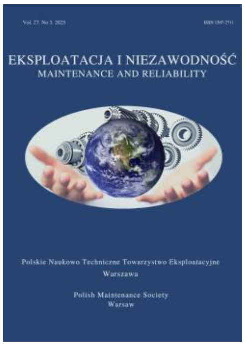

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Weibull Reliability Methodology based on Cumulated Vibration Damage

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Highlights

- We determine the reliability index of each damage element D_i and damage block B_i.
- We determined the Weibull shape and scale parameters for each D_i, and B_i element.
- The estimated shape parameter represents the spread of the cumulated damage until D=1.
- Fatigue vibration damage is cumulated by using a nonlinear model for AL6061-T6.
- The only input of the reliability methodology is the data from the cumulated damage analysis.

Abstract

In the paper, the formulated Weibull vibration reliability methodology is based on the cumulative vibration damage analysis. It lets us determine the reliability index of each damage element, each damage block, and the reliability of the analyzed element. Vibration damage is cumulated until $D = 1$, by using the addressed vibration stress and a nonlinear cumulative damage model. Based on static and modal analysis, the vibration stress is determined by incorporating to it, the geometry, weight and resonance effects. In the reliability analysis the Weibull shape (β) parameter is determined directly from the number of damage blocks, for which $D = 1$. The damage element, damage block, and element reliability are all determined based on the beta (β) value and D_i elements. Finally, based on the cumulated applied cycles n_i the Weibull scale (η_i) parameter is determined by each D_i elements, damage block, and analyzed element.

Keywords
random vibration, Weibull reliability, mechanical element, cumulative damage model, static and modal analysis

1. Introduction

Mechanical and structural elements subjected to random vibration cumulate damage due to fatigue (1). To avoid failing due to random vibration, a demonstration vibration zero failure test is performed. The test is based on a standard vibration profile, as those given in norm ISO16750-3 (2). And by considering the testing profile represents a stationary behavior, a probability density function is used to predict its behavior. And due to the GRMS (root mean square acceleration) is used to perform the analysis (3), then the Weibull distribution is used to model the random behavior. However, because in the demonstration test plan, no failure times are allowed, then in the analysis a supposed beta value in the range $[2.0 \leq beta \leq 2.5]$ is used (see appendices C and D in GMW3172) (4).

Therefore, the objective consists in determining the beta shape and eta scale parameters that represent the behavior of the analyzed profile. Among authors who have proposed methods to determine the element reliability by using the Weibull distribution we have (5,6), they improved the reliability and reduced the failure rate in motorized spindles in cycloid grinding machines using Monte Carlo simulation. In (7) they used the Weibull distribution and the traditional failure mode effects and criticality analysis tool to obtain the failure rate

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function and predict the remaining door life. In (8,9) authors provided methods to reduce the vibration in shearer drum cutting by applying a finite element model, to characterize road transport vibration levels. In (10), the author determined the reliability using the exponential distribution, he concludes that "for accurate results in reliability analysis, it is necessary to consider the effects of the randomness of the material properties."

On the other hand, authors who have used the cumulative damage and the Weibull distribution in the vibration analysis are (11,12). In (11), the beta parameter was not estimated from the damage, but from the minimum and maximum GRMS values of the used profile. Consequently, the estimated beta value does not represent the damage dispersion, but it represents the profile dispersion. In (12), the beta value was determined from the vibration bending stresses. Consequently, the estimated beta value does not represent the damage dispersion, but it represents the stress dispersion.

Thus, the novelty consists of the use of the number of damage blocks for which $D = 1$ to determine beta. Therefore, because the number of blocks at which $D = 1$ is random, then the estimated beta represents the randomness of the cumulated damage. Moreover, based on the cumulative n_i applied cycles of the cumulative damage analysis, the scale parameter that corresponds to each D_i element is determined. And due to the first cumulated block represents the application of the whole profile starting with zero damage, then we take the reliability of this first block as the reliability of the mechanical element. Additionally, observe that due to the only input of the proposed methodology is the cumulative damage analysis, then it can be applied to any analysis where the cumulative damage is available.

As a study case an AL6061-T6 aluminum cantilever beam is used. Data was published in (13). The simulation analysis was perform subjecting the beam to a weight of $2Nw$ mounted on the tip of the beam, with input PSD level of $0.475\ g\hat{}2/Hz$, and a frequency ranging between $20\ to\ 200Hz$, and it was tested by a period of $4.0\ hours$. From the vibration static and modal analysis, we found the natural frequency is $60Hz$, the dynamic factor is $2.42Mpa$, and the vibration bending stresses that contain the geometry, weight and resonance effect are in Table 3. The cycles to failure, and applied cycles are in Table 4 and le 5, respectively, and the corresponding cumulated damage is in Table 6. As reliability results, we have that $\beta = 2.1169$, and the element reliability is $R(t) = 0.9935$, with Weibull family given as $W(2.1169, 3.23E + 07\ cycles)$. The corresponding reliability index of each D_i element and their corresponding eta values are both given in Table 8.

The paper is structured as follows: section 2 presents the generalities of vibration analysis. In section 3, the numerical application of a cantilever beam is presented. Section 4 contains the formulation of the Weibull distribution and the steps of the proposed methodology. In section 5 the results of the application of the methodology are presented. Finally, the conclusions are given in section 6. The generalities of the performed vibration analysis are as follows.

