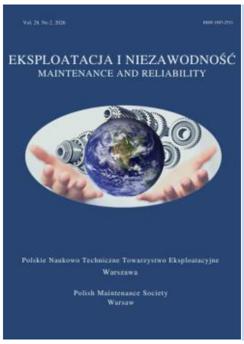

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A PM policy optimization for a two-stage failure process with multiple modes of external shocks in random environment

Indexed by: Web of Science Group

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Highlights

- Mode of Two-stage failure process with multiple modes of external shocks.
- Expressions of the reliability, expected availability and cost rate.
- Inspection-based PM optimization on first inspection time and inspection interval.
- A numerical example of airplane landing gears.

Abstract

In a random environment, the external shocks may conduct different impacts on the component. Besides, the failure process of the component can also affect the occurrence and impact of the external shock. Under the combined effect of external shocks and natural degradation, the failure process of the component follows a complex competing-risk mode. This paper proposes a mode of two-stage failure process based on delay-time mode (DTM) with multiple modes of external shocks and provides the models of the reliability, expected availability and cost rate in limited duration. An iterative algorithm combining with discretization method is proposed for approximate calculation. On this basis, a policy optimization method of preventive maintenance (PM) is proposed. Two decision variables, first inspection time and inspection interval, are determined by maximizing availability or minimizing cost rate. Finally, a numerical example of the airplane landing gear demonstrates the practicality and feasibility of our method.

Keywords
Delay-time model, Preventive maintenance, Policy optimization, Availability model, Competing-risk

1. Introduction

Preventive maintenance (PM) always plays a vital role in various industrial activities. Through implementing rational PM policies, the risk of unexpected failures, which can lead to significant economic losses and pose substantial hazards to personnel and environmental safety, can be mitigated [1-5]. There is a significant number of studies conducted on PM policy[6-9]. Inspection-based PM policy is the policy that recommends maintenance actions based on the information collected through inspections [10-12]. Besides, Inspection-based PM policy is effective in the scenarios where the state of a component is implicit and has received widely concerned about the prevention of sudden failure in degradation systems [13-18].

For Inspection-based PM activities, untimely inspections will lead to unexpected failures, while excessive inspections will result in a waste of time and resources. Therefore, it is crucial to determine an rational inspection schedule. There is a large amount of research conducted to optimize the inspection

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schedule [19-25]. A commonly used model to describe the failure process of a component is delay-time model (DTM) [26-28], which can help to capture the correlation between the failure process and inspections. DTM defines the failure process as a two-stage process: 1) the normal working stage, which starts from the new state and continues until an initial point at which a defect can be first identified through inspection, and 2) the delay-time stage, which occurs from that stage until failure if the defect remains unaddressed. Each stage is governed by the increment of its hazard rate. If an inspection takes place at a delay-time stage, the present defect should or could be identified and addressed. More importantly, there are numerous successful cases in industry with actual applications based on DTM [29-33].

In practical, numerous industrial systems operate in the random environment with external shocks[34-40]. Most relevant studies assume that only a single mode of shocks in random environment, which could cause same impact on the component. This is reasonable in many cases. For example, for some metal components, shocks are non-fatal, which can only increase the hazard rate of the components [41-43], while for some components of brittle materials, such as glass, shocks are fatal, which will cause the components to fail immediately [44-45]. However, these models above are not suitable for the components experiencing both normal shocks and fatal shocks in the environment. For example, for a micro-electro-mechanical component, if the strength of a shock is sight, it will not affect the condition of the component; if the strength of a shock is lower than its acceptable threshold, the shock can only cause a sudden increment of hazard rate; if the strength of a shock exceeds its acceptable threshold, the component will fail immediately[37, 46]. Besides, when an airplane lands and touches the ground, the landing gear may be impacted by external shocks. When the impact strength lower than the landing gear's acceptable threshold, the shocks may cause an increase in fatigue accumulation and result in subtle internal cracks, which is nonfatal. When the impact strength exceeds the landing gear's acceptable threshold, the landing gear may suddenly break, and the shocks can be seen as fatal shocks. Therefore, in this case, the external shocks should be divided into nonfatal shocks and fatal shocks based on their impacts on the landing gear. Unfortunately, few of studies consider multiple modes of shocks in the environment. As far as we know, [47] and [48] conducted studies on two modes of shocks, but the failure process of their systems does not follow DTM.

It should be noted that many components in engineering applications will be more fragile and more susceptible when being defective. For airplane landing gear, the failure process of airplane landing gear follows DTM and can be determined by detecting cracks in the landing gear. When cracks appears on the airplane landing gear, the landing gear will be more likely to be affected by external shocks, and the external shocks will be more likely to cause more fatigue accumulation or a sudden fracture of the landing gear. In recent years, some studies have noted the effects of failure process on external shocks. Among them, [49] formulated a competing-risk model, in which the occurrence of fatal shocks is influenced by the state of the component; [50] and [51] investigated the influence of the failure process on the sudden hazard rate increments caused by shocks. However, their studies most based on single mode of external shocks.

In this paper, the influence of the failure process on the occurrences and impacts of the multiple modes of external shocks in the random environment is considered. The modes of shocks include: 1) Nonfatal shock. The strength of the shock is lower than the component's acceptable threshold and the shock can only cause a sudden increment of hazard rate; 2) Fatal shock. The strength of the shock exceeds component's acceptable threshold and the component will fail immediately. Additionally, the influence of failure process on different modes of shocks include follow aspects: 1) the arrival of the shocks increases when the component enters the delay-time stage; 2) a shock at the delay-time stage are more likely to be fatal compared to the normal working stage; 3) the nonfatal shocks at a delay-time stage are more probable to cause more sudden hazard rate increment than normal working stage. As far as we know, no study has investigated the PM optimization based on the two-stage failure process and multiple modes of shocks. The main contribution of this paper lies in the following aspects:

- A novel mode of two-stage failure process based on DTM with multiple external shocks is proposed. The process failure can be divide into normal working stage and delay-time delay, and the shocks can be divided into two modes, i.e., nonfatal shocks and fatal shocks.

- The expressions of the reliability, expected availability and cost rate of the component in limited duration are provided, and an iterative algorithm combining with discretization method is proposed for approximate calculation.
- An Inspection-based PM policy optimization architecture is provided to maximize the expected availability and minimize cost rate of the component in limited time duration, where decision variables are the first inspection time and inspection interval.
- A numerical experiment of airplane landing gears along with sensitivity analysis demonstrates the practicality and feasibility of our method.

The remainder of this article is organized as follows. Section 2 gives the descriptions of proposed model and provides the derivation of the reliability, expected availability and cost rate. Section 3 presents the procedure of our PM policy optimization architecture. Section 4 provides a numerical example along with sensitivity analysis. Finally, Section 5 concludes the paper.

2. The model

2.1. Problem description

Assume a component operates within a mission. The failure process of a component can be divided into two stages based on DTM, i.e., the normal working stage and the delay-time stage. Besides, the failure process of the component is affected by multiple modes of external shocks, i.e., nonfatal shock and fatal shock. The defect of the component is implicit and can only be identified through inspections. The failure is self-announcing. When a component fails, it will stop working immediately. The periodic inspections are operated to determine the stage of the component. The inspections follow a pre-determined inspection schedule with fixed first inspection time and inspection interval. The first inspection is at time T_1 and the inspection interval is T. The inspections are scheduled at times $T_1, T_1+T, T_1+2T, T_1+3T, \cdots$. When a component is identified as defective or failed, it will be replaced by a new one. The purpose of this study is to determine the optimal first inspection time T_1^* and optimal inspection interval T^* that can optimize the expected availability and cost rate in the mission.

2.2. Assumptions and notation

The following assumptions are proposed either based on industrial practice and experience or from existing literatures:

1. The time duration of the mission is limited and the component is brand new at the beginning of the mission.
2. The normal working stage and failure delay-time stage of the component governs by corresponding baseline hazard rate functions independently [49], which indicate the failure process of the component in the environment without shocks.
3. The component is more likely to be impacted by external shocks when being defective. The arrival of the external shocks follows a Doubly Stochastic Poisson Process (DSPP) [49], where the external shocks arrive following a homogeneous Poisson Process (HPP) [52] in each stage independently,
4. The component can withstand specific strength of shocks in different states. When the strength of a shock exceeds the component's endurance, it can cause sudden failure of the component. Each upcoming shock could exceed the component's endurance and be a fatal shock by probability.
5. There are two competing failures of a component, i.e. defect-based failure (which is caused by hazard rate accumulation) and shock-based failure (which is caused by fatal shocks).
6. The shocks at the delay-time stage are more likely to cause more hazard rate increments or be fatal than at the normal working stage.
7. When a component is identified as defective, it will be replaced by a new one, which is referred as a preventive replacement.
8. When a component fails, it will be replaced by a new one, which is referred as a failure replacement.
9. The failure replacement is unexpected, which may result in postponed replacement and urgent maintenance. Therefore, a failure replacement requires more time and cost than a preventive replacement.

NOTATION	
T_1	First inspection time
T	Inspection interval
$N_X(t)$, $N_Y(t)$	Numbers of shocks arriving within duration t at normal working stage and delay-time stage.

$P_X(N_X(t)=n)$	Probability of *n* shocks arriving within duration *t* at normal working stage
$P_Y(N_Y(t)=n)$	Probability of *n* shocks arriving within duration *t* at delay-time stage
$\lambda_X(t)$,$\lambda_Y(t)$	Baseline hazard rate function at normal working stage and delay-time stage
φ_X,φ_Y	Probabilities of a shock being a fatal shock at normal working stage and delay-time stage
t_i	Arrival time of the *i*-th shock
μ_i,ν_i	Hazard rate increment for the *i*-th shock at normal working stage and delay-time stage
$h_X(t)$,$h_Y(t)$	Hazard rate during normal working stage and delay-time stage at time *t*
$g_X(\mu_i)(G_X(\mu_i))$	Probability density (distribution) function ofμ_i
$g_Y(\nu_i)(G_Y(\nu_i))$	Probability density (distribution) function ofν_i
ν_X,ν_Y	Intensities of the shock process at normal working stage and delay-time stage
$\tau_i, i=0,1,\dots,K$	Scheduled time point of the *i*-th inspection
T_i,T_p,T_f	Time consumption for each inspection, preventive replacement and failure replacement
C_i,C_p,C_f,C_n	Cost for per inspection, preventive replacement, failure replacement, and downtime
$D(t_1,t_2)$	Downtime within time duration(t_1,t_2)
$ED(T_1,T,t)$	Expected downtime of the component in the mission
$EA(T_1,T,t)$	Expected availability of the component in the mission
$EC(T_1,T,t)$	Expected cost rate of the component in the mission
$int(\ )$	Rounding down
EA_{min},EC_{max}	Minimum acceptable expected availability and maximum acceptable expected cost rate of the component

2.3. Random environment and reliability model

From Assumption 2, let the baseline hazard rate functions of normal working stage and delay-time stage be$\lambda_X(t)$ and$\lambda_Y(t)$, respectively. Without considering the fatal shocks, the hazard rate at time *t* during normal working stage is

$$h_X(t)=\lambda_X(t)+\sum_{i=1}^{N_X(t)}\mu_i,$$

Where $\mu_i, i=1,2,\dots,n$ represents the hazard rate increment caused by the *i*-th external shock at normal working stage and$N_X(t)$is the number of shocks arriving within duration *t* at normal working stage.

