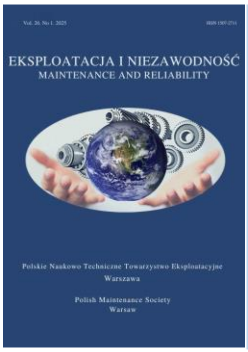

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Reliability Assessment Method Based on Small Sample Accelerated Life Test Data

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Highlights

- A method based on virtual augmentation augmentation fusion is proposed to expand the experimental data of small samples.
- The prior distribution is obtained by improving the empirical distribution function and combining bootstrap and kernel density estimation method.
- The posterior distribution was solved by Gibbs sampling combined with Bayes method and the reliability was evaluated.

Abstract

Aiming at the problems of small sample size, few test failure data and low reliability evaluation efficiency in accelerated life test of high-reliability and long-life products, an improved virtual augmentation Bootstrap-Bayes reliability evaluation method based on small sample accelerated life test data was proposed. Firstly, the test data under various stress conditions are augmented by virtual fusion. Secondly, the empirical distribution function and bootstrap sampling method are improved, and the kernel density estimation method is used to fit the density distribution of test data as prior information. Then, the parameter estimates of the test data are obtained by Gibbs sampling combined with Bayes formula. Finally, the reliability index under normal stress level is obtained by accelerating model extrapolation. The feasibility of the proposed method is verified by the accelerated life test data of a type of Ship communication equipment.

Keywords

reliability assessment, accelerated life test, virtual augmentation, bootstrap sampling, MCMC algorithm, bayes

1. Introduction

With the improvement of modern manufacturing process, many products show the characteristics of high reliability and long life, especially in aerospace and military equipment, there are many expensive, highly reliable and complex systems, affected by multiple factors such as test time and research cost, it is difficult to obtain enough life data within a short period of time, which makes the failure data show the significant characteristics of small samples, and it brings difficulties to the efficient and accurate reliability assessment of such products [1]-[3].

At present, reliability assessment techniques under small sample conditions have become a hot research topic. vvvvvvFor the small sample problem, Bayes method [4], Bootstrap method [5], Bayes Bootstrap method [6], Monte Carlo method [7], and Grey Model method [8] are the commonly used methods in practical engineering [9]-[12]. Many experts and scholars have done some research on the reliability assessment problem under small sample conditions: Literature [13] established a small-sample reliability assessment model for brake system anti-skid valve, heavy duty CNC milling machine, and spacecraft based on Bayes theory. Literature [14] solved the life distribution parameters of aerospace electric slip ring based on the idea of Bootstrap, and combined with the traditional reliability

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prediction methods to obtain the reliability index. Literature [15] integrated the Bayes method into the idea of Bootstrap, which solved the problem that the small-sample reliability model may have bias in practical applications. Literature [16] applied the grey theory to the prediction of small sample fault data, and improved the grey model from the fuzzy theory, the whitening equation itself, and the Bootstrap method, respectively, to improve the reliability of fault prediction. Literature [17] proposed a small data sample prediction method based on Least Squares Support Vector Machines, which has better generalisation and accuracy of life prediction when dealing with exponentially distributed small data samples.

At present, there is some research on the reliability assessment method under small sample conditions, and the accelerated life test is a common test data acquisition method in reliability assessment, the above paper does not involve the small sample test data in the accelerated life test. In the face of certain high-reliability, long-life and high-cost products, it is not practical to use a large number of test samples for testing. At present, fewer reliability assessment methods can be found for small-sample accelerated life test data. Literature [18] introduced the bayes theory into the small-sample reliability assessment of CNC systems, and investigated the reliability assessment technique under small-sample step accelerated life test; literature [19] investigated the method of constructing the exact confidence limit of acceleration factor under small sample; The literature [20] introduced the bayes neural network into the reliability assessment under small samples, and then evaluated the reliability life of the product after generating the virtual accelerated life test samples. However, the above studies have certain limitations, the scope of application is narrow and the methods are cumbersome; therefore, more research is needed on the reliability assessment of small-sample test data in the accelerated life test.

For the reliability assessment problem under small sample conditions, the reliability assessment based on the Bayes method is one of the most widely used methods [21]-[22], and the Bayes method can use the a priori information to determine the a priori distribution of the parameters, and then synthesise the experimental data to obtain the a posteriori distribution of the parameters, which can be used for reliability assessment. However, the current reliability assessment based on the Bayes method has two main difficulties, one is that the a priori distribution is difficult to determine; the second is that the constructed a posteriori distribution is more difficult to solve [23].

Aiming at the above problems, this paper proposes a reliability assessment method applicable to small-sample accelerated life test data, which introduces the idea of virtual augmentation and generalisation on the basis of the traditional Bayes Bootstrap method, expands and integrates the small-sample data of the accelerated life test, and converts the small-sample problem of the accelerated life test into a large-sample problem; and then transforms the empirical distribution function to carry out the Bootstrap sampling, and introduce the kernel density estimation method to determine the a priori distribution of the parameters, and finally use the Gibbs sampling algorithm in the Markov chain Monte Carlo method to calculate the a posteriori estimation of the model parameters. The feasibility of the method proposed in this paper is verified using simulation examples, and the reliability of a certain type of datalink equipment is evaluated by the accelerated life test data of this equipment.

2. Determination of prior distributions for small sample test data

2.1. Fusion Virtual Augmentation Data Expansion Model

Virtual Augmentation (VA) has a better application in the reliability assessment of products with very small sub-sample ($n \leqslant 2$), which can appropriately augment and expand the very small sample size and retain the statistical characteristics of the sample, which can then be better for the subsequent reliability assessment. The use of virtual augmentation needs to meet the following two theoretical bases:

(1) The sample mean of the post-enlargement sample is equal to the sample mean of the pre-enlargement or similar product;

(2) The standard deviation of the post-enlargement sample is equal to the standard deviation of the pre-enlargement or similar product sample.

In order to make the fault time data samples obtained from virtual augmentation more reasonable[24], the virtual augmentation formula (1) was used to augment the n fault data obtained from the test to m data:

$$T = T_0 \mp (a \times (i-1)^b + c)\sigma, i = 1,2,\ldots,\frac{m}{2} \quad (1)$$

Where T_0 is the sample mean; a, b are control coefficients describing the dispersion characteristics of the virtual broadening point (values based on engineering experience, the values of the parameters a, b vary in different cases and need to be weighed against the actual test results). c is the coefficient to be determined that satisfies the theoretical basis; σ is the sample standard deviation; and T is the augmented sample value.

According to the two bases of the virtual generalisation method, the equation can be formulated as follows.

$$\frac{1}{m}\sum_{i=1}^{m} T_i = T_0 \quad (2)$$

$$\frac{1}{m-1}\sum_{i=1}^{m}(T_i - T_0)^2 = \sigma^2 \quad (3)$$

The undetermined coefficient c can be solved by the above two arguments.

The applicability of the virtual augmentation method is better in the case of very small samples, and in order to apply it to the case of small samples, fusion processing is required. The specific approach is: the sample data $T_1{}^*, T_2{}^*, \ldots, T_m{}^*$ obtained after the virtual broadening and the original data $T_1, T_2, \ldots, T_n$ are compared and fused, the broadening data and the original data are sorted in order, the Euclidean distance between the original data and the broadening data is compared, and the original data is used to replace the broadening data with its closest distance, so as to realise the fusion of the broadening data and the original data and to preserve the original sample data at the same time as the broadening of the data and to ensure the credibility of reliability statistical assessment. The fusion of augmented data and original data is achieved.

2.2. A priori distribution determination based on Bootstrap sampling with kernel density estimation

2.2.1. Bootstrap Sampling

(1) Introduction to Bootstrap Sampling

Bootstrap method is a data processing method that approximates complex statistics through computer simulation[25]. In fact, the method is to re-sample within the original data, using the results of multiple simulated sampling to approximate the actual value.

