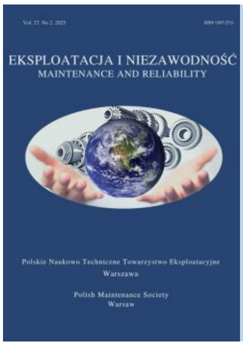

Eksploatacja i Niezawodnosc – Maintenance and Reliability

Volume 27 (2025), Issue 2

journal homepage: http://www.ein.org.pl

Article citation info:
C. Zhu, B. Yang, F. Jin, M. Li, J. Jiang, Optimal maintenance and pricing strategy for a periodic review production system with fixed maintenance costs and limited maintenance capacity, Eksploatacja i Niezawodnosc – Maintenance and Reliability 2025: 27(2)http://doi.org/10.17531/ein/195801

Optimal maintenance and pricing strategy for a periodic review production system with fixed maintenance costs and limited maintenance capacity

Indexed by: Web of Science Group

Chenbo Zhu[a], Baimei Yang[b,*], Fenghua Jin[b], Mi Li[a], Junjie Jiang[a]

[a]Zhejiang University of Technology, China
[b] Shanghai DianJi University, China

Highlights

- Integrate limited maintenance capacity and fixed maintenance costs into a unified strategy.
- Build a dynamic programming model and utilize strong CK-concavity to develop the strategy.
- Partially characterize the optimal maintenance strategy by two thresholds.

Abstract

In recent times, the escalating complexity of advanced production systems has led to increased exposure to various uncertainties, which impact systems' reliability. To maintain the reliability of systems, reduce maintenance costs and increase revenues, we establish an effective joint optimal maintenance and pricing strategy for a periodic review production system, which is of practical importance. Compared with the existing literature,we make a contribution to considering limited maintenance capacity and fixed maintenance costs simultaneously, and developingjoint optimal maintenance and pricing strategies. We initially construct a dynamic programming modelfor this problem, and prove the objective function is strongCK-concave. We then show that the optimal maintenance strategyis partially characterized by two thresholds,and the optimal pricing strategy depends on the optimal number of operational machines after repairs. Numerical results show that the optimal maintenance and pricing strategy is quite robust, and is not much affected by various parameters.

Keywords
strong CK -concavity, periodic review production system, fixed maintenance costs, limited maintenance capacity, optimal maintenance and pricing strategy

1. Introduction

The increasing intricacy of advanced production systems in contemporary times brings with it a host of uncertainties, drawing significant attention from both industry and academia. A pivotal aspect of enhancing system reliability lies in the development of effective maintenance strategies, particularly given the challenge of managing uncertainties and the associated costs. Notably, a common obstacle in maintenance strategy formulation is the limited capacity for repairs, as many factories can only fix a portion of malfunctioning machines within a given timeframe. Moreover, maintenance activities are costly, and fixed expenses incurred at the onset of repairs. To reduce maintenance costs and increase revenues, it is crucial for factories to implement an effective maintenance and pricing strategy for their production systems, which will be studied in this paper.

Our research is related to the field of the optimal maintenance for the maintainable deteriorating system. Previous studies have addressed various aspects of this issue.

(*) Corresponding author.
E-mail addresses: C. Zhu (ORCID: 0000-0001-8893-0648) chenbozhu@zjut.edu.cn,B. Yang (ORCID: 0000-0002-2815-0735) yangbm@sdju.edu.cn, F. Jin jinfh@sdju.edu.cn, M. Li 202105710104@zjut.edu.cn,J. Jiang 907205557@qq.com

For instance, Chiang and Yuan[1] explored a state-dependent maintenance policy for a multi-state continuous-time Markovian deteriorating system. Shi et al.[2] examined a system with multi-level preventive maintenance, and explored the optimal maintenance policy by solving a Markov Decision Process(MDP) model.Omshi et al.[3] proposed a dynamic auto-adaptive predictive maintenance policy for single-unit systems.Liu et al.[4] designed a selective maintenance strategy for multi-period and multi-state systems, and formulated the problem as a discrete-time MDP model.Chen et al.[5] investigated optimal maintenance rates for a single-server queueing system with server breakdowns. Özcan et al.[6] optimized the maintenance strategy for electrical equipment with multi-criteria by using integer programming integrated with Analytic Hierarchy Process(AHP) and Complex Proportional Assessment (COPRAS) methods. Jin et al.[7] investigated a large-scale maintainable system with uncertain maintenance time,and developed a novel optimal maintenance strategy.Hamzaoui et al.[8] studied a manufacturing system with multiple machines, and constructed an optimal planning for comprehensive non-periodic preventive maintenance by using a mixed integer linear programming. Qi et al.[9] developed a new maintenance strategy for a dual component warm standby system. To minimize maximum processing time, Zhang et al.[10] studied a scheduling problem for a replacement flow shop with predictive maintenance and production decisions by constructing a twin model, and solved the problem by using an improved genetic algorithm. Greiner and Cacereño[11] applied a surrogate assisted evolutionary algorithm to minimize the unavailability of a system and the strategy cost, and showed that the proposed algorithm performs well. Xu et al.[12] proposed a maintenance optimization model considering generation and operational risk costs for a power system, and solved the model efficiently by designing a generation-maintenance iterative algorithm. Lu et al.[13] constructed amulti-objective optimizationmodel to maximize operational reliability and minimize maintenance costs, and proved that the proposed model is better than the traditional models in terms of cost and operational reliability. Recently, UAVshave received increasing attention from researchers, in addition to researching how to optimize control strategies for UAVs, researchers are increasingly focusing on how to reduce maintenance costs of UAVs[14]. Zhang et al.[15] proposed a feasible solution to reduce maintenance and procurement costs and shorten the maintenance interval for UAV-operated delivery systems.There are also some papers studying the optimal maintenance strategy for leased equipment. For example, Pongpech and Murthy[16], Mabrouk et al.[17] and Liu et al.[18] proposed optimal maintenance strategies for leased equipment in different scenarios.Most of these researches used MDP, dynamic programming, or integer programming approaches to study the optimal maintenance problem, and designed exact or approximate algorithms to obtain the optimal maintenance strategy.

