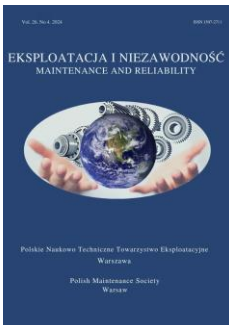


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# Optimal maintenance policy for a Markov deteriorating system under reliability limit

Indexed by: Web of Science Group

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## Highlights

- An imperfect preventive maintenance plan is created for multi-component complex machine.
- The use and age-related deterioration process of machine is evaluated.
- The reliability of machine-produced parts is sustained at a certain level.

## Abstract

In this study, failure data of computer numerical control machine used in defense industry was analyzed to develop maintenance algorithm with a Markov feature. An imperfect preventive maintenance model that minimizes long-term operational cost is created for the machine wearing down randomly over time. The reliability-centric preventive maintenance policy was developed where the system status was monitored instantaneously. The use and age-related deterioration process of system is defined as the failure rate increase factor and age reduction factor, and these variables are combined to create hybrid failure model. As result of the imperfect maintenance algorithm developed for the multi-component machine, minimum long-term total unit cost, optimum system reliability value, number of maintenance and times between sequential maintenance cycles are obtained as outputs. Furthermore, system sub-equipment was specified that needs to be maintained in each cycle. Moreover, imperfect maintenance activities are planned when the reliability level of subsystems drops to the predetermined R value.

**Keywords**
preventive maintenance, imperfect maintenance, age-related deterioration, optimization, reliability.



## 1. Introduction

The increasing importance and necessity of maintenance operations in industrial areas result in the development of maintenance models for systems that wear down randomly over time. Maintaining the proper level of readiness and reliability is crucial for any operating system. As the basis of maintenance policies widely covered in the literature, Barlow and Proschan [1] examined maintenance types and system reliability. Velmurugan et al. [32] focused on smart preventive maintenance with optimal planning and scheduling processes for small and medium sized enterprises. Preventive Maintenance (PM), which corresponds to all maintenance actions carried out during the operating period of a system before failure, is the leading one in maintenance technology. Fan et.al. [11] studied that preventive maintenance actions are fundamental to improve the overall reliability and availability of engineering systems. These tasks can be listed such as inspection, cleaning, lubrication, calibration or replacement of certain components or subsystems. Dongwei et al. [10] worked on preventive maintenance models by examining the dynamic fault data of systems. A number of studies have

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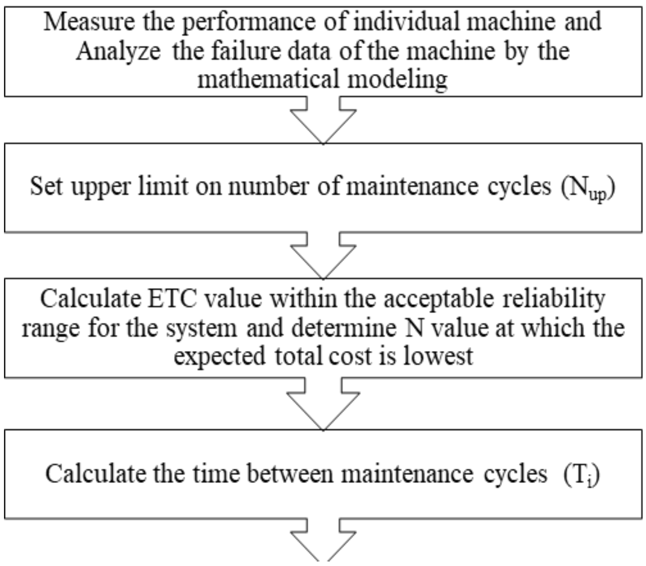
shown the methodology for tracking the condition of machines. Data capture and analysis, establishing a threshold limit for the parameter related to system condition, and developing the appropriate maintenance model for an effective solution to complex systems are the key applications in competitive industries Kolhatkar and Pandey [16]. Coria et al. [7] analyzed the fault data of the system with the parameter estimation method and improved the obtaining of shape and scale parameters. When these parameters are examined within the hybrid model while calculating the time between maintenance cycles and system reliability, an imperfect preventive maintenance policy is developed by Khatab [15]. There are many important studies in the literature, Raza and Ulansky [28] work on inspections when performing maintenance activities. Naderkhani and Makis [24] formulate the maintenance problem with partially observable data about the actual state of the system, and the formula changes based on the maintenance limit exceeded. Several maintenance policies are considered in order to create one that meets, with the lowest costs, the reliability constraints associated with the achievement of the machine. Li et al. [19] gave an improved reliability modeling method based on meta-action since traditional reliability modeling methods do not fully consider the randomness of the CNC machine tool failure. Additionally, there are studies in the literature that examine reliability problems using fuzzy logic Khaniyev et al. [14].

Reliability is one of indicators to system performance and quality. Reliability is defined as a quantity that degrades over time in a system life cycle. To keep performance, it is necessary to plan maintenance actions during the life time and restore the system to an improved state. Gurler and Kaya [12] made maintenance and quality evaluations for the systems and equipment, taking into account the wear and tear process. Sobaszek et al. [29] created preventive maintenance model based on time-based machine failure prediction. Lin et al. [22], developed an optimal model, in which system reliability is estimated and used as the condition variable, the system availability and cost are the main object. Preventive maintenance of systems has taken into account with considerable attention by reliability specialists as it is aimed to keep the systems in an optimum working state and operational expenditures in minimum. Liao et al. [20] and Dong et al. [9] examined cost calculation methods of imperfect maintenance models and cost balancing techniques. Das et al. [8] created a preventive maintenance model with a reliability and cost-based approach in order to maximize the performance of cell manufacturing system. Malik [23] developed the improvement factor to model the impact of maintenance activities on the system.