Basic idea of Bootstrap method: a known random sample $X = (X_1, \ldots, X_n)$ follows some unknown distribution F, $x = (x_1, \ldots, x_n)$ is its sample observation, θ is a parameter to be estimated for the population distribution F, Its theoretical truth value is $\theta(F)$.F_n is the empirical distribution function obtained from a given sample X, according to F_n, the estimated value of the parameter θ is $\hat{\theta} = \hat{\theta}(F)$. The estimated error between the parameter estimate $\hat{\theta}$ and the theoretical truth value is denoted as:

$$T_n = \hat{\theta}(F) - \theta(F) \quad (4)$$

Continue to draw sample $X^* = (X_1^*, \ldots, X_m^*)$ from the empirical distribution F_n, Call it a Bootstrap sample.F_n^* is the sample population of the regenerated sample X^*, and the parameter estimate obtained from F_n^* is $\hat{\theta}^* = \hat{\theta}(F_n^*)$, then the error between the parameter estimate obtained from the regenerated sample and the parameter estimate obtained from the sample X is as follows:

$$R_n = \hat{\theta}(F_n^*) - \theta(F_n) \quad (5)$$

By using the distribution of R_n to approximate the distribution of T_n, that is $R_n \approx T_n$, the parameters of the population distribution can be approximated by the parameter estimates of the regenerated samples, and group B of regenerated samples can be obtained by repeating the resampling process B times. The value B of θ_i population parameter θ can be obtained, and then the distribution model and statistical characteristic quantity of the population parameter can be obtained.

(2) Limitations of Bootstrap Sampling Methods

Although Bootstrap sampling can expand the sample data through multiple re-sampling, Bootstrap sampling will cause a relatively large bias when the sample size is small; in addition, Bootstrap sampling itself has certain theoretical limitations: First, there is a function construction error between the empirical distribution function constructed by the original samples and the true distribution; second, re-sampling the empirical distribution function constructed by the original samples will lead to sampling error; third, there is also an estimation error in the parameter estimation of the Bootstrap regeneration samples. The process of Bootstrap sampling and error analysis are shown in Fig.1.

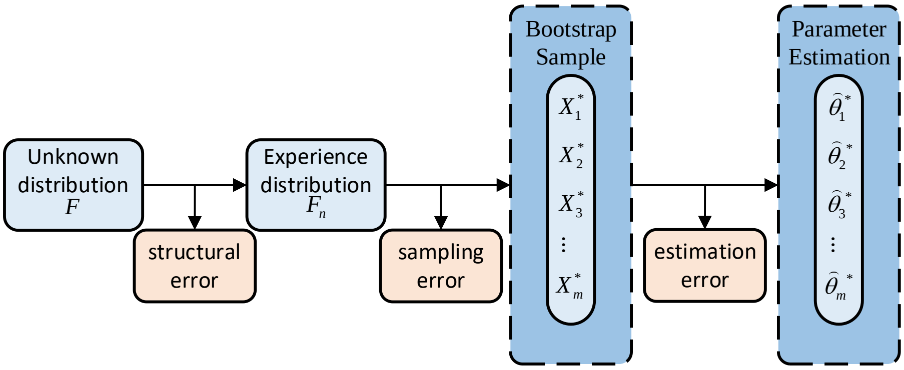

Fig.1. Bootstrap sampling process and error analysis.

The limitations of Bootstrap sampling are particularly prominent in small sample situations, which can limit the accuracy of Bootstrap sampling method. By analysing the process of Bootstrap sampling and the error analysis diagram, according to the large number theorem, the sampling error can be minimised by increasing the number of resampling; while the parameter estimation error is unavoidable. Therefore, we can start from the aspect of reducing the construction error between the unknown distribution and the empirical distribution to reduce the construction error and improve the accuracy of Bootstrap sampling.

2.2.2. Modification of the experience distribution function

The traditional empirical function is mostly constructed by the empirical formula dominated by the median rank formula, and its distribution function is as follows:

$$F_n(x) = \begin{cases} 0 & x < x_1 \\ \frac{i-0.3}{n+0.4} & x_{(i)} < x < x_{(i+1)} \\ 1 & x \geq x(n) \end{cases} \tag{6}$$

Where $x_{(i)}$ is sorted lifetime data, n is sample size.

This method constructs an empirical distribution function that is stepped, and the stepped empirical distribution function will deviate from the original theoretical distribution function in the case of small samples, and it will lead to a relatively more concentrated Bootstrap sample, resulting in a larger sampling error, which is not conducive to the subsequent statistical inference of reliability. In order to construct an empirical function suitable for small sample conditions, reduce the distribution construction error and sampling error, introduce an improved empirical distribution function construction method using the B-spline function, the specific empirical function distribution construction method is as follows:

(1) Reorder the invalid data from small to large. Construct the polyline function on the real number R as follows:

$$F_n(x) =$$
$$\begin{cases} 0, & x < x_{(1)} \\ \frac{2}{n(x_{(k+2)}-x_{(k)})}x + \frac{kx_{(k+2)}-x_{(k+1)}-(k+1)x_{(k)}}{n(x_{(k+2)}-x_{(k)})}, & \frac{x_{(k)}+x_{(k+1)}}{2} \leq x \leq \frac{x_{(k+1)}+x_{(k+2)}}{2} \\ 1, & x > x_{(n)} \end{cases} \tag{7}$$

Where $k = 0,1,\ldots,n-1, x_{(0)} = x_{(1)}, x_{(n+1)} = x_{(n)}$.

(2) The interval Δ is evenly divided by step size h:

$$x_{(1)} = a_0 < a_1 < \ldots < a_m < x_{(n)} \tag{8}$$

In uniformly divided intervals Δ, The maximum number of sample points in the interval is less than $nD_{n,\theta} - 1$, $D_{n,\theta}$ is the critical value of the Kolmogorov test for the statistic D_n at a given level.

(3) Extend the interval Δ:

$$\Delta_1: a_0 < a_1 < \ldots < a_m + h = a_{m+1} \tag{9}$$

Define a B-spline function $S_1(x)$ in the real field as:

$$S_1(x) = \begin{cases} 0, & x < a_0 \\ \sum_{i=0}^{m+1} F_n(a_i) M_2\left(\frac{x-a_0}{h} - i\right), & a_0 \leq x \leq a_{m+1} \\ 1, & x > a_{m+1} \end{cases} \tag{10}$$

where,

$$M_2(x) = \begin{cases} 1-x, & 0 \leq x < 1 \\ 1+x, & -1 \leq x < 0 \\ 0, & other \end{cases} \tag{11}$$

Similarly, the method of constructing the empirical distribution function with cubic B-spline function can be obtained [21]. Considering that a B-spline function constructs an empirical distribution function with good stability and a relatively simple formula; while using a cubic B-spline function to construct an empirical distribution function can be closer to the real distribution, but its construction formula is too complicated and less efficient, so a B-spline function is used to construct an empirical distribution function for subsequent Bootstrap sampling and reliability statistical assessment.

2.2.3. Kernel density estimate

Although multiple sets of estimates of parameter θ can be obtained by Bootstrap sampling method, the distribution expression of parameter θ cannot be obtained from these estimates alone, so the kernel density estimation method is introduced to fit the distributions of the multiple sets of parameter estimates in order to determine the prior distribution of the parameter.

Assuming $x_1, x_2, \ldots, x_n$ are samples from population X, the kernel density of the population density function $f(x)$ at any point x is estimated to be:

$$f_h^*(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right) \tag{12}$$

Where $K(\bullet)$is the kernel function and h is the window width.

In order to ensure the rationality of $f_h^*(x)$ as a density function estimation, kernel function $K(\bullet)$is required to satisfy:

$$\int_{-\infty}^{+\infty} K(x)\, dx = 1, K(x) \geq 0 \tag{13}$$

The window width h of the kernel function in the kernel density estimation will directly affect the smoothness of the kernel density estimation, a smaller window width h will cause the image of the kernel density estimation is not smooth; take a larger value will lose a certain amount of information on the data points, so it is necessary to make a reasonable choice of the window width h in order to obtain a good kernel density estimation.

