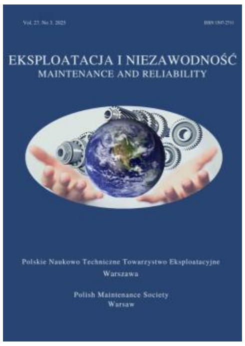


Eksploatacja i Niezawodnosc – Maintenance and Reliability
Volume 27 (2025), Issue 3
journal homepage: http://www.ein.org.pl

Article citation info:
Shangguan A, Feng N, Fei R, Hei X, Jin Y, Mu L, Reliability Assessment of Competitive Failure Systems Based on Three Parameter Weibull Distribution and Wiener Process, Eksploatacja i Niezawodnosc – Maintenance and Reliability 2025: 27(3) http://doi.org/10.17531/ein/199426

Reliability Assessment of Competitive Failure Systems Based on Three Parameter Weibull Distribution and Wiener Process

Indexed by: Web of Science Group

Anqi Shangguan[a,*], Nan Feng[b], Rong Fei[a], Xinhong Hei[a], Yongze Jin[a], Lingxia Mu[a]

[a] Xi'an University of Technology, China
[b] University of Science and Technology Beijing, China

Highlights

- The hard failure process is constructed by the three-parameter Weibull distribution.
- The unknown parameters of the reliability model are estimated by the MCMC-MH method.
- A nonlinear function is proposed to design the correlation of different failure processes.

Abstract

Considering the competitive failure that exists in the operation of industrial systems, including degradation failure and sudden failure, this paper presents a reliability assessment method based on the three-parameter Weibull distribution and the Wiener process. The Wiener process models the degradation failure process, while the three-parameter Weibull model describes the hard failure process. Nonlinear exponential functions are proposed to establish the relationship model between the different failure processes, and the reliability model for the competitive failure process is derived. The Metropolis-Hastings (MH) sampling algorithm of the Monte Carlo Markov Chain (MCMC) method is employed to estimate the parameters in this study. The reliability assessment results are obtained by the numerical and real degradation samples. The results show that the reliability model incorporating the three-parameter Weibull distribution produces more comprehensive and dependable results. Furthermore, MH sampling can solve the issues of complex likelihood functions that cannot directly obtain the evaluation results. Additionally, the sensitivity of proposed model parameters is analyzed, thereby offering theoretical support for enhancing the safe operation of the system.

Keywords
degradation failure, sudden failure, competing failure, wiener process, reliability assessment

1. Introduction

The demand for high reliability and extended useful life of key equipment has been progressively rising, commensurate with the growing complexity of industrial systems [1][2]. Ensuring the safe operation of these systems, it becomes significantly crucial to effectively evaluate their reliability. Currently, the evaluation of complex systems' reliability is primarily investigated based on their performance degradation process, which heavily relies on the monitored data regarding to the system's performance degradation. Once the system reaches a specific failure threshold, it experiences failure. Consequently, a probabilistic model is constructed based on the failure mechanism, and the analysis results provide a deeper understanding of the system's reliability.

Indeed, apart from failures caused by the system's own

(*) Corresponding autho
E-mail addresses: A. Shangguan (ORCID: 0000-0003-2848-322X) s_angelos@163.com, N. Feng (ORCID: 0009-0001-5615-0505) fengnan_fn@163.com, R. Fei (ORCID: 0000-0003-1892-8192) annyfei@xaut.edu.cn, X. Hei (ORCID: 0000-0002-6394-0492) heixinhong@xaut.edu.cn, Y. Jin (ORCID: 0000-0001-7942-4858) jinyongze@caut.edu.cn, L. Mu(ORCID: 0000-0002-6466-5277) mulingxia@xaut.edu.cn ,

performance degradation exceeding the specified failure threshold (designated as soft failures) [3-5], and those abruptly triggered by external shocks (termed sudden failures) [6]. As an illustrative case, the sliding spool valve in the hydraulic control system can fail not only due to wear degradation but also due to sudden stagnation [7]. Similarly, while prone to decline in their capacitive prowess, can experience a catastrophic short circuit when the operational voltage surpasses a critical juncture, causing the electric field to rupture the interstitial medium, a phenomenon attributed to the sudden onslaught of the external milieu. These two failure modes—natural degradation failure and sudden failure—are interdependent and competitive. Additionally, when the wear and tear of a tire transcend the failure thresholds enshrined in industry norms, its demise is inevitable. Similarly, sudden failure can occur when the tire is punctured by hard foreign objects on the road. Therefore, the failure mechanisms in complex system equipment encompass both natural degradation failure and sudden failure. Regardless of the type of failure process, the earliest failure process is the main cause of system failure. These two distinct failure modes together form the competitive failure process of the system. Studying system reliability modeling under competitive failure process conditions can enhance the safety and reliability of complex systems.

When considering the competitive failure conditions, the change in performance degradation of the system during operation will gradually affect the ability of a system to resist external shocks. Furthermore, external random shocks will also affect the performance degradation. Therefore, in the actual operation of complex system, it is necessary to consider the interaction between the degradation process and the sudden failure process [8-10]. Generally, many literatures mainly describe the inter-relationships between failure processes through two aspects: (1) external environmental shock accelerates system degradation; (2) The degradation process affects the failure rate of sudden failures. The former mainly focuses on the impact of sudden failures on the degradation process. For example, in [11], a decreasing random process was set as the sudden failure threshold, and a correlation between sudden failures and degradation failure was constructed. The reliability results of radar power amplification systems were obtained using component failure probability. Reference [12] adopts a gamma process to construct a system degradation process, considering the generation of degradation increments caused by external shock processes, and the Copula function is used to build a multi-failure-related model. In [13-15], a degradation failure process is constructed using a linear function, and the influence of external shock increments on the degradation rate and degradation increment are utilized to construct a competitive failure reliability model. In [16], the influence of continuous shocks and accelerated degradation is considered to model the reliability with competing failure processes. In [17], the changed threshold δ is proposed to construct the competing failure reliability model. A copula-based competing reliability model is proposed in [18], and the dependence structure is derived based on the lifetime data. In [19], a competing failure processes are conducted to analyze the complex system that constructed by multiple components, random shock model is used to model the sudden failure process, and degradation process follow a Gamma process. In [20], an age- and state-dependent competing risks model that considers random shocks is proposed. A linear degradation path is considered to generate degradation samples. In [21], a reliability model for the multi-component system subject to dependent competing failure processes considering multiple shock sources is presented.

However, most of the above literature analyzes the impact of external shocks on the degradation process, describes the relationship between degradation failure and sudden failure model through the degradation process, and further constructs a probability model that the degradation amount does not exceed the soft failure threshold. Additionally, when the sudden failure samples are difficult to obtain, how to solve the problem that the system reliability can only be conducted by the degradation samples. In the above research, the impact of the degradation process on sudden failure was not considered. Taking tires as an example, as the amount of tire wear increases, the probability of tires being punctured by foreign objects also increases. Therefore, the second type of correlation is more conducive to describing the competitive failure process of complex systems. In [19], by considering the impact of degradation processes on impact processes, the relevant factors γ and degradation are utilized to build a relationship model. In [20], considering that the magnitude of external shock loads will change with increasing degradation, a competitive failure reliability model is further proposed. Yang et al. [21] also considered a system that undergoes both two-stage degradation processes and random shocks, where the impact rate is influenced by the system state. In [22-23], the Weibull distribution is used to model the sudden failure process of tool wear, and the gamma process (Wiener process) is combined to construct a competitive failure reliability model. In [24], the aging failure rate of relay protection devices is estimated by considering the three-parameter Weibull distribution, and a reliability model through independent failure processes is established. In [25], q-Weibull distribution is proposed to solve the useful life prediction and fault diagnosis of system, and the maximum likelihood estimation (MLE) method and robust linear regression method is used to estimate the parameters. In [26], a novel flexible inverse modified Weibull model with a concave Weibull probability diagram is proposed to simulate the aging classes of life distributions, and the parameters of it is estimated by Weibull probability paper approach and the maximum likelihood method. In [27], the three-parameters (3-p) Weibull model is used to evaluate the lifetime distribution of critical wind turbine subassemblies, and an improved ergodic artificial bee colony algorithm is proposed to estimate the parameters of 3-p Weibull model. In [28], the two-parameter Weibull distribution is proposed to fit the reliability indexes of vibration component.

Based on the above research models, the three-parameter Weibull distribution is rarely used as a model for sudden failure in reliability analysis Moreover, most Weibull distributions are applied to lifetime prediction problems. Literature on reliability assessment using this distribution is scarce. For the sudden failure process modelling, only external shock models were considered, and the degradation process was described using a simplified path that further weakened the credibility of the model. Moreover, in building a sudden failure process model, the failure process model was too simplistic or did not consider competitive failure modes, which cannot solve the actual situation. On the other hand, in terms of parameter estimation for models, many literatures only rely on historical experience or parameter assumptions, without reasonable parameter estimation for the established model to improve its usability. When dealing with the parameters of the Weibull distribution or its combined models, the MLE method is predominantly employed. However, in scenarios where the reliability model becomes intricate, the corresponding likelihood function tends to be highly complex. This complexity often leads to suboptimal outcomes when utilizing the MLE method.

To address the shortcomings of the above methods, this article considers the interaction between performance degradation and sudden failure processes and uses the Wiener process to construct a degradation process model for complex systems. To accurately describe the sudden failure mode of system equipment, a three-parameter Weibull distribution is used as the sudden failure time distribution. By combining the nonlinear exponential function to characterize the relationship between degradation and sudden failure, a system reliability model based on the competitive failure process is constructed using the failure rate function. Given the complexity of the likelihood function corresponding to the reliability model, direct estimation of the parameter results using maximum likelihood estimation is not feasible. Consequently, to overcome this obstacle, an accurate estimation of the unknown parameters of the reliability model is achieved by combining the MCMC-MH sampling algorithm of the Bayes method. Subsequently, the reliability of different failure processes and the corresponding reliability of different sudden failure processes are compared using numerical samples and GaAs laser current as a measure of performance degradation. Furthermore, the sensitivity of different parameters to the reliability model is analyzed. The experimental results validate the rationality and advantages of the proposed method in evaluating the reliability of competitive failure processes. The proposed method offers theoretical support for the intelligent operation and maintenance of complex systems.

The contribution of this paper is mainly including the following points:

1). When it is not possible to obtain a set of sudden failure samples, this paper establishes a sudden failure process model using the failure time distribution and the three-parameter Weibull distribution.

2). Due to the interactions between different failure processes, this paper establishes a relational model using a nonlinear exponential function.

3). For the issue of the complex likelihood function of the model, which prevents direct parameter estimation, this paper employs the MCMC-MH sampling method to obtain parameter estimates, thereby enhancing the model's reliability.

The rest of this paper is organized as follows: Section 2 provides a detailed problem description of this article. In Section 3, the reliability probability of the system is modeled based on a competitive failure process, encompassing degradation process modeling, sudden failure modeling, and competitive failure reliability modeling. The estimation of unknown parameters in the proposed model is discussed in Section 4, employing the MCMC-MH sampling method. Section 5 presents the experimental verification conducted using numerical samples and actual degradation samples. The reliability results of the laser are obtained utilizing the MCMC-MH sampling algorithm and the proposed reliability model. Finally, Section 6 provides a summary of the overall paper.

