0.606 |
| E4 | Profiling + cutting out                           | 0.235  | 0.490 | 0.924 | 1.010 | 1.443 | 0.231 | 1.010 |
| E5 | Grinding  | 0.101  | 0.288 | 0.144 | 1.038 | 1.443 | 0.101 | 1.038 |
| E6 | Assembling, drilling, removing finishing grinding | 0.202  | 0.519 | 1.010 | 0.981 | 1.371 | 0.202 | 0.981 |
| E7 | Impregnation and painting                         | 0.173  | 0.188 | 0.721 | 0.865 | 1.298 | 0.173 | 0.866 |
|    | Total   | 1.000  |       |       |       |       |       |       |

In this paper, let us assume that the random variable  $\vartheta_{k,m}$ , that determines whether a part is in a state  $k$  with probability  $r_k$  has a normal distribution law. If the distribution density  $f_{k,m}(\vartheta_{k,m})$  of the random variable  $\vartheta_{k,m}$  is known, then the distribution density  $f_m(\vartheta_m)$  of the random variable  $\vartheta_m$ , can be constructed, which determines the processing time of the part at the  $m$ -th technological operation. The distribution density  $f_m(\vartheta_m)$  of a random variable  $\vartheta_m$  in accordance with the parameters of the technological process  $\vartheta_{k,m}$  (Table 2) and the



a) 1-th operation

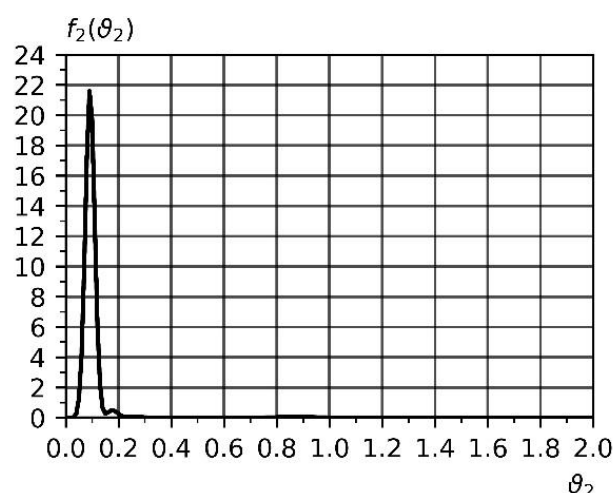
$$m_{\tau_{batch}} = \sum_{m=1}^M \vartheta_{mean\ m} + (N-1) \max(\vartheta_{mean})$$

$$= 1,0 + (60-1) \cdot 0,235$$

$$\approx 14,87$$

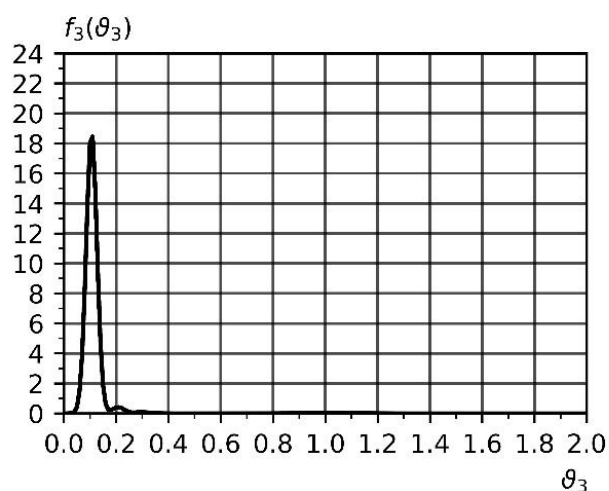
which approximately corresponds to a period equal to one month, represented in dimensionless form  $21.5 \cdot 8 \cdot 60 / 693 \approx 14,89$ .

probability  $r_k$  of the occurrence of the  $k$ -th state for each technological operation are presented in Fig. 1. Distribution functions are presented on the same scale for ease of analysis of the results. The distribution density  $f_m(\vartheta_m)$  for a technological operation is represented by a multimodal function, characterizing for each technological operation the risk of the  $k$ -th state occurring. The main mode is the state of processing of the part in accordance with the technological process.