2. Generalities of vibration analysis

Damage vibration analysis is performed on elements that are subjected to fatigue. Its accumulation damage is a complex process because it presents a nonlinear behavior. Among the models used to cumulate damage, we have the three-band technique (14), the Miner's rule and the nonlinear curve damage model. They generalities are.

2.1. Three-band technique generalities

The Steinberg three-band technique is a simplified method for analyzing fatigue failure due to random vibration by using Miner's rule approach. Unfortunately, although it let us a quick estimation of damage, as (15) mention it is not accurate. The method is based on the three sigma frequency bands of the normal distribution, 1σ (68.3%), 2σ (27.1%) and 3σ (4.33%) (4,14). Where σ is the standard deviation of the normal distribution. Therefore, to perform the fatigue damage analysis we determine the expected number of applied cycles n_i of each one of the three sigma bands, and their corresponding stress. Then the stresses values are used in the S-N curve to determine the corresponding cycles to failure (N) (14,16,17) that we use in the Miner rule approach.

2.2. Miner's Linear Rule Model

Miner (1945) (18) based on the work of Langer, applied the linear damage rule to axial stress-strain fatigue data of aircraft raw material. He found an agreement between the predictions of the linear damage rule and his experimental results (19). The principle of cumulated damage using Miner's rule is performed based on the assumption that fatigue strength is determined by applying different stress levels. Where each stress level contributes to a certain amount of damage. Thus, Miner's rule is used to predict the total life of a component subjected to a sequence of load levels, and it is given by eq.(1);

$$D_i = n_i/N_i \qquad (1)$$

where n_i is the number of apply cycles of a specific stress level, N_i is the number of cycles to failure for this stress level, and D_i is the damage that the material has cumulated during the application of the applied load. Thus, $D_i \leq 1$, means that the component or part does not fail yet. In general, for several stress levels the cumulated damage is as in eq.(2), and as in Figure 1;

$$D_c = \sum_{i=1}^{i=k} (n_i/N_i) \qquad (2)$$

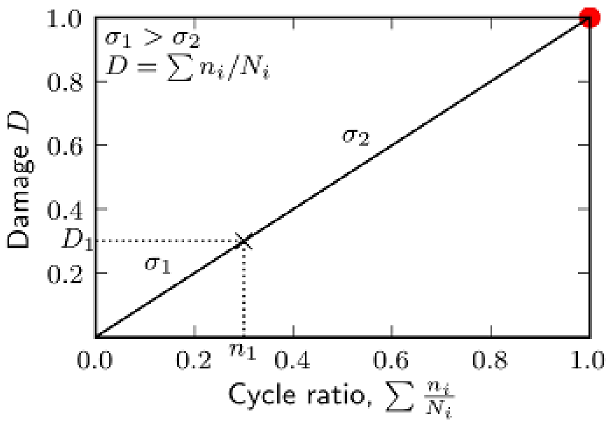

Figure 1. Miner's cumulative damage.

Therefore, the component failure is predicted when $D_c \geq 1$. From eq.(1), it is observed that the Palmgren-Miner rule is a simplistic model. It has no conclusive meaning because it does not allow us to evaluate probabilistically the selected design, which is fundamental in vibration fatigue analysis. Currently the Palmgren-Miner (1945) (18), and Palmgren (1924) approaches are still being used in most standards related to fatigue design to incorporate probability to the analysis. Moreover, from an engineering perspective, it is reasonable to consider damage as the probability of failure, derived from the field of the S-N curve, which agrees with the conventional concept of ultimate limit state (strength) (20). However, the nonlinear curve model was formulated to avoid the disadvantages of Miner's rule.

2.3. Nonlinear damage curve model

The double linear damage model by Manson and Halford rule (21) is used to determine the damage of an element subject to vibration (22). This model considers the interactions of the applied load and the nonlinear nature behavior of vibration. It considers equal damage for the two load levels based on the theory of elasticity and material properties (22), therefore, the equivalent damage radius cycle is represented by

$$\frac{n_1}{N_{f2}} = \left(\frac{n_2}{N_{f2}}\right)^{\left(\frac{N_{f2}}{N_{f1}}\right)^{0.4}} \qquad (3)$$

Consequently, the damage curve model is given by the power law equation,

$$D_i = \left(\frac{n_i}{N_{if}}\right)^{\left(\frac{N_{if}}{N_{if-1}}\right)^{0.4}} \qquad (4)$$

where the exponent 0.4 represents the cause-effect relationship of the material deformation, with the applied cycles. To consider the effect of the PSD (power spectral density) loads, the exponent of the previous model was modified by (23) to be a model that is in function of loads. It was formulated by substituting the exponent 0.4 by $(\sigma_i \pm 1_{vb}/\sigma i_{vb})$. Thus, the developed model is a nonlinear continuous damage function that incorporates the vibration-induced bending stress as follows

$$D = \sum_{i=1}^{2} D_2 = \left[\frac{n_2}{N_{2,f}}\right]^{\left(\frac{N_{2,f}}{N_{1,f}}\right)^{\left[\frac{\sigma 1_{vb}}{\sigma 2_{vb}}\right]}} \qquad (5)$$

In this paper we used eq.(5) to cumulate the damage generated by vibration. Where the cycles to failure N_i are determined by the Basquin equation

$$S = aN^b \qquad (6)$$

The numerical application is as follows.

