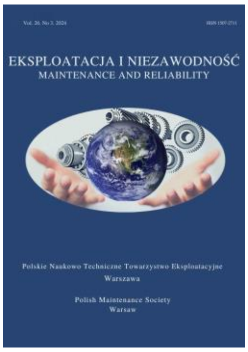

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Robust Design Based on Cost-Quality Model in Micro-Manufacturing

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Highlights

- We propose a novel economically quality design model under model parameter uncertainty.
- The model covers both pre-sale manufacturing and post-sale warranty costs.
- Warranty cost models considering model parameters uncertainty have been constructed.
- In modeling process trade-offs between cost and quality are considered.
- A micro-drilling manufacturing process validates the effectiveness of the method.

Abstract

This paper proposes a novel total cost model for the micro-products' entire life cycle that takes into account the uncertainty of the model parameters. The total cost includes pre-sale manufacturing and post-sale warranty costs. Additionally, different marketing strategies are also given based on the weight of internal and external costs. Furthermore, limited data and unknown effects in experiments may cause large errors in parameter estimates. This could prevent the achievement of reliable designs. To address this, robust optimization and interval estimation are used. This approach reduces the impact of uncertainty on parameter estimates. It ensures optimality and robustness in micro-manufacturing parameters. Example analysis and numerical simulation results show that the proposed method assists companies in selecting the optimal manufacturing parameter level that aligns with their marketing strategies. Besides, considering uncertainty factors can ensure that the optimization results remain guaranteed, even under the worst-case scenarios.

Keywords
model parameter uncertainty, interval estimation, warranty cost, economic quality design, manufacturing cost.

1. Introduction

Laser beam micro-machining is a precision machining technology that allows efficient processing and fabrication at the micron and nanometer scales by controlling the focusing position of the laser beam[1,2]. In recent years, with the rapid development of micro-hole machining technology, micro-precision products generated by micro-hole arrays have been widely applied. Quality improvement in micro-manufacturing process is one of the most challenging issues and a hot topic of research in the field of industrial engineering. Unlike traditional manufacturing processes, the micro-manufacturing process may exhibit many lower or even zero output levels in the experimental space[3]. The first issue to consider in the laser micro-machining process is the instability of laser output or the subsequent beam delivery system. That is, laser parameters may

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change with temperature, humidity, and material modification variations, thereby affecting the quality of machining. Minor manufacturing changes (such as variation in laser power or beam shape distortion) may also result in the final product quality falling below standards, thereby incurring additional time and cost. For instance, micro-holes produced during manufacturing that fall below or exceed specified limits can lead to rework or scrap costs, respectively. In addition, there are warranty costs associated with products sold to customers.

Robust parameter design, crucial for continuous quality improvement, is extensively used in product/process quality design, significantly improving product quality and reliability[4,5]. Most studies use response surface and optimization models for robust parameter design, focusing on Taguchi's quadratic loss function, rework, and scrap costs. For example, Ouyang et al.[6] constructed a robust design model for micro-manufacturing that includes quality loss, rework cost, and scrap cost from the perspective of the whole product life cycle. The quadratic loss function has a limitation in that it attempts to lump all the costs associated with a product's deviation from its ideal value into one vaguely defined parameter[7]. There is difficulty in accurately estimating the cost of external failures. Moreover, these losses are often related to many factors that may be interrelated. Therefore, consolidating all these losses into a single cost coefficient k is overly simplistic and restrictive[8].

Product quality design depends not only on the collected market information and customer preferences but also significantly influences subsequent process design, production, sales, and after-sales services. Neglecting the choice of production processes and the related costs of sales and after-sales services can lead to unnecessary costs and resource wastage[9]. Product warranty policy is an important means of connecting businesses with customers. On one hand, businesses can convey information about the product's quality and reliability through the warranty policy; on the other hand, quality issues experienced by customers during the product's use are reflected back to the business in the form of warranty claims data. By analyzing warranty claims data, businesses can identify defects in product design, manufacturing, and sales processes, and take measures to further improve product quality and reliability. To replace the vague concept of societal loss in the quadratic loss function, this paper uses warranty costs to represent the loss costs incurred after products are sold to customers. Therefore, for economic reasons, it is necessary to consider manufacturing costs, warranty costs, and marketing decisions in micro-product quality design to achieve maximum efficiency and profitability.