However, researches on the optimal maintenance problem for the maintainable, deteriorating system with limited maintenance capacity remain relatively scarce. De Smidt-Destombes et al.[19] considered a k-out-of-N system with limited maintenance capacity, and proposed an exact method to calculate the optimal strategy for the system.Chen et al.[20] studied a multi-state system with single maintenance capacity.Zhu et al.[21] then researched on the two-stage leased systems with finite maintenance capacity. Maquirriain et al.[22] studied a scheduling problem of machine maintenance activities with capacity constraints to minimize operation and maintenance costs, and solved the problem approximately by using some matheuristic algorithms. Moreover, studies integrating maintenance with other decision-making aspects for the maintainable and deteriorating system are also rare. Yao et al.[23] studied a production-inventory system with stochastic maintenance times, and then established joint maintenance and production policies by solving a discrete-time MDP model. Kuo[24] investigated a finite horizon multi-period batch production system with discrete time Markovian deterioration. In summary, these papers have not fully explored systems with both limited maintenance capacity and fixed maintenance costs as well as the joint optimization of the maintenance and product pricing strategy.

We find that although the maintenance optimization problem for a production system is not the same in detail as the inventory control optimization problem, they are similar to each other to a certain extent, i.e., the number of operational machines in a maintenance system can be compared to the inventory level in an inventory system. Therefore, our research also intersects with inventory control (and pricing) problems, particularly those with finite capacity and setup costs. This aspect of research has seen substantial contributions. Chen and Lambrecht[25] explored a stochastic periodic review inventory system with limited production capacity and setup costs.On this basis, Gallego and Scheller-Wolf[26] further studied the model through a generalization of K-concavity.Minner and Silver[27] analyzed a multi-product inventory replenishment system under a capacity constraint. Chao et al.[28] then examined a stochastic, periodic-review inventory problem with setup costs and limited ordering capacity by using the concept of strong CK-concavity.It is important to note the evolution of variables considered in these studies. Chen and Lambrecht[25], Gallego and Scheller-Wolf[26], and Minner and Silver[27] did not incorporate the product selling price as a decision variable, and Minner and Silver[27] did not theoretically construct an optimal inventory and pricing strategy for their systems. Chao et al.[28] incorporated the selling price in their decision-making process, and provided a more comprehensive understanding of dynamic inventory control under constraints of finite capacity and setup costs.

The main contribution of our work is considering limited maintenance capacity and fixed maintenance costs simultaneously, and integrating these factors into a unified maintenance and pricing strategy for production systems, which has not been studied in the literature. We investigate a joint maintenance and pricing problem for a production system with fixed maintenance costs and limited maintenance capacity, building upon the framework established by Chao et al.[28]. We construct a dynamic programming model and utilize the concept of strong CK-concavity to partially characterize optimal maintenance and pricing strategies in four distinct regions, based on the starting number of operational machines each period.

The rest of this paper is organized as follows. Section 2 introduces our model and assumptions. Section 3 characterizes the structural properties of the optimal maintenance and pricing strategy for the production problem with fixed maintenance costs and limited maintenance capacity.Section 4 presents numerical experiments to illustrate the impact of different parameters on the optimal strategy. Finally, Section 5 contains a conclusion.

2. Assumption and model

Consider a periodic review production system of a single product with N periods,and assume the maintenance cost is fixed, and the maintenance capacity is limited. Let M be the fixed maintenance cost, and the repair quantity per period cannot exceed Q. We assume when a repair decision is placed at the beginning of a period, the repair can be finished in that period. The timeline of each period is as follows: 1) review machines' status and make maintenance decisions; 2) repair broken machines and return them to a working condition; 3) set a selling price; 4) collect random demands; 5) some machines are broken again at the end of the period; and 6) compute the performance.

Let q_n^b be the quantity of operational machines before repairs and q_n^a be the quantity of operational machines after repairs in period n. Due to the maintenance capacity (denoted by Q), we have $q_n^b \le q_n^a \le q_n^b + Q$.We then let m_1 be the maintenance cost per unit, and let the production capacity be a linear function of q_n^a, i.e.,the production capacity is defined as $m_2 q_n^a$,$m_2 > 0$. We further assume the production cost per unit is m_3, and let $m_2 m_3 q_n^a$ denote the total production cost in period n, which is also linear w.r.t. the production capacity ($m_2 q_n^a$). Therefore, the total cost (the fixed maintenance cost, the variable repair cost, and the production cost) in period n is given as

$$M\mathbf{1}[q_n^a > q_n^b] + m_1(q_n^a - q_n^b) + m_2 m_3 q_n^a,$$

where $\mathbf{1}[\mathrm{A}] = 1$ if event A is true; $\mathbf{1}[\mathrm{A}] = 0$, if event A is false.