2. The problem description

During the operational lifespan of a system, it is inevitable to encounter diverse influences such as wear and environmental shocks, which can result in system failure or malfunctions. Assessing system reliability plays a crucial role in determining maintenance requirements and predicting performance to facilitate subsequent decision-making. Through timely monitoring and decision-making throughout the system's operational process, overall system reliability and health status can be enhanced. However, in the current process of system reliability assessment, sudden failures caused by stress or environmental shock during operation are overlooked. Additionally, when establishing the failure process model, multiple influencing factors and the absence of shock samples are not considered, thereby diminishing the accuracy of reliability assessment. Based on the competitive failure process and combined with sudden failure time distribution of system, a reliability evaluation method that incorporates a three-parameter Weibull distribution and the Wiener process of competitive failure is proposed. This method offers theoretical support for enhancing system safety, as well as intelligent operation and maintenance.

System degradation refers to the decline in operation performance, influenced by various factors during its usage, leading to a decrease in certain monitored physical quantities over time. When the performance values exceed the established failure threshold, the system will malfunction or fail, impacting operational efficiency. The notation used in this paper is described in the bellow.

Table 1. the different notation used in this paper.

Notation	Description	Notation	Description
$X(t)$	The degradation process of Wiener process	$\Lambda(t)$	Scale transformation function
$B(t)$	The standard Brownian motion	T_D	The failure time of degradation process
T_S	The failure time of sudden process	$R_S(t)$	The reliability that only affected by the sudden failures process
$R_D(t)$	The system reliability function that only have the degradation failure process	R(t)	The reliability model with competing failure processes
$K(t)$	The failure rate function of sudden failures	$K(S,X(S))$	The failure rate function of degradation and sudden failure processes
α	Scale parameter	γ	Position parameter
β	Shape parameter	$\lambda_s(t)$	The failure rate function based on the three-parameter Weibull distribution
$q(x_t)$	The density function of the degradation amount	(c_0, c_1)	The coefficient of nonlinear exponential function
$\boldsymbol{\theta}_1$	The parameters of Wiener process	$\boldsymbol{\theta}_2$	The parameters of three-parameters Weibull distribution
$\boldsymbol{\theta}_3$	The parameters of nonlinear exponential function	$\boldsymbol{L}$	The failure threshold
μ	Degradation rate	σ	Diffusion parameter
MCMC	Monte Carlo Markov Chain	MH	Metropolis-Hastings

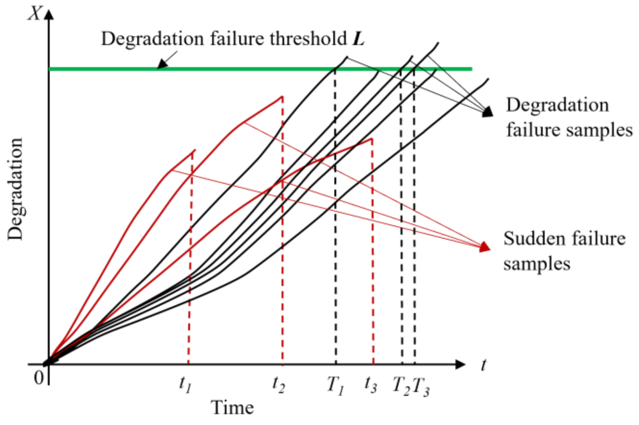


Fig. 1. The general degradation samples of different failure process.

To more clearly describe the failure process manifested in the degradation samples, it is represented by Fig.1, which shows the degradation amount changes. Since the degradation samples are time series data, the X-axis is the time *t* during the operation of the system, and Y-axis represents a specific physical quantity of the sample (such as electric current, wear amount or diameter, *etc.*). The amount by which the physical quantity of the system changes over time *t*. The green solid line represents the failure threshold of the device (generally determined by expert experience or industry standards). The black and red lines are the degradation data monitored by different samples of the same device. In the black line, since the degradation samples corresponding to $t=T_1$, $t=T_2$, and $t=T_3$ exceed the failure threshold ***L***, these black samples belong to the degradation failure process; when $t>t1$, $t>t2$, and $t>t3$, the degradation amount of the red line samples no longer changes, and it belongs to the sudden failure process. Hence, when it is not possible to directly monitor the external environmental shock samples on the system, degradation failure samples exceeding the failure threshold, sudden failure samples with no incremental degradation, and normal samples can be directly obtained from the degradation samples.

Given that complex systems are influenced by various factors during operation, such as the degradation of self-monitoring variables and external environmental shocks, it is crucial to consider the interaction between multiple failure modes when analyzing the system failure process. Hence, we employ the nonlinear exponential function to construct a framework that captures the relationship between degradation and sudden failure process. Subsequently, a reliability model based on competitive failure process for complex systems is derived. The detailed process is illustrated in Fig. 2.

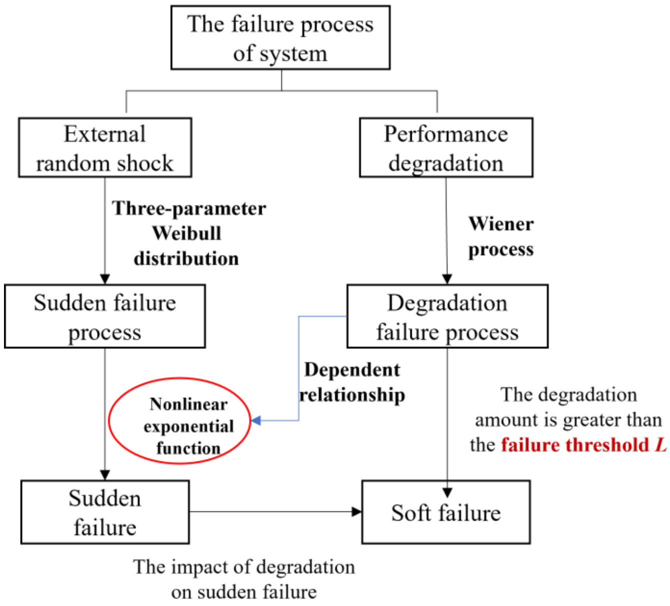


Fig. 2. The reliability modelling based on the competing failure of system.

In Fig. 2, the Wiener process is employed to represent the performance degradation failure process, while the sudden failure process is modeled by the three-parameter Weibull distribution and sudden failure time in this paper. When the performance degradation is greater than the failure threshold *L*, the system will fail (referred to as a soft failure). On the other hand, if the degradation remains below the failure threshold, a nonlinear exponential function is utilized to establish a relationship model between the degradation process and the sudden failure process. Then, the reliability model with the competing failure process can be obtained.

To enhance the alignment of the method proposed in this article with the actual state and facilitate model calculations, assumptions are incorporated into the reliability evaluation process, *i.e.*, assuming that the system is no longer usable after a sudden failure act on the system.

In the reliability assessment process, after obtaining the competitive failure reliability model of a complex system through a nonlinear exponential function, the likelihood function of the model is calculated, and the unknown parameters of the model are estimated using the MCMC-MH sampling algorithm. Based on the monitored real degraded samples and estimated parameters, the reliability evaluation results in the overall life cycle are obtained.

The overall framework for reliability evaluation of competitive failure systems based on the three-parameter Weibull distribution and Wiener process is shown in Fig. 3.

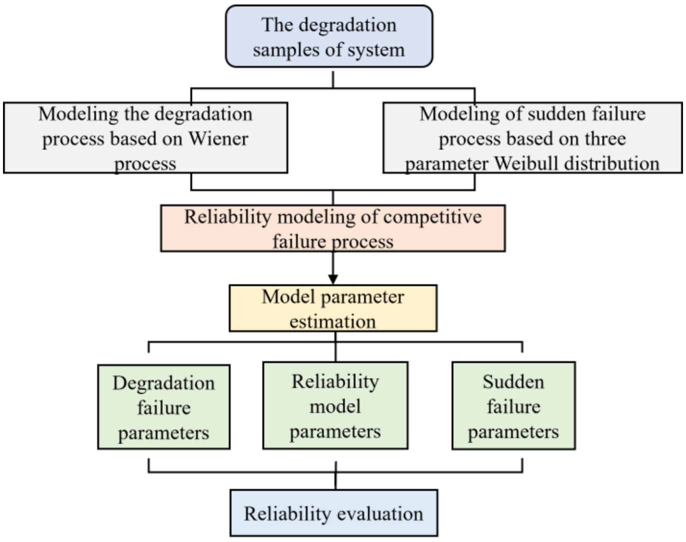


Fig. 3. Overall framework for reliability assessment of the system.

Through the process of reliability modeling and parameter estimation under competitive failure conditions, the reliability of the system during its service life can be evaluated, thereby improving its remaining useful life, and maintaining the stable operation of the system.

3. System Reliability Modeling Based on Competitive Failure Process

Since the operational reliability model of complex systems is established based on competitive failure processes, wherein both gradual degradation and sudden failure may occur, the system will experience corresponding failures correspondingly. Consequently, a reliability model can be constructed by assessing the probability of simultaneous occurrence of degradation and sudden failures, implying that throughout the entire lifespan of the system, the present moment has not yet reached the measurement time of either degradation or sudden failure.

If the failure time of degradation process in the system is T_D, the probability of it not experiencing the degradation failure process can be expressed as $P\{T_D>t\}$, where t is the current time; When a sudden failure process occurs in the operation process of system, the failure time is recorded as T_S, and the probability of no sudden failure occurring within time t is $P\{T_S>t\}$. If the degradation failure process and sudden failure process of system are determined independently, the reliability of the system can be calculated by multiplying the probabilities of these two events occurring. The relationship is illustrated by formula(1).

$$R(t) = P\{T_D > t\}P\{T_S > t\} = R_S(t) \cdot R_D(t) \quad (1)$$

where,$R_S(t)$ is the reliability of system that only affected by the sudden failures process, and$R_D(t)$ is the reliability function that only have the degradation failure process.

However, this situation ignores the correlation between the degradation process and the sudden failure process, thereby reducing the accuracy of system reliability. Hence, to account for the competitive failure process in complex systems, the reliability function of the system is the probability of both degraded and sudden failures occurring simultaneously, as represented by formula (2).

$$R(t) = P\{T_D > t, T_S > t\} = P\{T_S > t | T_D > t\} \cdot P\{T_D > t\} = \exp[-\mathrm{K}(t)] \cdot R_D(t) \quad (2)$$

where, R(t) is the reliability model with competing failure processes, $K(t)$ is the failure rate function of the system that experiences sudden failures.

