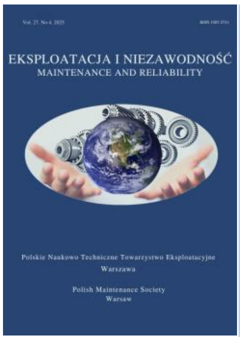

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Research on Dynamic Weighted FMECA for Reliability of CNC Machine Tools Based on Spherical Fuzzy Sets

Indexed by:

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Highlights

- This study performs FMECA based on SFS, providing results with higher rationality.
- This study integrates SFS-based entropy weight method and AHP to assign dynamic weights.
- SF-WASPAS is used to provide failure mode ranking for CNC machine tools in different ages.

Abstract

In this study, a novel dynamic FMECA method based on Spherical Fuzzy Sets (SFSs) is proposed to address the limitations of traditional FMECA in the reliability analysis of CNC machine tools, particularly the issue of neglecting the dynamic changes of CNC machine tools due to service age. The proposed method integrates objective and subjective weighting by combining an SFS-based entropy weighting method with SFS-AHP, allowing for the management of expert fuzzy evaluations and a multi-perspective weighting of risk factors. SFS-based WASPAS is used to generate dynamic rankings at expert-suggested time points (1,000 hours and 10,000 hours), incorporating service age to provide age-specific failure mode rankings. The effectiveness of the method is validated through a case study on T-model CNC machine tools. The results show that failure mode rankings change with service age, demonstrating that this method provides more valuable insights for reliability-related decision-making, such as design improvements and maintenance planning.

Keywords
CNC machine tool, dynamic FMECA, reliability analysis, spherical fuzzy sets

1. Introduction

With the rapid development of the manufacturing industry, both the standard of living and the global economy have continuously improved. To drive progress in the manufacturing industry and shift from a focus on quantity to a balance of both quality and quantity, it is essential to enhance the reliability of industrial machines. As CNC machine tools are the "mother machines" of modern industry, improving their reliability is crucial for the entire industrial transition, enabling higher precision and efficiency in production processes. Reliability analysis, as the initial step in reliability technology, serves as a prerequisite for improving equipment reliability. Failure Mode and Effects Analysis (FMEA) is a widely adopted reliability analysis technique with a long history of application. FMEA can be used during the design phase of a product to conduct

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a hierarchical analysis and identify potential failure modes, subsequently assessing the consequences of each failure mode as a basis for further product improvements. When combined with Criticality Analysis (CA), FMEA becomes Failure Mode, Effects, and Criticality Analysis (FMECA), incorporating an evaluation of failure criticality and enhancing its practical application. It has been increasingly applied in the field of CNC machine tools in recent years [1-4]. Existing studies conducted on the impact of time variation on failure mode ranking are limited.

The Risk Priority Number (RPN) method is a commonly used analysis technique within FMEA and FMECA, so works of literature for both analyses are referenced. In the RPN method, experts are invited to score each failure mode based on three risk factors: Occurrence (O), Severity (S), and Detection (D). The RPN for each failure mode is then calculated by multiplying these three risk factors together. By ranking the failure modes according to their RPN values, the weak points of the subsystems can be identified. The traditional FMEA/FMECA for CNC machine tools has the following shortcomings: ①It only considers the three risk factors of O, S, and D. Given that CNC machine tools are high-reliability products, neglecting some particular failure modes could cause significant economic losses, therefore, their economic impact is crucial in the analysis. Additionally, the traditional concept of severity is overly simplistic and may overlook certain potential failure modes; ② It fails to address the fuzziness in expert evaluations, which may lead to inaccuracies when merging evaluations; ③When multiple experts evaluate each failure mode, the traditional method does not consider the weights of the experts. Due to differences in industry experience and research background, different experts may have different levels of understanding of machine tools. Failing to assign weights could lead to biased outcomes; ④The traditional method assumes that all risk factors are equally important. However, the importance of each risk factor differs and changes with the service age of the CNC machine tools. Ignoring the weights of different factors may overlook critical failure modes under specific conditions; ⑤The RPN algorithm simply multiplies the values of all risk factors. It may not provide an optimal ranking, potentially resulting in inaccurate prioritization. Many studies have explored different methods to address some of the above issues with traditional FMEA/FMECA, most of which focus on better selection of risk factors, expert fuzziness expression, risk factor weight allocation, and improving RPN calculation. Nikhil M. Thoppil et al. [5] employed Gaussian fuzzy FMECA to assess CNC lathe subsystems, yielding rankings that are more closely aligned with industrial data and expert evaluations, thus providing more precise prioritization for the maintenance team. Chakhrit et al. [6] proposed a fuzzy resilience-based RPN model, this model extended risk factors beyond the traditional RPN method so that system cost, sustainability, and safety are comprehensively considered. The Fuzzy Analytical Hierarchy Process (F-AHP) and grey relational analysis are used to determine the subjective weights of different risk factors, while the entropy method is employed to calculate the objective weights. Lijuan Yu et al. [1] subdivided Severity S into severity for humans, machinery, the environment, and customer product satisfaction. They then used the cognitive Best-Worst Method (BWM) to weight risk factors and employed the Data Envelopment Analysis (DEA) method to adjust the efficiency of RPN evaluations, ensuring more reasonable rankings based on economic output. Several studies have been conducted on dynamic FMEA/FMECA in other areas of research, some of which provide valuable insights for this study on CNC machine tools. Gan et al. [7] proposed a computer-integrated fuzzy dynamic FMEA to optimize quality management in dynamic supply chains. The iterative approach provides adaptability in a flexible-based environment. Chakhrit et al., Chakhrit and Chennoufi, Chennoufi and Chakhrit [8-11] proposed a neuro-fuzzy inference system-based FMEA/FMECA, which can effectively, dynamically, and intelligently predict the ranking of failure modes. The method was applied to different industrial situations to demonstrate its effectiveness. Di Nardo et al. [12] propose the EN-B-ED Dynamic FMECA, which combines the Entropy Method and BWM Evaluation based on the Distance from Average Solution (EDAS) method while considering System Dynamics. The method is applied to an important Italian company in the agri-food sector which shows a reliable performance. While there are some studies on dynamic FMEA/FMECA, research on the impact of CNC machine tool service age on risk factor weights remains limited.

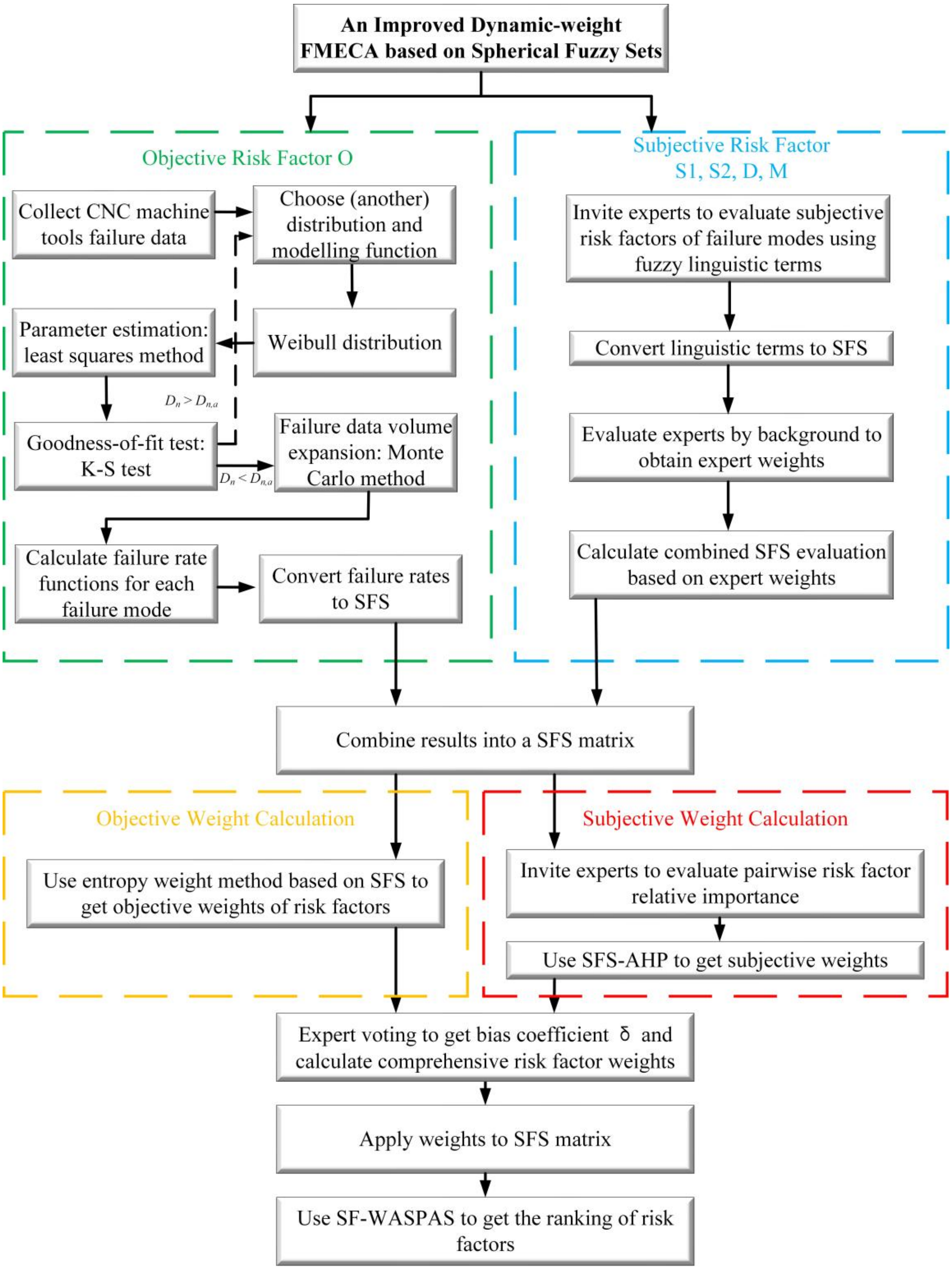

Fig. 1. Process flowchart of the proposed FMECA method.

In response to the five issues identified above, this paper proposes the following solutions:①Building on the traditional risk factors of Occurrence O, Severity S, and Detection D, this paper introduces Maintainability M to represent the economic loss associated with each failure mode. Additionally, Severity S is subdivided into two sub-factors: Severity to the machine tool S1 and Severity to the operational site S2, allowing for a more precise identification of potential failure modes. Among these, Occurrence O is an objective risk factor derived from historical failure data, while the remaining risk factors are subjective and determined through expert evaluation; ②Experts are scored based on their backgrounds, and their FMECA evaluation results are weighted according to these scores; ③SFS is used to express fuzzy information from experts. ④A dynamic weighting method based on SFS is proposed, which integrates the entropy weight method and AHP, both based on spherical fuzzy sets (SFSs), to provide dynamic risk factor weights that change according to the service age of the machine tool; ⑤ A Weighted Aggregated Sum Product Assessment (WASPAS) method based on SFSs is proposed to provide a more reasonable ranking of RPN, aiming to achieve an optimal prioritization of failure modes. The process of the improved dynamic-weight FMECA method is illustrated in Figure 1. Section 2 presents the detailed methodology of the proposed method. Section 3 provides a case study and subsequent discussion on T-model CNC machine tools to demonstrate the effectiveness of the proposed method. Finally, Section 4 concludes the study.

2. Methodology

2.1. Failure Mode Failure Rate Function Establishment

2.1.1. Reliability Function Modelling Based on Weibull Distribution

To analyze the reliability of CNC machine tools and derive failure mode failure rate functions, reliability modeling must first be conducted. Failures in CNC machine tools are random events with inherent uncertainty, making probabilistic models particularly suitable for this analysis. Common probabilistic models include the exponential distribution, normal distribution, and Weibull distribution models. Due to its strong adaptability, the Weibull model is widely used in the field of CNC machine tool failure analysis [13-17]. Therefore, this paper adopts the Weibull model for reliability modeling of CNC machine tools and their subsystems. The Weibull-based reliability modeling function used in this paper is as follows:

The cumulative distribution function *F(t)* is:

$$F(t) = 1 - exp\left(-\left(\frac{t}{\alpha}\right)^{\beta}\right) \tag{2.1.1}$$

The failure rate function *h(t)* is:

$$h(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \tag{2.1.2}$$

Where α is the scale parameter and β is the shape parameter.

Common methods for estimating the scale parameter α and shape parameter β of Weibull distribution include the least squares method, maximum likelihood method, and Bayesian methods. This paper employs the least squares method to estimate the parameters of the Weibull model.