In addition, the selling price in period n is defined as p_n, and the demand $D_n(p_n)$ is a decreasing linear function of p_n. When p_n increases from p_l to p_u, $D_n(p_n)$ decreases from $D_n(p_l)$ to $D_n(p_u)$. Moreover, $p_n D_n(p_n)$ is concave in p_n. When $p_n^* = p_0$, $p_n D_n(p_n)$ reaches the maximum value $p_0 D_n(p_0)$.Suppose surplus products will be lost, therefore, the revenue in period n then can be expressed by $p_n \min\{D_n(p_n), m_2 q_n^a\}$, when the demand $D_n(p_n)$ is larger than the production capacity $m_2 q_n^a$, all the products will be sold out; when the demand $D_n(p_n)$ is less than the production capacity $m_2 q_n^a$, all the demand is satisfied and surplus products are lost. Without loss of generality, we assume that the salvage value for surplus products at the end of each period is zero, and it is a common assumption in the literature, such as Qiu et al.[29]. We then assume the number of new broken machines at the end of period n is B_n, which is a random variable. For convenience we let $B_n = b_n + \epsilon_n$, where b_n is the average number of broken machines and ϵ_n is a random variable.Here $E(\epsilon_n) = 0$.

The objective is to maximize the expected total discounted profit over the N periods by optimizing the maintenance and pricing decisions simultaneously. Let $V_n(q_n^b)$ denote the maximum expected total discounted profit from period $n (n < N)$ to period N with the starting number of operational machines q_n^b. The optimality equation is given as

$$V_n(q_n^b) = \max_{q_n^b \le q_n^a \le q_n^b + Q} \max_{p_l \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\} - M\mathbf{1}[q_n^a > q_n^b] - m_1(q_n^a - q_n^b) - m_2 m_3 q_n^a + \alpha \mathrm{E}[V_{n+1}(q_n^a - b_n - \epsilon_n)]\}, \quad (1)$$

where α is the discount factor per period, $\alpha \in [0,1]$. The terminal condition $V_{N+1}(x_{N+1})$ means the salvage value, which is assumed to be linear w.r.t. x_{N+1}. For the sake of convenience, we define

$$R_n(q_n^a) = \max_{p_l \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\},$$

and

$$H_n(q_n^a) = R_n(q_n^a) - m_1 q_n^a - m_2 m_3 q_n^a + \alpha \mathrm{E}[V_{n+1}(q_n^a - b_n - \epsilon_n)].$$

Then the optimality equation could be rewritten as:

$$V_n(q_n^b) = m_1 q_n^b + \max_{q_n^b \le q_n^a \le q_n^b + Q} \{-M\mathbf{1}[q_n^a > q_n^b] + H_n(q_n^a)\}. (2)$$

3. Analytical results

Gallego and Scheller-Wolf [26] proposed an important concept named strong CK-convexity. Chao et al. [28] borrowed this concept and introduced a related concept called strong CK-concavity, which will be used in this paper to develop the optimal maintenance and pricing strategy for the system.We introduce the definition of strong CK-concave as follows.

Definition. Given non-negative constants C and K, we call the function G strong CK-concave if for all y, $a \ge 0$, $b > 0$ and $z \in [0, \mathrm{C}]$,

$$-K + G(y+z) \le G(y) + \frac{z}{b}\{G(y-a) - G(y-a-b)\}. (3)$$

We then derive the following two lemmas before characterizing the optimal maintenance and pricing strategy for the system.

Lemma 1.$R_n(q_n^a)$ is concave in q_n^a.

Proof. Firstly, note that $D_n(p_u) \le D_n(p_n) \le D_n(p_l)$. Because $R_n(q_n^a)$ is a piecewise continuous function of q_n^a, $R_n(q_n^a)$ can be analyzed in three cases.

Case 1: when $q_n^a < \frac{D_n(p_u)}{m_2}$, i.e., the production capacity is less than the lower bound of the demand, so the sales volume $D_n(p_n)$ equals production capacity, and when p_n is equal to p_u, $p_n m_2 q_n^a$ reaches its maximum value. Therefore, we have

$$R_n(q_n^a) = \max_{p_l \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\} = \max_{p_l \le p_n \le p_u} \{p_n m_2 q_n^a\} = p_u m_2 q_n^a.$$

It is clear that, $R_n(q_n^a)$ is concave in q_n^a in this case.

Case 2: when $\frac{D_n(p_u)}{m_2} \le q_n^a \le \frac{D_n(p_l)}{m_2}$,

Subcase 1: when $\frac{D_n(p_u)}{m_2} \le q_n^a < \frac{D_n(p_0)}{m_2}$, i.e., the production capacity is not less than the lower bound of the demand, but less than $D_n(p_0)$, which implies that $p_0 < D_n^{-1}(m_2 q_n^a) \le p_u$,and we have

$$\begin{aligned} R_n(q_n^a) &= \max_{p_l \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\} \\ &= \max\{\max_{p_l \le p_n < D_n^{-1}(m_2 q_n^a)} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\}, \\ &\quad \max_{D_n^{-1}(m_2 q_n^a) \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\}\} \\ &= \max\{D_n^{-1}(m_2 q_n^a) m_2 q_n^a, \max_{D_n^{-1}(m_2 q_n^a) \le p_n \le p_u} \{p_n D_n(p_n)\}\} \\ &= \max\{D_n^{-1}(m_2 q_n^a) m_2 q_n^a, D_n^{-1}(m_2 q_n^a) m_2 q_n^a\} \\ &= D_n^{-1}(m_2 q_n^a) m_2 q_n^a. \end{aligned}$$

Because $D_n^{-1}(z)z$ is concave in z, $R_n(q_n^a) = D_n^{-1}(m_2 q_n^a) m_2 q_n^a$ is concave in q_n^a in this subcase.