In other words, $K(t)$ represents the probability that the system has not failed before time t, but will experience failure after time t. Considering the impact of system performance degradation on the sudden failure process, the failure rate function$K(t)$ is a non-constant function that changes over time and is related to the degradation amount $X(t)$. Hence, the calculation process of the system reliability function is as follows.

$$R(t) = \exp[-\int_0^t \mathrm{K}(S, X(S))dS] \cdot R_D(t) \quad (3)$$

In formula (3), $K(S, X(S))$ is the failure rate function related to degradation and sudden failure. Therefore, it is obvious that the reliability model of the system based on competitive failure mainly needs to calculate the failure rate function$K(S, X(S))$ and performance degradation failure reliability$R_D(t)$.

3.1. Degenerate failure process model based on Wiener process

Since the system degradation amount $X(t)$ follows the Wiener process, its expression is shown in (4).

$$X(t) = \mu\Lambda(t) + \sigma B(\Lambda(t)) \quad (4)$$

where, μ is the drift parameter, which represent the degradation rate, such as the trend of laser current degradation rate; $\sigma$is a diffusion parameter that describes the heterogeneity of the samples, that is, the impact degree of random factors act on performance degradation; $\Lambda(t)$ is a scale transformation function that mainly describes the degradation path of the system. $B(t)$ is the standard Brownian motion and follows the normal distribution, i.e. $B(t) \sim N(0, t)$.

Considering that unary Wiener process can describe continuous time degradation paths and is suitable for describing various system degradation phenomena, such as the wear, corrosion, aging, etc. Hence, this paper chooses the scale transformation function $\Lambda(t) = t$ to model the degradation process in Eq. (4).

Since the condition of degradation failure process not occurred is that the degradation amount $X(t) = \mu t + \sigma B(t)$ does not exceed the soft failure threshold L, which is equivalent to the time T_d when the degradation amount of the system first reaches the soft failure threshold. Therefore, the probability of the system not experiencing soft failure within time t is equal to the reliability of the system considering only degradation failure, which is shown in Eq.(5).

$$R_D(t) = p(T_d > t) = p(X(t) < L) = \int_0^L f_d(x,t)dx \qquad (5)$$

where, T_d is the time when degradation failure occurs, L is the soft failure threshold.

Furthermore, based on the Kolmogorov forward equation, the expression of the density function $f_d(x,t)$ is obtained, as shown in formula (6).

$$f_d(x,t) = \frac{1}{\sqrt{2\sigma^2\pi t}}\left(exp\left(-\frac{(x-\mu t)^2}{2\sigma^2 t}\right) - exp\left(\frac{2\mu L}{\sigma^2}\right)exp\left(-\frac{(x-2L-\mu t)^2}{2\sigma^2 t}\right)\right) (6)$$

Combining formula (5) (6) [32], the system reliability function considering only degradation failure is obtained as follows.

$$R_D(t) = \Phi\left(\frac{L-\mu t}{\sigma\sqrt{t}}\right) - exp\left(\frac{2\mu L}{\sigma^2}\right)\Phi\left(\frac{-L-\mu t}{\sigma\sqrt{t}}\right) \qquad (7)$$

where, $\Phi(\cdot)$ represents the normal cumulative distribution function.

According to the properties of the Wiener process, the density function of the degradation amount $X(t)$ follows a normal distribution, as shown in formula (8):

$$q(x_t) = \frac{1}{\sqrt{2\sigma^2\pi t}}exp\left(-\frac{(x_t-\mu t)^2}{2\sigma^2 t}\right) \qquad (8)$$

3.2. Sudden failure process model based on three-parameter Weibull distribution

When the external shocks act on the system and cause sudden failure occurs, the external shock load is related to the degradation of the system. Therefore, it is necessary to construct a relationship model between degradation and external shocks. The Weibull distribution is widely used and is also suitable for small failure samples. Considering the cost issue of system monitoring, the number of external shock samples monitored is relatively small or even unable to be effectively collected, the Weibull distribution is suitable for analyzing the distribution of useful life. In the early stages, the failure rate of the system is relatively low. Furthermore, the two-parameter Weibull distribution will easily cause nonlinear Weibull transformations. But the positional parameters γ in the three-parameter Weibull distribution can accurately describe the life distribution of the system. Therefore, this article uses a three-parameter Weibull distribution to construct a sudden failure process model, and its cumulative failure distribution function is shown in formula (9).

$$F(t) = 1 - e^{-\left(\frac{(t-\gamma)^\beta}{\alpha}\right)} \qquad (9)$$

where, $\alpha > 0$, $\gamma >= 0$, $\beta > 0$ are the scale parameters, position parameters, and shape parameters in the failure distribution function, respectively.

Then, the reliability, probability density, and failure rate functions of sudden failures in complex systems are accordingly obtained, which are shown in (10).

$$\begin{cases} R_s(t) = e^{-\left(\frac{(t-\gamma)^\beta}{\alpha}\right)} \\ f_s(t) = \frac{dR_s(t)}{dt} \\ h_s(t) = \frac{f_s(t)}{R_s(t)} \end{cases} \qquad (10)$$

The failure rate function based on the three-parameter Weibull distribution is shown in formula (11) through the failure distribution function and failure density function.

$$\lambda_s(t) = \frac{f(t)}{1-F(t)} = \frac{\beta(t-\gamma)^{(\beta-1)}}{\alpha^\beta} \qquad (11)$$

By combining the three-parameter Weibull distribution failure rate function and the reliability $R_D(t)$, a probability model based on the competitive failure process can be further obtained.

3.3. Competitive failure reliability model

Considering that the degradation process of system will affect the failure rate function $K(t, X(t))$ of the sudden failure process, to reflect the impact degree of the system degradation process act on the sudden failure, this paper constructs a relationship model between the degradation process and the sudden failure process through the nonlinear exponential function and the failure rate function of the sudden failure process, as shown in formula (12).

$$\mathrm{K}(t, X(t)) = \mathrm{K}_s(t, x_t) = H(t, x_t) = \lambda_s(t)\, exp(c_0 + c_1 x_t) \qquad (12)$$

where $\lambda_s(t)$ is the failure rate function that the system only considers the existence of sudden failure processes, mainly described by the three parameter Weibull distribution failure rate function (formula (11)). $exp(c_0 + c_1 x_t)$ is the nonlinear exponential function, c_0, c_1 represents the model coefficients to describe the relationship between the degradation and sudden failure rate.

Therefore, by combining formulas (8) and (11), a relationship function model between degradation failure and sudden failure process is obtained, as shown in formula (13).

$$\mathrm{K}_s(t, X(t)) = \int_0^L H(t, x_t) q(x_t)\, dx_t = \int_0^L \lambda_s(t)\, exp(c_0 + c_1 x_t)\, q(x_t) dx_t = \int_0^L \frac{\beta(t-\gamma)^{(\beta-1)}}{\alpha^\beta} exp(c_0 + c_1 x_t) \cdot \frac{1}{\sqrt{2\sigma^2 \pi t}} exp\left(-\frac{(x_t-\mu t)^2}{2\sigma^2 t}\right) dx_t =$$

$$\frac{\beta(t-\gamma)^{(\beta-1)}}{\alpha^\beta} exp\left(c_0 + c_1 \mu t + \frac{1}{2} c_1 \sigma^2 t\right) \int_0^L \frac{1}{\sqrt{2\sigma^2 \pi t}} exp\left(-\frac{(x_t-\mu t - c_1\sigma^2 t)^2}{2\sigma^2 t}\right) dx_t = \frac{\beta(t-\gamma)^{(\beta-1)}}{\alpha^\beta} exp\left(c_0 + c_1 \mu t + \frac{1}{2} c_1 \sigma^2 t\right) \Phi\left(\frac{(L-\mu t - c_1 \sigma^2 t)^2}{\sigma\sqrt{t}}\right) \qquad (13)$$

where $\Phi(\ \ )$ is the standard normal distribution.

Based on the formula (3), the reliability of the system in time t is the probability of neither soft failure nor hard failure in the entire life cycle. Combining the failure rate function $K_s(t, X(t))$ and the reliability model $R_D(t)$ of Wiener process, the expression of the system reliability results with competitive failure process is shown in formula (14).

$$R_{inde}(t) = P(T_s > t, T_D > t) = P(T_s > t | T_D > t) P(T_D > t) = exp\left(-\int_0^t \mathrm{K}_s(t, X(s)) d_s\right) R_D(t)$$

$$= \exp\left(-\int_0^t \frac{\beta(\xi-\gamma)^{(\beta-1)}}{\alpha^\beta} exp\left(c_0 + c_1 \mu \xi + \frac{1}{2} c_1 \sigma^2 \xi\right) \Phi\left(-\frac{(L-\mu\xi - c_1\sigma^2\xi)^2}{\sigma\sqrt{\xi}}\right) d\xi\right) \cdot \left(\Phi\left(\frac{L-\mu t}{\sigma\sqrt{t}}\right) - exp\left(\frac{2\mu L}{\sigma^2}\right) \Phi\left(\frac{-L-\mu t}{\sigma\sqrt{t}}\right)\right) \qquad (14)$$

Based on the above calculation process, the trend of operational reliability variation of the system during its lifecycle can be obtained, which is the probability result that system does not fail in the service life. Then, to evaluate the operational reliability of the system, the parameters of the reliability model are evaluated by the bayes methods. The estimation process of unknown parameters will be described in the Section 4.

4. Parameter estimation

Assuming that N degradation samples of the system are collected through a sensor or radar, the system operation status without failure, with hard failure, or with soft failure are monitored in time. Specifically, the system status cannot be recorded after the hard failure occurs. Note that N=Z+V+M is the total number of samples, Z is the number of systems not failures, V is the number of system hard failures, and M is the number of soft failures occurs in the system. Therefore, the overall degradation of the system is described as bellow.

$$X_{ij} = \begin{bmatrix} X_{11}, X_{12}, \ldots, X_{1K_i} \\ X_{21}, X_{22}, \ldots, X_{2K_i} \\ \vdots \quad \ddots \quad \vdots \\ X_{i1}, X_{i2}, \ldots, X_{iK_i} \end{bmatrix}$$

where X_{ij} is the degradation amount at the j-th moment of the i-th sample, $j = 1, \ldots, K_i$ and K_i is the measurement duration of the i-th sample.

According to formula (14), the reliability function of the system includes three sets of parameters, including the Wiener process parameters $\boldsymbol{\theta}_1 = (\mu, \sigma^2)$ and the relationship model parameters $\boldsymbol{\theta}_2 = (\alpha, \beta, \gamma)$, $\boldsymbol{\theta}_3 = (c_0, c_1)$. To estimate the parameters accurately, the estimation process is mainly divided into two parts: $\boldsymbol{\theta}_1$ of Wiener process and $\boldsymbol{\theta}_2, \boldsymbol{\theta}_3$ of competing failure process. The specific estimation process is described in bellow.

