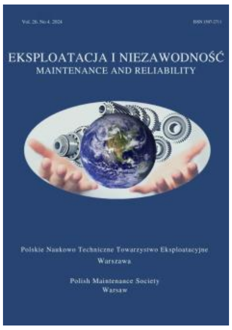

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Research on maintenance decision-making approach based on dynamic opportunistic window

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Highlights

- We propose a dynamic maintenance approach for the opportunistic window that enables real-time updates of service age, reliability, preventive maintenance intervals, and duration of the opportunistic window.
- We propose an improved expectation maximization algorithm to solve transcendental equations in traditional algorithms when estimating parameters of mixed Weibull distribution.
- By employing a case study of serial system, We substantiates the significance of the proposed method in terms of parameter estimation accuracy, reliability of maintenance schedule setting, and rationality of maintenance plans.

Abstract

We have developed a maintenance decision-making approach based on dynamic opportunistic window (OW), utilizing algorithms such as *k*-mean clustering, expected maximum and parameter estimation to address the lack of a reasonable basis for the duration and divided number of OWs in current maintenance decision-making. Firstly, we have comprehensively summarized the multi-stage opportunistic maintenance (OM) decision-making approach, with a particular focus on its current strengths and limitations. Secondly, the modeling concept of the dynamic OW is analyzed, and the underlying assumptions are established. Furthermore, it elaborates on the theoretical foundation of the maintenance decision-making approach based on the dynamic OW through a detailed modeling process. Finally, we validate the proposed model by conducting experiments on tandem components from the main combustion chamber in an aero-engine. The experimental results demonstrate the significant value of the proposed maintenance decision-making approach based on dynamic OW in enhancing equipment reliability and optimizing maintenance support resources.

Keywords
predictive maintenance, expectation maximization, reliability estimation, opportunistic maintenance

1. Introduction

Reaching a reasonable approach to maintenance decision-making is of paramount importance in the realm of cost reduction while ensuring dependable equipment operation. For example, according to Aviation Week, the total global demand for MRO (aviation maintenance, repair, and overhaul) is projected to reach $1.2 trillion between 2024 and 2033. However, current airline maintenance costs are only able to be constrained at approximately 12%. Consequently, effective control of maintenance costs has emerged as a pressing issue requiring urgent attention. There is a growing research interest in dynamically dividing the maintenance stage and implementing condition-based maintenance, particularly for the equipment with reliability correlation and service age correlation [1, 2]. The multi-stage OM approach aims to conduct preventive maintenance (PM) on one component and simultaneously perform maintenance operations on other components that meet specific criteria, such as reliability, degradation status, and service age. It aims to minimize

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downtime, mitigate the risk of excessive maintenance, disrepair and failures, enhance equipment availability, and optimize maintenance costs [3, 4].

Currently, numerous scholars have extensively investigated multi-stage OM decision-making approach, which can be broadly categorized into three groups: the maintenance logic-based approach, the component characteristics-based approach, and the opportunity window-based approach.

The maintenance logic-based approach effectively categorizes and rationally schedules the maintenance stages by investigating the interplay among the execution sequences of maintenance tasks [5] or among the faults [1], thereby establishing a logical relationship between pre- and post-tasks. Essentially, this maintenance approach entails dividing the overall OM into multiple two-stage maintenances without accounting for the influence of component health status on maintenance intensity. Consequently, there is a risk of excessively advancing component Maintenance Schedule (MS) and escalating associated costs.

The component characteristics-based approach is divided into multiple stages (such as normal, low vulnerability, medium vulnerability, high vulnerability, and fault), using health status [6, 7], degradation rule [8, 9], or damage grade [10]， which enables targeted condition-based maintenance. While this OM approach has made some progress in the categorization of multi-stage maintenance and condition-based maintenance, the classification of maintenance stages is primarily grounded in empirical knowledge, overlooking the incorporation of PM schedules to determine the optimal timing. As a result, it necessitates significant resources and real-time adjustments to the MS.

The opportunity window-based approach comprehensively considers PM schedules and significantly reduces the workload required for adjusting them based on the component characteristics-based approach. Currently, the predominant approach to OM involves evenly partitioning the imperfect maintenance window within the OW into a predetermined number and time period [8, 10-12]. When the scheduled maintenance of a component falls in the imperfect maintenance window, it is essential to conduct maintenance based on its current stage. This approach fixes the values of the OW and replacement window (RW), while the determination of the number of divisions in the imperfect maintenance window heavily relies on researchers' expertise, leading to the observed phenomenon illustrated in Fig. 1.

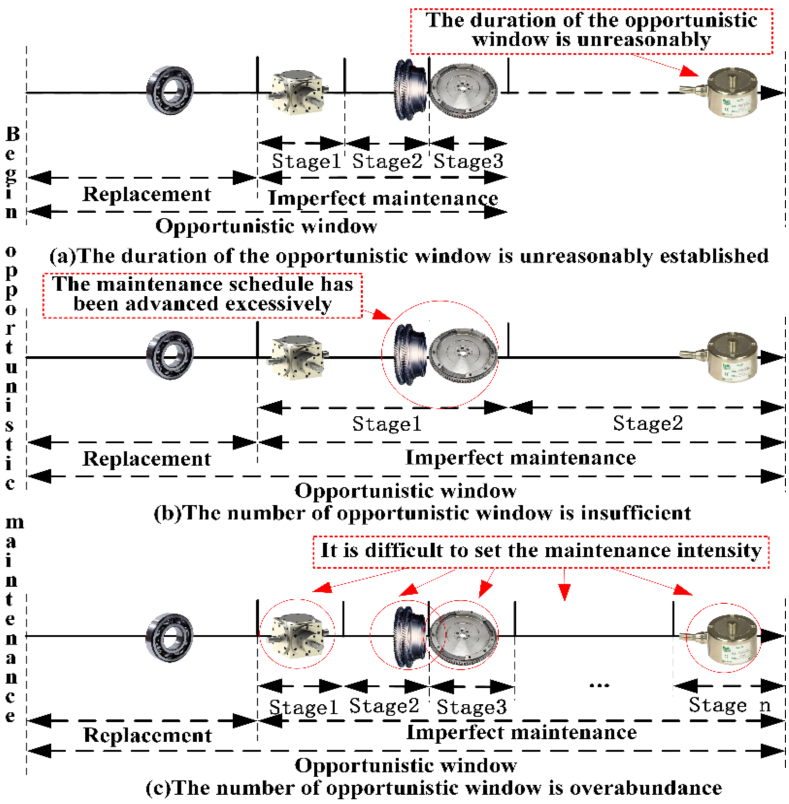


Fig. 1. Schematic diagram of various OM schemes.

(1) The duration of the OW is not reasonable, making it challenging to determine the quantity and timing of maintenance components, which results in frequent shutdowns for maintenance and wastage of maintenance support resources.

(2) The maintenance window is insufficiently divided, and the scheduling of maintenance intervals for components is excessively advanced, thereby potentially leading to inefficient utilization of maintenance support resources.

(3) The maintenance window is more divided, posing challenges in determining maintenance intensity, reducing maintenance duration, and enhancing equipment availability.

In summary, this paper proposes a dynamic maintenance decision-making approach based on the OW to address the limitations of the traditional approach [12]. The proposed approach utilizes K-means++ and Expectation Maximization (EM) algorithms, along with parameter estimation techniques, to update the service age, PM Interval, and OM timing of each component in real-time.