Subcase 2: when $\frac{D_n(p_0)}{m_2} \le q_n^a \le \frac{D_n(p_l)}{m_2}$, i.e., the production capacity is not less than $D_n(p_0)$, but not greater than the upper bound of the demand, which implies that $p_l \le D_n^{-1}(m_2 q_n^a) \le p_0$, and we have

$$
\begin{aligned}
R_n(q_n^a) &= \max_{p_l \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\} \\
&= \max\{ \max_{p_l \le p_n < D_n^{-1}(m_2 q_n^a)} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\}, \\
&\quad \max_{D_n^{-1}(m_2 q_n^a) \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\}\} \\
&= \max\{ D_n^{-1}(m_2 q_n^a) m_2 q_n^a, \max_{D_n^{-1}(m_2 q_n^a) \le p_n \le p_u} \{p_n D_n(p_n)\}\} \\
&= \max\{ D_n^{-1}(m_2 q_n^a) m_2 q_n^a, p_0 D_n(p_0)\} \\
&= p_0 D_n(p_0).
\end{aligned}
$$

Case 3: when $q_n^a > \frac{D_n(p_l)}{m_2}$, i.e., the production capacity is greater than the upper bound of the demand, so the sales volume equals the demand. Therefore, we have

$$R_n(q_n^a) = \max_{p_l \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\}$$

$$= \max_{p_l \le p_n \le p_u} \{p_n D_n(p_n)\} = p_0 D_n(p_0).$$

In summary, $R_n(q_n^a)$ is concave in q_n^a.□

To visualize that$R_n(q_n^a)$ is concave in q_n^a, we present the curve of $R_n(q_n^a)$ with respect to q_n^a in Figure 1. In this figure, we define the demand as $D_n(p_n) = 100 - p_n$, $\$45 \le p_n \le \$55$, $m_2 = 1$pc/set, $40\text{sets} \le q_n^a \le 60\text{sets}$.

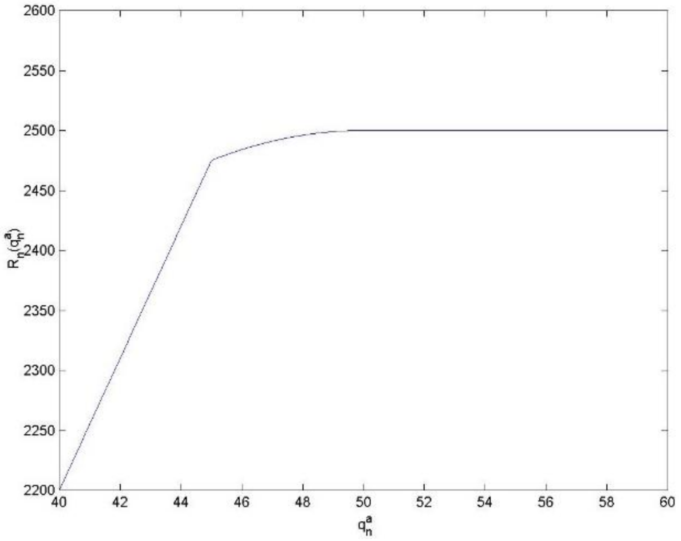

Figure 1. The curve of $R_n(q_n^a)$.

Lemma 2. Both $V_n(q_n^b)$ and $H_n(q_n^a)$ are strong CK-concave.

Proof. This lemma can be proved by induction. When $n = N+1$, it is clear that $V_{N+1}(x_{N+1})$is linear w.r.t.x_{N+1}. Now suppose $V_{n+1}(q_{n+1}^b)$ is strong CK-concave, and it is easy to show that $\alpha \mathrm{E}[V_{n+1}(q_n^a - b_n - \epsilon_n)]$ is strong CK-concave. Therefore, $H_n(q_n^a)$ is also strong CK-concave. Then according to the proof of Theorem 1 in Chao et al.[28], $V_n(q_n^b)$is also strong CK-concave.□

Given non-negative Q, M and strong CK-concave functions$H_n(q_n^a)$ $(n = 1, \dots, N)$, define T_n, t_n, and t_n' by

$$T_n = \inf\left\{q_n^a \in R \,\middle|\, H_n(q_n^a) = \sup_{q_n^a \in R} H_n(q_n^a)\right\},$$

$$t_n = \inf\left\{q_n^b \,\middle|\, -M + \sup_{q_n^b \le q_n^a \le q_n^b + Q} H_n(q_n^b) \le H_n(q_n^b)\right\},$$

$$t_n' =$$

$$\max\left\{q_n^b \le T_n \,\middle|\, -M + \sup_{q_n^b \le q_n^a \le q_n^b + Q} H_n(q_n^a) \ge H_n(q_n^b)\right\}.$$

It is obvious that, $-\infty \le t_n \le t_n' \le T_n$.

We then present the optimal maintenance and pricingstrategy of the production system in Theorem 1as follows.

Theorem 1 Suppose q_n^b is the number of operational machines at the beginning of period n before repairs. The optimal pricing strategy is characterized by the optimal price $p_n^*(q_n^a)$, which depends on q_n^a, and $p_0 \le p_n^*(q_n^a) \le p_u$. The optimal maintenance strategy is characterized by t_n and t_n' $(t_n \le t_n')$. If $t_n' - Q \le t_n$, then the optimal maintenance strategy is

(i)if $q_n^b < t_n' - Q$, then the optimal number of repairs is Q;

(ii)if $t_n' - Q \le q_n^b < t_n$, then the optimal number of repairs is $t_n - q_n^b$;

(iii)if $t_n \le q_n^b \le t_n'$, then either no to carry out repairs, or to carry out repairs and the optimal number of repairs is $t_n' - q_n^b$; and

(iv) if $q_n^b > t_n'$, then no to carry out repairs.

And if $t_n' - Q > t_n$, then the optimal maintenance strategy is

(i') if $q_n^b < t_n$, then the optimal number of repairs is Q;

(ii') if $t_n \le q_n^b < t_n' - Q$,then either no to carry out repairs, or to carry out repairs and the optimal number of repairs is$Q$;

(iii')if$t_n' - Q \le q_n^b \le t_n'$, then either no to carry out repairs, or to carry out repairs and the optimal number of repairs is$t_n' - q_n^b$; and

(iv')if$q_n^b > t_n'$, then no to carry out repairs.