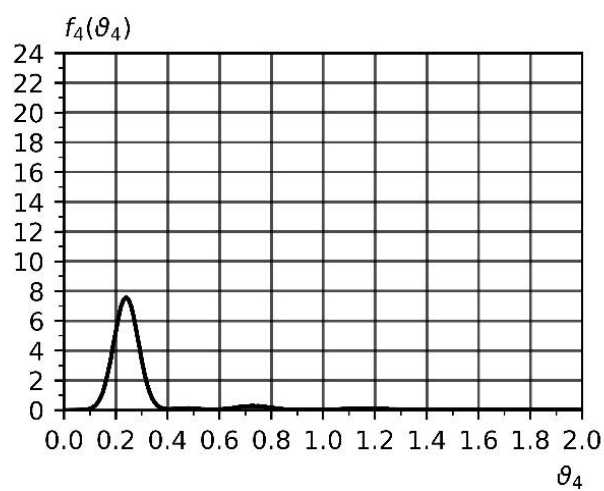


b) 2-th operation

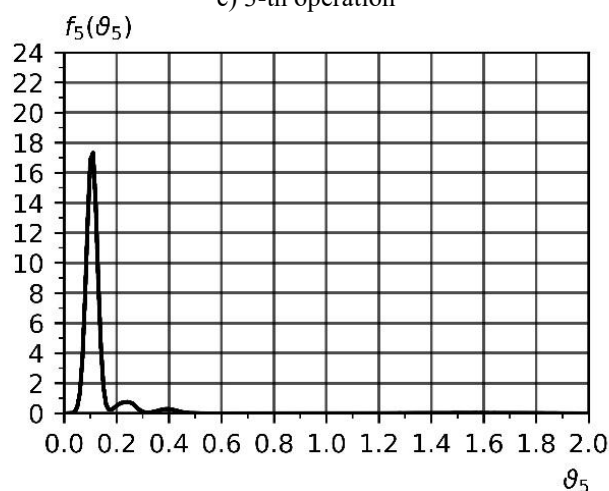




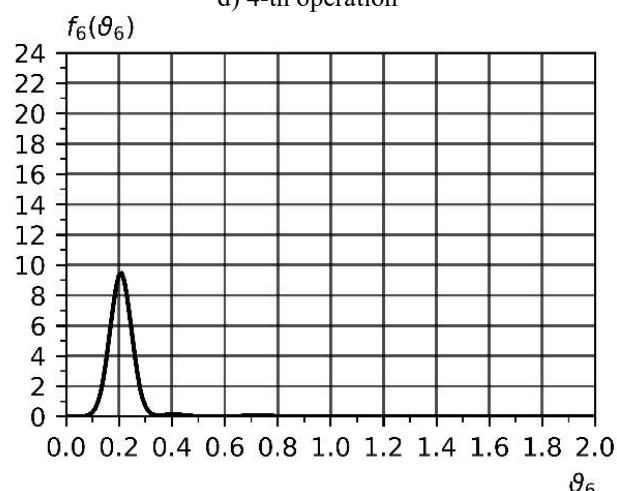
c) 3-th operation



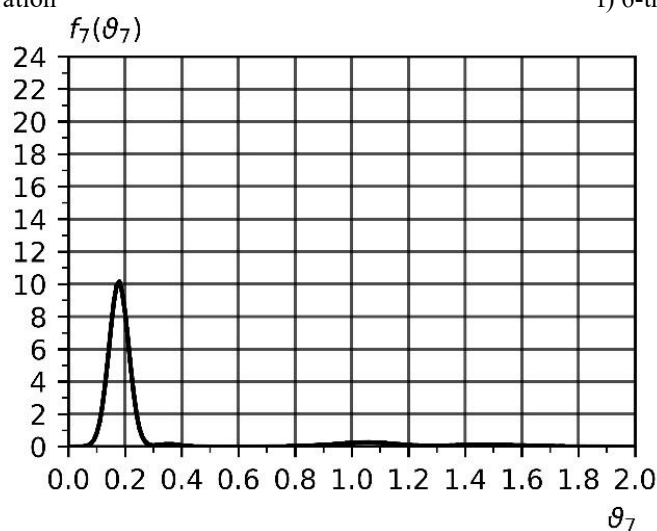
d) 4-th operation



e) 5-th operation



f) 6-th operation



g) 7-th operation

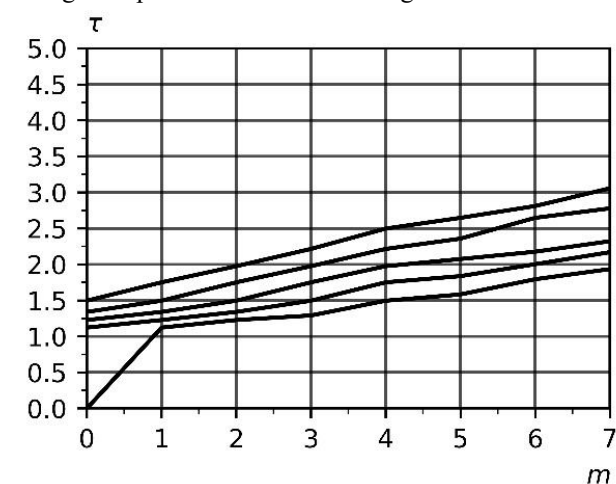
Figure 1. distribution density  $f_m(\theta_m)$  of the processing time of the part for the 1..7-th technological operation: a) – g).