The Mean Integrated Squared Error (MISE) method is a method for estimating the best window width for continuous variables. The MISE is defined as follows:

$$MISE(f_h^*) = E[\int\{f_h^*(x) - f_h(x)\}^2\, dx] \tag{14}$$

The expression is a function of window width h as variable, and h is selected to minimize the integral mean square error of $f_h^*(x)$, that is:

$$\frac{\partial MISE(f_h^*)}{\partial h} = 0 \tag{15}$$

To calculate the best window width estimate:

$$h^* = \left(\frac{4}{3}\right)^{\frac{1}{5}} \sigma n^{-\frac{1}{5}} \tag{16}$$

Where, h^* is the optimum window width, σ is the sample standard deviation and n is the sample number.

When using kernel density estimation to determine the prior distribution, choosing the appropriate type of kernel function is a critical step. Common kernel function types include Gaussian kernel, uniform kernel, triangular kernel and bimodal kernel. Each type of kernel function has its unique characteristics and applicable scenarios. The Gaussian kernel function is the most widely used. Choosing Gaussian kernel function as the kernel function for kernel density estimation can provide smooth and stable density estimation results, adapt to different data distributions, and have good performance in both theory and practical applications.

Fitting distributions to multiple sets of parameter estimates from Bootstrap sampling by kernel density estimation allows for more accurate determination of prior distributions from sampling results to improve the precision of subsequent posterior distributions.

2.2.4. Determining the prior distribution

So far, for the problem that the a priori distribution of products with less failure information is difficult to determine, this section proposes a method for determining the a priori distribution based on Bootstrap sampling with kernel density estimation, and the flowchart of the a priori distribution determination is shown in Fig. 2.

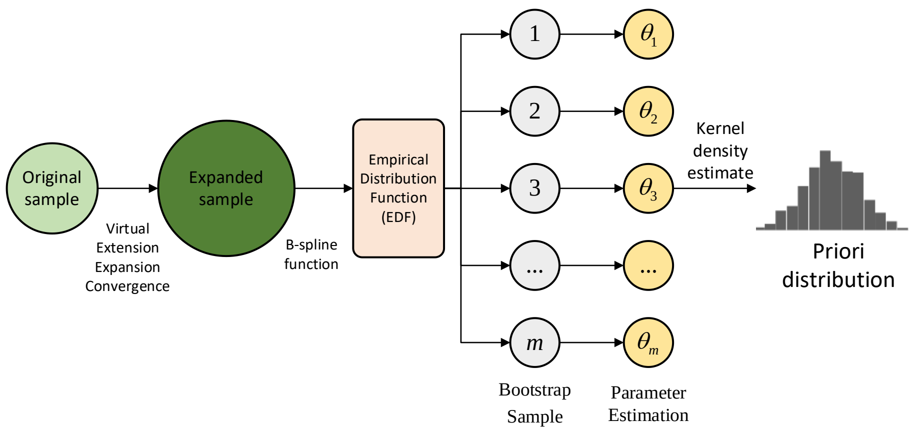

Fig. 2. A priori distribution determination process.

According to Fig. 2, a specific step for determining the prior distribution can be obtained:

1） A small number of *n* trial data obtained from the original trials are fused with virtual augmentation expansion to expand the sample size.
2） Construct an empirical distribution function with the expanded data using a one-time B-spline function to reduce the distribution function construction error.
3） Bootstrap sampling is performed on the constructed empirical distribution function, and the sampling is repeated B times in order to reduce the sampling error, and group B parameter estimates are obtained.
4） A kernel density estimation was performed to fit the distribution to the sampled group B parameter estimates to obtain the prior distribution of the parameters.

3. Solving the posterior distribution for small sample experimental data

Another difficulty in using Bayes method for reliability assessment is the calculation of the posterior distribution. The posterior distribution generally has no analytical solution, and it is usually necessary to use numerical integration to approximate the posterior expectation and posterior variance of the parameters to be estimated in the posterior distribution. However, the calculation of the posterior distribution of the model parameters often involves high-dimensional integration operations, and the error term will increase with the increase in the dimensionality, which causes practical difficulties in the application of Bayes method. Bayes method application caused difficulties in practical application. It was not until the emergence of Markov Chain Monte Carlo (MCMC) algorithm, which statisticians introduced into statistical analysis[26], that this difficulty was greatly improved, and the scope of application of the Bayes method was greatly broadened.

The essence of the MCMC algorithm is a Monte Carlo method of sample extraction by constructing a Markov chain, which approximates the sample expectation in terms of the mean value of the samples drawn from the desired distribution, and is capable of describing and solving complex models that are intractable to traditional statistical methods.

3.1. M-H algorithm and Gibbs algorithm

3.1.1. M-H algorithm

The M-H algorithm is based on a Markov chain and requires the aid of an auxiliary function $q(\theta^1|\theta^2)$, called the proposal distribution, which represents the probability of moving to the next value when the current value θ^1 is specified. Given the current state value θ^t, a random number θ^* is generated from the proposal distribution function and an acceptance probability is computed to decide whether to take θ^* as the next value in the sequence. This is done as follows:

(1) Generate a candidate value A from the proposed distribution θ^*;

(2) Calculate the probability of acceptance:

$$\alpha(\theta^t,\theta^*) = min\left\{1,\frac{p(\theta^*)q(\theta^t|\theta^*)}{p(\theta^t)q(\theta^*|\theta^t)}\right\} \tag{17}$$

(3) Accept $\theta^{t+1} = \theta^*$ with probability $\alpha(\theta^t,\theta^*)$, otherwise $\theta^{t+1} = \theta^t$, and return to (2);

(4) Based on the above steps, a Markov chain $\{\theta^0,\theta^1,\dots,\theta^n\}$ can be generated and then the mean can be obtained for the posterior sample.

3.1.2. Gibbs algorithm

Gibbs algorithm can continuously draw samples from the conditional distributions of the samples and utilize the samples from these conditional distributions to approximate the samples from the sampling distribution. Suppose the unknown parameter θcontains q elements $\theta = (\theta_1,\theta_2,\dots,\theta_q)$. Construct the full conditional posterior distribution for each element as:

$$p_1(\theta_1|\theta_2,\theta_3,\dots,\theta_q)$$
$$p_2(\theta_2|\theta_1,\theta_3\dots,\theta_q) \qquad \vdots\, p_q(\theta_q|\theta_1,\theta_2\dots,\theta_{q-1}) \tag{18}$$

Both the M-H algorithm and the Gibbs algorithm have their own characteristics in applications: the M-H algorithm is more advantageous in parameter estimation in low dimensions; compared with the M-H sampling method, the Gibbs sampling algorithm only needs to know the full conditional distributions of each parameter to be estimated in order to be sampled, so when dealing with complex integrals in high-dimensional spaces, the Gibbs method, which is more suitable for solving this kind of problems, is usually chosen. Since the problem of multiple integrals, which is difficult to compute, is encountered in the calculation of the posterior distributions in this paper, the Gibbs sampling method is chosen as the sampling method of the

MCMC method to construct a suitable Markov chain.

Then the sampling process of Gibbs algorithm can be depicted in Fig. 3:

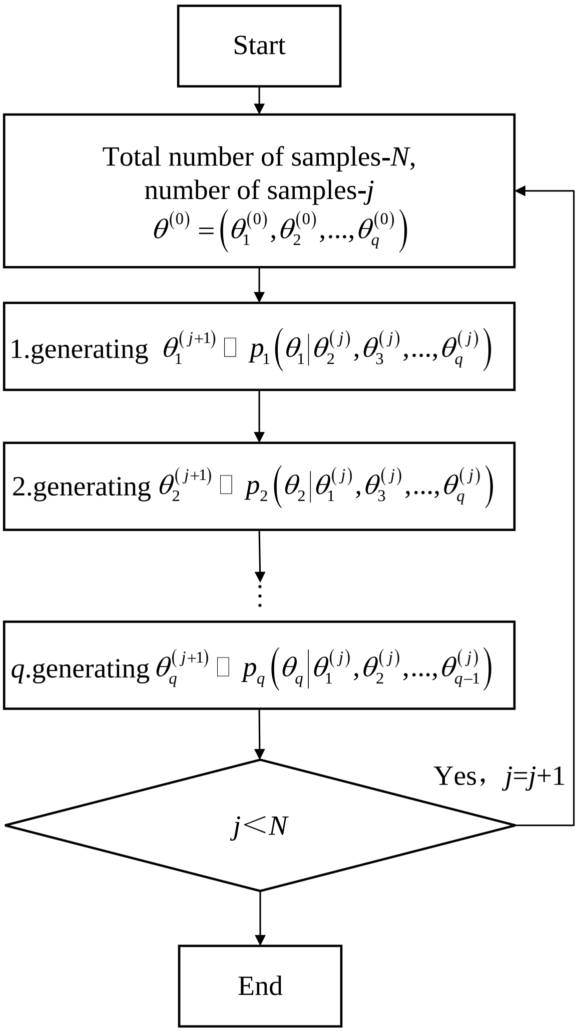

Fig. 3. Sampling process of Gibbs algorithm.