Apply a linear transformation to Equation 2.1.1, the linear equation obtained is:

$$\ln\{-\ln[1 - F(t)]\} = -\beta\ln\alpha + \beta\ln t \tag{2.1.3}$$

Sort the CNC machine tool failure interval times in descending order to obtain the interval sequence $\{t_1, t_2, \dots t_n\}$. Substitute these into Equation 2.1.3 to obtain:

$$\ln\{-\ln[1 - F(t_i)]\} = -\beta\ln\alpha + \beta\ln t_i \quad i = (1,2,\dots,n) \tag{2.1.4}$$

Use the median rank formula, and perform a point estimation for $F(t_i)$ to get $\hat{F}_i$:

$$\hat{F}_i = \frac{i - 0.3}{n + 0.4} \tag{2.1.5}$$

Thus, the objective function *Q(a,b)* for finding the scale parameter α and the shape parameter β is obtained:

$$Q(\alpha,\beta) = \sum_{i=1}^{n}\left\{\ln\left[-\ln\left(1 - \hat{F}_i\right)\right] - \left[-\beta\ln\alpha + \beta\ln t_i\right]\right\}^2 \tag{2.1.6}$$

Differentiate Q with respect to α and β to derive the estimated values $\hat{a}$ and $\hat{b}$:

$$\hat{a} = \exp\left(\bar{x} - \frac{\bar{y}}{b}\right) \tag{2.1.7}$$

$$\hat{b} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{2.1.8}$$

Where $x_i = \ln t_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$.

Common methods for goodness-of-fit testing include the χ^2 test and the Kolmogorov-Smirnov (K-S) test. This paper uses the K-S test, which is particularly suitable when the number of data points exceeds 50, to examine the goodness-of-fit of the failure data [18]. The steps for the K-S test are as follows:

Step 1: Formulate the null hypothesis H_0: The failure interval times of the CNC machine tools follow a Weibull distribution with parameters α and β, as determined by the least squares method.

Step 2: Calculate the test statistic D_n:

$$D_n = max\left\{\left|F(t_i) - \frac{i}{n}\right|, \left|F(t_i) - \frac{i-1}{n}\right|\right\} \tag{2.1.9}$$

Where n is the total number of failure data points, i is the rank of the failure data arranged in ascending order, $F(t_i)$ is the cumulative distribution function value obtained by substituting α and β, and the i-th failure data point into Equation 2.1.1.

Step 3: Select the significance level and refer to the K-S distribution table to obtain the critical value $D_{n,a}$. Compare $D_{n,a}$ and D_n. If $D_n < D_{n,a}$, accept the null hypothesis, confirming that the failure data conforms to the Weibull distribution. Otherwise, consider using another distribution.

2.1.2. Method for Obtaining Failure Rate Function for Low Data Volume

With the continuous improvement in the reliability of CNC machine tools, failure data has become increasingly limited. Furthermore, the increasing refinement of various components makes it progressively difficult to acquire failure data. As a result, fewer failure data are available within a limited time frame, posing challenges to the reliability analysis of individual failure modes. Jili Wang [19] proposed a Monte Carlo method to simulate the generation of failure data for machine tool subsystems based on CNC machine tool failure interval data. Using this method, reliability-related functions for subsystems of the machine tool are obtained.

The Weibull distribution is a commonly used failure model for CNC machine tools [15-16]. Based on the above studies, this paper assumes that the failure times of the entire machine follow a Weibull distribution, with the data for each failure mode derived from the overall machine failure data. Previous research has indicated that data for each failure mode also follow the Weibull distribution. Assuming the sample data Y of the CNC machine tool failure interval times is:

$$Y = (y_1, \dots, y_k, \dots, y_p) \quad k = (1,2,\dots,p) \tag{2.1.10}$$

Where y_k represents the k-th failure interval time sample of the entire machine, and the sample size is p.

Using the algorithm mentioned in Section 2.1.1, the parameters are estimated to determine that Y follows a Weibull distribution $W(\alpha, \beta)$. Based on this distribution model, generate a random sample $\hat{Y}$ with a sample size of m:

$$\hat{Y} = (\hat{y}_1, \dots, \hat{y}_k, \dots, m) \quad k = (1,2,\dots,m) \tag{2.1.11}$$

By considering the subscript k of the random sample $\hat{Y}$ as the failure sequence of the CNC machine tool, the converted failure times T:

$$T = (t_1, \dots, t_k, \dots, t_m) \quad k = (1,2,\dots,m) \tag{2.1.12}$$

Where $t_k = \sum_{i=1}^{k} y_i'$.

Assume the machine has n types of failure modes, labeled as FM1, FM2, …, FMq, …, FMn. A random sequence S is generated based on the frequency of occurrence of each failure mode for the entire CNC machine tool:

$$S = (s_1, \dots, s_i, \dots, s_n) \quad i = (1,2,\dots,n) \tag{2.1.13}$$

Where $s_i \in Z$ represents the frequency of occurrence of the i-th failure mode.

The failure times T of the CNC machine tool is converted into the occurrence times T_{FM} for each failure mode according to the values in S:

$$T_{FM} = \{T_1, \dots, T_i, \dots, T_n\} \quad i = (1,2,\dots,n) \tag{2.1.14}$$

$$T_i = (t_{i,1}, \dots, t_{i,r}, \dots, t_{i,N_i}) \quad r = (1,2,\dots,N_i) \tag{2.1.15}$$

The occurrence interval times X for each failure mode are obtained based on the occurrence times：

$$X_{FM} = \{X_1, \dots, X_i, \dots, X_n\} \quad i = (1,2,\dots,n) \tag{2.1.16}$$

$$X_i = (x_{i,1}, \dots, x_{i,r}, \dots, x_{i,N_i}) \quad r = (1,2,\dots,N_i) \tag{2.1.17}$$

Where $x_{i,r} = t_{i,r} - t_{i,r-1}$; N_i represents the number of occurrences of the i-th failure mode; i and r denote the r-th occurrence of the i-th failure mode.

Based on the above process, Monte Carlo simulations using MATLAB can be performed to obtain simulation results for the failure rates of various failure modes. Using the simulation data, the Weibull distribution $W(\alpha_i, \beta_i)$ for each failure mode can be estimated using the least squares method, where i=1, 2, …, n. Subsequently, the failure rate functions $h_i(t)$ for each failure mode can be obtained using the algorithm outlined in Section 2.1.1. According to Zhou et al. [20], the failure rates at the points of interest can be transformed into SFSs $\tilde{A}_i$, which are then incorporated into the scoring operations in Section 2.2. The transformation steps are as follows:

Step 1: Normalize the failure rates of each failure mode at the points of interest.

$$K_i(t) = \frac{\lambda_i(t)}{\sum_{i=1}^{n} \lambda_i(t)} \tag{2.1.18}$$

Step 2: Calculate the relative failure rate $A_i(t)$ for each failure mode.

$$A_i(t) = \frac{K_i(t) - \min\{K_i(t) | i = 1,2,\dots,n\}}{\max\{K_i(t) | i = 1,2,\dots,n\} - \min\{K_i(t) | i = 1,2,\dots,n\}} \tag{2.1.19}$$

where the value range of $A_i(t)$ is [0,1].

Step 3: Convert $A_i(t)$ into a SFS $\tilde{A}_i$ based on evaluation language:

$$\tilde{A}_i(t) = \left(\lfloor A_i(t) \rfloor + 1 - A_i(t)\right) \cdot U_c \oplus \left(A_i(t) - \lfloor A_i(t) \rfloor\right) \cdot U_{c+1} \tag{2.1.20}$$

Where $c = \lfloor A_i(t) \cdot (u-1) \rfloor + 1$, U_c (Where c = 1,2,…,u) represents the fuzzy set corresponding to the c-th level of importance as shown in Table 1 of Section 2.2.2, and u is the total number of evaluation language levels (in this case, 9). The fuzzy set-related algorithms in Equation 2.1.20 will be detailed in Section 2.2.1.

2.2. Dynamic Weight FMECA Method Based on Spherical Fuzzy Sets

2.2.1. Spherical Fuzzy Set Theory

In FMECA, experts are required to score the risk factors associated with each failure mode. Directly scoring these risk factors can be overly abstract and vague for experts. As a result, experts often provide linguistic evaluations, which are then converted into numerical scores. However, these linguistic evaluations introduce subjective fuzziness and uncertainty. To address this, fuzzy mathematics and interval values are often used to express them. Andrés A. Zúñiga et al. [21] applied a fuzzy mathematics-based FMECA method to analyze and rank failure modes in a power grid network, effectively addressing the fuzziness in expert evaluations and obtaining more reasonable failure mode rankings. Yangyang Zhang et al. [22] proposed an adaptive weighted information fusion model based on triangular fuzzy numbers to assign weights to the influencing factors in FMECA more reasonably. The method validated the feasibility of the method with a complex reciprocating machinery system. Qingji Zhou et al. [23] converted expert evaluations in FMECA into fuzzy numbers to assess failure modes in a submersible hydraulic pump system and used the Fuzzy Analytic Hierarchy Process (FAHP) and entropy theory to weight risk factors, achieving more reasonable analysis results. The latest advancement in fuzzy set theory, proposed by Kutlu Gündoğdu and Kahraman [24], is the spherical fuzzy set (SFS) based on spherical fuzzy distance. This method extends the two-dimensional fuzzy set to three dimensions, providing experts with a broader range of preferences and better handling of uncertain information. This study uses SFS to express fuzziness in expert evaluations.

An SFS is composed of three parts: membership degree, non-membership degree, and hesitancy degree. The values of these three parts range from 0 to 1. The basic definitions and operations of SFS are as follows:

Definition 1: Let $U \neq \emptyset$ be a universe of discourse. Then, an SFS $\tilde{A}_S$ over U can be expressed as:

$$\tilde{A}_S = \left\{ \langle u, \mu_{\tilde{A}_S}(u), \nu_{\tilde{A}_S}(u), \pi_{\tilde{A}_S}(u) \rangle \middle| u \in U \right\} \tag{2.2.1}$$

Where $\mu_{\tilde{A}_S}(u), \nu_{\tilde{A}_S}(u), \pi_{\tilde{A}_S}(u) \in [0,1]$ are the membership degree, non-membership degree, and hesitancy degree of u in U respectively. Additionally, these values satisfy $0 \leq \mu_{\tilde{A}_S}^2(u) + \nu_{\tilde{A}_S}^2(u) + \pi_{\tilde{A}_S}^2(u) \leq 1$.

Definition 2: Given two SFSs $\tilde{A}_1 = (\mu_{\tilde{A}_1}, \nu_{\tilde{A}_1}, \pi_{\tilde{A}_1})$ and $A_2 = (\mu_{\tilde{A}_2}, \nu_{\tilde{A}_2}, \pi_{\tilde{A}_2})$, the following operations apply:

$$\tilde{A}_1 \cup \tilde{A}_2 = \left\{ \begin{matrix} \max\{\mu_{\tilde{A}_1}, \mu_{\tilde{A}_2}\}, \min\{\nu_{\tilde{A}_1}, \nu_{\tilde{A}_2}\}, \\ \max\left\{ \left(1 - \left((\max\{\mu_{\tilde{A}_1}, \mu_{\tilde{A}_2}\})^2 + (\min\{\nu_{\tilde{A}_1}, \nu_{\tilde{A}_2}\})^2\right)\right)^{\frac{1}{2}}, \\ \min\{\pi_{\tilde{A}_1}, \pi_{\tilde{A}_2}\} \end{matrix} \right\} \right\} \tag{2.2.2}$$

$$\tilde{A}_1 \cap \tilde{A}_2 = \left\{ \begin{matrix} \min\{\mu_{\tilde{A}_1}, \mu_{\tilde{A}_2}\}, \max\{\nu_{\tilde{A}_1}, \nu_{\tilde{A}_2}\}, \\ \max\left\{ \left(1 - \left((\min\{\mu_{\tilde{A}_1}, \mu_{\tilde{A}_2}\})^2 + (\max\{\nu_{\tilde{A}_1}, \nu_{\tilde{A}_2}\})^2\right)\right)^{\frac{1}{2}}, \\ \min\{\pi_{\tilde{A}_1}, \pi_{\tilde{A}_2}\} \end{matrix} \right\} \right\} \tag{2.2.3}$$

$$\tilde{A}_1 \oplus \tilde{A}_2 = \left\{ \begin{matrix} \left(\mu_{\tilde{A}_1}^2 + \mu_{\tilde{A}_2}^2 - \mu_{\tilde{A}_1}^2 \mu_{\tilde{A}_2}^2\right)^{\frac{1}{2}}, \nu_{\tilde{A}_1}\nu_{\tilde{A}_2}, \\ \left(\left(1 - \mu_{\tilde{A}_2}^2\right)\pi_{\tilde{A}_1}^2 + \left(1 - \mu_{\tilde{A}_1}^2\right)\pi_{\tilde{A}_2}^2 - \pi_{\tilde{A}_1}^2 \pi_{\tilde{A}_2}^2\right)^{\frac{1}{2}} \end{matrix} \right\} \tag{2.2.4}$$

$$\tilde{A}_1 \otimes \tilde{A}_2 = \left\{ \begin{matrix} \mu_{\tilde{A}_1}\mu_{\tilde{A}_2}, \left(\nu_{\tilde{A}_1}^2 + \nu_{\tilde{A}_2}^2 - \nu_{\tilde{A}_1}^2 \nu_{\tilde{A}_2}^2\right)^{\frac{1}{2}}, \\ \left(\left(1 - \nu_{\tilde{A}_2}^2\right)\pi_{\tilde{A}_1}^2 + \left(1 - \nu_{\tilde{A}_1}^2\right)\pi_{\tilde{A}_2}^2 - \pi_{\tilde{A}_1}^2 \pi_{\tilde{A}_2}^2\right)^{\frac{1}{2}} \end{matrix} \right\} \tag{2.2.5}$$