The remainder of this study is structured as follows: Section 2 presents the theoretical framework and associated assumptions for modeling. Section 3 provides a comprehensive description of the proposed maintenance decision-making approach based on dynamic opportunistic window. Section 4 demonstrates the feasibility and effectiveness of the proposed approach through an example involving the main combustion chamber of an aircraft engine. Conclusions drawn from this study, along with further research, are then given in Section 5.

2. Dynamic OW modeling analysis

2.1. Dynamic OW analysis

The criteria for determining whether a component should be maintained based on the OW are as follows:

(1) If the reliability of the component falls below a specified threshold.

(2) If the component has reached its life expectancy.

(3) If the PM moment of the component is closer to that of another component undergoing replacements [13, 14].

Among them, (1) and (2) are primarily utilized for assessing whether the components should be replaced, while (3) is predominantly employed to determine if imperfect maintenance of the components is necessary. The concept of OM is a linear approach that is effective for components meeting conditions (1) or (2), requiring replacement, but exhibits slightly reduced efficacy for components meeting condition (3) and necessitating OM. It systematically schedules PM activities throughout the entire equipment life cycle and utilizes a predetermined OW to consecutively conduct OM on components that meet the conditions, as illustrated in Fig. 2.

Fig. 2. The thought chart of the opportunity window-based approach.

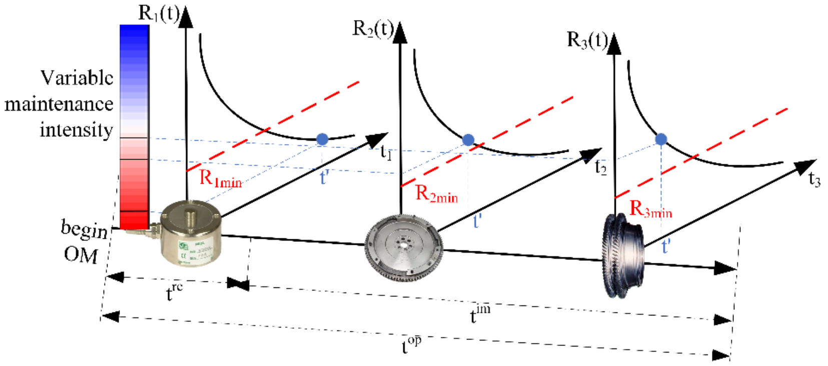


Fig. 3. The thought chart of the dynamic maintenance decision-making approach.

The dynamic maintenance decision-making approach based on the OW does not partition the imperfect maintenance window into equidistant. Instead, it employs clustering, evaluation, and parameter estimation algorithms to assess the reliability of other components when one component reaches its PM moment. Furthermore, it categorizes components with similar service age or reliability and conducts OM for components within the same category. The maintenance time associated with these components presents an OW, while the corresponding reliability represents a variable maintenance intensity. By systematically integrating the maintenance activities of multiple tandem components and optimizing their sequence, as depicted in Fig. 3, it can achieve the objective of minimizing downtime and total cost.

For OM reliability: although the components within the tandem system possess structural independence, they necessitate close interconnection during operation, and their reliability /failure distribution is bound to be coupled. Therefore, a component's reliability R /failure distribution f_k^{MR} should be comprised of the combined reliability R /failure distributions f_j^R of all components within the tandem system. Where A is the component number of the tandem system, ω_j is the weight.

$$f_k^{MR} \propto \sum_{j=1}^{A} \omega_j f_j^R \tag{1}$$

2.2. Modeling assumptions

The assumptions of the model should be established when constructing a dynamic OW.

(1) For the purpose of facilitating the study, we define opportunity maintenance as including all forms of imperfect maintenance (including minimum maintenance) but excluding replacement, which is reflected in the reduction of service age.

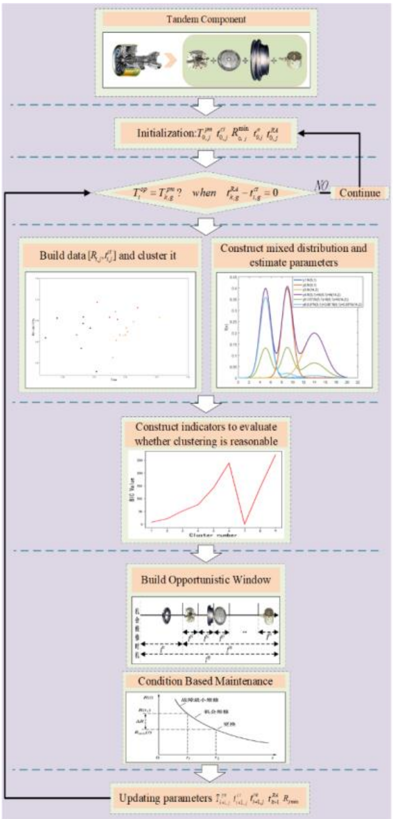
(2) When a component reaches its expected life expectancy, it will be replaced accordingly. Meanwhile, other components within the OW undergo imperfect maintenance, and the cumulative effect of multiple imperfect maintenance on the quality is denoted as factor λ.

(3) This paper focuses on studying maintenance decision-making, and the reliability of each component is a known condition.

(4) The components in the tandem system are all repairable and their failure probability distribution follows the Weibull distribution [15].

3. Maintenance decision-making modeling based on dynamic OW

The modeling procedure for the *i*-th OM in tandem system and its running pseudo-code are shown Fig. 4.



Input: Reliability data $\boldsymbol{D}_i$

Initialization: Preset preventive maintenance $T_{0,j}^{pm}$, service age $t_{0,j}^{ct}$, reliability threshold $R_{j\min}$, preventive maintenances interval $t_{0,j}^{RA}$, Run time $t=0$

While $t < 1000$:
  If $t_{0,j}^{RA} - t_{0,j}^{ct} == 0$:
    $T_{1,j}^{pm}\ t_{1,j}^{ct}\ R_j\ t_{1,j}^{RA} =$ Opportunitic_Maintenance($T_{0,j}^{pm}, t_{0,j}^{ct}, R_{j\min}, t_{0,j}^{RA}, \boldsymbol{D}_i$)

def Opportunitic_Maintenance($T_{0,j}^{pm}, t_{0,j}^{ct}, R_{j\min}, t_{0,j}^{RA}, \boldsymbol{D}_i$):
  # clusters are data with a cluster number of 1 to N
  Clusters = K-means++($\boldsymbol{D}$)
  Calculate the parameters of the distribution that follows
  θ = EM(Clusters)
  # The evaluation of the bayes information criterion in the number of clustering
  bic = $n$ln(Clusters)-2ln$L$(θ)
  # Calculate the current reliability of each component

$$R_j = 1 - \int_0^{t_{i-1,j}^{ct}+t_{k,g}^{RA}} f_{k,j}^{MR}(t)dt$$

  # Calculate the service age of each component after maintenance

$$t_{i+1,j}^{ct} = \begin{cases} 0, & T_i^{op} \le T_{k,j}^{pm} \le T_i^{op} + t_i^{re} \\ \lambda R_{i,j} t_{i,j}^{ct}, & T_i^{op} + t_i^{re} < T_{k,j}^{pm} \le T_i^{op} + t_i^{op} \\ t_{i,j}^{ct}, & T_i^{op} + t_i^{re} \le T_{k,j}^{pm} \end{cases}$$

Fig. 4. The thought chart of maintenance decision-making approach based on dynamic OW.

