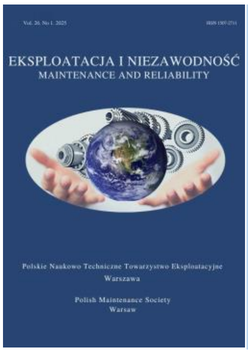

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Research on Equipment Reliability Modeling and Periodic Maintenance Strategies in Dynamic Environment

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Highlights

- The reliability model of equipment considering dynamic environment is established.
- The concept of maintenance cycle is redefined, and the periodic equipment maintenance decision model is established.
- The change trend of equipment maintenance strategy in different environments is discussed, and the adjustment scheme is provided for different regional environments.

Abstract

Equipment along the railway has different degradation paths and reliability in different environments. It is of great significance to study the reliability and maintenance strategy of equipment in dynamic environment to ensure the safety of train operation. In this paper, a method of equipment reliability modeling and periodic maintenance decision-making in dynamic environment is proposed. A reliability model considering the degradation and impact of environmental changes and considering the characteristics of the equipment is established. A new concept of maintenance cycle is defined, and a decision model of equipment periodic maintenance is established. Finally, through numerical simulation, it is concluded that under severe environment, the maintenance time is shortened by 29508h and 31639h respectively, and the maintenance cost is increased by 41.4% and 28.2% respectively. Furthermore, the change trend and adjustment scheme of the strategy under different operating environments are analyzed, which verifies the correctness and effectiveness of the model in this paper.

Keywords

dynamic environment, natural degradation, random shock, reliability, maintenance decision

1. Introduction

In industrial manufacturing and economic production, the degradation process of equipment ineffective service age during operation is not only determined by its own characteristics and working intensity, but also closely related to the operating environment [1]. In particular, some equipment across multiple geographical regions, such as trackside equipment along the railway, power transmission equipment, etc., have different degradation rates or failure modes in different regions due to various climatic environments in different geographical regions. Therefore, it is of great theoretical significance and practical value to study the impact of the environment on the equipment degradation process and formulate reasonable maintenance strategies.

According to different factors, the impact of environment on the equipment is mainly manifested in the natural degradation rate and random shock strength of the equipment. At the same time, it has a comprehensive effect on the degradation path of the equipment, such as wear, deformation, overload, corrosion, etc. The failure occurs when these degradation quantities accumulate to the equipment failure threshold. However, some

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equipment will also exhibit self-healing behavior after shock according to its own characteristics, reducing the increment of degradation. In a word, based on the state evolution trend of equipment, it is indispensable to establish a reliability model with solid generalization ability, which plays a supporting role in revealing the degradation process and failure mode of equipment, depicting the individual portrait of equipment, and formulating a reasonable maintenance strategy and adjustment scheme according to the dynamic environment.

Equipment degradation modeling is the critical method for analyzing equipment degradation path and establishing its reliability model. There are two main types of equipment degradation modeling. One is to treat the degradation process of equipment as several discrete states, which can generally be represented by Markov chains. The equipment transitions with a certain probability, and the residence time of each state follows a general distribution, so as to describe the degradation process of equipment and establish its failure model and reliability model [2,3,4]. Another way is to use the degradation path of equipment to directly model, such as the change of battery capacity and internal resistance, capacitance, bearing vibration amplitude, wear degree of electromechanical system, etc. Many scholars have conducted a lot of research on this kind of modeling method in different application scenarios. Chao Zhang et al. Used random process to simulate the degradation process of aircraft hydraulic system, and considered the influence of random shock on equipment reliability [5]; Minqiao Song et al. Described the degradation process of liquid coupling device by the Wiener process, and divided the degradation of equipment into two stages with different degradation rates to accurately describe the degradation process of equipment and formulate a reasonable maintenance plan [6]; Xiaoyang Ma et al. Described the temperature change of the cooler with a multi-stage Wiener process, and obtained the reliability model and state based maintenance strategy [7]; Anqi Shangguan et al. A competitive failure degradation model based on multiple Wiener processes was constructed to explain the heterogeneity of each unit in the system [8]. Some scholars use gamma and inverse Gaussian processes to describe equipment degradation processes. For example, Zhengqiang Pan et al. The gamma process was used to describe the degradation process of a two-component series parallel structure, and the Monte Carlo simulation was used to give the confidence interval of the failure probability of equipment [9]; Yixuan Geng et al. The inverse Gaussian process was used to establish a multi-stage degradation and shock model from the energy perspective [10]. Although there are different choices of stochastic processes to describe degradation, the research mainly focuses on the combination of natural degradation and random shock, which is divided into a variety of degradation stages, as well as the competitive relationship and interaction between shock and degradation. The competition between degradation and shock believes that the shock has general shock and fatal shock, which obey a certain probability distribution, and both of them may cause the final failure of the equipment [11-15]. The research on the interaction between degradation and shock believes that degradation will affect the strength of shock, and shock will also change the degradation rate. The two affect and interact with each other until the equipment fails. This situation can also be extended to the relationship between a variety of degradation and shock [16,17,18,19,]. While establishing the degradation model, some authors also discussed the corresponding maintenance decision-making model, and obtained a reasonable maintenance scheme.

The above research assumes that the equipment's operating environment will not change. However, for large-scale equipment arranged across regions, even if the environmental status of the same region is similar, the operating environment between different regions is quite different, resulting in the degradation model being unable to adapt to the changes of the equipment operating environment. For the degradation model of dynamic changes in the operating environment, Hasan Misaii and other scholars considered the impact of environment and shock and established a multiplicative failure model [20]. When the equipment is in a discrete degradation state, some scholars regard environmental information as the covariate of degradation information, use proportional hazard model to show the impact of environmental covariates on degradation, and discuss the working conditions such as environmental renewal [21,22,23]. It is worth noting that in the dynamic environment, the natural degradation and random shock are combined with the analysis of the self-healing characteristics of the equipment, and the reliability model and life distribution are established, making the model more consistent with the degradation path of the equipment with self-healing characteristics [24,25,26,27]. Although environmental factors are considered in the above study, it is still regarded as an environmental state with discrete state division, affecting equipment degradation according to different states. Therefore, the key to describe the degradation process of equipment is to establish the model of natural degradation and random shock of equipment under the influence of continuous environmental changes and combine the characteristics of equipment.

On the basis of equipment degradation path modeling, many scholars have developed imperfect preventive maintenance or replacement strategies. These studies mainly rely on the residual life or degradation threshold of equipment degradation process to determine the maintenance mode and interval, and measure the advantages and disadvantages of the strategy by the long-term maintenance cost or availability of equipment [7,15,21,22]. However, the equipment maintenance strategy and its adjustment scheme under the influence of continuous temperature and humidity are still blank. Based on the above research, the main problems are as follows:

(1) The continuous change model of environmental parameters should be established according to the actual operation environment of the equipment to provide refined reliability model support for the large-scale cross regional equipment degradation path;

(2) The above research is to consider the constraints of the actual operation and management cycle of the system, which makes the maintenance strategy difficult to reasonably implement in the management unit. It should be combined with the periodic constraints of the system operation and management unit to develop a reasonable maintenance strategy, and provide the maintenance strategy adjustment scheme for the equipment in the operation environment of different geographical regions.

This paper aims to explore the change process and reasonable scheme of equipment maintenance strategy based on the reliability model under the dynamic environment of equipment, so as to ensure the safe and reliable operation of equipment and fill in the blank of relevant research. It is worth mentioning that literature [2] proposed a failure risk model between maintenance intervals. Inspired by this, this paper expects to establish the periodic maintenance strategy of electronic devices in a dynamic environment, take the fixed cycle of maintenance and the number of imperfect preventive maintenance as the combined variables, combine the failure rate risk mapping model, and formulate the maintenance strategy and adjustment scheme that meets the large-scale cross regional deployment of equipment. To sum up, the main contributions of this paper are as follows:

(1) By introducing peck temperature and humidity acceleration factor and considering the self-healing characteristics of equipment, the natural degradation and random shock model of equipment in a continuous dynamic environment based on the Wiener process is established, and the reliability expression of equipment is derived;

(2) The concept of maintenance cycle is redefined, and the maintenance decision-making model under the dynamic environment of equipment is established based on the reliability of equipment;

(3) Through numerical simulation, the spatial distribution of the model solution is obtained, the combined maintenance strategy is selected, the changing trend of the strategy in different environments is analyzed, and the adjustment scheme in different regional environments is given.

