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Global-dynamic maintenance management of multi-component degrading plants with non-immediate replacement: a self-adaptive grouping approach

Shihan Zhou^a , Xiaobing Ma^a , Jingjing Wangb,* , Yi Chen^a , Li Yanga,*

^a School of Reliability and Systems Engineering, Beihang University, China ^b School of Management Engineering, Qingdao University of Technology, China

Highlights Abstract

- A global adaptive group maintenance strategy oriented to degrading systems is proposed.
- Maintenance is delayed to balance failure risk mitigation and resource preparation.
- A global dynamic union of group PM and OM is realized within an infinite time horizon.
- A heuristic reverse group search algorithm is devised to improve optimization efficiency.

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1. Introduction

As an important carrier of digital technology in the field of system health management, intelligent maintenance powered by autonomous decision-making plays an important role in reducing maintenance costs, improving service reliability, and enhancing comprehensive service efficiency of diverse industrial plants [1-4]. On the one hand, many failures stem from inadequate maintenance management, so that modern maintenance must transcend the traditional 'no damage, no repair' framework [5]. On the other hand, with the gradual

Group maintenance management is pivotal to ensure operational safety and performance of multi-component plants attributed to its capacity to share maintenance resources/time. Most group maintenance models, however, are globally/partially static following pre-specified maintenance sequences, with limited focus on the adaptability of group partition procedure. To fill this gap, we devise an innovative globaldynamic condition-based group maintenance policy. In contrast to existing methods, it allows for (a) postponement of component maintenance upon inspection to facilitate flexible resource allocation, and (b) automatic refinement of group maintenance structures to promote adaptivity. The proposed policy is shown to establish a global renewal mechanism for maintenance group partition over an infinite time horizon, which constitutes a dynamic union of both scheduled maintenance and opportunistic maintenance to mitigate downtime. A heuristic grouping algorithm is developed to realize efficient maintenance group planning, which verifies model effectiveness via numerical experiments.

Keywords

intelligent asset management, group maintenance, replacement scheduling, cost-effectiveness, dynamic decision-making.

> development of industrial systems such as high-speed rail, aircraft engines, wind farms, and inertial navigation towards scale and complexity, expenses involved in rental, development, procurement and maintenance are constantly increasing. In particular, the maintenance cost accounts for up to 40-45% of the entire lifecycle cost for complex equipment such as aircraft and wind turbines [6-10]. Therefore, enhancing the scientific rigor and intelligence of maintenance strategies holds practical importance in ensuring the reliability and performance of

(*) Co-corresponding author.

E-mail addresses: S. Zhou (ORCID: 0009-0007-3374-2851) zsh1220@buaa.edu.cn, X. Ma (ORCID: 0000-0002-0913-9012) maxiaobing@buaa.edu.cn, J. Wang (ORCID: 0000-0002-6617-5204) wangjingjing@qut.edu.cn, Y. Chen (ORCID: 0000-0002-0934-0787) chenyi1118@buaa.edu.cn, L. Yang (ORCID: 0000-0002-4738-0210) yanglirass@buaa.edu.cn.

diverse complex industrial systems.

Maintenance scheduling of multi-component systems is usually affected by resource sharing, structural correlation, or fault interaction between components [11-17]. Therefore, it is not practical to simply stack individual maintenance to form group maintenance. For example, the maintenance operations of offshore wind farms incur significant costs in personnel transportation, material preparation, system shutdown, dismantling, and other activities [18,19]. Likewise, maintenance works of railway trains require first waiting for train dispatch, vehicle cleaning, and structural dismantling, resulting in certain time delays and shutdown costs [20,21]. Such costs can be shared in a reasonable combination of maintenance activities with significant cost-effectiveness improvement, which emphasizes the importance of group maintenance planning [22].

Substantially, scheduled group maintenance and opportunistic maintenance are two typical maintenance strategies capturing economic relevance [23-26]. The former combines multiple preventive maintenance tasks together and executes them based on calendar time. The latter utilizes downtime window to execute extra maintenance activities, including shutdown due to preventive maintenance [27,28], failure maintenance [29-31], and environmental factors or production planning [32]. A common decision criterion is operational age or reliability [33,34], which converts health condition into reliability indicators to promote execution. However, such methods confront with the challenges when conducting analytical group structure, and the resolution velocity relies on the group size. Under a large group size, quantitative analysis is intractable by analytical model and can only be achieved through simulation methods. Another traditional group maintenance strategy is the block maintenance [35], according to which maintenance intervals of all components is set to a multiple of a certain basic interval, which can be seen as an extension of periodic maintenance. Although facilitating resource scheduling, such approach faces the challenges of adaptability to health condition variations or environmental disturbances, since most work are limited to static framework. More general preventive maintenance combination strategies are required to achieve global optimization. Martinod et al. [36] employed the clustering center as the maintenance execution time for similar components. Zheng et al. [37] proposed a multi-level PM decision-making approach, establishing a group maintenance strategy for multi heterogeneous NC machine tools using actual fault data. Ma et al. [38] proposed a maintenance policy for a two-unit warm standby cooling system via the joint optimization of temperature control limits and age thresholds. Park and Pham [39] proposed a fault delay replacement strategy for redundant systems, which replaces components uniformly after a certain number of accumulated failures. Moghaddam et al. [40] divided maintenance plans into discrete cycles for a repairable system with increasing failure rates, and comprehensively used dynamic programming and branch-andbound methods for solution.

The multi-stage rolling horizon approach (RHA) proposed by Wildeman et al. [41] is a good candidate for interpretable group maintenance optimization by exploring the structure of cost savings due to resource sharing. Its core idea is to organically combine maintenance activities based on individual maintenance plan [42], establishing an analytical functional relationship between component-level and system-level maintenance. This method realizes iterations of long-term maintenance plans by performing grouping operations within a given range and moving to subsequent windows to repeat operations until the planning period expires. Destombes et al. [43] explored a group maintenance strategy for a k-out-of-n installation base, and analyzed the impact of resource sharing on system availability and maintenance capabilities. Lu et al. [44] developed a cost-based maintenance operation decisionmaking approach based on quality and reliability evaluation. Nguyen et al. [45] proposed a grouping maintenance optimization strategy for offshore wind farms, taking into account factors such as weather conditions and equipment location, to determine the grouping maintenance plan. Bouvard et al. [46] and Van Horenbeek [47] extended this approach to degrading systems and proposed predictive maintenance strategies.

Despite the effectiveness of RHA in adapting to system state variation and improving maintenance interpretation, its application to complex degrading industrial systems confronts with a few challenges. First, RHA essentially belongs to static decision-making approach, although the scheduling horizon rolls iteratively. The correlation between adjacent maintenance activities is not considered, so that adaptively updating system plans is difficult. For example, when there is a time change in the current maintenance plan, subsequent plans are not able to be automatically modified, either advanced or delayed. Such ignorance of adjustments may increase the risk of failure and reduce economic benefits. Second, RHA is limited to iterating within a specified time span, confirming its regional statics, which increases the difficulties of maintenance planning for systems with long service periods. Third, the impact of random failures is difficult to estimate, as the economic dependence of preparation costs during failures, as well as economic benefits brought by maintenance opportunities are seldomly considered, limiting the scope for enhancing cost efficiency in complex systems.