This study investigates a data-driven predictive maintenance strategy that makes optimal maintenance decisions for complex repairable machines. Unlike the earlier maintenance plans, where maintenance actions can be initiated only at periodic inspection epochs, we assume that preventive maintenance actions are carried out just before sub-systems of the machine fail and stop production lines. Contrary to the previous works, in this study, an original model taking into account degradation assessment for the system and prediction of maintenance needs, discusses optimization processes considering a cost criterion, and a specified reliability level for each sub-system. An age-based preventive maintenance action is performed on the combined units if the reliability level of the units decreases under certain threshold. For maintenance decision-making, the perfect time for taking maintenance activities is determined by evaluating the system fully observable four-year failure data. Based on monitoring and current system check data, a new method enabling both early prediction of the subsystem's current status classification and its remaining availability is devised. For this given system, the study consists of determining the time intervals when the sub-components have to be preventively maintained. The choice of imperfect maintenance actions is decided, on the one hand, according to the available data about the current system state and, on the other hand, according to an objective of reliability for the next period. Additionally, in this study, it is aimed at increasing the availability of the system and making it better state than its current situation, reducing the number of unexpected failures, and providing product quality assurance. The imperfect maintenance application will improve the system status between as-bad-as-old and as-good-as-new when the system reliability drops to a certain level. And another focus point is to minimize the long-term average maintenance cost per time unit. When maintenance planning is carried out, the system state is determined according to the situation after the previous maintenance; therefore, it shows the Markov feature. The Markov model is best tool for Reliability Availability Maintainability (RAM) analysis of the systems and predicting the repairable system of the given working environment.Kumaresan et al. [18] worked through the use of the Markov Decision Model (MDM) approach to uncover optimal maintenance parameters for the best maintenance and service management systems. Velmurugan et al. [31] also used MDM to analyze the present variables of the production systems and forecast the optimal maintenance parameters, such as the failure rate and the repair rate, through the production system's availability analysis. Velmurugan et al. [33] study has additionally taken into consideration maintenance parameters among reliability, availability, maintainability, and dependability (RAMD) parameters to identify the critical subsystems and the effectiveness of the production system. Velmurugan et al. [34] explained in detail the availability analysis of the manufacturing systems by using the Markovian Birth-Death approach. And, Velmurugan et al. [35] used the Markov model as simplistic modeling approach for reliability measurement with respect to value of Reliability Availability Maintainability (RAM) of the system and proposed three functionfor the purpose of achieving high productivity and better customer satisfaction.

The period between maintenance cycles decreases depending on usage and age, and it is studied by Lie and Chun [21], Wang and Pham [36] in literature under an imperfect sequential maintenance policy. Jiang et al. [13] studied the semi-Markov decision process and investigated an optimal age-based replacement policy, where minimal repair, corrective replacement, and imperfect maintenance activities can be carried out upon a failure with different maintenance effects. Although many preventive maintenance studies have established models for multiple degradation processes, a significant number of them have overlooked the influence of system heterogeneity and complexity. After an unexpected misfunction, most studies assume that only one type of standard maintenance action is executed to restore the system to work, but modern systems are becoming more complex and need a plan tailored according to the situation. For instance, in the study of Noortwijk and Klatter [27], all parameters of the subsystems are considered equally. Additionally, in Nakagawa [26] and Block et al. [2] work, the system status was analyzed by adding an age reduction factor and a failure rate increase factor to the imperfect maintenance plan without accounting for dynamic reliability thresholds. Furthermore, the degradation process model is overly simplified, and a constant hard failure threshold is assumed, which undermines the credibility of the reliability function results. Previous authors have mainly studied non-perfect maintenance models of repairable systems, which generally apply to single failure modes for simple systems. Wang et al. [38] also studied the combination of maintenance policies for only a two-unit system. In this study, differently, Weibull parameters of the statistically independent failure modes of multi sub-parts were calculated, and the reliability value was obtained for each sub-system in the maintenance cycles by taking into account the system degradation rate coefficient and the decrease in the machine age after maintenance. During this study, instant system indicator outputs are included in the development of a hibrid model of complex multi component system. It enables the purposeful use of the maintenance potential in the utilization subsystems in order to maintain the facility's ability to continue operation. It is planned to implement maintenance activities by combining the parts with the lowest reliability (R) value that are the most likely to fail. Then, a reliability- based preventive model of the interaction between degradation processes that proves the existence of the stationary optimal policy is established in this paper. Proposed preventive maintenance approach is shared in Figure 1 below.



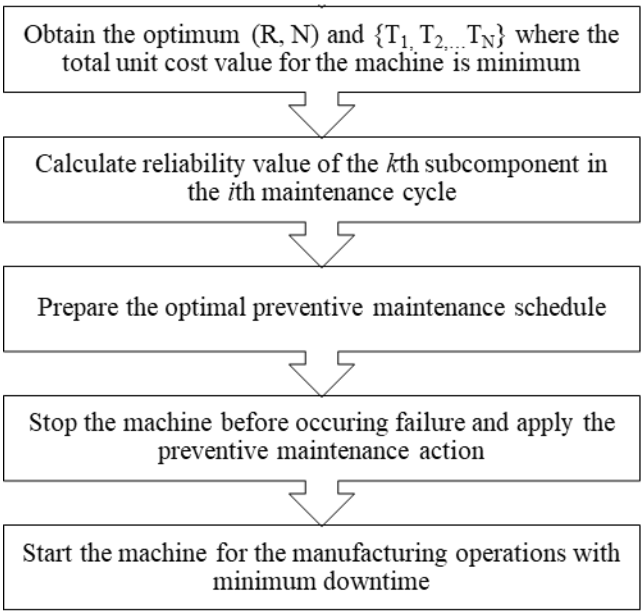



Fig. 1. Proposed preventive maintenance workflow.

Imperfect maintenance activities are planned to be implemented when the reliability level drops to the predeterminedthreshold R value. This workflow represents the cumulative risk of system failure in each maintenance cycle. The relationship between failure rate and reliability for systems that wear out over time is shared in Figure 3.The optimum time, number of maintenance cycles, and replacement are presented analytically and numerically. Failure data are analyzed to develop a new solution for real-time monitoring, and a case study for the CNC machine is conducted. Numerical cases and real cases are compared to illustrate the effectiveness of our proposed model, and the results indicate that the developed model aligns more closely with the actual operational state of the machine.

In the next section, preliminary information is provided for the reliability-centered optimum maintenance algorithm of the CNC machine with a high failure rate, which wears out randomly over time. Numerical studies are conducted in Section 3 to validate the performance of the proposed method. Finally, Section 4 draws the conclusions.