3.2. Posterior distribution solution based on Gibbs algorithm

Assume that the unknown parameter in the population distribution is θ and its prior distribution is $\pi(\theta)$, and the posterior distribution of θ is obtained by Bayes formula:

$$\pi(\theta|x) = \frac{f(x|\theta)\pi(\theta)}{\int_\theta f(x|\theta)\pi(\theta)d\theta} \tag{19}$$

Where, $f(x|\theta)$ is the joint density function of the unknown parameter θ, that is, the maximum likelihood function of the sample.

Sample $\theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ is drawn from the posterior distribution $\pi(\theta|x)$ using Gibbs' algorithm with the expectation of the parametric posterior distribution equal to the mean of the drawn sample:

$$E(\theta) = \int \theta\, \pi(\theta|x)d\theta = \frac{1}{n}\sum_{i=1}^{n}\theta_i \tag{20}$$

When the overall distribution of the sample is a Weibull distribution with unknown parameter $\theta = (\alpha, \beta)$, the steps of using Gibbs algorithm are as follows:

(1) $\alpha^{(j)}$ is sampled from the full conditional distribution $\pi(\alpha|X, \beta^{(j-1)})$.

(2) $\beta^{(j)}$ is sampled from the full conditional distribution $\pi(\beta|X, \alpha^{(j-1)})$.

Given any initial value of the parameter $\theta = (\alpha, \beta)$ for iteration, the above process is repeated and the samples when the Markov chain has converged and stabilised are used as the samples of the posterior distribution $\pi(\alpha, \beta|X)$ for calculating the results of the MCMC integrals.

When the overall distribution of the sample is a Weibull distribution, the posterior distribution of the unknown parameter θ is updated as:

$$\pi(\alpha,\beta|x) = \frac{f(x|\alpha,\beta)\pi(\alpha,\beta)}{\int_\Theta f(x|\alpha,\beta)\pi(\alpha,\beta)d\alpha d\beta}$$

$$= \frac{\pi(\alpha,\beta)\prod_{i=1}^{n}\frac{\beta x_i^{\beta-1}}{\alpha^\beta}exp\left[-\left(\frac{x_i}{\alpha}\right)^\beta\right]}{\int_\Theta \pi(\alpha,\beta)\prod_{i=1}^{n}\frac{\beta x_i^{\beta-1}}{\alpha^\beta}exp\left[-\left(\frac{x_i}{\alpha}\right)^\beta\right]d\alpha d\beta} \tag{21}$$

The updated posterior distribution is more complex, and in this chapter, the parameter estimation of the unknown parameter $\theta = (\alpha, \beta)$ is done using the OpenBUGS software [27], which has a built-in MCMC-Gibbs sampling method to facilitate the solution of the posterior parameter estimation. Using OpenBUGS language for model programming and checking, load the initial value of parameters and sample data information; set the number of iterations, the number of sophistication and other parameter settings after the execution of the procedure and the monitoring of the unknown parameter $\theta = (\alpha, \beta)$; at the end of the iteration to output the simulation value of the a posteriori distribution parameter, and can be generated to simulate the trajectory graph and autocorrelation coefficient change trend graph and other information to determine the convergence of MCMC algorithms, based on the parameter a posteriori estimation of the results of the subsequent reliability of the statistical inference.

So far, this chapter proposes a method for determining the a priori distribution based on virtual augmented expansion fusion and kernel density estimation, and combines the MCMC-Gibbs algorithm to achieve the a posteriori estimation of unknown parameters. The method is applicable to small sample test data, can effectively overcome the problems of small sample size and difficult to determine the a priori distribution, and can improve the efficiency and accuracy of reliability assessment. The complete flow of this proposed Va-BayesBootstrap method is shown in Fig. 4.

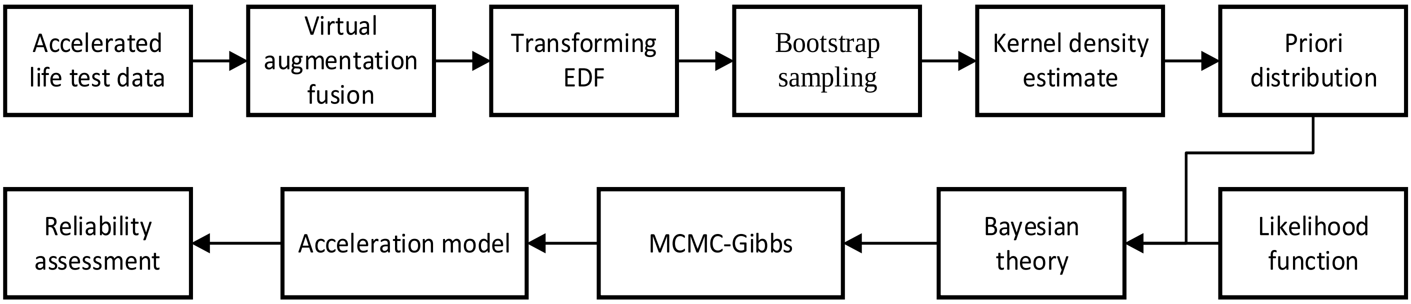

Fig. 4.Va-BayesBootstrap method reliability assessment flow.

4. Case Studies

4.1. Monte Carlo example validation

In order to verify the feasibility of the method proposed in this paper in the case of small samples, the Monte Carlo method is used for feasibility verification. Let α =300, β =3, using Matlab to randomly generate 10000 data conforming to the two-parameter Weibull distribution as the simulation failure data, in order to simulate the case of small samples, from which n sample sizes of the data are taken as the original samples, and the parameter estimation is carried out by the traditional great likelihood estimation method and the method proposed in this paper, respectively. In order not to lose the generality and reduce the error, the experiments under each group of sample size were conducted 100 times each, and the parameter estimation results were averaged, and the obtained calculation results are shown in Table 1 below.

Table 1.Comparison of parameter estimation results

Sample size	Parameters	MLE	Errors %	Va-BayesBootstrap	Errors %
n=4	α	294.5417	1.819%	295.1876	1.604%
	β	4.9025	63.41%	4.1052	36.84%
n=8	α	296.9888	1.004%	298.5624	0.479%
	β	3.7245	24.15%	3.2335	7.783%

The results of the calculations in Table 1 were transformed into a graphical visual representation, as in Fig. 5 and Fig. 6.

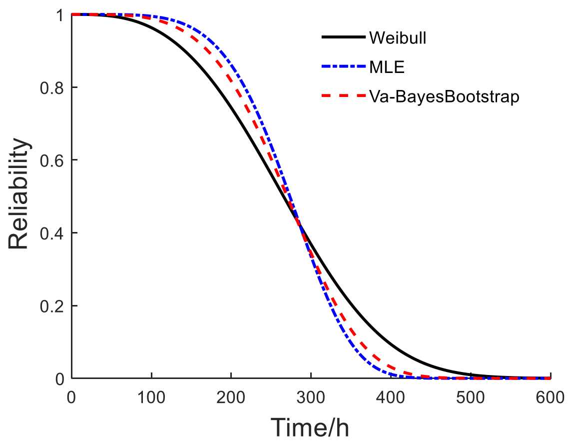

Fig.5. Comparison of results for sample size n = 4.