(1) Initially, we initialize various parameters for component j, including the PM moment $T_{0,j}^{pm}$, service age $t_{0,j}^{ct}$, reliability threshold R_{jmin}, the time $t_{0,j}^{a}$ required for PM and the interval between preventive maintenances $t_{0,j}^{RA}$.

(2) When the tandem component g reaches the k-th PM moment $T_{k,g}^{pm}$, indicating that its service age $t_{i,g}^{ct}$ has also reached the maximum service duration, it triggers OM. The i-th OM moment T_i^{op} can be determined using the following formula:

$$T_i^{op} = T_{k,g}^{pm}, \quad when \quad t_{k,g}^{RA} - t_{i,g}^{ct} = 0 \tag{2}$$

The component g that satisfies Equation (2) shall then be replaced.

(3) OM is conducted for other moment j that exhibit high similarity with component g and are in close proximity to the PM moment $T_{k,j}^{pm}$. The reliability $R_{i,j}$ and service age $t_{i,j}^{ct}$ of other components j in the i-th OM are composed into two-dimensional data $D_i = [D_{i,1}, D_{i,2}, \cdots, D_{i,j}] = [(R_{i,1}, t_{i,1}^{ct}), (R_{i,2}, t_{i,2}^{ct}), \cdots (R_{i,j}, t_{i,j}^{ct})]$ with data amount A. The K-means++ clustering algorithm is employed to partition dataset D_i into K clusters based on their shared characteristics. The fundamental concept of K-means++ clustering can be succinctly represented by Equation (3).

$$\begin{cases} p_{i,j}(D_{i,j}) = \dfrac{\sqrt{(R_{i,j}-\phi_R)^2+(t_{i,j}^{ct}-\phi_t)^2}}{\sum_{i=1}^{K}\sum_{D_{i,j}\in C_i}\sqrt{(R_{i,j}-\phi_R)^2+(t_{i,j}^{ct}-\phi_t)^2}} \\ SSE = \sum_{i=1}^{K}\sum_{D_{i,j}\in C_i}\sqrt{(R_{i,j}-\phi_R)^2+(t_{i,j}^{ct}-\phi_t)^2} \end{cases} \tag{3}$$

Where K represents the number of clusters; ϕ_R and ϕ_t represent the cluster centers of reliability dimension and service age dimension, respectively; C_i represents the cluster to which it belongs; $p_{i,j}(D_{i,j})$ represents the probability of each data belonging to the next cluster center, which is utilized for the selection of initial K cluster centers; The SSE represents the objective function, indicating that classification is considered accomplished once the SSE reaches its minimum value.

(4) According to Equation (1), a composite distribution $f_k^{MR}(t)$, consisting of A failure probability distributions $f_j(t)$, is constructed.

$$f_j(t) = \frac{\alpha_j}{\beta_j^{\alpha_j}} t^{\alpha_j-1} exp[-(\frac{t}{\beta_j})^{\alpha_j}], \quad t \ge 0 \tag{4}$$

$$f_{k,j}^{MR}(t) = \sum_{j=1}^{A}\omega_j \frac{\alpha_j}{\beta_j^{\alpha_j}} t^{\alpha_j-1} exp[-(\frac{t}{\beta_j})^{\alpha_j}], t \ge 0 \tag{5}$$

The EM algorithm is employed for parameter estimation of the mixed distribution $f_{k,j}^{MR}(t)$, as illustrated in Fig. 5.

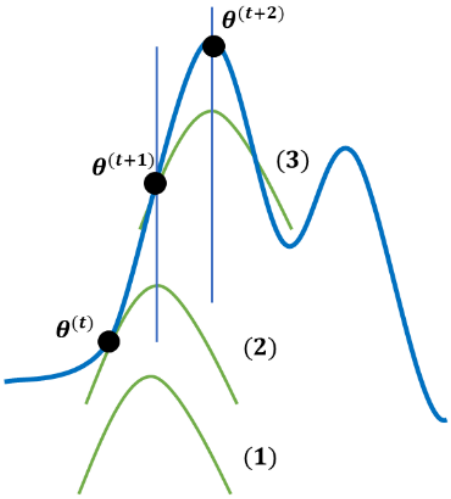


Fig. 5. Schematic diagram of EM algorithm parameter estimation.

Step1. We employ the gradient descent (GD) algorithm to estimate the parameters α_j and β_j of the failure probability distribution based on N reliability data, and to determine the failure probability at a given service age $t_{i,j}^{ct}$.

The objective function $J(\alpha_j, \beta_j)$ is:

$$J(\alpha_j, \beta_j) = \frac{1}{2N}\sum_{i=1}^{N}(R_{i,j}^{p} - R_{i,j}) \tag{6}$$

$$R_{i,j}^{p} = 1 - \int_0^{t_j^{ct}} f_j(t)dt \tag{7}$$

Where $R_{i,j}^{p}$ is the predicted reliability and N is the number of reliability.

According to Equation (6), the iterative formulas for parameters α and β are:

$$\alpha_j = \alpha_j - \frac{a_1}{N}\sum_{i=1}^{N}(R_{i,j}^{p} - R_{i,j})\int_0^{t_j^{ct}} \frac{\partial f_j}{\partial \alpha_j}dt \tag{8}$$

$$\beta_j = \beta_j - \frac{a_2}{N}\sum_{i=1}^{N}(R_{i,j}^{p} - R_{i,j})\int_0^{t_j^{ct}} \frac{\partial f_j}{\partial \beta_j}dt \tag{9}$$

$$\frac{\partial f_j}{\partial \alpha_j} = \frac{t^{\alpha_j-1}+\alpha_j t^{\alpha_j-1} ln\frac{t}{\beta_j}[1-(\frac{t}{\beta_j})^{\alpha_j}]}{\beta_j^{\alpha_j}} exp[-(\frac{t}{\beta_j})^{\alpha_j}] \tag{10}$$

$$\frac{\partial f_j}{\partial \beta_j} = \frac{t[-(\frac{t}{\beta_j})^{\alpha_j-1}]-\beta_j}{\beta^{\alpha_j+2}}\alpha_j^2 t^{\alpha_j-1} exp[-(\frac{t}{\beta_j})^{\alpha_j}] \tag{11}$$

Where a_1 and a_2 represent the respective learning rates.

Step2. Input the failure probabilities $F_j(t_j^{ct}) = \int_0^{t_j^{ct}} f_j(t)dt$ of component j, joint distribution $p(F_j, \mathbf{z}; \theta_j^{(m)})$, conditional distribution $p(\mathbf{z}|F_j; \theta_j^{(m)})$, and maximum iteration number M.

Where, $z = (z_1, z_2, \cdots, z_j)^T$ represents the implicit data which is the probability belonging to the failure probability distribution $f_j(t)$ of component j, and $\theta_j^{(m)}$ refers to $\alpha_j^{(m)}$, $\beta_j^{(m)}$ and $\omega_j^{(m)}$.

Step3. Randomly initialize the initial value θ_j^0 of the mixed distribution function parameter θ, that is, initialize the shape and position of the green curve (1).

Step4. Start the iteration of the EM algorithm:

a) E step. Calculate the conditional probability expectation of the m-th joint distribution, representing the increment from green curve (1) to green curve (2).

$$Q_{i,j}^{(m)}(z_i) = p(z_i = j | t_j; \theta_j^{(m)}) = \frac{\omega_j^{(m)} \cdot f_j(t)}{\sum_{j=1}^{A} \omega_j^{(m)} \cdot f_j(t)} \tag{12}$$

Where $Q_{i,j}^{(m)}(z_i)$ represents the probability distribution of the implied data z_i.

b) M step. Maximizing $L(\theta^m)$ yields θ^{m+1}, i.e. determining the parameter θ at the intersection of the green curve (2) and the blue curve.