The rest of this paper is structured as follows. Section 2 establishes the reliability model of natural degradation and random shock under dynamic environment, and carries out sensitivity analysis; Section 3 establishes the periodic equipment maintenance decision model; In Section 4, numerical simulation is carried out for large-scale equipment arranged across regions to analyze the impact of the environment on the strategy; Section 5 gives the conclusion and outlook.

2. Reliability Model

During the actual operation of the equipment, when the environment is stable, the natural degradation process approaches the fault threshold with a certain rule, and this change trend can be regarded as a steady incremental process. However, the degradation of the equipment is affected by the external environment as well as the interference of internal factors, such as voltage and current overload, vibration overrun, etc., which cause the shock on the equipment and lead to the sudden change of equipment degradation increment. Therefore, the equipment degradation process model is mainly composed of natural degradation process and shock process under environmental impact, which can be expressed as:

$$D(t) = X(t) + S(t) \tag{1}$$

where $X(t)$ represents the natural degradation part of the equipment, $S(t)$ represents the random shock degradation part, and $D(t)$ represents the total degradation amount of the equipment.

2.1. Natural Degradation Model

(1) Foundation degradation model

The natural degradation of equipment is generally an independent incremental process, which random processes, such as the Weiner process, gamma process and inverse Gaussian process can represent. Weiner process is widely used in the degradation simulation of electronic devices due to its good computational and statistical properties. Therefore, this paper selects the Weiner process as the basic model of equipment degradation, and its expression is:

$$X(t) = X_0 + \mu t + \sigma B(t) \tag{2}$$

where, X_0 represents the initial value of equipment degradation; μ is the Weiner process drift coefficient, which directly reflects the rate of equipment degradation; σ is the diffusion coefficient of the Weiner process, reflecting the internal differences between equipment; $B(t)$ is the standard Brownian motion, and $B(t) \sim N(0, t)$. When the device experiences failure from $X(0) = 0$ to $X(t) > L$, its probability density function at time t is expressed as Equation 3.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2 t}} exp\left(-\frac{(x-\mu t)^2}{2\sigma^2 t}\right) \tag{3}$$

Then the equipment reliability can be expressed as:

$$R(t) = P\{X(t) < L\} = \Phi\left(\frac{L-\mu t}{\sigma\sqrt{t}}\right) \tag{4}$$

where $\Phi(\cdot)$ is the standard normal distribution function.

(2) Degradation model in dynamic environment

The operating environment of the equipment is mainly affected by temperature and humidity. In contrast, the equipment arranged in different areas mainly brings about the difference in temperature and humidity, and the degradation rate in the Weiner Process is mainly controlled by the drift coefficient [28,29]. The operation environment of equipment along the railway is complex, and the impact of temperature and humidity on equipment operation is particularly prominent [30]. Therefore, in this paper, the temperature (*Ts*) and humidity (*RHs*) of the equipment operating environment are included in the degradation model. According to the peck model of the impact of temperature and humidity on equipment degradation, combined with the impact of shock on equipment degradation rate, the function of drift parameters of equipment degradation Wiener process is constructed, as shown in Equation 5.

$$\mu_i = f(\mu, Ts, RHs, \eta) \tag{5}$$

where, η is the increment of equipment degradation rate caused by shock, $\eta \sim N(\mu_\eta, \sigma_\eta^2)$. Since *Ts* and *RHs* are in dynamic change, the peck acceleration factor is defined as Λ function, as follows:

$$\Lambda(Ts, RHs) = \left(\frac{RHu}{RHs}\right)^{-n} exp\left(\frac{E_a}{k}\left(\frac{1}{Tu} - \frac{1}{Ts}\right)\right) \tag{6}$$

RHu is the humidity percentage of the equipment in normal use, and RHs is the average humidity percentage of the current area; n is the humidity acceleration constant, with a value range of 1-12, and the classical value is 3; E_a is the activation energy expressed in electron volts, also known as activation energy, which is usually interpreted as the energy required for molecules to change from normal state to active state prone to chemical reactions; The value of E_a is generally between 0.3 and 1.5, and the classical value is generally 0.9; k is the Boltzmann constant with a value of 8.617385×10^{-5} [28,29]; Tu is the Kelvin temperature of the equipment in normal use, and Ts is the average Kelvin temperature of the current area. The natural degradation model of equipment under a continuous dynamic environment can be obtained by combining Equations 2, 5 and 6, as shown in Equation 7.

$$X(t) = X_0 + \left[\left(\frac{RHu}{RHs}\right)^{-n} exp\left(\frac{E_a}{k}\left(\frac{1}{Tu} - \frac{1}{Ts}\right)\right)\right]\mu t + \sigma B(t)$$
$$= X_0 + \Lambda(Ts, RHs) \times \mu t + \sigma B(t) \tag{7}$$

2.2. Random Shock and Self-healing Mechanism

(1) Random shock model

Current, voltage and vibration will cause random mutation of equipment degradation. For the equipment along the railway, the shock mainly comes from the sudden change of load during the operation of the train. However, due to the effective safety interval of train operation, the two shocks are independent. Assuming that the number of shock arrivals at time [0, *t*] is *m*, the arrival rate is λ, and there is independence between two shocks, that is, the random shock can be expressed as a homogeneous Poisson process, then the probability distribution can be expressed as:

$$P = \frac{(\lambda t)^m}{m!} exp(-\lambda t) \tag{8}$$

The random shock intensity $W_i \sim N(\mu_W, \sigma_W^2)$, with α representing the coefficient of the shock on the device degradation increment, i.e., $Y_i = \alpha W_i$. Among the types of shocks within the time interval [0, *t*], non-fatal shocks indicate that the shock load Y is less than the device failure threshold L, with a probability of p_1, and the number of non-fatal shocks is denoted by $M_1(t)$; fatal shocks indicate that the shock load Y is greater than the device failure threshold L, with a probability of p_2, and the number of fatal shocks is denoted by $M_2(t)$. Therefore, the probability p_1 of no fatal shocks occurring to the device is:

$$p_1 = P\{Y_i < L\} = \Phi\left(\frac{L-\alpha\mu_W}{\alpha\sigma_W}\right) \tag{9}$$

(2) Self-healing mechanism

Considering the potential recovery behavior of devices after being shocked, some scholars have modeled the self-healing degree and self-healing time of device self-healing mechanisms, simulating the impact of different shock arrival times on self-healing [31-33]. Due to the independence and time interval of the shock, the device self-healing mechanism also assumes independence relative to time and only exists after the device is subjected to non fatal shock. For modeling the self-healing characteristics of continuous degradation processes, assuming an ideal time interval between shocks, an exponential model is generally used for description [26,27]. According to the self-healing characteristics of the device, $E_i = exp(-\gamma i)$ is used to describe the self-healing behavior of the device. The self-healing mechanism of the device is always accompanied by non fatal shocks to reduce the degradation increment caused by the shock. When $M(t)>0$, the self-healing model of the device can be obtained as:

$$S(t) = \sum_{i=1}^{M(t)} Y_i E_i = \sum_{i=1}^{M(t)} \alpha W_i \, exp(-\gamma i) \tag{10}$$

where γ represents the self-healing coefficient ($\gamma \geq 0$). It is worth noting that the self-healing mechanism is optional. When $\gamma = 0$, the device does not exhibit self-healing behavior. According to the geometric series $\sum_{i=0}^{\infty} exp(-i\gamma) = \frac{exp(-\gamma)}{1-exp(-\gamma)}$, Equation 10 can be rewritten as[26]:

$$S(t) \approx \frac{exp(-\gamma i)}{1-exp(-\gamma i)} \sum_{i=1}^{M(t)} \alpha W_i \tag{11}$$

When the device does not experience fatal shocks, i.e., when $M_1(t)=m_1$ and $M_2(t)=0$, the probability distribution function of the device shock model $S(t)$ is as shown in Equation 12.

$$F_S(s) = \Phi\left(\frac{s - m_1\alpha\mu_W\left(\frac{exp(-\gamma)}{(1-exp(-\gamma))}\right)}{\sqrt{m_1\alpha^2\sigma_W^2\left(\frac{exp(-2\gamma)}{(1-exp(-2\gamma))}\right)}}\right) \tag{12}$$

Combining Sections 2.1 and 2.2 regarding the model of natural degradation, random shocks, and self-healing characteristics of devices in dynamic continuous environments, the schematic diagram of the degradation of large-scale cross-regional deployment of devices in dynamic environments is illustrated in Fig. 1.