Some studies suggest that building a comprehensive model incorporating manufacturing and marketing information during the product development can help companies achieve higher profits (e.g., Karmarkar et al.[10], Kirkizoğlu and Karaer[11], Park et al.[12]). Typically, manufacturers produce products and then ask the marketing team to sell them. In another scenario, products designed to meet market demands and customer needs, including functional requirements, parameters, and tolerances, often overlook manufacturing costs. These practices can result in unnecessary costs, resource waste, lower investment returns, missed opportunities, and failure to meet customer expectations. Therefore, a more rational approach involves integrating marketing decisions and customer expectations with manufacturing capabilities and costs during product development to maximize efficiency and profitability. Karmarkar et al.[10] noted that although quality in manufacturing is often equated with specification conformance, in marketing, it is defined by the product's performance or grade. Acknowledging the different conceptions of quality in manufacturing and marketing, this research bridges the gap by developing a customer utility function and a manufacturing cost optimization objective. Although manufacturing costs are related to the mean and variance of product characteristics, the form of the manufacturing cost function in the model is overly simplistic. Some researchers, such as Zhang et al.[13], Wang[14], and Qiao et al.[15], have indicated that product warranty can serve as a major influencing factor of product quality. That is, longer warranty periods and better warranty policies are seen by customers as signs of higher quality and more reliable products. The selling price is tied to reliability and warranty; as reliability increases, so does the selling price, potentially improving warranty terms for the customer. However, higher reliability means higher manufacturing costs, leading to lower warranty-related costs. Higher manufacturing costs might result in higher selling prices, which in turn could negatively impact sales volumes. Zhang et al.[13] proposed a total cost optimization model that addresses manufacturing issues affecting product reliability as well as marketing issues related to price and warranty to maximize profits. To enhance the reliability and robustness of products, Wang et al.[16] developed a new expected loss function from the perspectives of manufacturers and customers, considering product warranty period, design life, and customer satisfaction. Cheng et al.[17] constructed a comprehensive life cycle cost model, incorporating capital, annual operating, and risk costs, to assess product quality's robustness and reliability throughout its life cycle. However, their research did not consider the impact of product warranty strategies on aspects such as product quality level and qualified product rate. Despite Hassain[18] considering tolerance, rework, scrap, and warranty costs in constructing a total cost model for products, the approach had limitations. The models for mean and variance relied on the simplest linear and nonlinear forms with constant parameters, overlooking the uncertainty of these model parameters. And no marketing strategies were developed based on internal and external costs.

Due to uncertainty factors like environmental instability (temperature, humidity, etc.), variability in material batches, and inaccuracy of measuring instruments, the accuracy of the experimental data required for modeling is impacted. Peterson[19] once pointed out that neglecting the uncertainty factors in modeling could lead to an overestimation of the reliability of the optimal input levels. Ng[20] also believes that overlooking the uncertain disturbances in modeling could lead to incorrect parameter estimation, resulting in irrational process design and a decrease in product pass rates. In recent years, many scholars have been focusing on the issue of uncertainty factors interference and have also achieved some very meaningful research results. Tan and Wu[21] adopted an improved quadratic loss in optimization design, enhancing the credibility of confidence intervals for controllable factors under parameter uncertainty compared to traditional methods. Feng et al.[22] employed Markov Chain Monte Carlo sampling to establish simulation models and quantify uncertainty. They introduced interval analysis to traditional quality loss functions, devising an optimization strategy that reduces the effect of model uncertainty on solutions. To reduce the fluctuation problems in micro-manufacturing processes, Han et al.[23] established new quality loss functions, rework costs, and scrap costs through Monte Carlo simulation and quantified the model's uncertainty using interval analysis theory. Ouyang, Dey, and Park[24] noted that sampling variations often lead to inaccuracies in assessing Prediction Confidence Intervals (PCIs). Given the widespread use but notable time consumption of the bootstrap method, the delta method has been employed as an alternative to construct robust confidence intervals for PCIs. Nevertheless, applying traditional normal theory methods to skewed data and overlooking uncertainties in parameters can lead to a progressive decline in optimization quality. Zeybek[25]introduced an innovative method that utilizes confidence interval (CI) response modeling for the process mean. This new interval robust design approach effectively addresses the challenges posed by both the skewed nature of the data and data contamination. Li, He, and Zhang[26] addressed prediction variability by integrating regression model confidence intervals into a robust desirability function. This approach maintains solution viability amidst parameter uncertainty. Wilcox[27] proposed a new method for calculating confidence intervals for the population mean in situations involving small sample sizes. Shah and Abdeljawad[28], Khan et al.[29], Sher et al.[30], and Ahmed et al.[31] employed numerical simulation methods to analyze the robustness and effectiveness of the proposed model. However, this method did not consider information related to marketing.