Proof. We first characterize the optimal pricing strategy based on analyzing equation (1) in three cases as follows.

Case 1: If $D_n(p_n) \ge m_2 q_n^a$ for any p_n $(n = 1, \dots, N)$, which means the demand always exceeds the supply in any period, then the optimal price $p_n^*(q_n^a)$ should equal to p_u, and equation (1) can be rewritten as

$$V_n(q_n^b) = \max_{q_n^b \le q_n^a \le q_n^b+Q} \max_{p_l \le p_n \le p_u} \{p_n m_2 q_n^a - M\mathbf{1}[q_n^a > q_n^b] - m_1(q_n^a - q_n^b) - m_2 m_3 q_n^a + \alpha \mathrm{E}[V_{n+1}(q_n^a - b_n - \epsilon_n)]\}$$
$$= \max_{q_n^b \le q_n^a \le q_n^b+Q} \{p_u m_2 q_n^a - M\mathbf{1}[q_n^a > q_n^b] - m_1(q_n^a - q_n^b) - m_2 m_3 q_n^a + \alpha \mathrm{E}[V_{n+1}(q_n^a - b_n - \epsilon_n)]\}.$$

Case 2: If $D_n(p_n) < m_2 q_n^a$ for any p_n ($n = 1, \dots, N$), which means the supply always exceeds the demand in any period, then the optimal price $p_n^*(q_n^a)$ should equal to p_0, which is obtained by maximizing $p_n D_n(p_n)$. Equation (1) can be rewritten as

$$V_n(q_n^b) = \max_{q_n^b \le q_n^a \le q_n^b+Q} \max_{p_l \le p_n \le p_u} \{p_n D_n(p_n) - M\mathbf{1}[q_n^a > q_n^b] - m_1(q_n^a - q_n^b) - m_2 m_3 q_n^a + \alpha \mathrm{E}[V_{n+1}(q_n^a - b_n - \epsilon_n)]\}$$
$$= \max_{q_n^b \le q_n^a \le q_n^b+Q} \{-M\mathbf{1}[q_n^a > q_n^b] - m_1(q_n^a - q_n^b) - m_2 m_3 q_n^a + \alpha \mathrm{E}[V_{n+1}(q_n^a - b_n - \epsilon_n)] + p_0 D_n(p_0)\}.$$

Case 3: If $\mathrm{D_n(p_n)} \ge \mathrm{m_2 q_n^a}$ for some $\mathrm{p_n}$ and $\mathrm{D_n(p_n)} < \mathrm{m_2 q_n^a}$ for the other $\mathrm{p_n}(\mathrm{n} = 1, \dots, \mathrm{N})$, then equation (1) can be rewritten as:

$$V_n(q_n^b) = \max_{q_n^b \le q_n^a \le q_n^b+Q} \{-M\mathbf{1}[q_n^a > q_n^b] - m_1(q_n^a - q_n^b) - m_2 m_3 q_n^a + \alpha \mathrm{E}[V_{n+1}(q_n^a - b_n - \epsilon_n)] + \max_{p_l \le p_n \le p_u} \{p_n \min\{D_n(p_n), m_2 q_n^a\}\}\}.$$

In this case, we have to jointly optimize p_n and q_n^a. And according to the proof of Lemma 1, we have $p_0 \le p_n^*(q_n^a) \le p_u$.

In summary, the optimal pricing strategy is characterized by the optimal price $p_n^*(q_n^a)$, which depends on q_n^a, and $p_0 \le p_n^*(q_n^a) \le p_u$. According to Lemma 1 and Lemma 4.1 in Gallego and Scheller-Wolf[26], and similar to the proof of Theorem 1 in Chao et al.[28], it is easy to prove the structure of the optimal maintenance strategy, therefore, the subsequent proof is omitted here.☐

Theorem 1 indicates that the optimal maintenance strategy proposed in Theorem 1 can only be partially characterized. Theorem 1 shows that, when the number of operational machines before repairs (q_n^b) is fewer than $\min\{t_n' - Q, t_n\}$, the optimal number of repairs is the full capacity Q, and when q_n^b is larger than t_n', the optimal number of repairs is 0, and these two conclusions are intuitive. However, when $\min\{t_n' - Q, t_n\} \le q_n^b \le t_n'$, the optimal maintenance strategy is no longer simple: (1) When $\max\{t_n' - Q, t_n\} \le q_n^b \le t_n'$, the optimal maintenance strategy is either not to carry out repairs or to carry out repairs until q_n^a reaches t_n'. (2)When $\min\{t_n' - Q, t_n\} \le q_n^b < \max\{t_n' - Q, t_n\}$, there would be two cases, if $t_n' - Q \le t_n$, then the optimal strategy is to carry out repairs until q_n^a reaches t_n, else if $t_n' - Q > t_n$, then the optimal strategy is either not to carry out repairs or to carry out repairs by using all the maintenance capacity.

4. Numerical experiments

In this section, we present several numerical experiments in a4-period production system to investigate the effects of the fixed maintenance cost, the variable maintenance cost, and the maintenance capacity on the optimal strategy and the profit.In period n (n=1,2,3,4), we define the demand as $D_n(p_n) = 100 - p_n$, where p_n is the unit price (in dollars) of the product and $D_n(p_n)$ is the number of units that customers would purchase at the price p_n, and $\$45 \le p_n \le \$55$. Set $b_n = 5$sets, andthe probability mass function of ϵ_n is given as $\mathrm{P}\{\epsilon_n = 0\} = 0.5$ and $\mathrm{P}\{\epsilon_n = -1\} = \mathrm{P}\{\epsilon_n = 1\} = 0.25$. The salvage value is equal to $1 \times q_5^b$, where $q_5^b = q_4^a - B_4$. Other parameters are given as follows: $\alpha = 0.9$, $M = \$100$, $Q = 10$ sets, $m_1 = \$5/$ set, $m_2 = 1$ pc/set, and $m_3 = \$35/\mathrm{pc}$. Without loss of generality, we will illustrate the sensitivity analysis by using the data obtained from period 1.