To address the foregoing research gaps, this paper innovatively introduces a globally dynamic group maintenance approach for multi-component degrading systems with selfadaption mechanism, which serves as a dynamic union of (a) delayed condition-based maintenance and (b) immediate opportunity maintenance. The most prominent advantage of this method is its *global-dynamic self-adaption updating ability*, which is free of time horizon limitation presumed in most group maintenance models. This real-time updating feature plays a pivotal role in cost containment and availability enhancement, as it enables the swift adjustment of component health status and maintenance plans, thereby enhancing the timeliness, agility, and precision of system-level maintenance. To fulfill this capacity, the first group is selected and the remaining groups are discarded at every grouping decision, so as to ensure policy flexibility and reduce redundancy. **Second,** unlike previous group maintenance models, we consider *delayed maintenance during worn-out stages* of system polymorphism degradation. As such, the matched maintenance and support resources can be fully prepared between the state identification and actual maintenance execution, contributing to (a) reducing downtime losses due to immediate maintenance, and (b) exploring the potentials of remaining life. **Third**, this is the first to formulate a global dynamic union of both postponed predictive replacement and unscheduled opportunistic replacement, so as to sufficiently improve downtime utilization capacity, in particular from unexpected failures. **Finally**, an efficient

heuristic algorithm is developed to achieve analysis of group partition dynamic programming, which is effective to reduce computational complexity and alleviate analytical dimension explosion due to system scale. The effectiveness of the proposed maintenance framework is verified by comparative numerical experiment conducted on train bogie.

To sum up, this study contributes to group maintenance planning of complex industrial systems from the following four perspectives:

- A globally dynamic group maintenance strategy oriented to degrading systems is proposed for the first time, which automatically adjust maintenance plans based on selfadapting updating of both component health estimation and maintenance plans, so as to capture the correlation between adjacent maintenance plans, enhancing the agility and precision of decision-making;
- Preventive maintenance is allowed during the worn-out stage to be delayed, which realizes a balance between failure risk mitigation and maintenance resource preparation, and provides sufficient flexibility for group maintenance partition;
- Delayed group maintenance and immediate opportunity maintenance are firstly integrated in a global dynamic manner, which fully captures both positive and negative effect of unexpected failures and their economic dependencies to improve downtime loss control;
- An efficient heuristic dynamic programming and reverse search algorithm is developed for sequentially updating maintenance groups and execution time. It effectively reduces model complexity and improves optimization efficiency through automatic dimensional reduction mechanism.

The rest of the paper is organized as follows. Section 2 introduces the innovative group maintenance policy. Section 3 constructs component-level condition-based maintenance model. Section 4 formulates the group maintenance model. A high-speed rail bogie is taken as an example to demonstrate the applicability in Section 5. Section 6 concludes the paper.

Acronyms and notations

2. Global Dynamic Group Maintenance Policy

We innovatively devise a global dynamic group maintenance policy oriented to a multi-component system subject to continuous degradation. As shown in Fig. 1, the policy enables a global dynamic union of (a) scheduled group maintenance and (b) unscheduled opportunistic maintenance within an infinite time horizon, through devising the adaptive updating mechanism upon inspection and group maintenance. In other words, the maintenance clustering process is always selfadaptive following the latest component-level health information and maintenance optimization outcome, iterating without time limitation.

- **(1) Inspection-driven adaptive updating.** Inspections are equally spaced to reveal the underlying component degradation, whose health are assessed adaptively once acquiring the latest degradation observation;
- **(2) Maintenance-empowered adaptive updating.** Whenever a group maintenance is completed, the health status as well as individual CBM plans of all components are updated immediately. Accordingly, subsequent maintenance group sequence is re-scheduled. *In other words, only the first group of each group set is chosen for implementation, while the remaining ones are abandoned after the implementation***.** This process continuously iterates in infinite time horizon.

Figure 1 illustrates the realization process of global-dynamic group maintenance procedure following tentative scheduling of component maintenance plans. Subsequent to accessing the most recent system inspection, a subset of components potentially necessitating maintenance is selected to constitute a tentative *delayed preventive group replacement (DPGR)* group, which contains component experiencing delayed replacement for life extension purpose. This is an ideal situation as the unpredictability and abruptness of system malfunctions prompt an immediate conversion from a DPGR plan to the *immediate*

opportunistic replacement (IOR) plan. To ensure real-time decision-making and strategy adaptability, the updated status upon the time of each (a) maintenance decision, (b) DPGR execution, and (c) IOR execution are documented to offer feedback to the subsequent inspection epoch for comparison. In the absence of system status alterations, the original maintenance plan will be executed as planned; conversely, any modifications will trigger a renewal of the plan. In this regard, the effectiveness of maintenance postponement and the promptness of opportunistic maintenance can be ensured.

Figure 1. Flowchart of the proposed strategy.

Through such self-adaption updating mechanism, the proposed maintenance policy possesses owes the superiorities of globality, dynamicity and cost-effectiveness, as outlined below.

- a) **Globality**: The maintenance sequence updating process is no longer limited to pre-set time interval, which can be extended to infinite time horizon;
- b) **Dynamicity**: Component conditions are dynamically updated through both (a) health inspection and (b) group maintenance execution;
- c) **Cost-effectiveness**: The proposed framework realizes

the global dynamic union of scheduled maintenance and unscheduled maintenance, so as to sufficiently share maintenance resources and control maintenance downtime.

2.1. Component-Level Postponed Maintenance Planning

A degradation-centered postponed replacement policy is employed at the component level, such that to set a benchmark for system group maintenance. To be specific, inspections are equally spaced with an interval τ to reflect the underlying degradation of each component. If the degradation of

a component $i, i = 1, 2, \dots, n$ at an inspection exceeds a control limit ξ_i , a preventive replacement (PR) is scheduled $j_i \tau$ time units later; otherwise if the degradation exceeds the failure threshold $L_i(L_i > \xi_i)$, corrective replacement (CR) is immediate.

The reason to postpone replacement is two-fold: (a) allowing more sufficient time for abundant resource preparation compared to immediate replacement; and (b) offering windows for opportunistic maintenance, which facilitates the scheduling and updating of group maintenance, as we will show in the rest of this paper.

2.2. System-level Dynamic Group Scheduling

Following the component-level maintenance optimization outcomes, the global dynamic group maintenance policy is devised. Within an infinite time horizon, all component information are iteratively updated upon the completion of each group maintenance, whereas the subsequent group sequence are rescheduled following the updated information.

To be specific, multiple components with similar time are clustered into a maintenance group to minimize maintenance costs. Note that the triggering of maintenance relies on the variation of system status indicators. Specifically, we categorize the state set of a component into three types $\{0,1,2\}$. State 0 indicates that the component is in a normal state, state 1 indicates that it has been detected as exceeding ξ_i , and state 2 indicates that it has been detected as exceeding L_i . If the state of any a component changes, the new group maintenance planning is initiated immediately; while other the original plan remains unchanged.

Recall that the proposed maintenance policy is a dynamic union of two types of group maintenance: *delayed preventive group replacement (DPGR)* and *immediate opportunistic replacement (IOR)*. In the following, we specify these two replacement types.

◼ **Delayed preventive group replacement (DPGR) planning**

As shown in Fig. 2, at each group decision point, the DPGR determines: a) the size of the maintenance group and b) the execution time. To ensure the flexibility and timeliness of the proposed strategy, only the first group is chosen to be implemented. After a replacement is completed, we update the status information of the components. Notably, if a component failure is detected at any inspection prior to the planned maintenance point, the current DPGR plan is abandoned immediately and transferred to the IOR plan.

◼ **Immediate opportunistic replacement (IOR) planning**

As shown in Fig. 3, IOR grouping is immediately implemented when a component is found failed. Selective preventive replacement is executed on surviving components while corrective replacement of the failed components is carried out. Similar to DPGR planning, the status information is updated after IOR is completed. Subsequently, the aforementioned DPGR method is employed to plan the subsequent DPGR groups, facilitating the joint optimization of IOR and DPGR. Therefore, at each IOR decision point, a) the size of the current IOR group and b) the size and execution time of the next DPGR maintenance group are determined.

Figure 3. Illustration of the opportunity replacement grouping.

3. Component-Level Delayed Maintenance Optimization

In this section, we characterize the degradation behaviors of system components, and formulate the maintenance optimization model at the component level. The optimization outcome serves as a benchmark for subsequent group maintenance planning.