3. Application case

In the numerical case, we used the analyzed cantilever beam element published by Kumar. Thus, here we present only the generalities, for deeper analysis see (13). The material proprieties are, aluminum Al 6061-T6, elasticity modulus $E = 68.9\, Gpa$, Poisson's ratio $\gamma = 0.3$, yield strength $Sy = 276\, Mpa$, ultimate tensile strength $Sut = 310\, Mpa$, fatigue strength $Se = 96.5\, Mpa,$ density $\rho = 0.0975\, lb/pul^3$, and $150\, mm$ length by $15\, mm$ wide and $7\, mm$ high, as shown in Figure 2. An overall damping ratio of 5 percent is considered in the analysis. The beam supports a weight of $2Nw$ mounted on the tip of the beam, and its movement is restricted to only vertical direction. The mechanical element must be capable of operating in a white noise random vibration environment with an input PSD level of 0.475 g^2/Hz, and frequency range of 20 to 200Hz for a period of 4.0 $hours$.

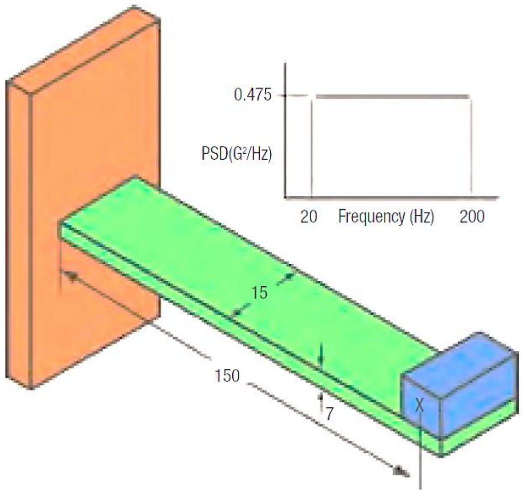

Figure 2. Aluminum cantilever beam, (13).

Base on the material characteristic and applied environment the nonlinear analysis is as follows.

3.1. Nonlinear analysis

The input test profile in the range frequency of $20-200Hz$ with accelaration of 0.475 g^2/Hz is given in Table 1. It presents a maximum of 9.25 $GRMS$ with a velocity of 31.5 in/s and a displacement of 0.3 in.

Table 1. Input testing profile.

Frequency (HZ)	Gravities (G)	Acceleration [G^2/Hz]
20	3.082	0.475
50	4.873	0.475
80	6.164	0.475
120	7.550	0.475
150	8.441	0.475
200	9.747	0.475

The corresponding simulation in Matlab is as follows.

3.1.1. Incorporation of time effect to the test profile

For simulation, we use a testing time of 4 hrs. (14400 sec) in the Matlab Vibrationdata library. The acceleration output response is shown in Table 2, and in Figure 3.

Table 2. Matlab response acceleration

Frequency (Hz)	Response acceleration in G units	Response acceleration in Grms (G^2/Hz)
20	8.88	3.94
50	19.21	7.38
80	24.33	7.39
120	29.77	7.38
150	33.17	7.33
200	29.03	4.21

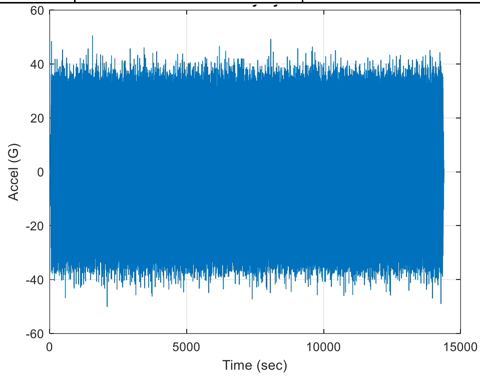

Figure 3. Matlab acceleration synthesis

The difference between data of Table 1 and Table 2 is because of the testing time effect. Similarly, the effect in the analysis of geometry, weight, and resonance is as follows.

3.1.2. Static and modal analysis

To incorporate the effect of geometry, weight, and resonance into the analysis the corresponding angular natural frequency W_n in Rad/Sec, is determinated as.

$$W_n = \sqrt{\frac{3EI}{ml^3}} \tag{7}$$

where E is the elasticity modulus in Lb/in^2, I is the inertia moment in in^4, m is the effective mass of the load in $Lb-seg^2/in$, and l is the length of the component in in. Numerically, W_n is estimated as

$$W_n = \sqrt{\frac{3(9993100.12\ \frac{lb}{in^2})(0.0010976\ in^4)}{(0.000937757\ \frac{lb-sec^2}{in})(6.3^3)}} = 374.60\frac{Rad}{Sec}$$

Thus, based on W_n the corresponding natural frequency is,

$$f_n = \frac{W_n}{2\pi} \tag{8}$$

$$f_n = \frac{374.60}{2\pi} \approx 60\ Hz$$

On the other hand, the dynamic factor used to determine the generated bending vibration stress is determined as

$$\sigma_{dynamic} = \left(\frac{Km_e\hat{L}C}{I}\right)A \tag{9}$$

where, $k=1$, is the stress concentration factor, $C=0.1377in$ is the distance to the neutral axis, $\hat{L}=5.9\ in$ is the distance from the fixed point of the component to the point of application of the mass, $A=386\ in/sec^2$ is the constant of gravity, $I=0.0010976in^4$ is the inertia moment, and $m_e=0.001232311\ lb-sec^2/in$ is the effective mass . Therefore, numerically, its value is $\sigma_{dynamic} = 352.08\ Psi\ (2.42\ MPa)$. With this dynamic factor value, (11), the corresponding bending stress is determined as follows.