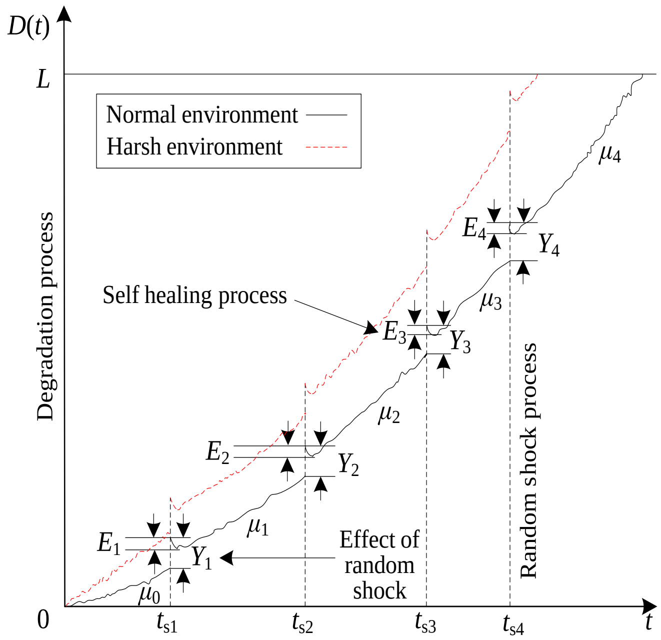


Fig. 1. Schematic diagram of the degradation model.

2.3. Equipment Reliability

The failure of a device can be caused by natural degradation, fatal shocks, or the combination of natural degradation and non-fatal shocks. When a fatal shock occurs, the device fails directly. Therefore, the reliability corresponding to different types of shocks is discussed separately. Here, the standard normal distribution is denoted as Φ.

(1) Reliability in the absence of shocks

When $M(t)=0$ ($M_1(t)=0$, $M_2(t)=0$), the device is only affected by the natural degradation under the current operating environment. The reliability of the device, denoted as R_1, is derived as follows:

$$R_1(t) = P\{D(t) < L|M(t) = 0\} \times P\{M(t) = 0\} = P\{X(t) < L|M_1(t) = 0, M_2(t) = 0\} \times P\{M_1(t) = 0\} \times P\{M_2(t)0\}$$
$$= \int_0^L \frac{1}{\sqrt{2\pi\sigma^2 t}} exp\left(-\frac{(x-\Lambda(Ts,RHs)\times\mu t)^2}{2\sigma^2 t}\right) dx \times exp(-\lambda p_1 t) \times exp(-\lambda p_2 t) = \Phi\left(\frac{L-\Lambda(Ts,RHs)\times\mu t}{\sigma\sqrt{t}}\right) \times exp(-\lambda t) \tag{13}$$

(2) Reliability in the presence of non-fatal shocks

When $M(t)>0$ ($M_1(t)=m_1$, $M_2(t)=0$), the device is affected by both natural degradation and non-fatal shocks under the current operating environment. The reliability of the device, denoted as R_2, is given by Equation 14, and the derivation process is provided in Appendix A.

$$R_2(t) = \sum_{m_1=1}^{\infty}\left[\left[\int_0^L \Phi\left(\frac{L-x-m_1\alpha\mu_W\left(\frac{exp(-\gamma)}{1-exp(-\gamma)}\right)}{\sqrt{m_1\alpha^2\sigma_W^2\left(\frac{exp(-2\gamma)}{1-exp(-2\gamma)}\right)}}\right) \times \frac{1}{\sqrt{2\pi\sigma^2 t}} exp\left(-\frac{(x-\Lambda(Ts,RHs)\times\mu t)^2}{2\sigma^2 t}\right) dx - \Phi\left(\frac{-\sqrt{m_1}\mu_W\left(\frac{exp(-\gamma)}{1-exp(-\gamma)}\right)}{\sqrt{\sigma_W^2\left(\frac{exp(-2\gamma)}{1-exp(-2\gamma)}\right)}}\right) \times \Phi\left(\frac{L-\Lambda(Ts,RHs)\times\mu t}{\sigma\sqrt{t}}\right) \times \frac{(\lambda p_1 t)^{m_1}}{m_1!} exp(-\lambda t)\right]\right] \tag{14}$$

Then the reliability expression of equipment in dynamic environment is shown in Equation 15.

$$R(t) = R_1(t) + R_2(t) \tag{15}$$

Equation 15 involves a complex function integration problem that is difficult to compute analytically. Monte Carlo simulation can be employed to obtain numerical solutions for this integral, providing a practical alternative.

2.4. Reliability Analysis

Rail transit signal system is a large-scale cross regional complex system. Taking its ground subsystem ZPW-2000A uninsulated frequency shift track circuit trackside equipment compensation capacitor and interlocking operation equipment relay as examples, the reliability numerical simulation is carried out to verify the model.

(1) Compensating capacitors

Compensating capacitors are extensively deployed alongside railway tracks, often spanning multiple geographical regions along certain long-distance railway lines, leading to variations in operating environments. The fault modes of compensating capacitors in railway signal systems mainly include capacitance drop and wire breakage. The decrease in capacitance is the main degradation mechanism of compensating capacitors, so the degradation process of compensating capacitors is modeled. Railway signal trackside capacitors are made of polypropylene film and are subject to significant degradation influenced by temperature, humidity, and current shocks. When a capacitor experiences a current shock that punctures the dielectric layer, a self-healing mechanism is activated to mitigate the impact of the shock, although it does not reverse the degradation in capacitance caused by the shock. Hence, the degradation process can be effectively described using the reliability model established in this paper, which considers both shocks and self-healing mechanisms based on the Wiener process. For capacitors configured with a capacitance value of 50μF at a carrier frequency of 2000Hz, in accordance with references [26,29,34], their parameter settings are detailed in the left column of Table 1. The subscript 1 in the parameter represents the compensation capacitance.

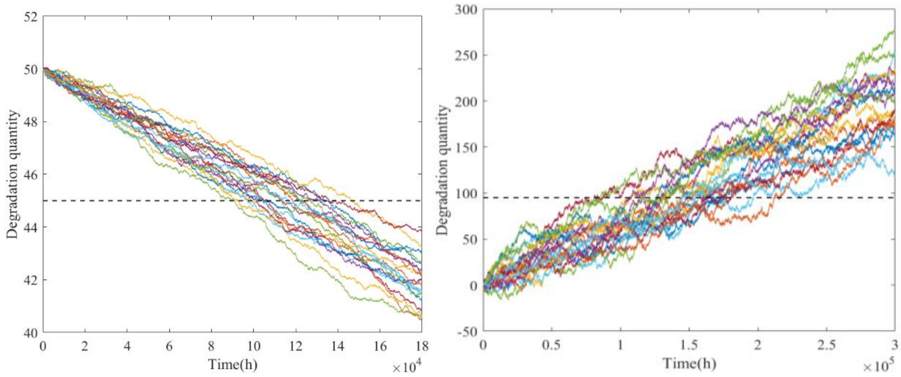
(2) Relay

As a key interlocking operation and output device in the railway signal system, relays are important safety equipment for controlling and inspecting the operation and status of outdoor signal machines, switch machines, and track circuits in railway stations. The main reason for the degradation of relays during use is the oxidation of the contact heads, which leads to an increase in resistance and poor contact, resulting in failure. Under external factors such as applied current and vibration, relays can experience degradation shocks, while external environments such as temperature and humidity can increase oxidation and corrosion of contacts, accelerating their degradation. According to references [34-38], the degradation parameters of relays can be configured as shown on the right side of Table 1, the subscript 2 in the parameter represents the relay.

Table 1. Numerical Simulation Parameters.