3.1. Degradation Process Modelling

Define the degradation of component $i, i = 1,2,3,\dots, n$ as $X_i(t)$, which leads to failure if the degradation attains a specific threshold L_i . We characterize the underlying degradation process via a generalized stochastic process with Brownian motion process error, as structured by $X_i(t) = X_i(0) +$ $v_i \Lambda(t; \beta_i) + \sigma_i B(\Lambda(t; \beta_i)),$ which can effectively explain the degradation characteristics of nonlinear and non-monotonic of practical systems [48,49]. Here $v_i A(t; \beta_i)$ is the drift function; $\Lambda(t;\beta_i)$) is the space-time scale transform function; $B(t) \sim N(0,t)$ is the standard Brownian motion. The advantage of this model lies in that: (a) it scales well in fitting non-linear trajectory due to the adaptively adjustable parameter β_i ; and (b) the remaining useful life model can be analytically calculated. Following the statistical independence property, the degradation increment $X_i(t + \Delta t) - X_i(t)$ yields

$$
g_i(x,\Delta t) = \frac{1}{\sigma_i \sqrt{2\pi \zeta(\Delta t)}} e^{-\frac{1}{2} \left(\frac{x - \nu_i \zeta(\Delta t)}{\sigma_i \sqrt{\zeta(\Delta t)}}\right)^2},
$$
(1)

where $g_i(x, \Delta t)$ represents the probability density function of the degradation increment over time Δt , and $\zeta(\Delta t) = \Lambda(t +$ Δt ; β_i) – $\Lambda(t; \beta_i)$. Since the time to hit the failure threshold L_i for the first time follows the Inverse Gaussian distribution, the remaining life l given the latest observation x_0 follows

$$
f_i(l|x_0) = \frac{L_i - x_0}{\sigma_i \sqrt{2\pi \zeta(l)^3}} e^{-\frac{\left(\nu \zeta(l) - L_i - x_0\right)^2}{2\sigma^2 \zeta(l)}} \frac{d\zeta(l)}{dl}, l > 0.
$$
 (2)

Prior to the maintenance modeling, we define some basic settings. Inspections are non-destructive and perfect, whose execution time is negligible compared to maintenance intervals. Each inspection incurs a cost $C_{i,l}$. In this study, the focus is solely on replacement actions, and we interchangeably use the terms maintenance and replacement throughout the remainder of the study. As for spare replacement actions, both the shareable set-up maintenance cost $C_{s,R}$ (including personnel scheduling, material scheduling, and other support costs) and the independent component maintenance cost $C_{i,R}$ are involved which yields $C_{i,R} \gg C_{i,I}$. In addition, corrective replacement brings additional losses, including: downtime losses per unit time $C_{i,d}$ resulted from failure concealment; and $C_{i,f}$ caused by the untimely logistics support.

3.2. Maintenance Interval Optimization

Now we begin the component-level maintenance modeling. Remember that, when the degradation level of the component i is detected in (ξ_i, L_i) , preventive replacement is carried out after a delayed time $j_i \tau$. When, however, the degradation is found to exceeds L_i , corrective replacement is immediate. Accordingly, the density function of the average detections' frequency k between two preventive replacements is

$$
p_i^{PR}(k) = P(N_i^{PR} = k) =
$$

\n
$$
\begin{cases}\n0 & k \le j_i + 1, \\
\int_{\xi_i}^{L_i} \int_0^{L_i - x} g_i^{j_i}(u) \cdot g_i(x) \, du dx & k = j_i + 2, \\
\int_0^{\xi_i} \int_{\xi_i - x}^{L_i - x} \int_0^{L_i - x - u} g_i^{j_i}(s) g_i(u) g_i^{k - j_i - 2}(x) \, ds du dx & k \ge j_i + 3.\n\end{cases}
$$
\n(3)

where $g_i^q(x) = g_i(x, q\tau)$ represents the probability density function of incremental degradation over time $q\tau$. Specially, when $q = 1$, $g_i(x) = g_i(x, \tau)$ yields.

We make some interpretations about Eq. (3). When $k \leq j_i$ + 1, this situation was not allowed because the initial degradation level was zero; Both two subcases in $k \ge j_i + 2$ indicate that

the component was found to be exceeding the control limit, and still remained in this state even after a delay time $j_i \tau$. Only through this approach can the initiation of PR planning be facilitated. The difference between these two subcases lies in the historical detection times prior to attaining the control limit, as shown in Fig. 4 (a).

Similarly, the probability density function of the average detections' frequency k between two corrective replacements can be determined as

$$
p_i^{CR}(k) = P(N_i^{CR} = k) = \begin{cases} 0 & k = 1, \\ \bar{G}_i(L_i) & k = 2, \\ \sum_{m=1}^{j_i} \sum_{r=1}^{+\infty} \int_0^{\xi_i} \int_{\xi_i - x}^{L_i - x} \int_0^{L_i - x - u} \int_{L_i - x - u - s}^{+\infty} g_i(v) \cdot g_i^{j_i - m - 1}(s) \cdot g_i(u) \cdot g_i^{n - 1}(x) \, dvdsdu dx & k \ge 3, \\ & + \int_0^{\xi_i} g_i^{k - 2}(x) \bar{G}_i(L_i - x) dx & \end{cases}
$$
(4)

where $G_i(x) = \int_0^x g_i(u, \tau) du$ represents the distribution function of the degradation increment over τ . Accordingly, if a component is found to be faulty at the next detection point after the time starting or replacement ended point, $k = 2$ is met. There are two subcases when $k \geq 3$. The former scenario signifies that the previous detection was normal, but a sudden failure occurred subsequently, as shown in the above of Fig. 4 (a). While the latter represents the original PR plan has encountered failure, as shown in the below of Fig. 4 (b).

The average downtime T_d caused by failure concealment is obtained from the following equation. Notably, k cannot increase to $+\infty$ because (a) the actual number of state detections

is limited, and (b) it is meaningless to calculate the downtime once $k\tau$ severely exceed the actual failure detection points

$$
T_d = E(t_d) = \sum_{k=1}^{+\infty} \left[\int_{(k-1)\tau}^{k\tau} (k\tau - t) f(t|0) \, dt \cdot \left(1 - \int_0^{(k-1)\tau} f(t|0) \, dt \right) \right] . \tag{5}
$$

According to the renewal-reward theory, the long-term maintenance cost rate η_i is defined as

$$
\eta_i(j_i) = \frac{c_{i,i} \sum_{k=1}^{+\infty} k \cdot (p_i^{PR}(k) + p_i^{CR}(k)) + c_{i,R} + (c_{i,f} + c_{i,d} \cdot T_d) \cdot \sum_{k=1}^{+\infty} p_i^{CR}(k)}{\sum_{k=1}^{+\infty} (k-1) \cdot \tau \cdot (p_i^{PR}(k) + p_i^{CR}(k))}.
$$
 (6)

The essence of our proposed maintenance strategy hinges on effectively partitioning the maintenance activities of components into suitable maintenance groups to distribute setup costs $C_{s,R}$. In the following, we investigate the group maintenance modeling approach that sufficiently share set-up cost and downtime.

4. Dynamic Group Maintenance Scheduling

This section focuses on the global dynamic scheduling of the entire multi-component system, following the component-level maintenance optimization outcome. The dynamic predictive group maintenance model, as well as the dynamic opportunistic maintenance model are formulated separately, and a heuristic grouping algorithm is developed for sequential and efficient maintenance group partition.

4.1. Penalty Modelling

Despite the importance of sharing resources/downtime, group maintenance may cause the synchronous maintenance time to deviate from its individual optimal execution time. We define a cost penalty function to characterize such loss, which is further partitioned into: (a) penalty by advancing maintenance, and (b) penalty by postponing maintenance.

(a) advancing situation (b) postponing situation

Figure 5. Cost penalty scenarios when the maintenance time is deviated.