4.1.Effects of the fixed maintenance cost

We first assumethe fixed maintenance cost $M \in \{50, 150, 250, 350, 450\}$, and then illustrate the optimal maintenance and pricing strategy for different M, which are shown in Figure 2-4.

Figure 2 shows that, keeping the other parameters fixed, as Mincreases,$V_1^*(q_1^b)$decreases. And in general, as q_1^b increases, $V_1^*(q_1^b)$ first increases and then decreases, however, when q_1^b in some interval, e.g., $q_1^b \in [51,55]$ given $M = \$50$, $V_1^*(q_1^b)$ fluctuates irregularly with q_1^b. The fluctuation happens because when q_1^b in such an interval, the optimal maintenance strategy is always changing.

Figure 3 shows that, as q_1^bincreases, 1) when $M = \$50$, the optimal maintenance strategy is to carry out repairs by using all the maintenance capacity (full-capacity-repair strategy) first, then part of the capacity (part-capacity-repair strategy), then the full capacity, and then part of the capacity again, and finally not to carry out repairs (no-repair strategy); 2) when $M = \$150$ and $M = \$250$, the optimal maintenance strategy is first to carry out repairs by using the full capacity, and then to repair machines by using part of the capacity, and finally not to carry out repairs; 3) when $M = \$350$ and $M = \$450$, the optimal maintenance strategy is first to carry out repairs by using all the maintenance capacity, and then not to carry out repairs. It implies that, when the value of the fixed maintenance cost (M) is small, the decision makers would like to do maintenance frequently, but when M is large, it is better to do maintenance less frequently to reduce the fixed maintenance cost, and they would like to not to carry out repairs until the number of broken machines reaches the maintenance capacity.

Because the optimal selling price depends on the optimal maintenance strategy, Figure 4 shows a similar pattern as observed from Figure 3. Figure 4 shows that, when q_1^b in the interval before the no-repair strategy area, e.g., $q_1^b \leq 42$sets given $M = \$450$, p_1^* fluctuates with q_1^b(p_1^* first equals to \$55, and then decreases to \$50); when q_1^b in the interval related to the no-repair strategy, p_1^* is non-increasing inq_1^b. In addition, $p_1^*$is not much affected by M and mainly changes depending on q_1^{a*}, from Figure 3 and 4, it is observed that, as q_1^{a*} increases, p_1^* is first equal to \$55,and then decreases to \$50, and afterwards increases to \$55 again, and finally gradually decreases to \$50. Note that $p_0 = \$50$ in this parameter setting.

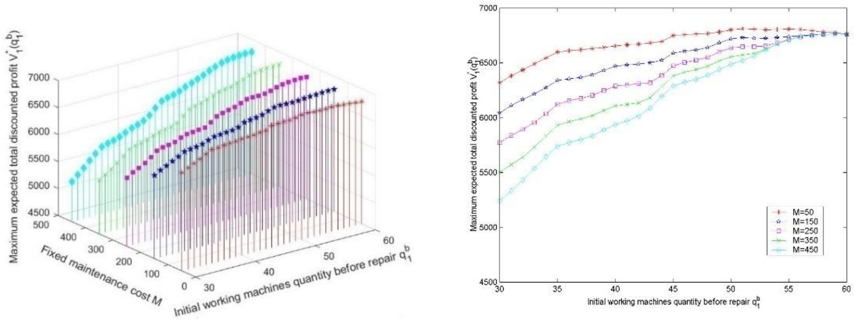

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Figure 2. $V_1^*(q_1^b)$ for different M.

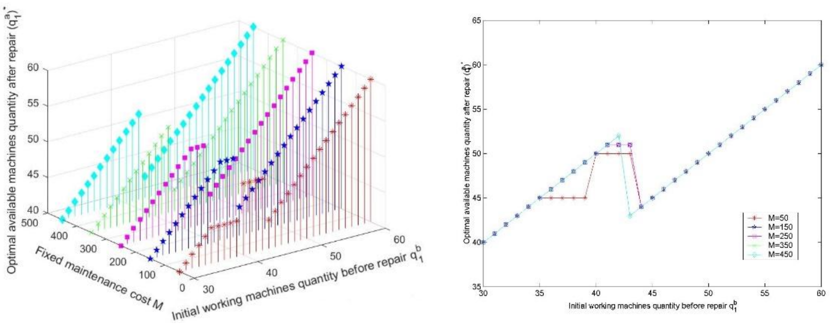

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Figure 3. q_1^{a*} for different M.

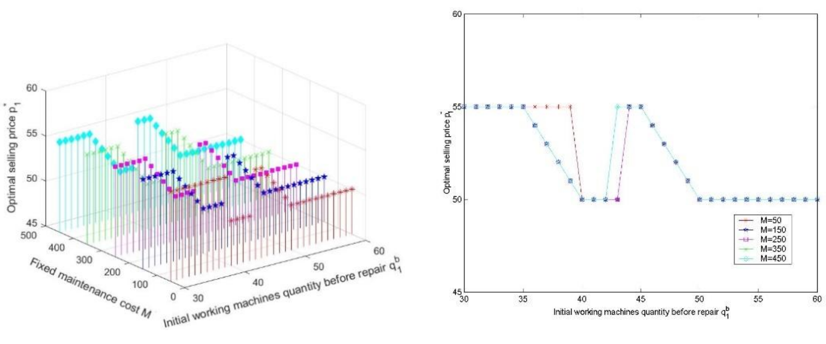

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Figure 4. p_1^* for different M.