Definition 3: if $\lambda > 0$ and $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}}, \pi_{\tilde{A}})$, then:

$$\lambda \cdot \tilde{A} = \left\{ \left(1 - \left(1 - \mu_{\tilde{A}}^2\right)^{\lambda}\right)^{\frac{1}{2}}, \nu_{\tilde{A}}^{\lambda}, \left(\left(1 - \mu_{\tilde{A}}^2\right)^{\lambda} - \left(1 - \mu_{\tilde{A}}^2 - \pi_{\tilde{A}}^2\right)^{\lambda}\right)^{\frac{1}{2}} \right\} \tag{2.2.6}$$

$$\tilde{A}^{\lambda}=\left\{\mu_{\tilde{A}}^{\lambda},\left(1-\left(1-v_{\tilde{A}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}},v_{\tilde{A}}^{\lambda},\left(\left(1-\mu_{\tilde{A}}^{2}\right)^{\lambda}-\left(1-v_{\tilde{A}}^{2}-\pi_{\tilde{A}}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}\right\} \tag{2.2.7}$$

Definition 4: Given two SFSs $\tilde{A}_1=(\mu_{\tilde{A}_1},v_{\tilde{A}_1},\pi_{\tilde{A}_1})$ and $\tilde{A}_2=(\mu_{\tilde{A}_2},v_{\tilde{A}_2},\pi_{\tilde{A}_2})$, and λ, λ_1, $\lambda_2>0$, then:

$$\tilde{A}_1\oplus\tilde{A}_2=\tilde{A}_2\oplus\tilde{A}_1 \tag{2.2.8}$$

$$\tilde{A}_1\otimes\tilde{A}_2=\tilde{A}_2\otimes\tilde{A}_1 \tag{2.2.9}$$

$$\lambda(\tilde{A}_1\oplus\tilde{A}_2)=\lambda\tilde{A}_1\oplus\lambda\tilde{A}_2 \tag{2.2.10}$$

$$\lambda_1\tilde{A}_1\oplus\lambda_2\tilde{A}_1=(\lambda_1+\lambda_2)\tilde{A}_1 \tag{2.2.11}$$

$$\tilde{A}_1^{\lambda_1}\otimes\tilde{A}_1^{\lambda_2}=\tilde{A}_1^{\lambda_1+\lambda_2} \tag{2.2.12}$$

Definition 5: The Spherical Weighted Arithmetic Mean (SWAM) and Spherical Weighted Geometric Mean (SWGM) of SFSs are defined as follows:

$$\begin{aligned}&SWAM_{\omega}(\tilde{A}_{S1},\dots,\tilde{A}_{Sn})\\&=\omega_1\tilde{A}_{S1}+\omega_2\tilde{A}_{S2}+\cdots+\omega_n\tilde{A}_{Sn}\\&=\left\{\begin{array}{c}\left[1-\prod_{i=1}^{n}\left(1-\mu_{\tilde{A}_{Si}}^{2}\right)^{\omega_i}\right]^{1/2},\prod_{i=1}^{n}v_{\tilde{A}_{Si}}^{\omega_i},\\\left[\prod_{i=1}^{n}\left(1-\mu_{\tilde{A}_{Si}}^{2}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\mu_{\tilde{A}_{Si}}^{2}-\pi_{\tilde{A}_{Si}}^{2}\right)^{\omega_i}\right]^{1/2}\end{array}\right\}\end{aligned} \tag{2.2.13}$$

$$\begin{aligned}&SWGM_{\omega}(\tilde{A}_{S1},\dots,\tilde{A}_{Sn})\\&=\tilde{A}_{S1}^{\omega_1}+\tilde{A}_{S2}^{\omega_2}+\cdots+\tilde{A}_{Sn}^{\omega_n}\\&=\left\{\begin{array}{c}\prod_{i=1}^{n}\mu_{\tilde{A}_{Si}}^{\omega_i},\left[1-\prod_{i=1}^{n}\left(1-v_{\tilde{A}_{Si}}^{2}\right)^{\omega_i}\right]^{1/2},\\\left[\prod_{i=1}^{n}\left(1-v_{\tilde{A}_{Si}}^{2}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-v_{\tilde{A}_{Si}}^{2}-\pi_{\tilde{A}_{Si}}^{2}\right)^{\omega_i}\right]^{1/2}\end{array}\right\}\end{aligned} \tag{2.2.14}$$

Where n is the number of SFSs, $\tilde{A}_{Sn}=\{\mu_{\tilde{A}_{Sn}},v_{\tilde{A}_{Sn}},\pi_{\tilde{A}_{Sn}}\}$, $\omega_i=(\omega_1,\omega_2,\dots,\omega_n)$ are the weights corresponding to $\tilde{A}_{Sn}$ satisfying $\omega_i\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$.

2.2.2. Expert Evaluation Method Based on Spherical Fuzzy Sets

In traditional FMECA, all risk factors (O, S, D) are scored by experts. The FMECA method proposed in this paper introduces maintainability (M) as an economically related risk factor and divides Severity S into two sub-factors Severity to the machine (S1) and Severity to the worksite (S2). Experts are required to score the subjective risk factors S1, S2, D, and M, while O is calculated based on failure rates. The scores, expressed as fuzzy linguistic sets, are then converted into SFSs to more effectively represent uncertainty. The relationship between fuzzy linguistic terms and SFSs is shown in Table 1.

Table 1. Conversion Table between Fuzzy Linguistic Terms and SFS

Importance	Fuzzy Linguistic Terms	SFS
1	Absolutely Low Importance (ALI)	(0.1, 0.9, 0.1)
2	Very Low Importance (VLI)	(0.2, 0.8, 0.2)
3	Low Importance (LI)	(0.3, 0.7, 0.3)
4	Slightly Low Importance (SLI)	(0.4, 0.6, 0.4)
5	Equally Importance (EI)	(0.5, 0.5, 0.5)
6	Slightly More Importance (SMI)	(0.6, 0.4, 0.4)
7	High Importance (HI)	(0.7, 0.3, 0.3)
8	Very High Importance (VHI)	(0.8, 0.2, 0.2)
9	Absolutely more Importance (AMI)	(0.9, 0.1, 0.1)

Experts' subjective evaluations are influenced by factors such as personal background, decision-making ability, and individual characteristics. To enhance the accuracy of these evaluations, each expert should be weighted according to these influencing factors. Various methods for allocating expert weights have been proposed, including background-based weighting [25], confidence-based weighting [26], and dynamic adjustment based on group consensus [27]. In the context of CNC machine tools, and considering the practical feasibility of the research, this paper uses a background-based method for weighting experts.

The background evaluation includes three criteria: "educational background," "years of experience," and "frequency of equipment interaction". The sum of these three criteria forms the final score for each expert. The expert background evaluation table is shown in Table 2.

Table 2. Expert Background Evaluation Table.

Education Level	Years of Experience	Frequency of Equipment Interaction	Score
Below Associate	<3	Rarely	0
Associate's	3-6	Occasionally	1
Bachelor	7-10	Generally	2
Master	10-20	Frequently	3
Doctor	>20	Very Frequently	4

The weight ω_k for each expert is calculated using Equation 2.2.18:

$$\omega_k = \frac{S_k}{\sum_{k=1}^{l} S_k} \quad (2.2.15)$$

Where S_k represents the final score of the k-th expert, l is the total number of experts, and $\sum_{k=1}^{l} S_k$ is the total score of all experts.

2.2.3. Determination of Dynamic Weights Based on Spherical Fuzzy Sets

This paper proposes a dynamic weighting method for FMECA risk factors based on SFSs, allowing for weight variations based on the service age of CNC machine tools, thus enhancing the rationality of FMECA evaluation results. The method integrates the entropy weight method based on SFSs and the Spherical Fuzzy Set-Analytic Hierarchy Process (SFS-AHP). The entropy weight method determines attribute weights based on attribute values [23, 28], in this case, the value of risk factors. While it can determine objective weights based on given values, its weight allocation heavily relies on those values and fails to account for inter-attribute influences. On the other hand, AHP is a subjective weighting method, where the obtained weights do not depend on time variations [23, 29], though its pairwise comparison matrix is constrained by the subjective judgment of the decision-maker. This paper combines these two methods to complement each other, proposing a comprehensive fuzzy dynamic weighting method, enabling the dynamic update of risk factor weights over time and identifying changes in risk focus as service age progresses.

2.2.4. Entropy Weight Method Based on Spherical Fuzzy Sets

The entropy weight method is an objective weighting method based on attribute values. In this paper, the entropy weight method based on SFSs proposed by Ali Aydoğdu [30] is used to estimate the objective weights of risk factors. The steps are as follows:

Step 1: Calculate the SFS entropy measure.

Assume that the evaluation matrix for n failure modes and h risk factors is:

$$\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \cdots & \tilde{A}_{1h} \\ \tilde{A}_{21} & \tilde{A}_{22} & \cdots & \tilde{A}_{2h} \\ \vdots & \vdots & \tilde{A}_{ij} & \vdots \\ \tilde{A}_{n1} & \tilde{A}_{n2} & \cdots & \tilde{A}_{nh} \end{bmatrix}$$

Where i = (1,2,…,n), j = (1,2,…,h). $\tilde{A}_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$ is the fuzzy set converted from the linguistic evaluation according to Table 1.

The SFS entropy measure E_j for the j-th risk factor is:

$$E_j = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{4}{5}\left[\left|\mu_{ij}^2 - \nu_{ij}^2\right| + \left|\pi_{ij}^2 - 0.25\right|\right]\right) \quad (2.2.16)$$

Step 2: Calculate the divergence div_j of the intrinsic information of risk factor j:

$$div_j = 1 - E_j \quad (2.2.17)$$

Step 3: Calculate the objective SFS entropy weight ω_j^E for the j-th risk factor:

$$\omega_j^E = \frac{div_j}{\sum_{j=1}^{n} div_j} \quad (2.2.18)$$

2.2.5. Spherical Fuzzy Set-Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) is a decision-making method that decomposes decision elements into three parts: objectives, criteria, and alternatives. Its systematic structure and straightforward process have made it widely adopted in decision-making across various fields. To handle the fuzzy information inherent in the subjective qualitative components, various fuzzy set-based AHP methods have been developed, such as neutrosophic fuzzy set AHP [31] and Pythagorean fuzzy set AHP [32]. Recent studies [33-34] have proposed the SFS-based AHP (SFS-AHP), which has shown excellent performance due to its broader applicability across different scenarios. In this study, SFS-AHP is applied to the FMECA of CNC machine tools to derive subjective weights of risk factors. The specific steps of SFS-AHP are as follows:

Step 1: Construct a decision hierarchy model based on objectives, criteria, and alternatives. In this context, it includes the objective, risk factors, and failure modes.

Step 2: Construct the judgment matrix. This step involves using expert evaluation language to construct the judgment matrix. Unlike traditional AHP's numerical comparisons, SFS-AHP uses SFSs converted from evaluation language for pairwise comparisons. The conversion relationship between evaluation language and SFSs is shown in Table 3.

Table 3. Correspondence between Evaluation Language, SFS, and SI in the SFS-AHP Judgment Matrix.