### 1.1. Notations and Symbols

$\alpha$ :Weibull scale parameters

$\beta$ :Weibull shape parameters

$a$ :Age reduction factor

$b$ :Failure rate increase factor

$C_{bd}$ :The cost of breakdown

$C_{mr}$ :Minimal repair cost

$C_o$ :Operational cost

$C_{oo}$ :The sum of fixed operating cost

$C_{vi}$ :Variable cost according to the number of maintenance cycles

$C_{vt}$ :Variable cost according to time

$C_{ir}$ :Imperfect maintenance cost

$C_r$ :Renewal and/or perfect maintenance cost

CNC: Computer numerical control

ETC :Expected Total Cost

RAM: Reliability Availability Maintainability

$i$ :Number of maintenance cycles

$N$ :The number of maintenance cycles

$T_i$ :Times between maintenance cycles

## 2. CNC Machine Maintenance Policy

### 2.1.Problem Description and Recommended Maintenance Algorithm

The Belotti MDL Series is an advanced high-speed 5-axis CNC machining center that ensures maximum productivity in milling models and prototypes and finishing large-scale molds and aluminum/composite parts for the aerospace industry. As traditional maintenance strategies are becoming less effective, the importance of considering the acquisition of information on future technology is highlighted by numerous studies when formulating a maintenance and replacement policy. The main expectations regarding CNC machine tools pertain to their higher availability in terms of the possibility of service and repair optimization and intelligent maintenance. CNC machine tools have various types of sensors, as well as measurement and diagnostic systems used in their design, which allow the control of many different parameters. In modern manufacturing systems, monitoring based on the use of appropriate sensors is key importance. Some common sensors include position, pressure, and flow sensors to provide feedback on the position of the machine's moving parts, temperature and humidity sensors to monitor the temperature of the machine and workpiece; and tool wear sensors to detect the condition of cutting tools. The use of intelligent sensors provides a reliable solution for system monitoring in real time.With considerable amounts of various types of data about the manufacturing process, the wear of the production line's key elements or the state of hardware resources are devised. An effective transition from raw industrial data to knowledge-based actions requires the development of new analytical tools. This serves as a basis for determining the optimal time of repair or part replacement to prevent catastrophic failure and improve the production flow in Kozłowski et al. [17] work.

In the current application, the traditional periodic maintenance policy is enforced. The system operates based on periodic maintenance activities, which are performed for eight hours every three months. The work order opens for unexpected failures that occur between periodic planned maintenance cycles. The machine does not operate during the administrative period required to identify the fault details, supply the necessary labor and equipment for fault repair, modification period and put it back into use. Due to the high failure rate, disruptions occur in the production plan and the quality of the product decreases. Considering the purchase and installation costs, it is not possible to replace the machine in a short time. With the current maintenance model, it would fail to complete the operation or increase the risk of system failure during the working duration. The major accidents, unimaginable economic losses, limited usability and finite durability and maintenance, material and energy might occur. Therefore, an appropriate preventive maintenance policy should be proposed to guarantee the required reliability and maintainability of the systems.

The proposed maintenance model in this study, a maintenance schedule for independent component subsystems was developed by creating a single and gated quantitative fault tree with Bayesian probability theory. Its failure and normal properties were distinguished as well. In this structure, which is referred to as the single and gate fault tree in the literature, the main event, including *n* system sub-components, can only occur if all of the basic events $E_1,E_2,\dots,E_n$occur simultaneously. If all the CNC machine subcomponents discussed in this study work, the system is operational. If any sub-parts fails, the system stops. Minimal maintenance is applied to make the system work if an unexpected failure with stopping effect occurs within the planned maintenance cycle. Minimal maintenance does not lead to an improvement in the system state, equipment is returned to the state it was in just before failure, and time spent on minimal repair is disregarded. While determining the optimum reliability level (R), the total number of maintenance (N), the time between maintenance cycles (T) and the parts to be repaired in the maintenance cycles, it is aimed to increase system availability, quality of product and minimize long-term costs. With the acceptable failure probability, this model would provide machine operators with an expert system that is easy to implement and use at the operational level, thus allowing them to perform technological processes.

It is assumed that a change that may occur in the operating conditions of the machine is detected immediately and the system status is determined immediately. Failure rate and operating cost were considered as variables depending on the maintenance process and system condition, and were used to decide the optimum reliability level and number of maintenance cycles (Boland [3]). When T is the continuous random variable showing the time for the unit to fail and h(t) is the probability density function, H(t) is the cumulative distribution function and the mean time to repair (MTTR) is calculated as:

$$MTTR = \int_0^\infty t h(t)dt = \int_0^\infty \big(1 - H(t)\big)dt \qquad (1)$$

Variance of the repair distribution:

$$\sigma^2 = \int_0^\infty (t - MTTR)^2 h(t)dt \qquad (2)$$

In this study, it is stated that preventive maintenance activities reduce the age of the system, that is, they rejuvenate the system by increasing its reliability to a certain level. The "recovery factor" concept developed by Malik [23] is used to measure the improvement in system age following maintenance activities. According to this concept, when the recovery factor is defined as k ($1 \le k \le \infty$), when maintenance is performed at the system age t, the equipment returns to *t/k* age. k=1 represents minimal maintenance between two cycles, and k=∞ indicates perfect maintenance. The age reduction factor ($a$, $0<a\le 1$) of the system is equal to $a=1/k$. While the time between two maintenance cycle is $T_i$, the pre-maintenance system failure rate function is $\lambda_i(t)$. After *i*th maintenance, the system failure rate is calculated below as $t\epsilon(0,T_i)$, $0<a\le 1$, $i=1,2\dots.N$:

$$\lambda_{i+1}(t) = \lambda_i\left(t + \frac{T_i}{k}\right) = \lambda_i(t + a_i T_i) \qquad (3)$$

The effect of age reduction factor is illustrated and shared as graphic in Figure 2.

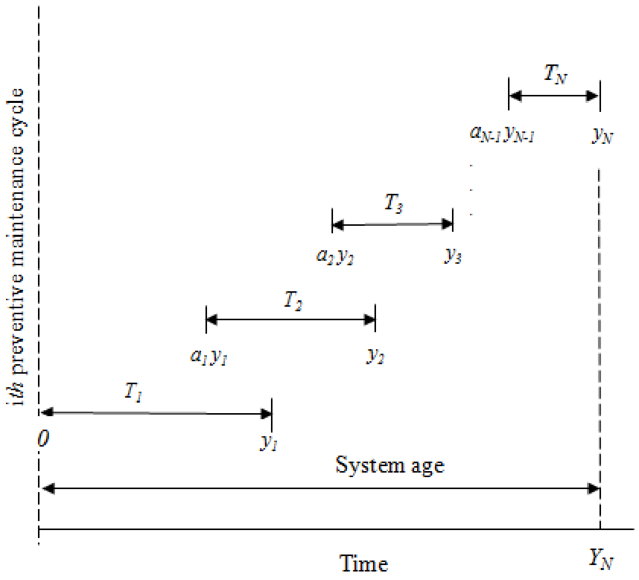



Fig. 2. The effect of age reduction factor.

Here $a_i$ indicates the age reduction factor in the $i$th cycle. If $a_i=0$, it refers to perfect maintenance, as $a_i=1$ means minimal maintenance. In order to measure the effect of the maintenance frequency, the failure rate increase factor ($b$, $b\geq 1$) which is another improvement factor developed in the study of Nakagawa [25], is used. Given this factor, the failure rate function is expressed as $b_i\lambda_i(t)$, $t\epsilon\ (0,T_{i+1})$ after $i$th maintenance cycle. Failure rate increase factor greater than 1 ($b_i>1$) indicates that the system's failure rate has increased due to wear down.