Technological trajectories  $\tau_{n,m}$  ( $m = 1..7$ ) of movement of the along the technological route are presented in Fig. 2. The processing time of a part in a technological operation is a random variable. When modeling the technological trajectory, it was assumed that the production line did not contain parts from other batches in processing. It was also assumed that there

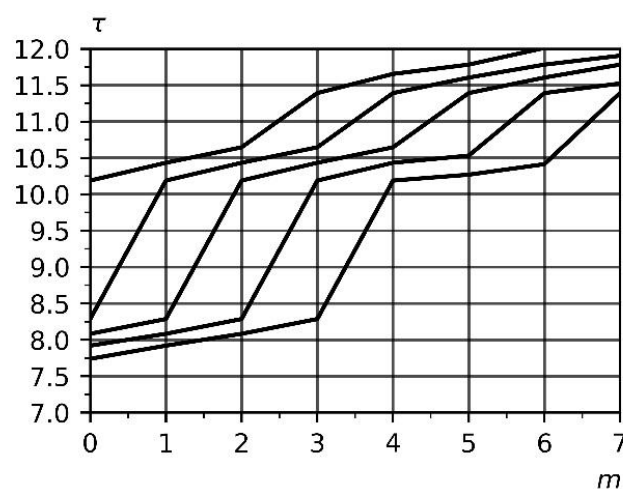
were no inter-operational backlogs between operations. At each technological operation, only one part can be in the processing state (with processing time  $\vartheta_{n,m}$ ) or in the waiting state  $m = 1..7$  (with waiting time  $\theta_{n,m}$ ). This assumption does not affect the total processing time of a batch of parts, but affects the structure of technological trajectories, which are interdependent.

Changing the trajectory of the  $(n - 1)$ -th part affects the structure of trajectory of the  $n$ -th part. The introduced assumption leads to the formation of a backward wave of propagation of the delay, that arose at the  $m$ -th operation for the  $n$ -th part to the parts following it, which is determined by equations (12)-(15). Technological trajectories of parts 1-5, parts 30-35 and parts 55-60 are presented in Fig. 2a-2c. A comparative analysis of Fig. 2a-2c demonstrates the difference in the structure of technological trajectories, which is explained by the stochastic process of processing parts in technological operations. Each of the figures demonstrates the

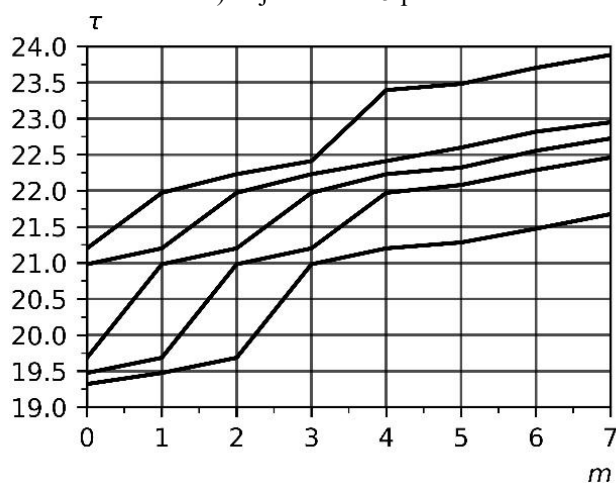
propagation of waves of delays in the execution of a technological operation. Moreover, the delay value is different for Fig. 2a-2c. This is explained by the finite value of the propagation speed of the delay wave and the formation of a new wave after the previous wave has reached the first technological operation. The waves that stand out most in terms of delay are presented as rarefactions on the graph of the trajectories of a batch of products (Fig. 2 d). The rarefactions are formed as a result of the occurrence of  $k$ -state ( $k > 0$ ) with the probability of the state  $r_k$  (Table.1).



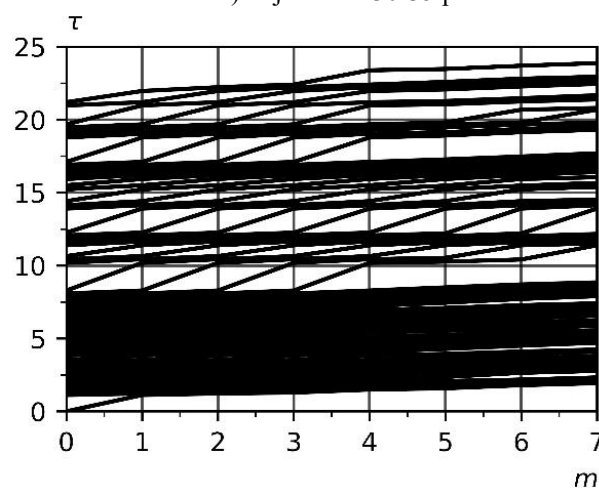
a) trajectories 1-5 parts



b) trajectories 30-35 parts



c) trajectories 55-60 parts



d) trajectories 1-60 parts

Figure 2. Technological trajectories of movement of parts along a technological route (realization of a random process of manufacturing parts).