4.1. Parameter $\boldsymbol{\theta}_1$ estimation of Wiener process

In this article, the system degradation failure process follows the Wiener process. According to the properties of the Wiener process, its degradation increment also follows a normal distribution, that is, $\Delta X_{ij} \sim N[\mu\Delta t_{ij}, \sigma^2\Delta t_{ij}]$. The parameters that need to be estimated are $\boldsymbol{\theta}_1 = (\mu, \sigma^2)$. Hence, the parameter likelihood function of *n* systems in K_i measurement durations is:

$$L(\mu,\sigma^2) = \prod_{i=1}^{n}\prod_{j=1}^{K_i}\frac{1}{\sqrt{2\sigma^2\pi\Delta t_{ij}}}exp\left[-\frac{(\Delta x_{ij}-\mu\Delta t_{ij})2}{2\sigma^2\Delta t_{ij}}\right] \quad (15)$$

Through the maximum likelihood estimation, the estimated results $\widehat{\boldsymbol{\theta}}_1 = (\hat{\mu}, \hat{\sigma}^2)$ can be obtained, as the formula (16) shown.

$$\hat{\mu} = \frac{\sum_{i=1}^{n} X_{iK_i}}{\sum_{i=1}^{n} t_{iK_i}}, \hat{\sigma}^2 = \frac{1}{\sum_{i=1}^{n} K_i}\left[\sum_{i=1}^{n}\sum_{j=1}^{K_i}\frac{(\Delta X_{ij})^2}{\Delta t_{ij}} - \frac{(\sum_{i=1}^{n} X_{iK_i})^2}{\sum_{i=1}^{n} t_{iK_i}}\right] \quad (16)$$

4.2 Parameters $\boldsymbol{\theta}_2, \boldsymbol{\theta}_3$ estimation of competing failure process

Considering the different number of failure samples and of the measurement duration, the parameters $\boldsymbol{\theta}_1$ are obtained (the model parameters $\boldsymbol{\theta}_1 = (\mu, \sigma^2)$ have been estimated by maximum likelihood), the parameters estimation of $\boldsymbol{\theta}_2, \boldsymbol{\theta}_3$ can be mainly divided into the following three situations:

A. When the system has not failed, the relationship between the useful life and the measurement time is $T_i > t_{iK_i}$. Hence, the system reliability function is $R(t_{iK_i})$ (combining formula (14));

B. Once a hard failure of the system occurs, the operation state is not monitored after the hard failure. The measurement time of the system ist_{ih}. So, the distribution function of the system life is:

$$F_s(t_{ih}) = 1 - exp\left[-\int_0^{t_{ih}} K_s(\xi, X(\xi)|\hat{\mu}, \hat{\sigma}^2)d\xi\right] \quad (17)$$

C. When a soft failure of the system occurs, the measurement time of the system ist_{id}, and its life distribution function is:

$$F_d(t_{id}) = exp\left[-\int_0^{t_{id}} K_s(\xi, X(\xi)|\hat{\mu}, \hat{\sigma}^2)d\xi\right] F_d(t_{id}|\hat{\mu}, \hat{\sigma}^2) \quad (18)$$

Therefore, based on the above description, the likelihood function of the system operation reliability model parameters is shown in (19).

$$L(\alpha,\beta,\gamma,c_0,c_1|\hat{\mu},\hat{\sigma}^2) = \prod_{i=1}^{Z} R(t_{iK_i}|\hat{\mu},\hat{\sigma}^2)\cdot\prod_{i=1}^{V}\frac{dF_s(t_{ih}|\hat{\mu},\hat{\sigma}^2)}{dt_{ih}}\cdot\prod_{i=1}^{M} F_d(t_{id}|\hat{\mu},\hat{\sigma}^2) = \prod_{i=1}^{N} exp\left[-\int_0^{\delta_i}\mathrm{K}(\xi,X(\xi)|\hat{\mu},\hat{\sigma}^2)d\xi\right]\cdot \prod_{i=1}^{V}\mathrm{K}(t_{ih},X(t_{ih})|\hat{\mu},\hat{\sigma}^2) = \prod_{i=1}^{N} exp\left[-\int_0^{\delta_i}[\frac{\beta(\xi-\gamma)^{(\beta-1)}}{\alpha^\beta}exp\left(c_0+c_1\hat{\mu}\xi+\frac{1}{2}c_1\hat{\sigma}^2\xi\right)\Phi\left(\frac{(L-\hat{\mu}\xi-c_1\hat{\sigma}^2\xi)^2}{\hat{\sigma}^2\sqrt{\xi}}\right)]|\hat{\mu},\hat{\sigma}^2)d\xi\right]\cdot \prod_{i=1}^{V}\frac{\beta(t_{ih}-\gamma)^{(\beta-1)}}{\alpha^\beta}exp\left(c_0+c_1\hat{\mu}t_{ih}+\frac{1}{2}c_1\hat{\sigma}^2 t_{ih}\right)\Phi\left(\frac{(L-\hat{\mu}t_{ih}-c_1\hat{\sigma}^2 t_{ih})^2}{\hat{\sigma}\sqrt{t_{ih}}}\right)|\hat{\mu},\hat{\sigma}^2) \quad (19)$$

where $\delta_i = \{t_{iK_i}, t_{ih}, t_{id}\}$is the measurement duration of the *i*-th sample.

Since the integral function existed in the likelihood function of the system reliability model, which are difficulty to estimate its maximum likelihood, this paper uses the MCMC method in Bayesian area to obtain the estimated parameters$\widehat{\boldsymbol{\theta}}_2, \widehat{\boldsymbol{\theta}}_3$. The MCMC method mainly obtains the posterior distribution results through prior distribution knowledge and uses sampling methods to obtain parameter convergence results.

Therefore, the transformation form of the Likelihood function of the model parameters can be obtained based on the Bayesian formulas.

$$\pi(\boldsymbol{\theta}_2,\boldsymbol{\theta}_3|\mathbf{X}) = \frac{L(\mathbf{X}|\boldsymbol{\theta}_2,\boldsymbol{\theta}_3)\pi(\boldsymbol{\theta}_2,\boldsymbol{\theta}_3)}{\iint_{\boldsymbol{\theta}_2,\boldsymbol{\theta}_3}\pi(\boldsymbol{\theta}_2,\boldsymbol{\theta}_3)L(\mathbf{X}|\boldsymbol{\theta}_2,\boldsymbol{\theta}_3)d\boldsymbol{\theta}_2 d\boldsymbol{\theta}_3} \propto L(\mathbf{X}|\boldsymbol{\theta}_2,\boldsymbol{\theta}_3)\pi(\boldsymbol{\theta}_2,\boldsymbol{\theta}_3) \quad (20)$$

where $\boldsymbol{X}$ is the monitoring sample matrix of system, $\pi(\boldsymbol{\theta}_2, \boldsymbol{\theta}_3)$ is a prior distribution, and $\pi(\boldsymbol{\theta}_2, \boldsymbol{\theta}_3|\boldsymbol{X})$ represents a posterior distribution. This paper adopts an uninformed prior distribution as the prior distribution.

In Eq. (20), it is necessary to calculate the posterior distribution results of unknown parameters for the estimated parameters $\widehat{\boldsymbol{\theta}}_2, \widehat{\boldsymbol{\theta}}_3$. Based on the likelihood function $L(\alpha, \beta, \gamma, c_0, c_1|\hat{\mu}, \hat{\sigma}^2)$ (formula(19)), the posterior distribution results of parameters $\alpha, \beta, \gamma, c_0, c_1$are described as bellows.

$$\pi(\alpha|\beta,\gamma,c_0,c_1,\mathrm{Data}) \propto \pi(\alpha)\cdot L(\alpha|\beta,\gamma,c_0,c_1,\mathbf{X}) \propto \pi(\alpha)\cdot exp\left[-\sum_{i=1}^{N}\frac{1}{\alpha^\beta}\cdot\delta_i\right]\cdot\frac{1}{\alpha^\beta} \quad (21)$$

$$\pi(\beta|\alpha,\gamma,c_0,c_1,\mathrm{Data}) \propto \pi(\beta)\cdot L(\beta|\alpha,\gamma,c_0,c_1,\mathbf{X}) \propto \pi(\beta)\cdot exp\left[-\sum_{i=0}^{N}\int_0^{\delta_i}\frac{\beta(\xi-\gamma)^{(\beta-1)}}{\alpha^\beta}d\xi\right]\cdot\prod_{i=1}^{V}\frac{\beta(t_{ih}-\gamma)^{(\beta-1)}}{\alpha^\beta} \quad (22)$$

$$\pi(\gamma|\alpha,\beta,c_0,c_1,\mathrm{Data}) \propto \pi(\gamma)\cdot L(\gamma|\alpha,\beta,c_0,c_1,\mathbf{X}) \propto \pi(\gamma)\cdot exp\left[-\sum_{i=0}^{N}\int_0^{\delta_i}\beta(\xi-\gamma)^{(\beta-1)}d\xi\right]\cdot\prod_{i=1}^{V}\beta(t_{ih}-\gamma)^{(\beta-1)} \quad (23)$$

$$\pi(c_0|\alpha,\beta,\gamma,c_1,\mathrm{Data}) \propto \pi(c_0)\cdot L(c_0|\alpha,\beta,\gamma,c_1,\mathbf{X}) \propto \pi(c_0)\cdot exp[-exp(c_0)\sum_{i=1}^{N}\delta_i]\cdot exp(c_0) \quad (24)$$

$$\pi(c_1|\alpha,\beta,\gamma,c_0,\text{Data}) \propto \pi(c_1)\cdot L(c_1|\alpha,\beta,\gamma,c_0,\mathbf{X}) \propto \pi(c_1)\cdot exp\left[-\sum_{i=1}^{N}\int_0^{\delta_i} exp\left(c_1\hat{\mu}\xi+\frac{1}{2}c_1\hat{\sigma}^2\xi\right)\Phi\left(\frac{(L-\hat{\mu}\xi-c_1\hat{\sigma}^2\xi)^2}{\hat{\sigma}\sqrt{\xi}}\right)d\xi\right]$$

$$\cdot\, exp\sum_{i=1}^{V}\left(c_1\hat{\mu}t_{ih}+\frac{1}{2}c_1\hat{\sigma}^2t_{ih}\right)\cdot\prod_{i=1}^{V}\Phi\left(\frac{(L-\hat{\mu}t_{ih}-c_1\hat{\sigma}^2t_{ih})^2}{\hat{\sigma}\sqrt{t_{ih}}}\right) \quad (25)$$

The MCMC method mainly obtains a stationary distribution by establishing a Markov chain. In formulas (21) - (25), the posterior distribution that needs to be sampled is not a standard distribution, that is, the conditional probability density function of the parameters cannot be directly obtained. Therefore, the Gibbs sampling method cannot be used to estimate the parameters. In this paper, the MH sampling method is used to obtain the estimated parameters $\hat{\boldsymbol{\theta}}_2, \hat{\boldsymbol{\theta}}_3 = (\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{c}_0, \hat{c}_1)$. Record that u follows a uniform distribution $u \sim U(0,1)$; To simplify the parameter estimation process, the parameters to be estimated are recorded as $\boldsymbol{\Theta}$, the recommended distribution is $Q(\boldsymbol{\Theta}^{[t]}|\boldsymbol{\Theta}^{*})$ and the acceptance probability is

$\alpha(\boldsymbol{\Theta}^{*}|\boldsymbol{\Theta}^{[t]})=\min\left\{\frac{p(\boldsymbol{\Theta}^{*}|t)Q(\boldsymbol{\Theta}^{[t]}|\boldsymbol{\Theta}^{*})}{p(\boldsymbol{\Theta}^{[t]}|t)\alpha(\boldsymbol{\Theta}^{*}|\boldsymbol{\Theta}^{[t]})},1\right\}$. Hence, the specific steps of MH sampling are as follows:

a) Determining the prior distribution $\pi(\boldsymbol{\Theta})$ and the initial value $\boldsymbol{\Theta}^{(0)}$ of the parameter to be estimated based on the prior distribution without information;

b) Sampling u in a uniform distribution U (0,1) and obtaining alternative parameter values through formulas (19) - (22) and suggested distributions $\boldsymbol{\Theta}^{*}$;

c) By comparing the magnitude of u and the acceptance probability value, if $u \le \alpha(\boldsymbol{\Theta}^{*}|\boldsymbol{\Theta}^{[t]})$, assign the candidate parameter value $\boldsymbol{\Theta}^{*}$ to the estimated value of the parameter $\boldsymbol{\Theta}^{[t]}$ at the current time; otherwise, the current value remains unchanged $\boldsymbol{\Theta}^{[t]} = \boldsymbol{\Theta}^{[t\text{-}1]}$;

d) Repeating the second step m times to obtain m sampling samples $(\boldsymbol{\Theta}^{(t)}), t = 1, \ldots, m$;

e) Until the function of the parameter to be estimated converges to the function $\alpha, \beta, \gamma, c_0, c_1$ according to the distribution, the converged model parameter estimation value $(\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{c}_0, \hat{c}_1)$ is obtained.