4.2.Effects of the variable maintenance cost

We first assume the variable maintenance cost $m_1 \in \{5,10,15\}$, andthen illustrate the optimal maintenance and pricing strategy for different m_1, which are shown in Figure 5-7. Figure 5-7 show similar patterns as observed from Figure 2-4 respectively, but there's a small difference between Figure 3 and Figure 6, as well as between Figure 4 and Figure 7, i.e., when q_1^b in the interval before the no-repair strategy area, 1) if m_1 becomes larger, then the decision makers would like to repair fewer machines (see Figure 6), and the optimal selling price increases (see Figure 7); 2)if Mbecomes larger, then the decision makers would like to do maintenance less frequently (see Figure 3), and the optimal selling price decreases (see Figure 4).

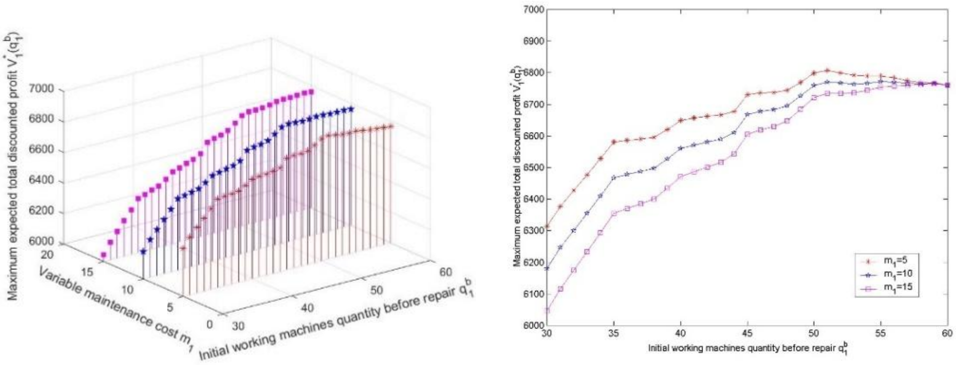

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Figure 5. $V_1^*(q_1^b)$ for different m_1.

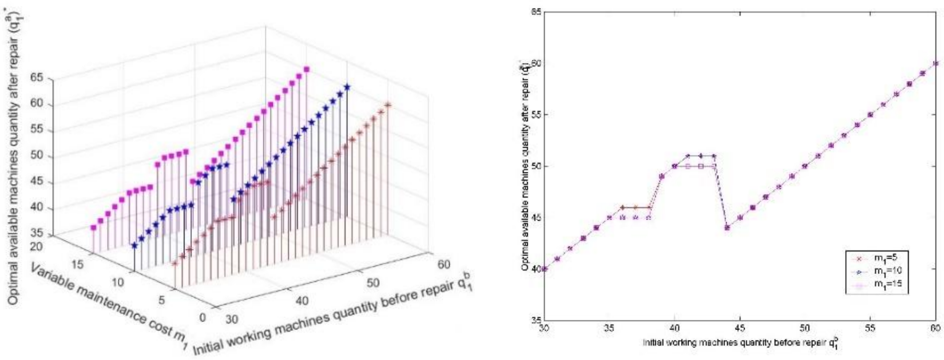

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Figure 6. q_1^{a*} for different m_1.

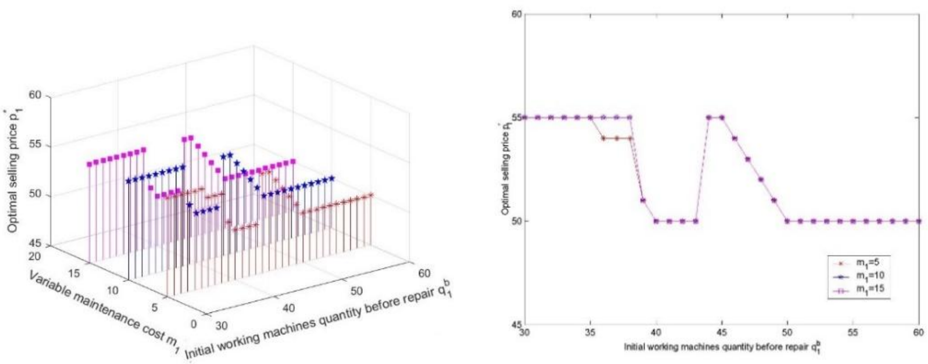

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Figure 7. p_1^* for different m_1.

4.3.Effects of the maintenance capacity

We first assume the maintenance capacity $Q \in \{8, 10, 12\}$, andthen illustrate the optimal maintenance and pricing strategy for different Q, which are shown in Figure 8-10.Figure 8 shows that, keeping the other parameters fixed, as Q increases, $V_1^*(q_1^b)$ increases. The effect of q_1^b on $V_1^*(q_1^b)$ is similar to that observed from Figure 2 or Figure 5, and we won't repeat it here.

Figure 9 shows that, as q_1^b increases, 1) when $Q = 8$sets, the optimal maintenance strategy is first to carry out repairs by using all the maintenance capacity, and then to repair machines by using part of the capacity, and afterwards to repair machines by using the full capacity again, and finally not to carry out repairs; 2) when $Q = 10$sets and $Q = 12$sets, the optimal maintenance strategy is to do maintenance by using the full capacity first, then part of the capacity, then the full capacity, and then part of the capacity again, and finally not to carry out repairs. Figure 10 shows similar pattern as observed from Figure 4, and we also observe that p_1^* is not much affected by$Q$, and mainly changes depending on q_1^{a*} from Figure 9 and Figure 10.