For comparative analysis, Fig. 3 shows the technological trajectories of parts for the deterministic case. In this case, at each technological operation, the parts can be in only one state  $r_0 \equiv 1$  with a constant processing time  $\vartheta_m = \vartheta_{mean\ 0,m}$ . The probability of occurrence of the remaining states is zero:  $r_k \equiv 0$  for  $k > 0$ . As in the case of a stochastic process (Fig. 2a), at the

initial moment of time a reverse delay wave of duration. This wave affects and modifies the trajectories of the first three parts. The reason for the wave propagation is that the processing time of parts in each of the  $M$  technological operations differs (Table 4). The wave occurs when the first part passes through the fourth technological operation, the technological operation with the

maximum processing time  $\vartheta_{mean\ 4} = 0,235$  (Table 4). This leads to the fact that parts leave processing after a time interval equal to  $\vartheta_{mean\ 4} = 0,235$ . At the same interval, after the wave has reached the first technological operation, subsequent parts are sent for technological processing, which corresponds to the results presented by formulas (27), (28). In this regard, it should be noted that starting from the fourth part, technological trajectories are similar and can be constructed by parallel transfer of the fourth trajectory along the axis with a step  $\max(\vartheta_{mean\ m}) = \vartheta_{mean\ 4}$ . Additionally, it should be noted that Fig. 3a clearly demonstrates formula (28) for calculating the total production time of a batch of parts  $\tau_{batch}$ .

The formula consists of two parts: the first part determines the production time of the first part; the second part represents  $(N - 1)$  successive trajectory shifts with the above step  $\max(\vartheta_{mean\ m})$ . For a deterministic process, there are no rarefactions (Fig. 3d), which are characteristic of a stochastic process (Fig. 2d). In addition, (Fig. 3d) can be used to estimate the total time  $\tau_{batch}$ , required to process a batch of  $N$  parts. For the production of a batch of 60 wooden single-leaf windows on a small carpentry, the time  $m_{\tau_{batch}}$  can be determined visually and amounts  $\sim 15$  dimensionless units. The deterministic process allows us to give a lower estimate of the time required to manufacture a batch of products.