(1) Penalty by advancing maintenance

The deviation of maintenance time may lead to an extension (or reduction) of the average service life, an avoidance (or saving) of failure downtime, and urgent needs for logistics support. Correspondingly, it results in loss functions on cost parameter dimensions η_i^* , $C_{i,d}$, $C_{i,f}$. Due to the concealment and uncertainty of failure, the penalty function can be solved in three subcases under the premise of advancing maintenance, as shown in the Fig. 5 (a). Let the original delayed maintenance time be $j_i \tau$, the offset maintenance time be $k_i \tau(k_i \leq j_i)$. The failure time variable is set as S_i .

a1). In this case, $s_i < k_i \tau$. Since corrective replacement is immediate upon failure identification, only CR will be performed, which incurs no extra loss Therefore, the loss function yields

$$
CoP_{a1} = 0.\t\t(7)
$$

a2). In this case, $k_i \tau \leq s_i \leq j_i \tau$. The advancing of maintenance execution time avoids possible additional downtime cost $C_{i,d} \cdot t_d$ and untimely logistics support cost $C_{i,f}$, but it also leads to a shortened service lifetime, whose losses are manifested by η_i^* . The penalty function is expressed as

$$
CoP_{a2} = -C_{i,d} \cdot \int_{k_i\tau}^{j_i\tau} f(s) \cdot (\left[\frac{s}{\tau}\right] \tau - s) ds
$$

\n
$$
-C_{i,f} \cdot [F(j_i\tau) - F(k_i\tau)]
$$

\n
$$
+ \int_{k_i\tau}^{j_i\tau} \eta_i^* \cdot (\left[\frac{s}{\tau}\right] - k_i) \cdot \tau \cdot f(s) ds.
$$
 (8)

a3). In this case, $j_i \tau \leq s_i$. It indicates that failure occurrence will not affect the deviation of component's maintenance task as it belongs to a future event after maintenance execution. Therefore, only the punishment of shortening service lifetime should be introduced. Consequently, the penalty function is

$$
CoP_{a3} = \eta_i^* \cdot (j_i - k_i) \cdot \tau \cdot R(j_i \tau). \tag{9}
$$

To conclude, the penalty function in advancing maintenance case can be summarized as

$$
CoP_a = CoP_{a1} + CoP_{a2} + CoP_{a3}
$$

\n
$$
= -C_{i,d} \cdot \int_{k_i\tau}^{j_i\tau} f(s) \cdot (\left[\frac{s}{\tau}\right] \tau - s) ds - C_{i,f} \cdot [F(j_i\tau) - F(k_i\tau)]
$$

\n
$$
+ \int_{k_i\tau}^{j_i\tau} \eta_i^* \cdot (\left[\frac{s}{\tau}\right] - k_i) \cdot \tau \cdot f(s) ds + \eta_i^* \cdot (j_i - k_i) \cdot \tau \cdot R(j_i\tau)
$$

\n
$$
= \int_{k_i\tau}^{j_i\tau} f(s) \cdot \left[-C_{i,d} \cdot (\left[\frac{s}{\tau}\right] \tau - s) + \eta_i^* \cdot (\left[\frac{s}{\tau}\right] - k_i) \cdot \tau\right] ds
$$

\n
$$
-C_{i,f} \cdot [F(j_i\tau) - F(k_i\tau)] + \eta_i^* \cdot (j_i - k_i) \cdot \tau \cdot R(j_i\tau). \tag{10}
$$

(2). Penalty by postponing maintenance

The postponing maintenance case is also partitioned into three subcases, as shown in Fig. 5 (b). Let the originally scheduled time be $j_i \tau$, the offset maintenance time be $k_i \tau(k_i >$ j_i).

b1). In this case, $s_i < j_i \tau$. The failure occurrence pulls the delayed maintenance back to the original scheduled time without any losses. Therefore,

$$
CoP_{b1} = 0.\t\t(11)
$$

b2). In this case, $j_i \tau \leq s_i \leq k_i \tau$. Although postponement of maintenance execution extends the service life (also manifested by η_i^*), it inevitably leads to downtime cost $C_{i,d} \cdot t_d$ and untimely logistics support cost $C_{i,f}$. Hence, the penalty function

$$
CoP_b = CoP_{b1} + CoP_{b2} + CoP_{b3}
$$

is expressed as

$$
CoP_{b2} = C_{i,f} \cdot [F(k_i\tau) - F(j_i\tau)]
$$

+
$$
C_{i,d} \cdot \int_{j_i\tau}^{k_i\tau} f(s) \cdot (\left|\frac{s}{\tau}\right| \tau - s) ds
$$

-
$$
\int_{j_i\tau}^{k_i\tau} \eta_i^* \cdot (\left|\frac{s}{\tau}\right| - j_i) \cdot \tau \cdot f(s) ds.
$$
 (12)

b3). In this case, $k_i \tau \leq s_i$. The failure occurrence has no impact on the scheduled maintenance work, but indirectly extends the service life. Thus

$$
CoP_{b3} = -\eta_i^* \cdot (k_i - j_i) \cdot \tau \cdot R(k_i \tau). \tag{13}
$$

To sum up, the penalty function for postponed maintenance is

$$
= \int_{j_i\tau}^{k_i\tau} f(s) \cdot \left[C_{i,d} \cdot \left(\left[\frac{s}{\tau} \right] \tau - s \right) - \eta_i^* \cdot \left(\left[\frac{s}{\tau} \right] - j_i \right) \cdot \tau \right] ds + C_{i,f} \cdot \left[F(k_i\tau) - F(j_i\tau) \right] - \eta_i^* \cdot (k_i - j_i) \cdot \tau \cdot R(k_i\tau). \tag{14}
$$

It can be easily found that the mentioned penalty functions have a similar form, because the two cases are time-reversal forms of each other both mathematically and physically. We

therefore integrate Equations (10) and (14) into a unified penalty function

time deviation, a major advantage of combining multicomponent maintenance activities is that maintenance costs $C_{S,R}$ can be shared. We define the maintenance gain function as the difference between the saved and penalty cost, to quantitatively analyze the resource savings by grouping. For a certain

maintenance group $|G|$, there is a gain function $C(G)$:

$$
CoP_i(k_i|\tau_i) = sgn(j_i - k_i) \cdot \begin{cases} \int_{\min\{k_i\tau, j_i\tau\}}^{\max\{k_i\tau, j_i\tau\}} f(s) \cdot \left[-C_{i,d} \cdot \left(\left[\frac{s}{\tau} \right] \tau - s \right) + \eta_i^* \cdot \left(\left[\frac{s}{\tau} \right] - \min\{k_i, j_i\} \right) \cdot \tau \right] ds \\ + \eta_i^* \cdot |j_i - k_i| \cdot \tau \cdot R(\max\{k_i\tau, j_i\tau\}) - C_{i,f} \cdot |F(k_i\tau) - F(j_i\tau)| \end{cases} \tag{15}
$$

where $sgn(j_i - k_i)$ is a sign function defined as

$$
sgn(j_i - k_i) = \begin{cases} -1 & j_i < k_i, \\ 0 & j_i = k_i, \\ +1 & j_i > k_i. \end{cases}
$$

4.2. DPGR Scheduling in CBM Framework

Setting aside the negative impact caused by the aforementioned

$$
C(G) = (|G| - 1) \cdot C_{S,R} - \sum_{i \in G} CoP_i(k_i | \tau_i) = (|G| - 1) \cdot C_{S,R} - \sum_{i \in G} sgn(j_i - k_i) \cdot \begin{cases} \n\max\{k_i \tau, j_i \tau\} f(s) \cdot \left[-C_{i,d} \cdot \left(\left[\frac{s}{\tau} \right] \tau - s \right) + \eta_i^* \cdot \left(\left[\frac{s}{\tau} \right] - \min\{k_i, j_i \} \right) \cdot \tau \right] ds \right) \\
+ \eta_i^* \cdot \left[j_i - k_i \right] \cdot \tau \cdot R(\max\{k_i \tau, j_i \tau\}) - C_{i,f} \cdot |F(k_i \tau) - F(j_i \tau)| \end{cases} \tag{16}
$$

The size and execution time of the maintenance group should be optimized, so as to obtain the maximum $C(G)$ within given range

$$
\{G^*, k_i^*\} = argmax C(G)
$$

 $= argmax[(|G| - 1) \cdot C_{S,R} - \sum_{i \in G} CoP_i(k_i|\tau_i)].$ (17)

Dynamic programming is a typical optimization algorithm for solving multi-stage decision problems, which has been widely applied in production scheduling and resource allocation. Within the hierarchical framework of dynamic programming,

boundary conditions are used as the start of the algorithm, and the optimal solution is sought step by step through recursion. This effect makes the entire strategy generation constantly changing based on state. In our framework, the status is updated after maintenance or detection, and each maintenance grouping structure relies on the previous one, continuously rolling and iterating throughout the lifespan. Therefore, our structure is similar to the issues that dynamic programming can solve.

