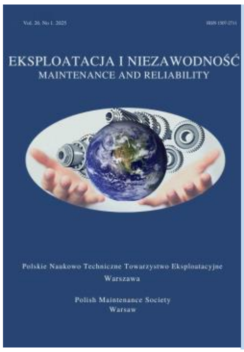

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Random renewable replacements to manage product reliability through a random renewable repair warranty

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Highlights

- Random renewable repair back and front repair warranties are proposed earlier.
- Two random renewable replacements are offered to manage the post-warranty reliability.
- The approaches developed are innovative solutions to overcome the existing limitations.
- The introduced solutions can advance random maintenance theory.

Abstract

Advanced digital technologies facilitate real-time and seamless monitoring of diverse data types throughout the operational lifespan of a product. Consequently, utilizing monitored mission data to devise and model novel approaches for managing the product's reliability over its operational lifespan represents an innovative topic. This paper proposes two random warranties from the fresh perspectives, namely random repair back warranty with renewable coverage (RRBW-RC) and random repair front warranty with renewable coverage (RRFW-RC). Extending the concepts from RRBW-RC, this study presents two random renewable replacements based on the RRBW-RC framework for managing product reliability during the post-warranty coverage: bivariate random renewable back replacement (BRRBR) and univariate random renewable back replacement (URRBR). Finally, the numerical results reveals that as mission cycles statistically elongate, the warranty-service cost increases and the related time lengthens for RRFW-RC, while it shows opposite trends for RRBW-RC; the proposed URRFR is unique and feasible.

Keywords
mission data, reliability, repair back warranty, repair front warranty, renewable replacement

1. Introduction

In the context of utilizing advanced digital technologies to revolutionize multiple industries, there is a growing need to reform the management of reliability throughout its entire operational lifespan. As a result, it is essential to leverage state-of-the-art digital technologies that enable real-time acquisition of diverse data types with utmost accuracy, devoid of any gaps or limitations. The incorporation of this gathered data into the overall operational lifespan for devising and modeling reliability management approaches or controlling system failure risk have emerged as a crucial topic in both industry and academia. For example, in the case of advanced sensor technology monitoring the operational state data, Yang et al. (2023a) constructed a dynamic age-state-dependent intelligent opportunistic maintenance framework for ensuring the safe and reliable operation of wind turbines; Yang et al. (2023b) constructed a prognosis-centered intelligent maintenance framework for enhancing the operational safety and availability of diverse mechatronic systems; using data

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collected by condition monitoring as partial informationa, Yang et al. (2024) investigates controlling system failure risk via combined optimization of sampling and mission abort decisions.

The coverage of the operational lifespan can be categorized into two distinct phases: warranty coverage and post-warranty coverage. Warranty models/strategies/policies aim to effectively manage reliability during the warranty coverage. Conversely, the post-warranty maintenance policies/strategies/models are designed to tackle reliability management during the post-warranty coverage. The pioneers have proposed various solutions that incorporate essential information, such as mission data and degradation data, to effectively manage reliability in terms of warranty coverage and post-warranty coverage. After conducting an extensive review of existing literature on warranty models, it has been observed that advanced digital technologies can be utilized to categorize warranty models into two groups: mission-driven warranty models and degradation-driven warranty models. In this paper, these are also referred to as random warranty models and condition-based warranty models respectively.

The utilization of random warranty models is primarily confined to products that exhibit self-declaring malfunctions, where the loss of one or more functionalities serves as an indication for these failures. The limited missions are implemented as warranty terms to effectively manage warranty expenses and provide flexible services for this specific type of warranty. For instance, Shang et al., (2022) devised and simulated two two-dimensional random repair warranties.

The primary focus of condition-based warranty models based on product conditions revolves around addressing degradation failures in products. These failures occur when the performance level either surpasses or falls below a specified threshold (refer to Ye et al., 2015; Qiu and Cui, 2019a; Zhu et al., 2015; Qiu et al., 2019a; Zhao et al., 2021; Zhao et al., 2020; and Zhang et al., 2023). For instance, Shang et al. (2018) proposed a renewable free replacement warranty model based on an inverse Gaussian process to characterize the degradation failure process. Similarly, Zhang et al. (2018) introduced and modeled a nonrenewable replacement warranty using gamma processes to describe the degradation failure process. Furthermore, recent studies by Wang et al. (2021), Zhao at el. (2023), and Liang et al. (2024), have also put forth alternative warranty models. Additionally, there have been suggestions for different warranty models that do not incorporate advanced digital technologies as well (refer to Qiao at el., 2022; Wang at el., 2020; Banerjee and Bhattacharjee, 2012; Wang at el., 2018; Wang et al., 2017; and Wang et al., 2020).

In the area of warranty mentioned above, it is assumed that mathematical functions predicting changes in reliability (or performance) and failure occurrence, as well as bearing failure modes, are deterministic expressions. This implies that these mathematical functions need to be determined using mathematical methods, machine learning methods, or other approaches. Several scholars and practitioners have been developing reliability methodologies for addressing these needs. For instance, Song et al., (2021) introduced a unified framework by incorporating fuzzy-neuro surrogate into a hierarchical modeling strategy to enhance fatigue reliability efficiently and accurately; Song et al., (2024) proposed a cascade ensemble learning method by integrating the cascade synchronous strategy with wavelet neural network-based AdaBoost ensemble learning; Li et al., (2023) suggested physics-informed modeling approaches for complex reliability evaluations; other advancements in the field of reliability methodologies have been offered in Song et al., (2020) and Li et al., (2021).

After conducting an extensive literature review on the post-warranty maintenance policies, it has been observed that advanced digital technologies have the potential to classify this type of policies into two distinct categories: mission-based and degradation-based warranty models. Similarly, Shang et al., (2021) proposed the former category of policies that utilize limited missions as maintenance terms to effectively manage costs, lengthening remaining lifespan or both. In line with the latter category of maintenance policies, Shang et al., (2018) introduced a condition-based maintenance policy known as a replacement policy, which incorporates degradation data into the post-warranty coverage while considering preventive maintenance. Other studies conducted by Xie et al., 2014; Karar et al., 2023; and Liu and Wang 2023) have also put forth alternative policies without fully acknowledging the value of advanced digital technologies.