Analogously, the hazard rate function at time *t* during delay-time stage is

$$h_Y(t)=\lambda_Y(t)+\sum_{i=1}^{N_Y(t)}\nu_i,$$

where$\nu_i, i=1,2,\dots$ represents the hazard rate increment caused by the *i*-th external shock at delay-time stage and$N_Y(t)$is the number of shocks arriving within duration *t* at delay-time stage.

As mentioned, the probability distribution function and probability density function of a defect conditional on *n* nonfatal shocks during normal working stage are given by

$$F_X^{(n)}(t)=1-exp\left(-\int_0^t\lambda_X(u)du\right)\cdot\left(t^{-1}\cdot\int_0^t G_X^*(t-x)dx\right)^n, \tag{1}$$

$$f_X(t|N_X(t)=n)=exp\left(-\int_0^t\lambda_X(u)du\right)\cdot\left(t^{-1}\int_0^t G_X^*(t-x)dx\right)^n\cdot\left(\lambda_X(t)+n\cdot\left(t^{-1}-G_X^*(t)/\int_0^t G_X^*(t-x)dx\right)\right). \tag{2}$$

where$G_X^*(t-x)$ is Laplace transform of$G_X(\mu_i)$.

Similarly, the probability distribution function and probability density function of a failure conditional on *n* norma shocks during delay-time stage are given by

$$F_Y^{(n)}(t)=1-exp\left(-\int_0^t\lambda_Y(u)du\right)\cdot\left(t^{-1}\cdot\int_0^t G_Y^*(t-x)dx\right)^n, \tag{3}$$

$$f_Y^{(n)}(t)=exp\left(-\int_0^t\lambda_Y(u)du\right)\cdot\left(t^{-1}\int_0^t G_Y^*(t-x)dx\right)^n\cdot\left(\lambda_Y(t)+n\cdot\left(t^{-1}-G_Y^*(t)/\int_0^t G_Y^*(t-x)dx\right)\right), \tag{4}$$

where$G_Y^*(t-x)$ is Laplace transform of$G_Y(\lambda_i)$.The derivation of Eqs. (1-4) refers to [53] and is given in Appendix A.

As described in Assumption 4, each shock could be fatal by probability. Let each shock could be a fatal shock by probability of φ_X at normal working stage and φ_Y at delay-time stage. Assume that there is no failure before time *t*, and one of the following events will occur:

1) There is no defect before time *t*, and the survival function

of the component is given by

$$\bar{F}_R^{(1)}(t) = \sum_{m=0}^{\infty} P_X(N_X(u) = m)(1-\varphi_X)^m \left(1 - F_X^{(m)}(t)\right), \tag{5}$$

2) A defect occurs at time u and $u < t$. The survival function of the component is given by

$$\bar{F}_R^{(2)}(t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int_0^t \left(P_X(N_X(u) = m)P_Y(N_Y(t-u) = n)(1-\varphi_X)^m(1-\varphi_Y)^n f_X^{(m)}(u)\left(1 - F_Y^{(n)}(t-u)\right)\right)du, \tag{6}$$

Combining Eqs. (5, 6), the reliability function of the component is

$$F_R(t) = \bar{F}_R^{(1)}(t) + \bar{F}_R^{(2)}(t) = \sum_{m=0}^{\infty} P_X(N_X(t) = m)(1-\varphi_X)^m\left(1 - F_X^{(m)}(t)\right) + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int_0^t \left(P_X(N_X(u) = m)P_Y(N_Y(t-u) = n)(1-\varphi_X)^m(1-\varphi_Y)^n f_X^{(m)}(u)\left(1 - F_Y^{(n)}(t-u)\right)\right)du, \tag{7}$$

whereas describes in Assumption 3, external shocks arrive following a follows a DSPP. Thus,

$$P_X(N_X(t) = m) = exp(-v_X t)\cdot(v_X t)^m/m!$$

$$P_Y(N_Y(t) = n) = exp(-v_Y t)\cdot(v_Y t)^n/n!.$$

We adopt an Inspection-based PM policy with the first inspection time T_1 and inspection interval *T*. If a component is replaced, it is inevitable that there exists a first replacement in the mission. Depending on what the first replacement is caused by, one of the following events will occur:

(1) The first replacement is a failure replacement caused by defect-based failure. Let the initial time point of the first replacement is at time *t* and a defect occurs at time $u, u \in (\tau_K, t)$. Assume that the component experiences *m* nonfatal shocks at normal working stage, and *n* nonfatal shocks at delay-time stage before time *t*, and none of shock is fatal. The probability density function of the first replacement caused by defect-based failure is

$$p_b(T_1, T, t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int_{\tau_K}^t \left(P_X(N(u) = m)f_X^{(m)}(u)(1-\varphi_X)^m P_Y(N(t-u) = n)f_Y^{(n)}(t-u)(1-\varphi_Y)^n\right)du, \tag{8}$$

where $\tau_K = T_1 + int\left((t-T_1)/T\right)\cdot T$, which represents the time of the *k*-th inspection.

(2) The first replacement is a failure replacement caused by shock-based failure. Let the initial time point of the first replacement is at time *t*. There are two possibilities: 1) There is no defect before time *t*, and the component experiences *m*-1 nonfatal shocks at normal working stage with the last shock being a fatal shock at time *t*; 2) There is a defect which occurs at time $u, u < t$, and the component experiences *m* nonfatal shocks at normal working stage, and *n*-1 nonfatal shock at delay-time stage, with the last shock being a fatal shock at time *t*. Taking into account the possibilities above, the probability density function of the first replacement caused by shock-based failure is

$$p_a(T_1, T, t) = v_X \cdot \sum_{m=1}^{\infty} P_X(N(t) = m-1)\left(1 - F_X^{(m-1)}(t)\right)(1-\varphi_X)^{m-1}\varphi_X + v_Y \cdot \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\int_{\tau_K}^t \left(P_X(N(u) = m)P_Y(N(t-u) = n-1)f_X^{(m)}(u)\left(1 - F_Y^{(n-1)}(t-u)\right)(1-\varphi_X)^m(1-\varphi_Y)^{n-1}\varphi_Y\right)du, \tag{9}$$

where $\tau_K = T_1 + int\left((t-T_1)/T\right)\cdot T$.

(3) The first replacement is a preventive replacement. Let the initial time point of the first replacement is at time τ_i and a defect occurs at time *u*, and we have $u \in (\tau_{i-1}, \tau_i)$. Assume that the component experiences *m* nonfatal shocks at normal working stage and *n* nonfatal shocks and nonfatal shocks at delay-time stage, and none of shock is fatal. The probability of the first replacement being a preventive replacement at time τ_i is

$$P_s(T_1, T, \tau_i) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int_{\tau_{i-1}}^{\tau_i}\left(P_X(N(u) = m)(1-\varphi_X)^m f_X^{(m)}(u)P_Y(N(\tau_i-u) = n)(1-\varphi_Y)^n\left(1 - F_Y^{(n)}(\tau_i-u)\right)\right)du. \tag{10}$$

where$\tau_i = T_1 + (i-1)\cdot Tand\tau_0 = 0$.

(4) No replacement occurs in the mission. Let the mission duration is t. There are two possibilities: 1) There is no defect in the mission and the component experiences m shocks at normal working stage; 2) There is a defect which occurs at time u,$u \in (\tau_K, t)$, and the component experiences m nonfatal shocks at normal working stage and n nonfatal shocks at delay-time stage, and none of shock is fatal. Considering the possibilities above, the probability of no replacement before time t is

$$P_n(T_1, T, t) = \sum_{m=0}^{\infty} P_X(N(t) = m)(1-\varphi_X)^m \left(1 - F_X^{(m)}(t)\right) + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int_{\tau_K}^{t}\left(P_X(N(u) = m)P_Y(N(t-u) = n)(1-\varphi_X)^m(1-\varphi_Y)^n f_X^{(m)}(u)\left(1 - F_Y^{(n)}(t-u)\right)\right)du. \tag{11}$$

where$\tau_K = T_1 + int\big((t - T_1)/T\big)\cdot T$.

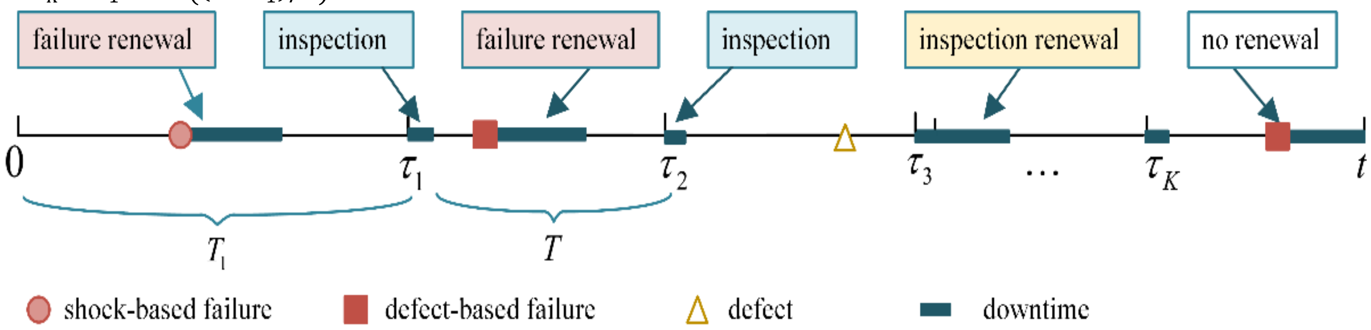


Fig. 1. Process of a mission.