(1)Parameters	Value	(2) Parameters	Value
μ_1	4.58×10^{-5}	μ_2	7×10^{-4}
σ_1	0.2×10^{-2}	σ_2	1×10^{-1}
L_1	45μF	L_2	95mΩ
μ_{W1}	0.7×10^{-5}	μ_{W2}	0.7×10^{-2}
σ_{W1}	0.1×10^{-3}	σ_{W2}	0.1×10^{-3}
α_1	0.1	α_2	0.2
λ_1	0.25×10^{-5}	λ_2	0.25×10^{-5}
p_1	0.7	p_1	0.7
p_2	0.3	p_2	0.3

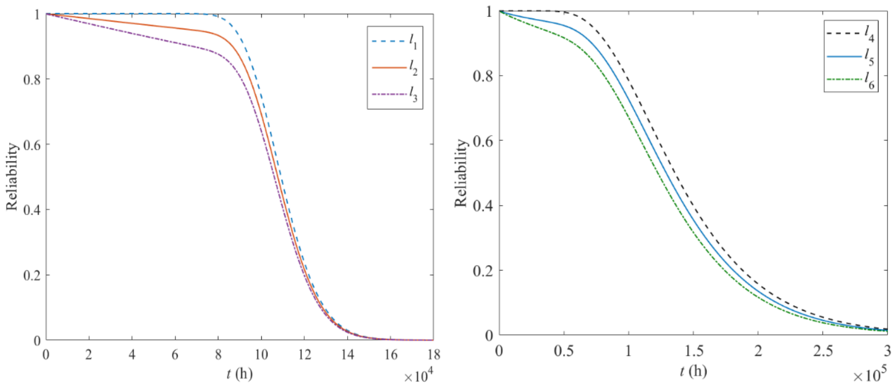
Draw the basic degradation curves for two types of equipment based on the parameters in Table 1, as shown in Fig. 2.



(a) Compensation capacitor (b) Relay

Fig. 2. Schematic diagram of equipment foundation degradation path.

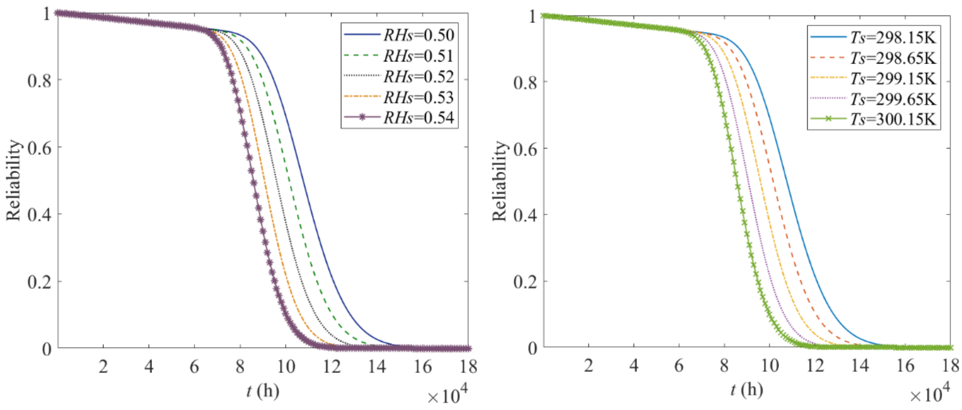
Assuming that the operating environment temperature and humidity of the equipment are consistent with the rated temperature and humidity, reliable results can be obtained based on Equations 13, 14, and 15. The reliability of the compensating capacitor is shown in Fig. 3 (a), and the reliability curve of the relay is shown in Fig. 3 (b).



(a) Compensation capacitor (b) Relay

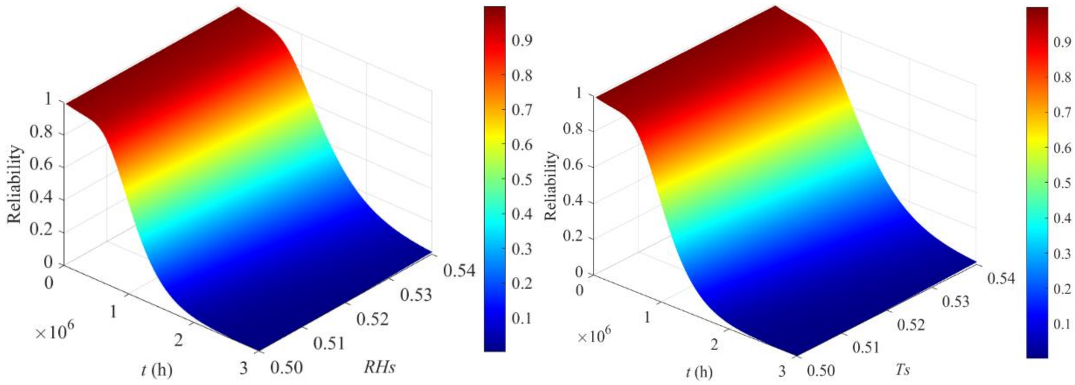
Fig. 3. Reliability numerical simulation results.

In Fig. 3, l_1 and l_4 represent the reliability curves of devices without considering random shock self-healing effects, while l_2 and l_5 represent the reliability curves established in this paper considering random shocks and including device self-healing effects. Additionally, l_3 and l_6 indicate the reliability curves considering random shocks but without including device self-healing effects. Fig. 3 shows that the reliability of l_1 and l_4, without considering random shocks, is significantly higher than when considering random shocks. Furthermore, in the initial stages of degradation, the reliability values remain at 1, which does not align with the actual operational conditions of devices. The slight elevation of l_2 and l_5 compared to l_3 and l_6 suggests that device self-healing properties mitigate degradation, confirming the effectiveness of the model. Additionally, l_2, l_5, l_3, and l_6 exhibit lower reliability than l_1 and l_4 in the initial stages of degradation, indicating the increased impact of random shocks on device degradation. However, this impact gradually diminishes in the later stages of degradation, as natural degradation becomes the predominant factor. The trends in reliability variation shown in Fig. 4 suggest that reasonable maintenance should be performed before the rapid decline in device reliability. However, reliability models that do not consider random shocks may fail to accurately represent the correct trend in reliability variation during the initial stages of degradation, leading to potential deviations in maintenance strategies. Therefore, utilizing the degradation and reliability model proposed in this paper, which considers random shocks and includes device self-healing properties, effectively reflects the degradation process and reliability variation trends, thus providing a basis for formulating rational maintenance strategies. When changes occur in the operating environment of devices, the device reliability curve variation trends are illustrated in Fig. 4 and Fig. 5.



(a) Variation of reliability under different humidity (b) Variation of reliability under different temperature

Fig. 4. Variation trend of equipment reliability under dynamic environment of the compensation capacitor.



(a) Variation of reliability under different humidity (b) Variation of reliability under different temperature

Fig. 5. Variation trend of equipment reliability under relay dynamic environment.

From Fig. 4 and Fig. 5, it is evident that as the operating environment of the equipment deteriorates, its reliability also changes. This indicates a strong correlation between device reliability and the operating environment, with device reliability gradually decreasing as the environment worsens. Therefore, the formulation of maintenance strategies for devices not only requires accurate modeling of the degradation process and reliability but also consideration of the impact of the operating environment on devices. Only by doing so can rational and cost-effective maintenance strategies be developed for devices.

3. Periodic Maintenance Strategy Model

The trend of equipment reliability reflects the extent of equipment degradation and does not rely on a large amount of degradation data. For equipment deployed on a large scale across regions, such as devices installed along railway lines, a combination of regular inspections and fault repairs is typically employed. However, this approach often results in high maintenance costs and delayed maintenance activities. Therefore, this paper combines the maximum maintenance lifespan of the system where the equipment is located to define a new equipment maintenance cycle. Subsequently, a periodic maintenance decision model based on equipment reliability in dynamic environments is established to enhance equipment operational reliability and reduce maintenance costs. This section first introduces the improvement of imperfect maintenance strategies on equipment degradation and then presents a periodic maintenance cost model as the basis for maintenance decision-making.

3.1. Maintenance Cycle

The maintenance process based on equipment reliability is conducted with the threshold of reliability degradation as the node for maintenance. However, maintenance activities can affect the subsequent degradation process of the equipment. Therefore, the following points regarding maintenance and its impacts are outlined:

- The operational environment of the equipment is only related to the geographical region and does not change with maintenance activities.

- Imperfect preventive maintenance and replacement maintenance are both optional. That is, at the reliability threshold Rc node, one can choose between imperfect preventive maintenance and replacement maintenance.
- After replacement maintenance, the equipment begins to degrade from time 0. After imperfect preventive maintenance, the reliability of the equipment is restored, but this may lead to the accumulation of initial equipment degradation and an increase in the degradation rate in the subsequent stages.