Proposed advancements, despite being pioneered by experts from academia and industry, still encounter unresolved critical challenges as outlined below: ① In the scenario where limited missions are used as a constraint for warranties, the warranty models lack flexibility as they offer less coverage to users with shorter mission durations. This can lead to situations of unequal treatment or disparity in warranties. These have been implicitly observed in the context of two-dimensional random repair warranties by Shang et al., (2022). ② The inclusion of limited missions as a warranty restriction broadens the range of random warranty models applicable to users with statistically longer mission durations, resulting in both increased warranty expenses incurred by manufacturers and unequal treatment or disparity. This phenomenon is similarly observed in both two-dimensional random repair warranties last in Shang et al., (2022). ③ The utilization of limited missions as an alternative constraint for post-warranty maintenance policies may result in increased expenses, a reduction in the remaining lifespan of products, or both. Similar observations have been made regarding random periodic replacement last and random periodic replacement first (refer to Shang et al., 2022).

The paper presents innovative approaches for addressing these challenges by classifying operational lifespan into two distinct types of coverage: warranty coverage and post-warranty coverage. From the manufacturer's perspective, two random warranty models are proposed, namely the random repair back warranty with renewable coverage (RRBW-RC) and the random repair front warranty with renewable coverage (RRFW-RC). In these approaches, the defined coverage is renewable for eliminating potential instances of unequal treatment or disparity in existing warranty models, called the renewable coverage. From the consumer's standpoint, two post-warranty maintenance policies are recommended, namely the bivariate random renewable back replacement (BRRBR) and the univariate random renewable back replacement (URRBR). The aforementioned policies also encompass the renewable coverage formed by two decision variables or one decision variable, with the aim of reducing post-warranty expenses or flexibly using the remaining lifespan. The proposed approaches are derived and modeled using probability theory. Numerical analysis on a number of representative approaches can be used to identify the underlying mechanisms and provide guidance for industrial applications.

The advancements discussed in this paper encompass the following aspects: 1) the proposed innovative random warranties can enhance the flexibility of both warranty service and coverage, while addressing concerns associated with existing random models, which include but are not limited to: eliminating potential instances of warranty discrimination or unfairness and reducing manufacturers' increased expenses related to warranties; 2) this paper proposes and models post-warranty maintenance policies, which belong to novel policies for effectively managing post-warranty product reliability, because of reducing post-warranty expenses, flexibly using the remaining lifespan or both; and 3) the inclusion of specific elements in this paper is a contribution to the advancement of warranty theory, maintenance theory or both.

The structure of the paper is as follows: In Section 2, a comprehensive inventory of assumptions is provided, along with the definition and modeling of random warranty models and their associated variants. The BRRBR and URRBR approaches are established in Section 3 through definition and modeling. To examine the underlying mechanisms of these solutions, numerical examinations are carried out in Section 4. Finally, a summary encompassing the entire paper is presented in Section 5.

2. Random renewable repair warranties

The used assumptions include: the product works for missions at mission cycles Y_i ($i = 1,2,\cdots,\infty$), which are random variables subject to a memory-less distribution function given by $G(y)$; the distribution function of the arrival time X of the first failure is expressed as $F(x) = 1 - exp\left(-\int_0^x r(u)\,d\,u\right)$, where $r(u)$ is a failure rate function; and the time to repair/replacement is negligible.

2.1. The definition and modeling of warranty A

As mentioned in above, when limited missions are used as a constraint for warranties (such as the two-dimensional random repair warranties in Shang et al., 2022), unequal treatment or disparity in warranties, increased warranty expenses or both will appear. To address these limitations, a novel random warranty is proposed as follows.

2.1.1. The definition of warranty A

Denoting m by a non-negative integer and letting ϖ be a time span, a coverage is defined as a time duration that is formed by the end of the m^{th} mission cycle or the time span ϖ, whichever occurs first. Applying them, a random warranty is described as:

- *If the m^{th} mission cycle ends before the time span ϖ, then all failures before the end of the m^{th} mission cycle are removed by means of minimal repairs and the coverage is renewed from the end of the m^{th} mission cycle, acting as a starting point;*
- *In the process of renewing the coverage, if the first case where the m^{th} mission cycle doesn't end until the time span ϖ occurs, then all failures before the time span ϖ are removed by means of minimal repairs and the warranty expiries at the time span ϖ.*

In this warranty, minimal repairs are applied to manage reliability; the coverage is renewed as the event that the m^{th} mission cycle ends before the time span ϖ takes place; and the expiry of the warranty service is triggered by an event that the m^{th} mission cycle doesn't end until the time span ϖ, which implies that the m^{th} mission cycle ends completely after the time span ϖ. Considering the reality that the keyword 'after' implies the meaning of the keyword 'back', thus this warranty is called a random repair back warranty with renewable coverage (RRBW-RC).

Compared to the initial study conducted by Shang et al., (2022) on two-dimensional random repair warranty first, our RRBW-RC introduces a renewable coverage that takes into account two specific limitations, and therefore the RRBW-RC ensures fair treatment and eliminates existing disparities by renewing the defined coverage. Namely, the defined coverage in the RRBW-RC is renewable, called the renewable coverage.

For the sake of convenience in the subsequent description, the terms "defined coverage" and "renewable coverage" will be used interchangeably.