While the literature mentioned above have discussed uncertainty issues and yielded significant outcomes, research on robust design that accounts for uncertainty factors has not been optimized from an economic perspective. There is scant attention to the issue of model parameter uncertainty in integrated models considering information related to manufacturing and marketing. Taking into account the economy and robustness of the laser micro-manufacturing process, the quality evaluation of micro-holes is closely related to manufacturing costs, rework costs, scrap costs, and warranty costs. Meanwhile, model parameters are treated as interval numbers during modeling to increase the reliability of parameter estimation. To address the aforementioned issues, this paper introduces a robust optimization design method for a cost-quality model that considers uncertainty in model parameters. The goal is to find optimal micro-manufacturing parameters, ensuring high-quality, low-cost products in laser processing. First, the relationship between quality characteristics (such as radius) and input parameters (such as average power, switching frequency, and cutting) in the micro-manufacturing process is studied using the response surface model. Next, interval estimation is performed on the model parameters. Then, utilizing the method of minimizing the worst-case scenario, a two-layer nested optimization model is constructed. Finally, intelligent algorithms are employed to seek the optimal economic parameter settings for the micro-manufacturing process.

The structure of this paper is organized as follows. we introduce the assumptions required for the model parameters. Section 3 establishes a total cost model considering the uncertainty of model parameters from a whole lifecycle perspective. In Section 4, we employ micro-manufacturing examples and simulation analysis to validate the effectiveness of the proposed model. Section 5 provides some concluding remarks. The proposed method has three possible significances:

(1) Manufacturers can vary weights for expected loss costs based on marketing strategy type, choosing optimal input levels for more flexible outcomes.

(2) Incorporating interval estimation theory into robust design reduces uncertainty impacts on micro-product performance, enhancing process robustness and reliability.

(3) Creating a cost model from a lifecycle perspective improves quality cost-effectively in complex manufacturing, providing a reliable strategy for optimizing parameters.

2. Interval estimation of model parameters

Assume that in the micro-manufacturing process, the output quality characteristics follow a normal distribution. The set of controllable processing parameters that affect the quality characteristic Y is $(x_1, x_2, \cdots, x_v)$. Utilizing the design of experiments based on response surface methodology yields the corresponding experimental data, as shown in Table 1. The controllable parameters are subjected to n treatments, with each treatment being replicated m times for experimentation. y_{pq} is the observed value $(p = 1,2,\cdots,n; q = 1,2,\cdots,m)$ for the qth replicate trial of the pth treatment. The experiment aims to identify the optimal processing parameters $(x_1^*, x_2^*, \cdots, x_v^*)$ to maximize processing performance and minimize deviation from the target value.

Table1. Experimental Design Framework.