In practical equipment operation management, periodic inspections are primarily conducted, followed by a comprehensive system overhaul after several inspection stages to ensure the safe and reliable operation of the system. Therefore, based on the actual equipment operation and maintenance management strategies, this paper formulates a realistic periodic maintenance strategy.

From a system perspective and considering the fulfillment of equipment functions, this paper defines the concept of a maintenance cycle as follows: starting from the installation and operation of new equipment, after undergoing several imperfect maintenance or replacement maintenance cycles, the maintenance cycle T ends at the full-system overhaul lifespan determined by the operational management unit. The schematic diagram of the time division for equipment within a maintenance cycle is illustrated in Fig. 6.

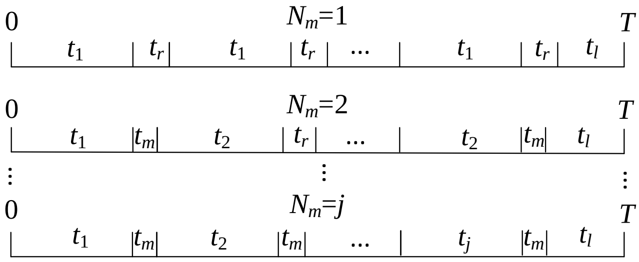


Fig. 6. Schematic diagram of maintenance behavior and maintenance cycle time nodes.

In Fig. 6, t_j represents the running time of the device; N_m represents the sum of preventive maintenance and replacement repairs performed by the equipment during its service life cycle. It is worth noting that there may be multiple equipment replacements within a maintenance cycle as defined in the manuscript. At this point, the total number of preventive maintenance times and replacement maintenance times of the equipment in a maintenance cycle is $N_r \times N_m + N_s$, (the calculation of N_r is shown in Equation 16 of the manuscript, and the calculation of N_s is shown in Equation 17 of the manuscript). Among them, N_r represents the number of times the equipment is replaced in a maintenance cycle, which can also be understood as the cycle of the equipment's life cycle in a maintenance cycle. t_r means the time required for equipment replacement, and the replacement cost is c_r; t_{mb} represents the basic time required for preventive maintenance of imperfect equipment. The current preventive maintenance time t_m changes in a negative correlation with the change of reliability threshold R_c, i.e. $t_m = t_{mb} \times (1 + exp(-R_c))$; The basic cost of preventive maintenance is c_{mb}, and the current preventive maintenance cost c_m is positively correlated with the selection of N_m, that is, $c_m = c_{mb}(1 + \varepsilon N_m)$. Of which, ε Indicates the preventive imperfect maintenance cost coefficient. At the end of the maintenance cycle, it should also include the effective service age cost c_x of the equipment balance. It can be seen from Fig. 6 that the equipment may undergo multiple replacement and repair in a maintenance cycle. When N_m=1, it means that the equipment is non repairable. When N_m>1, it means that the equipment is repairable.

In order to reduce the failure probability of the equipment during operation, inspired by literature [2], this paper maps the failure probability of the equipment during operation to the penalty cost c_f in the equipment cycle. Because the equipment is shut down during fault maintenance, there will be a shutdown loss cost c_s; It is used to increase fault perception ability and improve equipment operation reliability. Therefore, the total maintenance cost of the equipment in the maintenance cycle is composed of preventive maintenance cost, replacement cost and fault penalty cost. The decision variable is the reliability threshold R_c of the equipment and the number of preventive maintenance nm. Then, the number of replacements of the equipment in a maintenance cycle N_r is:

$$N_r = \begin{cases} \frac{T}{T_{life}} = \frac{T}{t_1 + t_r} & N_m = 1 \\ \frac{T}{T_{life}} = \frac{T}{\sum_{j=1}^{N_m} t_j + (N_m - 1) \times t_m + t_r} & N_m > 1 \end{cases} \quad (16)$$

When N_m>1, N_s indicates the number of times the equipment has been subject to imperfect preventive maintenance in the remaining time after the last replacement in [0, T]. The calculation process is:

$$N_s = \left\{ j \left| \left\lfloor \frac{T - T_{life}}{\sum_{j=1}^{k} t_j} \right\rfloor = 0, k = 1,2,\ldots,N_m \right. \right\} \tag{17}$$

Then the time t_l from the end of the last maintenance task performed by the equipment in a maintenance cycle can be expressed as:

$$t_l = \begin{cases} T - \lfloor N_r \rfloor T_{life} & N_m = 1 \\ T - \lfloor N_r \rfloor T_{life} - \sum_{j=1}^{N_s} t_j & N_m > 1 \end{cases} \tag{18}$$

3.2. Periodic Maintenance Cost

It is assumed that the initial degradation of equipment after imperfect preventive maintenance is $D_j \sim N(\mu_m, \sigma_m^2)$, because the superposition of the number of imperfect maintenance of equipment will lead to the increase of initial value of equipment degradation and drift coefficient [5,13,33,]. For the convenience of calculation, the initial degradation increment is converted into the decline of the fault threshold, and after N_r replacement and N_m-1 imperfect maintenance, the equipment degradation fault threshold is $L_j = L - \sum_{j=2+(N_r-1)\times(N_m-1)}^{N_r\times(N_m-1)} D_j$. Because the number of imperfect repairs is positively correlated with the equipment degradation rate, the degradation rate increase factor η is defined, and the equipment degradation drift coefficient after N_m times of imperfect repairs is:

$$\mu_{N_m} = \Lambda(Ts, RHs)\mu\left(1 + \eta \times \sum_{j=2+(N_r-1)\times(N_m-1)}^{N_r\times(N_m-1)} D_j\right),\ \eta>0 \tag{19}$$

The schematic diagram of equipment degradation and imperfect maintenance process is shown in Fig. 7.

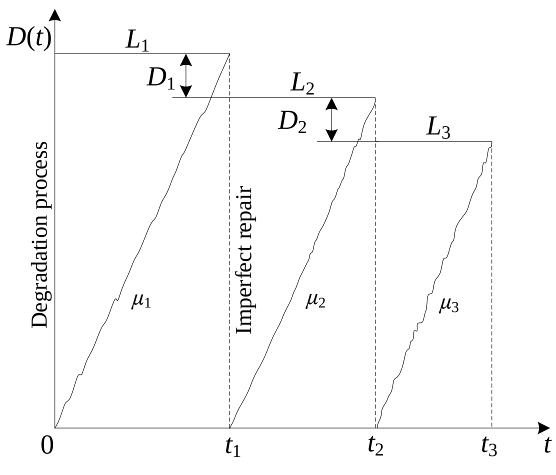


Fig. 7. Schematic diagram of equipment degradation and imperfect maintenance.

In Fig. 7, the imperfect maintenance time node t={t_1, t_2,..., t_j} is determined by the equipment reliability reaching the reliability threshold. Based on the results of the composition of the maintenance strategy, the equipment maintenance strategy is formulated with the main maintenance cost in the process of equipment operation and maintenance as the goal. Then the maintenance cost objective function C_z of the equipment in a maintenance cycle is shown in Equation 20, and the derivation process is shown in Appendix B.

$$argmin\, C_z\,(N_m, R_c) =$$

$$\begin{cases} \lfloor N_r \rfloor \left[c_r + c_f \times (-\ln R_c)\right] + \left(c_f \times \int_0^{t_l} \lambda(t)dt\right) & \\ +c_s[\lfloor N_r \rfloor t_r] + c_r\left[1 - \frac{\lfloor N_r \rfloor T_{life}}{T}\right] & N_m = 1 \\ \lfloor N_r \rfloor\left[(N_m - 1)c_m + c_r + N_m c_f(-\ln R_c)\right] & \\ +N_s\left[c_m + c_f(-\ln R_c)\right] + \left(c_f \times \int_0^{t_l} \lambda(t)dt\right) & \\ +c_s[\lfloor N_r \rfloor((N_m - 1)t_m + t_r) + N_s t_m] + & N_m > 1 \\ +c_r\left[1 - \frac{\left(\lfloor N_r \rfloor \times T_{life} - \sum_{j=1}^{N_s}(t_j + t_m)\right)}{T}\right] & \end{cases} \tag{20}$$

In Equation 20, the decision variable is the combination of the number of times of selecting imperfect preventive maintenance and the reliability threshold. For non repairable equipment, the decision variable only has the reliability threshold.