2.1.2. The modeling of the RRBW-RC

Let S_m be a random sum produced by m mission cycles; then, the random sum S_m can be measured as $S_m = \sum_{i=1}^{m} Y_i$. When the m^{th} mission cycle ends before the time span ϖ, the renewal of the defined coverage will be triggered. This event can be modeled as an inequality $S_m \le \varpi$. When the event that the m^{th} mission cycle ends after the time span ϖ occurs, the RRBW-RC expiries at the time span ϖ. Such an event can be modeled as an inequality $S_m > \varpi$. The occurrence probabilities of such two events can be expressed as $G^{(m)}(\varpi) = Pr\{S_m \le \varpi\}$ and $\bar{G}^{(m)}(\varpi) = Pr\{S_m > \varpi\}$, respectively. Let κ_b be a renewal number until the m^{th} mission cycle doesn't end at the time span ϖ; then, the renewal number κ_b is a variable following a geometric distribution p_b:

$$p_b = Pr\{\kappa_b = k\} = \left[G^{(m)}(\varpi)\right]^{k-1} \bar{G}^{(m)}(\varpi) \tag{1}$$

where $k = 1, 2, \cdots, \infty$.

Until the m^{th} mission cycle doesn't end at the time span ϖ, $k-1$ renewals have been completed, and the related warranty-servicing time can be expressed as $(k-1)S_m + \varpi$. The total repair cost $TRC_b(k, S_m)$ in the interval from zero to $(k-1)S_m + \varpi$ can be obtained as

$$TRC_b(k, S_m) = c_m \int_0^{(k-1)S_m+\varpi} r(u)\, d\, u \tag{2}$$

where c_m is the unit cost of minimal repair.

When the m^{th} mission cycle ends before the time span ϖ, the defined coverage will be renewed. The random sum S_m under this case satisfies the following distribution $H(s)$:

$$H(s) = Pr\{S_m < s | S_m \le \varpi\} = \frac{G^{(m)}(s)}{G^{(m)}(\varpi)} \tag{3}$$

where s satisfies $s \le \varpi$.

By means of the distribution p_b and $H(\cdot)$, the warranty-servicing cost WSC_b of the RRBW-RC can be obtained as

$$\begin{aligned} WSC_b &= \sum_{k=1}^{\infty}\left([G^{(m)}(\varpi)]^{k-1}\bar{G}^{(m)}(\varpi)\cdot E[TRC_b(k,S_m)]\right) = c_m \sum_{k=1}^{\infty}\left([G^{(m)}(\varpi)]^{k-1}\bar{G}^{(m)}(\varpi)\cdot \int_0^{\varpi}\left(\int_0^{(k-1)s+\varpi} r(u)\, d\, u\right) dH(s)\right) \\ &= c_m \sum_{k=1}^{\infty}\left([G^{(m)}(\varpi)]^{k-2}\bar{G}^{(m)}(\varpi)\cdot\left(\int_0^{\varpi}\bar{G}^{(m)}(s) d\left(\int_0^{(k-1)s+\varpi} r(u)\, d\, u\right) - \bar{G}^{(m)}(\varpi)\int_0^{k\varpi} r(u)\, d\, u\right)\right) + c_m \int_0^{\varpi} r(u)\, d\, u \end{aligned} \tag{4}$$

When the RRBW-RC expiries at the time span ϖ, the warranty-servicing time $WST_b(k, S_m)$ can be expressed as $(k-1)S_m + \varpi$, which has been mentioned above. By means of the distribution p_b and $H(s)$, the warranty-servicing time

WST_b of the RRBW-RC can be obtained as

$$WST_b = \sum_{k=1}^{\infty}\left(\left[G^{(m)}(\varpi)\right]^{k-1}\bar{G}^{(m)}(\varpi)\cdot E[(k-1)S_m+\varpi]\right)$$
$$= \sum_{k=1}^{\infty}\left(\left[G^{(m)}(\varpi)\right]^{k-1}\bar{G}^{(m)}(\varpi)\cdot\left((k-1)\int_0^{\varpi} s dG^{(m)}(s)\,/G^{(m)}(\varpi)+\varpi\right)\right) = \int_0^{\varpi}\bar{G}^{(m)}(s)\,ds/\bar{G}^{(m)}(\varpi) \quad (5)$$

2.2. The definition and modeling of warranty B

In the RRBW-RC, the defined coverage is renewed as the event that the m^{th} mission cycle ends before the time span ϖ takes place. In this section, revising such a term, the second random warranty is proposed, as shown below.

2.1.1. The definition of warranty B

By means of m, ϖ and the defined coverage in the RRBW-RC, another random warranty is summarized as:

• *If the m^{th} mission cycle doesn't end until the time span ϖ, then all failures before the time span ϖ are removed by means of minimal repairs and the defined coverage is renewed from the time span ϖ;*

• *In the process of renewing the defined coverage, if the first case where the m^{th} mission cycle ends before the time span ϖ occurs, then all failures before the end of the m^{th} mission cycle are removed by means of minimal repairs and the warranty expiries at the end of the m^{th} mission cycle.*

The expiry of this warranty is triggered by an event that the m^{th} mission cycle ends before the time span ϖ, which differs from the RRBW-RC. Considering the reality that the keyword 'before' implies the meaning of the keyword 'front', thus this warranty is called a random repair front warranty with renewable coverage (RRFW-RC).

Compared to the initial study conducted by Shang et al., (2022) on two-dimensional random repair warranty last, our research introduces renewable coverage that takes into account two specific limitations in the RRFW-RC. Hence, through the process of renewing the specified coverage, the RRFW-RC can guarantee equitable handling, eradicate prevailing discrepancies, and minimize escalated warranty costs.