In this study, a hybrid model that takes into account wear and tear due to use and age was created with the combination of age reduction factor ($a$) and failure rate increase factor ($b$). In this case, the system failure rate function is expressed as follows (Liao et al.[20]):

$$\lambda_{i+1}(t) = b_i\lambda_i\left(t+\frac{T_i}{k}\right) = b_i\lambda_i(t+a_iT_i) \qquad (4)$$

$t\ \epsilon(0,T_{i+1})$ , $(b_i\geq 1)$ and $(0<a_i\leq 1)$; $i=1,2,..,N,$

System reliability increases after maintenance cycles. The age reduction ($a_i$) and failure rate increase factor ($b_i$) of randomly worn equipment over time during the system life cycle are different in each maintenance cycle. The ratio of $a_i$ and $b_i$ factors in maintenance applications was determined with the existing system failure data and expert evaluation. The representation of these factors, which vary according to the maintenance effectiveness and the number of cycles, is as follows when the maintenance cycle number is $i= 1,2\ldots N,$ (Khatab [15]):

$$a_i = \frac{i}{7i+1} \quad , \ A_i = \sum_{k=1}^{i-1} a_kT_k \quad (i\geq 2) \qquad (5)$$

$$b_i = \frac{12i+1}{11i+1} \ , \ B_i = \prod_{k=1}^{i-1} b_k \qquad (i\geq 2) \qquad (6)$$

In this case, $A_1=0$ and $B_1=1$ for the first maintenance cycle. Considering equations (5) and (6), the failure ratio function is expressed as $\lambda_i(t) = B_i\lambda_i(A_i+t)$. Each maintenance activity reduces the age of the system by $a_i$, and the age reduction factor should be taken into account to measure the system efficiency after maintenance. Otherwise, the wear and tear of the equipment due to environmental conditions, use and age will be ignored. In this case, the system age $y_i$ is obtained as follows when maintenance cycle is $i= 1,2\ldots N$, and the interval between two maintenance cycles is $Ti$, $0<a_1<a_2<\ldots<1$, $y_0 = 0$, $y_1 = T_1$ ,$y_i = T_i + a_{i-1}y_{i-1}$ (Lin et. al.[22]).

Theoretically, the relationship between the system failure rate function and system reliability is as follows:

$$\exp\left[-\int_0^{T_i} \lambda_i(t)dt\right] = R_i \qquad (7)$$

Imperfect maintenance activities are planned to be implemented when the reliability level drops to the predetermined $R_{th}$(*threshold R*) value. System reliability equality for each maintenance cycle under the $R_{th}$criteria is as follows, while the number of maintenance cycles is $i$, $i\epsilon\{1,2,\ldots,N\}$:

$$\exp\left[-\int_0^{T_1} \lambda_1(t)dt\right] = \cdots = \exp\left[-\int_0^{T_i} \lambda_i(t)dt\right] =$$

$$= \exp\left[-\int_0^{T_N} \lambda_N(t)dt\right] = R_{th} \qquad (8)$$

Equation (8) shows the relationship between the system failure rate function, system reliability and the time $T_i$ between maintenance cycles. Equation (8) can be written with another arrangement as follows (Liao et al. [20]):

$$\int_0^{T_1} \lambda_1(t)dt = \cdots = \int_0^{T_i} \lambda_i(t)dt = \cdots = \int_0^{T_N} \lambda_N(t)dt = -\ln R_{th} \qquad (9)$$

This formula represents the cumulative risk of system failure in each maintenance cycle. The relationship between failure rate and reliability for systems that wear out over time is shared in Figure 3 (Liao et.al.[20]).

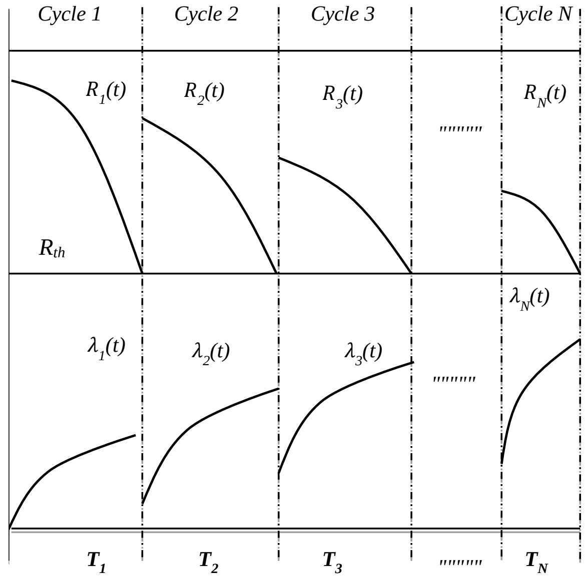



Fig. 3. Relationship between system failure rate and reliability for the systems.

When maintenance planning is made, the system state is determined according to the situation after the previous maintenance, thus it shows the Markov feature.This analysis technique is used for predicting the maximum RAM of the system and helps to rearrange the future preventive maintenance sequence with respect to value of the the repair and failure rate of the given subsystem. See also studies given in [31-35]. Considering the (8) and (9) equations, the relationship between the system reliability $R_i(t)$ and the failure rate function for the *i*. preventive maintenance cycle is shown as follows:

$$R_i(t) = \exp\left(-\int_0^t \lambda_i(x)dx\right) \qquad (10)$$

Considering equation (4),system reliability can be rewritten as:

$$R_i(t) = \exp\left(-B_i \int_{A_i}^{t+A_i} \lambda_1(x)dx\right) \qquad (11)$$

Planned preventive maintenance times $T_i$ occur when system reliability reaches predetermined $R_{th}$level. The time of the *i*th maintenance activity $T_i$ is achieved using the above equation (10) for each *i=1,2,…N,* (Khatab [15]):

$$T_i = \lambda_1^{-1}\left(\frac{\lambda_1(A_i)-\log(R_{th})}{B_i}\right) - A_i \qquad (12)$$

When the four-year failure data of the CNC machine considered in this study was examined, it was accepted that it is conformed to the Weibull distribution. The Weibull distribution, one of the most commonly used distributions in reliability analysis, is a flexible distribution that can model many different types of failure rate behavior with appropriate adjustment of parameters. In this analysis, the weibull distribution function is used to solving the mathematical problem. If the Weibull shape ($\beta$) and scale ($\alpha$) parameters are included when calculating the time $T_i$ between maintenance cycles and system reliability ($R_i(t)$), the following equations will be obtained for the proposed imperfect preventive maintenance policy (Khatab [15]).

$$T_i = \alpha\left(\left(\frac{A_i}{\alpha}\right)^\beta - \frac{\log(R_{th})}{B_i}\right)^{1/\beta} - A_i \;; \qquad (13)$$

$$R_i(t) = \exp\left[B_i\left(\left(\frac{A_i}{\alpha}\right)^\beta - \left(\frac{t+A_i}{\alpha}\right)^\beta\right)\right] \qquad (14)$$