4. Numerical Simulation

This paper continues to take the compensation capacitor and relay in Section 2.4 as numerical examples to verify the maintenance decision-making model and adjustment scheme of large-scale cross regional deployment equipment in a dynamic environment. Based on the literature [34,40] and actual on-site research, the parameter settings of the maintenance model are shown in Table 2. Among them, the maintenance cycle T is the actual maintenance cycle time of 15 years for Chinese railway operation and management units. The subscript 1 of the parameters in Table 2 represents repairable equipment, while subscript 2 represents repairable equipment.

Table 2. Maintenance parameter settings.

Non repairable parameter	Value	Repairable parameter	Value
μ_{m1}	0.04	μ_{m2}	0.45
σ_{m1}	0.06	σ_{m2}	0.65
t_{r1}	2h	t_{r2}	0.2h
t_{mb1}	-	t_{mb2}	0.2h
t_{f1}	0.5h	t_{f2}	0.8h
c_{r1}	80CNY	c_{r2}	480CNY

Non repairable parameter	Value	Repairable parameter	Value
c_{mb1}	-	c_{mb2}	60CNY
c_{f1}	80CNY	c_{f2}	480CNY
c_{s1}	50CNY	c_{s2}	1000CNY
η_1	0.2	η_2	0.2
ε_1	-	ε_2	0.1
T_1	1.314×10^5h	T_2	1.314×10^5h

4.1. Non Repairable Equipment

(1) Replacement maintenance strategy for rated environment

In the railway field, the compensation capacitor is a non repairable equipment, and the direct replaceable maintenance is generally selected, while the relay is a repairable equipment, which can carry out many times of imperfect preventive maintenance. According to the parameter configuration in Table 2, the variation trend of the cost of the compensation capacitor of non repairable equipment with the reliability threshold in a maintenance cycle is shown in Fig. 8.

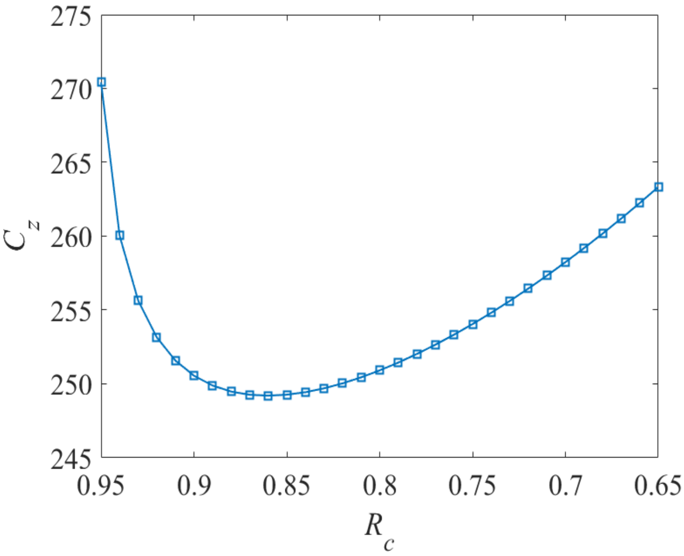


Fig. 8. Optimization results of compensation capacitor replacement strategy for non repairable equipment.

It can be seen from Fig. 8 that when selecting time nodes with different reliability thresholds, the periodic maintenance cost of equipment shows a trend of first decreasing and then increasing. When R_c=0.86, C_z has a minimum value of 249.2 CNY, and the corresponding replaceable maintenance time node of the equipment is about 81125 hours. At this time, the comprehensive maintenance cost of the equipment is the lowest.

(2) Replacement maintenance strategy in dynamic environment

When the operating environment of non repairable equipment changes, the decreasing trend of equipment reliability increases, resulting in the reduction of the time when the equipment reaches the reliability threshold. Therefore, the time node of the optimal replaceable maintenance strategy and the minimum comprehensive maintenance cost of the equipment in different operating environments are shown in Fig. 9 and Fig. 10. t_{Ts} represents the temperature change influence curve, t_{RHs} represents the humidity percentage change influence curve, t_{RHs},t_{Ts} represents the equipment replacement time node change curve when the temperature and humidity change at the same time.

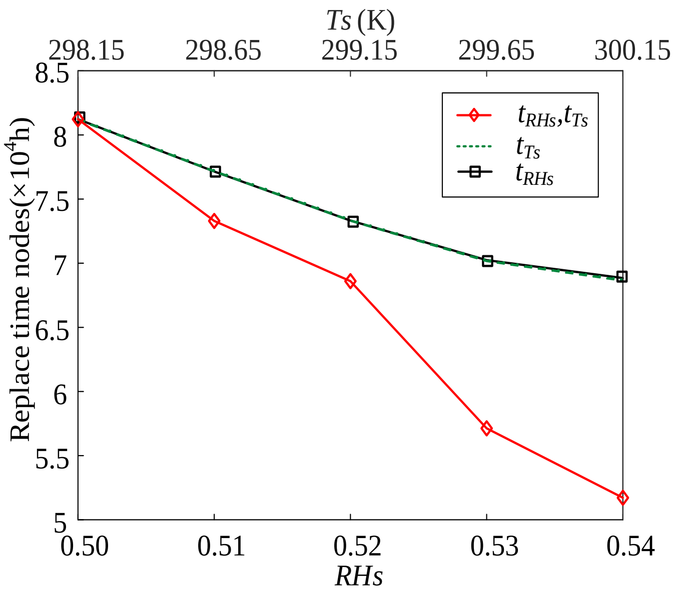


Fig. 9. Change of equipment replaceable maintenance time node in dynamic environment.

Combining Fig. 4 and Fig. 9 reveals that as the equipment operating environment deteriorates, including the increase in environmental temperature and humidity percentage, the degradation rate of the equipment increases. When the equipment reaches the same reliability threshold, the corresponding time nodes for t_{Ts} and t_{RHs} decrease, leading to an accelerated decline in equipment reliability. Concurrently, the time node for replacement maintenance of non-repairable equipment also decreases. In more harsh environments, the first replacement time of the device was shortened by 29508 hours. The geographical region where the equipment is located generally experiences simultaneous changes in temperature and humidity, as depicted by the red line in Fig. 9. The significant decrease in the replacement node time for equipment compared to a single temperature or humidity factor indicates that the environment has a significant impact on the replacement maintenance strategy for non-repairable equipment.

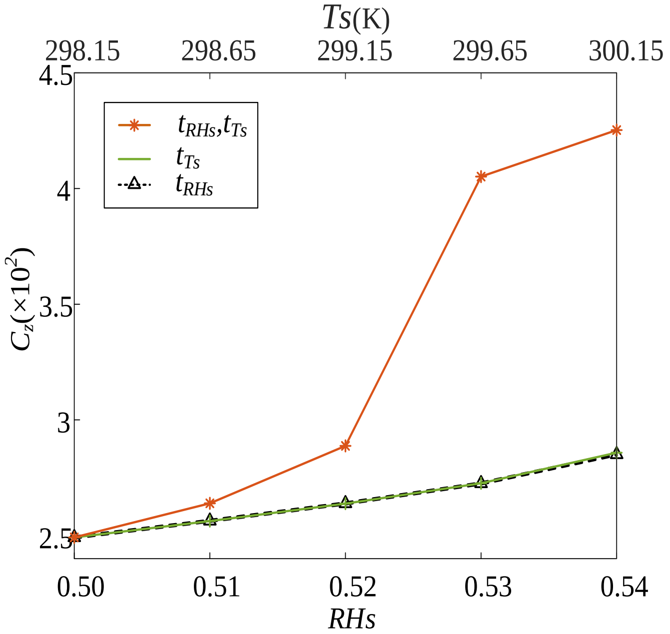


Fig. 10. Minimum comprehensive maintenance cost of equipment in dynamic environment.

In Fig. 10, as the temperature and humidity of the equipment operating environment gradually increase, even with the optimal selection of reliability thresholds and replacement maintenance time nodes, the overall maintenance cost of the equipment still shows a gradual increase. When the temperature and humidity of the equipment change simultaneously (during the process of humidity 0.52 to 0.53 and temperature 299.15K to 299.65K), C_z shows a significant increase of approximately 41.4%. This indicates that under the combined influence of environmental factors, the growth trend of C_z is not a linear summation of single-factor environmental effects. This could be attributed to the acceleration of equipment degradation, leading to an increase in penalty costs for failures and subsequently shifting the reliability threshold forward, thereby increasing the replacement cost.