We relax the maintenance optimization problem to the

global-level and further utilize the 'continuous grouping' assumption to efficiently approach the optimal solution. However, due to (a) the correlation between successive groups; and (b) the updated system information, we only take the first set of solutions from the optimal sequence. This global optimization method actually trades efficiency for partial optimality. Here, GL is defined as all components which have been detected to have exceeded ξ_i and are not assigned to any current maintenance group. These components are divided into mutually exclusive groups $GL = \{G_1, G_2, \dots, G_m\}$, in which $\bigcup_{i \in m} G_i = G$, $\forall i \neq j \in G$, $G_i \cap G_j = \emptyset$. Hence, the optimal group structure should be obtained by

$$
\{k^*, GS^*\} = argmax C(GL) = argmax \left\{ \sum_{G_j \in GL} (|G_j| - 1) \right\}
$$

$$
C_{S,R} - \sum_{i \in G_j} CoP_i(k_j|\tau_i) \big\}.
$$
 (18)

To be specific, let $\{j_1^*, j_2^*, j_3^*, \cdots, j_n^*\}$ be the optimal independent maintenance time at one detection point, which is arranged in ascending order, corresponding to the component index $\{i_1, i_2, i_3, \dots, i_n\}$. (If $j_i^* = j_j^*$, the component with higher sensitive penalty function should be arranged ahead). When better-performed components are included in maintenance while the worse ones are still in operation, it will inevitably lead to a more negative penalty but have no impact on saving $C_{S,R}$.

Figure 6. Flowchart for solving the optimal maintenance structure.

To clarify the maintenance group process, let s_p denote the beginning state at stage p, indicating the group size where i_{p-1} was in. $x_p = 0$ means that i_p starts a new group at stage $p. x_p =$ 1 means i_p joins the current group. Correspondingly, the state transition equation and the introduced cost saving function are decided by Eq. (19), and the Bellman Equation is determined by Eq. (20). We obtain the maintenance group structure in reverse order. Notably, only the first group should be taken as the optimal maintenance group to ensure the dynamism of the strategy.

$$
\begin{cases}\nS_{p+1} = 1 \text{ and } v_p(S_p, x_p) = 0, & x_p = 0, \\
S_{p+1} = S_p + 1 \text{ and } v_p(S_p, x_p), & x_p = 1.\n\end{cases}
$$
\n(19)

$$
\begin{cases}\nf_p(s_p) = \max_{x_p \in D_p(s_p)} \{v_p(s_p, x_p) + f_{p+1}(s_{p+1})\}, p = n, n-1, \dots, 2, \\
f_{n+1}(s_{n+1}) = 0.\n\end{cases}
$$
\n(20)

Through the aforementioned steps, we effectively transform the grouping problem into a multi-stage decision-making

framework, where the sequential decision-making process involves determining whether to incorporate components into existing groups. This process is illustrated in the backward Dynamic Programming Algorithm (DPA) depicted in Fig. 6.

4.3. IOR Scheduling in CBM Framework

Recalling that IOR is immediate corrective replacement on failed components and preventive replacement on survival parts. Here, the optimized range has no need to relax to the same as DPGR (components have been detected to exceed ξ_i). Assuming that component h is detected to have failed before τ_h , it will be certainly removed from the "continuous grouping" optimization structure. To ensure maintenance effectiveness, those surviving components G_h , whose independent time was before τ_h , were directly incorporated into this IOR group, so there is no need to be included into 'continuous grouping'. Notably, after the IOR completion, status has been updated (the unreplaced components have been updated in detection). As a result, the IOR and subsequent DPGR plans can be updated synchronously. Joint optimization is achieved by maximizing the total cost savings

$$
C_{O\&P}(G_{OR}) = C_f(G_{OR}) + C^*(G_{PR}|G^*_{OR}),
$$
 (21)

$$
G_{OR}^* = \operatorname{argmax} C_{O\&P} (G_{OR}) = \operatorname{argmax} \left[|G_{OR}| \cdot C_{S,R} - \right]
$$

$$
\sum_{i \in G_{OR}} C o P_i \left(\frac{\tau_h - \tau_i}{\tau} | \tau_i \right) + C^* (G_{PR} | G_{OR}^*) \Big], \tag{22}
$$

where G_{OR} represents the IOR group that excluded G_h and h , $C^*(G_{PR} | G_{OR}^*)$ is the maximum cost savings of the DPGR combination under the optimal IOR group G^*_{OR} , and τ_i is set as the time first detected to exceed ξ_i . Ultimately, the optimal IOR is determined by $G_{OM}^* = G_h \cup G_{OR}^*$.

5. Numerical Experiment

In this section, the proposed intelligent maintenance framework is applied to a high-speed railway bogie, to validate its effectiveness and verify its superiority through comparisons with other strategies.

5.1. Experimental Background

As the important part of rail vehicles, the bogie bears the roles of (a) supporting the vehicle body and distributing external loads from the wheel rail and vehicle body; (b) guiding vehicles to smoothly pass through bends; (c) buffering the vibration and impact between railways and vehicles; and (d) transmitting traction and braking forces to ensure normal operation. The bogie is composed of various components, as shown in Fig. 7.

As important rotating mechanical components, due to the high load, high frequency of curved driving, uneven road surface, and fast/frequent starting/braking working conditions, the axle box bearing bears the peeling from the rolling working surface. Likewise, the gearbox bears the tooth breakage and surface damage from the gear. And the wheelset bears the cracks caused by the wear of the wheel tread and rail, which have a high failure rate. These fault modes are hidden, and can only be detected through inspections. The intelligent maintenance engineering for bogies is of great significance for ensuring the safe operation and effectively controlling operating costs. The health status of the bogie is revealed in structural monitoring, and the priority of group maintenance is determined by analyzing the current degradation stage of each component. To clarify the proposed model, we used maintenance data provided by a railway operation & maintenance company, covering detection and maintenance records (including fault time, fault frequency, detection data, maintenance time, etc.) of three key components, including wheel tread, bearings, and gears.

5.2. Component-Level Optimization Outcome

By fitting the historical degradation and maintenance data of the aforementioned components, we have established that the time-

space scale transform function is a power function, with the relevant parameters detailed in Table 1. Here, we have taken six components, namely wheel tread, gears, and axle box bearings, as illustrative examples. The planning steps remain consistent even with the inclusion of additional components within our strategy. Our measurement coordinate is based on train mileage, maintaining the same significance *t* as previously mentioned. At present, high-speed rail can achieve relatively real-time vehicleground detection. The single service cycle of high-speed rail is long, so we choose $5 \times 10^4 km$ to be the detection time (for maintenance) interval. For rolling bearings, the root mean square data (unit: dB) of the outer ring vibration signal after denoising is selected as the degradation indicator. Similarly, for wheel tread and gearbox gears, vibration signals (unit: dB) are also used as indicators.

Table 1. Degradation parameters setting of the relevant bogie components.