Maintenance activities move the system to a better state, but do not return it to the initial $\lambda_1(0)$ state. If there is no upper limit to the number of maintenance cycles, *N* takes an infinite value with a reasonable repair cost. To avoid an infinite number of maintenance applications without replacement, the upper limit of the number of maintenance cycles is set in this maintenance model. (N-1) imperfect maintenance is planned for the considered CNC machine. In the *N*th cycle, perfect maintenance policies have been studied to restore the system to its initial good condition. The upper limit for the number of maintenance cycles was determined by examining the available data. This limit also implies a constraint on the cost purpose function. In the proposed maintenance model, operational cost is determined as a value that varies depending on maintenance activities and usage. Operational cost is the sum of fixed operating cost ($C_{oo}$), variable cost according to the number of maintenance cycles ($C_{vi}$), and variable cost according to time ($C_{vt}$), as follows:

$$C_o(i,t) = C_{oo} + C_{vi}i + C_{vt}t \qquad (15)$$

When complex systems fail, there is a loss of production and time because the equipment stops. This cost of loss is expressed as the cost of breakdown ($C_{bd}$). Other cost factors in this maintenance model include: minimal repair cost ($C_{mr}$), imperfect maintenance cost ($C_{ir}$), renewal and/or perfect maintenance cost ($C_r$). To minimize the total long-term cost of the system, the cost function must be fully defined. The potential cost of minimal repair for each cycle, the cost of repair, the reliability of the predetermined system and the likelihood of a failure within the $T_i$cycle time are placed in the formula as $C_{mr}\left(\int_0^{T_i}\lambda_i(t)dt\right)R_{th}$ . The planned imperfect maintenance cost $C_{ir}$ for each cycle is added to total cost function in proportion to the time, as one maintenance is planned for the $T_i$period. Since the operational cost function varies with time and cycle time $T_i$, it is expressed as $\int_0^{T_i} C_o(i,t)dt$. The probability of breakdown cost ($C_{bd}$) is assumed to be one as the system will stop during minimal repair or scheduled maintenance, and is added to the expected total cost proportionately to the time. In this case, the expected total cost (Expected Total Cost (ETC)) for 0<i<N:

$$ETC_i = \frac{1}{T_i} \text{ x } \begin{bmatrix} C_{mr}\left(\int_0^{T_i}\lambda_i(t)dt\right)\exp\left(-\int_0^{T_i}\lambda_i(t)dt\right) + \\ C_{ir} + \left(\int_0^{T_i} C_0(i,t)dt\right) + C_{bd} \end{bmatrix} \tag{16}$$

According to equation (8);

$$\exp\left(-\int_0^{T_i}\lambda_i(t)dt\right) = R_{th} \tag{17}$$

$$\int_0^{Ti} C_o(i,t)dt = \int_0^{T_i}(C_{oo} + C_{vi}i + C_{vt}t)dt$$

$$= (C_{oo} + C_{vi}i)T_i + C_{vt}\frac{T_i^2}{2} \tag{18}$$

In this maintenance model, perfect maintenance or equipment replacement is applied to the system when the system reliability reaches the $R_{th}$ value for the N*th* time, that is, when i=N and the system reaches its initial good state. Total expected cost for the N*th* maintenance cycle is calculated as follows i=N:

$$ETC_N = \frac{1}{T_N} \text{x} \begin{bmatrix} C_{mr}\left(\int_0^{T_N}\lambda_N(t)dt\right)\exp\left(-\int_0^{T_N}\lambda_N(t)dt\right) + \\ C_r + \left(\int_0^{T_N} C_0(N,t)dt\right) + C_{bd} \end{bmatrix} \tag{19}$$

From system installation through equipment renewal (0<i≤ N), the expected long-term total cost formula is (Liao et.al. [20]):

$$ETC = \frac{\sum_{i=1}^{N-1} T_i x ETC_i + T_N x ETC_N}{\sum_{i=1}^{N} T_i} \tag{20}$$

N is determined as the upper limit of the number of maintenance cycles. The local optimum long-term total cost comparatively determines the minimum cost value, optimal reliability, number of cycles and times for the different threshold Rin the acceptable reliability range for the system. The optimum ETC*, the relevant system reliability and the number of maintenance cycles ($R_{th}^*$,$N^*$)and the times between two cycles {$T_1$, $T_2$,...$T_N$} are obtained as the output of the developed algorithm.

### 2.2. Parameter Estimation of Weibull Distribution

The analytical method developed by Coria et al. [7] studies the closed form of the shape parameter obtained by Maximum Likelihood Estimation (MLE). Four years of failure data collected for the maintenance and repair of the CNC machine was analyzed by statistical methods and it was observed that the data fit the Weibull distribution. By using the hypothesis testing method and applying the chi-square test, it was tested on MATLAB that it fits Weibull distribution with p-value about %1. By analyzing the fault data of the considered system, its compliance with the Weibull distribution was accepted, and the shape parameter $\beta$ and scale parameter α were obtained. If the system fails at times $t_1 \le t_2 \le \ldots \le t_n$, the probability density function of n failures is calculated with following equations:

$$\tau \equiv [\prod_{k=1}^{n} t_k]^{(1/n)}\,, \tag{21}$$

$$\theta(\beta) \equiv \left[\frac{1}{n}\sum_{k=1}^{n} t_k^{\beta}\right]^{\left(\frac{1}{\beta}\right)}, \tag{22}$$

$$\frac{1}{\beta_1} \equiv \frac{1}{n}\sum_{k=1}^{n}\frac{\ln t_n}{t_k} = \frac{\ln t_n}{\tau} \tag{23}$$

$$Z(\beta) \equiv \frac{1}{\beta_1} - \frac{1}{\beta} - r(\beta) = 0 \tag{24}$$

$$r(\beta) = \frac{\sum_{k=1}^{n}\left(\frac{\ln t_n}{t_k}\right)t_k^{\beta}}{\sum_{k=1}^{n} t_k^{\beta}} = \ln(t_n) - \frac{\sum_{k=1}^{n}\left(\ln(t_k)\, t_k^{\beta}\right)}{\sum_{k=1}^{n} t_k^{\beta}}$$

According to the method developed Coria et. Al. [8], system failure time is $t_1 \le t_2 \le \ldots \le t_n$ and there is $m$ in the set$\{1,2,\ldots,n-1\}$: $t_m < t_n$ and $t_{m+1} = t_n$. In this case, there is a number $\beta^* > 0$ such that let $z(\beta^*) = 0$. In addition, $\beta^*$ is achieved by:

$$\frac{1}{\beta_1} - r(\beta_1) < \frac{1}{\beta^*} < \frac{1}{\beta_1} \tag{25}$$