3.1.3. Bending stress analysis.

The bending stresses used to calculate the cycles to failure, based on the response acceleration from Table 2, and the dynamic factor given in eq.(9) are presented in Table 3. It is determined as

$$\sigma_{vb} = \sigma_{dinamic} * A_{res} \tag{10}$$

Table 3. Vibration profile with bending stress.

Frequency (Hz)	Response Acceleration A_{res} (G^2/Hz)	Scale Factor $\sigma_{dynamic}$(Mpa)	Bending Stress σ_{vib} (Mpa)
20	8.88	2.42	21.5784
50	19.21		46.6803
80	24.32		59.0976
120	29.77		72.3411
150	33.17		80.6031
200	29.03		70.5429

Therefore, by using the bending stress of Table 3 in the Basquin's formula defined in eq.(6), the corresponding failure cycles are determined as

$$N_i = \left(\frac{\sigma_{vib/bending}}{a}\right)^{\frac{1}{b}} \tag{11}$$

The N_i data is presented in Table 4. The material coefficient is $b = -\frac{1}{3} log\left[\frac{0.92*310\ Mpa}{96.5\ MPa}\right] = -0.1568$ and $a = \frac{(0.92*310\ MPa)^2}{96.5\ MPa} = 842.89$.

Table 4. Results of cycles to failure (N_i).

Frequency (HZ)	Bending Stress σ_{vib} (Mpa)	N_i(Cycles to Failure)
20	21.5784	1.40E+10
50	46.6803	1.02E+08
80	59.0976	2.28E+07
120	72.3411	6.28E+06
150	80.6031	3.15E+06
200	70.5429	7.37E+06

On the other hand, the applied cycles of each damage block are determined by using the Rainflow algorithm (ASTM E 1049-85) of Matlab. They are given in Table 5.

Table 5. Applied cycles (n_i) Rainflow.

Frequency (Hz)	n_i (applied cycles)
20	110437
50	74883.5
80	114802
120	137055
150	249066
200	156561.5

Therefore, based on the N_i values of Table 4 and n_i values of Table 5, the cumulative damage is performed as fallows.

3.1.4. Determination of the accumulated damage

The accumulation of the vibration damage is performed by using the nonlinear curve model (23) as.

$$D_i = \left[\frac{n_i}{N_{i,f}}\right]^{\left(\frac{N_{i,f}}{N_{i-1,f}}\right)^{\left(\frac{\sigma_{vf(i-1)}}{\sigma_{vf(i)}}\right)}} \tag{12}$$

The accumulated damage until $D = 1$ is given in Table 6.

Table 6. Results of the accumulated damage calculation.

		20 Hz		50 Hz		80 Hz		120 Hz		150 Hz		200 Hz
Block No.	$\mathbf{n_i}$	$\mathbf{D_1}$	$\mathbf{n_{eq}+n_2}$	$\mathbf{D_{1+2}}$	$\mathbf{n_{eq}+n_3}$	$\mathbf{D_{1+2+3}}$	$\mathbf{n_{eq}+n_4}$	$\mathbf{D_{1+2+3+4}}$	$\mathbf{n_{eq}+n_5}$	$\mathbf{D_{1+2+3+4+5}}$	$\mathbf{n_{eq}+n_6}$	$\mathbf{D_{1+2+3+4+5+6}}$
1	1.10E+05	7.88E-06	9.84E+06	8.18E-06	2.07E+06	1.07E-05	1.46E+05	1.36E-03	2.53E+05	8.04E-02	2.99E+06	9.27E-02
2	1.10E+05	9.27E-02	6.37E+07	9.32E-02	1.40E+07	9.70E-02	1.80E+06	1.11E-01	6.00E+05	1.91E-01	4.09E+06	2.11E-01
3	1.10E+05	2.11E-01	7.51E+07	2.12E-01	1.66E+07	2.19E-01	2.78E+06	2.40E-01	1.00E+06	3.19E-01	4.94E+06	3.47E-01
4	1.10E+05	3.47E-01	8.30E+07	3.49E-01	1.84E+07	3.59E-01	3.64E+06	3.84E-01	1.46E+06	4.63E-01	5.66E+06	4.99E-01
5	1.10E+05	4.99E-01	8.92E+07	5.01E-01	1.98E+07	5.15E-01	4.44E+06	5.44E-01	1.96E+06	6.23E-01	6.32E+06	6.66E-01
6	1.10E+05	6.66E-01	9.45E+07	6.69E-01	2.11E+07	6.87E-01	5.20E+06	7.19E-01	2.52E+06	7.99E-01	6.92E+06	8.48E-01
7	1.10E+05	8.48E-01	9.92E+07	8.51E-01	2.21E+07	8.73E-01	5.95E+06	9.09E-01	3.11E+06	9.88E-01	7.49E+06	1.05E+00

From Table 6, we notice $D = 1.0$ (fatigue failure) is reached in block 7. The corresponding nonlinear cumulated damage behavior is shown in Figure 4.