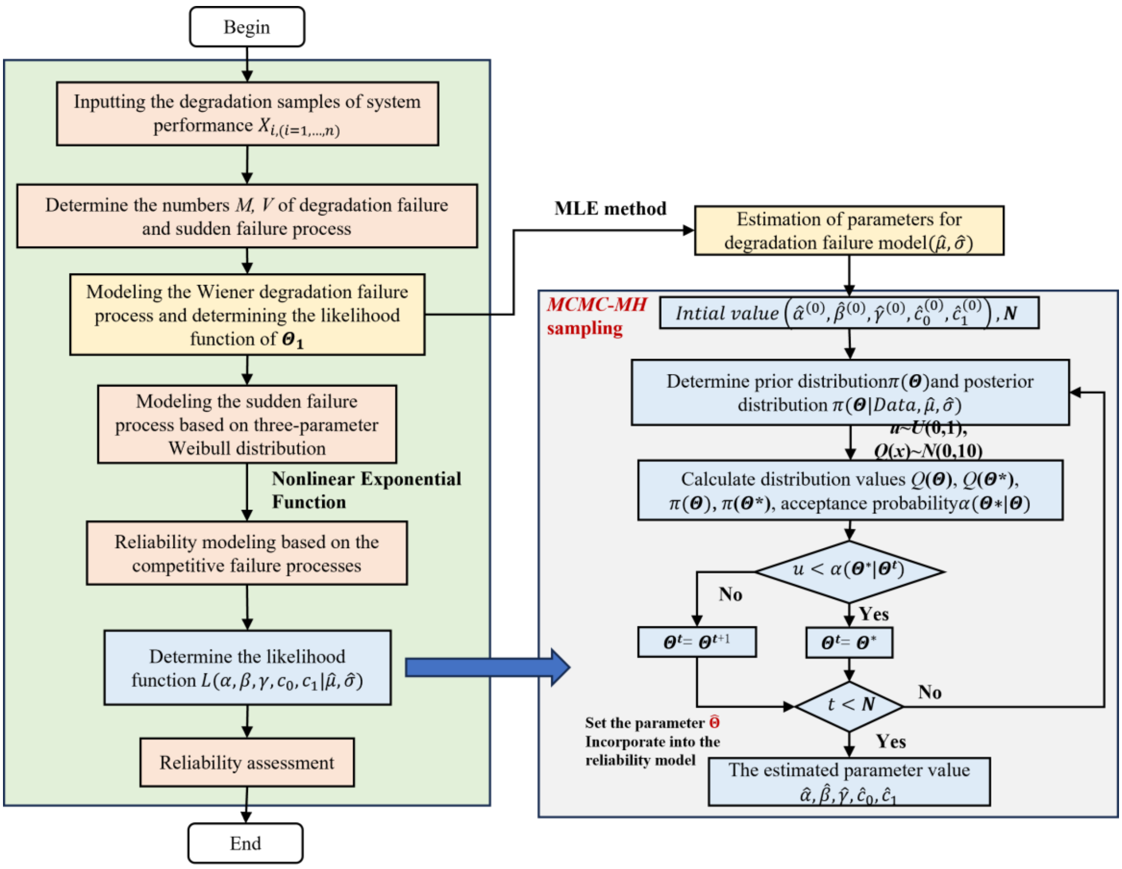


Fig. 4. The flow chart of system reliability evaluation based on competitive failure process.

Based on the above steps, combined with the estimated parameter values $(\hat{\mu}, \hat{\sigma}^2, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{c}_0, \hat{c}_1)$ and the monitored degradation samples, the reliability evaluation results of the system under competitive failure process can be further obtained. By constructing a relationship model between system soft failure and hard failure, the reliability assessment results of system based on three parameter Weibull distribution and Wiener process are obtained. The overall process is shown in Fig. 4. According to the overall evaluation process in Fig.4, it is possible to obtain the operational reliability results of the system effectively in its entire lifecycle. the real degradation samples are used as the validation indicator, and the specific experimental results are shown in the following section.

5. Experiment evaluation and analysis

To evaluate the effectiveness of the methods proposed in this paper, the numerical example and real samples are used for experience verification. The degradation process is the Wiener process, and the sudden failure process follows the Three-parameter Weibull distribution.

Based on the reliability function $R_{inde}(t)$ and the estimated parameters, the reliability evaluation results in the entire life cycle are further obtained.

5.1. The numerical example

In this subsection, a simulation experiment is carried out to verify the reliability model and the estimation process. The degradation samples are generated by the Wiener process, while the sudden failure samples are obtained by the three-parameter Weibull distribution. We set the /mu=0.05, /sigma=0.009, the time length is 100, and the samples size is 20. The degradation samples are shown in Fig.5.

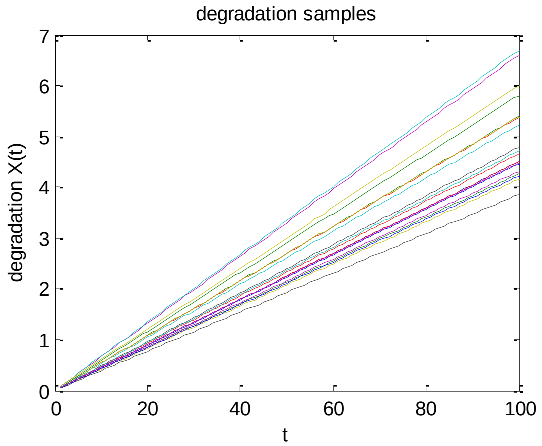


Fig. 5. The original degradation samples.

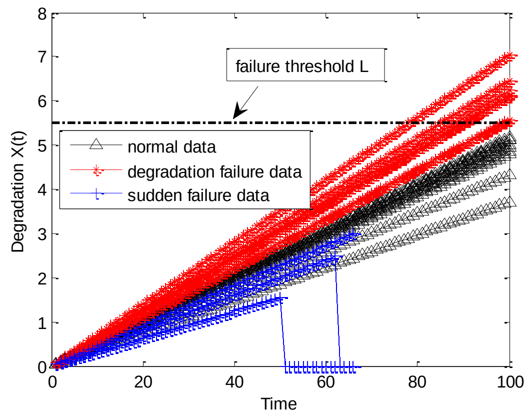


Fig.6. The distribution of different failure process.

To obtain the sudden failure data and degradation failure data, the failure threshold is determined as L=5.5. The sudden failure time is set three points (t_s=50,76,79). Hence, the different failure data is shown as the Fig. 6.

In. Fig.6, the black line is the failure threshold, the blue line are the sudden failure samples, the red line represents the degradation failure samples, and the others are the non-failure samples. The black rot line represents the failure threshold, and it is obvious that the degradation samples exceed the threshold is 7. Hence, the total number N of samples is 20 (N=20), the number of non-failure samples Z=10, the number of sudden failures V=3, and the number of degradation failures M is 7 (M=7).

Based on the Formulas(15)-(16), the Wiener process parameters$\boldsymbol{\theta}_1 = (\mu, \sigma^2)$can be obtained. $\hat{\mu}$=0.04265, $\hat{\sigma}^2$=0.009234, which represent the estimation results of the MLE method and indicate its accuracy. Hence, according to the non-informative prior distribution, the positional parameter γ of $\boldsymbol{\theta}_2$ is assumed as follow the uniform distribution(γ ~U[0,4000]), shape parameter β follow the uniform distribution(β ~U[0,1]), and the proportional parameters α follow the normal distribution(α~N(0,100)).

Table 1. The parameter estimation results of MCMC-MH sampling method.

Parameter	Mean	Variance	MCE
c_0	-0.5073	1.6223	0.00016
c_1	1.8189	0.0855	0.00022
α	54.9363	1.7995e+03	0.00001
β	505.0679	5.2243e+04	0.00042
γ	-0.9931	68.9973	0.00006

Furthermore, the prior distribution of parameter$\boldsymbol{\theta}_3$follows the normal distribution ($c_0 \sim N(0,100)$, $c_1 \sim N(0,100)$). The initial value of parameters $\alpha, \beta, \gamma, c_0, c_1$ $(\alpha^{(0)}, \beta^{(0)}, \gamma^{(0)}, c_0{}^{(0)}, c_1{}^{(0)}) = (0,0,0,0,0)$.

Based on the formulas (21)-(25), the MCMC-MH sampling algorithm is used to estimate the parameters of the competitive failure reliability model. To evaluate the steady ability of the estimation process, the samples are extracted 10000 times repeatedly, and the estimation results of the numerical samples are shown in Table 1.

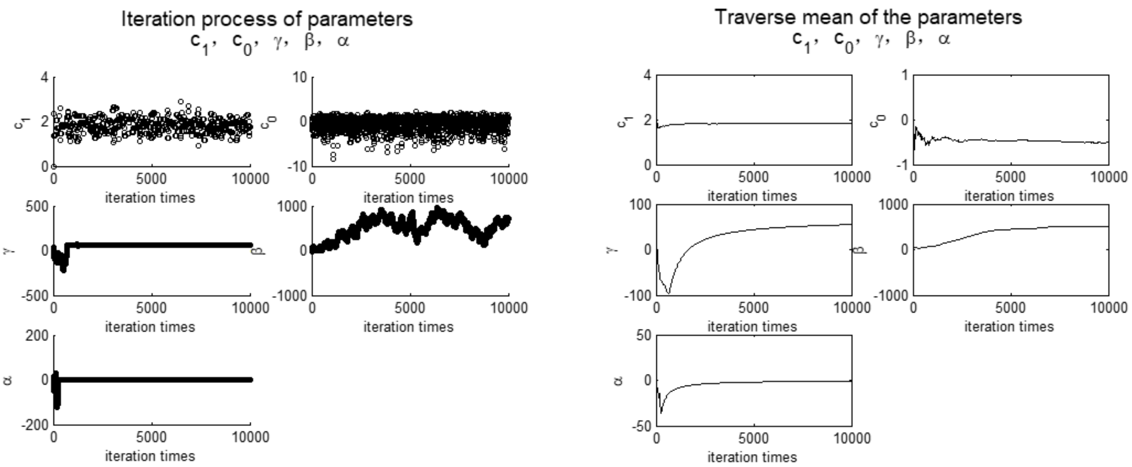


Fig. 7. the parameters estimation results of numerical samples.