2.1.2. The modeling of the RRFW-RC

The case where the m^{th} mission cycle ends after the time span ϖ can be characterized as an inequality $\varpi \le S_m$; the case where the m^{th} mission cycle ends before the time span ϖ can be modeled as an inequality $S_m < \varpi$. The occurrence probabilities of both can be given by $\bar{G}^{(m)}(\varpi)$ and $G^{(m)}(\varpi)$, respectively. Let κ_f be a renewal number until the m^{th} mission cycle doesn't end until the time span ϖ; then, the renewal number κ_f is a variable following a geometric distribution p_f:

$$p_f = Pr\{\kappa_f = k\} = \left[\bar{G}^{(m)}(\varpi)\right]^{k-1}G^{(m)}(\varpi) \quad (6)$$

Similar to Eq. (2), the total repair cost $TRC_f(k,S_m)$ until the RRFW-RC expiring can be obtained as

$$TRC_f(k,S_m) = c_m\int_0^{(k-1)\varpi+S_m} r(u)\,d\,u \quad (7)$$

By means of the distribution p_f and $H(\cdot)$, the warranty-servicing cost WSC_f of the RRFW-RC can be obtained as

$$WSC_f = \sum_{k=1}^{\infty}\left([\bar{G}^{(m)}(\varpi)]^{k-1}G^{(m)}(\varpi)\cdot E\left[TRC_f(k,S_m)\right]\right) = \sum_{k=1}^{\infty}\left([\bar{G}^{(m)}(\varpi)]^{k-1}G^{(m)}(\varpi)\cdot\int_0^{\varpi}\left(\int_0^{(k-1)\varpi+s_m} r(u)\,d\,u\right)dG^{(m)}(s)\,/G^{(m)}(\varpi)\right)$$
$$= c_m\sum_{k=1}^{\infty}\left([\bar{G}^{(m)}(\varpi)]^{k-1}\cdot\left(\int_0^{\varpi}\bar{G}^{(m)}(s)d\left(\int_0^{(k-1)\varpi+s} r(u)\,d\,u\right)-\bar{G}^{(m)}(\varpi)\int_0^{k\varpi} r(u)\,d\,u+G^{(m)}(\varpi)\int_0^{(k-1)\varpi} r(u)\,d\,u\right)\right) \quad (8)$$

Until the RRFW-RC expires, the warranty-servicing time $WST_f(k,S_m)$ can be expressed as

$$WST_f(k,S_m) = (k-1)\varpi + S_m \quad (9)$$

where has been used in Eq. (7).

By means of the distribution p_f and $H(\cdot)$, the warranty-servicing time WST_f of the RRFW-RC can be obtained as

$$WST_f = \sum_{k=1}^{\infty}\left(\left[\bar{G}^{(m)}(\varpi)\right]^{k-1}G^{(m)}(\varpi)\cdot E\left[WST_f(k,S_m)\right]\right)$$
$$= \sum_{k=1}^{\infty}\left(\left[\bar{G}^{(m)}(\varpi)\right]^{k-1}G^{(m)}(\varpi)\cdot\left((k-1)\varpi+\int_0^{\varpi}sdG^{(m)}(s)\,/G^{(m)}(\varpi)\right)\right) = \int_0^{\varpi}\bar{G}^{(m)}(s)ds\,/G^{(m)}(\varpi) \quad (10)$$

2.3. Analysis and discussion

When $m \to \infty$, it is obvious for $\lim_{m\to\infty}\bar{G}^{(m)}(\varpi) = 1$ and $\lim_{m\to\infty}G^{(m)}(\varpi) = 0$ to hold. This signals that $m \to \infty$ makes that the m^{th} mission cycle ends after the time span ϖ. Therefore, $m \to \infty$ reduces the RRBW-RC to a classic repair warranty (CRW). By means of $\lim_{m\to\infty}\bar{G}^{(m)}(\varpi) = 1$ and $\lim_{m\to\infty}G^{(m)}(\varpi) = 0$, the warranty-servicing cost of the CRW can be obtained as

$$\lim_{m\to\infty} WSC_b = c_m\int_0^{\varpi} r(u)\,d\,u \quad (11)$$

Likewise, by means of $\lim_{m\to\infty}\bar{G}^{(m)}(\varpi) = 1$ and $\lim_{m\to\infty}G^{(m)}(\varpi) = 0$, the warranty-servicing time of the CRW can be obtained as

$$\lim_{m\to\infty} WST_b = \int_0^{\varpi}\lim_{m\to\infty}\bar{G}^{(m)}(s)\,ds/\lim_{m\to\infty}\bar{G}^{(m)}(\varpi) = \varpi \quad (12)$$

In the case of $\lim_{m\to\infty}\bar{G}^{(m)}(\varpi) = 1$ and $\lim_{m\to\infty}G^{(m)}(\varpi) = 0$, $m \to \infty$ reduces the RRFW-RC to a life-time repair warranty (LTRW). By means of $\lim_{m\to\infty}\bar{G}^{(m)}(\varpi) = 1$ and $\lim_{m\to\infty}G^{(m)}(\varpi) = 0$, the warranty-servicing cost of the LTRW can be obtained as

$$\lim_{m\to\infty} WSC_f = \infty \quad (13)$$

Likewise, by means of $\lim_{m\to\infty}\bar{G}^{(m)}(\varpi) = 1$ and $\lim_{m\to\infty}G^{(m)}(\varpi) = 0$, the warranty-servicing time of the LTRW can be obtained as

$$\lim_{m\to\infty} WST_f = \int_0^{\varpi}\lim_{m\to\infty}\bar{G}^{(m)}(s)ds\,/\lim_{m\to\infty}G^{(m)}(\varpi) = \infty \quad (14)$$

3. Random maintenance policies

As mentioned above, each of the RRBW-RC and RRFW-RC can manage the warranty-coverage reliability. After the RRBW-RC or RRFW-RC expires, the corresponding product will enter into the post-warranty coverage. In this section, two random maintenance policies are designed and modeled for managing the products' reliabilities after the warranty coverage (i.e., the reliabilities over the post-warranty coverage or the post-warranty reliabilities), taking the RRBW-RC as an example.

3.1. The definition and modeling of random maintenance policies

By applying the principles derived from RRBW-RC to the post-warranty coverage, two random maintenance policies are formulated and simulated in the following subsections.