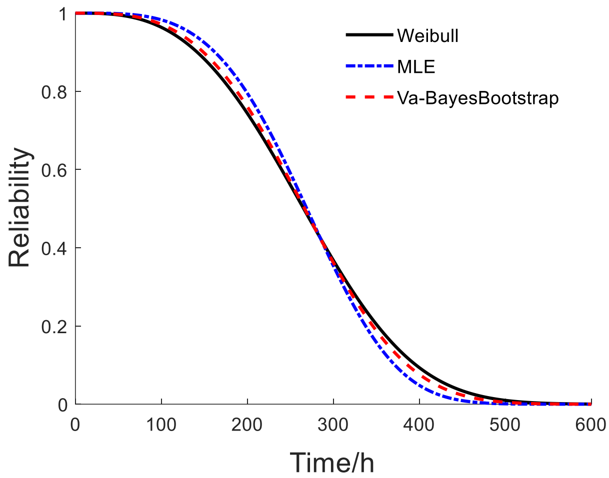

Fig.6. Comparison of results for sample size n = 8.

As can be seen from Fig. 5 and Fig. 6, the accuracy of parameter estimation is lower in the case of lower sample size, but with the increase of the sample size, the accuracy of parameter estimation increases, and the error between the calculation result and the true value is smaller. In the case of the same sample size, the proposed method in this paper has a higher computational accuracy than the traditional great likelihood method to find the parameters. Simulation examples show that the method in this paper can effectively improve the accuracy of small sample conditional reliability assessment.

In order to compare the effect of reliability assessment more intuitively, a sample point is taken as a sample point at an interval of 1 in time [0, 600], and the root mean square error(RMSE), between the reliability estimates of the two methods and the set true value is calculated separately, which is given by the formula:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{R} - R)^2} \tag{22}$$

Where N is the number of sample points, R is the set reliability truth value and $\hat{R}$ is the estimated reliability value.

From equation (22), the smaller the RMSE is, the smaller the error between the estimated value and the true value is, indicating a better assessment.The results of RMSE are shown in Table 2.

Table 2.Comparison of RMSE for different estimation methods.

Sample size *n*	MLE	Va-BayesBootstrap
4	0.069212	0.046405
8	0.03271	0.011791

As can be seen from Table 2, the RMSE of the proposed method in this paper is smaller with the same sample size, which again proves that the proposed method in this paper is feasible.

4.2. Reliability assessment of Ship communication equipment

4.2.1. Accelerated life test

Ship communication equipment plays an important role in effective communication and information transfer between ships, and its reliability will directly affect the ability of safe navigation. However, for such products, the time and manpower costs required for traditional life tests are too high, so accelerated life tests can be used to improve the reliability assessment efficiency of ship communication equipment.

The test takes into account the sensitive stresses during normal operation of the device, and takes temperature and humidity as the two accelerating stresses of the accelerated life test, which are noted as T and S respectively, and the rated temperature T_0 and humidity S_0 during normal operation are 25°C and 40% RH respectively. For this type of ship communication equipment, comprehensive engineering experience and expert opinion, set the temperature accelerated stress range of 50℃~80℃, humidity accelerated stress range of 60%RH~90%RH, that in this stress range of the test produced by the failure are all belong to the same failure mechanism.

To synthesise the actual situation, 20 test samples were used for the test, and the number of test groups for the accelerated life test was divided into 4 groups, with 4 accelerated stress levels selected for each stress, and the stress division was carried out by using a uniform and equal spacing, and the test grouping was carried out in accordance with the idea of uniform orthogonal design. The specific stress combinations for the accelerated life test were: (50°C, 70%RH), (60°C, 90%RH), (70°C, 60%RH), and (80°C, 80%RH).

An accelerated life test platform was built as shown in Fig.7, which mainly consists of a high and low temperature test chamber, a DC regulated power supply, a computer and several ship communication devices. The schematic diagram of the accelerated life test platform is shown in Fig.8.

Fig.7. Accelerated life test platform.

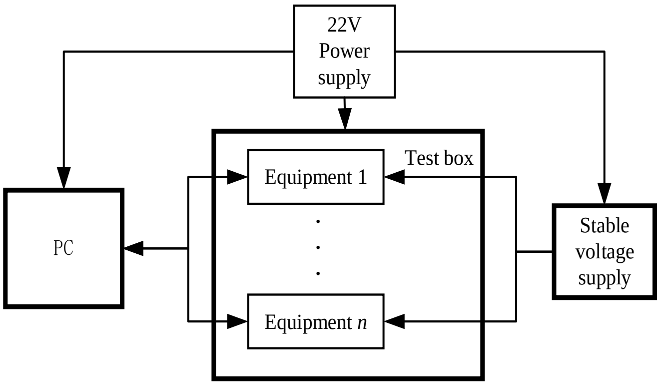

Fig.8. Schematic diagram of accelerated life test platform.

This section provides a reliability assessment of a Ship communication device based on accelerated life test data for a particular type of Ship communication device, which is shown in Table 3.

Table 3.Accelerated life test data for a type of Ship communication equipment.

Class number	Stress level（℃，RH%）	Failure time（h）
1	50,70	285.78,396.30,449.50,500.70,609.45,776.01
2	60,90	102.74,150.60,179.57,210.83,350.30
3	70,60	125.85,215.95,275.98,317.62,498.73
4	80,80	63.43,118.50,153.98,176.81

4.2.2. Reliability assessment

Generally speaking, the life distribution of electronic devices obey the Weibull distribution, in order to judge the data obtained from the Weibull distribution of the goodness of fit, you can use the double logarithmic linear transformation method of judgement, the life distribution function on both sides of the logarithm of the two times transformed into the following formula of the simpler linear form:

$$lnln\frac{1}{1-F_i(t)} = m_i \ln t_{ij} - m_i \ln \eta_i \tag{23}$$

Let $y = lnln\frac{1}{1-F_i(t)}$， $x = \ln t_{ij}$， $b = m_i \ln \eta_i$, then the equation becomes the general standard form shown in the following equation:

$$y = m_i x + b \tag{24}$$

For the failure times of the equipment for each set of test conditions, $F_i(t_{ij})$ can be calculated from the empirical distribution equation：

$$F_i(t_{ij}) = \frac{j-0.3}{n+0.4} \tag{25}$$

Taking the test data under the first set of stress conditions as an example, the $F_i(t_{ij})$ for this set of samples as well as the general standard form of each parameter were obtained as shown in Table 4 below.

Table 4.Test data and standard form parameter values for the first set of stress conditions.

Failure sequence	$t_{ij}(h)$	$F_i(t_{ij})$	$y = lnln\frac{1}{1-F_i(t)}$	$x = \ln t_{ij}$
1	285.78	0.109	-2.15562	5.65522
2	396.3	0.266	-1.17527	5.98217
3	449.5	0.422	-0.60154	6.10814
4	500.7	0.578	-0.14729	6.21601
5	609.45	0.734	0.28192	6.41256
6	776.01	0.891	0.79434	6.65417

According to the data obtained in Table 4, equation (24) can be fitted by the least squares method to obtain $m_i = 3.0327$, $b = -19.2162$, and similarly the fitted values of the parameters and the correlation coefficients can be calculated to obtain the other sets of stress conditions as shown in Table 5.

Table 5.Parameter fitting values and correlation coefficients under each group of test levels.

Stress level	Fitting of m_i	Fitting of b	correlation coefficient
1	3.0327	-19.2162	0.9906
2	2.2389	-12.1547	0.9730
3	2.0272	-11.7634	0.9910
4	2.1708	-10.8635	0.9785

Based on the test data under each set of stress conditions a fitted plot of the Weibull distribution for the four sets of test conditions can be obtained as shown in Fig. 9.

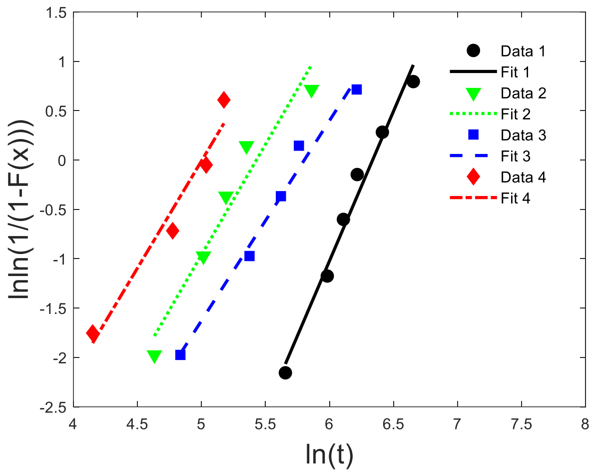

Fig. 9. Fitted plot of the Weibull distribution of the test data.