Intuitive Language	SFS	SI
Absolutely strong important (AS)	(0.9, 0.1, 0.0)	9
Very strong important (VS)	(0.8, 0.2, 0.1)	7
Fairly strong important (FS)	(0.7, 0.3, 0.2)	5
Slightly strong important (SS)	(0.6, 0.4, 0.3)	3
Equal important (E)	(0.5, 0.4, 0.4)	1
Slightly low important (SL)	(0.4, 0.6, 0.3)	1/3
Fairly low important (FL)	(0.3, 0.7, 0.2)	1/5
Very low important (VL)	(0.2, 0.8, 0.1)	1/7
Absolutely low important (AL)	(0.1, 0.9, 0.0)	1/9

The transformed judgment matrix $\tilde{P}$ is as follows:

$$\widetilde{\boldsymbol{P}} = \left[\widetilde{\boldsymbol{P}}_{ji}\right]_{h\times h} = \begin{bmatrix} \tilde{S}_{11} & \tilde{S}_{12} & \cdots & \tilde{S}_{1h} \\ \tilde{S}_{21} & \tilde{S}_{22} & \cdots & \tilde{S}_{2h} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{S}_{h1} & \tilde{S}_{h2} & \cdots & \tilde{S}_{hh} \end{bmatrix}$$

Where $j = i = 1,2,\ldots,h$, and h is the number of risk factors to be compared. $\tilde{S}_{ji} = (\mu_{ji}, v_{ji}, \pi_{ji})$ is SFS of the importance of the j-th risk factor relative to the i-th risk factor.

Step 3: Consistency Check.

After obtaining the matrix from Step 2, a consistency check is required. This involves converting the SFS obtained from the expert evaluation into a score index (SI).

For evaluation languages AS, VS, FS, SS, E, the SI calculation is as follows:

$$SI = \sqrt[2]{\left|100 \times \left((\mu_{\tilde{S}} - \pi_{\tilde{S}})^2 - (v_{\tilde{S}} - \pi_{\tilde{S}})^2\right)\right|} \qquad (2.2.19)$$

For the remaining evaluation languages SL, FL, L, AL, the SI calculation is:

$$SI = \frac{1}{\sqrt[2]{\left|100 \times \left((\mu_{\tilde{S}} - \pi_{\tilde{S}})^2 - (v_{\tilde{S}} - \pi_{\tilde{S}})^2\right)\right|}} \qquad (2.2.20)$$

Replacing all fuzzy sets in $\widetilde{\boldsymbol{P}}$ with the corresponding SI values results in a crisp number matrix, which can then be subjected to a consistency check using the standard method for crisp numbers. The consistency check formulas are as follows:

$$CI = \frac{\lambda_{max} - n}{n - 1}, CR = \frac{CI}{RI} \qquad (2.2.21), (2.2.22)$$

Where λ_{max} is the maximum eigenvalue of the matrix, n is the dimension of the matrix, RI is the random consistency index, and CR is the consistency ratio. The matrix satisfies the consistency requirement if CR<0.1. The RI values for different matrix dimensions are shown in Table 4.

Table 4. RI Values for Different Matrix Dimensions.

Matrix Dimension (n)	1	2	3	4	5	6	7
RI	0	0	0.52	0.89	1.12	1.26	1.36

Step 4: Calculate Spherical Fuzzy Weights.

This step utilizes the Spherical Weighted Arithmetic Mean (SWAM) mentioned in Equation 2.2.13 to determine the spherical fuzzy weights of the risk factors. Since the pairwise comparison results equally influence the final weight determination, set $\omega = 1/n$ in Equation 2.2.13. Thus, the spherical fuzzy weight $\widetilde{\omega}_j^S$ for the j-th risk factor is obtained as follows:

$$\begin{aligned} \widetilde{\omega}_j^S &= SWAM_\omega(\tilde{S}_{j1}, \tilde{S}_{j2}, \ldots, \tilde{S}_{jh}) \\ &= \omega\tilde{S}_{j1} + \omega\tilde{S}_{j2} + \cdots + \omega\tilde{S}_{jh} \\ &= \left\{ \begin{array}{c} \left[1 - \prod_{j=1}^{h}\left(1 - \mu_{\tilde{S}_{jh}}^2\right)^\omega\right]^{1/2}, \prod_{j=1}^{h} v_{\tilde{S}_{jh}}^\omega, \\ \left[\prod_{j=1}^{h}\left(1 - \mu_{\tilde{S}_{jh}}^2\right)^\omega - \prod_{j=1}^{h}\left(1 - \mu_{\tilde{S}_{jh}}^2 - \pi_{\tilde{S}_{jh}}^2\right)^\omega\right]^{1/2} \end{array} \right\} \\ &= (\mu_j^S, v_j^S, \pi_j^S) \end{aligned} \qquad (2.2.23)$$

Step 5: Calculate Subjective Weights.

Defuzzify the spherical fuzzy weight ω_j^S to obtain:

$$S(\widetilde{\omega}_j^S) = \sqrt[2]{\left|100 \times \left[\left(3\mu_j^S - \frac{v_j^S}{2}\right)^2 - \left(\frac{v_j^S}{2} - \pi_j^S\right)^2\right]\right|} \qquad (2.2.24)$$

Normalize $S(\widetilde{\omega}_j^S)$ to get the subjective weight ω_j^A for the j-th risk factor:

$$\omega_j^A = \frac{S(\omega_j^S)}{\sum_{j=1}^{h} S(\omega_j^S)} \qquad (2.2.25)$$

As the service life of CNC machine tools progresses, the relative importance of different risk factors may change. Therefore, experts are asked to perform pairwise comparisons of risk factors for CNC machine tools at different stages of their service life, generating multiple judgment matrices. By applying the above algorithms, the weights of each risk factor under different service ages can be derived, enabling dynamic weighting.

2.2.6. Dynamic Integrated Weights Based on Spherical Fuzzy Sets

Sections 2.2.4 and 2.2.5 introduced the methods for obtaining objective weights using the entropy weight method based on SFSs and subjective weights using SFS-AHP, respectively. This section presents a combined weighting method that integrates both objective and subjective weights. The calculation method for the combined weight ω_j is as follows:

$$\omega_j = \delta\omega_j^E + (1-\delta)\omega_j^A \tag{2.2.26}$$

Where δ is the bias coefficient and $0 < \delta < 1$. If the expert considers the objective weight more important than the subjective weight, then $\delta > 0.5$; otherwise, $\delta < 0.5$.

2.2.7. WASPAS Based on Spherical Fuzzy Sets

For multi-criteria decision-making (MCDM) problems such as FMECA, identifying the optimal alternative has been a central focus of research. Various methods have been developed for ranking alternatives in MCDM problems, including TOPSIS, VIKOR, EDAS, MULTIMOORA, and WASPAS. To address the uncertainties inherent in practical decision-making, many researchers have applied these methods in fuzzy environments. Gundogdu and Kahraman extended the WASPAS method to the spherical fuzzy environment, developing the Spherical Fuzzy Weighted Aggregated Sum Product Assessment (SF-WASPAS). This method has been successfully applied to select the best industrial robot, yielding promising results [35]. SF-WASPAS combines the Weighted Sum Model (WSM) and the Weighted Product Model (WPM). The specific steps for calculating scores are as follows:

Step 1: Obtain the Evaluation Matrix.

Obtain the evaluation matrix from experts for n failure modes and h risk factors (same as the evaluation matrix in Section 2.2.4).

Step 2: Calculate WSM and WPM Weights.

Calculate the WSM weight $\tilde{Q}_i^1$ and the WPM weight $\tilde{Q}_i^2$ using Equations 2.2.27 and 2.2.28, respectively:

$$\tilde{Q}_i^1 = \sum_{j=1}^{h} \tilde{A}_{ij}\omega_j\,, \tilde{Q}_i^2 = \sum_{j=1}^{h} \tilde{A}_{ij}{}^{\omega_j} \qquad (2.2.27), (2.2.28)$$

Where ω_j is the weight of the risk factor obtained in Section 2.2.6.

Step 3: Combine WSM and WPM Weights.

Combine $\tilde{Q}_i^1$ and $\tilde{Q}_i^2$ to obtain the final spherical fuzzy score $\tilde{Q}_i$ for the *i*-th failure mode:

$$\tilde{Q}_i = \lambda\tilde{Q}_i^1 + (1-\lambda)\tilde{Q}_i^2 \tag{2.2.29}$$

Where λ and $(1-\lambda)$ represent the contributions of $\tilde{Q}_i^1$ and $\tilde{Q}_i^2$ to the final spherical fuzzy score $\tilde{Q}_i$ respectively.

Step 4: Defuzzify the Spherical Fuzzy Score.

Defuzzify $\tilde{Q}_i$ based on the method in the literature [36] to obtain the final score Q_i for the i-th failure mode and the ranking of Q_i.

$$Q_i = \frac{(1+\mu_i{}^2 - v_i{}^2 - \pi_i{}^2)(\mu_i{}^2 + v_i{}^2 + \pi_i{}^2)}{2} \tag{2.2.30}$$

The ranking of Q_i represents the final failure mode ranking in the SFS FMECA.

3. Case Study

To validate the proposed improved FMECA method, this study takes the T-model CNC machine tools from Company A as a case study. Failure data of T-model CNC machine tools were professionally collected by well-trained on-site workers at the factories of Company A. The failure data is processed and summarized in Section 3.1.1. To assess the effectiveness of the dynamic weighting approach proposed in Section 2.2.5, Three experts were invited to perform pairwise comparisons and evaluate the CNC machine failure risk factors at 1000 hours and 10000 hours. These experts included one doctor with ten years of experience in the field and two master research fellows. The evaluations yield dynamic weights at different service ages, enabling a comparative analysis of the final failure mode rankings. Failure data of ten T-model CNC machine tools from Company A were collected from February 1 to August 31, 2014. Various failure modes were identified, and their frequencies were recorded. The application of the improved FMECA method is detailed in Sections 3.1-3.4.

3.1. Failure Data Analysis of T-Model CNC Machine Tools

3.1.1. Reliability Modelling of T-Model CNC Machine Tools

The failure data of ten T-model CNC machine tools were combined and processed to obtain the Time Between Failures (TBF) data. The TBF data are then sorted in ascending order, as shown in Table 5. Under the assumption of "as good as new" after repairs, the TBF data is modeled using the Weibull distribution reliability modeling method mentioned in Section 2.1.1. The least squares method was used to estimate the scale parameter α and the shape parameter β , resulting in $\alpha = 510.835$ and $\beta = 1.143$. Based on these calculations, the cumulative distribution function *F(t)* is:

$$F(t) = 1 - exp\left(-\left(\frac{t}{510.835}\right)^{1.143}\right) \tag{3.1.1}$$

The goodness-of-fit was tested using the K-S test mentioned in Section 2.1.1. The null hypothesis H_0 was set as: the TBF data follows a Weibull distribution with $\alpha = 510.835$ and $\beta = 1.143$. The test yielded $D_n = 0.1159$. At a significance level of 0.1, the critical value $D_{n,\alpha} = 0.1252$ (for $n = 95$). Since $D_n < D_{n,\alpha}$, the null hypothesis is accepted.

Table 5. T-Model CNC Machine Tools TBF Data.

TBF for T-model of CNC Machine Tools(hours)													
17	20	20	23	24	46	47	49	51	52	52	54	73	79
94	96	112	115	125	153	154	162	173	178	181	185	212	251
262	274	310	314	320	328	329	333	335	351	364	369	373	376
378	383	391	397	402	405	426	429	430	437	446	461	462	474
491	516	524	535	554	569	573	575	577	601	612	643	647	651
652	654	664	673	700	723	724	767	768	769	777	794	801	817
837	838	897	906	1005	1046	1105	1112	1232	1321	1861			

3.1.2. Failure Mode Analysis

Based on the failure data of T-model CNC machine tools and the judgment of experts and on-site personnel, 21 failure modes were identified. The occurrence frequency of each failure mode is shown in Table 6.

Table 6. Failure Mode Frequency.