The traditional parameter estimation approach generally solves model parameters through Equation (13), such as maximum likelihood estimation.

$$\begin{aligned} L(\theta_j^{(m)}) &= \sum_{i=1}^{N}\sum_{j=1}^{A} Q_{i,j}^{(m)}(z_i) \ln \frac{P(t_j \mid z_i; \alpha_j, \beta_j) P(z_i; \omega_j)}{Q_{i,j}^{(m)}(z_i)} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{A} Q_{i,j}^{(m)}(z_i) \ln \frac{\omega_j \cdot f_j(t)}{Q_{i,j}^{(m)}(z_i)} \end{aligned} \tag{13}$$

However, Equation (13) is not expressed in closed form and the maximum likelihood estimation necessitates solving the transcendental equation, which poses challenges in terms of complexity and accuracy. Therefore, this study adopted the following solution method:

The weighted mean and variance are iteratively computed based on Equation (14).

$$\begin{cases} \hat{\mu}_j^{(m+1)} = \frac{\sum_{i=1}^{N} Q_{i,j}^{(m)} t_j}{N_j^{(m+1)}} \\ \hat{\sigma}^{2(m+1)}_j = \frac{\sum_{i=1}^{N} Q_{i,j}^{(m)} (t_j - \hat{\mu}_j)^2}{N_j^{(m+1)}} \\ \omega_j^{(m+1)} = \frac{\sum_{i=1}^{N} Q_{i,j}^{(m)}}{N} \\ N_j^{(m+1)} = \sum_{i=1}^{N} Q_{i,j}^{(m)} \end{cases} \tag{14}$$

In Equation (14), $\hat{\mu}_j$ and $\hat{\sigma}^2$, $\hat{\alpha}_j$ and $\hat{\beta}_j$ exist transcendental equation (15), which makes it difficult to obtain the analytical expression.

$$\begin{cases} \mu = \beta_j \Gamma(1 + \frac{1}{\alpha_j}) \\ \sigma^2 = \beta_j^2 [\Gamma(1 + \frac{2}{\alpha_j}) - \Gamma^2(1 + \frac{1}{\alpha_j})] \end{cases} \tag{15}$$

Let $\lambda = \mu/\sigma$, and the formula is:

$$\lambda = \frac{\Gamma(1+\frac{1}{\alpha_j})}{[\Gamma(1+\frac{2}{\alpha_j}) - \Gamma^2(1+\frac{1}{\alpha_j})]^{1/2}} \tag{16}$$

The solution of Equation (16) is complex; however, due to the monotonic relationship between λ and α, as well as the known value of α, it becomes feasible to employ the least square method for parameter estimation of α and subsequent determination of β based on Equation (15).

$$\beta = \frac{\mu}{\Gamma(1+\frac{1}{\alpha_j})} \tag{17}$$

c) If θ^{m+1} converges, the algorithm terminates; otherwise, it returns to step a) for iteration. In other words, when the green curve intersects with the blue curve, another parameter of the green curve is modified and the iteration continues.

Step5. Output model parameters.

(5) The parameter values of the mixed distribution function f for the *k*-th cluster, obtained from equation (4), are utilized to assess the clustering method with varying clustering numbers K, employing Bayesian Information Criterion (BIC). This evaluation aims to identify the optimal number of clusters k that minimizes the information content in each cluster, thereby maximizing data similarity (reliability or service age) and minimizing cluster complexity.

$$BIC_k = n\, ln(A) - 2\, ln\, L\,(\theta) \tag{18}$$

Where BIC_k represents the estimated value of cluster number k, n denotes the number of unknown parameters, $L(\theta)$ refers to the maximum likelihood function of the mixed distribution function $f_{k,j}^{MR}(t)$, and $n = 3A$.

When $k > 1$, i.e., when the number of clusters exceeds one, let $L(\theta_k)$ denote the likelihood function of the mixed distribution $f_{k,j}^{MR}(t)$ for the *k*-th cluster. The expression for $L(\theta)$ is as follows:

$$L(\theta) = \prod_{k=1}^{K} \prod_{i=1}^{N_k} \sum_{j=1}^{A} \omega_j \cdot f_j \tag{19}$$

Where $\sum_{j=1}^{A} \omega_j \cdot f_j$ represents a mixed distribution and n_k denotes the data of the *k*-th cluster, in accordance with Equation (18) and Equation (19).

$$BIC_i = n\, ln(A) - 2 \sum_{k=1}^{K} ln \prod_{i=1}^{N_k} \sum_{j=1}^{A} \omega_j \cdot f_j \tag{20}$$

(6) When component g satisfies Equation (2), it triggers

replacement, while the remaining components j undergo imperfect maintenance. At this time, the service age of component j is $t_{i-1,j}^{ct} + t_{k,g}^{RA}$, and its reliability R_j is:

$$R_j = 1 - \int_0^{t_{i-1,j}^{ct}+t_{k,g}^{RA}} f_{k,j}^{MR}(t)dt \tag{21}$$

(7) We compose the imperfect maintenance window t_i^{im} with the PM time $t_{i,s}^{a}$ of all components in the cluster where component g (the component that trigger PM) is located. $t_{i,j}^{a} \in [t_{i,j,0}^{a}, t_{i,j,1}^{a}, t_{i,j,2}^{a}]$ respectively represents the time corresponding to the disrepair (when not in the OW), imperfect maintenance and replacement of component j, and the imperfect maintenance window t_i^{im} is:

$$t_i^{im} = \sum_{j=1\&j\neq g}^{A} t_{i,j}^{a}, \quad when \quad R_j > R_{jmin} \tag{22}$$

Given the randomness of reliability degradation, it is essential to compare the reliabilities of other components j with their respective reliability threshold R_{jmin}, while also addressing the replacement needs of component g that triggers OM. If $R_j < R_{jmin}$, replacement will be conducted; if $R_j > R_{jmin}$, imperfect maintenance will be conducted. Therefore, the replacement window t_i^{re} for the i-th OM is determined.

$$t_i^{re} = t_{i,g}^{re} + \sum_{j=1\&j\neq g}^{A} t_{i,j}^{re}, \quad when \quad R_j < R_{jmin} \tag{23}$$

The i-th dynamic OW t_i^{op} is

$$t_i^{op} = t_i^{re} + t_i^{im} \tag{24}$$

(8) For condition-based maintenance: When the reliability of a component is less than or equal to the reliability threshold R_{jmin}, it necessitates replacement; conversely, if the component falls outside the designated maintenance window, it does not require maintenance. Additionally, the components j of the k-th cluster should be subjected to OM based on its reliability $R_{i,j}$, with λ, $\lambda > 1$, being an adjustment factor used to represent the impact of imperfect maintenance on quality. Update the $k+1$-th service age $t_{i+1,j}^{ct}$ of component j:

$$t_{i+1,j}^{ct} = \begin{cases} 0, & T_i^{op} \le T_{k,j}^{pm} \le T_i^{op} + t_i^{re} \\ \lambda R_{i,j} t_{i,j}^{ct}, & T_i^{op} + t_i^{re} < T_{k,j}^{pm} \le T_i^{op} + t_i^{op} \\ t_{i,j}^{ct}, & T_i^{op} + t_i^{re} \le T_{k,j}^{pm} \end{cases} \tag{25}$$

Then the reliability $R_{i+1,j}$ of component j is:

$$R_{i+1,j} = 1 - \int_0^{t_{i+1,j}^{ct}} f_{k,j}^{MR}(t)dt \tag{26}$$

The reliability $R_{i+1,j}$ of component j for the i+1-th OM is given, and the time interval between the k-th and k+1-th PM is denoted as $t_{k+1,j}^{RA}$.

$$R_{i+1,j} - R\int_0^{t_{k+1,j}^{RA}} f_{k,j}^{MR}(t)dt_{jmin} \tag{27}$$

The k+1-th PM moment $T_{k+1,j}^{pm}$ of component j is

$$T_{k+1,j}^{pm} = T_{k,j}^{pm} + t_{i,s}^{a} + t_{k+1,j}^{RA} + t_{k,g}^{RA} - t_{i,g}^{ct} \tag{28}$$

(9) Return (2) to update the i+1-th OM moment T_{i+1}^{op}, and then make a decision on whether to continue running or initiate OM.