From Assumptions 7 and 8, a component will be replaced in each replacement. Therefore, the end of each replacement can be regarded as the start of a new mission with a new component. For example, as shown in Fig. 2, the first inspection time, inspection interval and duration of the original mission areT_1, T, and t. Assume a failure replacement ends at time u. We can regard the time u as the start of a new mission. The duration of new mission is the remaining time of the original mission after the replacement, i.e.$t-u$. Besides, the first inspection of the new mission is the closest scheduled inspection of the original mission after time u. The first inspection time of the new mission is

$$T_1{}' = T_1 + \left(int\left(\frac{(u-T_1)}{(T+T_i)}\right) + 1\right)\cdot (T+T_i) - u.$$

Since the inspection interval remains unchanged, the expected downtime of the new mission can be expressed as

$$E\big(D(u,t)\big) = ED(T_1{}', T, t-u).$$

Then we have the following theorem for the downtime within$(0,t)$.

Theorem 1. If a replacement is completed at time u,$u \in (0,t)$, the downtime within$(0,t)$can be expressed as

$$D(0,t) = D(0,u) + ED(T_1{}', T, t-u).$$

Proof. As $u \in (0,t)$, the downtime $D(0,t) = D(0,u) + E\big(D(u,t)\big) = D(0,u) + ED(T_1{}', T, t-u)$, within $(0,t)$ can be expressed as

$$D(0,t) = D(0,u) + D(u,t).$$

As mentioned earlier, the remaining time horizon after the replacement,(u,t), can be regarded as a new mission with the first inspection time, inspection interval and mission duration being $T_1{}'$, T and $t-u$, respectively. As the downtime within (u,t) is uncertain, we adopt the expected downtime

2.4. Availability model

As shown in Fig. 1, a component works in a mission with limited duration t. The downtime in the mission includes the time consumption for inspections, failure replacements, preventive replacements, and the duration of no replacement after a failure in the mission. Specially, no replacement after a failure occurs when there is no need for replacement after a failure because the mission will end before the replacement is completed. The time consumption for each inspection, preventive replacement and failure replacement isT_i,$T_p$andT_f, respectively. The expected downtime of the component in the mission is given as

$$ED(T_1, T, t) = E\big(D(0,t)\big).$$

Specifically, it should be noted that because of the time consumption for inspections, the initial point of i-th inspection in this subsection is at time $\tau_i = T_1 + (i-1)\cdot(T+T_i)$. However, this does not affect subsequent calculations.

within$(u,t)$for calculation. Thus, the downtime within$(0,t)$can be expressed as

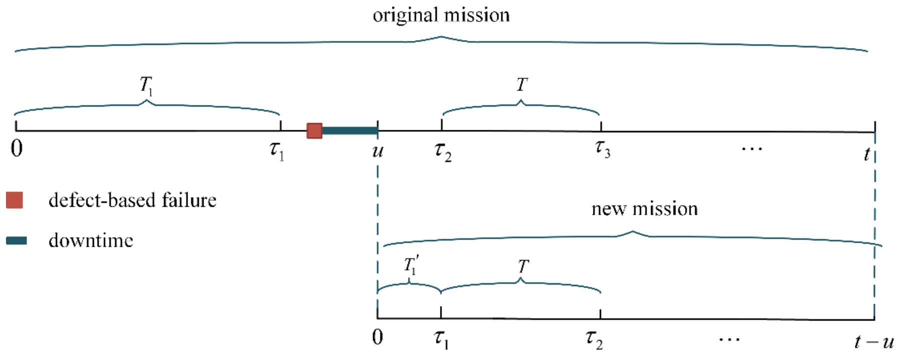


Fig. 2. The relationship between original mission and iterative new mission.

Based on the influence of mission duration, there are three possible scenarios:

Scenario 1. If$0 \le t \le T_p$, there is no need for inspections as the mission will end before a preventive replacement is completed. Therefore, there will be no inspection in the mission and the downtime can only be caused by failure. Assume that the first failure occurs at time $u, u \in (0,t)$. As$T_p < T_f$ (from Assumptions 9), the failure replacement is also unnecessary because failure replacement cannot be completed before the end of mission. Thus, the downtime in the mission is the duration of no replacement after a failure. The expected downtime in this scenario is

$$ED(T_1,T,t) = \int_0^t (t-u)(p_b(T_1,T,u) + p_a(T_1,T,u))du. \tag{12}$$

Scenario 2. If $T_p < t \le T_1 + T_p$, like Scenario 1, no preventive replacement can be completed before the end of the mission, so there is no need for inspection during the mission. Assuming that the first failure occurs at time $u, u \in (0,t)$, consider following possibilities: 1) If$t - T_f < u \le t$, as the mission will end before the replacement is completed, there is no need for a failure replacement, and the downtime in the mission is the duration of no replacement after failure; 2) If$u \le t - T_f$, there is enough time for a replacement before the end of mission and downtime is the time consumption for the failure replacement. Considering all the possibilities above, the expected downtime in this scenario is

$$\begin{aligned} ED(T_1,T,t) = &\int_{t-T_f}^{t} (t-u)\cdot\big(p_b(T_1,T,u) + p_a(T_1,T,u)\big)du \\ &+ \int_0^{t-T_f} \Big(T_f + ED\big(T_1',T,t-u-T_f\big)\Big)\cdot\big(p_b(T_1,T,u) + p_a(T_1,T,u)\big)du. \end{aligned} \tag{13}$$

Scenario 3. If$t > T_1 + T_p$, unlike Scenarios 1 and 2, there is enough time for a preventive replacement after an inspection, and a component will be inspected in the mission. Let$K = int\big((t-T_1)/(T+T_i)\big) + 1$be the number of inspections in the mission. Consider the following events:

(1) If there is no replacement in the mission, the downtime could be only caused by inspections. According to Eq. (11), the expected downtime caused for this event is

$$ED^{(3-1)}(T_1,T,t) = KT_iP_n(T_1,T,t-KT_i). \tag{14}$$

(2) If the first replacement in the mission is a failure replacement occurring at time u, consider following possibilities based on u:

a. if$u < t - T_f$, the failure replacement can be completed before the end of mission, and the downtime includes the time consumption for the failure replacement and inspections before time u.

b. if$t - T_f < u < t$, there is no need for a replacement as the failure replacement cannot be completed before the end of mission, and the downtime in the mission includes the duration of no replacement after the failure and the time consumption for inspections.

According to the possibilities above and integrating over the feasible range of u, the expected downtime caused for this event is

$$ED^{(3-2)}(T_1,T,t) = \sum_{i=1}^{K}\int_{\tau_{i-1}+T_i}^{\tau_i}\Big(\big((i-1)T_i + T_f + ED\big(T_1{}',T,t-u-T_f\big)\big)\cdot\big(p_b(T_1,T,u-(i-1)T_i) + p_a(T_1,T,u-(i-1)T_i)\big)\Big)du + \int_{\tau_K+T_i}^{t-T_f}\Big(\big(T_f + KT_i + ED\big(T_1{}',T,t-u-T_f\big)\big)\cdot\big(p_b(T_1,T,u-KT_i) + p_a(T_1,T,u-KT_i)\big)\Big)du + \int_{t-T_f}^{t}\Big((KT_i + t - u)\cdot\big(p_b(T_1,T,u-KT_i) + p_a(T_1,T,u-KT_i)\big)\Big)du. \tag{15}$$

For convenience, $\tau_0 = -T_i$.

(3) If the first replacement in the mission is a preventive replacement with the initial point at time τ_i, the downtime includes the time consumption for the preventive replacement and inspections. Considering all the possibilities of τ_i, the expected downtime for this event is

$$ED^{(3-3)}(T_1,T,t) = \sum_{i=1}^{K}\Big(iT_i + T_p + ED\big(T_1{}',T,t-\tau_i-T_p\big)\Big)\cdot P_s(T_1,T,\tau_i-(i-1)\cdot T_i). \tag{16}$$

Combining all the possible events above, the expected downtime in scenario is given by

$$ED(T_1,T,t) = ED^{(3-1)}(T_1,T,t) + ED^{(3-2)}(T_1,T,t) + ED^{(3-4)}(T_1,T,t) \tag{17}$$

where $ED^{(3-1)}(T_1,T,t), ED^{(3-2)}(T_1,T,t), ED^{(3-3)}(T_1,T,t)$ can be obtained from Eqs. (14, 15, 16).

Summarizing Scenarios 1, 2 and 3, the expected availability of the component in a mission with limited time duration *t* is given by

$$EA(T_1,T,t) = 1 - \frac{ED(T_1,T,t)}{t}. \tag{18}$$

$$ED(T_1,T,t) = \begin{cases} \int_0^t (t-u)(p_b(T_1,T,u) + p_a(T_1,T,u))du \\ \int_{t-T_f}^{t}(t-u)\cdot\big(p_b(T_1,T,u) + p_a(T_1,T,u)\big)du + \int_0^{t-T_f}\Big(T_f + ED\big(T_1{}',T,t-u-T_f\big)\Big)\cdot\big(p_b(T_1,T,u) + p_a(T_1,T,u)\big)du \\ \sum_{i=1}^{K}\Big(iT_i + T_p + ED\big(T_1{}',T,t-\tau_i-T_p\big)\Big)\cdot P_s(T_1,T,\tau_i-(i-1)\cdot T_i) + KT_iP_n(T_1,T,t-KT_i) + \\ \sum_{i=1}^{K}\int_{\tau_{i-1}+T_i}^{\tau_i}\Big(\big((i-1)T_i + T_f + ED\big(T_1{}',T,t-u-T_f\big)\big)\big(p_b(T_1,T,u-(i-1)T_i) + p_a(T_1,T,u-(i-1)T_i)\big)\Big)du + \\ \int_{\tau_K+T_i}^{t-T_f}\Big(\big(T_f + KT_i + ED\big(T_1{}',T,t-u-T_f\big)\big)\big(p_b(T_1,T,u-KT_i) + p_a(T_1,T,u-KT_i)\big)\Big)du + \\ \int_{t-T_f}^{t}\Big((KT_i + t - u)\big(p_b(T_1,T,u-KT_i) + p_a(T_1,T,u-KT_i)\big)\Big)du \end{cases} \tag{19}$$