It is obvious that the parameters are convergence by the MCMC-MH sampling method. Based on the estimation result, the reliability results can be obtained by the formulas(14). Furthermore, the reliability of the independent degradation process is used to compare with the model proposed in this paper, the comparison results are shown in Fig.8.

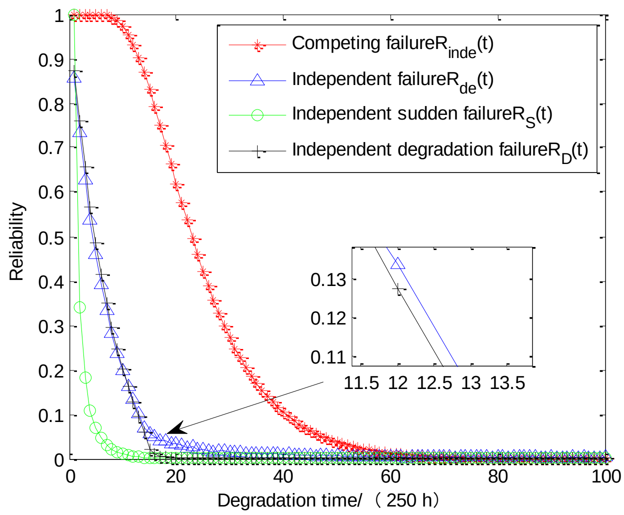


Fig.8. The comparison results of different reliability model under the numerical samples.

In Fig.8, the X-axis is the variable *t* (no physical meaning), the Y-axis represents the degradation amount. It is obvious that the reliability results under competing failure process is higher than other failure process, and the failure time is less than the independent failure process. The results of independent sudden failure process are far lower than the $R_{inde}(t)$, Hence, the competing failure process can reflect the real state, and the constructed reliability model has been proven to be useful in the process of reliability assessment.

To analyze the influence of failure threshold L on the reliability model, the sensitivity of it is calculated and shown in Fig. 9.

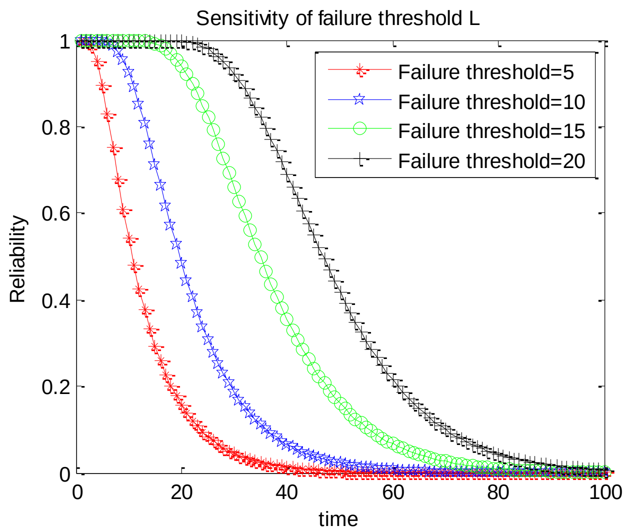


Fig. 9. The sensitivity results of failure threshold.

In Fig.9, the higher the value of failure threshold L, the higher the reliability results. In other words, a higher failure threshold indicates that the conditions for system failure are more stringent, resulting in higher reliability outcomes, which aligns with the actual state.

5.2. The real case application

GaAs lasers are widely used in industrial systems, the failure process includes two parts. On the one hand, excessive current density can cause device overheating and may cause excessive carrier density, leading to gain saturation. Generally, the current threshold is set to 10%. When this threshold is exceeded, the laser will fail due to degradation. In addition, the driving current of the laser directly affects its working state, and even in extreme cases, it can cause thermal breakdown of the material, resulting in sudden failure. Therefore, the failure process of the laser is very consistent with the failure model established in this article.

A. Dataset description

The laser degradation process is established by the Wiener process, and the sudden failure time is determined which follows a three-parameter Weibull distribution. The performance degradation of GaAs lasers is mainly due to the percentage change in operating current, and the current degradation data is shown in Fig.5 [33]. The total number of samples is 15, which is the number of monitored lasers, with a time interval of 250 (*h*). The laser is tested at a temperature of 80℃ and becomes ineffective when the working current increases to 10% of the initial value. Hence, according to expert experience and historical data, the soft failure threshold *L* of GaAs lasers is 10, that is, when the current percentage exceeds 10%, the GaAs laser will fail.

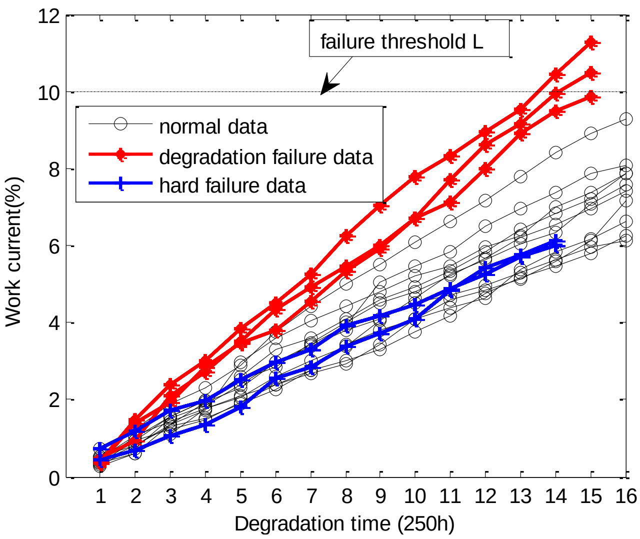


Fig. 10. The current degradation trajectory of GaAs lasers.

In Fig. 10, the degradation of samples 1, 6, 10 are exceeded the soft failure threshold *L* within the measurement time 4000 (*h*), causing laser degradation failure; Due to the degradation of the working current of laser samples 3,14 are no longer changes after the measurement time 3500 (*h*), it is determined that the laser has experienced sudden failure caused by the external environmental shocks; The remaining samples are not failed during the measurement time. Therefore, the total number of samples *N*=15, the number of not failure samples *Z*=10, the number of hard failures *V*=2, and the number of soft failures *M*=3. Based on expert experience and historical data, it is known that the soft failure threshold *L* of the laser is *L*=10 (%).

B. The parameter estimation of constructed reliability model

Since the degradation amount of the laser degradation sample during the measurement time is not monotonic increasing, and the performance degradation at a certain monitoring time will decrease and then increase, the laser degradation data are the non-monotonic samples. The physical performance variables of laser degradation only include the working current. Hence, using the one-dimensional Wiener process to describe the degradation process is in line with actual working conditions.

In this paper, the sudden failure time distribution of GaAs laser follows a three-parameter Weibull distribution, while the position parameter represents the product will not fail within γ, the shape parameter β represents the failure rate of the laser will change with the measurement time, and the scale parameter α indicate the scaling of the failure rate. Therefore, based on the formula (14), the reliability model of the GaAs laser can be obtained.

Hence, same as the parameter estimation process in the subsection 5.2-A, the estimation results of the GaAs samples are shown in Table 2.

Table 2. The parameter estimation results of MCMC-MH sampling method.

Parameter	Mean	Variance	MCE
c_0	-0.6130	1.6323	0.00027
c_1	-0.5765	0.3466	0.00002
α	0.0053	0.1360	0.00001
β	0.6564	0.2073	0.0001
γ	14.0610	7.5329	0.00036

The Monte Carlo error (MCE), the iterative process as well as the traversal mean results of the parameters $\boldsymbol{\theta}_2$,$\boldsymbol{\theta}_3$ are used to describe the convergence degree of the MCMC-MH algorithm for the constructed reliability model, which are shown in Fig. 11 and Fig. 12.

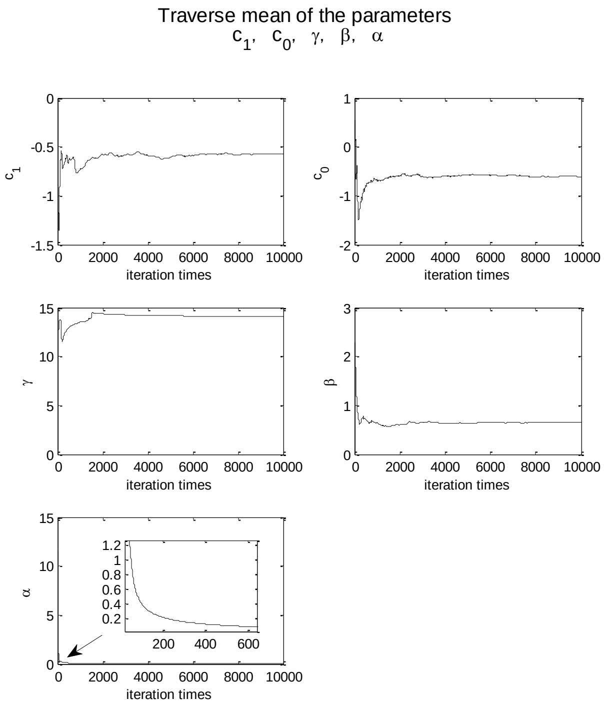


Fig. 11. Traversal mean of estimated parameters.

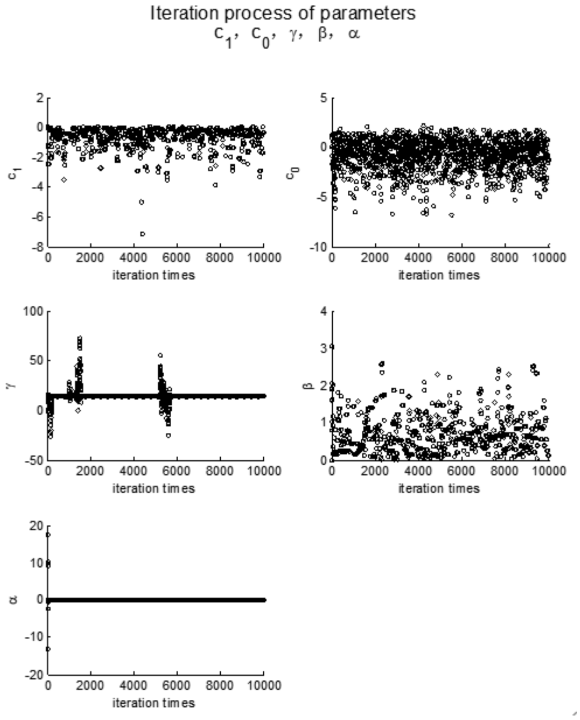


Fig. 12. Estimation parameter iteration process

From the experience results, the Markov chain process of the parameters has reached convergence, and the parameters value are obtained. Based on the estimation results, the reliability operation curve of the constructed competitive failure reliability model can be obtained. The life distribution density function GaAs lasers over measurement time can be further calculated. Based on the formulas (14-15), the parameters$\boldsymbol{\theta}_1 = (\mu, \sigma^2)$of the Wiener process is estimated, and the reliability analysis and experimental results are described in the next section.