$r(\beta)$ is a continuously and monotonically decreasing function. When $\beta$>0, there is $\beta^*$ for $\lim_{\beta\to 0} r(\beta) = \frac{1}{\beta_1}$ and $\lim_{\beta\to+\infty} r(\beta) = 0$, $z(\beta^*) = 0$ and the interval inequality is stated as follows:

$$\beta_1 < \beta^* < \frac{\beta_1}{1-\beta_1 r(\beta_1)} \tag{26}$$

Using this inequality, the closed form of $\beta^*$ and $\alpha^*$parameters is obtained.

$$\beta^* = \beta_1 \frac{1-\frac{1}{2}\beta_1 r(\beta_1)}{1-\beta_1 r(\beta_1)} \;; \tag{27}$$

$$\alpha^* = \theta(\beta^*) \tag{28}$$

The Weibull probability density function is:

$$f(t) = \frac{\beta}{\alpha}\left(\left(\frac{t}{\alpha}\right)^{\beta-1}\right) exp\left(-\left(\frac{t}{a}\right)^{\beta}\right) \tag{29}$$

A numerical study is provided to illustrate the effectiveness of our proposed model. The analysis of the data of the system considered is carried out in the next section.

## 3. Numerical Analysis of Data and Results

The details of the failure data kept for the CNC machine are as follows: failure occurrence time, fault category and sub-detailed fault information, time spent by the system in malfunction, information on whether the malfunction has a stopping effect on the system, opinion of the personnel using the equipment and maintenance specialist, malfunction repair time, and prioritization information to determine the repair order of simultaneous faults. With the help of the malfunction data of the machine, information on the time between two failures, the time until the failure is repaired, that is, the time the system remains idle, can be obtained. The average interval between failures is listed from the system installation date and the number of times ($t_i$) that the failures occur is accumulated. Parameter estimation was made using analytical method over these periods. Weibull distribution parameter values for system data are shape parameter $\beta^*$=1,3545 and scale parameter $\alpha^*$=60,387 days.

The operational cost values of the machine are calculated according to the cost formula $C_o = C_{oo} + C_{vi}\, i + C_{vt}\, t$ , depending on time and maintenance cycle. In order to avoid an infinite number of maintenance applications without replacement of parts, the upper limit of the number of maintenance cycles ($N_{up}$) in this maintenance model has been determined as $N_{up} \leq 15$ by examining the data we have. In order to calculate the failure rate function λ(t) value throughout the system life cycle with parameters suitable for real production systems, the past maintenance activities of the CNC machine are examined and the values $a_i = i/(7i+1)$ and $b_i=(12i+1)/(11i+1)$ are determined. Maintenance cycle numbers are $i=1,\ldots N$, $0<a_i\leq 1$ and $b_i \geq 1$. The reason why the age reduction and failure rate increase factor depends on the number of maintenance cycles is that equipment failure increases due to wear and tear of the system over time. As the system ages, more frequent maintenance will be required and the time between maintenance cycles will decrease. The duration between the maintenance cycles is expressed as $T_i$ (i=1,…N), whereas $T_1>T_2>\ldots>T_N$ .

Failure data and operation, fault, repair and maintenance costs of the machine are inputs; optimum reliability level (R), number of maintenance (N), times between maintenance cycles ($T_i$) and information about the system subcomponent to be maintained were evaluated as outputs. The aim is to create a maintenance policy that minimizes the cost of maintaining, repairing, and operating systems. The assumptions accepted in the proposed reliability-based imperfect maintenance plan are as follows:

- A new system was initially established,
- System faliures are stochastic and can only be described by the system error rate function.
- System status can be observed continuously and accurately,
- Unexpected failures are noticed as soon as they occur and the failures are independent,
- The relationship between costs is $C_{ir}<C_r$ and $C_{mr}<C_r$,
- Part renewal brings the system to its initial good state,
- The system wear process is modeled with a non-homogeneous Poisson process and a stationary Weibull distribution suitable for this process is used,
- It is assumed that the system wears out with the same Weibull parameter values during the periods between maintenance cycles,
- The times spent for failure repair and planned maintenance activities are assumed to be equal for each maintenance cycle and are ignored,
- $N_i$-1 imperfect maintenance will be applied to the system and perfect maintenance or part replacement activities will be planned in the $N_i$th step.

The algorithm created for the recommended maintenance plan, accepting the assumptions, is given in Figure 4.

**Initiate**

**1-** Set upper limit on number of maintenance cycles to $N_{up}$ =15 ; $i=1,\ldots,N_{up}$,

**2-** Assign maintenance costs large enough to be beyond $N_{up}$ cycle costs, initial value $C_{ir} = 10^7$

**3-** Assign initial ETC* = $10^7$ as the smallest of expected total costs,

**4-** [$R_{1=}$ 0,50; $R_{2=}$ 0,99] , Assume $R_1$<$R_2$ as $R_{th}$=$R_1$,

**5-** For each $i \in \{1, \dots N_{up}\}$, calculate

$ETC = \frac{\sum_{i=1}^{N-1} ETC_i . T_i + ETC_N . T_N}{\sum_{i=1}^{N} T_i}$ and determine the value of $N$ at which the expected total cost is smallest (Liao et.Al.[20]),

**6-** For each $\forall\, i \in \{1, \dots N\}$, calculate

$$T_i = \alpha \left( \left( \frac{A_i}{\alpha} \right)^{\beta} - \frac{\log(R_{th})}{B_i} \right)^{\frac{1}{\beta}} - A_i$$

**7-** If *ETC<ETC**, consider the smaller value as valid and update it as *ETC*=ETC*, update the relevant *N*=N* maintenance cycle number as the local optimum value,

**Purpose** Minimum cost

**8-** Update as $R_{th}$ = $R_{th}$ + Δr (Δr = 0.05 or 0.01 to find local best value). If $R_{th} \le R_2$, return Step-5, otherwise terminate the algorithm.

**Output** *ETC*, (R*, N*)* and *$\{T_1, T_{2,\dots} T_N\}$*

**End**

Fig. 4. Proposed maintenance algorithm.

The algorithm outputs are the optimum *ETC**, the relevant system reliability and the number of maintenance cycles (Rth*, N*) and the number of days between two cycles $\{T_1, T_{2,\dots} T_N\}$. T values, which are the output of the algorithm run with real data, are given in the Figure 5.

| | | | | | |
|---|---|---|---|---|---|
| $T_1$=94,7 | $T_2$=81,6 | $T_3$=71,5 | $T_4$=63,2 | $T_5$=56,3 | $T_6$=50,3 |
| $T_7$=45,2 | $T_8$=40,7 | $T_9$=36,7 | $T_{10}$=33,2 | $T_{11}$=30,1 | $T_{12}$=27,3 |

Fig. 5. Days between maintenance cycles.