Run	$(x_1, x_2, \cdots, x_v)$			Number of replications	Mean	Standard deviation
1	x_{11}	x_{12} $\cdots$	x_{1v}	$y_{11}, y_{12}, \cdots, y_{1m}$	$\bar{y}_1$	s_1
2	x_{21}	x_{22} $\cdots$	x_{2v}	$y_{21}, y_{22}, \cdots, y_{2m}$	$\bar{y}_2$	s_2
$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\vdots$
n	x_{n1}	x_{n2} $\cdot$	x_{nv}	$y_{n1}, y_{n2}, \cdots, y_{nm}$	$\bar{y}_n$	s_n

During the pth treatment $(p = 1,2,\cdots,n)$, the estimates of the mean and standard deviation are given by Equations (1) and (2), respectively:

$$\bar{y}_p = \frac{\sum_{q=1}^{m} y_{pq}}{m} \tag{1}$$

$$\sigma_p = \sqrt{\frac{\sum_{q=1}^{m}(y_{pq} - \bar{y}_p)^2}{m-1}} \tag{2}$$

The regression model is expressed as: $\boldsymbol{Y} = \boldsymbol{X\theta} + \boldsymbol{\varepsilon}$, where $\boldsymbol{Y}$ denotes a set of vectors of response values, $\boldsymbol{X}$ is an $n \times k$ model matrix, $\boldsymbol{\theta}$ is a $k \times 1$ model parameter, $\boldsymbol{\varepsilon}$ is an $n \times 1$ vector of random errors, and $k - 1$ is the number of predictor variables of the model. Ignoring the uncertainty of model parameters, the unknown model parameters can be estimated from the acquired experimental data using the least squares method:

$$\widehat{\boldsymbol{\theta}} = (\hat{\theta}_0, \hat{\theta}_1, \cdots, \hat{\theta}_{k\text{-}1}) = (\mathbf{X'X})^{-1}\mathbf{X'Y} \tag{3}$$

where, $\boldsymbol{X} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1,k\text{-}1} \\ 1 & X_{21} & \cdots & X_{2,k-2} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & X_{n1} & \cdots & X_{n,k-1} \end{bmatrix}, \boldsymbol{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}$.

In practice, the complexity of the process, experimental errors and other factors lead to uncertainty in the estimated model parameters $\boldsymbol{\theta}$, which makes the constructed polynomial model unable to accurately estimate the real process output value at a certain design point. At this point, the uncertainty in the model parameters can be quantified using interval estimation methods. Since the model parameter vector $\widehat{\boldsymbol{\theta}}$ estimated by least squares is a linear combination of the response y, $\widehat{\boldsymbol{\theta}}$ obeys a normal distribution $N(\boldsymbol{\theta}, \sigma^2(\boldsymbol{X}'\boldsymbol{X})^{-1})$. Based on the distribution of the model parameter vector, denoted as $\widehat{\boldsymbol{\theta}} \sim N(\boldsymbol{\theta}, \sigma^2(\boldsymbol{X}'\boldsymbol{X})^{-1})$, the $1 - \alpha$ confidence interval for $\boldsymbol{\theta}(j)(j = 1,2,\ldots,k)$ can be obtained:

$$[\boldsymbol{\theta}^L(j), \boldsymbol{\theta}^U(j)] = \left[\widehat{\boldsymbol{\theta}}(j) - t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}^2(\mathbf{X'X})^{-1}},\ \ \widehat{\boldsymbol{\theta}}(j) + t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}^2(\mathbf{X'X})^{-1}}\right] \tag{4}$$

where, k represents the number of model parameters to be estimated in the model, $t_{\frac{\alpha}{2},n-k}$ is the t distribution quantile at the significance level of $\frac{\alpha}{2}$ and degrees of freedom of $n-k$, where n is the number of experiments.

The mean and standard deviation responses are modelled using response surface methodology as follows:

$$\mu(x) = \beta_0 + \sum_{i=1}^{v} \beta_i x_i + \sum_{i=1}^{v} \beta_{ii} x_i^2 + \sum_{i<j}^{v} \sum \beta_{ij} x_i x_j + \varepsilon_\mu \quad (5)$$

$$\sigma(x) = \gamma_0 + \sum_{i=1}^{r} r_i x_i + \sum_{i=1}^{v} r_{ii} x_i^2 + \sum_{i<j}^{v} \sum r_{ij} x_i x_j + \varepsilon_s \quad (6)$$

where $\beta_\mu = \left(\beta_0, \beta_1, \dots, \beta_v, \beta_{11}, \dots, \beta_{vv}, \beta_{12}, \dots, \beta_{v-1,v}\right)^T$, $\beta_\sigma = (\gamma_0, \gamma_1, \dots, \gamma_v, \gamma_{11}, \dots, \gamma_{vv}, \gamma_{12}, \dots, \gamma_{v-1,v})^T$, ε_μ and ε_s are the random error terms, respectively; and $\varepsilon_\mu \sim N(0, \sigma_\mu^2)$, $\varepsilon_s \sim N(0, \sigma_s^2)$.