3.1.1. The definition of random maintenance policies

Let T be a post-warranty time span and N be a non-negative integer, which are two decision variables; it is defined that a bivariate coverage is formed by the end of the N^{th} mission cycle or the post-warranty time span T, whichever occurs first; then, the terms of the first random maintenance policy include:

- *If the N^{th} mission cycle ends before the post-warranty time span T, then the bivariate coverage is renewed from the end of the N^{th} mission cycle;*
- *In the process of renewing the bivariate coverage, if the first case where the N^{th} mission cycle doesn't end until the post-warranty time span T occurs, then the product is replaced at the post-warranty time span T;*
- *All failures are removed by means of minimal repairs.*

Obviously, this random maintenance policy is a bivariate replacement policy because the mission number N and the post-warranty time span T are two decision variables. Similar to naming the RRBW-RC, this policy is named as a bivariate random renewable back replacement (BRRBR) based on the reality that the keyword 'before' implies the meaning of the keyword 'front'.

The measure $N \to 1$ makes decision variable N reduced to a constant, which implies that decision variable number is reduced. Therefore, $N \to 1$ reduces the BRRBR to an univariate random renewable back replacement (URRBR). The URRBR can effectively reduce post-warranty costs due to the shorter coverage formed by the end of the first mission cycle or the post-warranty time span T, compared to the BRRBR.

Clearly, by renewing the designated coverage with the constraint of two decision variables or one decision variable (namely the defined coverage is renewable), both BRRBR and URRBR have the potential to decrease post-warranty costs, effectively utilize the remaining lifespan, or both.

3.1.2. The modeling of the BRRBR

The policies encompass both BRRBR and URRBR. Since URRBR is a specific instance of BRRBR in the case of $N = 1$, we solely focus on modeling BRRBR, subsequently modeling URRBR by setting $N = 1$, as illustrated below.

The case where the N^{th} mission cycle ends before the post-warranty time span T can be modeled as an inequality $S_N \le T$, where S_N is the total length produced by N mission cycles and the case where the N^{th} mission cycle ends after the post-warranty time span T can be modeled as an inequality $T < S_N$. The occurrence probabilities of both can be listed as $G^{(N)}(T)$ and $\bar{G}^{(N)}(T)$, respectively. Let N_b be a renewal number until the N^{th} mission cycle doesn't end at the post-warranty time span T; then, the renewal number N_b is a random variable with geometric distribution Q_b given by

$$Q_b = Pr\{N_b = n\} = \left[G^{(N)}(T)\right]^{n-1}\bar{G}^{(N)}(T) \tag{15}$$

According to Eq. (6), when the RRBW-RC expires, the age of the product is equal to $(k-1)S_m + \varpi$. By means of the reliability theory, the failure rate function at age $(k-1)S_m + \varpi$ can be expressed as $r((k-1)S_m + \varpi + u)$. Obviously, the post-warranty repair cost $PWRC_{b_1}(N,T|n,S_N,S_m,k)$ formed by $n-1$ renewals can be obtained as

$$\begin{aligned} &PWRC_{b_1}(N,T|n,S_N,S_m,k) \\ &= c_m\left(\begin{aligned}&\int_0^{S_N} r((k-1)S_m + \varpi + u)\,d\,u + \\ &\int_{S_N}^{2S_N} r((k-1)S_m + \varpi + u)\,d\,u + \cdots + \\ &\int_{(n-2)S_N}^{(n-2)S_N+S_N} r((k-1)S_m + \varpi + u)\,d\,u\end{aligned}\right) \\ &= c_m\int_0^{(n-1)S_N} r((k-1)S_m + \varpi + u)\,d\,u \end{aligned} \tag{16}$$

If the case where the N^{th} mission cycle doesn't end until the post-warranty time span T occurs, the product will be replaced. The post-warranty repair cost $PWRC_{b_2}(N,T|n,S_N,S_m,k)$ resulting from such a replacement can be obtained as

$$\begin{aligned} &PWRC_{b_2}(N,T|n,S_N,S_m,k) \\ &= c_m\int_{(n-1)S_N}^{(n-1)S_N+T} r((k-1)S_m \\ &+ \varpi + u)\,d\,u \end{aligned} \tag{17}$$

By means of the distribution Q_b and $H(\cdot)$, the post-warranty cost $PWC_b(N,T|k)$ of the BRRBR can be obtained as

$$\begin{aligned} PWC_b(N,T|k) &= \sum_{n=1}^{\infty}\left(\left[G^{(N)}(T)\right]^{n-1}\bar{G}^{(N)}(T)\cdot E\left[PWRC_{b_1}(N,T|n,S_N,S_m,k) + PWRC_{b_2}(N,T|n,S_N,S_m,k)\right]\right) + C_R \\ &= c_m\sum_{n=1}^{\infty}\left(\left[G^{(N)}(T)\right]^{n-1}\bar{G}^{(N)}(T)\cdot\int_0^T\left(\int_0^{(n-1)s+T} r((k-1)S_m + \varpi + u)\,d\,u\right)d\,J(s)\right) + C_R \\ &= c_m\sum_{n=1}^{\infty}\left(\left[G^{(N)}(T)\right]^{n-2}\bar{G}^{(N)}(T)\cdot\int_0^{\varpi}\left(\int_0^T\left(\int_0^{(n-1)s+T} r((k-1)s_m + \varpi + u)\,d\,u\right)d\,G^{(N)}(s)\right)dH(s_m)\right) + C_R \end{aligned} \tag{18}$$

where $J(s) = Pr\{S_N < s|S_N \le T\} = G^{(N)}(s)/G^{(N)}(T)$ and C_R is the unit replacement cost.