Component	β_i	σ_i^2	L_i v_i		ξ_i
(1) tread #1	1.06	0.00000248	15.4351	48	32
2 tread #2	1.06	0.00000248	15.1564	48	32
$\circled{3}$ gear #1	1.35	0.00000952 10.0756		90	61
(4) gear #2	1.35	0.00000952 10.1102		90	61
\circledS bearing #1	1.17	0.0000152	3.9283	26	20
bearing $#2$ (6)	1.17	0.0000152	4.0259	26	20

The relationships between the delayed maintenance time j_i and the maintenance cost rate η_i of the above components are shown in Fig. 8, with the optimal results summarized in Table 2. Moreover, Table 3 shows the relevant cost setting. It can be easily seen that the rate of decrease in maintenance cost rate basically maintains stability before reaching the optimal. Afterwards, the rate of increase in maintenance cost rate as j_i increases is relatively fast. On one hand, CR operation is not triggered at a relatively small j_i , so its corresponding cost item is almost zero. At this time, the average maintenance interval slowly increases with the increase of j_i , steadying the magnitude of changes in η_i . On the other hand, as j_i grows larger, the delayed maintenance time diminishes the significance of PR. The corresponding CR cost is much greater than PR, resulting in a sudden spike in overall costs.

Table 2. Optimal solution of component-level maintenance.

	Component	J_i time	η_i^* / (CNY/Km)	Component	\mathbf{I} i time	η_i^* (CNY/Km)
	tread $#114$		1.9323	(4) gear #2 32		1.1183
(2	tread #2 16		1.7522	$\circled{5}$ bearing #125		1.3137
3.	gear #1 36		0.9590	6 bearing #222		1.3236

Table 3. Cost parameters setting of the relevant bogie components.

Figure 8. Variations of maintenance cost rate to delayed maintenance time.

5.3. System-Level Optimization Outcome

Assuming the shareable maintenance cost is $50 \times 10^4 CNY$. Table 4 presents the scheduled maintenance without any failure in $310 \times 5 \times 10^4$) km. After the first maintenance, the replacement time for all components is {96, 107, 110, 119, 142, 148}, with the corresponding sequence being {tread # 1, tread # 2, bearing # 2, bearing # 1, gear # 2, gear # 1}. It is reasonable to conduct the second combination DPGR on {tread # 1, tread # 2, bearing # 2} for their close repair time. Similarly, the third maintenance group only includes bearing # 1. If viewed in chronological order, the next maintenance group should be $\{\text{ gear } \# 2, \text{ gear } \# 1\}$. However, since tread $\# 1$ and tread $\# 2$ have already been replaced before, and their next replacement time is ${143, 156}$, {tread # 1, tread # 2, gear # 2, gear # 1} will form the next group. The above analysis results meet the predetermined "continuous grouping".

Assuming that component Φ suddenly fails at 138(\times 5 \times $10⁴$) km , the originally planned { $(3, 4)$, $(1), 2)$ } PGM will be temporarily suspended and immediately switched to the IOR triggered by component ④. Joint optimization of (a) the current

IOR and (b) the next DPGR planning will be carried out, and subsequent planning will proceed normally, as shown in Table 5. Fig. 9 provides a schematic diagram of the first five group. By comparing Table 4 and Table 5, it can be found that the overall maintenance frequency increases, and the maintenance groups of components $(1, 2, 6)$, (6) have a great change. The sudden IOR have a significant impact on the global maintenance plan.

Figure 9. Diagram of the system-level planning (with sudden failure).

Order	Maintenance group	DPGR time $(\times 5 \times 10^4 \text{ km})$	Cost saving $(\times 10^4 \text{ CNY})$	
1	$\circled{2}$ {(1),	55	29.4618	
2	(2, 6) $\{ \circled{1},$	104	47.7731	
3	$\{\circledS\}$	119	θ	
	$\{\circled{3}, \circled{4}, \circled{1}\}$	138	56.3166	
4	(2)	156	θ	
5	$\{\textcircled{1}\}$	180	θ	
6	$\{ \mathcal{D}, \mathcal{D}, \mathcal{D} \}$	223	63.6743	
7	{(1)}	279	θ	
8	(З), (4)	282	26.0224	

Table 5. Optimal IOR scheduling with sudden failure.

5.4. Maintenance Strategies Comparison

This section compares the proposed strategy with three maintenance strategies widely used in industry to demonstrate the superiority of in reducing maintenance costs. The optimization target of all policies is to minimize the average maintenance cost during a certain service period. The strategies are outlined below:

Policy A. Independent maintenance without grouping strategy. The strategy outlined in Section 3 is used for all components, and an immediate CR should be carried out upon failure;

- **Policy B.** Time-based Maintenance Policy. The maintenance cycles of all components are set to be integer multiples of a benchmark interval, automatically combined at overlapping maintenance points.
- **Policy C.** The classic rolling horizon approach. It realizes static combination of PR activities during the planning period, while CR is carried out immediately upon failure without opportunity replacement.
- **Policy D**. The group maintenance policy proposed in this paper.

Notably, strategies A, B, and C are actually fixed maintenance plans. The average maintenance cost of Policy A is easy to be determined, while the average maintenance cost of B and C can be defined as:

 $C(G_{all}) = \sum_{G \in G_{all}} ((C_{S,R} + \sum_{i \in G} C_{i,R}) + \sum_{i \in G_{all}} (C_{S,R} + C_{i,f}) \cdot p_i(t_G)),$ (23) where G_{all} represents all the fixed PR groups during the service period. G indicates a certain maintenance combination. $p_i(t_G)$ describes the failure probability of component i within t_G (time between the maintenance group G and its previous group).

Fig. 10 shows the total maintenance costs of the above strategies within different service cycles. It can be found that the proposed strategy is significantly more cost-effectiveness than the other three cases. Due to the large enough magnitude of $C_{S,R}$, Policy A, the non-group strategy, is significantly inferior to the others, while Policy B and C are somewhat similar because they are both fixed schedules, lacking dynamism and flexibility. Such disadvantages need to be replaced by an increase in cost.

Figure 10. Variations of maintenance cost rate to delayed maintenance time.

Table 6 shows the sensitivity of each strategy to the set-up cost $C_{S,R}$ within 300(\times 5 \times 10⁴) km . At lower values, the economic benefits of grouping are not significant, resulting in a narrower gap. With the increase of $C_{S,R}$, this gap has a growing trend, indirectly supporting the important economic benefits of group maintenance, which coincides with the findings of Zheng [37]. Also, when the cost of corrective maintenance is too high, the delay in maintenance time will have a trend of decreasing, as shown in Table 7. This is because under the premise of uncertainty in failure, if the after-failure cost is too high, the preventive maintenance task will be more inclined to perform earlier than the possible failure point. The economic losses caused by advancing the maintenance time and not fully utilizing the remaining life are much lower than those caused by corrective maintenance.

Table 6. Sensitivity of each strategy to C_{SR} within 300(\times 5 \times 10^4) km .

Maintenance	$C_{S,R}(\times 10^4)$						
Policy	30	40	50	60	70		
Policy A	4.05	4.19	4.36	4.49	4.57		
Policy B	4.03	4.10	4.17	4.22	4.26		
Policy C	3.96	4.03	4.06	4.08	4.09		
Policy D	3.94	4.02	3.88	3.86	4.01		

Table 7 Sensitivity of each strategy to $C_{S,R}$ within 300(\times 5 \times 10^{4}) km .

$C_{i,f}(\times 10^4)$	200	300	400	500	600	700
Postponement interval	Δ		14			

6. Conclusion

In this paper, a global-dynamic group maintenance strategy with self-adaptive information renewal is devised, which is applicable to generic multi-components degrading systems. Unlike previous studies, this policy allows for group information renewal upon both (a) health inspection and (b) group maintenance execution, so as to promote decisionmaking agility and precision. Moreover, the policy successfully integrates postponed preventive maintenance and immediate opportunistic maintenance into a unified decision-making framework, significantly improving the flexibility of downtime control and resource allocation. In the comparative experiments on high-speed train bogies, the heuristic maintenance strategy reveals the superior model compared to some widespread adopted strategies.