2.5. Cost rate model

The cost of the component in the mission includes two parts: maintenance cost and losses of downtime. The expected maintenance cost is express as $EC_m(T_1,T,t)$, which includes the cost for inspections, preventive replacements, and failure replacements in the mission. As the derivation of $EC_m(T_1,T,t)$ is like that of $ED(T_1,T,t)$, it is not repeated in this subsection (see Appendix B for details). The expected cost rate in a mission is given by

$$EC(T_1,T,t) = \frac{EC_m(T_1,T,t) + C_n\cdot ED(T_1,T,t)}{t} \tag{20}$$

$$EC_m(T_1,T,t) =$$

$$\begin{cases} 0 \\ \int_0^{t-T_f}\left(C_f + EC_m(T_1',T,t-u-T_f)\right)\cdot\left(p_b(T_1,T,u)+p_a(T_1,T,u)\right)du & T_1 \\ \sum_{i=1}^{K}\left(\left(iC_i + C_p + EC_m(T_1',T,t-\tau_i-T_p)\right)\cdot P_s(T_1,T,\tau_i-(i-1)\cdot T_i)\right) + KC_iP_n(T_1,T,t-KT_i) + \\ \sum_{i=1}^{K}\int_{\tau_{i-1}+T_i}^{\tau_i}\left(\left((i-1)C_i + C_f + EC_m(T_1',T,t-u-T_f)\right)\left(p_b(T_1,T,u-(i-1)T_i)+p_a(T_1,T,u-(i-1)T_i)\right)\right)du + \\ \int_{\tau_K+T_i}^{t-T_f}\left(\left(C_f + KC_i + EC_m(T_1',T,t-u-T_f)\right)\left(p_b(T_1,T,u-KT_i)+p_a(T_1,T,u-KT_i)\right)\right)du + \\ \int_{t-T_f}^{t} KC_i\cdot\left(p_b(T_1,T,u-KT_i)+p_a(T_1,T,u-KT_i)\right)du \end{cases} \tag{21}$$

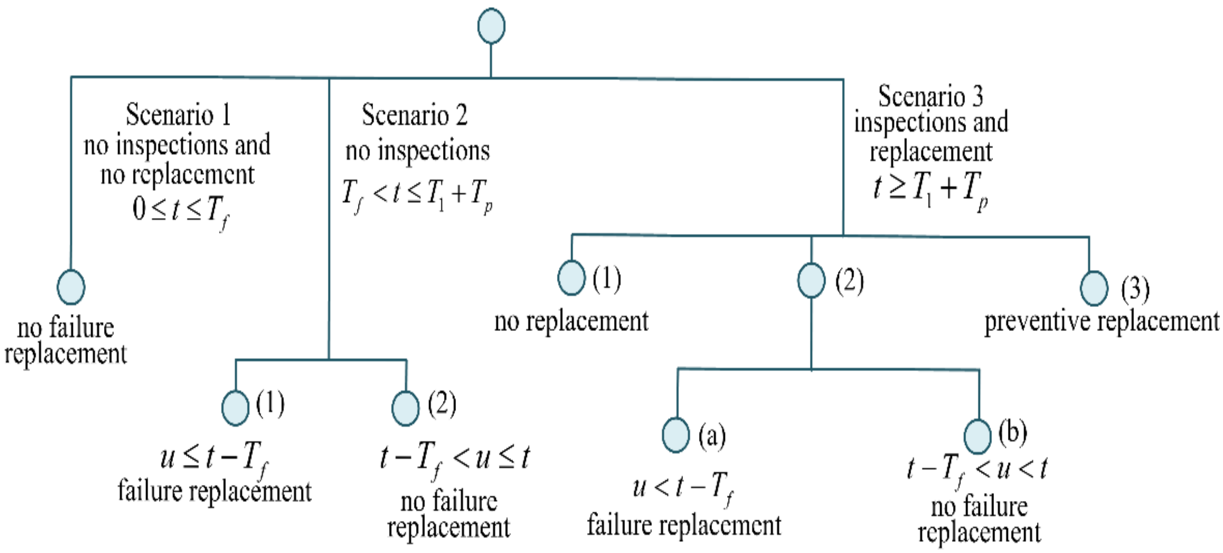


Fig. 3. The probability tree of different scenarios

3. PM policy optimization

In this paper, the objective combines the expected availability and expected cost rate in the mission, as follow:

$$Max\ Z(T_1,T,t) = a_1\cdot EA(T_1,T,t) + a_2\cdot EC(T_1,T,t)$$
$$s.t.\ \begin{cases} EA(T_1,T,t) \ge EA_{min} \\ EC(T_1,T,t) \le EC_{max} \end{cases} \tag{22}$$

where a_1 and a_2 are weight parameters, EA_{min} and EC_{max} are restriction parameters. When, $Z(T_1,T,t) = EA(T_1,T,t)$, the optimization objective is transformed into maximizing the expected availability. Similarly, when $a_2 = -1, a_1 = 0$, $Z(T_1,T,t) = -EC(T_1,T,t)$, the optimization objective is transformed into minimizing the expected cost rate.

From Eqs. (18-22), the expressions above are very difficult to solve directly because of their recurrence relations. Hence, we introduce an iterative algorithm with discretization method to calculate the value of $EA(T_1,T,t)$ and $EC(T_1,T,t)$ approximately (as shown in Algorithm 1). Take $EA(T_1,T,t)$ as an example:

Algorithm1. Iterative algorithm with discretization method to calculate $EA(T_1,T,t)$

Step 1. Let $a_1 = 1, a_2 = 0$ $t t_0 = t$

Step 2. If $0 \le t \le T_p$

$$ED(T_1,T,t) = \int_0^t (t-u)\left(p_b(T_1,T,u)+p_a(T_1,T,u)\right)du \tag{23}$$

Step 3. If $T_p < t \le T_1 + T_p$

Discretize the mission duration by step Δt according to the calculation accuracy requirement, and $\Delta t = 1$ in this paper. We have

$$ED(T_1,T,t)=\int_{t-T_f}^{t}(t-u)\cdot\left(p_b(T_1,T,u)+p_a(T_1,T,u)\right)du+\int_{0}^{\Delta t}\left(p_b(T_1,T,u)+p_a(T_1,T,u)\right)du\cdot\Big(\\ +\int_{\Delta t}^{2\cdot\Delta t}\left(p_b(T_1,T,u)+p_a(T_1,T,u)\right)du\cdot\left(T_f+ED\left(T_1{}',T,t-2\cdot\Delta t-T_f\right)\right)+\ldots+ \\ \cdot\left(T_f+ED(T_1{}',T,0)\right). \tag{24}$$

If the value of $ED\left(T_1{}',T,*\right)$ during the calculation process of Eq. (24) is unknown,

Turn to Step 5.

Else

Save the value of $ED\left(T_1{}',T,t\right)$.

Turn to Step 6.

Step 4. If $t>T_1+T_p$

Discretize the mission duration by steps $\Delta t=1$. We have

$$ED(T_1,T,t)= \\ \sum_{i=1}^{K}\left(\left(iT_i+T_p+ED\left(T_1{}',T,t-\tau_i-T_p\right)\right)\cdot P_s(T_1,T,\tau_i-(i-1)\cdot T_i)\right)+KT_iP_n(T_1,T,t-KT_i)+ \\ \sum_{i=1}^{K}\left(\begin{array}{l}\int_{\tau_{i-1}+T_i}^{\tau_{i-1}+T_i+\Delta t}\left(p_b(T_1,T,u-(i-1)T_i)+p_a(T_1,T,u-(i-1)T_i)\right)du\cdot\left((i-1)T_i+T_f+ED\left(T_1{}',T,t-T_f-(\tau_{i-1}+T_i+\Delta t)\right)\right)+ \\ \int_{\tau_{i-1}+T_i+\Delta t}^{\tau_{i-1}+T_i+2\Delta t}\left(p_b(T_1,T,u-(i-1)T_i)+p_a(T_1,T,u-(i-1)T_i)\right)du\cdot\left((i-1)T_i+T_f+ED\left(T_1{}',T,t-T_f-(\tau_{i-1}+T_i+2\Delta t)\right)\right)+\ldots \\ +\int_{\tau_i-\Delta t}^{\tau_i}\left(p_b(T_1,T,u-(i-1)T_i)+p_a(T_1,T,u-(i-1)T_i)\right)du\cdot\left((i-1)T_i+T_f+ED\left(T_1{}',T,t-T_f-(\tau_i-\Delta t)\right)\right)\end{array}\right)+ \\ \left(\begin{array}{l}\int_{\tau_K+T_i}^{\tau_K+T_i+\Delta t}\left(p_b(T_1,T,u-KT_i)+p_a(T_1,T,u-KT_i)\right)du\cdot\left(T_f+KT_i+ED\left(T_1{}',T,t-T_f-(\tau_K+T_i+\Delta t)\right)\right)+ \\ \int_{\tau_K+T_i+\Delta t}^{\tau_K+T_i+2\Delta t}\left(p_b(T_1,T,u-KT_i)+p_a(T_1,T,u-KT_i)\right)du\cdot\left(T_f+KT_i+ED\left(T_1{}',T,t-T_f-(\tau_K+T_i+2\Delta t)\right)\right)+\ldots \\ +\int_{t-T_f-\Delta t}^{t-T_f}\left(p_b(T_1,T,u-KT_i)+p_a(T_1,T,u-KT_i)\right)du\cdot\left(T_f+KT_i+ED(T_1{}',T,0)\right)\end{array}\right)+ \\ \int_{t-T_f}^{t}\left((KT_i+t-u)\left(p_b(T_1,T,u-KT_i)+p_a(T_1,T,u-KT_i)\right)\right)du. \tag{25}$$

If the value of $ED\left(T_1{}',T,*\right)$ during the calculation process of Eq. (25) is unknown

Turn to Step 5.

Else

Save the value of $ED\left(T_1{}',T,t\right)$.

Turn to Step 6.

Step 5. Let $t=*$, repeat Step 2 to Step 4.

Step 6. If $t=t_0$

Calculate $EA(T_1,T,t)$ according to Eq. (18).

Else

Let $t=t_0$, repeat Step 2 to Step 4.

(**Note:** There are two situations where the value of $ED(T_1{}',T,*)$ is known: 1) the value of $ED(T_1{}',T,*)$ has been obtained and saved by previous calculations; 2) the value of $ED(T_1{}',T,*)$ can be directly obtained through the calculation. The value of $ED(T_1,T,t)$ can be derived backwards by iterations from Step2 to Step 6, until all the values of $ED(T_1{}',T,*)$ during the calculation process are known. It should be noted that all the obtained values of $ED\left(T_1{}',T,*\right)$ should be saved for subsequent calculations.)