As can be seen from Table 5, although the fitted values of the experimental data under each group of conditions are slightly different, the correlation coefficients are high; at the same time, according to the Fig. 9, it can be seen that the fitted straight lines obtained from each group of experimental data have a similar trend and are nearly parallel, so it can be regarded as conforming to the Weibull distribution.

Since temperature and humidity are the main factors affecting the failure of datalink equipment, the relationship between the characteristic life of datalink equipment and temperature and humidity can be expressed by a generalised log-linear model [28]:

$$ln\,\alpha = \mu(x,y) = \gamma_0 + \gamma_1 x_i + \gamma_2 y_j \tag{26}$$

Where x_i-transformed thermal stress level; y_j-transformed non-thermal stress level; $\gamma_0, \gamma_1, \gamma_2$-parameters to be estimated.

According to the fusion virtual augmentation data expansion model mentioned in the previous section, comprehensive engineering practical experience to determine the parameters a, b, the virtual augmentation formula can be approximated by the following formula:

$$T = T_0 \mp (0.09 \times (i-1)^2 + c)\sigma, i = 1,2,\dots,\frac{m}{2} \tag{27}$$

The sample data is expanded to 10 and solved for $c = 0.25$, which in turn yields the expanded data after fusion through this model as shown in Table 6.

Table 6.Experimental data after integration of the virtual augmentation expansion.

Class number	Stress level（℃，RH%）	Failure time（*h*）
1	50,70	213.74,285.78,396.30,449.50,500.70,545.01,560.46,609.45,684.04,776.01
2	60,90	41.15,102.74,150.60,179.57,175.88,210.83,230.15,255.41,297.53,350.30
3	70,60	53.18,125.85,215.95,240.38,275.98,317.62,333.28,370.72,433.11,498.73
4	80,80	44.97,63.43,98.30,111.64,118.50,140.28,144.72,153.98,176.81,211.39

Taking the first set of experimental data as an example, the classical empirical distribution function is improved with a one-time B-spline function to obtain $F_{10}(x)$ through equation (7) above:

$$F_{10}(x) = \begin{cases} 0, & x < 213.74 \\ \frac{2}{720.38}x - \frac{427.48}{720.38}, & x \in \left[213.74, \frac{213.74+285.78}{2}\right] \\ \dots & \vdots \\ \frac{2}{919.7}x - \frac{632.32}{919.72}, & x \in \left[\frac{684.04+776.01}{2}, 776.01\right] \\ 1, & x > 776.01 \end{cases} \tag{28}$$

For a given level $\theta = 0.05$, query the Kolmogorov test threshold table to get $D_{10,\theta} = 0.4093, 10D_{10,\theta} - 1 = 3.093$. It is required that in the uniformly divided interval Δ, the maximum number of sample points in the interval is less than 3.093, so the maximum number of sample points is 3.

According to the above requirements, Δ of step length $h = 28.11$ is evenly divided on the interval [213.74,776.01], and $a_{21} = 804.12$ is supplemented. The corresponding partition point values and function values are shown in Table 7.

Table 7.Divides point values and function values.

Number	Numerical value	Function value	Number	Numerical value	Function value
a_0	213.7417	0.0000	a_{11}	522.9893	0.5077
a_1	241.8551	0.0781	a_{12}	551.1027	0.5945
a_2	269.9686	0.1221	a_{13}	579.2161	0.6822
a_3	298.0820	0.1529	a_{14}	607.3295	0.7362
a_4	326.1954	0.1837	a_{15}	635.4429	0.7817

Number	Numerical value	Function value	Number	Numerical value	Function value
a_5	354.3088	0.2162	a_{16}	663.5563	0.8202
a_6	382.4222	0.2506	a_{17}	691.6698	0.8539
a_7	410.5356	0.2763	a_{18}	719.7832	0.8877
a_8	438.6490	0.3302	a_{19}	747.8966	0.9389
a_9	466.7625	0.3840	a_{20}	776.0100	1.0000
a_{10}	494.8759	0.4414	a_{21}	804.1234	1.0000

Then the constructed empirical distribution function is shown in Fig. 10, from which it can be clearly seen that the empirical distribution function constructed by one-time B-spline function is more close to the theoretical real distribution than the traditional empirical distribution function, which can reduce the empirical distribution function construction error to a certain extent.

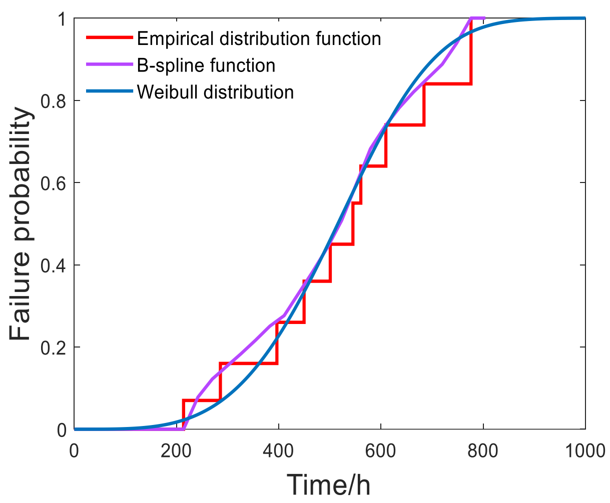

Fig. 10. Comparison of empirical distribution functions.

Bootstrap sampling was performed on the constructed empirical distribution function, and 10,000 groups of samples with a capacity of 10 were drawn in order to minimise the sampling error; each group of samples was subjected to parameter estimation and the distribution of the 10,000 groups of parameter estimates was fitted to the distribution using kernel density estimation as the prior distribution of the parameters.

The fitted plots of the distributions of the Weibull distribution parameters A and B obtained by the proposed method for the first set of test data are shown in Fig. 11 and Fig. 12.

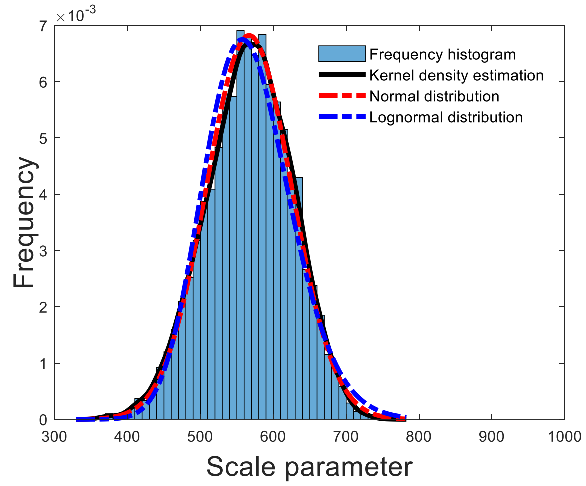

Fig. 11. Fitting of scale parameter distributions.

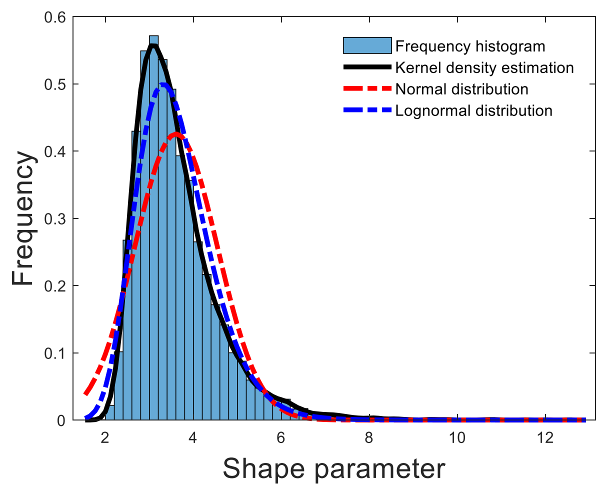

Fig. 12. Fitting of shape parameter distributions.