4. Simulation analysis

In this study, we present an analysis of the tandem components of the main combustor in a specific type of aeroengine, which includes the inner/outer sleeve, main swirler, radial swirler, fuel nozzle, flame tube, and front sealing ring. A maintenance decision-making model based on dynamic OW is validated using 100 sets of reliability data. The preset EM algorithm iteration times are 100 and the minimum allowable error is 10^{-10}. For the GD algorithm, the iteration number is 1000, the learning rate is 0.01, and the minimum allowable error is 10^{-10}. The life expectancy of the main combustor is 1000 hours. Tab. 1 shows the initial parameters of each component for OM .

Table. 1. Initial Parameters for OM of Components in Main Combustor.

No.	components	t^{ct}/h	t^{RA}/h	R_{min}	t^{a}/h		maintenance cost / dollar	replacement cost / dollar
					OM	replacement		
1	inner/outer sleeve	369	600	0.38	12	6	50	500
2	main swirler	109	400	0.2	20	10	200	1200
3	radial swirler	149	800	0.22	8	2	150	800
4	fuel nozzle	239	500	0.58	8	2	50	500
5	flame tube	79	1400	0.31	10	4	150	800
6	front sealing ring	129	250	0.67	6	2	80	600

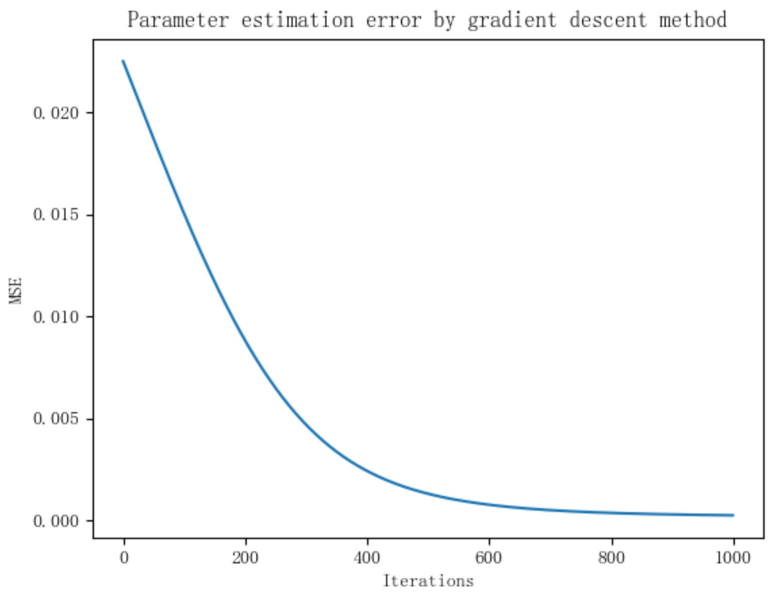


Fig. 6. Parameter estimation error variation graph of GD algorithm.

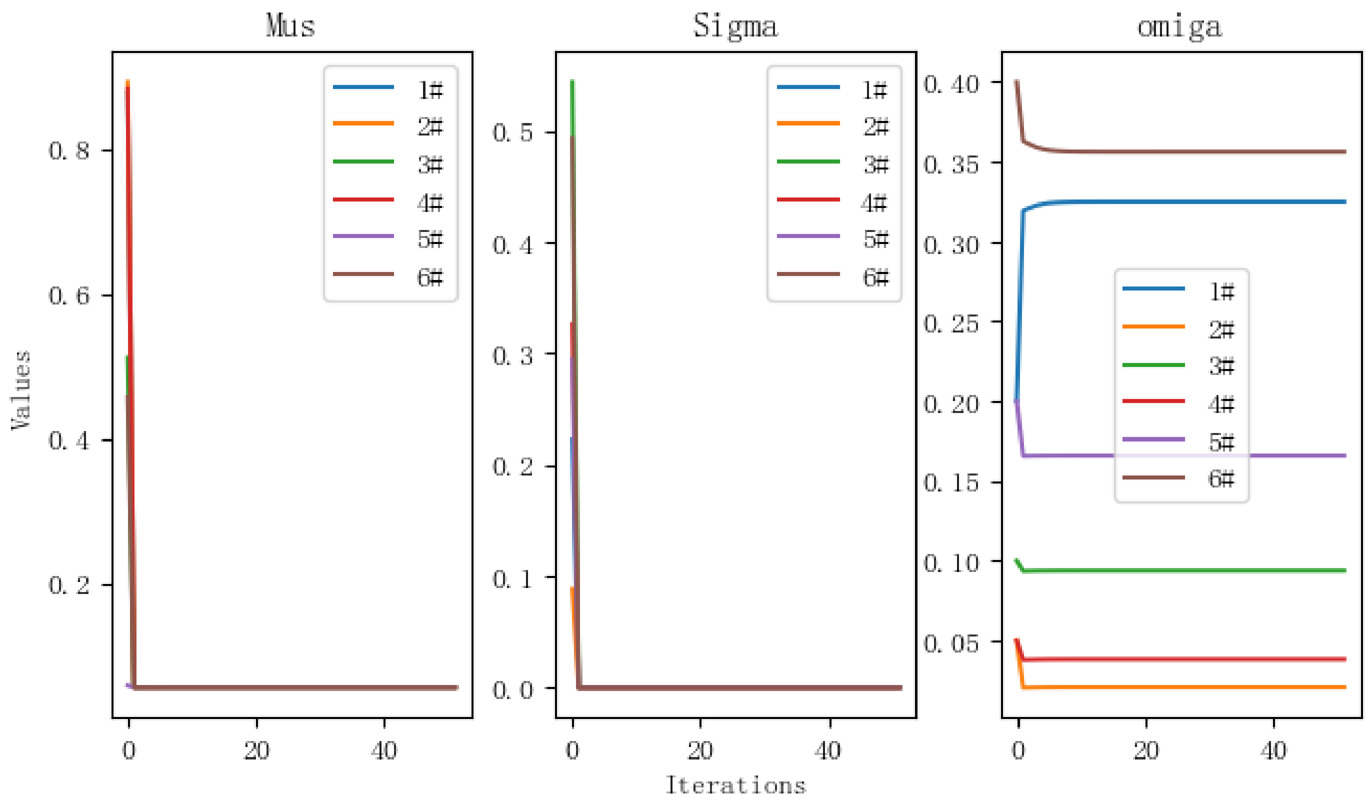


Fig. 7. Iteration diagram of Weibull parameters.

Fig. 6 and Fig. 7 are the variation graphs of each parameter of the GD algorithm and EM algorithm for the first OM, respectively.