Until the replacement occurs at the post-warranty time span T, the post-warranty servicing time $PWST_b(N,T|n,S_N)$ can be obtained as

$$PWST_b(N,T|n,S_N) = (n-1)S_N + T \tag{19}$$

By means of the distribution Q_b and $H(\cdot)$, the post-warranty time $PWT_b(N,T)$ of the BRRBR can be given by

$$\begin{aligned} PWT_b(N,T) &= \sum_{n=1}^{\infty}\left([G^{(N)}(T)]^{n-1}\bar{G}^{(N)}(T)\cdot E[PWST_b(T|n,S_N)]\right) \\ &= \frac{\int_0^T \bar{G}^{(N)}(s)\,ds}{\bar{G}^{(N)}(T)} \end{aligned} \tag{20}$$

The total failure cost TFC_b produced by the RRBW-RC is obtained by replacing c_m in the warranty-servicing time WSC_b in Eq. (5) with the unit failure cost c_f. Then, using the distribution p_b, the expected cost rate $CR_b(N,T)$ of the

BRRBR can be calculated as

$$CR_b(N,T) = \frac{TFC_b + \sum_{k=1}^{\infty}\left([G^{(m)}(\varpi)]^{k-1}\bar{G}^{(m)}(\varpi)\cdot PWC_b(N,T|k)\right)}{WST_b + PWT_b(N,T)}$$

$$= \frac{TFC_b + C_R + c_m\sum_{k=1}^{\infty}\left([G^{(m)}(\varpi)]^{k-2}\bar{G}^{(m)}(\varpi)\cdot\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}[G^{(N)}(T)]^{n-2}\bar{G}^{(N)}(T)\cdot\\ \int_0^{\varpi}\left(\int_0^{T}\left(\int_0^{(n-1)s+T} r((k-1)s_m+\varpi+u)\,d\,u\right)d\,G^{(N)}(s)\right)dG^{(m)}(s_m)\end{array}\right)\right)\right)}{WST_b + \int_0^T \bar{G}^{(N)}(s)\,ds/\bar{G}^{(N)}(T)} \quad (21)$$

In this paper, the proposed policies are modeled using cost rate, a widely-used assessment criterion in the field of reliability (refer to Wang et al., 2021a; Wang et al., 2021b; Wang et al., 2023; and Chen et al., 2022). Additionally, availability criteria have been employed by Qiu et al., (2017a); Qiu et al., (2017b); Qiu et al., (2018a); Qiu et al., (2018b); Qiu and Cui (2019b); Qiu and Cui (2019c); Qiu and Cui (2019d); Qiu et al., (2019b); Qiu et al., (2019c); Qiu et al., (2021); Qiu et al., (2020); and Lin et al., (2023) as another means of evaluation.

As mentioned in above, $N \to 1$ reduces the BRRBR to an univariate random renewable back replacement (URRBR). Hence, utilizing $N \to 1$, the expected cost rate $CR_b(T)$ of the URRBR can be calculated as

$$CR_b(T) = \frac{TFC_b + C_R + c_m\sum_{k=1}^{\infty}\left(\left[G^{(m)}(\varpi)\right]^{k-2}\bar{G}^{(m)}(\varpi)\cdot\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}[G(T)]^{n-2}\bar{G}(T)\cdot\\ \int_0^{\varpi}\left(\int_0^{T}\left(\int_0^{(n-1)s+T} r((k-1)s_m+\varpi+u)\,d\,u\right)d\,G(s)\right)dG^{(m)}(s_m)\end{array}\right)\right)\right)}{WST_b + \int_0^T \bar{G}(s)\,ds/\bar{G}(T)} \quad (22)$$

4. Numerical investigation

The previous section introduced two random warranty models and two random replacements policies for managing the operation-lifespan reliabilities of products under mission cycles. However, extracting the hidden mechanisms from an analytical perspective has proven challenging. Therefore, in order to explore these hidden mechanisms numerically, this section will examine specific examples such as RRBW-RC, RRFW-RC, and URRBR.

Excavator robots are becoming increasingly prevalent in the mining and construction sectors. These robots incorporate an integrated management system that utilizes advanced digital technologies, enabling real-time and seamless monitoring of various types of data throughout their operational lifespan, including mission data and work area information. By utilizing the monitored data, managers can effectively implement proposed methods for excavator robot operations. Therefore, excavator robots are selected as the subject for a case study.

The failure rate functions of certain excavator robots exhibit an increasing trend with respect to their servicing time, which is a practical reality. To accurately model this behavior, the failure rate functions are defined as an increased function: $\gamma(u) = a(u)^b$, where $a > 0$ and $b > 0$. Additionally, the mission cycles Y_i have been considered as random variables following a memory-less distribution function denoted by $G(y)$. In reliability theory, the exponential distribution function is commonly used to represent memory-less distributions. Therefore, $G(y) = 1 - exp(-y\lambda)$ is employed as the memory-less distribution function for modeling the mission cycles Y_i, where $\lambda > 0$. Some of parameters are assigned as: $a = 0.8$, $b = 1$, and $c_m = 0.1$, and other parameters will be assigned wherever used.

4.1. Numerical analysis of the RRBW-RC and RRFW-RC

In order to overcome the difficulty of displaying it in figures, the mathematical symbol ϖ will be substituted with either ω or w.

4.1.1. Numerical analysis of the RRBW-RC

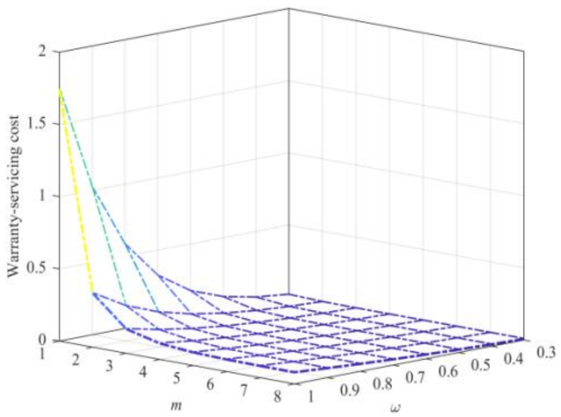

Figure 1. The effects of ϖ and m on the warranty-servicing cost of the RRBW-RC

In the case of using $\lambda = 2$, Figure 1 is used to investigate the impact of time span ϖ and mission number m on the warranty-servicing cost of the RRBW-RC. Figure 1 clearly indicates a positive correlation between the time span ϖ and the warranty-servicing time of the RRBW-RC, while there is an inverse correlation with respect to mission number m. The findings indicate that the expansion of the renewable coverage in the RRBW-RC does not necessarily lead to an increase in warranty-servicing costs.