The trends observed in Fig. 9 and 10 validate the correctness of the reliability model established in this paper and the maintenance decision-making process for non-repairable equipment. This provides a reference for adjusting the replacement time nodes in different regions for equipment deployed on a large scale across regions, thereby reducing the overall cost.

4.2. Repairable Equipment

(1) Preventive maintenance strategy for rated environment

According to the parameter configuration in Table 2, the cost of repairable equipment in one maintenance cycle varies with the reliability threshold and the number of imperfect preventive repairs, as shown in Fig. 11.

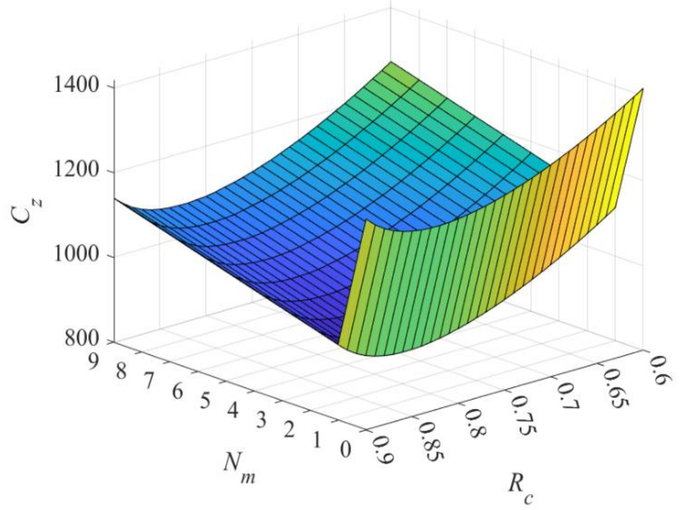


Fig. 11. Optimization results of maintenance strategy for repairable equipment relays.

It can be seen from Fig. 11 that the comprehensive maintenance cost of repairable equipment decreases first and then increases in the process of gradual increase of R_c, which has the same trend as that of non repairable equipment. When the equipment is repairable, the maintenance cost first decreases significantly, and then gradually increases with the increase of the number of imperfect preventive maintenance. This is because the cost of equipment replacement is greater than the cost of preventive maintenance, resulting in the cost of equipment in the stage of preventive maintenance being less than the cost of replacement maintenance. When N_m gradually increased, the synchronous increase of C_z mainly came from the increase of t_l and c_x. When R_c=0.83, N_m=1, the minimum comprehensive maintenance cost of the equipment is 916.42 CNY, and the corresponding preventive maintenance time node is about 85123 hours.

(2) Preventive maintenance strategies in dynamic environments

The time nodes of imperfect preventive maintenance under the dynamic environment of repairable equipment are shown in Table 3.

Table 3. Imperfect preventive maintenance time nodes in dynamic environment (h).

t	*Ts*=298.15	*Ts*=298.65	*Ts*=299.15	*Ts*=299.65	*Ts*=300.15
RHs=0.50	85123	81782	81188	78963	64073,128044
RHs=0.51	81739	81131	77709	64016,127971	61598,123136
RHs=0.52	81155	78906	64016,127931	61590,123120	57803,115540
RHs=0.53	78995	64071,128081	61636,123212	57838,115615	55605,111145
RHs=0.54	64178,128255	61735,123409	57923,115780	55681,111297	53484,106909

According to Table 3, in more severe environments, the first preventive maintenance time of the equipment was shortened by 31639 hours. When the operation environment of the repairable equipment changes, the optimal number of imperfect preventive maintenance and the minimum comprehensive maintenance cost of the equipment in different operation environments are shown in Fig. 12 and Fig. 13.

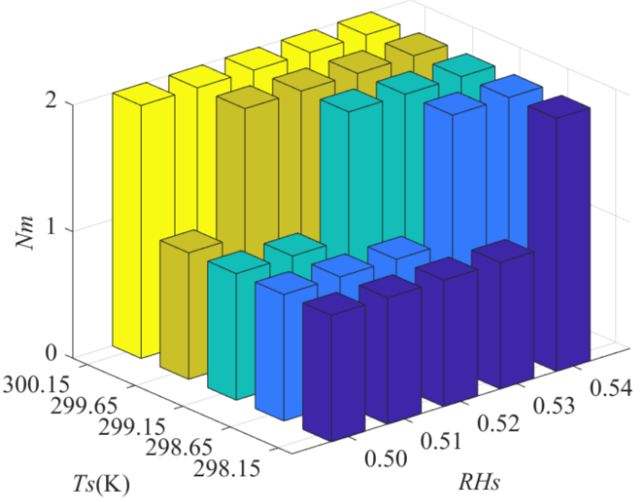


Fig. 12. Optimization results of imperfect preventive maintenance times of repairable equipment.

Fig. 12 is a histogram of the number of imperfect preventive maintenance under the dynamic environment of repairable equipment. When the temperature and humidity gradually increase, N_m will not increase directly, but when the environment deteriorates to a certain extent, the number of preventive maintenance will increase from one to two. However, when the temperature and humidity change synchronously, the change process will be accelerated, indicating that the periodic maintenance decision model established in this paper is stable to environmental changes. At this time, the adjustment process of the environment to the equipment maintenance strategy is measured by the comprehensive maintenance cost, as shown in Fig. 13.

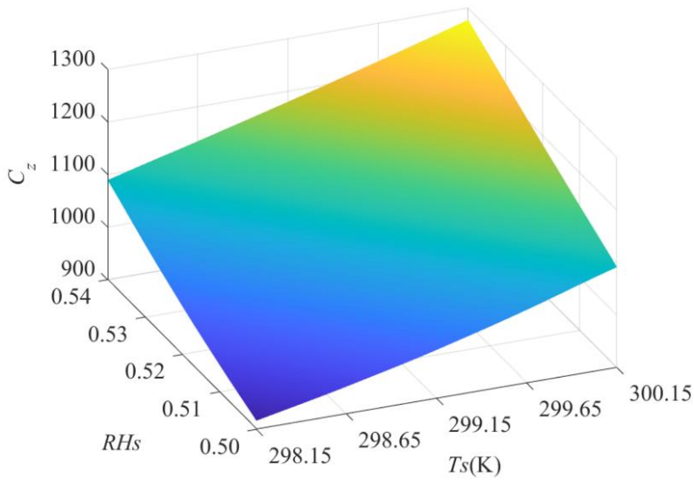


Fig. 13. Optimization results of minimum comprehensive maintenance cost for repairable equipment.

Fig. 13 shows the trend of the minimum comprehensive maintenance cost under the dynamic environment of repairable equipment. When the environment changes from mild to severe, C_z will gradually increase by about 28.2%, which is consistent with the actual trend of equipment degradation and reliability changes. Combining Fig. 12 and Fig. 13, although the selection of imperfect preventive maintenance frequencies for repairable equipment is discrete, the minimum comprehensive maintenance cost of the equipment does not exhibit a step-like change. This is mainly because the penalty cost c_f for fault perception and the residual effective service life cost c_x of repairable equipment compensate for this gap during environmental changes.

Compared to non-repairable equipment, the minimum comprehensive cost for repairable equipment also does not exhibit linear summation and does not show significant increases as observed in Fig. 10. Instead, it remains lower than the above two scenarios even in the harshest environment. The main reason for this is that the preventive maintenance cost for repairable equipment is lower than the replacement cost, reflecting better long-term economic benefits of repairable equipment.