From Fig. 11 and Fig. 12, it can be clearly seen that the group of test data by Bootstrap sampling 10,000 times and parameter estimation, respectively, after the kernel density estimation fitted to obtain the distribution is more in line with its true distribution than directly fitted to obtain the normal distribution, lognormal distribution, etc., and then get the group of experimental data unknown parameter of the a priori distribution for:

$$\pi(\alpha) = \Phi\left(\frac{x-556.72}{58.44}\right) \tag{29}$$

$$\pi(\beta) = \Phi\left(\frac{\ln x-1.22}{0.219}\right) \tag{30}$$

The posterior distribution of the distribution parameters of the first set of test data can be obtained from equation (21). Gibbs sampling is performed on the obtained posterior distribution to solve the posterior distribution of the parameters α and β. The results are shown in Table 8.

Table 8. Posterior statistics of parameter α and β.

Mean		Standard deviation		Error of MCMC		Start the sample	Number of samples
α	β	α	β	α	β		
566.7	2.67	0.1296	1.82	0.00622	0.177	2000	8001
566.7	3.416	0.1288	1.787	0.00378	0.1332	2000	18001
566.7	3.637	0.1293	1.765	0.00303	0.109	2000	28001
566.7	3.774	0.1282	1.745	0.002429	0.09292	2000	38001
566.7	3.884	0.1294	1.759	0.002009	0.08329	2000	48001

As can be seen from Tables 8, as the number of iterations increases, the mean value of the parameters gradually tends to be constant, the MC sampling error keeps decreasing, and the Markov chain tends to be stable. Discarding the first 2000 iterations of the annealing period of the algorithm, when the number of iterations is 50000, the posterior probability densities of the parameters are shown in Fig. 13 and Fig. 14.

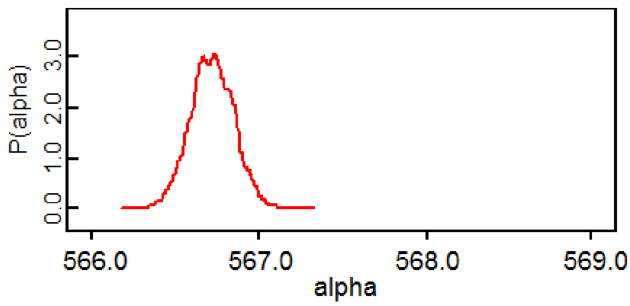

Fig. 13. Scale-parameter a posteriori probability density plot.

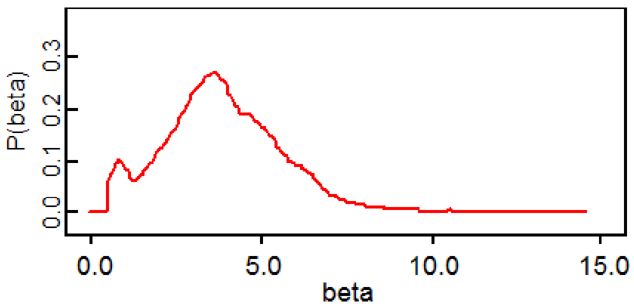

Fig. 14. Posterior probability density plot of shape parameters.

The mean value of the posterior estimate statistics of the parameters of the Ship communication communication system under the first set of stress conditions was taken as the point estimate of the parameters, and the Weibull distribution parameters α、β of the Ship communication communication system under the first set of stress conditions were 566.7 and 3.884 respectively when the number of iterations was 50000.

The VA-Bayes Bootstrap method in this paper was used to solve the Weibull distribution parameters of the Ship communication communication system under the stress conditions of each group. The mean value of the posterior statistics of the Ship communication communication system parameters under each stress condition was used as the estimated value of the parameters to evaluate the reliability of the Ship communication communication system under the normal use environment.

The parameter estimates of Ship communication equipment under each group of stress conditions are shown in Table 9.

Table 9. Parameter estimates for Ship communication equipment.

Class number	Stress level (℃, RH)	Scale parameter α	Shape parameter β
1	50,70%	566.7	3.884
2	60,90%	229.0	2.378
3	70,60%	322.3	2.256
4	80,80%	148.3	3.427

Firstly, the scaling parameter α is used as the characteristic life of the Ship communication equipment, and the relationship between scaling parameter and accelerating stress is obtained according to the acceleration model, and the scaling parameter value under normal environment is obtained. Then, based on the principle of invariability of failure mechanism, the mean value of shape parameter β under each stress level is used to represent the shape parameter value under normal environment. Finally, the reliability of the Ship communication communication system under normal environment is obtained by substituting the distributed parameter values under normal temperature into the reliability function.

The scale parameter α of Weibull distribution is taken as the characteristic life of the Ship communication equipment, and the model parameter $\gamma_0 = -7.4136, \gamma_1 = 4.2279, \gamma_3 = -1.7585$ are obtained by solving equation (26) with matlab.

Therefore, the acceleration model equation between its characteristic life and acceleration stress is as follows:

$$ln\,\alpha = -7.4136 + 4.2279x_i - 1.7585y_j \quad (31)$$

By substituting the ambient temperature and humidity of the Ship communication equipment in normal operation into equation (31), the point estimated value of the scale parameter under normal environmental conditions is 4348.38.

The shape parameters of Weibull distribution can reflect the failure mechanism of the Ship communication communication system. Since the failure mechanism of the Ship communication equipment remains unchanged under the accelerated life test and normal use environment, the shape parameters obtained in Table 8 are averaged to obtain the shape parameter of the Ship communication communication system under normal use environment as 2.986.

The reliability function of Ship communication equipment under normal use environment is:

$$R(t) = exp\left(-\frac{t}{4348.38}\right)^{2.986} \quad (32)$$

The change curve of its reliability is shown in the Fig. 15:

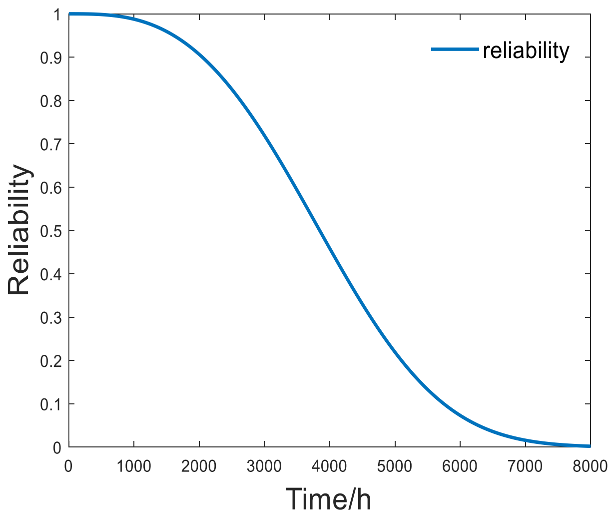

Fig. 15. Reliability curve.

For such high reliability and long life products, people pay more attention to the reliability information in the high reliability range, so this paper focuses on the reliability evaluation of a certain type of Ship communication equipment in the high reliability range. The reliability evaluation results are shown in Table 10:

Table 10. Reliability evaluation results.

Reliability	Time/h
0.99	931.2
0.95	1612
0.90	2083
0.80	2640
0.70	3084

As can be seen from Figure 15 and Table 10, the reliability level of this type of Ship communication communication system began to decline rapidly after about 1500 hours. At around 2083 hours its reliability level is about 0.9. According to the reliability information of the Ship communication communication system, special inspection and maintenance personnel can be arranged to check and maintain the Ship communication communication system in time before the fault occurs to ensure the normal and stable operation of the Ship communication communication system equipment.

5. Conclusion

Aiming at the problems such as the small number of test samples in accelerated life test of high-cost and long-life products and the low efficiency of reliability assessment, and making better use of the small amount of existing test data, a reliability assessment method of the virtual augmentation and expansion fusion of BayesBootstrap method for accelerated life test of small samples is put forward; and a certain type of datalink equipment is taken as an example for the validation of the method, the main The main results of this paper are as follows:

1. On the basis of small-sample test data, through the method of virtual expansion and fusion, make full use of the existing test data, expand the amount of sample information, and fully excavate the overall information of life and reliability carried in small-sample test data.