C. The reliability analysis and results

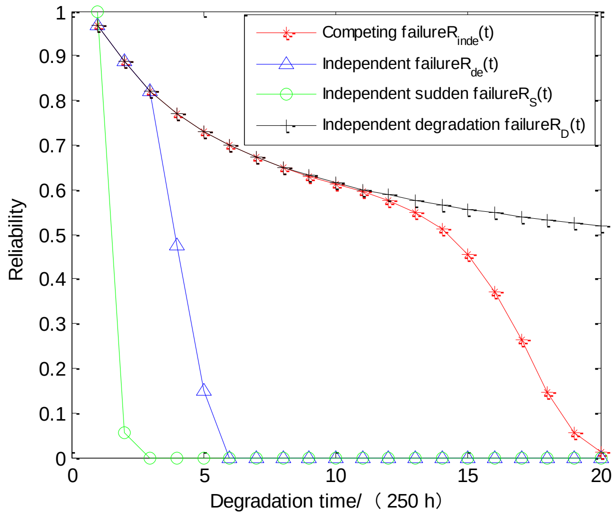


Fig. 13. The reliability under different failure process.

By incorporating the estimated values into formula (14), the lifetime distribution density function and reliability curve of GaAs lasers over time can be obtained. To verify the accuracy of the established reliability model, the comparison results of lasers corresponding to individual failure processes, independent failure processes, and competitive failure processes were compared, as shown in Fig.13.

In Fig. 13, the red line R_{inde} (t) is the reliability change curve of the model constructed in this paper, the blue line R_{de} (t) represents the reliability results corresponding to independent failure. The green line R_S (t) and black line R_D (t) are the reliability results of GaAs laser that suffer from single failure process, respectively. It is evident that the proposed reliability result R_{inde} (t) based on the three-parameter Weibull and Wiener processes is in line with the actual operating state mostly. The failure rate in the early usage stage of the laser is relatively low, but when the failure threshold L is exceeded by 10%, the reliability of the laser rapidly decreases and with a higher failure rate. At this moment, replacement or intelligent maintenance strategies should be considered to improve the usage performance. When the different failure processes independently existed in the operation process, the reliability results $R_{de}(t)$ with independent failure process underestimate the actual reliability of the laser seriously, and the failure time is earlier (the reliability is already lower than 0.024 at the time (500h)), which will affect the normal operation and maintenance strategy of the laser.

Furthermore, when only considering a single failure state, there will be a significant impact on the reliability results of the laser. For example, when only the degradation failure process is considered in the reliability modeling, the reliability $R_D(t)$ of the laser will continue to be high, which will overestimate the reliability of the laser and cause misjudgment for normal maintenance. On the other hand, when only sudden failure $R_S(t)$ is considered, the high-reliability time of the laser is too short, resulting in the inability of the laser to operate normally. Therefore, the reliability results also indicate the effectiveness of the model established in this paper in the system reliability evaluation process.

Furthermore, to demonstrate the advantages of the three-parameter Weibull distribution in the reliability modeling under competing failure process, this article compares the different sudden failure process (three-parameter Weibull distribution and two-parameter Weibull distribution), the comparison results are as shown in Fig. 14.

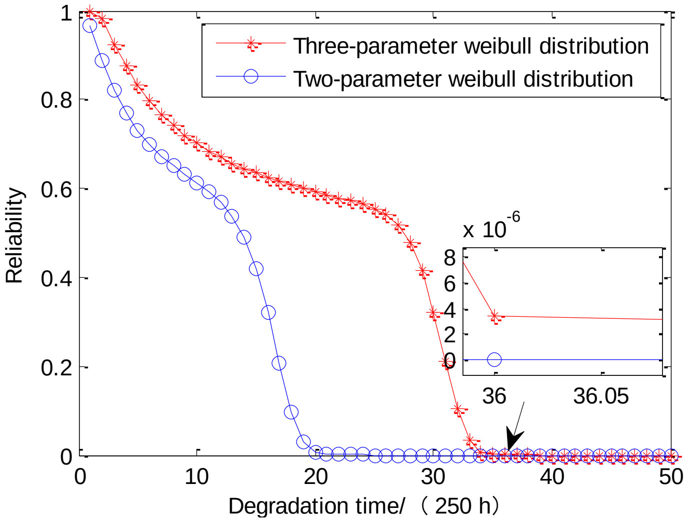


Fig. 14. The reliability under different sudden failure process.

In Fig. 14, due to the positional parameters γ, with the extension of monitoring time, it can be seen that the reliability results corresponding to the competition failure process based on the three-parameter Weibull distribution are better than those corresponding to the two-parameter Weibull distribution, and can maintain higher reliability within the entire operation time. This indicates that the reliability model method based on the three-parameter Weibull distribution is feasible and has better analytical results.

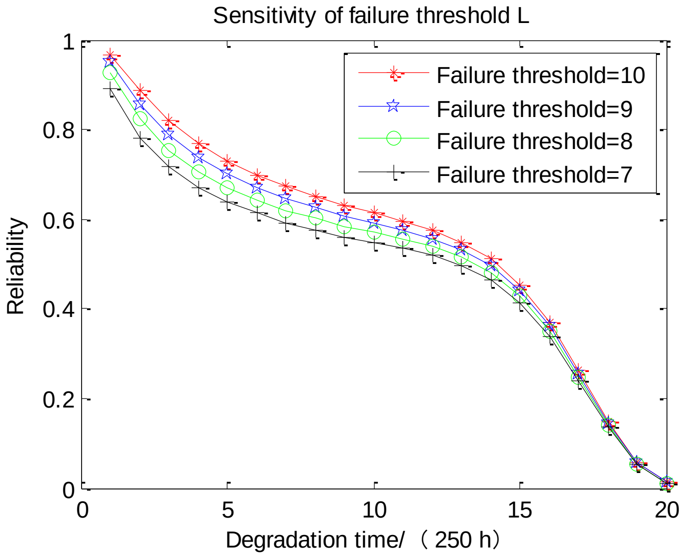
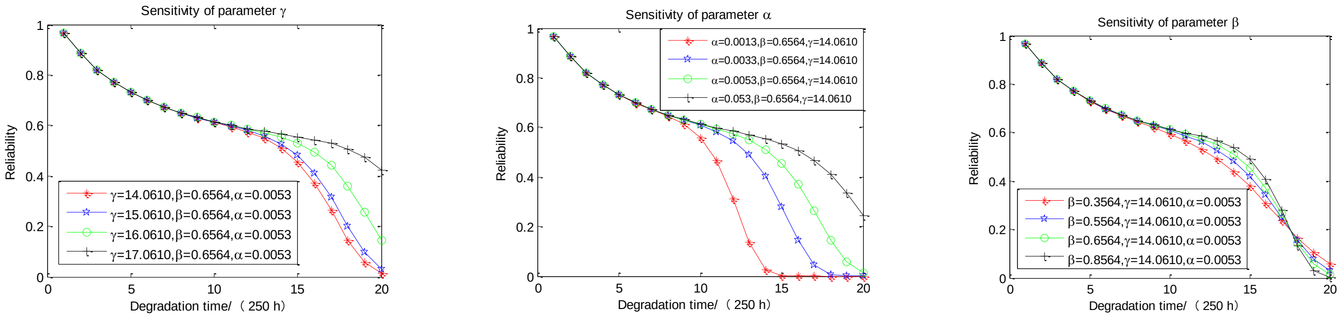


Fig. 15. The sensitivity of failure threshold.

For the sensitivity evaluation of model parameters to the reliability model, the laser reliability with different parameter values is set to compare the performance. Since the different failure thresholds correspond to different failure processes of lasers. First, the different failure threshold is changed L=[7,10], and the corresponding reliability results are as shown in Fig. 10. The red, blue, green, and black lines correspond to reliability results with failure thresholds of L=10%, L=9%, L=8%, and L=7%, respectively. The larger the failure threshold L, the longer the degradation time corresponding to the current degradation failure of the laser, the fewer failure samples, and the higher the reliability of the laser. This is in line with the true operating state of the laser. Therefore, the specified failure threshold of the laser can be set based on the relationship between the failure threshold and the operation reliability.

Furthermore, to analyze the influence of reliability models on different parameters in the three-parameter Weibull distribution, two parameters in the Weibull distribution are fixed, and the value range of one parameter is changed to obtain the operation reliability results. In the three-parameter Weibull distribution, the reliability results under different positional parameters γ, scale parameters α, and shape parameters β are shown in Fig. 16.



(a) (b) (c)

Fig. 16. Sensitivity of different Weibull parameters. (a). positional parametersγ (b). scale parametersα (c). shape parametersβ

In Fig. 16, the X-axis represents the laser degradation time, and the Y-axis is the reliability results corresponding to different parameters. In Fig. 16 (a), before the monitoring duration of 3000 (h), the reliability corresponding to different position parameters remained almost unchanged. However, after the measurement time 3000 (h) of laser, the position parameters are proportional to the laser operation reliability, with the larger the value, the higher the reliability $R(t)$; Fig. 16(b) shows the laser reliability corresponding to different scale parameters. When the measurement time is less than 2000 hours (*i.e.* T<2000 (h)), the scale parameters will not affect the reliability results of the laser. However, when the running time is greater than 2000 (h), the scale parameters$\alpha$are proportional to the laser reliability. The larger the scale parameters, the higher the reliability results $R(t)$, and the longer the failure time of laser; In Fig. 16 (c), the influence of shape parameters on laser reliability will changes with the measurement time. When the operation time less than 2250(h) (*i.e.* T<2250 (h)), the shape parameterβdoes not affect the laser reliability. When the operation time T is greater than 2250(h)and less than 4450(h) (*i.e.* 2250(h)<T<4450(h)), the larger the shape parameters β, the greater the operation reliability of laser. When operation time T>4450 (h), the shape parameters$\beta$are inversely proportional to the reliability of the laser. Therefore, when setting shape parameters, it is necessary to determine the effective size of shape parameters based on different maintenance needs.