4.3. Result Analysis

In this paper, the equipment reliability model under dynamic environment is established considering the different operating environment of the equipment in different geographical regions. Compared with the literature [39,41], it is more in line with the actual degradation path of the equipment. Due to the differences in the operating environment of large-scale cross regional deployment equipment, it makes up for the lack of equipment degradation path and reliability modeling in the literature [26,27] in the dynamic environment. However, it is worth considering that the degradation mechanism of equipment is often diverse, and different degradation mechanisms correspond to different paths. This paper selects the main degradation mechanism of equipment for modeling, but some relatively minor degradation mechanisms also affect the reliability of equipment, and even bring irreversible damage. The continuous modeling of the equipment operation environment reduces the competent dependence of the literature [22] on the evaluation of the environment, so that the model can not only objectively and accurately describe the operation of the equipment when the gradient of environmental change in the geographical region is small, but also effectively evaluate the equipment maintenance strategies in different operation environments. In terms of maintenance strategy, this study establishes a periodic maintenance strategy model based on the actual operation management requirements. Compared with the existing literature [39,40], the maintenance decision-making model has theoretical time wireless continuity because the actual operation deadline of the equipment is not set. This result does not conform to the actual operation of the equipment, and it is difficult to effectively guide the maintenance activities on site. In this paper, not only the repairable equipment and non repairable equipment are simulated and analyzed, but also the adjustment process of the scheme is given for different operating environments. In terms of maintenance cost, this paper considers the maintenance node and the time balance cost from maintenance to the end of the maintenance cycle to minimize the waste of cost, which is more effective than the existing research [40,41]. During the maintenance activities, the fault perception module was added to reduce the hidden risk of system operation caused by equipment maintenance delay.

5. Conclusion

This paper considers the degradation process of equipment in dynamic environments, taking into account the random shock model and self-healing characteristics during equipment operation. A reliability model of equipment under variable temperature and humidity conditions is established and compared with models that do not include shock and self-healing, providing a basis for reliability analysis of equipment deployed on a large scale across regions. The concept of maintenance cycle is redefined according to the actual operational management of equipment in the system. Periodic maintenance decision models are established for repairable and non-repairable equipment, and numerical simulations are conducted to obtain periodic optimal maintenance strategy recommendations for both types of equipment. As the environment deteriorates, the initial maintenance time for equipment generally decreases, preventive maintenance frequency increases when the environment deteriorates to a certain extent, and comprehensive maintenance costs gradually increase. Furthermore, this paper discusses the equipment maintenance decision-making process in depth in dynamic environments, analyzing comprehensive maintenance costs, time nodes, and strategy changes for repairable and non-repairable equipment during environmental changes. Adjustment schemes for equipment maintenance strategies in dynamic environments are proposed, providing corresponding maintenance decision recommendations for equipment operating in different geographic regions.

In future work, we can consider the time-varying characteristics of environmental changes, as well as the interaction and relationship of shock, self-healing and drift coefficient, and establish a degradation process and reliability model more in line with the trend of equipment deterioration. The current research focuses on equipment and does not consider the structural and constraint relationships between equipment and equipment or equipment and systems. The next step could involve establishing a multi-objective periodic maintenance decision model at the system level to formulate comprehensive system maintenance strategies.

Appendix A

$R_2(t)$ derivation process:

$$R_2(t)=\sum_{m_1=1}^{\infty}\left[P\{(X(t)+S(t))<L|M_1(t)=m_1,M_2(t)=0\}\times P\{M_1(t)=m_1\}\times P\{M_2(t)=0\}\right]$$

$$=\sum_{m_1=1}^{\infty}\left[P\left\{\left(X(t)+\frac{exp(-\gamma)}{1-exp(-\gamma)}\sum_{i=1}^{m_1}\alpha W_i\right)<L|M_1(t)=m_1,M_2(t)=0\right\}\right.$$
$$\left.\times\frac{(\lambda p_1 t)^{m_1}}{m_1!}exp(-\lambda p_1 t)\,exp(-\lambda p_2 t)\right]$$

$$=\sum_{m_1=1}^{\infty}\left[\int_0^L f_X(x)dx\int_0^{L-x}f_S(s)ds\times\frac{(\lambda p_1 t)^{m_1}}{m_1!}exp(-\lambda t)\right]$$

$$=\sum_{m_1=1}^{\infty}\left[\int_0^L f_X(x)dx(F_S(L-x)-F_S(0))\times\frac{(\lambda p_1 t)^{m_1}}{m_1!}exp(-\lambda t)\right]$$

$$=\sum_{m_1=1}^{\infty}\left[\left(\int_0^L F_S(L-x)f_X(x)dx-\int_0^L F_S(0)f_X(x)dx\right)\times\frac{(\lambda p_1 t)^{m_1}}{m_1!}exp(-\lambda t)\right]$$

$$=\sum_{m_1=1}^{\infty}\left[\left[\int_0^L\Phi\left(\frac{L-x-m_1\alpha\mu_W\left(\frac{exp(-\gamma)}{1-exp(-\gamma)}\right)}{\sqrt{m_1\alpha^2\sigma_W^2\left(\frac{exp(-2\gamma)}{1-exp(-2\gamma)}\right)}}\right)\times f_X(x)dx-\right.\right.$$
$$\left.\left.\int_0^L\Phi\left(\frac{-\sqrt{m_1}\mu_W\left(\frac{exp(-\gamma)}{1-exp(-\gamma)}\right)}{\sqrt{\sigma_W^2\left(\frac{exp(-2\gamma)}{1-exp(-2\gamma)}\right)}}\right)f_X(x)dx\right]\times\frac{(\lambda p_1 t)^{m_1}}{m_1!}exp(-\lambda t)\right]$$

$$=\sum_{m_1=1}^{\infty}\left[\left[\int_0^L\Phi\left(\frac{L-x-m_1\alpha\mu_W\left(\frac{exp(-\gamma)}{1-exp(-\gamma)}\right)}{\sqrt{m_1\alpha^2\sigma_W^2\left(\frac{exp(-2\gamma)}{1-exp(-2\gamma)}\right)}}\right)\times\right.\right.$$
$$\frac{1}{\sqrt{2\pi\sigma^2 t}}exp\left(-\frac{(x-\Lambda(Ts,RHs)\times\mu t)^2}{2\sigma^2 t}\right)dx$$
$$-\Phi\left(\frac{-\sqrt{m_1}\mu_W\left(\frac{exp(-\gamma)}{1-exp(-\gamma)}\right)}{\sqrt{\sigma_W^2\left(\frac{exp(-2\gamma)}{1-exp(-2\gamma)}\right)}}\right)\times$$
$$\left.\left.\Phi\left(\frac{L-\Lambda(Ts,RHs)\times\mu t}{\sigma\sqrt{t}}\right)\right]\times\frac{(\lambda p_1 t)^{m_1}}{m_1!}exp(-\lambda t)\right]$$

Appendix B

C_z derivation process:

$$
\operatorname{argmin} C_z\left(N_m, R_c\right)=\begin{cases}
\begin{cases}
\lfloor N_r \rfloor\left(c_r+c_f \times \int_0^{t_1} \lambda(t) d t\right)+\left(c_f \times \int_0^{t_l} \lambda(t) d t\right) \\
+c_s\left[\lfloor N_r \rfloor t_r\right]+c_x \\
=\lfloor N_r \rfloor\left(c_r+c_f \times\left(-\ln R_c\right)\right)+\left(c_f \times \int_0^{t_l} \lambda(t) d t\right) \\
+c_s\left[\lfloor N_r \rfloor t_r\right]+c_r\left[1-\dfrac{\lfloor N_r \rfloor\left(t_1+t_r\right)}{T}\right]
\end{cases} & N_m=1 \\
\begin{cases}
\lfloor N_r \rfloor\left(N_m c_m+c_r+c_f \times \sum_{j=1}^{N_m} \int_0^{t_j} \lambda(t) d t\right) \\
+N_s\left(c_m+c_f \times \int_0^{t_s} \lambda(t) d t\right)+\left(c_f \times \int_0^{t_l} \lambda(t) d t\right) \\
+c_s\left[\lfloor N_r \rfloor\left(N_m t_m+t\right)_r+N_s t_m\right]+c_x \\
=\lfloor N_r \rfloor\left[N_m c_m+c_r+N_m c_f\left(-\ln R_c\right)\right] \\
+N_s\left[c_m+c_f\left(-\ln R_c\right)\right]+\left(c_f \times \int_0^{t_l} \lambda(t) d t\right) \\
+c_s\left[\lfloor N_r \rfloor\left(N_m t_m+t\right)_r+N_s t_m\right] \\
+c_r\left[1-\dfrac{\left(\lfloor N_r \rfloor \times T_{life}-\sum_{j=1}^{N_s}\left(t_j+t_m\right)\right)}{T}\right], \ s=1,2,\ldots,N_s
\end{cases} & N_m>1
\end{cases}
$$

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