Additionally, the calculation methods of $EC(T_1,T,t)$ are similar to $EA(T_1,T,t)$ (as shown in Algorithm 2 in Appendix C).

Now we can obtain the value of $EA(T_1,T,t)$ and $EC(T_1,T,t)$. Then, the optimal first inspection time T_1^* and optimal inspection interval T^* can be obtained by enumeration algorithm with steps Δt.

4. Numerical example

Assume a set of identical airplane landing gears operate in an airplane fleet. The service time duration of the landing gear is $t=200$. The baseline hazard rate at normal working stage and delay-time stage follow $\lambda_X(t)=\partial_X\cdot t$ and $\lambda_Y(t)=\partial_Y\cdot t$, where $\partial_X=5\times10^{-4}$ and $\partial_Y=1\times10^{-3}$. According to the frequency of aircraft landing, the arrival of random shocks follows a DSPP and the intensities at normal working stage and delay-time stage are $\nu_X = 0.05$ and $\nu_Y = 0.1$. The sudden hazard rate increments for nonfatal shocks at normal working stage and delay-time stage follow independent normal distributions with different parameters, i.e. $\mu_i \sim N(\mu_X, {\sigma_X}^2)$ and $v_i \sim N(\mu_Y, {\sigma_Y}^2)$, where $\mu_X = 0.005$, $\sigma_X = 0.001$, $\mu_Y = 0.01$ and $\sigma_Y = 0.002$. The probabilities of a shock being a fatal shock at normal working stage and delay-time stage are $\varphi_X = 0.01$ and $\varphi_Y = 0.02$. For Inspection-based PM policy, the time consumptions for each inspection, preventive replacement and failure replacement are $T_i = 0.2$, $T_p = 2$ and $T_f = 12$, and the costs for each inspection, preventive replacement, failure replacement and per-unit downtime are $C_i = 5$, $C_p = 200$, $C_f = 10$ and $C_n = 5$. The object of PM to maximize expected availability or minimize expected cost rate practical requirements. According to practical requirements, we set EA_{min} and EC_{max}.

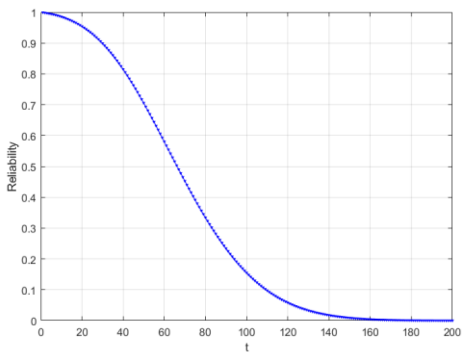
4.1. Reliability analysis

We first conduct the reliability analysis of the system in the proposed environment in term of time *t*, as shown in Fig. 4(a). Under the current parameter settings, the reliability becomes approach to 0 at $t = 160$. In Figs. 4(b-d), we further provide the sensitivity analysis of the reliability on some critical parameters.

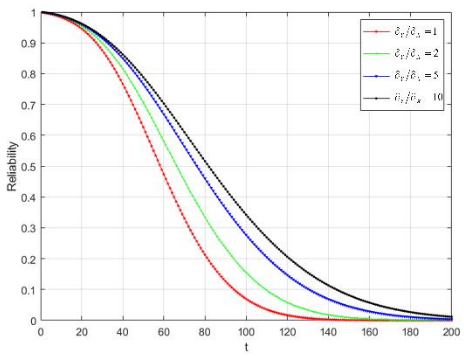
Figs. 4(b) and 4(c) depict the impact of $\frac{\partial_Y}{\partial_X}$ and $\frac{\nu_Y}{\nu_X}$ on reliability, respectively. The values of $\frac{\partial_Y}{\partial_X}$ and $\frac{\nu_Y}{\nu_X}$ are both between 1 and 10. We can see that both $\frac{\partial_Y}{\partial_X}$ and $\frac{\nu_Y}{\nu_X}$ have significant effect on reliability. This means that both internal degradation and external shocks are significant influencing factors of the failure process.

As shown in Fig. 4(d), the influence of $\frac{\varphi_Y}{\varphi_X}$ on reliability is studied, where $\frac{\varphi_Y}{\varphi_X}$ also takes value from 1 to 10. One can see that the impact of this proportion on reliability is smaller than that of $\frac{\partial_Y}{\partial_X}$ and $\frac{\nu_Y}{\nu_X}$. This can be explained as follows: although a fatal shock can cause immediate failure, the arrival probability of fatal shocks is relatively small. As a result, the impact of fatal shock parameters on reliability is relatively small.

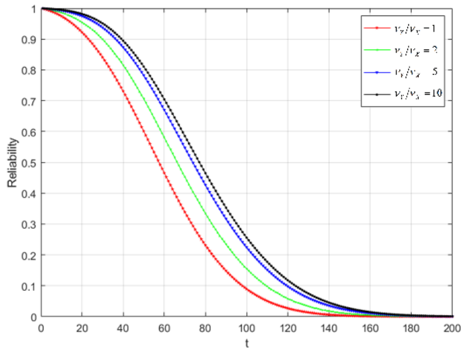
Figs. 4(e) and 4(f) describe the effect of the drift parameters of the sudden hazard rate increments caused by normal shocks at normal working stage and delay-time stage, respectively. We select μ_X and μ_Y for sensitivity analysis as they represent the expected values of sudden hazard rate increments caused by normal shocks at various stages. It can be observed that the drift parameter has a greater impact on reliability at normal working stage than delay time stage. It can be explained that the system may stay at normal working stage for a longer duration than delay time stage, which makes the parameters based on normal working stage more influential.



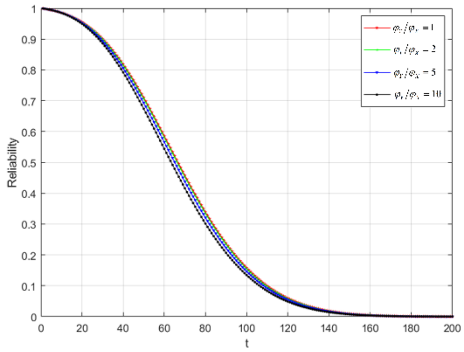
a. reliability versus time *t*.



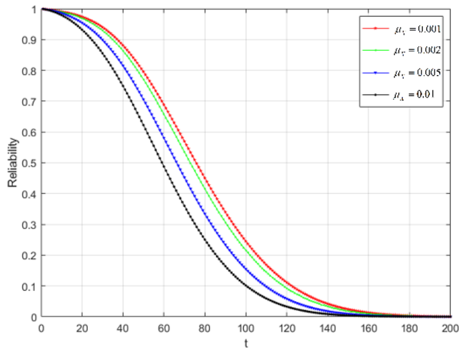
b. sensitivity of reliability on $\frac{\partial_Y}{\partial_X}$.



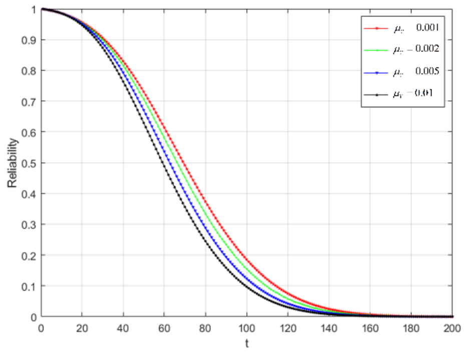
c. sensitivity of reliability on$\frac{\nu_Y}{\nu_X}$.



d. sensitivity of reliability on$\frac{\varphi_Y}{\varphi_X}$.



e. sensitivity of reliability onμ_X.



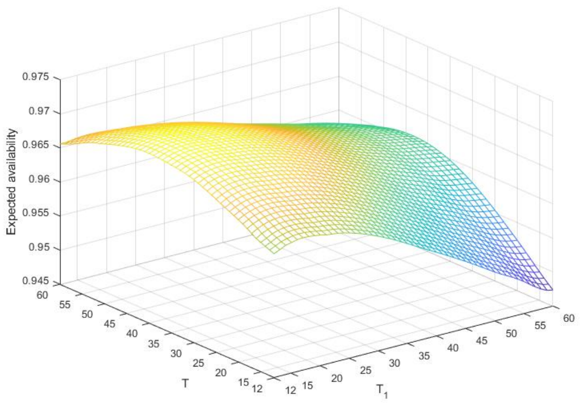
f. sensitivity of reliability onμ_Y.

Fig. 4. Sensitivity analysis of reliability on critical parameters

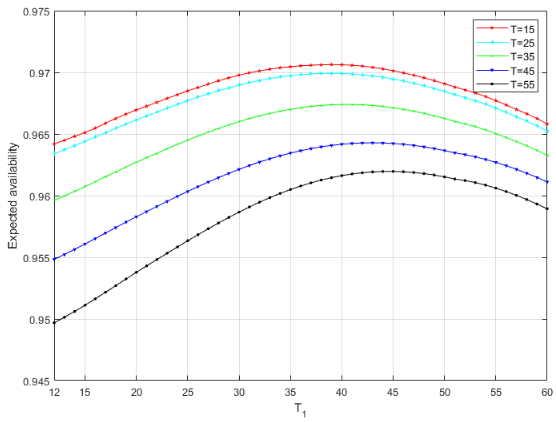
4.2. Expected availability and cost rate

As show in Fig. 5 and Fig. 6, we analyze the curve behaviors of the expected availability and expected cost rate as Case 1 and Case 2 by varying decision variables,$T_1$and T, within[12,60] in steps of 1, as follows. Then, we can obtain an optimal Inspection-based PM policy.

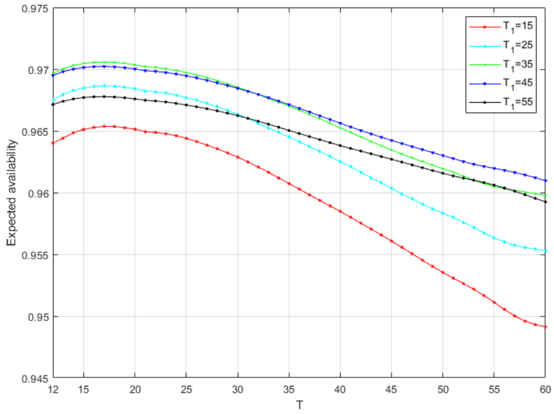
Case 1: In Fig. 5a, we can see that the 3D curved surface is convex and exists a maximum point of expected availability. We can obtain the maximum point, i.e.$T_1^* = 40$and$T^* = 18$, and the maximum expected availability isEA_{max}^{*max}. From Figs. 5b and 5c, we can observe that the expected availability initially increases and then decreases as $T(T_1)$ increases whenT_1 (T) is constant. The result is reasonable. Excessive inspections can result in more inspections and preventive replacements, and increased downtime for inspections and preventive replacements, while untimely inspections may result in more failures and increased downtime for failure replacements. Therefore, the optimal Inspection-based PM policy with maximum expected availability should be able to balance inspection frequency and hazard rate.



a. T_1-T-expected availability 3D plotting.