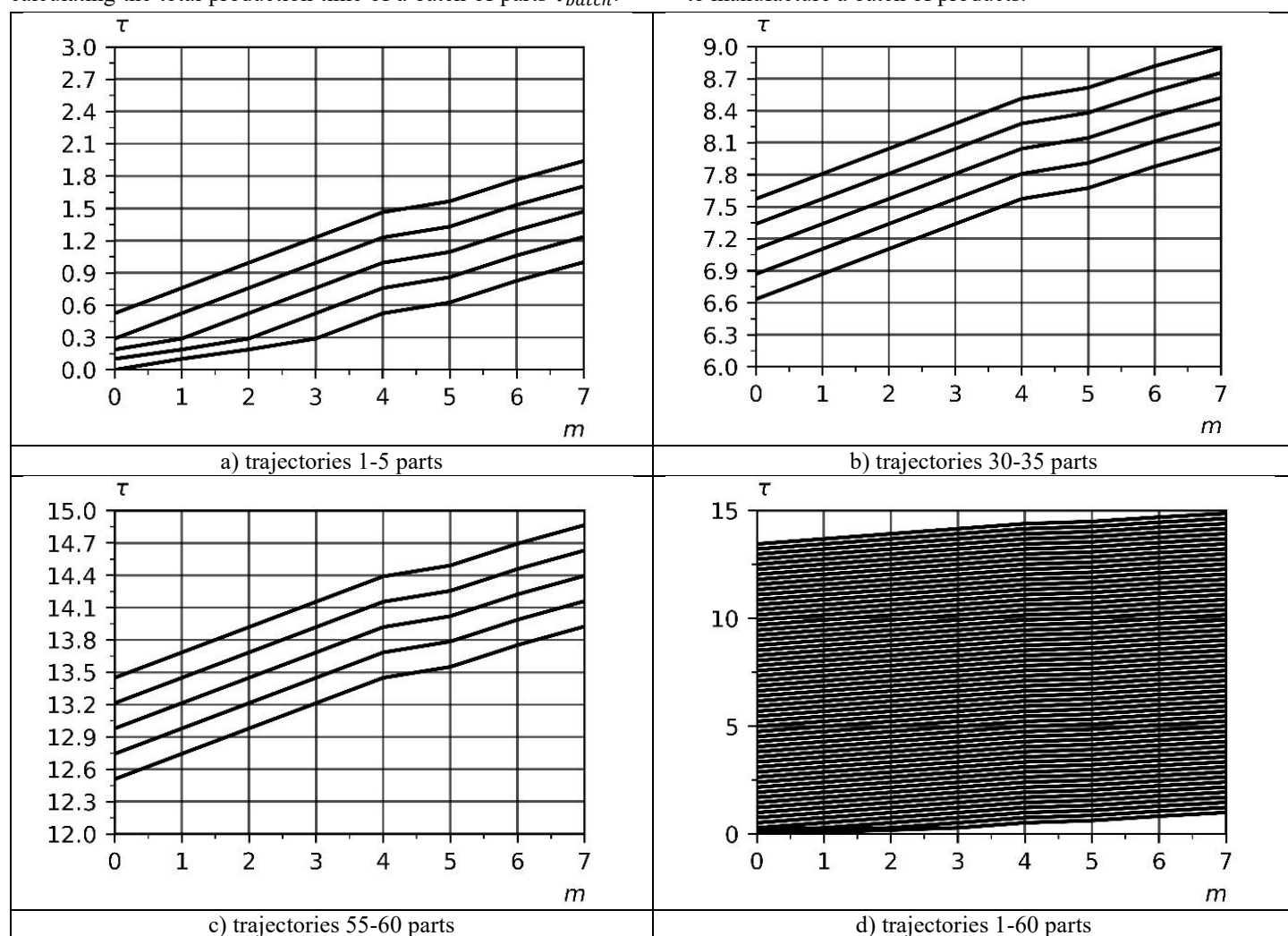


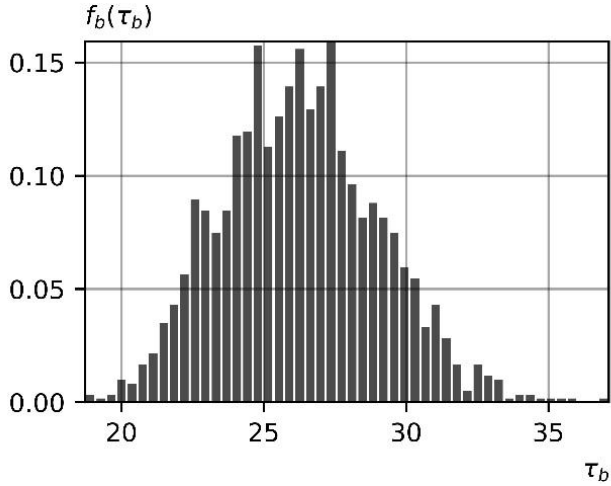
Figure 3. Technological trajectories of movement of parts along the technological route (realization of a deterministic process of manufacturing parts).

The next step after determining the parameters of  $N = 60$  technological trajectories of the parts is to calculate the total time  $\tau_{batch}$ , required to process a batch of  $N = 60$  parts. For the calculation, the formula (22) is used, which contains random variables  $\vartheta_{1,m}$ ,  $\theta_{n,m}$ . For a deterministic approximation, the

result of an analytical calculation of the average production time for a batch of wooden single-leaf windows at a small carpentry enterprise is given in (29) and amounts to  $m_{\tau_{batch}} = 14,87$ . A close value for the total time  $\tau_{batch}$  was obtained above as a result of the analysis of Fig. 3 (the value  $\sim 15$  dimensionless

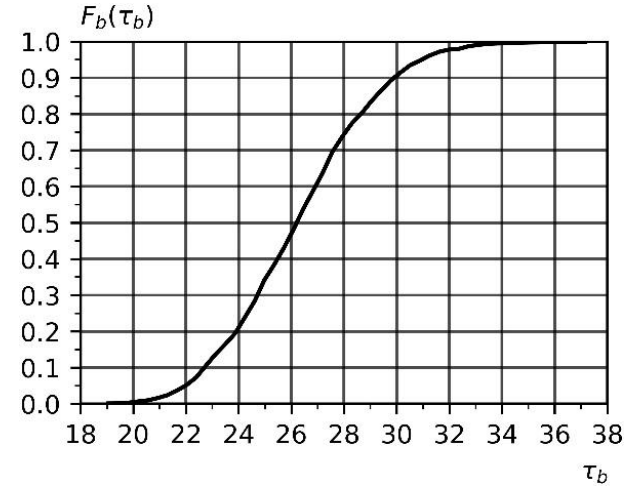
units).

Let's also consider a method for calculating the total production time  $\tau_{batch}$  of a batch of parts, based on the statistical characteristics of random variables  $\vartheta_m$ , Fig. 1. When constructing the distribution density and distribution function, it was assumed that only one part can be in a technological operation (either in a processing state or in a waiting state). The distribution density and distribution function of the order processing time  $\tau_{batch}$  for 60 windows calculated in this way



a) distribution density  $\tau_{batch}$

are presented in Fig. 4. To construct the distribution density and distribution function,  $10^5$  realizations of the random process  $\tau_{batch}$  were used, Fig. 2d. To build each implementation, a system of equations is solved that determines the structure of the technological trajectories of each part in accordance with (12)-(15). To calculate the production time of a batch of parts, simulation modeling can be used as an alternative approach, which allows taking into account interoperational backlogs at each technological operation.



b) distribution function  $\tau_{batch}$

Figure 4. Distribution law of the total processing time  $\tau_{batch}$  for the batch of 60 wooden single-leaf windows.