Code	Failure Mode	Failure Reason	Failure Effect	Frequency
FM1	Worktable cannot move	Metal shavings and cutting fluid accumulation	Operation unable to proceed	2
FM2	Coolant circuit anomaly	Corrosion of pipeline coating	Overheating of machine tool moving components	2
FM3	Handwheel cannot start	1. Failure of handwheel start switch 2. External wiring of the handwheel broken	Reduced production efficiency	2
FM4	Hydraulic pump and circuit leakage	1. Oil can rupture 2. Excessive pressure 3. Excessive wear of hydraulic pump	Insufficient pressure on machining components	2
FM5	Protective cover detachment	Protective cover edge lifting	Potential for chip splashing, posing risks to the machine and onsite personnel	2
FM6	Abnormal noise from worktable	Accumulation of metal shavings under protective cover	Affecting machining accuracy of components	2
FM7	Arc extinguisher damage	1. Overload operation 2. Excessive humidity at the worksite	Increasing on-site safety risks, with potential machine damage	3
FM8	Chip conveyor malfunction	Incomplete separation of chips and cutting fluid	Excessive chip accumulation, causing machining abnormalities	3
FM9	Lubricating oil leakage	1. Oil gun rupture 2. Oil outlet pipe rupture	Affecting on-site personnel operation and judgment	3
FM10	Lighting failure	Prolonged operation of the lighting fixture	Affecting the tool changer tools, preventing further cutting operations	3
FM11	Tool magazine motor temperature anomaly	1. Tool magazine fan unable to rotate with the spindle 2. Prolonged high-load operation	Affecting machine tool operation performance	4
FM12	Robot arm fails to return	Tool changer sensor signal error	Increased tool vibration; potential tool damage and ejection; noise and wear; potential hazards to equipment and on-site personnel	4
FM13	Spindle looseness and wear	1. Loosening of bolts securing the spindle to the bed 2. Deformation of balls in the support bearing	Impacts machining precision of components	5
FM14	Unacceptable machining accuracy error	1. Corrosion and loosening of the nut securing the ball screw 2. Wear of the screw bearing 3. Metal shavings and cutting fluid accumulation	Affecting the efficiency of normal machining, potentially leading to product scrap	5
FM15	Water tank leakage	1. Metal shavings blocking the mesh opening	Damaging machine tool components while potentially posing a hazard to the worksite environment.	5

		2. Water tank welds detached		

Table 6-Continued

FM16	Tool jam	1. Abnormal downward movement of the spindle in the Z-axis direction 2. Robot arm angle deviation	Interrupting machining process and damaging the tool	6
FM17	Air pressure circuit leakage	1. Air pressure circuit damage 2. Sealing component wear	Insufficient air supply to the machine tool, causing machining abnormalities	6
FM18	Program error and alarm triggered	1. Circuit system failure 2. Connection issues between system hardware and software 3. Loose wiring	Interrupting normal machining process	7
FM19	Tool change failure	1. Loss of clamping force in the tool holder spring 2. Excessive gap at the spindle pull stud, causing loose tool-spindle fit 3. Robot arm misalignment 4. Robot arm tool change angle deviation 5. Spindle upper cylinder shifting 6. Spindle downward movement in the Z-axis, causing tool collision during robot arm tool change 7. Deformation of robot arm tool slot 8. Sensor detection error	Potentially causing tool breakage and damage to components	8
FM20	Tool drop	1. Air leakage in the cylinder 2. Deformation and angle deviation in the robot arm tool gripping position 3. Air leakage in the tool change cylinder 4. Loose bolts connecting the cylinder to the bed 5. Excessive gap at the spindle pull stud	Tool breakage, which in severe cases, can cause significant harm to the machine and on-site personnel	9
FM21	Start switch failure	Long-term impact from hard objects	Unable to start the machine tool properly	12

As shown in Table 6, the failure modes "Worktable cannot move (FM1)," "Coolant circuit anomaly (FM2)," "Handwheel cannot start (FM3)," "Hydraulic pump and circuit leakage (FM4)," "Protective cover detachment (FM5)," and "Abnormal noise from worktable (FM6)" have the same lowest frequency of occurrence, appearing twice. In contrast, the failure modes "Program error and alarm triggered (FM18)," "Tool change failure (FM19)," "Tool drop (FM20)," and "Start switch failure (FM21)" have higher frequencies of 7, 8, 9, and 12 occurrences, respectively. Generally, the CNC systems, spindle systems, and tool carriage systems of CNC machine tools exhibit higher failure frequencies than other subsystems [37, 38]. The start switch of this model is made of plastic, and due to the impact of the on-site working environment, it is frequently struck by hard objects, making FM21 the most frequent failure mode.

3.1.3. Failure Rate Function

Based on the data presented in Table 6, the failure data for individual failure modes is limited. Therefore, the Monte Carlo simulation method outlined in Section 2.1.2 is employed to model the failure rates of those failure modes. Using the reliability function of the T-model CNC machine tool, as calculated in Section 3.1.1, and the simulation method described in Section 2.1.2, a simulation was conducted to model the failure rates of each failure mode. The number of CNC machine tool failures in the simulation was set to 5000 to minimize errors. The results from the MATLAB simulation provided the Weibull distribution shape and scale parameters for the time between failures of 21 different failure modes, as shown in Table 7.

Table 7. Simulated Weibull Parameters for Failure Modes.

Code	α	β	**Code**	α	β	**Code**	α	β
FM1	22304.8	0.831	FM8	15118.5	0.974	FM15	9202.3	1.025
FM2	23144.5	1.013	FM9	15725.0	1.135	FM16	7692.8	1.051

FM3	24647.5	0.899	FM10	15056.6	0.952	FM17	7558.0	0.941
FM4	23417.0	1.037	FM11	11738.8	0.956	FM18	6732.0	0.973
FM5	23504.6	1.165	FM12	11250.2	0.913	FM19	6103.2	1.128
FM6	22200.3	0.959	FM13	9154.4	1.019	FM20	5279.3	0.974
FM7	16014.0	0.914	FM14	9112.5	1.077	FM21	4023.1	1.090

All failure rate functions for the failure modes are shown in Figure 2.

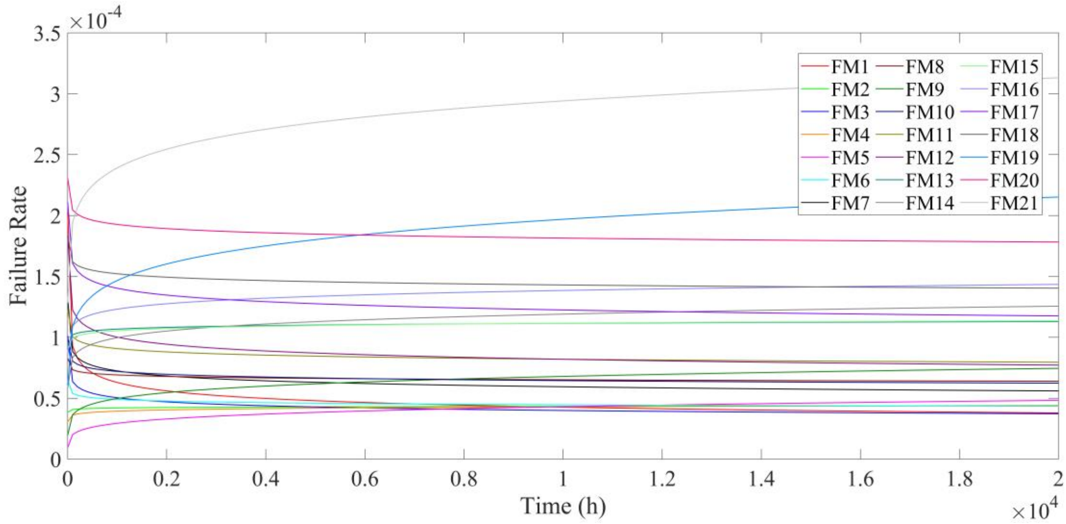

Fig.2. Failure Rate Functions of Various Failure Modes.

As shown in Figure 2, the failure rates and rankings for various failure modes change with the service ages of the CNC machine tool. This suggests that the key failure modes to focus on may vary at different stages of the CNC machine tool's life. To more reasonably rank the failure modes, this paper will conduct a dynamic FMECA study on the machine's failure modes at 1000 hours and 10,000 hours of operation. The failure rates for each failure mode at 1000 hours and 10,000 hours are shown in Table 8.

Table 8. Failure Rates of Various Failure Modes at 1000h and 10000h.

Code	Failure Rate /10^{-5}(1000h)	Failure Rate /10^{-5} (10000h)	Code	Failure Rate /10^{-5} (1000h)	Failure Rate /10^{-5} (10000h)
FM1	6.30	4.27	FM12	10.0	8.20
FM2	4.20	4.33	FM13	10.7	11.1
FM3	5.04	4.00	FM14	9.9	11.9
FM4	3.94	4.29	FM15	10.5	11.2
FM5	2.94	4.30	FM16	12.3	13.9
FM6	4.91	4.26	FM17	14.0	12.2
FM7	7.25	5.94	FM18	15.2	14.3
FM8	6.91	6.51	FM19	14.7	19.7
FM9	4.97	6.79	FM20	19.3	18.1
FM10	7.20	6.45	FM21	23.9	29.4
FM11	9.08	8.20			

3.2. Failure Mode Risk Factor Evaluation Matrix

3.2.1. Expert Weight Allocation

For the subjective risk factors S1, S2, D, and M, three experts from Company A were invited to evaluate their importance for all failure modes. Those experts have different levels of seniority. The evaluation results were weighted using the method mentioned in Section 2.2.2. The background information of the three experts is shown in Table 9.

Table 9. Expert Background Information.

Expert Code	Educational Level	Years of Experience	Equipment Contact Frequency	Score
TM1	Doctor	10	Frequent	10
TM2	Master	3	Moderate	6
TM3	Master	2	Occasional	4

Based on equation 2.2.15, the expert weight vector $\omega = [0.5,0.3,0.2]$.

3.2.2. Subjective Evaluation Matrix Considering Expert Weights

Based on the evaluation language provided in Table 1, three experts were invited to evaluate the importance of the four subjective risk factors S1, S2, D, and M for 21 failure modes at the points of 1000h and 10000h. The evaluation results are shown in Table 10 (1000h) and Table 11 (10000h).

Table 10: Expert Subjective Risk Factor Evaluation Language Table for 1000h

Code	S1	S2	D	M
FM1	EI SLI SMI	SLI LI SMI	ALI LI ALI	EI LI SMI
FM2	EI HI SLI	EI VLI SLI	LI VHI LI	SLI SLI EI
FM3	EI HI SLI	LI EI ALI	VLI VLI LI	VLI ALI SMI
FM4	EI SMI EI	SLI EI EI	LI VLI SLI	EI VLI SLI
FM5	EI SMI SLI	EI EI LI	LI LI VLI	LI LI LI
FM6	LI SMI ALI	EI LI LI	VLI LI VLI	EI VLI VHI
FM7	SMI EI VLI	EI LI LI	LI LI ALI	EI SLI SMI
FM8	EI EI LI	EI LI LI	LI SLI EI	SLI SLI EI
FM9	HI HI EI	SLI SMI EI	LI SLI SLI	EI SLI LI
FM10	LI VLI SLI	LI LI EI	VLI ALI LI	ALI ALI LI
FM11	HI SMI AMI	HI LI AMI	ALI EI ALI	HI EI VHI
FM12	SMI SMI SLI	SLI SLI SLI	LI LI SLI	EI EI HI
FM13	HI SMI VHI	SMI SMI SMI	SMI SMI SMI	HI HI VHI
FM14	SMI EI SMI	SLI LI EI	EI SLI HI	LI SLI LI
FM15	EI LI SMI	EI SLI EI	VLI VLI LI	SMI LI SMI
FM16	SMI HI EI	LI EI EI	LI LI SLI	EI EI SLI
FM17	SMI HI EI	HI SMI EI	ALI LI HI	EI EI SLI
FM18	HI AMI SMI	SMI EI SMI	VLI VLI VLI	HI SMI VHI
FM19	VHI AMI VHI	SMI SMI HI	VLI LI LI	VHI VHI HI
FM20	VHI HI AMI	VHI VHI AMI	VLI LI SLI	HI VHI SMI
FM21	ALI VLI ALI	ALI ALI ALI	VLI ALI VLI	ALI VLI ALI

Table 11. Expert Subjective Risk Factor Evaluation Language Table for 10000h.

Code	S1	S2	D	M
FM1	EI SLI EI	SLI VLI SMI	VLI LI VLI	SMI SLI SMI
FM2	EI HI SLI	EI VLI SLI	LI VHI LI	SLI SLI EI
FM3	EI HI SLI	LI SLI VLI	VLI VLI LI	LI VLI SMI
FM4	EI SMI EI	SLI EI EI	ALI VLI SLI	SMI LI SLI
FM5	SMI SMI EI	EI EI LI	LI LI VLI	LI LI LI
FM6	LI EI ALI	EI LI LI	LI VLI SLI	SMI LI VHI
FM7	SMI EI VLI	EI LI LI	LI LI ALI	EI SLI SMI
FM8	EI EI LI	EI LI SLI	LI SLI EI	EI EI HI
FM9	HI HI EI	EI SMI SMI	LI SLI SLI	EI SLI LI
FM10	LI VLI EI	LI LI EI	VLI ALI LI	VLI VLI LI
FM11	VHI SMI AMI	HI LI AMI	ALI EI ALI	HI EI VHI
FM12	SMI SMI SMI	SLI SLI SLI	LI LI SLI	SMI EI EI
FM13	HI SMI VHI	SMI SMI SMI	SMI SMI SMI	VHI VHI AMI
FM14	SMI EI SMI	EI SLI EI	SMI EI EI	LI SLI SLI
FM15	EI SLI SMI	SLI SLI EI	VLI VLI VLI	HI EI SMI

FM16	SMI HI EI	LI EI EI	LI LI SLI	SMI EI SLI
FM17	HI HI SMI	HI SMI EI	ALI LI HI	EI SMI EI
FM18	VHI AMI HI	SMI EI SMI	LI VLI VLI	HI SMI VHI
FM19	VHI AMI VHI	HI HI HI	SLI SLI LI	AMI AMI VHI
FM20	VHI HI AMI	VHI VHI AMI	VLI VLI VLI	HI VHI SMI
FM21	ALI ALI ALI	ALI ALI ALI	ALI ALI ALI	ALI VLI VLI

The evaluation language in Tables 10 and 11 is converted into SFS based on Table 1. The results are weighted using the expert weight vector ω from Section 3.2.1 and the SWAM operator mentioned in Equation 2.2.13. The weighted matrices are shown in Table 12 (1000h) and Table 13 (10000h).