2. On the basis of the traditional empirical distribution by constructing a 1-time B-spline function distribution, combined with Bootstrap theory, multiple sampling on the newly constructed distribution function, further expanding the amount of sample information, approximating the overall true

distribution of the unknown parameter, and using the kernel density estimation method to fit the parameters of the sample model of the Bootstrap sampling, to the many times Bootstrap sampling after the The sample parameter distribution information is used as the a priori distribution information, which improves the accuracy of the a priori distribution in the reliability assessment of small-sample test data.

3. Using Gibbs sampling in MCMC combined with Bayes formula to get the a posteriori estimates of the model parameters avoids the cumbersome and complex high-dimensional integral operations in the posterior distribution, simplifies the solution process, and improves the efficiency of reliability assessment.

4. According to the acceleration model to extrapolate the normal use conditions of a certain type of Ship communication equipment reliability assessment indicators of the point estimate, and then get its reliability function and its reliability assessment information, can effectively guide the staff to regular inspection and maintenance, to ensure that the equipment works normally.

Reference

1. Cui Y , Zhang Y , Wang Z ,et al.Interval statistic based reliability analysis method on small sample hot test of satellite thruster[J].Applied Mathematical Modelling, 2018, 60.DOI:10.1016/j.apm.2018.03.044.
2. Zhao D, Liu Y, Ren X, et al. Fatigue life prediction of wire rope based on grey particle filter method under small sample condition. Eksploatacja i Niezawodność – Maintenance and Reliability. 2021;23(3):454-467. DOI:10.17531/ein.2021.3.6.
3. Wang H, Zhang Z, LI J, Xin W, Han C. Fault analysis and reliability evaluation for motorized spindle of cycloidal gear grinding machine based on multi-source Bayes. Eksploatacja i Niezawodność – Maintenance and Reliability. 2024;26(1). DOI:10.17531/ein/175010.
4. Fan J W , Zhou Z Y , Wang Z L ,et al.Research on the Evaluation of Small Sample Reliability for CNC Grinding Machine Tools Based on Bayes Theory[J].Advanced Materials Research, 2014, 971-973:688-692.DOI:10.4028/www.scientific.net/AMR.971-973.688.
5. Al-Mutairi D K , Ghitany M E , Gupta R C .Estimation of reliability in a series system with random sample size[J].Computational Statistics & Data Analysis, 2011, 55(2):964-972.DOI:10.1016/j.csda.2010.07.027.
6. Yin M X , Zhu T , Xu J T ,et al.Service Reliability of a Heavy-Haul Wagon Coupler Body Based on the SMOTE-Bootstrap-Bayes Method[J].Engineering Failure Analysis, 2020, 118(3):104836.DOI:10.1016/j.engfailanal.2020.104836.
7. Patowary A N , Hazarika J , Sriwastav G L .Reliability estimation of multi-component cascade system through Monte-Carlo simulation[J].International journal of systems assurance engineering and management, 2018, 9(6):1279-1286.DOI:10.1007/s13198-018-0716-y.
8. Zhao D , Gao C , Zhou Z ,et al.Fatigue life prediction of the wire rope based on grey theory under small sample condition[J].Engineering Failure Analysis, 2019, 107:104237.DOI:10.1016/j.engfailanal.2019.104237.
9. Coburger A , Likar J , Smith C .Bayesian Radiation Modeling for Spacecraft Reliability Prediction[C]//2019 IEEE Aerospace Conference.IEEE, 2019.DOI:10.1109/AERO.2019.8741783.
10. Zhang L , Wang T , Hu Z .Assessment method of heavy NC machine reliability based on Bayes theory[J].Transactions of Tianjin University, 2016.DOI:10.1007/s12209-016-2567-4.
11. Talafus T P,Pohl E A.Small sample reliability growth modeling using a grey systems model [J].Quality Engineering,2017,29(3):455-467. DOI: 10.1080/08982112.2017.1318920.
12. Xia X .Reliability Analysis of Zero - Failure Data with Poor Information[J].Quality & Reliability Engineering International, 2012, 28(8):981-990.DOI:10.1002/qre.1279.
13. Yang J , Qiu X , Chen Z .The Reliability Evaluation of Antiskid Valve of CRH2 EMUs Based on Bayes Method[J].Periodica Polytechnica Transportation Engineering, 2018.DOI:10.3311/PPtr.11377.
14. Liu X J,Sun Y H,Wang Y S,et al.Reliability evaluation of slip ring based on multi-field coupling modeling and Bootstrap method [J].Journal of Beijing University of Aeronautics and Astronautics,2019,45(11):2301-2311.DOI: 10.13700/j.bh.1001-5965.2019.0106(in Chinese)
15. Chen C H , Yang Z J , Chen F ,et al.Reliability modeling of machining center spindle based on Boosttrap-Bayes[J].Journal of Jilin University(Engineering and Technology Edition), 2014, 44(1):95-100.DOI:10.13229/j.cnki.jdxbgxb201401017.
16. Zhou Q , Thai V V .Fuzzy and grey theories in failure mode and effect analysis for tanker equipment failure prediction[J].Safety Science, 2016, 83:74-79.DOI:10.1016/j.ssci.2015.11.013.
17. Xin-Yao Z .Lifetime Prediction for Exponential Distribution with Small Sample Data[J].Lecture Notes in Electrical Engineering, 2014,

273:505-512.DOI:10.1007/978-3-642-40640-9-64.

18. You, D., Pham, H. Self-adaptive stress accelerated life testing scheme. J Braz. Soc. Mech. Sci. Eng. 39, 2095–2103 (2017). DOI:10.1007/s40430-016-0683-7.
19. Zheng D , Fang X .Exact Confidence Limits for the Acceleration Factor Under Constant-Stress Partially Accelerated Life Tests With Type-I Censoring[J].IEEE Transactions on Reliability, 2017:1-13.DOI:10.1109/TR.2017.2747762.
20. Cai BP , Sheng CY , Gao CT ,et al.Artificial Intelligence Enhanced Reliability Assessment Methodology With Small Samples[J].IEEE Transactions on Neural Networks and Learning Systems, 2021, 34(9).DOI:DOI10.1109/TNNLS.2021.3128514.
21. Guo J, Li Z J, Keyser T. A Bayesian approach for integrating multilevel priors and data for aerospace system reliability assessment [J]. Chinese Journal of Aeronautics, 2018, 31(1): 41-53. DOI:10.1016/j.cja.2017.08.020.
22. Young T J D A .Objective Bayes and conditional inference in exponential families[J].Biometrika, 2010, 97(2):497-504.DOI:10.1093/biomet/asq002.
23. Efron B. Bayesian inference and the parametric Bootstrap. Ann Appl Stat. 2012 Oct 1;6(4):1971-1997. DOI: 10.1214/12-AOAS571.
24. J. N. Corcoran, U. Schneider. Pseudo-perfect and adaptive variants of the Metropolis-Hastings algorithm with an independent candidate density[J]. Journal of Statistical Computation and Simulation, 2005, 75(6): 459-475. https://doi.org/10.1080/00949650410001729463
25. Chen Q , Huang N E , Riemenschneider S D ,et al.A B-spline approach for empirical mode decompositions[J].ADVANCES IN COMPUTATIONAL MATHEMATICS, 2006(1).DOI:10.1007/S10444-004-7614-3.
26. Mingas,Grigorios,Bottolo,et al.Particle MCMC algorithms and architectures for accelerating inference in state-space models.[J].International Journal of Approximate Reasoning, 2017.DOI:10.1016/j.ijar.2016.10.011.
27. Baek E , Ferron J M .Bayesian Analysis for Multiple-baseline Studies Where the Variance Differs across Cases in OpenBUGS[J].Developmental Neurorehabilitation, 2021:1-14.DOI:10.1080/17518423.2020.1858455.
28. Lee J H , Cho Y H , Kim D S ,et al.Lifetime estimation of harmonic reducer for manufacturing robot using accelerated life test[J].Journal of Mechanical Science and Technology, 2022(6):36.DOI:10.1007/s12206-022-0520-0.