By employing the least squares method, a fitted response model concerning the mean μ and standard deviation σ (containing $k-1$ predictive variables) can be obtained:

$$\hat{\mu}(x) = \mathbf{X}\widehat{\boldsymbol{\theta}}_\mu \quad (7)$$

$$\hat{\sigma}(x) = \mathbf{X}\widehat{\boldsymbol{\theta}}_\sigma \quad (8)$$

where, model parameter $\widehat{\boldsymbol{\theta}}_\mu = \left[\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_v, \hat{\beta}_{11}, \dots, \hat{\beta}_{v-1,v}\right]^T$, $\widehat{\boldsymbol{\theta}}_\sigma = \left[\hat{\gamma}_0, \hat{\gamma}_1, \dots, \hat{\gamma}_v, \hat{\gamma}_{11}, \dots, \hat{\gamma}_{v-1,v}\right]^T$.

Based on Equation (4), the $1-\alpha$ confidence intervals for the model parameters of the mean response and variance response can be obtained respectively:

$$\mathrm{CI}_{\widehat{\boldsymbol{\theta}}_\mu} = \left[\widehat{\boldsymbol{\theta}}_\mu(j) - t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}_1^2(\mathbf{X}'\mathbf{X})^{-1}},\ \widehat{\boldsymbol{\theta}}_\mu(j) + t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}_1^2(\mathbf{X}'\mathbf{X})^{-1}}\right] \quad (9)$$

$$\mathrm{CI}_{\widehat{\boldsymbol{\theta}}_\sigma} = \left[\widehat{\boldsymbol{\theta}}_\sigma(j) - t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}_2^2(\mathbf{X}'\mathbf{X})^{-1}},\ \widehat{\boldsymbol{\theta}}_\sigma(j) + t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}_2^2(\mathbf{X}'\mathbf{X})^{-1}}\right] \quad (10)$$

where, $\widehat{\boldsymbol{\theta}}_\mu(j)$ and $\widehat{\boldsymbol{\theta}}_\sigma(j)$, $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are the mean response and variance response j th model parameter and variance, respectively.

3. Optimization method considering the confidence intervals of model parameters

3.1 Analysis of total cost structure considering model parameters uncertainty

For economic reasons, it is necessary to reduce the costs for both customers and manufacturers in the micro-manufacturing process. Therefore, production economics and product quality should be simultaneously incorporated into the optimization process. The process of implementing continuous quality improvement includes two economic issues. One is the manufacturing cost incurred before the product is sold to the customer, and the other is the warranty cost after the product is sold to the customer. Warranty costs typically encompass manufacturer liabilities, repair or replacement inconveniences and time wasted for customers, loss of future sales, and market share decline, etc.[8]. In the literature of manufacturing and quality engineering, manufacturing costs are commonly modeled as a function of the natural process tolerance of product quality characteristics. The smaller the tolerance, the more robust the product quality characteristics. However, this requires more precise machining equipment and more skilled operators, thus resulting in higher manufacturing costs.

3.1.1 manufacturing cost

Tolerance costs associated with manufacturing costs aid enterprises in assessing and optimizing product quality control strategies. The manufacturing expense C_m of a product is related to the magnitude of the product's quality characteristic values, which in turn are related to the tolerance of the design function. The tolerance of the design function can be calculated using the standard deviation of the product quality characteristics, with the formula as follows:

$$\mathrm{C}_m = \mathrm{d}_0 + \frac{d_1}{\Delta y} \quad (11)$$

$$\Delta y = 3\sigma_y \quad (12)$$

where, d_0 and d_1 are the coefficients of the tolerance cost model; Δy represents the width of the specification limit for quality characteristic y; and σ_y is the standard deviation of the design function y.