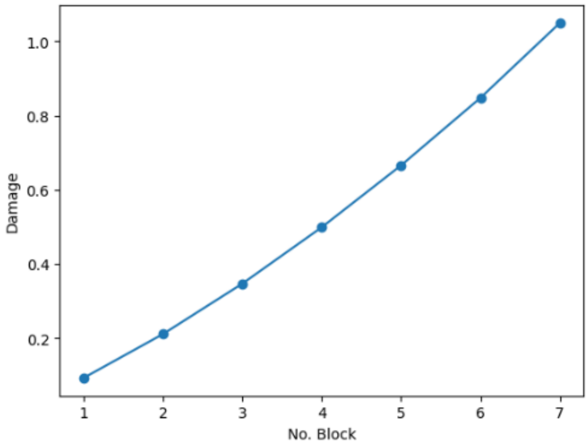

Figure 4. Damage curve behavior.

At this point, it is important to mention that although the cumulative damage already contains the effect of test time, geometry, weight, and resonance, it is still impossible to determine neither the reliability of the element nor the reliability that each damage blocks presents. Thus, in this paper, the novelty is that based on the Weibull distribution and on the cumulated damage, we determine those reliabilities indices.

4. Weibull distribution

Due to in the vibration analysis, the effect is represented by the vibration stress eq.(9), then the two parameter Weibull distribution (22) is used to perform the reliability analysis (11). The Weibull density function is:

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\}} \tag{13}$$

where beta (β) is the shape parameter and eta (η) is the scale parameter. Thus, the Weibull reliability function is given by:

$$R(t) = e^{\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\}} \tag{14}$$

With cumulative risk function given by

$$H(t) = \left(\frac{t}{\eta}\right)^{\beta} \tag{15}$$

Consequently, since eq.(15) represents the Weibull cumulative risk, and because the D_i elements of the accumulated damage analysis already represent the time, geometry, weight and resonance effects, and they also are accumulated, then for the reliability estimation, we only need to find an equivalence between eq.(15) and the addressed D_i elements, and an alternative way of estimating the Weibull shape parameter, as they are formulated in section 5. However, before describing the methodology, because in vibration the generated stress has a variant behavior (11), then in the determination of the reliability index of the analyzed element, it is advisable to use the cumulative damage model for variant stress given in the following section.

4.1. Cumulative damage variant stress model

The formulation of the cumulative damage model for variant stress that could be used in the reliability vibration analysis using the Weibull distribution, is as follows (24,25),

$$R\big(t, x(t)\big) = e^{-\left[\int_0^t \frac{1}{\eta(u)} du\right]^{\beta}} = e^{-\left(\frac{b_i}{\eta(t)}\right)^{\beta}} \tag{16}$$

where b_i is the ith block of the cumulative damage analysis generated by vibration. In eq.(16) the scale parameter eta η in function of the variant stress $\big(x(t)\big)$, could be estimated as

$$\eta(t) = \left[\frac{a}{x(t)}\right]^{n} \tag{17}$$

where a and n are the model parameters to be estimated. Therefore, the Weibull/cumulative damage stress variant model is

$$f\big(t, x(t)\big) = \left\{\beta\left[\frac{x(t)}{a}\right]^{n}\left[\int_0^t \left[\frac{x(u)}{a}\right]^{n} du\right]^{\beta-1}\right\} e^{-\left[\int_0^t \left[\frac{x(u)}{a}\right]^{n} du\right]^{\beta}} \tag{18}$$

However, notice that to apply eq.(18) to vibration data, it is necessary to know the value of beta (β), as well as known how the vibration affects behaves, or how these effects are related to the corresponding $\eta(t)$ value (26). Also notice the complexity of this relationship is that, it must be a function of the testing profile and the vibration stress given by eq.(10). And that for the estimated model parameters a and n, it must be possible to predict the correct value of eta (η). On the other hand, also observe that although the parameters of eq.(18) could be determined by using the maximum likelihood method, currently it is not possible because in the vibration analysis we have not failure times, therefore more research must be undertaken.

Thus, in this paper instead of use the time to failure which are unknown, we use the accumulated damage element (D_i) as the variable that represents the accumulated damage in the Weibull reliability function (27) (see eq.(15)) up to time t_i to determine the reliability of the element. This is performed based on the following formulation. By linearizing eq.(14), we get $y = \beta[ln(t) - ln(\eta)]$, implying that $\frac{y}{\beta} = ln(t) - ln(\eta) \rightarrow$

$\frac{y}{\beta} = ln\left(\frac{t}{\eta}\right)$. Thus, from eq. (15), $e^{\frac{y}{\beta}} = \left(\frac{t}{\eta}\right) = H(t)^{\frac{1}{\beta}}$. Therefore, the damage element $D_i = \frac{n_{ieq}}{N_i}$, by considering $n_{ieq} = t_i$ and $N_i = \eta_i$, in terms of the Weibull cumulative hazard function is given by.

$$D_i = e^{\frac{y}{\beta}} = \left(\frac{t}{\eta}\right) \tag{19}$$

Since D_i in eq.(19) represents the cumulative damage at t_i in the Weibull cumulative risk function (eq.(15)), then using the element D_i in eq.(15), and by replacing it in eq.(14), the reliability of the damage element is given by.

$$R(t_i) = e^{\left\{-1*D_{ik}{}^{\beta}\right\}} \tag{20}$$

Where the beta parameter is determined according to the methodology presented in section 4.2 (see step 6). Additionally, since D_i is a function of $n_{i,eq}$ which from eq.(12) is given as.

$$n_{i,eq} = N_{i,f} * (D_{i-1})^{(N_{i,f}/N_{ref})^{\frac{1}{\left(\sigma_{vf(i-1)}/\sigma_{vf(i)}\right)}}} \tag{21}$$

Then since the relation between $n_{i,eq}$ and D_i is unique, the reliability of the element in terms of $n_{i,eq}$ is also given as.

$$R(t_i) = \left(\frac{n_{ieq}}{\eta}\right)^{\beta} \tag{22}$$

Consequently, from eq.(22) we have that the reliability of the element as well as the reliability of each accumulated damage blocks, can be determined based on either the D_i, or the $n_{i,eq}$ elements. Based on this, the steps of the proposed methodology are as follows.