There are four promising extensions to the current model framework. Firstly, imperfect repair with random effects can be considered to support more flexible source allocation [50,51]. Secondly, the joint optimization of production and maintenance for manufacturing systems is worth exploration, which strives to seek a balance between reducing inventory cost, ensuring production batches, and improving system reliability [52]. Thirdly, group maintenance models oriented to multiple failure modes (including but not limited to degradation-centered failure, shock-induced failures) are potential interesting topics worth examination [53-55]. Ultimately, it is highly valuable to delve into the impact of resource constraint, such as spare, repair tools and maintenance teams on group condition-based maintenance, which is realistic in practice [56,57].

Reference

- 1. Gao W, Wang Y, Zhang X, et al. Quasi-periodic inspection and preventive maintenance policy optimisation for a system with wiener process degradation. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2023; 25(2). https://doi.org/10.17531/ein/162433
- 2. Zhang C, Qian Y, Dui H, et al. Opportunistic maintenance strategy of a Heave Compensation System for expected performance degradation. Eksploatacja i Niezawodnosc - Maintenance and Reliability. 2021; 23(3): 512–521. https://doi.org/10.17531/ein.2021.3.12
- 3. Nguyen K A, Do P, Grall A. Multi-level predictive maintenance for multi-component systems. Reliability Engineering & System Safety. 2015; 144: 83-94. https://doi.org/10.1016/j.ress.2015.07.017
- 4. Wu D, Han R, Ma Y, et al. A two-dimensional maintenance optimization framework balancing hazard risk and energy consumption rates. Computers & Industrial Engineering. 2022; 169: 108193. https://doi.org/10.1016/j.cie.2022.108193
- 5. de Pater I, Mitici M. Predictive maintenance for multi-component systems of repairables with Remaining-Useful-Life prognostics and a limited stock of spare components. Reliability Engineering & System Safety. 2021; 214: 107761. <https://doi.org/10.1016/j.ress.2021.107761>
- 6. Huynh K T, Vu H C, Nguyen T D, et al. A predictive maintenance model for k-out-of-n: F continuously deteriorating systems subject to stochastic and economic dependencies. Reliability Engineering & System Safety. 2022; 226: 108671. <https://doi.org/10.1016/j.ress.2022.108671>
- 7. Yang L, Chen Y, Ma X. A State-age-dependent Opportunistic Intelligent Maintenance Framework for Wind Turbines Under Dynamic Wind Conditions. IEEE Transactions on Industrial Informatics. 2023; 19(10): 10434-10443.<https://doi.org/10.1109/tii.2023.3240727>
- 8. Kowalski M, Izdebski M, Zak J, et al. Planning and management of aircraft maintenance using a genetic algorithm. Eksploatacja i Niezawodnosc - Maintenance and Reliability. 2021; 23(1): 143–153. https://doi.org/10.17531/ein.2021.1.15
- 9. Peng H, van Houtum G J. Joint optimization of condition-based maintenance and production lot-sizing. European Journal of Operational Research. 2016; 253(1): 94-107[. https://doi.org/10.1016/j.ejor.2016.02.027](https://doi.org/10.1016/j.ejor.2016.02.027)
- 10. Zhang C, Zhang Y, Dui H, et al. Importance measure-based maintenance strategy considering maintenance costs. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2022; 24(1): 15-24. https://doi.org/10.17531/ein.2022.1.3
- 11. Ramirez IS, Mohammadi-Ivatloob B, Marqueza FP G. Alarms management by supervisory control and data acquisition system for wind turbines. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2021; 23(1): 110-6. https://doi.org/10.17531/ein.2021.1.12
- 12. Wang L, Song L, Qiu Q, et al. Warranty Cost Analysis for Multi-State Products Protected by Lemon Laws. Applied Sciences. 2023, 13(3): 1541. https://doi.org/10.3390/app13031541
- 13. Olde Keizer MCA, Flapper S D P, Teunter R H. Condition-based maintenance policies for systems with multiple dependent components: A review. European Journal of Operational Research. 2017; 261(2): 405-420. https://doi.org/10.1016/j.ejor.2017.02.044
- 14. Thomas L C. A survey of maintenance and replacement models for maintainability and reliability of multi-item systems. Reliability Engineering. 1986; 16(4): 297-309. [https://doi.org/10.1016/0143-8174\(86\)90099-5](https://doi.org/10.1016/0143-8174(86)90099-5)
- 15. Zheng R, Zhou Y. A dynamic inspection and replacement policy for a two-unit production system subject to interdependence. Applied Mathematical Modelling. 2022; 103: 221-237[. https://doi.org/10.1016/j.apm.2021.10.028](https://doi.org/10.1016/j.apm.2021.10.028)
- 16. Cao X, Shi X, Zhao J, et al. Dynamic grouping maintenance optimization by considering the probabilistic remaining useful life prediction of multiple equipment. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2024. https://doi.org/10.17531/ein/187793
- 17. Yang L, Zhou S, Ma X, et al. Group machinery intelligent maintenance: Adaptive health prediction and global dynamic maintenance decision-making. Reliability Engineering & System Safety. 2024; 110426. https://doi.org/10.1016/j.ress.2024.110426
- 18. Gutierrez-Alcoba A, Hendrix EMT, Ortega G, et al. On offshore wind farm maintenance scheduling for decision support on vessel fleet composition. European Journal of Operational Research. 2019; 279(1): 124–131. https://doi.org/10.1016/j.ejor.2019.04.020
- 19. Leigh J M, Dunnett S J. Use of Petri nets to model the maintenance of wind turbines. Quality and Reliability Engineering International. 2016; 32(1): 167-180.<https://doi.org/10.1002/qre.1737>
- 20. Liu J, Jiang Z, Zhou H. Integrated operation and maintenance optimization for high-speed train fleets considering passenger flow. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2022; 24(2): 297-305[. https://doi.org/10.17531/ein.2022.2.11](https://doi.org/10.17531/ein.2022.2.11)
- 21. D'Ariano A, Meng L, Centulio G, et al. Integrated stochastic optimization approaches for tactical scheduling of trains and railway infrastructure maintenance. Computers & Industrial Engineering. 2019; 127: 1315-1335. https://doi.org/10.1016/j.cie.2017.12.010