In order to follow the system movement at different reliability values, the algorithm was studied by accepting the reliability initial value as $R_1$=0,50. The upper reliability value $R_2$=0,99 was used to terminate the algorithm. The system has advanced Δr steps in the range [0,50; 0,99]. Although the upper limit for the number of maintenance cycles is $N_{up}$≤15, the minimum expected cost for R=0.95 and R=0.99 values was obtained in N=17 and N=20 cycles, respectively. Since it is more costly than other reliability values and the number of circuits exceeds the $N_{up}$ value determined as the upper limit, it is not considered within the maintenance policy of the CNC machine.

The total unit cost value for the CNC cutting machine is minimum at (R*=0.66; N*=12). If a lower value is set for system reliability, the equipment will take longer to wear out to that level and the time between maintenance cycles will increase. If equipment reliability is fixed at a higher level, the time between maintenance cycles will decrease, and maintenance will be required in a shorter time, as the system will reach the specified high reliability level in a shorter time. With the help of Weibull shape and scale parameters, the reliability value of the sub-components at each maintenance cycle time was calculated, and using Bayes' theorem, maintenance was planned for the sub-components with the lowest reliability value, that is, those with the highest probability of failure.

The maintenance cycle times at the point where the total unit cost of the system is smallest (R*=0.66; N*=12), the failure rate increase factor and age reduction factor for each cycle are examined below.

The age reduction factor ($a_i$), the failure rate increase factor ($b_i$), and its value at each cycle time ($A_i$,$B_i$) are obtained by equations (5) and (6). The subcomponents are independent of each other, and if any of them fail, the system stops. In order to classify CNC machine failure types and to distinguish relatively more important failure types from others, pareto analysis was carried out according to the failure rates of the subparts forming the system. Pareto analysis was made by calculating the duration of malfunctions and the number of malfunctions for all subparts of the machine. As a result of the analysis, it was calculated that if the maintenance planning of 10 of the 23 sub-parts was improved, an 87% saving would be achieved in system malfunctions. Therefore, the proposed preventive maintenance algorithm was run for 10 sub-parts and those subcomponents listed in Figure 6.

| ***k: system subcomponent*** | | ***Number of failures*** |
|---|---|---|
| 1 | Pogo system | 85 |
| 2 | Control system | 72 |
| 3 | Water supply system | 50 |
| 4 | Manual fault status | 27 |
| 5 | Nozzle and nozzle pistons | 22 |
| 6 | Vacuum system | 21 |
| 7 | Abrasive grit system | 14 |
| 8 | Pneumatic system | 14 |
| 9 | Cutting fluid system | 14 |
| 10 | Axes | 12 |

Fig. 6. List of 10 sub-systems as the result of Pareto analysis.

*k* : system subcomponent *(k=1,2,…10)*

$i$: number of maintenance cycles *(i=1,….,12)*

$R_{ki}$ : Reliability value of the $k$th subcomponent in the $i$th maintenance cycle

$$R_{ki}(t) = \exp\left[B_i\left(\left(\frac{A_i}{\alpha_k}\right)^{\beta_k} - \left(\frac{t+A_i}{\alpha_k}\right)^{\beta_k}\right)\right] \tag{30}$$

Subcomponents with a high probability of failure have been identified to be maintained during cycle times ($T_i$). Systems with the lowest reliability values during the maintenance cycle period pose risks and are interpreted as subcomponents with a high probability of failure.

***1th Cycle***

After the system is used at t = 0, the reliability value reaches the lowest level of R = 0.66 and the need for maintenance increases. $R_{k1}(T_1)$is calculated for each subsystem.

$$R_{k1}(T_1) = 1 - F_k(T_1) = \exp\left[-\left(\frac{T_1}{\alpha_k}\right)^{\beta_k}\right] \tag{31}$$

***2nd Cycle***

Since the system reaches the reliability value R=0.66 after the first maintenance cycle, the next maintenance application is planned. While the reliability value of the sub-parts that are maintained in the first cycle is calculated for the $T_2$ time interval, the components that are not maintained from the system start-up are examined for the $T_1$+$T_2$ time interval.

$$R_{k2} = \begin{cases} R_k(T_1+T_2), If no maintenance planned in the\ 1st cycle \\ R_k(T_2), If maintenance was done in the\ 1st cycle \end{cases}$$

$$R_k(T_2) = \exp\left[B_2\left(\left(\frac{A_2}{\alpha_k}\right)^{\beta_k} - \left(\frac{T_2+A_2}{\alpha_k}\right)^{\beta_k}\right)\right]; \tag{32}$$

$$R_k(T_1+T_2) = \exp\left[B_2\left(\left(\frac{A_2}{\alpha_k}\right)^{\beta_k} - \left(\frac{T_1+T_2+A_2}{\alpha_k}\right)^{\beta_k}\right)\right] \tag{33}$$

The reliability value of subsystems that are not maintained decreases until the next cycle.

***3rd Cycle***

In order to determine the sub-parts to be repaired in the third maintenance cycle of the system, $R_{k3}$ iscalculated as follows: $R_{k3} =$

$$\begin{cases} R_k(T_1+T_2+T_3),\ If\ no\ maintenance\ has\ been\ done, \\ R_k(T_2+T_3),\ If\ maintenance\ was\ carried\ out\ in\ 1st\ cycle, \\ R_k(T_3),\ If\ maintenance\ has\ been\ done\ on\ 2nd\ cycle \end{cases}$$

The reliability value of the system sub-parts that were maintained in the first and second maintenance cycle is related to the time interval after the maintenance is applied. A subsystem may have been maintained both in the first cycle and in the second cycle, but when calculating the reliability value, the time of the last cycle in which maintenance was performed is always taken into account.

***11st Cycle***

Taking into account the maintenance practices of the system subcomponents in previous cycles, $R_{k11}(t)$ values of the parts after $T_{11}$ days are listed as follows:

$$R_{k11} = \begin{cases} R_k(T_7+T_8+T_9+T_{10}+T_{11}), If\ maintenanced\ on\ 6th\ cycle, \\ R_k(T_8+T_9+T_{10}+T_{11}), If\ maintenanced\ on\ 7th\ cycle, \\ R_k(T_9+T_{10}+T_{11}), If\ maintenanced\ on\ 8th\ cycle, \\ R_k(T_{10}+T_{11}), If\ maintenanced\ on\ 9th\ cycle, \\ R_k(T_{11}), If\ maintenanced\ on\ 10th\ cycle \end{cases}$$

In the proposed maintenance plan, reliability values of the system subsystems calculated according to equation (30) are given in the Figure 7.