4.2. Steps of the methodology for vibration reliability analysis

1. Determine the accelerated testing profile, and by simulation, incorporate the effect that the testing time has on the initial input acceleration of the testing profile. From this simulation, obtain the corresponding output response acceleration data (see sections 3.1, and 3.1.1). In our case, we use the vibration library of the Matlab software.
2. Perform the static and modal analysis, to incorporate the effect of geometry, weight and resonance (see section 3.1.2).
3. Determine the dynamic factor defined in section 3.1.2, eq.(9) and by using it with the response acceleration of step 1 in eq.(10), determine the corresponding vibration stresses (see section 3.1.3).
4. Determine the cycles to failure N_i corresponding to each vibration stress using the Basquin's formula given in eq.(11). And from the rainflow analysis, determine the applied cycles n_i (see section 3.1.3).
5. Based on eq.(12) perform the cumulative damage analysis, and identify the element of the block for which $D = 1$. In our case, we perform the cumulative damage analysis using the nonlinear curve model given in eq.(12) (see section 3.1.4).
6. Determine the Weibull beta (β) value by using the number of blocks (b_i) at which $D = 1$, as the variable t $(b_i = t_i)$ in eq.(13), and solve it by applying the maximum likelihood method (retain only the beta value (β), the eta value (η) is not used) (27).
7. Determine the reliability of each damage D_i element of the cumulative damage analysis by using the D_i value as the variable t $(D_i = t_i)$ with the beta value (β) of step 6 in eq.(20).
8. Determine the value of eta η_i that corresponds to each damage D_i element by using the n_{eq} value in eq.(23).

$$\eta_i = \frac{n_{ieq}}{\left[-Ln\left(R\left(n_{ieq}\right)\right)\right]^{1/\beta}} \tag{23}$$

9. In each block, take the reliability of the last D_i element as the reliability of this block.
10. Take the element's reliability as the first block's reliability index.

In the next section, the numerical case is presented.

5. Results of numerical application case

The step-by-step analysis of the numerical application is as follows.

1. See section 3.1.1 The output response acceleration is given in Table 2.
2. See section 3.1.2.
3. See section 3.1.3. The dynamic factor and the corresponding vibration stresses are given in Table 3.
4. See section 3.1.3. The cycles to failure N_i are given in Table 4, and the applied cycles n_i are given in Table 5.
5. From eq.(12), the cumulated data is given in Table 7 (see also section 3.1.4).

Table 7. Cumulated damage data.

	0 - 20 Hz	20 - 50 Hz	50 - 80 Hz	80 - 120 Hz	120 - 150 Hz	150 - 200 Hz
Block No.	**D_1**	**D_{1+2}**	**D_{1+2+3}**	**$D_{1+2+3+4}$**	**$D_{1+2+3+4+5}$**	**$D_{1+2+3+4+5+6}$**
1	7.88E-06	8.18E-06	1.07E-05	1.36E-03	8.04E-02	9.27E-02
2	9.27E-02	9.32E-02	9.70E-02	1.11E-01	1.91E-01	2.11E-01
3	2.11E-01	2.12E-01	2.19E-01	2.40E-01	3.19E-01	3.47E-01
4	3.47E-01	3.49E-01	3.59E-01	3.84E-01	4.63E-01	4.99E-01
5	4.99E-01	5.01E-01	5.15E-01	5.44E-01	6.23E-01	6.66E-01
6	6.66E-01	6.69E-01	6.87E-01	7.19E-01	7.99E-01	8.48E-01
7	8.48E-01	8.51E-01	8.73E-01	9.09E-01	9.88E-01	**1.05E+00**

From Table 7 observe $D = 1$ occurred on the seventh block.

6. By using the maximum likelihood method, the addressed Weibull parameters are $\beta = 2.1169$, $\eta = 4.5184$, with $Lk\ value = -14.5769$. (Remember, the eta value will not be used).
7. The reliability index that corresponds to each one of the damages D_i elements are given in Table 8.
8. The eta η_i elements that correspond to each one of the damages D_i elements are given in Table 8.

Table 8. Summary of cumulative blocks damage nalysis $\beta = 2.1169$.