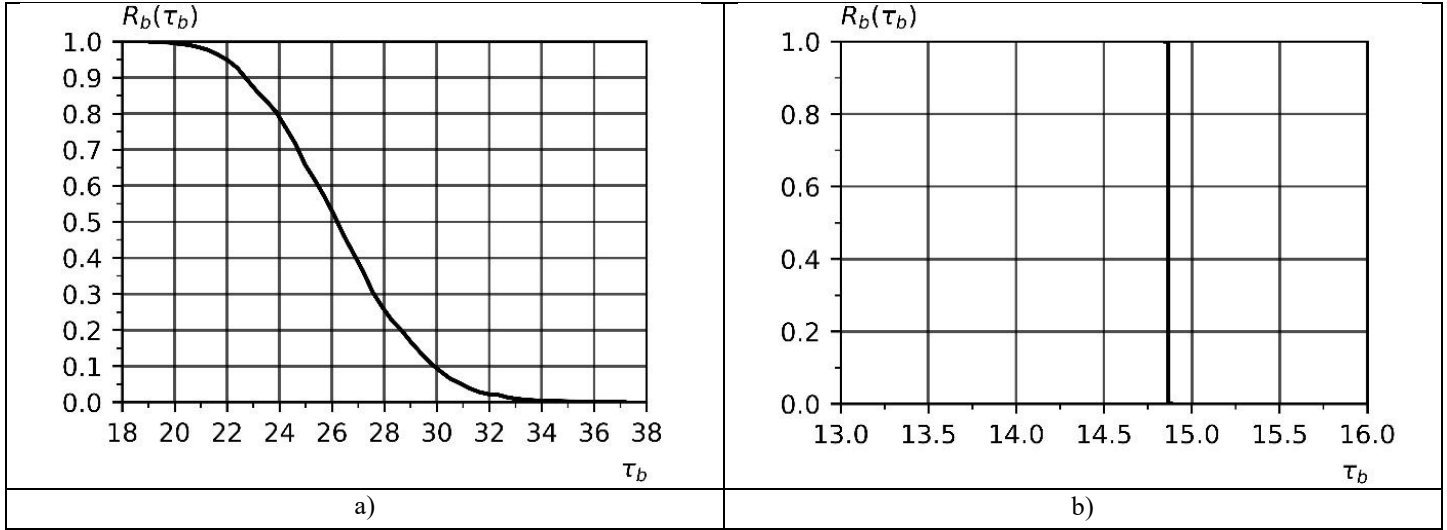


Figure 5. Risk function  $R_b(\tau_b)$  of exceeding production time for a batch of 60 wooden single-leaf windows: a) stochastic process; b) deterministic process.

The distribution function  $F_b(\tau_b)$  determines the probability that the production time  $\tau_{batch}$  for a batch of 60 wooden single-hung windows will not exceed time  $\tau_b$ . So, for example, with a probability of 0.9 it can be stated that the production time for a batch of 60 wooden single-leaf windows will not exceed 30, while only with a probability of 0.2 it can be stated that the time  $\tau_{batch} \leq 24$ . This approach allows you to flexibly estimate the

duration of production of a batch of parts depending on a given probability. Since the production time  $\tau_{batch}$  of a batch of 60 wooden single-hung windows is determined by a large number of random variables, it should be expected that the random variable  $\tau_{batch}$  is distributed according to a law close to the normal law. The results presented in Fig. 3 confirm this assumption. The probability that the production time  $\tau_{batch}$  for

a batch of products will exceed  $\tau_b$  can be defined as the probability that a batch of products will not be produced within the agreed time  $\tau_b$ :

$$R_b(\tau_b) = 1 - F_b(\tau_b). \quad (30)$$

The function  $R_b(\tau_b)$  can be interpreted as the risk of the production system exceeding the agreed production time for a batch of parts. Fig. 5 shows the risk functions  $R_b(\tau_b)$  exceeding the production time  $\tau_b$  for a batch of 60 wooden single-leaf windows for a stochastic process, a separate realization of which is represented by a set of technological trajectories in Fig. 2, and for a deterministic process, a separate realization of which is represented by a set of technological trajectories in Fig. 3.

The risk function for a deterministic process can be represented by the Heaviside function  $R_b(\tau_b) = H(m_{\tau_{batch}} - \tau_b)$ , where  $H(x) = 1$ , for  $x \geq 0$  и  $H(x) = 0$ , for  $x < 0$ . It should be noted that to construct the risk function  $R_b(\tau_b)$  for both the stochastic process and the deterministic process,  $10^5$

realizations of  $\tau_{batch}$  were used. Each point in time  $\tau_b$  corresponds to losses of the production system presented in the form of unproduced parts  $N_L$ . If  $N_{det}$  parts can be produced over a time interval  $\tau_{det}$  for a deterministic process, then losses in the form of unproduced products  $N_L$  over the time period  $\tau_{det}$  are calculated as follows:

$$N_L = N \left( 1 - \frac{\tau_{det}}{\tau_b} \right)_{det}. \quad (31)$$

where  $\tau_{det}$  is the time required to produce  $N_{det}$  parts for the deterministic case of operation of the production system (Figure 4), when there are no risks  $r_1, \dots, r_6$ , and the processing time  $\vartheta_{mean\ 0,m}$  of the part for each  $m$ -th technological operation  $m$ -ой is deterministic time. Using the transformation rule (31), we calculate the distribution function  $F_L(N_L)$  and risk function  $R_L(N_L)$  of the random variable  $N_L$ :

$$F_L(N_L) = F_b(\tau_b), \quad R_L(N_L) = 1 - F_L(N_L) \quad (32)$$

The distribution function  $F_L(N_L)$  determines the probability that over time  $\tau_{det}$  (29) number of unproduced parts from a batch  $N_{det}$  will not exceed the value  $N_L$ , Fig. 6.