It can be observed from Fig. 6 that the mean square error (MSE) of the Gradient Descent (GD) algorithm approaches 0 after approximately 800 iterations, indicating a highly effective parameter estimation effect. Furthermore, it is evident from Fig. 7 that during the iterative process of parameter estimation using the EM algorithm, both the $\hat{\mu}$ and $\hat{\sigma}^2$ of the mixed Weibull distribution converge rapidly within about 5 iterations, demonstrating excellent convergence performance. Additionally, it is noted that the weight ω stabilizes quickly within approximately 5 iterations as well, providing further evidence for the high efficiency of our proposed model's parameter estimation.

Table. 2. Parameter estimates of different algorithms.

Algorithms	ω	α	β
Theoretical distribution parameter	$\omega_0 = 0.35, \omega_1 = 0.02, \omega_2 = 0.08$ $\omega_3 = 0.05, \omega_4 = 0.15, \omega_5 = 0.35$	$\alpha_0 = 5, \alpha_1 = 5, \alpha_2 = 5$ $\alpha_3 = 5, \alpha_4 = 5, \alpha_5 = 5$	$\beta_0 = 1, \beta_1 = 1, \beta_2 = 1$ $\beta_3 = 1, \beta_4 = 1, \beta_5 = 1$
Improved EM algorithm	$\omega_0 = 0.332, \omega_1 = 0.023, \omega_2 = 0.095$ $\omega_3 = 0.042, \omega_4 = 0.156, \omega_5 = 0.352$	$\alpha_0 = 5.04, \alpha_1 = 5.05, \alpha_2 = 5.05$ $\alpha_3 = 5.05, \alpha_4 = 5.04, \alpha_5 = 5.04$	$\beta_0 = 0.920, \beta_1 = 0.918, \beta_2 = 0.919$ $\beta_3 = 0.920, \beta_4 = 0.920, \beta_5 = 0.921$
GA	$\omega_0 = 0.285, \omega_1 = 0.037, \omega_2 = 0.064$ $\omega_3 = 0.115, \omega_4 = 0.077, \omega_5 = 0.422$	$\alpha_0 = 5.13, \alpha_1 = 5.32, \alpha_2 = 4.85$ $\alpha_3 = 5.01, \alpha_4 = 5.16, \alpha_5 = 4.94$	$\beta_0 = 0.913, \beta_1 = 0.908, \beta_2 = 0.909$ $\beta_3 = 0.902, \beta_4 = 0.922, \beta_5 = 0.911$
LSM	$\omega_0 = 0.285, \omega_1 = 0.037, \omega_2 = 0.064$ $\omega_3 = 0.115, \omega_4 = 0.077, \omega_5 = 0.422$	$\alpha_0 = 5.33, \alpha_1 = 4.78, \alpha_2 = 4.96$ $\alpha_3 = 5.15, \alpha_4 = 5.12, \alpha_5 = 4.82$	$\beta_0 = 1.163, \beta_1 = 0.898, \beta_2 = 0.939$ $\beta_3 = 0.971, \beta_4 = 1.222, \beta_5 = 0.981$

To validate the accuracy of the parameter estimation method, we employed a genetic algorithm (GA) and least square method (LSM) to estimate the parameters using the same dataset, and presented the results in Tab. 2. Compared with the GA and LSM, the mean values of the parameter estimates of the proposed improved EM algorithm are $\bar{\alpha} = 5.045$ and $\bar{\beta} = 0.9197$, which are closer to the theoretical distribution parameter value than the mean values of the parameter estimates of the GA ($\bar{\alpha} = 5.068$, $\bar{\beta} = 0.9108$.) and LSM ($\bar{\alpha} = 5.070$, $\bar{\beta} = 1.1207$). The variances of the parameter estimates obtained from the improved EM algorithm, denoted as $Std(\alpha) = 2.5 \times 10^{-5}$ and $Std(\beta) = 8.9 \times 10^{-7}$, exhibit greater stability compared to those obtained from the GA ($Std(\alpha) = 2.385 \times 10^{-2}$, $Std(\beta) = 3.647 \times 10^{-5}$) and LSM ($Std(\alpha) = 2.613 \times 10^{-2}$, $Std(\beta) = 3.069 \times 10^{-2}$). The above results confirm the hypothesis that the proposed mixed reliability or failure rate of components is a composition of the reliability or failure rate of each component in the tandem system, thereby validating the accuracy of our proposed model.

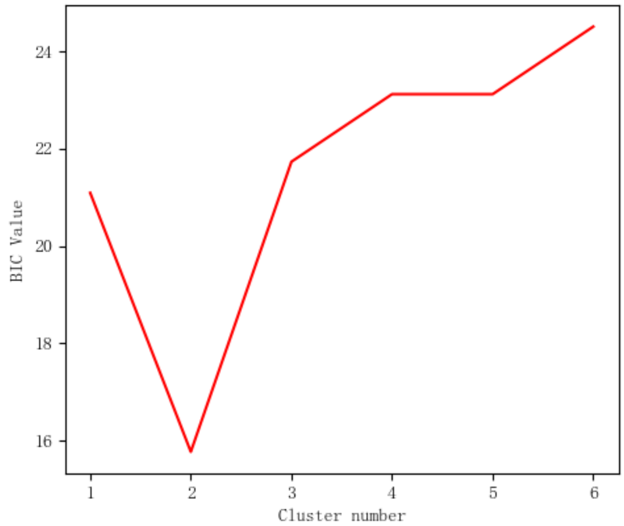


Fig. 8. BIC value change graph of each cluster number.

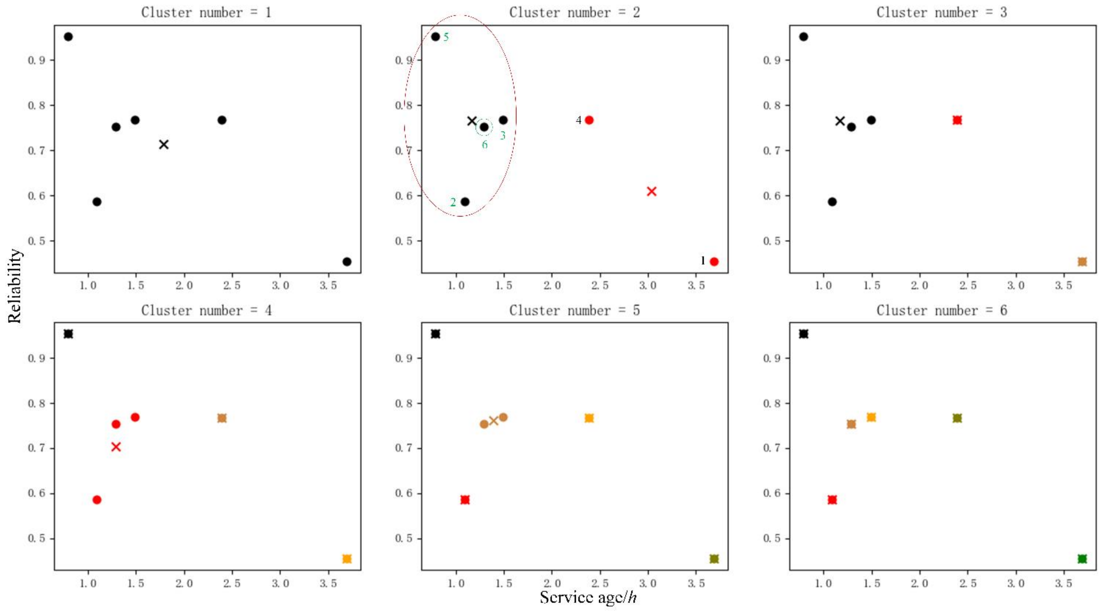


Fig. 9. Schematic diagram of k-means classification.