Block 1	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**
$\boldsymbol{D}$	0.000008	0.000008	0.000011	0.001359	0.080424	0.092679
$\boldsymbol{R(n_{ieq})}$	1.000000	1.000000	1.000000	0.99999915	0.995194	0.993517
$\boldsymbol{n_{ieq}}$	1.10E+05	9844194.1745	2070793.2004	146354.8597	253347.6474	2993717.2565
$\boldsymbol{\eta}$	1.40E+10	1.20E+12	1.93E+11	1.08E+08	3.15E+06	3.23E+07
Block 2	D1	D2	D3	D4	D5	D6
$\boldsymbol{D}$	0.092687	0.093234	0.096978	0.111455	0.190519	0.211205
$\boldsymbol{R(n_{ieq})}$	0.993516	0.993435	0.992866	0.99043427	0.970540	0.963490
$\boldsymbol{n_{ieq}}$	1.10E+05	63741504.4362	13967924.1449	1799452.5535	600167.1037	4089575.3686
$\boldsymbol{\eta}$	1.19E+06	6.84E+08	1.44E+08	1.61E+07	3.15E+06	1.94E+07
Block 3	D1	D2	D3	D4	D5	D6
$\boldsymbol{D}$	0.211213	0.212269	0.219426	0.239773	0.318838	0.347161
$\boldsymbol{R(n_{ieq})}$	0.963487	0.963107	0.960477	0.95251251	0.914899	0.898975
$\boldsymbol{n_{ieq}}$	1.10E+05	75141298.5128	16574955.0381	2783438.5685	1004391.9805	4936402.8615
$\boldsymbol{\eta}$	5.23E+05	3.54E+08	7.55E+07	1.16E+07	3.15E+06	1.42E+07
Block 4	D1	D2	D3	D4	D5	D6
$\boldsymbol{D}$	0.347169	0.348740	0.359323	0.384367	0.463432	0.499038
$\boldsymbol{R(n_{ieq})}$	0.898970	0.898051	0.891765	0.87624095	0.821764	0.794848
$\boldsymbol{n_{ieq}}$	1.10E+05	82984683.4795	18379932.7636	3641478.5470	1459885.8765	5663617.8214
$\boldsymbol{\eta}$	3.18E+05	2.38E+08	5.12E+07	9.47E+06	3.15E+06	1.13E+07
Block 5	D1	D2	D3	D4	D5	D6
$\boldsymbol{D}$	0.499046	0.501146	0.515229	0.544399	0.623463	0.666171
$\boldsymbol{R(n_{ieq})}$	0.794842	0.793214	0.782188	0.75878705	0.692242	0.654946

n_{ieq}	1.10E+05	89225225.3764	19821958.4851	4439710.4883	1964011.3842	6318324.1153
η	2.21E+05	1.78E+08	3.85E+07	8.16E+06	3.15E+06	9.48E+06
Block 6	D1	D2	D3	D4	D5	D6
D	0.666179	0.668824	0.686505	0.719452	0.798517	0.848231
$R(n_{ieq})$	0.654939	0.652608	0.636981	0.60770433	0.537366	0.493722
n_{ieq}	1.10E+05	94526806.4543	21050840.0254	5203561.2843	2515459.4615	6923692.6626
η	1.66E+05	1.41E+08	3.07E+07	7.23E+06	3.15E+06	8.16E+06
Block 7	D1	D2	D3	D4		D6
D	0.848239	0.851448	0.872835	0.909325	0.988389	1.045066
$R(n_{ieq})$	0.493715	0.490926	0.472450	0.44143080	0.376974	0.333603
n_{ieq}	1.10E+05	9.92E+07	2.21E+07	5.95E+06	3.11E+06	7.49E+06
η	1.30E+05	1.17E+08	2.54E+07	6.54E+06	3.15E+06	7.17E+06

9. The reliability index of each block is given in Table 9.

Table 9. Block's reliability indices.

Block	1	2	3	4	5	6	7
$R(t)$	0.993517	0.963490	0.898975	0.794848	0.654946	0.493722	0.333603

10. From the first row of Table 9, the reliability of the analyzed element is $R(t) = 0.993517$.

6. Conclusions

1. The given methodology could be used as a guideline to perform reliability vibration analysis because it presents the step from the testing profile simulation to the reliability indices estimation.
2. In the reliability analysis only the cumulative damage (Table 6) analysis is needed.
3. Due to beta (β) is estimated directly from the number of damage blocks until $D = 1$, then its value represents the dispersion for the blocks until $D = 1$ occurs. In case, $D = 1$ occurs in the first or second block, then we can use the recommended beta value mentioned in norm GMW3172 which is in the range of $[2.0 \leq \beta \leq 2.5]$.
4. The proposed methodology lets us determine the reliability index for each one of the cumulated D_i elements and damage blocks b_i.
5. The reliability of the first block is selected as the reliability of the analyzed element because it represents the first complete application of the testing profile. This is because only the first block starts with zero damage.
6. By applying the proposed methodology, it will be always possible to calculate the eta η_i value that corresponds to each one of the D_i elements. This occurs because, from the accumulated damage analysis, the corresponding n_{eq} values used in eq.(23), are always determined.
7. In section 4.1, the formulation to relate the Weibull cumulative risk function and the cumulated vibration D_i elements is given.
8. Since in eq.(21) a unique relationship between the n_{eq} and D_i elements exists, then eq.(21) can be used as the life/stress model in vibration analysis and accelerated vibration test analysis. This by taking the D_i elements as the accelerated stress, which in practice can be accelerated by increasing either the frequency and/or the generating bending stress. However, because it could not be evident, more research must be undertaken.