6. Conclusion

In the operation of the complex system, when external environmental shock samples cannot be directly recorded, a reliability evaluation method based on a three-parameter Weibull distribution and Wiener process is proposed in this paper. The degradation failure process and sudden failure process of the system are described using a univariate Wiener process and a three-parameter Weibull distribution, respectively, representing competing failure processes. A nonlinear exponential function is employed to establish the relationship model between the degradation process and the sudden failure process, and the corresponding reliability model under the competitive failure process is derived. To accurately obtain the reliability evaluation results, the Bayesian method is combined with the MCMC-MH sampling algorithm to estimate the values of the proposed reliability model. Subsequently, the established model is validated using real performance degradation samples and compared with different reliability models. The impact and sensitivity of the proposed model on various parameters are analyzed. The simulation results demonstrate that the constructed model in this study better aligns with the actual operating state and can enhance the operational reliability of the system. The proposed reliability evaluation method provides an analytical basis for the maintenance strategy of repairable systems and enables the calculation of the remaining life of the system within its lifecycle based on this model, thus further extending the service life of the system.

In the future work, there are 3 points should be considered. 1) a dynamic failure threshold will be considered to modeling the failure process. Since the system or equipment is affected by environmental shocks, fixed failure thresholds do not conform to the actual operating state. 2) Due to the complexity of the actual operating environment of the system, the simple three-parameter Weibull distribution may not accurately describe sudden failures. In the future, a combination of multiple distributions will be considered to establish a sudden failure model. 3) How to combine the reliability assessment process with model fault-tolerant control is an important research direction.

Acknowledgment

This work was supported by the National Science Foundation of China (No. 62403376, No. U2034209, No. 62120106011). The Research startup foundation of Xi'an University of Technology (No. 451124001)

Reference

1. Jiawen Hu, Qiuzhuang Sun, Zhisheng Ye, Condition-Based maintenance planning for systems subject to dependent soft and hard failures[J], IEEE Transactions on Reliability, vol. 70, no. 4, pp. 1468-1480, 2021. https://doi.org/10.1109/TR.2020.2981136
2. Anqi Shangguan, Nan Feng, Lingxia Mu, Rong Fei, Xinhong Hei, Guo Xie[J], Quality and Reliability Engineering International, vol. 39, no. 7, pp. 2851-2868, 2023. https://doi.org/10.1002/qre.3394
3. Anqi Shangguan, Guo Xie, Rong Fei, Lingxia Mu, Xinhong Hei, Train wheel degradation generation and prediction based on the time series generation adversarial network[J], Reliability Engineering & System Safety, vol.229. 2022. https://doi.org/10.1016/j.ress.2022.108816
4. Lina Bian, Guanjun Wang, Peng Liu, Reliability analysis for k-out-of-n(G) systems subject to dependent competing failure processes, Computers & Industrial Engineering, Volume 177, 2023, 109084, 2023.
5. Jianing Man, Qiang Zhou,Prediction of hard failures with stochastic degradation signals using Wiener process and proportional hazards model, Computers & Industrial Engineering, Volume 125, Pages 480-489, 2018. https://doi.org/10.1016/j.cie.2018.09.015
6. Miaoxin Chang, Xianzhen Huang, Frank P.A. Coolen, Tahani Coolen-Maturi, Reliability analysis for systems based on degradation rates and hard failure thresholds changing with degradation levels[J], Reliability Engineering & System Safety, vol.216. 108007, 2021. https://doi.org/10.1016/j.ress.2021.108007
7. Riccardo Amirante, Elia Distaso, Paolo Tamburrano, sliding spool design for reducing the actuation forces in direct operated proportional directional valves: Experimental validation[J], Energy Conversion and Management, Vol. 119, pp. 399-410, 2016. https://doi.org/10.1016/j.enconman.2016.04.068
8. YanHui Lin, YanFu Li, Enrico Zio, Reliability assessment of systems subject to dependent degradation processes and random shocks[J], IIE Transactions, vol. 48, no. 11, pp. 1072–1085, 2016. https://doi.org/10.1080/0740817X.2016.1190481
9. Chuanxi Jin, Yan Ran, Zhichao Wang, Guangquan Huang, Liming Xiao, Genbao Zhang, Reliability analysis of gear rotation meta-action unit based on Weibull and inverse Gaussian competing failure process, Engineering Failure Analysis, vol 117, 104953, 2020. https://doi.org/10.1016/j.engfailanal.2020.104953
10. Guangze Pan, Guangkuo Guo, Dan Li, Yaqiu Li, Qian Li, Wenwei Liu, A reliability analysis method based on the mixed correlated competition model considering multi-performance degradation and sudden failures, Engineering Failure Analysis, vol 146, 107126, 2023. https://doi.org/10.1016/j.engfailanal.2023.107126
11. Gang Pan, Chaoxuan Shang, Yuying Liang, Jinyan Cai, Yafeng Meng, Reliability evaluatioin of radar power amplifier system in case of related competing failures[J], Acta Electronica Sinica, vol. 45, no. 4, pp.805-812, 2017. (in Chinese)
12. Zhiyuan Yang, Jianmin Zhao, Zhonghua Cheng, Liying Li, Chi Kuo, Reliability model of competing failure system with dependent degradation[J], Acta Armamentarii, vol. 41, no. 7, pp. 1424-1433, 2020. (in Chinese)
13. Xingang Wang, Lin Li, Miaoxin Chang, Kaizhong Han, Reliability modeling for competing failure processes with shifting failure thresholds under severe product working conditions[J], Applied Mathematical Modelling, Vol. 89, pp. 1747-1763, 2021. https://doi.org/10.1016/j.apm.2020.08.032
14. Anqi Shangguan, Guo Xie, Lingxia Mu, Rong Fei, Xinhong Hei, Reliability modeling: combining self-healing characteristics and competing failure process[J], Quality Technology & Quantitative Management, vol. 21, no.3, pp. 363-385, 2023. https://doi.org/10.1080/16843703.2023.2202955
15. Hongda Gao, Lirong Cui, Qingan Qiu, Reliability modeling for degradation-shock dependence systems with multiple species of shocks[J], Reliability Engineering & System Safety, vol. 185, pp. 133-143, 2019. https://doi.org/10.1016/j.ress.2018.12.011
16. Yankai Qin, Xiaohong Zhang, Jianchao Zeng, Guannan Shi, Bin Wu, Reliability Analysis of Mining Machinery Pick Subject to Competing Failure Processes with Continuous Shock and Changing Rate Degradation[J], IEEE Transactions on Reliability, vol. 72, no.2, pp.795-807, 2023. https://doi.org/10.1109/TR.2022.3192060

17. Hao Lyu, Hongchen Qu, Zaiyou Yang, Li Ma, Bing Lu, Michael Pecht, Reliability analysis of dependent competing failure processes with time-varying δ shock model[J], Reliability Engineering & System Safety, vol. 229, 108876, 2023. https://doi.org/10.1016/j.ress.2022.108876
18. Chunfang Zhang, Liang Wang, Xuchao Bai, Jianan Huang, Bayesian reliability analysis for Copula based step-stress partially accelerated dependent competing risks model[J], Reliability Engineering & System Safety, vol. 227, 108718, 2022. https://doi.org/10.1016/j.ress.2022.108718
19. Wenjie Dong, Sifeng Liu, Suk Joo Bae, Yingsai Cao, Reliability modelling for multi-component systems subject to stochastic deterioration and generalized cumulative shock damages[J], Reliability Engineering & System Safety, vol. 205, 107260, 2021. https://doi.org/10.1016/j.ress.2020.107260
20. Jia Wang, Zhigang Li, Guanghan Bai, Ming J. Zuo, An improved model for dependent competing risks considering continuous degradation and random shocks, Reliability Engineering & System Safety, vol. 193, 106641, 2020. https://doi.org/10.1016/j.ress.2019.106641
21. Hao Lyu, Shuai Wang, Zaiyou Yang, Hongchen Qu, Li Ma, Reliability modeling for multi-component system subject to dependent competing failure processes with phase-type distribution considering multiple shock sources, Quality Engineering, vol. 35, no. 1, pp. 95-109, 2023. https://doi.org/10.1080/08982112.2022.2098043
22. Mengfei Fan, Zhiguo Zeng, Enrico Zio, Rui Kang, Modeling dependent competing failure processes with degradation-shock dependence[J], Reliability Engineering & System Safety, vol. 165, pp. 422–430, 2017. https://doi.org/10.1016/j.ress.2017.05.004
23. Haiyang Che, Shengkui Zeng, Jianbin Guo, Yao Wang, Reliability modeling for dependent competing failure processes with mutually dependent degradation process and shock process[J], Reliability Engineering & System Safety, vol. 180, pp. 168–178, 2018. https://doi.org/10.1016/j.ress.2018.07.018
24. Li Yang, Xiaobing Ma, Rui Peng, Qingqing Zhai, Yu Zhao, A preventive maintenance policy based on dependent two-stage deterioration and external shocks[J], Reliability Engineering & System Safety, vol. 160, pp. 201–211, 2017. https://doi.org/10.1016/j.ress.2016.12.008
25. Xingang Wang, Xinyao Zhang, Lujie Yang, Ruimin Ma, Tool reliability analysis for wear degradation data under competitive failure conditions[J], China Mechanical Engineering, vol. 31, no. 14, pp. 1672-1677, 2020. (in Chinese)
26. Nooshin Yousefi, David W. Coit, Xiaoyan Zhu, Dynamic maintenance policy for systems with repairable components subject to mutually dependent competing failure processes, Computers & Industrial Engineering, Volume 143, 2020, 106398, https://doi.org/10.1016/j.cie.2020.106398
27. Ancheng Xue, Lin Luo, Qi Jing, Junhao Wang, Xuankun Song, Ying Liu, Jun Li, ShaoFeng Huang Tianshu Bi, Research on aging failure rate estimation of protective relay based on three-parameter Weibull distribution[J], Power System Protection and Control, vol. 42, no. 24, pp. 72-78, 2014. (in Chinese)
28. Meng Xu, Huachao Mao, q-Weibull Distributions: Perspectives and Applications in Reliability Engineering[J], IEEE Transactions on Reliability, https://doi.org/10.1109/TR.2024.3448289
29. Mohamed Kayid, Salah Djemili, Reliability Analysis of the Inverse Modified Weibull Model with Applications[J], Mathematical Problems in Engineering, vol. 2022, 4005896, 2022. https://doi.org/10.1155/2022/4005896
30. Fucheng Han, Xin Li, Shengwenjun Qi, Wenhua Wang, Wei Shi, Reliability analysis of wind turbine subassemblies based on the 3-P Weibull model via an ergodic artificial bee colony algorithm[J], Probabilistic Engineering Mechanics, vol. 73, 103476, 2023. https://doi.org/10.1016/j.probengmech.2023.103476
31. Jesus M. Barraza-Contreras, Manuel R. R. Pina-Monarrez, Roberto C. C. orres-Villasenor, Vibration Fatigue Life Reliability Cable Trough Assessment by Using Weibull Distribution[J], Applied Sciences-Basel, vol. 13, no.7,4403, 2023. https://doi.org/10.3390/app13074403
32. Cox D R, Miller H D, The theory of stochastic processes[M]. London: chapman and Hall, 1965
33. Meeker W Q, Escobar L A, Statistical methods for reliability data[M], John Wiley & Sons, New York, 1998.