Table. 3. The initial opportunity maintenance parameter table.

components	1	2	3	4	5	6
Reliability before maintenance	0.356	0.273	0.548	0.697	0.803	0.668
Reliability after maintenance	1	0.455	0.699	0.697	0.840	1
Service age before maintenance	490	230	270	360	200	250
Service age after maintenance	0	104.7	188.7	360	168.4	0
PM interval	600	400	800	500	1400	250
Service age change value	490	125.3	81.3	0	31.6	250
Reliability change value	0.644	0.182	0.151	0	0.037	0.332

Analysis from the rationality of the MS. As can be seen from Fig. 8, Fig. 9 and Tab. 3, when the number of clusters is 2, the BIC value reaches the minimum. Notably, the first cluster components consist of components 2, 3, 5, and 6. At this time, the service age of component 6 reaches 250 hours, meeting the condition for replacement, and components 2, 3 and 5 trigger OM. After maintenance, the reliability is 0.455, 0.699 and 0.840 respectively, the service age change value is 125.3, 81.3 and 31.6 hours respectively, and the service age change rate is 31.3%, 10.2% and 2.3% respectively. The second cluster of components are 1 and 4. Component 4 has been in service for 360 hours, with 140 hours remaining before the PM interval. Its reliability is 0.687, surpassing the minimum allowable reliability threshold of 0.58. Moreover, it does not belong to the same cluster as component 6 that necessitates replacement. Hence, no maintenance operation is required for component 4. Despite having a service age of only 490 hours, component 1 exhibits an excessively rapid decline in reliability, resulting in a pre-maintenance reliability value of merely 0.356 which falls below the minimum allowable level of 0.38, thereby triggering its replacement. Based on the analysis of reliability, service age change value, and service age change rate, it is evident that components with lower reliability exhibit a higher degree of service age update, whereas those with higher reliability demonstrate a lower degree of service age update.

Based on the analysis of reliability, service age change value, and service age change rate, it is evident that components with lower reliability demonstrate a higher degree of service age update, whereas those with higher reliability exhibit a lower degree of service age update. At the same time, this model has been validated to effectively determine maintenance intensity, minimize support resource consumption, reduce maintenance duration, and enhance the scientific of maintenance.

Analysis from the reliability of MS. As can be seen from Table 3 and Table 4, the reliability of component 1 is only 0.356 when the cluster number is 1. This value falls below its minimum allowable reliability threshold of 0.38, necessitating replacement. However, relying solely on OM yields limited improvements in its reliability, indirectly leading to increased downtime and reduced equipment availability. When the cluster number exceeds 2, the reliability of component 1 is merely 0.356, falling below its minimum allowable threshold of 0.38, necessitating replacement. Failure to perform OM or replacement will significantly compromise equipment safety. The MS can achieve high reliability only when the number of clusters is 2. At the same time, it also verifies the effectiveness of the proposed OM approach based on dynamic OW.

Table. 4. The maintenance schedule for the first opportunity maintenance of each cluster.

Cluster	Replaced components	Components for OM	Maintenance time /h
1	6	1、2、3、4、5	60
2	1、6	2、3、5	46
3	6	2、3、5	40
4	6	2、3	30
5	6	3	30
6	6	/	2

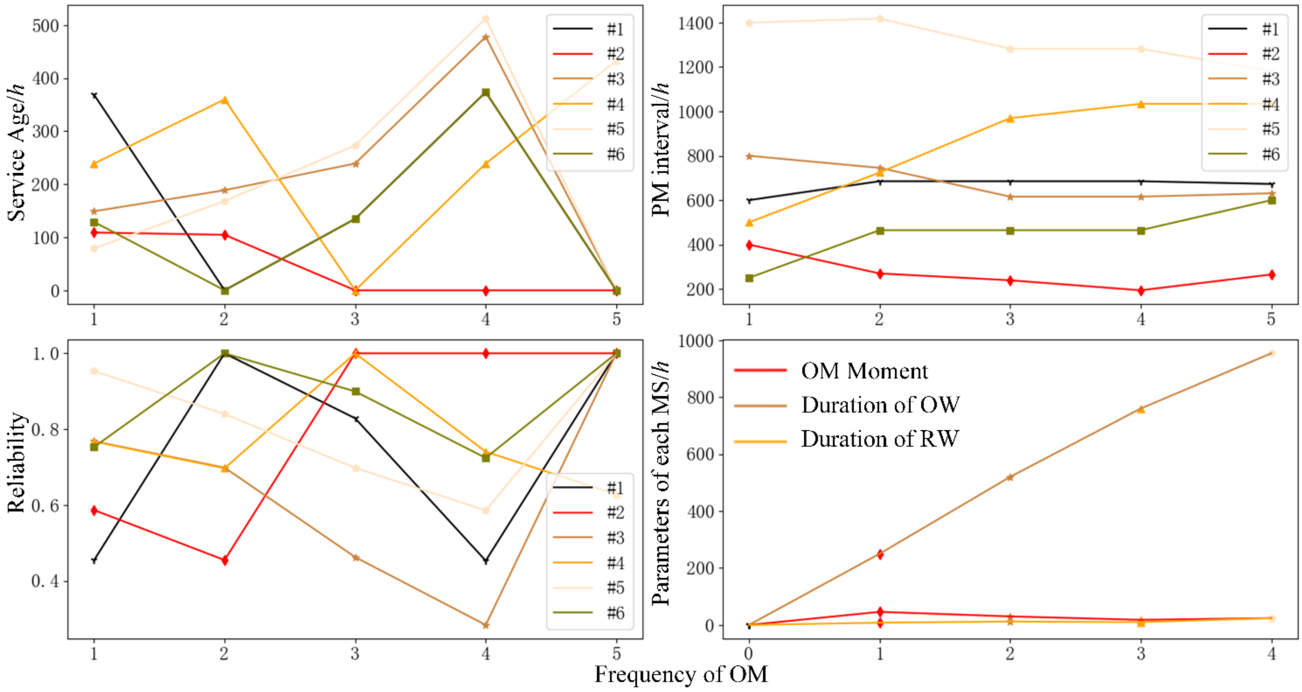


Fig. 10. Parameter change diagram of each OM scheme (initial moment).