| *k* | *1* | *2* | *3* | *4* | *5* | *6* | *7* | *8* | *9* | *10* |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_{k1}$ | **0,61** | **0,66** | **0,62** | 0,73 | 0,81 | 0,85 | 0,87 | 0,82 | 0,85 | 0,84 |
| $R_{k2}$ | **0,60** | 0,65 | 0,63 | **0,55** | **0,59** | 0,69 | 0,72 | 0,70 | 0,73 | 0,74 |
| $R_{k3}$ | 0,65 | **0,28** | **0,36** | 0,78 | 0,83 | **0,55** | 0,60 | 0,61 | 0,63 | 0,66 |
| $R_{k4}$ | **0,23** | 0,68 | 0,67 | 0,61 | 0,65 | 0,89 | **0,50** | **0,54** | 0,56 | 0,60 |
| $R_{k5}$ | 0,61 | **0,36** | **0,42** | **0,49** | 0,50 | 0,77 | 0,92 | 0,88 | 0,50 | 0,55 |
| $R_{k6}$ | **0,25** | 0,71 | 0,70 | 0,82 | **0,39** | 0,66 | 0,82 | 0,79 | **0,45** | **0,50** |
| $R_{k7}$ | 0,64 | **0,42** | **0,47** | 0,69 | 0,87 | 0,57 | 0,74 | 0,72 | 0,92 | **0,47** |
| $R_{k8}$ | **0,31** | 0,72 | 0,72 | **0,58** | 0,76 | **0,49** | 0,66 | 0,66 | 0,85 | 0,92 |
| $R_{k9}$ | 0,65 | **0,46** | **0,52** | 0,86 | 0,65 | 0,93 | **0,59** | 0,61 | 0,79 | 0,86 |
| $R_{k10}$ | **0,35** | 0,74 | 0,75 | 0,74 | **0,55** | 0,85 | 0,95 | **0,57** | 0,73 | 0,81 |
| $R_{k11}$ | 0,67 | **0,50** | **0,55** | **0,65** | 0,91 | 0,78 | 0,89 | 0,93 | 0,69 | 0,77 |

Fig. 7. Reliability values of subsystems in maintenance cycles.

In the first maintenance cycle, maintenance is applied to the k = 1,2 and 3 subcomponents with the smallest reliability value. These sub-parts are likely to fail $T_1$=94,7days after the machine starts working. If maintenance is not applied to k=1,2 and 3 subcomponents at the specified time, stoper failures ocur due to these parts. These faults cause the cost of breakdown ($C_{bd}$) and the cost of repairing the fault ($C_{mr}$). The reliability of the subsystems that are not maintained decreases until the next cycle. The reliability value of sub-parts that are maintained in the first cycle is related to the time until the next maintenance. In the second maintenance cycle, the reliability values of the sub-parts are calculated according to the (32) and (33) formulas. After the reliability calculation, maintenance planning is made for the k=1,4 and 5 sub-parts with the lowest value. In each maintenance cycle, the part to be maintained is decided by calculating the reliability value of the sub-parts.

***12th Cycle***

In the recommended maintenance plan, after N-1 imperfect maintenance application, when the system reliability drops to R=0.66 in the *N*th circuit, perfect maintenance and parts replacement are performed to bring the system status to the initial level. At an acceptable value of R=0.66 for the system, the ideal number of maintenance cycle is calculated as N=12, where the objective function takes the best value. The total unit cost value of the system will increase, if more than twelve maintenance circuit is performed.

In the proposed model, the average number of failures decreased with planned preventive maintenance activities. While it is known that the distribution of system failures occurs with the parameters W($\beta$=1.3545; $\alpha$=60.3877), the average time between failures and the number of failures expected in each cycle interval in the proposed maintenance plan were calculated with the help of the incomplete gamma function. In order to evaluate the effectiveness of the developed maintenance schedule, the total duration of maintenance cycles, frequency of occurrence of faults and total failure, and maintenance cost values of the current maintenance plan for the working year are compared in Figure 8.

| Values | Proposed plan | Current plan |
|---|---|---|
| Total duration of maintenance cycles | 631,53 day | 365 day |
| Frequency of occurrence of fault | 25,95 day/failure | 8,7 day/failure |
| Total failure cost | 9662,57 $ | 16663,50 $ |
| Total maintenance cost | 3336,60 $ | 4245,70 $ |

Fig. 8. Comparison of proposed maintenance plan and current plan.

The number of system failures has been reduced with preventive actions implemented during maintenance cycles. It is anticipated that there will be a 56% reduction in the total malfunction and maintenance costs of the system. With an imperfect preventive maintenance policy, the number of unexpected failures will decrease and accordingly, the failure repair costs and the cost incurred by the system when it is out of use will decrease proportionally. In addition, the duration of the system's availability will increase.

## 4. Conclusion

In this study, an imperfect sequential preventive maintenance policy that minimizes long-term operating and maintenance costs was created for the machines used in the defense and aerospace industries as a result of increasing requirements for reliability, availability, maintainability and safety of systems. A maintenance algorithm with Markov properties was developed by analyzing past failure data of the machine, which wears out randomly over time. Completely different from the current maintenance plan, the wear and tear process due to use and age has been studied in the system maintenance algorithm as a failure rate increase factor and age reduction factor after each maintenance cycle. The effect of imperfect maintenance is interpreted in terms of how the hazard rate function and the effective age are changed by maintenance actions. The decision variables in the maintenance policy, such as the number of maintenance actions to be performed, the interval between successive maintenance actions, and the reliability level of the multi component machine, can be recursively updated when new monitored information arrives. According to the recommended reliability-centered preventive maintenance policy for the machine, N-1 imperfect maintenance application has improved the system from its current state, increasing the usable time and reducing the number of unexpected failures. In the developed maintenance policy, not only maintenance costs but also long-term operational costs of the system are minimized. The planned maintenance schedule depends on both the predicted number of future failures and the minimization of the expected maintenance cost rate defined in the long term. In addition to Figure 7, which compares the effectiveness of the proposed plan, the number of expected and actual failures can also be considered for evaluation. In the current situation the number of failures is 42, while in the proposed planaverage number of failures expected is calculated as 24,34. Hence, there will be 42% decrease in total number of failures. This reduction in the number of failures will also reduce the labor force allocated for the repair of failures in direct proportion. Additionally, by performing maintenance on equipment with a high probability of failure, unnecessary maintenance on all system subcomponents is prevented. Thus, unnecessary costs and time expenses for maintenance activities are prevented. More efficient use of the engineering system capacity, manufacturing continuity and obtaining products at a certain reliability and quality level are ensured.

The proposed sequential preventive maintenance policy is also applicable to other production machines with a critical failure rate.

This study is open to improvement on some issues. The parameters, defined as failure rate increase factor and age reduction factor in the literature, were obtained by making inferences from expert opinion and past situations, since the environment in which the system is located is very dynamic. Additionally, different methods can be developed to determine system subcomponents for imperfect maintenance activities.

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