Table 12. Expert Subjective Risk Factor SFS Table for 1000h

Code	**S1**	**S2**	**D**	**M**
FM1	(0.50, 0.51, 0.46)	(0.43, 0.58, 0.38)	(0.19, 0.83, 0.19)	(0.48, 0.53, 0.44)
FM2	(0.57, 0.42, 0.43)	(0.43, 0.57, 0.44)	(0.56, 0.46, 0.28)	(0.42, 0.57, 0.41)
FM3	(0.56, 0.44, 0.42)	(0.36, 0.67, 0.38)	(0.22, 0.78, 0.22)	(0.33, 0.72, 0.26)
FM4	(0.53, 0.47, 0.47)	(0.45, 0.55, 0.46)	(0.30, 0.71, 0.30)	(0.42, 0.60, 0.43)
FM5	(0.52, 0.48, 0.45)	(0.47, 0.53, 0.48)	(0.28, 0.72, 0.28)	(0.30, 0.70, 0.30)
FM6	(0.41, 0.62, 0.33)	(0.42, 0.59, 0.43)	(0.24, 0.77, 0.24)	(0.55, 0.48, 0.39)
FM7	(0.52, 0.49, 0.42)	(0.42, 0.59, 0.43)	(0.27, 0.74, 0.27)	(0.50, 0.51, 0.46)
FM8	(0.47, 0.53, 0.38)	(0.42, 0.59, 0.43)	(0.38, 0.62, 0.38)	(0.42, 0.58, 0.43)
FM9	(0.67, 0.33, 0.34)	(0.49, 0.51, 0.43)	(0.35, 0.65, 0.36)	(0.44, 0.56, 0.45)
FM10	(0.30, 0.71, 0.30)	(0.35, 0.65, 0.36)	(0.20, 0.81, 0.20)	(0.16, 0.86, 0.17)
FM11	(0.74, 0.26, 0.28)	(0.71, 0.31, 0.25)	(0.30, 0.75, 0.30)	(0.68, 0.32, 0.34)
FM12	(0.57, 0.43, 0.40)	(0.40. 0.60, 0.40)	(0.32, 0.68, 0.33)	(0.55, 0.45, 0.46)
FM13	(0.70, 0.30, 0.31)	(0.60, 0.40, 0.40)	(0.60, 0.40, 0.40)	(0.72, 0.20, 0.28)
FM14	(0.57, 0.43, 0.43)	(0.40, 0.61, 0.41)	(0.53, 0.48, 0.43)	(0.33, 0.67, 0.34)
FM15	(0.48, 0.53, 0.44)	(0.47, 0.53, 0.48)	(0.22, 0.78, 0.40)	(0.54, 0.47, 0.38)
FM16	(0.62, 0.38, 0.39)	(0.42, 0.59, 0.43)	(0.32, 0.68, 0.33)	(0.48, 0.52, 0.49)
FM17	(0.62, 0.38, 0.39)	(0.64, 0.36, 0.37)	(0.39, 0.67, 0.25)	(0.48, 0.52, 0.49)
FM18	(0.78, 0.23, 0.25)	(0.57, 0.43, 0.43)	(0.20, 0.80, 0.20)	(0.70, 0.30, 0.31)
FM19	(0.84, 0.16, 0.17)	(0.62, 0.38, 0.38)	(0.26, 0.75, 0.26)	(0.78, 0.22, 0.22)
FM20	(0.81, 0.20, 0.21)	(0.83, 0.17, 0.18)	(0.28, 0.73, 0.29)	(0.72, 0.28, 0.29)
FM21	(0.14, 0.87, 0.14)	(0.10, 0.90, 0.10)	(0.18, 0.83, 0.18)	(0.14, 0.78, 0.14)

Table 13. Expert Subjective Risk Factor SFS Table for 10000h.

Code	**S1**	**S2**	**D**	**M**
FM1	(0.47, 0.53, 0.48)	(0.41, 0.60, 0.37)	(0.24, 0.77, 0.24)	(0.55, 0.45, 0.40)
FM2	(0.57, 0.42, 0.43)	(0.43, 0.57, 0.44)	(0.56, 0.46, 0.28)	(0.42, 0.57, 0.41)
FM3	(0.56, 0.44, 0.42)	(0.32, 0.69, 0.32)	(0.22, 0.78, 0.23)	(0.37, 0.65, 0.32)
FM4	(0.53, 0.47, 0.47)	(0.45, 0.55, 0.46)	(0.30, 0.71, 0.30)	(0.50, 0.51, 0.38)
FM5	(0.58, 0.42, 0.42)	(0.47, 0.53, 0.48)	(0.28, 0.72, 0.28)	(0.30, 0.70, 0.30)
FM6	(0.36, 0.67, 0.38)	(0.42, 0.59, 0.43)	(0.30, 0.71, 0.30)	(0.61, 0.41, 0.34)
FM7	(0.52, 0.49, 0.42)	(0.42, 0.59, 0.43)	(0.27, 0.74, 0.28)	(0.50, 0.51, 0.46)
FM8	(0.47, 0.53, 0.48)	(0.43, 0.57, 0.44)	(0.38, 0.62, 0.39)	(0.55, 0.45, 0.46)
FM9	(0.67, 0.33, 0.34)	(0.55, 0.45, 0.45)	(0.35, 0.65, 0.36)	(0.44, 0.56, 0.45)
FM10	(0.33, 0.68, 0.35)	(0.35, 0.65, 0.36)	(0.20, 0.81, 0.21)	(0.22, 0.78, 0.23)
FM11	(0.79, 0.21, 0.23)	(0.71, 0.31, 0.25)	(0.30, 0.75, 0.33)	(0.68, 0.32, 0.34)
FM12	(0.60, 0.40, 0.40)	(0.40, 0.60, 0.40)	(0.32, 0.68, 0.33)	(0.55, 0.45, 0.45)
FM13	(0.70, 0.30, 0.31)	(0.60, 0.40, 0.40)	(0.60, 0.40, 0.40)	(0.83, 0.17, 0.18)
FM14	(0.57, 0.43, 0.43)	(0.47, 0.53, 0.48)	(0.55, 0.45, 0.45)	(0.35, 0.65, 0.36)
FM15	(0.50, 0.51, 0.46)	(0.42, 0.58, 0.43)	(0.20, 0.80, 0.20)	(0.63, 0.37, 0.38)
FM16	(0.62, 0.38, 0.39)	(0.42, 0.59, 0.43)	(0.32, 0.68, 0.33)	(0.54, 0.46, 0.43)
FM17	(0.68, 0.32, 0.32)	(0.64, 0.36, 0.37)	(0.39, 0.67, 0.25)	(0.53, 0.47, 0.47)

FM18	(0.83, 0.18, 0.19)	(0.57, 0.43, 0.43)	(0.26, 0.75, 0.26)	(0.70, 0.30, 0.31)
FM19	(0.84,0.16, 0.17)	(0.70, 0.30, 0.30)	(0.38, 0.62, 0.38)	(0.84, 0.16, 0.17)
FM20	(0.81, 0.20, 0.21)	(0.83, 0.17, 0.18)	(0.20, 0.80, 0.20)	(0.72, 0.28, 0.29)
FM21	(0.10, 0.90, 0.10)	(0.10, 0.90, 0.10)	(0.10, 0.90, 0.10)	(0.15, 0.85, 0.15)

3.2.3. Objective Evaluation Matrix

Based on the failure rates calculated in Section 3.1.4 for the 1000h and 10000h and using the SFS transformation method mentioned in Section 2.1.2, the spherical fuzzy evaluations for the objective risk factor O of each failure mode are obtained. The transformation results are shown in Table 14.

Table 14. SFS Table of Failure Mode Occurrence.

Code	SFS(1000h)	SFS(10000h)	Code	SFS(1000h)	SFS(10000h)
FM1	(0.22, 0.78, 0.22)	(0.10, 0.90, 0.10)	FM12	(0.34, 0.66, 0.34)	(0.22, 0.78, 0.22)
FM2	(0.11, 0.89, 0.11)	(0.10, 0.90, 0.10)	FM13	(0.34, 0.66, 0.34)	(0.33, 0.67, 0.34)
FM3	(0.11, 0.89, 0.11)	(0.10, 0.90, 0.10)	FM14	(0.34, 0.66, 0.34)	(0.33, 0.67, 0.33)
FM4	(0.11, 0.89, 0.11)	(0.10, 0.90, 0.10)	FM15	(0.34, 0.66, 0.34)	(0.33, 0.67, 0.33)
FM5	(0.10, 0.90, 0.10)	(0.10, 0.90, 0.10)	FM16	(0.45, 0.55, 0.45)	(0.44, 0.56, 0.45)
FM6	(0.11, 0.89, 0.11)	(0.10, 0.90, 0.10)	FM17	(0.57, 0.44, 0.45)	(0.34, 0.67, 0.34)
FM7	(0.22, 0.78, 0.23)	(0.11, 0.89, 0.11)	FM18	(0.57, 0.44, 0.44)	(0.45, 0.56, 0.45)
FM8	(0.22, 0.78, 0.22)	(0.11, 0.89, 0.11)	FM19	(0.57, 0.44, 0.44)	(0.58, 0.44, 0.44)
FM9	(0.11, 0.89, 0.11)	(0.12, 0.89, 0.12)	FM20	(0.82, 0.22, 0.22)	(0.57, 0.44, 0.44)
FM10	(0.22, 0.78, 0.23)	(0.11, 0.89, 0.11)	FM21	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)
FM11	(0.33, 0.67, 0.34)	(0.22, 0.78, 0.22)			

3.3. Dynamic Weights for Risk Factors

After obtaining the SFSs for each risk factor of each failure mode, this section will address subjective and objective weights to these risk factors at different CNC machine tool service ages (1000h and 10000h) to improve the accuracy of the evaluation results. The objective weighting method employed is the entropy weight method based on SFSs, while the subjective weighting approach is the SFS-AHP.

3.3.1. Risk Factor Weights Based on Spherical Fuzzy Set Entropy Weight Method

Using Equation 2.2.16 to Equation 2.2.18 and data from Table 12 to Table 14, the objective weights of the risk factors for 1000h and 10000h are calculated based on the SFS entropy weight method. The calculation results are shown in Table 15.

Table 15. Objective Weights of Each Risk Factor.

Risk Factor	O	S1	S2	D	M
Objective Weight (1000h)	0.2834	0.1643	0.1411	0.2454	0.1658
Objective Weight (10000h)	0.2976	0.1682	0.1403	0.2300	0.1639

3.3.2. Risk Factor Weights Based on SFS-AHP

As mentioned in Section 2.2.3, the entropy weight method is sensitive to data errors and may overlook the actual significance of the evaluated attributes when assigning weights to risk factors of CNC machine tool failure modes. To provide more practically meaningful risk factor weight assignments, this section adopts the SFS-AHP method to reflect expert opinions in weight allocation. The process and calculation example of the SFS-AHP method are as follows.

Step 1: Establish Hierarchical Structure.

The overall goal is to select the most hazardous failure mode; criteria include risk factors O, S1, S2, D, and M; alternatives are 21 failure modes. The program flowchart is shown in Figure 3.