According to Fig. 10, we take component 2 as an example to analyze the rationality of the proposed approach for setting MSs within the life expectancy. For the first time, component 2 conduct OM. Its service age is updated from 230 (before maintenance) to 104.673, and its reliability was improved from 0.273 to 0.455. Since its reliability (0.273) is relatively close to its minimum required reliability (0.2), and the reliability drops significantly (from 1 to 0.273), the preventive maintenance interval is adjusted from 400 hours to 269.709 hours. In this case, the replacement maintenance window and OM window are updated to 8 hours and 46 hours, respectively. For the second time, component 2 conduct replacement. Its service age is updated from 270 hours to 0 hours, and its reliability is increased from 0.191 to 1. Since its reliability (0.191) is relatively close to its minimum allowable reliability (0.2), the preventive maintenance interval has been updated from 269.709 hours to 239.068 hours. In this case, the replacement maintenance window is updated to 12 hours and the OM window to 30 hours. For the third time, component 2 conduct replacement. Its service age is updated from 240 hours to 0 hours, and its reliability is increased from 0.186 to 1. Since minimum allowable reliability (0.2) is higher than 0.186, and the reliability drops significantly (from 1 to 0.186), the preventive maintenance interval has been updated from 239.068 hours to 193.892 hours. In this case, the replacement maintenance window is updated to 10 hours and the OM window to 18 hours. For the fourth time, component 2 conduct replacement. Its service age is updated from 194 hours to 0 hours, and its reliability is increased from 0.221 to 1. Although its reliability (0.221) is slightly greater than the minimum allowable reliability (0.2), due to the fact that it reaches the PM time (194 hours for the third update) and the reliability drops significantly (from 1 to 0.221), the PM interval is updated from 239.068 hours to 193.892 hours. In this case, the replacement maintenance window is updated to 10 hours and the OM window to 18 hours. Based on the variations in service age, reliability, PM intervals, and the duration of OWs, the proposed approach enables updating of MSs using maintenance data, thereby further validating the rationality and timeliness of the proposed approach.

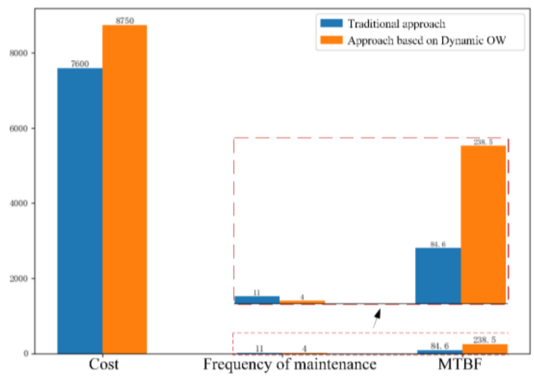


Fig. 11. Comparison chart of the proposed scheme and the traditional scheme.

Analysis from the economy and availability of MS. As shown in the figure, the proposed MS exhibits a significant improvement over the traditional approach. It reduces the frequency of maintenance from 11 times to 4 times, resulting in an approximately 2.75-fold increase. Moreover, it enhances the Mean Time Between Failure (MTBF) from 84.6 hours to 238.5 hours, yielding a roughly 2.82-fold improvement.

In summary, the proposed maintenance approach enables real-time updates of the MS based on equipment reliability data. This holds significant implications for streamlining maintenance tasks and improving the reliability, availability, and economy.

5. Conclusion

Addressing the limitations of the traditional maintenance decision-making approach in tandem system, we propose a maintenance decision-making approach based on dynamic OW.

(1) By analyzing the modeling concept based on the maintenance decision-making approach of the OW, this paper proposes a dynamic maintenance approach for the OW that enables real-time updates of service age, reliability, PM interval, and duration of the OW.

(2) To address the problem of estimating reliability due to the coupling of each component in the tandem system's structure, failure rate, and reliability, this paper ingeniously utilizes the monotonic relationship between two parameters of the Weibull distribution. It proposes an improved EM algorithm to solve transcendental equations in traditional algorithms when estimating mixed Weibull distribution parameters. This simplifies parameter estimation for mixed Weibull distributions and enhances algorithm convergence performance.

(3) This study uses a specific tandem system of aircraft engines as an example to demonstrate the significance of the proposed approach in enhancing equipment reliability and optimizing maintenance support resources through accurate parameter estimation, reliable MS setting, and rational and timely adjustments to real-time maintenance strategies.

This study does not currently include factors such as maintenance costs and equipment importance, as well as the issue of randomly selecting the first initial clustering center in the k-means++ algorithm, which is the direction for future research to be improved.

References

1. Su, Chun, and Lin Wu. "Opportunistic maintenance optimisation for offshore wind farm with considering random wind speed." *International Journal of Production Research*, vol. 62, 2024, pp. 1862-1878, https://doi.org/10.1080/00207543.2023.2202280.
2. Dinh, Duc, et al. "Reliability Modeling and Opportunistic Maintenance Optimization for a Multicomponent System with Structural Dependence." *Reliability Engineering & System Safety*, vol. 241, 2023, pp. 109708, https://doi.org/10.1016/j.ress.2023.109708.
3. Zhao, Baorong, et al. "Research on opportunistic maintenance strategy of series system considering fault correlation and component importance." *QR2MSE 2022*， 2022， pp. 642-647, https://10.1049/icp.2022.2937.
4. Shi, Haohao, et al. "Opportunistic maintenance policies for multi-machine production systems with quality and availability improvement." *Reliability Engineering & System Safety*, vol. 234, 2023, pp. 109183, https://doi.org/10.1016/j.ress.2023.109183.
5. Xiao, Lei, et al. "Joint Optimization of Opportunistic Maintenance and Production Scheduling considering Batch Production Mode and Varying Operational Conditions." *Reliability Engineering & System Safety*, vol. 202, 2020, pp. 107047, https://doi.org/10.1016/j.ress.2020.107047.
6. Meng, D, et al. (2022). "Fault analysis of wind power rolling bearing based on EMD feature extraction. " *CMES*, vol 130, 2022, pp. 543-558, https://doi.org/10.32604/cmes.2022.018123.
7. Rasay, H, et al. "Opportunistic maintenance integrated model for a two-stage manufacturing process. " *Int J Adv Manuf Technol*, vol. 119, 2022, pp. 8173–8191, https://doi.org/10.1007/s00170-021-08571-5.
8. Ma, Hui, et al. "Residual useful life prediction of the vehicle isolator based on Bayesian inference." *Structures*. vol. 58, 2023, pp. 105518, https://doi.org/10.1016/j.istruc.2023.105518.
9. Meng, Debiao, et al. "Kriging-assisted hybrid reliability design and optimization of offshore wind turbine support structure based on a portfolio allocation strategy." *Ocean Engineering*, vol. 295, 2024, pp. 116842.
10. Yang, Shiyuan, et al. "A novel learning function for adaptive surrogate-model-based reliability evaluation." *Philosophical Transactions of*

the Royal Society A, vol. 382, 2024, pp. 20220395, http://doi.org/10.1098/rsta.2022.0395.

11. J. Huang, et al. "A Real-Time Maintenance Policy for Multi-Stage Manufacturing Systems Considering Imperfect Maintenance Effects." *IEEE Access*, vol. 6, 2018, pp. 62174-62183, https://doi.org/10.1109/ACCESS.2018.2876024.
12. Yang, Xiuzhen, et al. "Mission Reliability-centered Opportunistic Maintenance Approach for Multistate Manufacturing Systems." *Reliability Engineering & System Safety*, vol. 241, 2023, pp. 109693, https://doi.org/10.1016/j.ress.2023.109693.
13. Xia, Tangbin, et al. "Dynamic Maintenance Decision-making for Series–Parallel Manufacturing System Based on MAM–MTW Methodology." *European Journal of Operational Research*, vol. 221, 2012, pp. 231-240, https://doi.org/10.1016/j.ejor.2012.03.027.
14. Liu, Gehui, et al. "Optimum Opportunistic Maintenance Schedule Incorporating Delay Time Theory with Imperfect Maintenance." *Reliability Engineering & System Safety*, vol. 213, 2021, p. 107668, https://doi.org/10.1016/j.ress.2021.107668.
15. Zhang, Baoshan, et al. "A fault prediction model of adaptive fuzzy neural network for optimal membership function." *IEEE Access*, vol. 8, 2020, pp. 101061-101067, https://doi.org/10.1109/ACCESS.2020.2997368.