For examining the influence of time span ϖ and mission number m on the warranty-servicing time of the RRBW-RC, Figure 2 has been offered using $\lambda = 2$. It is evident from Figure 2 that time span ϖ exhibits a positive correlation with the warranty-servicing time of the RRBW-RC while mission number m exhibits an inverse correlation with the warranty-servicing time of the RRBW-RC. These are identical to those mentioned Figure 2.

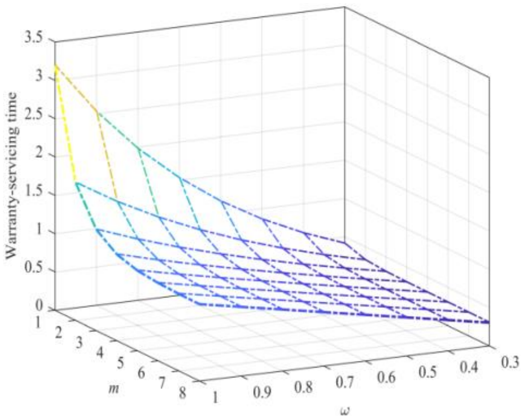

Figure 2. The effects of m and ϖ on the warranty-servicing time of the RRBW-RC

The findings from both Figure 1 and Figure 2 indicate that the cost and time measures of the RRBW-RC exhibit identical changes.

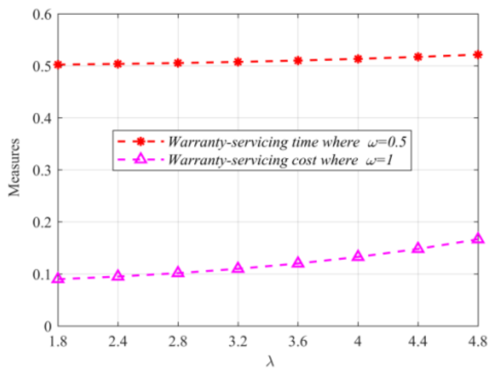

Figure 3. The effects of λ on the measures of the RRBW-RC

The findings suggest that when mission cycles exhibit statistical elongation, there is a simultaneous reduction in both the cost and time measures of the RRBW-RC.

4.1.2. Numerical analysis of the RRFW-RC

For examining the influence of mission number m and time span ϖ on the warranty-servicing cost of the RRFW-RC, Figure 4 has been obtained using $\lambda = 5$. It is evident from Figure 4 that time span ϖ exhibits a negative correlation with the warranty-servicing cost of the RRFW-RC; mission number m exhibits a positive correlation with the warranty-servicing cost of the RRFW-RC.

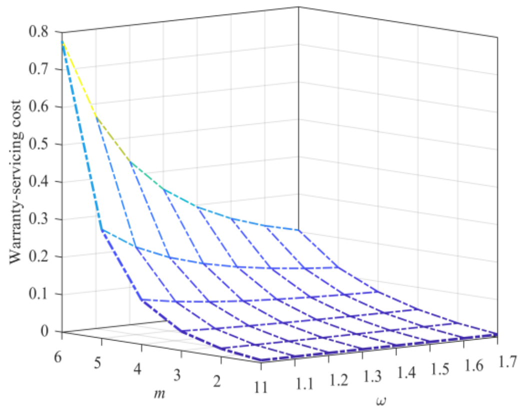

Figure 4. The effects of m and ϖ on the warranty-servicing cost of the RRFW-RC

The findings in Figure 4 differ from that in Figure 1, yet they both support the same conclusion that the expansion of renewable coverage does not necessarily result in an increase in warranty-servicing costs.

In the case of using $\lambda = 5$, Figure 5 is used to examine the influence of mission number m and time span ϖ on the warranty-servicing time of the RRFW-RC. It is evident from Figure 5 that mission number m exhibits a positive correlation with the warranty-servicing time of the RRFW-RC while time span ϖ exhibits a negative correlation with the warranty-servicing time of the RRFW-RC.

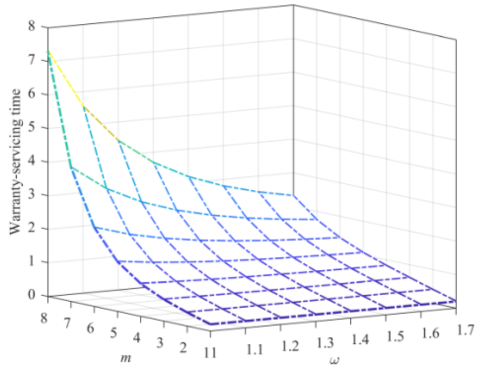

Figure 5. The effects of m and ϖ on the warranty-servicing time of the RRFW-RC

In the case of using $\lambda = 5$, Figure 5 is used to examine the influence of mission number m and time span ϖ on the warranty-servicing time of the RRFW-RC. It is evident from Figure 5 that mission number m exhibits a positive correlation with the warranty-servicing time of the RRFW-RC while time span ϖ exhibits a negative correlation with the warranty-servicing time of the RRFW-RC.

The findings suggest that the variations in warranty-servicing time for the RRFW-RC are consistent with the fluctuations in warranty-servicing cost for the same product.

For examining the influence of parameter λ on the cost and time measures of the RRFW-RC, Figure 6 is got using $\varpi = 1$. It is evident from Figure 6 that parameter λ exhibits a negative correlation with the measures of the RRFW-RC. The variation suggests that if the mission cycles exhibit statistical elongation, then the warranty-servicing cost of the RRFW-RC will be increased while the warranty-servicing time will be prolonged.