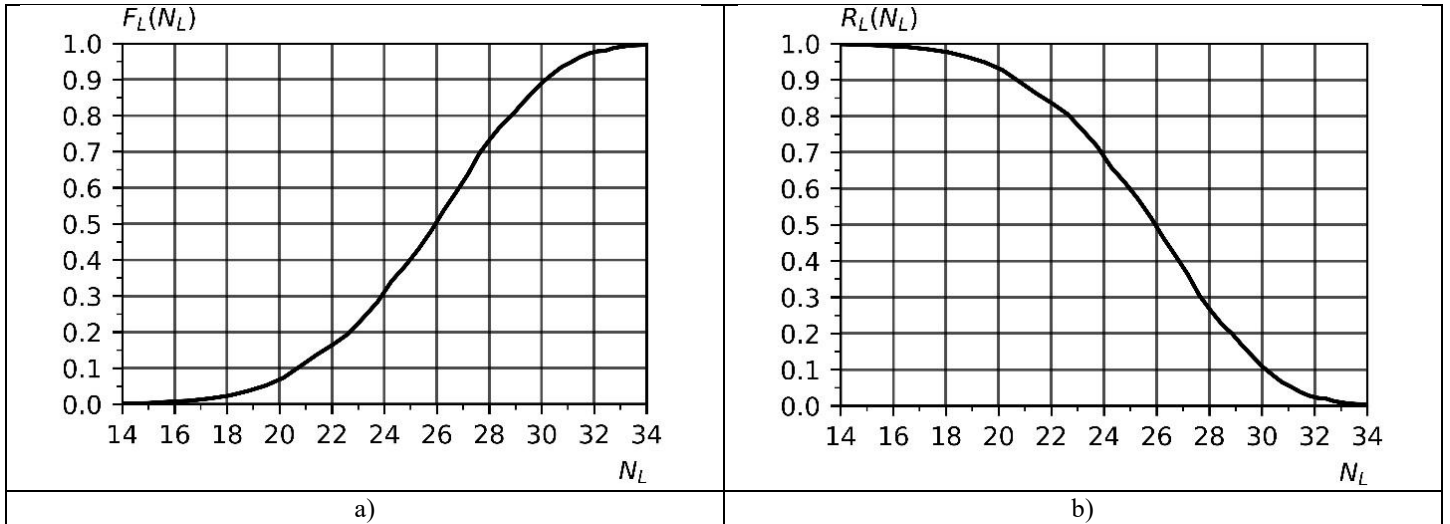


Figure 6. Distribution function  $F_L(N_L)$  a) and loss risk function  $R_L(N_L)$  b) for a batch of 60 wooden single-leaf windows per time interval  $\tau_{det}$ .

The results presented in Figure 6 allow us to make important conclusions regarding production line losses as a result of the presence of risks in a stochastic production process. The minimum amount of loss is 14 windows, namely  $N_L \rightarrow 14$  dimensionless units for  $R_L(N_L) \rightarrow 1$ . Similarly, the maximum value of production losses is  $N_L \rightarrow 34$  for  $R_L(N_L) \rightarrow 0$ . Of course, there is a possibility that there will be no losses in the production system, but the value of this probability is so small that such an event cannot be realized in production conditions. Thus, with a fairly high degree of probability, losses of the

production system under study are contained in the interval  $N_L \in [14; 34]$ , which is approximately 25%~50% of the planned production volume for the time interval  $\tau_{det}$ . Based on the risk function  $R_L(N_L)$  of production losses are: a)  $N_L \cong 24$  dimensionless units corresponds to the value of the risk function  $R_L(24) \cong 0.7$ ; b)  $N_L \cong 28$  dimensionless units corresponds to the value of the risk function  $R_L(28) \cong 0.27$ ; c)  $N_L > 34$  correspond to the values of the risk function  $R_L(N_L) < R_L(34) \rightarrow 0$ . Values  $N_L$  are calculated using formula (31).

In order to conduct a comparative analysis of the proposed

approach to risk assessment using the technological model of the production system, the same initial data (Table 1) considered in [13] were used as initial data for the risk analysis of the production system. The first important addition to the original model [13] is the addition of statistical characteristics describing the functioning of the technological equipment used to perform the technological operation, as well as the fact that the technological route for manufacturing the product is specified. This made it possible to unambiguously determine the sequence of movement of products through technological operations, taking into account the stochastic nature of the time it takes to complete a technological operation. The second important addition is that to determine the probability  $r_{k,m}$  of finding technological equipment in one of the  $k$ -states, the distribution density of the time the equipment is in the  $k$ -state is introduced.