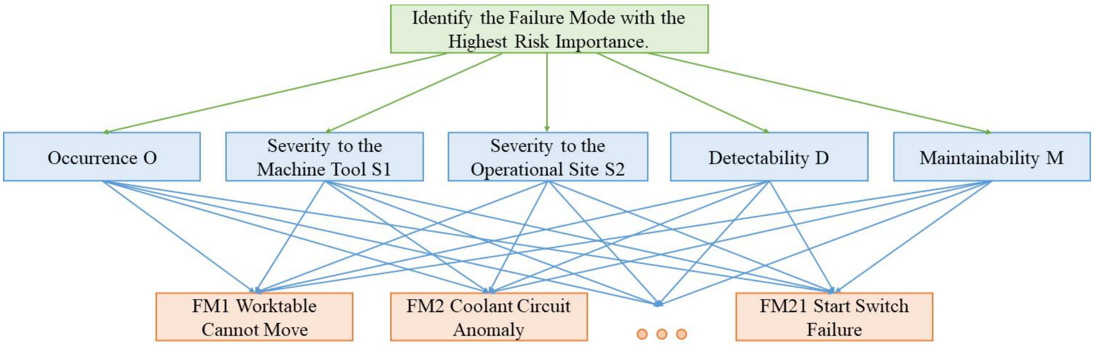

Fig.3. SFS-AHP Hierarchical Structure.

Step 2: Construct the Judgment Matrix.

Considering the expert's background level, the first expert (with a background score of 10) from Section 3.2.1 was invited to compare the relative importance of the risk factors for 1000h and 10000h. The 1000h results of the pairwise comparisons based on intuitive language from Table 3 are as follows.

$$\boldsymbol{J}_{1000h} = \begin{vmatrix} E & FL & VL & SS & SS \\ FS & E & E & VS & FS \\ VS & E & E & AS & FS \\ SL & VL & AL & E & SL \\ SL & FL & FL & SS & E \end{vmatrix}$$

Convert into SFS：

$$\boldsymbol{J}_{1000h} = \begin{vmatrix} (0.5,0.4,0.4) & (0.3,0.7,0.2) & (0.2,0.8,0.1) & (0.6,0.4,0.3) & (0.6,0.4,0.3) \\ (0.7,0.3,0.2) & (0.5,0.4,0.4) & (0.5,0.4,0.4) & (0.8,0.2,0.1) & (0.7,0.3,0.2) \\ (0.8,0.2,0.1) & (0.5,0.4,0.4) & (0.5,0.4,0.4) & (0.9,0.1,0.0) & (0.7,0.3,0.2) \\ (0.4,0.6,0.3) & (0.2,0.8,0.1) & (0.1,0.9,0.0) & (0.5,0.4,0.4) & (0.4,0.6,0.3) \\ (0.4,0.6,0.3) & (0.3,0.7,0.2) & (0.3,0.7,0.2) & (0.6,0.4,0.3) & (0.5,0.4,0.4) \end{vmatrix}$$

Step 3: Consistency Verification.

Based on Table 3, the converted judgment matrix for 1000h is shown below:

$$\boldsymbol{J}_{1000h,SI} = \begin{vmatrix} 1 & \frac{1}{5} & \frac{1}{7} & 3 & 3 \\ 5 & 1 & 1 & 7 & 5 \\ 7 & 1 & 1 & 9 & 5 \\ \frac{1}{3} & \frac{1}{7} & \frac{1}{9} & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{5} & \frac{1}{5} & 3 & 1 \end{vmatrix}$$

The maximum eigenvalue was found to be $\lambda_{max} = 5.282$. By substituting λ_{max} into Equation 2.2.24 (where n is 5), the calculation result showed that CI = 0.07 .

Referring to the consistency check table, the average random consistency index RI = 1.12. Using Equation 2.2.25 got CR = 0.0625. Since 0.0625 > 0.10, the matrix meets the consistency requirement.

Step 4: Calculate Fuzzy Weights for Risk Factors.

Using the SWAM operator mentioned in Equation 2.2.13, the spherical fuzzy weights for each risk factor were calculated. The results are shown in Table 16.

Table 16. Subjective Spherical Fuzzy Weights for Risk Factors at 1000h.

Risk Factor	**O**	**S1**	**S2**	**D**	**M**
Spherical Fuzzy Weights	(0.48,0.51,0.30)	(0.67,0.31,0.24)	(0.74,0.25,0.23)	(0.36,0.64,0.28)	(0.44,0.54,0.30)

Step 5: Defuzzify the Fuzzy Weights to Obtain Subjective Weights.

The fuzzy weights for the risk factors were defuzzified using Equation 2.2.24 and normalized using Equation 2.2.25. The results are shown in Table 17.

Table 17. Subjective Weights for Risk Factors at 1000h.

Risk Factor	**O**	**S1**	**S2**	**D**	**M**
Subjective Weight	0.1755	0.2537	0.2843	0.1265	0.1600

The intuitive language evaluation results for the relative importance of risk factors at 10000h are as follows:

$$J_{10000h} = \begin{vmatrix} E & FL & VL & SS & E \\ FS & E & E & VS & SS \\ VS & E & E & VS & SS \\ SL & VL & VL & E & SL \\ E & SL & SL & SS & E \end{vmatrix}$$

Repeating Steps 1-5, it is found that the matrix at 10000h also meets the consistency requirements. The subjective weights at 10000h are shown in Table 18.

Table 18. Subjective Weights for Risk Factors at 10000h.

Risk Factor	O	S1	S2	D	M
Subjective Weight	0.1684	0.2517	0.2655	0.1337	0.1807

Comparing Table 17 and Table 18, it can be observed that at relatively higher service ages, experts assigned a higher weight to maintainability M. This suggests that at higher ages, greater attention should be given to maintainability, which can help reduce economic losses caused by machine downtime or costly repairs.

3.3.3. Comprehensive Risk Factor Weights

To determine the bias coefficient δ for the subjective and objective weights of the risk factors, ten experts (distinct from the three experts involved in the weight evaluation. Five of them are from related school of University B and the rest of them are from Company A) were invited to assess the importance of the objective weights derived from the entropy weight method and the subjective weights based on evaluations from three experts. Among the values ranging from 0 to 1, with a step size of 0.1, most of the experts (9 out of 10) selected 0.5. The main reasons for this choice are summarized below:

- Given the limited failure data available for CNC machine tools, expert evaluations can supplement the information for decision-making. At the same time, expert judgments based on experience combined with data can enhance the reasonableness and accuracy of the results for the target CNC machine tools, making both aspects equally important.
- Using 0.5 as the bias coefficient allocates equal importance to both expert evaluations of risk factors and the weights of those factors, ensuring consistency between the expert evaluations.

Therefore, the most frequently voted bias coefficient δ in Equation 2.2.26 is set to 0.5. The comprehensive weights are calculated using Equation 2.2.26, based on the subjective and objective weights from Sections 3.3.1 and 3.3.2. The results are shown in Table 19.

Table 19. Comprehensive Weights of Risk Factors.

Risk Factor	O	S1	S2	D	M
Comprehensive Weight (1000h)	0.2294	0.2090	0.2127	0.1860	0.1629
Comprehensive Weight (10000h)	0.2330	0.2099	0.2029	0.1819	0.1723

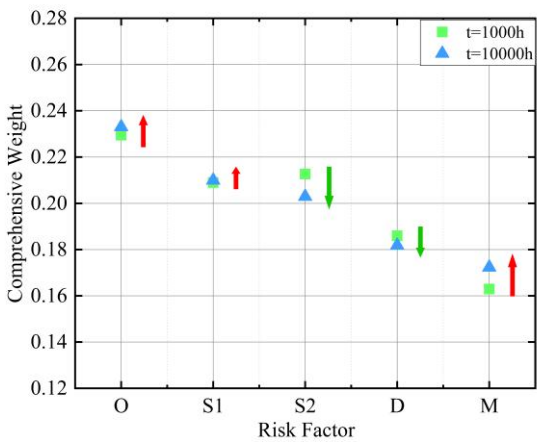

Fig. 4. Change of Comprehensive Weight between 1000h and 10000h.

Figure 4 shows the changes in comprehensive weights of risk factors. From 1000h to 10000h, the weights for risk factors O, S1, and M increase. Based on expert judgment and objective data, these three risk factors should receive more attention as the service age of machine tools increases. Meanwhile, the weights of S2 and D decrease. These results demonstrate the proposed dynamic weighting method improves the rationality of evaluation. The dynamic weighting approach allows for more dynamic insights for reliability-related decision-making.

3.4. Failure Mode Hazard Ranking Based on SF-WASPAS

This section uses the SF-WASPAS to rank failure modes. Using the SFSs from Tables 12–14 and the comprehensive weights from Table 19, the hazard degree SFSs of each failure mode at 1000h and 10000h are calculated using Equations 2.2.27 and 2.2.28. Experts consulted in this study believe that the contributions of WSM and WPM are equal. Therefore, with $\lambda = 0.5$ as per Equation 2.2.29, the final spherical fuzzy hazard degrees for each failure mode are calculated. Using the defuzzification method outlined in Section 2.2.7 (Equation 2.2.30), the final hazard scores Q_i are obtained. The results are summarized in Table 20.

Table 20. SF-WASPAS Fuzzy Hazard Degree and Hazard Score for Each Failure Mode.

Code	$\widetilde{Q}_i$(1000h)	Q_i(1000h)	$\widetilde{Q}_i$(10000h)	Q_i(10000h)
FM1	(0.39, 0.64, 0.37)	0.21095	(0.39, 0.64, 0.36)	0.21298
FM2	(0.46, 0.57, 0.37)	0.24957	(0.45, 0.57, 0.37)	0.24866
FM3	(0.36, 0.68, 0.32)	0.19607	(0.36, 0.68, 0.32)	0.19665
FM4	(0.40, 0.63, 0.40)	0.21409	(0.41, 0.62, 0.39)	0.22391
FM5	(0.37, 0.65, 0.38)	0.20302	(0.40, 0.63, 0.37)	0.21556
FM6	(0.37, 0.67, 0.33)	0.20353	(0.39, 0.65, 0.34)	0.21219
FM7	(0.41, 0.62, 0.38)	0.22045	(0.40, 0.63, 0.38)	0.21533
FM8	(0.39, 0.62, 0.38)	0.21245	(0.41, 0.61, 0.41)	0.22488
FM9	(0.47, 0.56, 0.36)	0.25709	(0.48, 0.55, 0.37)	0.26635
FM10	(0.26, 0.75, 0.27)	0.15245	(0.26, 0.76, 0.28)	0.15019
FM11	(0.61, 0.42, 0.31)	0.35346	(0.62, 0.42, 0.28)	0.36048
FM12	(0.45, 0.56, 0.39)	0.24654	(0.45, 0.57, 0.38)	0.24487
FM13	(0.61, 0.38, 0.35)	0.35472	(0.65, 0.37, 0.33)	0.38683
FM14	(0.45, 0.56, 0.40)	0.24773	(0.47, 0.54, 0.42)	0.26097
FM15	(0.43, 0.59, 0.42)	0.23299	(0.45, 0.57, 0.39)	0.24434
FM16	(0.48, 0.53, 0.42)	0.26372	(0.49, 0.52, 0.41)	0.27011
FM17	(0.56, 0.46, 0.40)	0.32267	(0.55, 0.47, 0.36)	0.30896
FM18	(0.62, 0.40, 0.35)	0.36896	(0.63, 0.39, 0.34)	0.37286
FM19	(0.68, 0.34, 0.31)	0.41896	(0.72, 0.30, 0.29)	0.46136
FM20	(0.76, 0.27, 0.23)	0.50488	(0.70, 0.32, 0.28)	0.44128
FM21	(0.57, 0.53, 0.14)	0.32164	(0.57, 0.53, 0.12)	0.32251

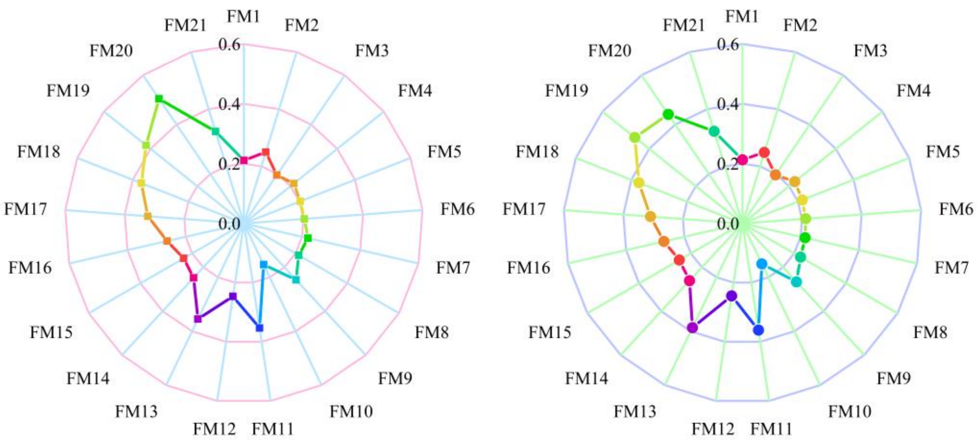

Fig. 5. Hazard Score Radar Charts at 1000 Hours and 10000 Hours.