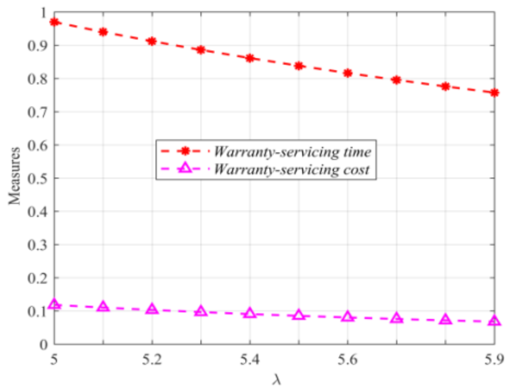

Figure 6. The effects of λ on the measures of the RRFW-RC

Obviously, the findings in Figure 6 differ from that in Figure 3.

4.2. Numerical analysis of the URRBR

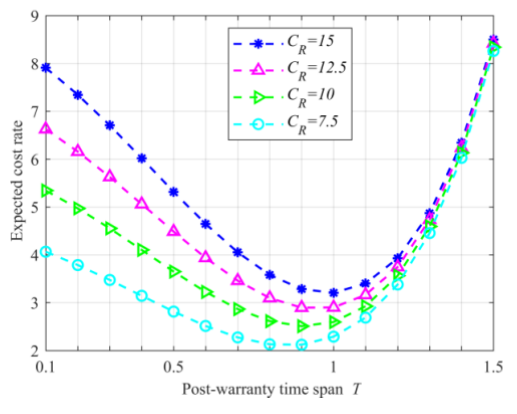

Figure 7. The uniqueness and feasibility of optimal URRFR

For validating the uniqueness and feasibility of optimal URRFR, **Figure 7** has been provided using $\lambda = 1$, $m = 3$, $\varpi = 1$, and $c_f = 0.1$. It is evident from Figure 7 that: the uniqueness of optimal URRFR exists and is feasible; the optimal post-warranty time span T^* increases as the unit replacement cost C_R increases, and the optimal cost rate increases with respect to the increase in this measure.

In order explores how the limitations m and ϖ of the renewable coverage in the RRBW-RC affect the optimal URRFR, Figure 8 has been obtained using $\lambda = 2$, $C_R = 15$, and $c_f = 0.1$. It is evident from Figure 8(A) that: the optimal post-warranty time span T^* increases as the limitation m increases and the optimal cost rate also increases with respect to the increases in this measure. Moreover, It is evident from Figure 8(B) that: the optimal post-warranty time span T^* increases as the limitation ϖ decreases, and the optimal cost rate also increases with respect to the decrease in this measure.

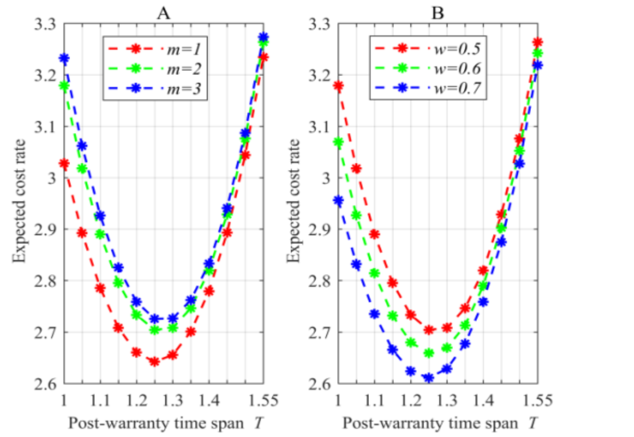

Figure 8. The effects of m and ϖ on the optimal URRFR

To explore how both parameter λ and the unit failure cost c_f affect the optimal URRFR, Figure 9 has been presented using where $\varpi = 0.5$, $m = 2$, and $C_R = 15$.

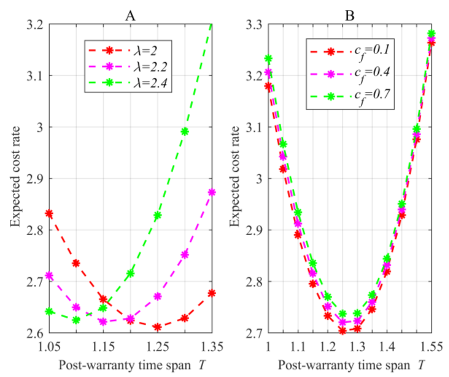

Figure 9. The effects of λ and c_f on the optimal URRFR

It is evident from Figure 9(A) that: the optimal post-warranty time span T^* decreases as parameter λ decreases while the optimal cost rate increases with respect to the increase in this measure. Moreover, It is evident from Figure 9(B) that: the optimal post-warranty time span T^* decreases as the unit failure cost c_f increases and the optimal cost rate increases with respect to the increase in this measure.

5. Conclusions

The advancement of digital technologies enables practical support for monitoring, recording and transferring lifespan mission cycles. As a result, the use of monitored mission cycles to manage the lifespan reliabilities of products becomes an inevitable trend. In view of this, this paper proposes and formulates two random warranty models called random repair back warranty with renewable coverage (RRBW-RC) and random repair front warranty with renewable coverage (RRFW-RC). In these approaches, the coverage formed by limited missions and time span are renewed based on the triggered events to control costs and provide flexible service. Bivariate random renewable back replacement (BRRBR) and univariate random renewable back replacement (URRBR) are two random maintenance policies introduced within the RRBW-RC framework to effectively manage product reliability through the RRBW-RC. These policies are extensions of ideas from the proposed RRBW-RC and include also the renewable coverage formed by decision variables. Using some of the proposed approaches as typical examples, the management insights are gained through numerical analysis, while the uniqueness and feasibility of the proposed URRFR are also validated through such an analysis.

The proposed approaches, although novel methodologies for addressing critical limitations in existing multi-dimensional random warranty models, raise concerns regarding the accuracy and validity of their practical engineering applications. These concerns will be addressed in future research endeavors.

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