References

1. Kundu P, Darpe AK, Kulkarni MS. Weibull accelerated failure time regression model for remaining useful life prediction of bearing working under multiple operating conditions. Mech Syst Signal Process [Internet]. 2019;134:106302. Available from: https://doi.org/10.1016/j.ymssp.2019.106302
2. Standardization IO for. INTERNATIONAL STANDARD Road vehicles — Environmental conditions and testing for electrical. 2012;
3. Wang LJ, Wang ZW. FEM verification of accelerated vibration test method based on Grms - T curve. Advances in Mechanical Engineering. 2022;14(2):1–12. https://doi.org/10.1177/16878132221078487

4. Edson L. The GMW3172 Users Guide. 2008;
5. Wang H, Zhang Z, Li J, Xin W, Han C. Fault analysis and reliability evaluation for motorized spindle of cycloidal gear grinding machine based on multi-source bayes. Eksploatacja i Niezawodnosc. 2024;26(1). https://doi.org/10.17531/ein/175010
6. Wang W, Shen G, Zhang Y, Zhu Z, Li C, Lu H. Dynamic reliability analysis of mechanical system with wear and vibration failure modes. Mech Mach Theory. 2021 Sep 1;163. https://doi.org/10.1016/j.mechmachtheory.2021.104385
7. Wang W, Liu W, Fang Y, Zheng Y, Lin C, Jiang Y, et al. Reliability analysis of subway sliding plug doors based on improved FMECA and Weibull distribution. Eksploatacja i Niezawodnosc. 2024;26(2). https://doi.org/10.17531/ein/178275
8. Wan L, Jiang K, Zeng Q, Gao K. Dynamic response and reliability analysis of shearer drum cutting performance in coal mining process. Eksploatacja i Niezawodnosc. 2022;24(1):123–9. https://doi.org/10.17531/ein.2022.1.14
9. Rouillard V, Lamb MJ. Using the Weibull distribution to characterise road transport vibration levels. Packaging Technology and Science [Internet]. 2020;33(7):255–66. Available from: https://doi.org/10.1002/pts.2503
10. Hosseini SAA, Khadem SE. Vibration and reliability of a rotating beam with random properties under random excitation. Int J Mech Sci. 2007 Dec;49(12):1377–88. https://doi.org/10.1016/j.ijmecsci.2007.04.008
11. Barraza-Contreras JM, Piña-Monarrez MR, Torres-Villaseñor RC. Reliability by Using Weibull Distribution Based on Vibration Fatigue Damage. Applied Sciences (Switzerland). 2023 Sep 1;13(18). https://doi.org/10.3390/app131810291
12. Piña-Monarrez MR. Weibull stress distribution for static mechanical stress and its stress/strength analysis. Qual Reliab Eng Int. 2018 Mar 1;34(2):229–44. https://doi.org/10.1002/qre.2251
13. Kumar SM. Analyzing Random Vibration Fatigue. ANSYS Advantage [Internet]. 2008;II(3):39–42. Available from: www.ansys.com
14. Steinberg DS. Vibration analysis for electronic equipment. John Wiley & Sons; 2000. 0–432 p.
15. Al-Yafawi A, Patil S, Yu D, Park S, Pitarresu J. Random Vibartion Test for Electronic Assemblies Fatigue Life Estimation. IEEE. 2010; https://doi.org/10.1109/ITHERM.2010.5501295
16. Qin S, Li Z, Chen X, Shen H. Comparing and Modifying Estimation Methods of Fatigue Life for PCBA under Random Vibration Loading by Finite Element Analysis. IEEE. 2015;
17. Zheng M, Shen F, Luo P. Vibration Fatigue Analysis of the Structure under Thermal Loading. In: Advanced Materials Research. 2014. p. 559–64. https://doi.org/10.4028/www.scientific.net/AMR.853.559
18. Miner MA. Cumulative Damage in Fatigue. J Appl Mech [Internet]. 1945 Sep 1 [cited 2023 Mar 13];12(3):A159–64. Available from: https://asmedigitalcollection.asme.org/appliedmechanics/article/12/3/A159/1103202/Cumulative-Damage-in-Fatigue, https://doi.org/10.1115/1.4009458
19. Lee YL, Pan J, Hathaway R, Barker M. Fatigue Testing and Analysis. 2005. 5–417 p.
20. Castillo E, Fernández-Canteli A. A Unified Statistical Methodology for Modeling Fatigue Damage. 2009. 0–235 p.
21. Manson SS, Halford GR. Practical implementation of the double linear damage rule and damage curve approach for treating cumulative fatigue damage. Vol. 17, International Journal of Fracture. 1981. https://doi.org/10.1007/BF00036194
22. Weibull W. A Statistical theory of the strength of materials. 1939;1–45.
23. Barraza-Contreras JM, Piña-Monarrez MR, Molina A, Torres-Villaseñor RC. Random Vibration Fatigue Analysis Using a Nonlinear Cumulative Damage Model. Applied Sciences (Switzerland). 2022;12(9). https://doi.org/10.3390/app12094310
24. ReliaSoft. Two Methods for Analyzing Time-Varying Stress Data in ALTA. 2016.
25. ReliaSoft. Topics A Look Under the Hood at the Cumulative Damage Model. 2003.
26. Živković P, Milutinović M, Tica M, Trifković S, Čamagić I. Reliability Evaluation of Transmission Planetary Gears “bottom-up” approach. Eksploatacja i Niezawodnosc. 2023;25(1). https://doi.org/10.17531/ein.2023.1.2
27. Hernández-Ramos MM, Piña-Monarrez MR, Barraza-Contreras JM, Monclova-Quintana O. Weibull Random Vibration Reliability Based on Cumulated Damage [Internet]. 2024. Available from: https://www.preprints.org/manuscript/202408.0634/v1