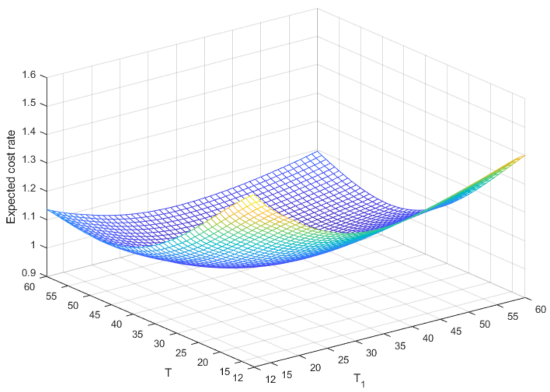
b. effect of$T_1$with different selected Ts.



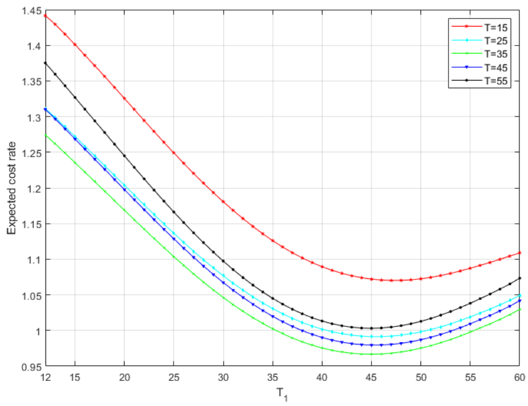
c. effect of T with different selectedT_1s.

Fig. 5. Expected availability under differentT_1s and inspection Ts.

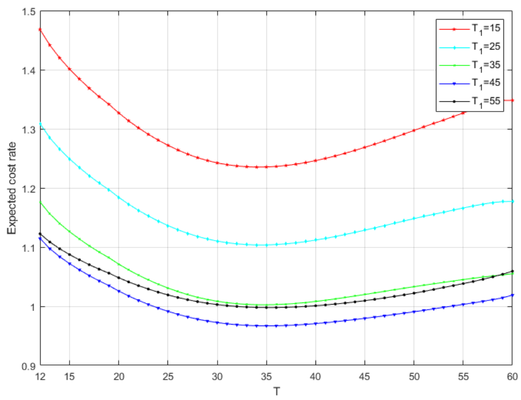
Case 2: The 3D curved surface in Fig. 6a is concave and exists a minimum point of expected cost rate. The minimum expected cost rate is 0.9671 and corresponding optimal first inspection time and inspection interval are$T_1^* = 44$and$T^* = 39$. From Figs. 6b and 4c, we can see that the expected cost rate first decreases and then increases as $T(T_1)$ increases whenT_1 (T) is constant. It can be explained that increased inspection frequency increases inspections and preventive replacements, which results in more cost. Conversely, increased inspection frequency can also reduce the probability of failures and subsequent costs for failure replacements. Therefore, the optimal Inspection-based PM policy with minimum expected cost rate should be able to achieve a balance between inspection frequency and hazard rate.



a. T_1-T-expected cost rate 3D plotting.



b. effect of$T_1$with different selected Ts.



c. effect of T with different selectedT_1s.

Fig. 6. Expected cost rate under differentT_1s and inspection Ts.

Combining Figs.5 and 6, we can observe that the expected availability and expected cost rate of the component are not the competing objectives we have always considered. In fact, the expected availability and expected cost rate will both improve with the optimization of T_1 and T in most cases. For example, when $T_1 \in (15,40)$ and $T = 15$, the expected availability will increase and expected cost rate will decrease with T_1 increasing (as shown in Figs. 5b and 6b). This suggests that when considering policy optimization, we can focus more on multi-objective optimization.

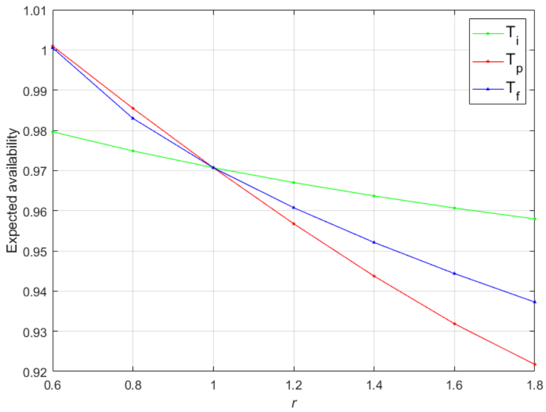
4.3. Sensitivity analysis

In this subsection, we analyzed the sensitivity of parameters T_i, T_p, T_f and C_i, C_p, C_f, C_n in Case 1 and Case 2 by multiplying them with ratio r (.$r \in [0.6,1.8]$.with a step size of 0.2). This setting is due to the significant numerical differences among these parameters, making it challenging to objectively compare their impact through simple changes. By utilizing a unified ratio variation, we aim to objectively reflect the sensitivity of the expected availability and expected cost rate to different parameters through the observation of curve slopes.

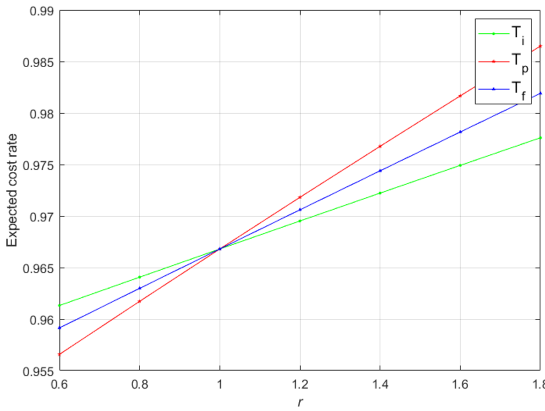
As shown in Fig. 7a, T_P has the greatest impact on the maximum expected availability, with T_f being homogeneous, while T_i is much smaller than the former two. Therefore, when increasing the expected availability of the component, it is important to reduce time consumption of preventive replacement and failure replacement.

Fig. 7b shows the impact of T_i, T_p, T_f on the minimum expected cost rate. As the time consumption (ratio r) increases, the minimum expected cost rate of the component will also increase, but it is not significant.

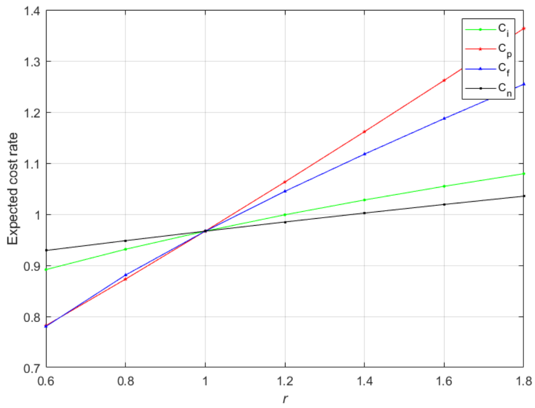
Obviously, compared to time consumption, the cost of the inspection or replacement has a more significant impact on the minimum expected cost rate of the component (as shown in Fig. 7c). Among them, C_P has the greatest impact, homogeneous C_f, while C_i and C_n have relatively small impacts. Therefore, the cost of the preventive replacement is the key influencing factor of the expected cost rate and should be focused on when reducing the expected cost rate of the component.



a. sensitivity analysis on time consumption in Case 1



b. sensitivity analysis on time consumption in Case 2



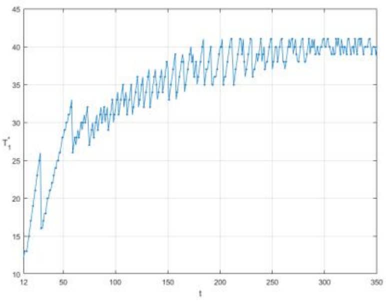
c. sensitivity analysis on cost in Case 2

Fig. 7. Sensitivity analysis on time consumption and cost in Case 1 and Case 2.

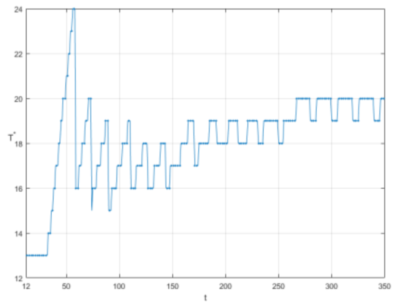
4.4. Mission duration and inspection interval

In Fig. 8, we use Case 1 and Case 2 above as example for examining the effect of mission duration on the optimal inspection-based PM policy. The mission duration is changed with steps of 1. As shown in Figs. 8(a) and 8(c), the curves of the optimal first inspection time in Case 1 and Case 2 are accompanied by high-frequency and small-amplitude fluctuations during the ascending process. In Figs. 8(b) and 8(d), the curves of the optimal inspection interval in Case 1 and Case 2 experience multiple sudden decreases in the rising processes. We notice that these sudden decreases correspond to the changes in the optimal number of inspections in the mission. It is worth noting that the mission duration has greater impact on the optimal inspection-based PM policy when the mission duration is small. However, the change ranges of the optimal first inspection time and optimal inspection interval continue to decrease and the values of them tend to be stable with the mission duration continuously increasing. The result is consistent with previous studies [51, 54].