The hazard score radar charts at 1000h and 10000h are shown in Figure 5.

The hazard degree ranking for each failure mode is listed in Table 21.

Table 21. Hazard Degree Ranking for Each Failure Mode.

1000h Ranking	Code	Failure Mode	10000h Ranking	Code	Failure Mode
1	FM20	Tool drop	1	FM19	Tool change failure
2	FM19	Tool change failure	2	FM20	Tool drop
3	FM18	Program error and alarm triggered	3	FM13	Spindle looseness and wear
4	FM13	Spindle looseness and wear	4	FM18	Program error and alarm triggered
5	FM11	Tool magazine motor temperature anomaly	5	FM11	Tool magazine motor temperature anomaly
6	FM17	Air pressure circuit leakage	6	FM21	Start switch failure
7	FM21	Start switch failure	7	FM17	Air pressure circuit leakage
8	FM16	Tool jam	8	FM16	Tool jam
9	FM9	Lubricating oil leakage	9	FM9	Lubricating oil leakage
10	FM2	Coolant circuit anomaly	10	FM14	Unacceptable machining accuracy error
11	FM14	Unacceptable machining accuracy error	11	FM2	Coolant circuit anomaly
12	FM12	Robot arm fails to return	12	FM12	Robot arm fails to return
13	FM15	Water tank leakage	13	FM15	Water tank leakage
14	FM7	Arc extinguisher damage	14	FM8	Chip conveyor malfunction
15	FM4	Hydraulic pump and circuit leakage	15	FM4	Hydraulic pump and circuit leakage
16	FM8	Chip conveyor malfunction	16	FM5	Protective cover detachment
17	FM1	Worktable cannot move	17	FM7	Arc extinguisher damage
18	FM6	Abnormal noise from worktable	18	FM1	Worktable cannot move
19	FM5	Protective cover detachment	19	FM6	Abnormal noise from worktable
20	FM3	Handwheel cannot start	20	FM3	Handwheel cannot start
21	FM10	Lighting failure	21	FM10	Lighting failure

3.5. Result Discussion

This section analyses the top 9 failure modes based on their hazard rankings. The changes in rankings with respect to machine age are illustrated in Figure 6.

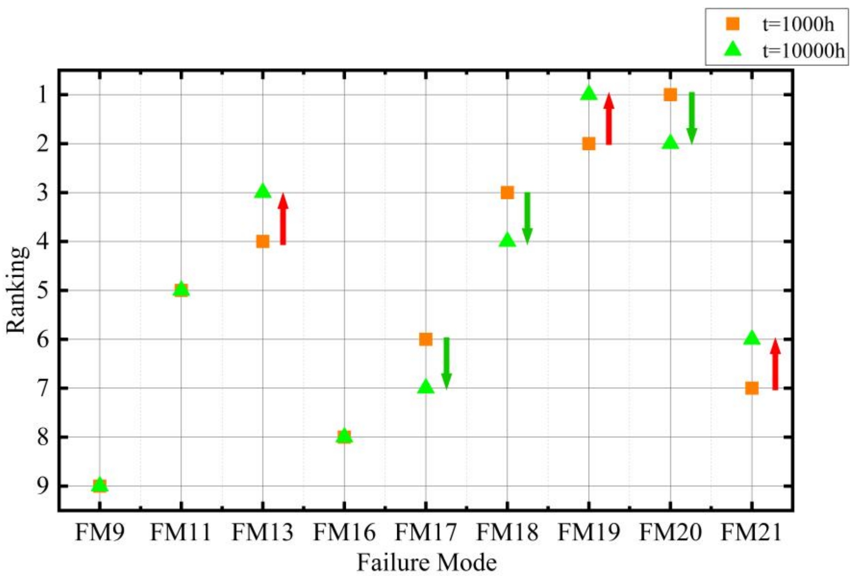

Fig. 6. Hazard Ranking Changes of the Top 9 Failure Modes.

Overall, for the two different service ages of the CNC machine tools, the failure modes "Tool Drop", "Tool change failure", "Program error and alarm triggered", "Spindle looseness and wear" and "Tool magazine motor temperature anomaly" are consistently ranked among the top five in terms of hazard levels. From an engineering perspective, the failure rates of moving components, automatic tool changer systems, and CNC systems are relatively high. Consequently, failure modes associated with these components generally have higher hazard levels.

Specifically, the hazard ranking of the failure mode "Tool change failure" increased from second place at 1000 hours to first place at 10000 hours. This shift can be explained by the increasing frequency of spindle wear faults as the CNC machine tool operates, which results in a higher hazard level compared to other failure modes. Similarly, "Spindle looseness and wear" rose from fourth place at 1000 hours to third place at 10000 hours and "Start switch failure" rose from seventh place at 1000 hours to sixth place at 10000 hours, driven by an increase in their respective occurrence frequencies. Therefore, as time progresses, greater attention should be given to these three failure modes.

This case demonstrates that the proposed method not only identifies high-hazard failure modes but also dynamically determines their hazard ranking over time. In the field of CNC machine tool reliability analysis, integrating the consideration of service age, especially at the subsystem level, is critical for achieving accurate reliability rankings. As CNC machines undergo prolonged usage, various subsystems experience unique degradation patterns, leading to changes in failure probability and severity that are specific to each subsystem. Initial reliability assessments based on experts' subjective reviews often fail to capture these shifts over time. As the machine's operational duration extends beyond the original evaluation, the relevance and accuracy of these initial rankings deteriorate, reducing the reliability of early risk predictions and leading to potentially suboptimal reliability-related decision-making.

Adopting the proposed age-sensitive method for reliability analysis significantly enhances the alignment of rankings with the actual condition of machine subsystems. By periodically updating expert evaluations and failure data according to the specific service ages, reliability-related decision-making can be refined to target the most vulnerable areas. This not only optimizes resource allocation but also maximizes the production efficiency of CNC machine tools. The proposed method supports the long-term reliability of CNC machine tools over its lifecycle.

3.6. Comparative Analysis

To further demonstrate the effectiveness and superiority of the proposed method, this section compares it with the methods proposed in [1] and [4]. The comparison of failure mode rankings is shown in the following Table 22 and Figure 7.

Table 22. Failure Mode Ranking Comparison Table of Proposed Method, [1] and [4]

Code	1000h Ranking	10000h Ranking	[1] Ranking	[4] Ranking
FM1	17	18	16	12
FM2	10	11	11	14
FM3	20	20	19	19
FM4	15	15	15	16
FM5	19	16	17	18
FM6	18	19	18	10
FM7	14	17	14	11
FM8	16	14	13	15
FM9	9	9	10	13
FM10	21	21	21	20
FM11	5	5	6	5
FM12	12	12	8	7
FM13	4	3	4	4
FM14	11	10	9	17
FM15	13	13	12	9
FM16	8	8	7	8
FM17	6	7	5	6
FM18	3	4	3	3
FM19	2	1	2	1
FM20	1	2	1	2
FM21	7	6	20	21

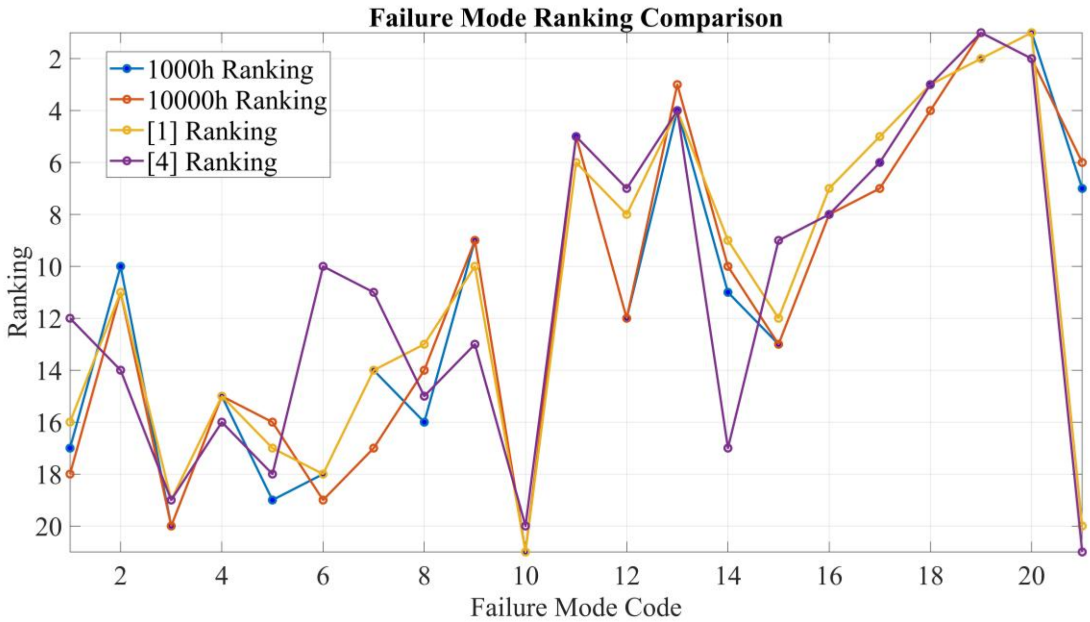

Fig. 7. Failure Mode Ranking Comparison Graph of Proposed Method, [1] and [4].

It can be seen in Figure 7 that the overall trend of the proposed method at two different service ages is similar to the methods in [1] and [4], demonstrating the effectiveness of the proposed method. The ranking of the 18 failure modes shows comparable results, with three out of four values for each failure mode having closely aligned rankings. Specifically, the maximum difference between any of the three closed values does not exceed one rank compared to at least one of the remaining two values. The failure modes that are not closely ranked are FM7 “Arc extinguisher damage”, FM12 “Robot arm fails to return” and FM21 “Start switch failure”. The differences in the rankings of FM12 and FM21 are because methods in [1] and [4] rely on DEA, which tends to assign low priority to failure modes that occur frequently but are easy to repair. In contrast, the method proposed in this paper treats them equally, recognizing that better design or maintenance resource allocation can improve overall production efficiency. The method in [4] heavily emphasizes economic impacts. On the other hand, the proposed method shows the same ranking as the method in [1] for FM7 at 1000 hours of service but a lower ranking at 10,000 hours. This can be explained by the fact that, as the service age increases, the CNC machine tool's electrical system is better adjusted due to previous maintenance, resulting in a significant reduction in the probability of FM7 occurring. This demonstrates that our method provides a unique perspective on failure mode ranking, further highlighting the advantages of the proposed approach.

4. Conclusion

This paper develops a dynamic weighted FMECA method based on SFS, which enables the identification of the hazard ranking of failure modes over time, providing a reliable foundation for on-site personnel to allocate resources more effectively, ultimately enhancing the reliability of CNC machine tools. The main strengths of the proposed FMECA methods are as follows:

1. By extending the FMECA risk factor set to include maintainability M, the proposed method offers a comprehensive evaluation that incorporates the financial aspect of failure modes, providing a more reasonable approach for CNC machine tool companies.

2. By using the expert background weighting method, different weights are assigned to each expert's evaluation results, making the final ranking results more reasonable.

3. By introducing the concept of SFS, the subjective fuzziness in expert evaluations is effectively captured and expressed, which helps enhance the accuracy of subjective information during evaluation.

4. A comprehensive dynamic weighting model based on SFS is established, achieving dynamic updates of comprehensive weights of risk factors with service ages of CNC machine tools.

5. The SF-WASPAS method is used to rank the hazard levels of failure modes at different service ages of CNC machine tools, thereby identifying key failure modes and highlighting shifts in failure mode ranking over time, providing an optimal ranking

result.

In conclusion, integrating service age into the reliability analysis of CNC machine tools is critical for obtaining accurate failure mode reliability rankings. As machines age, the wear and degradation of subsystems can significantly alter failure probabilities, which are often overlooked in static and expert-based evaluations. Relying on initial rankings without considering the evolving condition of the machine leads to inaccurate risk assessments and inefficient allocation of resources. By adopting a dynamic, age-aware approach, reliability analysis can better reflect the true state of each subsystem, allowing for more precise ranking. This not only improves resource allocation but also maximizes the production efficiency of CNC machines, enhancing overall reliability and cost-effectiveness in industrial operations.

It is recommended that future research can focus on the following areas:

1. Investigating the impact of variations in the bias coefficient δ on the final failure mode ranking.
2. Examining the influence of expert consensus issues on the final ranking of failure modes.
3. Developing an application to facilitate dynamic evaluations of CNC machine tools or other mechanical systems by experts.

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