- 22. Wang C, Xu J, Wang H, et al. A criticality importance-based spare ordering policy for multi-component degraded systems. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2018; 20(4): 662-670. https://doi.org/10.17531/ein.2018.4.17
- 23. Nguyen T A T, Chou S Y. Maintenance strategy selection for improving cost-effectiveness of offshore wind systems. Energy Conversion and Management. 2018; 157: 86-95. https://doi.org/10.1016/j.enconman.2017.11.090
- 24. Dinh D H, Do P, Iung B. Multi-level opportunistic predictive maintenance for multi-component systems with economic dependence and assembly/disassembly impacts. Reliability Engineering & System Safety. 2022; 217: 108055. https://doi.org/10.1016/j.ress.2021.108055
- 25. Zheng R, Zhao X, Hu C, et al. A repair-replacement policy for a system subject to missions of random types and random durations. Reliability Engineering & System Safety. 2023; 232: 109063[. https://doi.org/10.1016/j.ress.2022.109063](https://doi.org/10.1016/j.ress.2022.109063)
- 26. Yang L, Chen Y, Ma X, et al. A Prognosis-Centered Intelligent Maintenance Optimization Framework Under Uncertain Failure Threshold. IEEE Transactions on Reliability. 2024; 73(1): 115-130. https://doi.org/10.1109/tr.2023.3273082
- 27. Zhang X, Zeng J. Joint optimization of condition-based opportunistic maintenance and spare parts provisioning policy in multiunit systems. European Journal of Operational Research. 2017; 262(2): 479-498[. https://doi.org/10.1016/j.ejor.2017.03.019](https://doi.org/10.1016/j.ejor.2017.03.019)
- 28. Shang L, Liu B, Gao K, et al. Random Warranty and Replacement Models Customizing from the Perspective of Heterogeneity. Mathematics. 2023; 11(15): 3330. https://doi.org/10.3390/math11153330
- 29. Shafiee M, Finkelstein M. An optimal age-based group maintenance policy for multi-unit degrading systems. Reliability Engineering & System Safety. 2015; 134: 230-238. https://doi.org/10.1016/j.ress.2014.09.016
- 30. Zhu W, Fouladirad M, Bérenguer C. A multi-level maintenance policy for a multi-component and multifailure mode system with two independent failure modes. Reliability Engineering & System Safety. 2016; 153: 50-63. https://doi.org/10.1016/j.ress.2016.03.020
- 31. Babishin V, Hajipour Y, Taghipour S. Optimisation of non-periodic inspection and maintenance for multicomponent systems. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2018; 20(2): 327-42. https://doi.org/10.17531/ein.2018.2.20
- 32. Yang L, Wei F, Qiu Q. Mission Risk Control via Joint Optimization of Sampling and Abort Decisions. Risk Analysis. 2024; 44(3): 666- 685. https://doi.org/10.1111/risa.14187
- 33. Song S, Li Q, Felder F A, et al. Integrated optimization of offshore wind farm layout design and turbine opportunistic condition-based maintenance. Computers & Industrial Engineering. 2018; 120: 288–297[. https://doi.org/10.1016/j.cie.2018.04.051](https://doi.org/10.1016/j.cie.2018.04.051)
- 34. Qu L, Liao J, Gao K, et al. Joint Optimization of Production Lot Sizing and Preventive Maintenance Threshold Based on Nonlinear Degradation. Applied Sciences. 2022; 12(17): 8638. https://doi.org/10.3390/app12178638
- 35. Radouane Laggoune, Chateauneuf A, Djamil Aissani. Impact of few failure data on the opportunistic replacement policy for multicomponent systems. Reliability Engineering & System Safety. 2010; 95(2): 108–119. https://doi.org/10.1016/j.ress.2009.08.007
- 36. Martinod RM, Bistorin O, Castaneda LF, et al. Maintenance policy optimisation for multi-component systems considering degradation of components and imperfect maintenance actions. Computers & Industrial Engineering. 2018; 124: 100-112. <https://doi.org/10.1016/j.cie.2018.07.019>
- 37. Zheng R, Qian X, Gu L. Group Maintenance for Numerical Control Machine Tools: A Case Study. IEEE Transactions on Reliability. 2023; 72 (4): 1407–19[. https://doi.org/10.1109/tr.2022.3233893](https://doi.org/10.1109/tr.2022.3233893)
- 38. Ma X, Liu B, et al. Reliability analysis and condition-based maintenance optimization for a warm standby cooling system. Reliability Engineering & System Safety. 2020, 193: 106588. https://doi.org/10.1016/j.ress.2019.106588
- 39. Park M, Pham H. A generalized block replacement policy for a k-out-of-n system with respect to threshold number of failed components and risk costs. IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans. 2011; 42(2): 453-463. https://doi.org/10.1109/tsmca.2011.2162499
- 40. Moghaddam K S, Usher J S. Preventive maintenance and replacement scheduling for repairable and maintainable systems using dynamic programming. Computers & Industrial Engineering. 2011; 60(4): 654-665. https://doi.org/10.1016/j.cie.2010.12.021
- 41. Wildeman R E, Dekker R, Smit A C J M. A dynamic policy for grouping maintenance activities. European Journal of Operational Research. 1997; 99(3): 530–51. [https://doi.org/10.1016/s0377-2217\(97\)00319-6](https://doi.org/10.1016/s0377-2217(97)00319-6)
- 42. Chen Y, Wu T, et al. System Maintenance Optimization Under Structural Dependency: A Dynamic Grouping Approach. IEEE Systems Journal. 2024; 18(3): 1605-1616. https://doi.org/10.1109/JSYST.2024.3422284
- 43. de Smidt-Destombes K S, van der Heijden M C, van Harten A. Availability of k-out-of-N systems under block replacement sharing limited

spares and repair capacity. International Journal of Production Economics. 2007; 107(2): 404-421. https://doi.org/10.1016/j.ijpe.2006.08.013

- 44. Lu B, Zhou X. Quality and reliability oriented maintenance for multistage manufacturing systems subject to condition monitoring. Journal of Manufacturing Systems. 2019; 52: 76-85. https://doi.org/10.1016/j.jmsy.2019.04.003
- 45. Nguyen T A T, Chou S Y. Maintenance strategy selection for improving cost-effectiveness of offshore wind systems. Energy Conversion and Management. 2018; 157: 86-95. https://doi.org/10.1016/j.enconman.2017.11.090
- 46. Bouvard K, Artus S, Bérenguer C, et al. Condition-based dynamic maintenance operations planning & grouping. Application to commercial heavy vehicles. Reliability Engineering & System Safety. 2011; 96(6): 601–610. https://doi.org/10.1016/j.ress.2010.11.009
- 47. Van Horenbeek A, Pintelon L. A dynamic predictive maintenance policy for complex multi-component systems. Reliability Engineering & System Safety. 2013; 120: 39-50[. https://doi.org/10.1016/j.ress.2013.02.029](https://doi.org/10.1016/j.ress.2013.02.029)
- 48. Li K, Ren L, Li X, et al. Remaining useful life prediction of equipment considering dynamic thresholds under the influence of maintenance. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2024; 26(1)[. https://doi.org/10.17531/ein/174903](https://doi.org/10.17531/ein/174903)
- 49. Song M, Zhang Y, Yang F, et al. Maintenance policy of degradation components based on the two-phase Wiener process. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2023; 25(4). https://doi.org/10.17531/ein/172537
- 50. Wang J, Zhou S, Peng R, et al. An inspection-based replacement planning in consideration of state-driven imperfect inspections. Reliability Engineering & System Safety. 2022; 232: 109064.<https://doi.org/10.1016/j.ress.2022.109064>
- 51. Qiu Q, Cui L, et al. Maintenance Policies for Energy System Subject to Complex Failure Processes and Power Purchasing Agreement. Computers & Industrial Engineering. 2018; 119: 193-203. https://doi.org/10.1016/j.cie.2018.03.035
- 52. Zheng R, Zhou Y, Gu L, et al. Joint optimization of lot sizing and condition-based maintenance for a production system using the proportional hazards mode. Computers & Industrial Engineering. 2021; 154: 107157. https://doi.org/10.1016/j.cie.2021.107157
- 53. Wang J, Zheng R, Lin T. Maintenance modeling for balanced systems subject to two competing failure modes. Reliability Engineering & System Safety. 2022; 225: 108637. https://doi.org/10.1016/j.ress.2022.108637
- 54. Meng Y, Lin M, Xu Z, et al. Joint optimization of condition-based maintenance policy and buffer capacity for a two-component selfrepairable serial system. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2024; 26(2).<https://doi.org/10.17531/ein/185581>
- 55. Ma J, Cai L, Liao G, et al. A multi-phase Wiener process-based degradation model with imperfect maintenance activities. Reliability Engineering & Systems Safety. 2023; 232: 109075[. https://doi.org/10.1016/j.ress.2022.109075](https://doi.org/10.1016/j.ress.2022.109075)
- 56. Wang C, Xu J, Wang H, Zhang, Z. A criticality importance-based spare ordering policy for multi-component degraded systems. Eksploatacja i Niezawodnosc – Maintenance and Reliability. 2018; 20(4): 662-670. https://doi.org/10.17531/ein.2018.4.17
- 57. Shang L, Liu B, Qiu Q, et al. Three-dimensional warranty and post-warranty maintenance of products with monitored mission cycles. Reliability Engineering & System Safety. 2023; 239:109506. https://doi.org/10.1016/j.ress.2023.109506