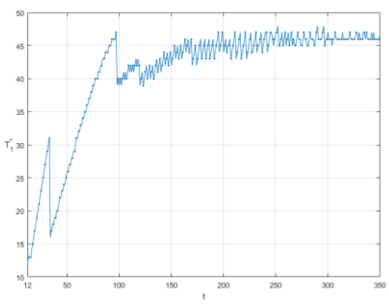
Fig. 9 describes the comparisons of the optimal expected availability and optimal expected cost rate between infinite duration method (which considers the mission duration is limited) and limited duration method (which is the method proposed in this paper that considering the mission duration is limited), where mission duration t takes values from 20 to 200 with step size 1. Firstly, we approximate the solution at $t = 350$ in the limited duration method as the optimal solution in the infinite time method (which is close enough to the solution of the infinite duration method based on the previous analysis,), i.e. $(T_1^*, T^*) = (40,20)$ in Case 1 and $(T_1^*, T^*) = (47,37)$ in Case 2. We can see that when mission duration is small, the limited duration method has a significantly advantage over the infinite duration method. For example, when $t = 60$, the expected availability increases from 0.9671 in infinite duration method to 0.9701 in limited duration method, an increase of 0.65%; the expected cost rate decreases from 0.9612 in infinite duration method to 1.1874 in limited duration method, a decrease of 45.30%. However, the advantage of the limited duration method will continue to decrease with the mission duration increasing. When $t = 200$, the results obtained by the limited duration method and the infinite duration method are basically the same. However, due to the use of mission iteration method, the calculation complexity of the final method proposed will also continue to increase with time increasing. Therefore, our method is more suitable for the mission when the mission duration is small, and the simpler infinite duration method which commonly bases on the well-known renewal-reward theory [55], such as [49, 51], can be considered for approximate calculation when the mission duration is large enough.



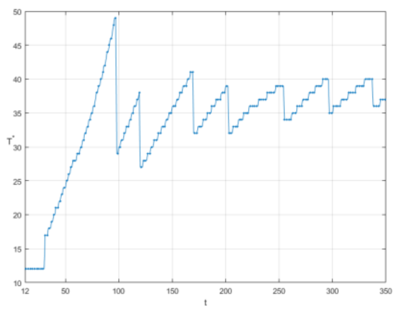
a. optimal first inspection time in Case 1.



b. optimal inspection interval in Case 1.

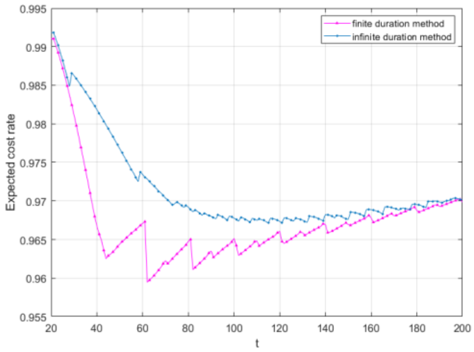


c. optimal first inspection time in Case 2.

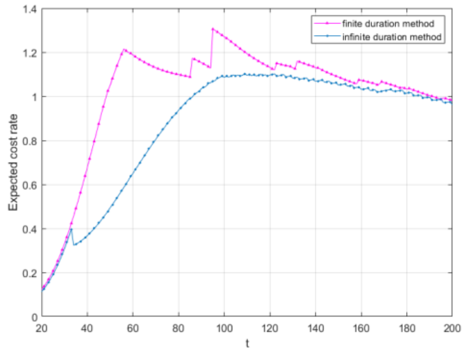


d. optimal inspection interval in Case 2.

Fig. 8. Analysis on the optimal first inspection and inspection interval.



a. comparison between infinite duration method and limited duration in Case 1.



b. comparison between infinite duration method and limited duration in Case 2.

Fig. 9. Comparisons between infinite duration method and limited duration.

5. Conclusions

In this paper, we propose an Inspection-based PM policy optimization model based on DTM for the component system under random environment in a mission with limited duration. There are two modes of external shocks: nonfatal shocks and fatal shocks. Besides, the failure process is baes on DTM, the arrival and impact of the external shocks will be affected by the failure process. Based on an Inspection-based PM policy, mathematical models are derived for the expected availability and expected cost rate of the component in limited time duration. An iterative algorithm combining with discretization method is proposed to solve proposed models. On this basis, the optimal first inspection time and optimal inspection interval for maximum expected availability or minimum expected cost rate of the PM policy are determined. A numerical example of airplane landing gear is provided to prove the accuracy of our method. The analysis of cases illustrates serval useful suggestions:

1)The impact of the normal shocks at normal working stage is one of the most important influencing factors of the component's reliability.

1) Multi-objective optimization including the expected availability and cost rate will be feasible.

2) Reducing the time consumption of preventive maintenance and fault maintenance is a key factor in increasing expected availability.

3) The key determining factor in reducing the expected cost rate is to lower the cost of preventive maintenance.

4) The method proposed is more effective when the mission duration is small.

Several interesting research directions remain to be explored in the future. Firstly, in term of PM policy, non-periodic inspections, and imperfect repairs rather than replacements may be more practical. In addition, the expected availability and expected cost rate models can be also extended to a multi-component system where each component is subject to dependent degradation and random shocks. Finally, in terms of reliability prediction and maintenance decision-making, the integration with some of the latest methods including deep learning and reinforcement learning needs further research and application.

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Appendix A

The derivation process of Eqs. (1-4) is inferred from [53], and simple adjustments have been made based on assumptions. The detailed derivation procedure is provided as follows:

Without considering fatal shocks, the joint conditional distribution function of no defect during normal working stage is given by

$$\bar{F}_X(t|N_X(t)=n,t_1,t_2,t_3\dots,t_n,\mu_1,\mu_2\dots,\mu_n)$$
$$=exp\left(-\left(\int_0^{t_1}\lambda_X(u)du+\int_{t_1}^{t_2}(\lambda_X(u)+\mu_1)du+\dots+\int_{t_n}^{t}\left(\lambda_X(u)+\sum_{i=1}^{N}\mu_i\right)du\right)\right)$$
$$=exp\left(-\int_0^t\lambda_X(u)du\right)\cdot exp\left(-\sum_{i=1}^{n}\mu_i(t-t_i)\right).$$

Denoting by the probability density (distribution) function ofμ_i, i.e.$g_X(\mu_i)$ $(G_X(\mu_i))$, we have

$$\bar{F}_X(t|N_X(t)=n,t_1,t_2,t_3\dots,t_n)$$
$$=\int_0^\infty\dots\int_0^\infty\int_0^\infty\left(exp\left(-\int_0^t\lambda_X(u)du\right)\cdot exp\left(-\sum_{i=1}^{n}\mu_i(t-t_i)\right)\cdot g_X(\mu_1)g_X(\mu_2)\dots g_X(\mu_n)\right)d\mu_n\dots d\mu_2d\mu_1$$
$$=exp\left(-\int_0^t\lambda_X(u)du\right)\cdot\prod_{i=1}^{n}\int_0^\infty(g_X(\mu_i)\,exp(-\mu_i(t-t_i)))d\mu_i$$
$$=exp\left(-\int_0^t\lambda_X(u)du\right)\cdot\prod_{i=1}^{n}G_X^*(t-t_i),$$

where$G_X^*(t-t_i)$ is the Laplace transform of$G_X(\mu_i)$.

Assume that a defect occurs at time u and the joint probability of$(u>t,N_X(t)=n,t_1,t_2\dots,t_n)$is

$$\bar{F}_X(u>t,N_X(t)=n,t_1,t_2\dots,t_n)$$
$$=v_X{}^n\cdot exp(-v_Xt)\cdot exp\left(-\int_0^t\lambda_X(u)du\right)\cdot\prod_{i=1}^n G_X^*(t-t_i)=exp\left(-\int_0^t(v_X+\lambda_X(u))du\right)\cdot\prod_{i=1}^n v_XG_X^*(t-t_i).$$

By integrating the arrival time of the external shocks, we have the joint distribution function of $(u>t,N_X(t)=n)$ as

$$\bar{F}_X(u>t,N_X(t)=n)$$
$$=\int_0^t\dots\int_0^{t_3}\int_0^{t_2}exp\left(-\int_0^t(v_X+\lambda_X(u))du\right)\cdot\prod_{i=1}^n v_XG_X^*(t-t_i)dt_1dt_2\dots dt_n$$
$$=exp\left(-\int_0^t(v_X+\lambda_X(u))du\right)\cdot\int_0^t\dots\int_0^{t_3}\int_0^{t_2}\prod_{i=1}^n v_XG_X^*(t-t_i)dt_1dt_2\dots dt_n$$
$$=\frac{exp\left(-\int_0^t(v_X+\lambda_X(u))du\right)\cdot\left(v_X\int_0^tG_X^*(t-x)dx\right)^n}{n!}.$$

From Assumption 3, the arrival of external shocks follows a HPP in each stage independently. Thus, $P_X(N_X(t)=m)=exp(-v_Xt)\cdot(v_Xt)^m/m!$. According to the conditional probability formula, the probability of no defect before time *t* conditional on *n* nonfatal shocks during normal working stage is

$$\bar{F}_X(t|N_X(t)=n)=\frac{\overline{F}_X(u>t,N_X(t)=n)}{P_X(N_X(t)=n)}=exp\left(-\int_0^t\lambda_X(u)du\right)\cdot\left(t^{-1}\cdot\int_0^tG_X^*(t-x)dx\right)^n,$$

For simplicity, we represent $F_X^{(n)}(t)=1-\bar{F}_X(t|N_X(t)=n)$ and $f_X^{(n)}(t)=dF_X^{(n)}(t)/dt$. The probability distribution function and probability density function of a defect conditional on *n* nonfatal shocks during normal working stage are given by

$$F_X^{(n)}(t)=1-\bar{F}_X(t|N_X(t)=n)=1-exp\left(-\int_0^t\lambda_X(u)du\right)\cdot\left(t^{-1}\cdot\int_0^tG_X^*(t-x)dx\right)^n,$$
$$f_X(t|N_X(t)=n)=dF_X(t|N_X(t)=n)/dt$$
$$=exp\left(-\int_0^t\lambda_X(u)du\right)\cdot\left(t^{-1}\int_0^tG_X^*(t-x)dx\right)^n\cdot\left(\lambda_X(t)+n\cdot\left(t^{-1}-G_X^*(t)/\int_0^tG_X^*(t-x)dx\right)\right).$$

Similarly, represent $F_Y^{(n)}(t)=1-\bar{F}_Y(t|N(t)=n)$ and $f_Y^{(n)}(t)=dF_Y^{(n)}(t)/dt$, and the probability distribution function and probability density function of a failure conditional on *n* nonfatal shocks during delay-time stage are

$$F_Y^{(n)}(t)=1-\bar{F}_Y(t|N(t)=n)=1-exp\left(-\int_0^t\lambda_Y(u)du\right)\cdot\left(t^{-1}\cdot\int_0^tG_Y^*(t-x)dx\right)^n,$$
$$f_Y^{(n)}(t)=dF_Y^{(n)}(t)/dt$$
$$=exp\left(-\int_0^t\lambda_Y(u)du\right)\cdot\left(t^{-1}\int_0^tG_Y^*(t-x)dx\right)^n$$
$$\cdot\left(\lambda_Y(t)+n\cdot\left(t^{-1}-G_Y^*(t)/\int_0^tG_Y^*(t-x)dx\right)\right),$$

where $P_Y(N_Y(t)=n)=exp(-v_Yt)\cdot(v_Yt)^n/n!$.