In the limiting case, when the distribution density  $f_m(\vartheta_m)$  is represented by the Dirac function (6), (7), a model is obtained for determining the probability  $r_{k,m}$  of finding process equipment in one of the  $k$ -states, presented in the work [13]. The use of these two additions made it possible to represent the process of manufacturing a batch of products as a random process, in which the trajectory of movement of an individual product along the technological route is considered as a separate implementation of the stochastic process of processing the product. The use of the Dirac function in the distribution density  $f_m(\vartheta_m)$  allows us to perform a limit transition from the model studied in this paper to the model presented in [13], demonstrating the fact that the FMEA model for a production line with a sequential arrangement of technological operations [13] is a special case of the proposed approach. Thus, the proposed approach links the technological level of production description, at which the technological process of processing an individual product is considered, and the macroscopic level, at which the flow parameters of the production system are considered, as well as such macroparameters as the duration of production of a batch of products. This made it possible not only to determine the potential losses that may occur, as was excellently done in [13], but also to estimate the probability values with which these losses may occur. For example, in work [13] for a batch of 60 units, losses were estimated at 28 units. At the same time, the developed model for the same batch of 60

units estimates losses at 28 units with a probability of  $R_L(28) \cong 0.27$ . Additionally, the final formula (32), which is demonstrated in Fig. 6, allows us to indicate for the same batch losses in the amount of 24 units with a probability of  $R_L(24) \cong 0.7$ , and losses in the amount of 34 units with a probability close to zero:  $R_L(34) \rightarrow 0$ . Probabilistic assessment of production losses, based on the technological approach to the description of the production line, allows to expand the horizons of planning and control of the production process. The company's management has an opportunity to compare the income from additional orders received and the penalties in the event of losses incurred while fulfilling these orders. Also, calculations using the developed model show that maximum losses will not exceed 34 units, while the enterprise will produce at least 26 products.

On the other hand, the calculation shows that with an unchanged production process, the losses will be at least 14 units:  $R_L(14) \cong 1.0$ , which determines the maximum permissible volume of production. However, despite the above additional capabilities for assessing production risks, the proposed model has a number of limitations. The first limitation is that the model describes a production line with sequential product processing operations. Further improvement of the model is required to analyze complex production lines with a process flow containing both parallel and sequential process operations. As a second limitation, it should be noted that the model does not assume interaction between product batches, and does not take into account priorities in processing product batches that arrived at the input of the production line at different times, while production is a complex dynamic system with a stochastic flow of orders for the manufacture of product batches. These issues will be addressed in separate studies.

## 6. Conclusion

The paper addresses the problem of assessing the risk associated with the production of a batch of products within a specified time frame. The study employs the statistical theory to model a production line defined as a linear sequence of technological operations. Each technological operation is represented by a set of states, including: technological processing of the product, reconfiguration of technological parameters, equipment breakdown, downtime due to a lack of materials, changes in

processing parameters due to low-quality raw materials, and employee absence from the workstation. The duration of each state is characterized by distribution functions, mean time, and the standard deviation of product processing time (or equipment downtime). The sequence of technological operations through which a product passes forms the realization of a stochastic manufacturing process. Multiple technological trajectories for producing a batch of products create the realization of a stochastic batch production process, characterized by the batch processing time. It is demonstrated that the processing time of a product batch is a random variable following a distribution close to normal. This time is determined by the stochastic parameters of technological operations, particularly when the batch contains a large number of products. The distribution function of the total batch processing time serves as the foundation for constructing a risk model. A comparative analysis of deterministic and stochastic processes in batch production is also conducted.

The key advantages of the proposed approach include the consideration of the technological route structure and the statistical characteristics of technological operations. When calculating risk, the method accounts not only for the probability of an operation being in a specific state (e.g., equipment breakdown or adjustment) but also for the distribution of the time the product spends in that state, characterized by the expected value and standard deviation. A technological description of the production system is used to model the production line. As an example, the time interval for

processing a batch of parts on a production line consisting of seven sequential technological operations is analyzed. Risk is defined as the probability that the production time for a batch of parts will exceed the planned order fulfillment time. The proposed method allows for estimating the total processing time of a product batch for a given risk indicator, as well as the production system's losses in the form of unproduced parts. It is shown that the production time of a batch of parts follows a distribution close to normal.

The practical significance of this study lies in the developed methodology for quantitative risk assessment in operational terms, which allows managers to assess potential losses and accordingly allocate buffers or strategies to mitigate these losses, which corresponds to ISO 31000 standards [5] in risk management based on production data.

Future research directions include: 1) assessing the risk of exceeding the agreed production time for a batch of parts in a production line with a technological route consisting of parallel operations; 2) developing a risk model for a production line processing multiple batches of parts simultaneously; and 3) modeling the processing of product batches with a stochastic order flow.

This paper makes a contribution to the development of risk management methods in production systems, offering tools for precise assessment and control of risks related to production time. The proposed methodology provides a structured framework for improving production efficiency and supports decision-making processes in manufacturing enterprises.

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