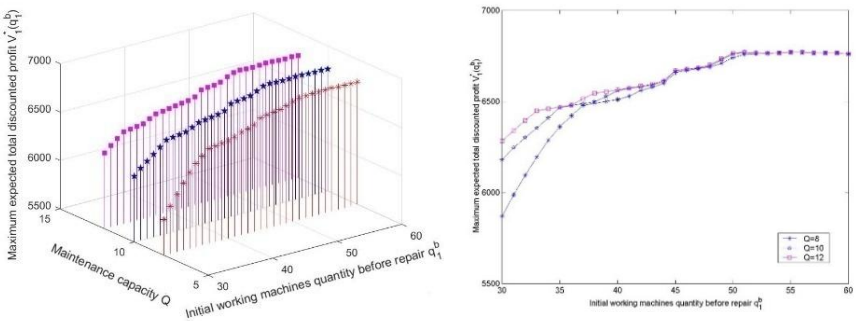

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Figure 8. $V_1^*(q_1^b)$ for different Q.

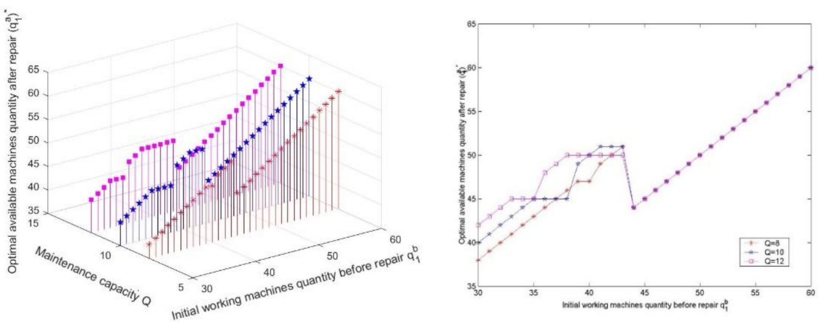

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Figure 9. q_1^{a*} for different Q.

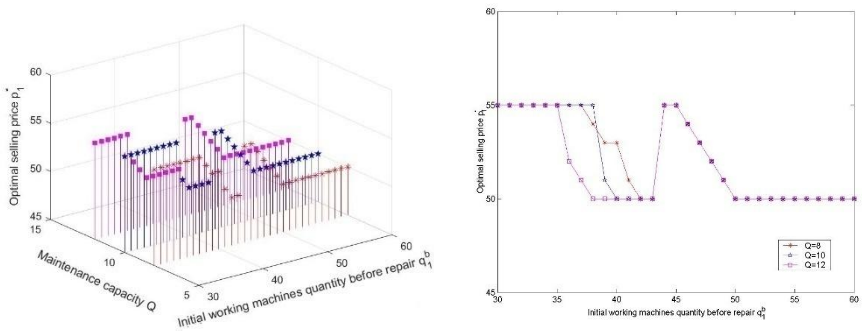

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Figure 10. p_1^* for different Q.

5. Conclusion and discussion

In this study, we investigate a periodic review production system with fixed maintenance costs and limited maintenance capacity, and develop an optimalmaintenance and pricing strategy for the system. Using the concept of strong CK-concavity, the optimal maintenance strategy for the production system can be partially characterized by t_n and t_n'. On the other hand, the optimal pricing strategy depends on the optimal number of operational machines after repairs. Numerical results show that the optimal maintenance and pricing strategy is quite robust, and is not much affected by the fixed and variable maintenance costs, as well as the maintenance capacity.

In the current increasingly competitive market environment,the optimal maintenance and pricing strategy proposed in this study can help industrial companies to better reduce maintenance costs and improve profits. This work is motivated by the author's previous experience while working withSamsung Electronics on its semiconductor wafer fabrication system. On the one hand, machines for wafer fabrication in one of Samsung's factories are maintained by a limited number of technicians, i.e., the maintenance capacity is limited. Once maintenance is initiated,it will not only incur variable maintenance costs associated withmachines, but also fixedmaintenance costs associated with maintenance technicians and maintenance equipment, i.e., there exist fixed maintenance costs. In this case, factory managers want to find the optimal maintenance strategy to minimize maintenance costs. On the other hand, the maintenance strategy affects production capacity, which in turn affects the selling price of wafers and profits of the factory, which means that the machine maintenance strategy is linked to the wafer pricing strategy. Therefore, it is of great importance for Samsung and other similar companies to establisheffective joint maintenance and pricing strategies in order to reduce maintenance costs and increase profits.

In the specific implementation process,because fixed and variable maintenance costs and the maintenance capacity vary from industry to industry,the maintenance strategy is also adjusted according to different industries. In some industries, such as the energy power industry, the fixed maintenance cost is very high compared to the variable maintenance cost, in this case, it is better to do maintenance less frequently to reduce the fixed maintenance cost, by either using the full maintenance capacity or not to carry out repairs. However, in some other industries, such as the consumer goods industry, the fixed maintenance cost is not that high compared to the variable maintenance cost, in this case, the maintenance frequency could be much higher compared to the case with high fixed maintenance costs, and it may often happen that only part of the maintenance capacity is utilized when the repair is carried out. Future research directions include considering the case when the salvage value for surplus products at the end of each period is not zero. Note that the salvage value for surplus products can be greater than zero (when the recycling price of the product is greater than the waste disposal fee), or less than zero (when the recycling price of the product is less than the waste disposal fee).

Funding

This work is supported in part by "Pioneer" and "Leading Goose" R&D Program of Zhejiang (2024C01208), NSFC (Grant 72271221, 71720107003), and Shanghai Educational Science Research Project (No.C2022406).

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