Appendix B

The detailed derivation procedure of Eq. (21) is provided as follows. The maintenance cost includes the cost for failure replacements, preventive replacements, and inspections in the mission. The expected maintenance cost of the component in the mission is expressed as

$$EC_m(T_1,T,t)=E\big(C_m(0,t)\big).$$

Theorem 2. If a replacement is completed at time u,$u\in(0,t)$, the maintenance cost within $(0,t)$ can be expressed as

$$C_m(0,t)=C_m(0,u)+EC_m\big(T_1{}',T,t-u\big).$$

Proof. If $u\in(0,t)$, the maintenance cost within $(0,t)$ is

$$C_m(0,t)=C_m(0,u)+C_m(u,t).$$

Similar to Theorem 1, as a component is replaced at time u, the remaining time horizon(u,t), can be regarded as a new mission with the first inspection time, inspection interval and mission duration being$T_1{}'$, T and$t-u$, respectively. Thus, the maintenance cost within$(0,t)$can be expressed as

$$C_m(0,t)=C_m(0,u)+E\big(C_m(u,t)\big)=C_m(0,u)+EC_m\big(T_1{}',T,t-u\big),$$

where $T_1{}'=T_1+\left(int\left(\frac{(u-T_1)}{(T+T_i)}\right)+1\right)\cdot(T+T_i)-u$.

End

Consider the following scenarios:

Scenario 1. If$0\le t\le T_p$, there is no inspection and replacement in the mission. The expected maintenance cost in this scenario is

$$EC_m(0,t)=0. \tag{26}$$

Scenario 2. If$T_p<t\le T_1+T_p$, there is no inspection in the mission. Assuming that the first failure occurs at time u, consider following possible events: 1) If$t-T_f<u\le t$, there is no need for replacement; 2) If$u\le t-T_f$, the maintenance cost in the mission is the cost for the failure replacement. Considering all the events above, the expected maintenance cost in this scenario is

$$EC_m(\tau,T,t)=\int_0^{t-T_f}\Big(C_f+EC_m\big(T_1{}',T,t-u-T_f\big)\Big)\cdot\big(p_b(T_1,T,u)+p_a(T_1,T,u)\big)du. \tag{27}$$

Scenario 3. If$t>T_1+T_p$, let K be the number of inspections before t,$K=int\big((t-T_1)/(T+T_i)\big)+1$. Consider following events:

(1) If there is no replacement in the mission, the maintenance cost is only caused by inspections. Considering the possibility of no replacement before time t, the expected maintenance cost for this event is

$$EC_m^{(3-1)}(T_1,T,t)=KC_iP_n(T_1,T,t-KT_i). \tag{28}$$

(2) If the first replacement is a failure replacement at time u, consider following several possibilities:

a. if$u<t-T_f$, the maintenance cost includes the cost for failure replacement and inspections before time u.

b. if$t-T_f<u<t$, there is no need for a replacement as the failure replacement cannot be completed before the end of mission, and the maintenance cost in the mission includes the cost for inspections and the duration of no replacement after failure.

According to the above possibilities and integrating over the feasible range of u, the expect maintenance cost for this event is

$$\begin{aligned}&EC_m{}^{(3-2)}(T_1,T,t)=\\&\sum_{i=1}^{K}\int_{\tau_{i-1}+T_i}^{\tau_i}\Big(\Big((i-1)C_i+C_f+EC_m\big(T_1{}',T,t-u-T_f\big)\Big)\big(p_b(T_1,T,u-(i-1)T_i)+p_a(T_1,T,u-(i-1)T_i)\big)\Big)du+\\&\int_{\tau_K+T_i}^{t-T_f}\Big(\Big(C_f+KC_i+EC_m\big(T_1{}',T,t-u-T_f\big)\Big)\big(p_b(T_1,T,u-KT_i)+p_a(T_1,T,u-KT_i)\big)\Big)du+\\&\int_{t-T_f}^{t}KC_i\cdot\big(p_b(T_1,T,u-KT_i)+p_a(T_1,T,u-KT_i)\big)du,\end{aligned} \tag{29}$$

where$\tau_0=-T_i$.

(4) if the first replacement is a preventive replacement at timeτ_i, the maintenance cost includes the cost for the preventive replacement and inspections. Considering all the possibilities ofτ_i, the expected maintenance cost for this event is

$$EC_m^{(3-3)}(T_1,T,t)=\sum_{i=1}^{K}\Big(iC_i+C_p+EC_m\big(T_1{}',T,t-\tau_i-T_p\big)\Big)\cdot P_s(T_1,T,\tau_i-(i-1)\cdot T_i). \tag{30}$$

Combining all the possible events above, the expected maintenance cost in this scenario is given by

$$EC_m(T_1,T,t)=EC_m^{(3-1)}(T_1,T,t)+EC_m^{(3-2)}(T_1,T,t)+EC_m^{(3-3)}(T_1,T,t) \tag{31}$$

where$EC_m^{(3-1)}(T_1,T,t)$, $EC_m^{(3-2)}(T_1,T,t)$, $EC_m^{(3-3)}(T_1,T,t)$can be obtained from Eqs. (28, 29, 30).

Summarizing Scenarios 1, 2 and 3, we can obtain the expected maintenance cost in the mission with limited duration t as Eq. (21).

Appendix C

Algorithm2. Iterative algorithm with discretization method to calculate $EC(T_1, T, t)$

Step 1. Let $t_0 = t$

Step 2. $0 \le t \le T_p$

$$EC_m(T_1, T, t) = 0$$

Step 3. If $T_p < t \le T_1 + T_p$

Discretize the mission duration by step Δt according to the calculation accuracy requirement, and $\Delta t = 1$ in this paper. We have

$$\begin{aligned} &EC_m(T_1, T, t) = \\ &\int_0^{\Delta t} \big(p_b(T_1, T, u) + p_a(T_1, T, u)\big) du \cdot \Big(C_f + ED\big(T_1', T, t - \Delta t - T_f\big)\Big) + \\ &\int_{\Delta t}^{2\cdot\Delta t} \big(p_b(T_1, T, u) + p_a(T_1, T, u)\big) du \cdot \Big(C_f + ED\big(T_1', T, t - 2\cdot\Delta t - T_f\big)\Big) + \ldots \\ &+ \int_{t-T_f-\Delta t}^{t-T_f} \big(p_b(T_1, T, u) + p_a(T_1, T, u)\big) du \cdot \Big(C_f + ED(T_1', T, 0)\Big). \end{aligned} \tag{32}$$

If the value of $EC_m\left(T_1{}', T, t\right)$ during the calculation process of Eq. (32) is unknown,

Turn to Step 5.

Else

Save the value of $EC_m\left(T_1{}', T, t\right)$.

Turn to Step 6.

Step 4. If $t > T_1 + T_p$

Discretize the mission duration by steps $\Delta t = 1$. We have

$$\begin{aligned} &EC_m(T_1, T, t) = \\ &\sum_{i=1}^{K} \left(\begin{aligned} &\int_{\tau_{i-1}+T_i}^{\tau_{i-1}+T_i+\Delta t} \big(p_b(T_1, T, u-(i-1)T_i) + p_a(T_1, T, u-(i-1)T_i)\big) du \cdot \Big((i-1)C_i + C_f + EC_m\Big(T_1', T, t - T_f - (\tau_{i-1} + T_i + \Delta t)\Big)\Big) + \\ &\int_{\tau_{i-1}+T_i+\Delta t}^{\tau_{i-1}+T_i+2\Delta t} \big(p_b(T_1, T, u-(i-1)T_i) + p_a(T_1, T, u-(i-1)T_i)\big) du \cdot \Big((i-1)C_i + C_f + EC_m\Big(T_1', T, t - T_f - (\tau_{i-1} + T_i + 2\Delta t)\Big)\Big) + \ldots \\ &+ \int_{\tau_i-\Delta t}^{\tau_i} \big(p_b(T_1, T, u-(i-1)T_i) + p_a(T_1, T, u-(i-1)T_i)\big) du \cdot \Big((i-1)C_i + C_f + EC_m\Big(T_1', T, t - T_f - (\tau_i - \Delta t)\Big)\Big) \end{aligned} \right) \\ &+ \\ &\left(\begin{aligned} &\int_{\tau_K+T_i}^{\tau_K+T_i+\Delta t} \big(p_b(T_1, T, u-KT_i) + p_a(T_1, T, u-KT_i)\big) du \cdot \Big(C_f + CT_i + EC_m\Big(T_1', T, t - T_f - (\tau_K + T_i + \Delta t)\Big)\Big) + \\ &\int_{\tau_K+T_i+\Delta t}^{\tau_K+T_i+2\Delta t} \big(p_b(T_1, T, u-KT_i) + p_a(T_1, T, u-KT_i)\big) du \cdot \Big(C_f + CT_i + EC_m\Big(T_1', T, t - T_f - (\tau_K + T_i + 2\Delta t)\Big)\Big) + \ldots \\ &+ \int_{t-T_f-\Delta t}^{t-T_f} \big(p_b(T_1, T, u-KT_i) + p_a(T_1, T, u-KT_i)\big) du \cdot \Big(C_f + KC_i + EC_m(T_1', T, 0)\Big) \end{aligned} \right) + \\ &\int_{t-T_f}^{t} KC_i \cdot \big(p_b(T_1, T, u-KT_i) + p_a(T_1, T, u-KT_i)\big) du. \end{aligned} \tag{33}$$

If the value of $EC_m\left(T_1{}', T, t\right)$ during the calculation process of Eq. (33) is unknown

Turn to Step 5.

Else

Save the value of $EC_m\left(T_1{}', T, t\right)$.

Turn to Step 6.

Step 5. Let $t = *$, repeat Step 2 to Step 4.

Step 6. If $t = t_0$

Calculate $EC\left(T_1{}', T, t\right)$ according to Eq. (20).

Else

Let $t = t_0$, repeat Step 2 to Step 4.