Many studies define manufacturing costs solely as a function related to the tolerance of the design function. Other studies make it more comprehensive by introducing an independent internal failure cost item into the manufacturing cost relationship, namely scrap and rework costs. These studies consider that these costs are influenced not only by the tolerance of the design function but also by the means of production. If the product's main quality characteristic mean is near the design tolerance midpoint, internal failure costs can be modeled as directly related to this tolerance and included in the earlier mentioned cost relationship.

In the production process, when the quality characteristics of a product do not meet the predetermined specification limits, additional costs are incurred. These costs may include the extra labor costs for reworking products, the material and service costs for scrapped products, or the cost of additional material purchases. For those products that do not meet specification limits, products that qualify for rework are reprocessed, otherwise, they are scrapped. The extent of rework and scrap costs depends on the type of quality characteristic observed and the predefined specification limits. The rework and scrap costs are represented as follows:

$$\mathrm{E(Rework\ Cost)} = \mathrm{C}_{\mathrm{rework}}(P[y \leq LSL] = \mathrm{C}_{\mathrm{rework}} \int_{-\infty}^{LSL} f(\mathrm{y})dy \quad (13)$$

$$\mathrm{E(Scrap\ Cost)} = \mathrm{C}_{\mathrm{scrap}}(P[y \geq USL] = \mathrm{C}_{\mathrm{scrap}} \int_{USL}^{\infty} f(\mathrm{y})dy \quad (14)$$

where, C_{rework} represents the unit cost of rework; C_{scrap} represents the unit cost of scrap; $f(y)$ is the probability density function of the quality characteristic *Y*. *LSL* is the lower specification limit of the product, referring to the minimum allowable value or lower limit of the range accepted for the product or process. *USL* represents the upper specification limit of the product, referring to the maximum allowable value accepted for the product or process.

Based on the mean and standard deviation estimated by the design of experiments, the probability density function of the quality characteristic ***Y*** is as follows:

$$f(y) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} exp\left\{-\frac{1}{2}\left[\frac{(y-\hat{\mu})^2}{\hat{\sigma}^2}\right]\right\} \quad (15)$$

3.1.2. Warranty cost

Due to the complexity and repair costs of products such as construction machinery, customers increasingly prioritize warranty policies for complex items to mitigate losses from uncertain product quality[32]. The formulation of warranty policies involves many processes, including product design, manufacturing, marketing, and after-sales service. However, departments within a company often operate independently, lacking coordination and cooperation. This results in an inability to systematically design and optimize quality assurance policies from a product lifecycle perspective. The lack of systematic decision-making can lead to numerous problems. For example, a mismatch between warranty policies and product reliability can damage a company's profits. When the design reliability of a product is not high, the sales department may establish aggressive quality assurance policies to boost product sales. This leads to a significant number of product claims during the after-sales service phase due to low product reliability in the field, causing excessively high warranty costs for the manufacturer.

The warranty cost of a product refers to the expenses incurred for quality assurance activities carried out to guarantee product quality and meet customer needs within a specified period. The warranty can be limited or lifetime, allowing consumers to request repairs, replacements, or a one-time rebate from the manufacturer. In addition to formulating the warranty policies provided to customers, manufacturers must also decide on the duration of the warranty period and calculate the warranty costs.

Warranty costs can be calculated using different formulas, but the basic approach is to evaluate the total cost of warranty activities based on the probability density function or the cumulative probability distribution function. This paper considers only a simple warranty model, namely the minimal cost warranty model. This model emphasizes minor repairs, suggesting that a repair either maintains the product's failure rate, restores it to its original state, or replaces a small component within a larger system. Because of the degradation of other components, the system's reliability essentially remains unchanged. The minimal warranty cost model is as follows[33]:

$$M(w) = \int_0^w h(\tau)\, d\tau = -\ln R\,(w) \quad (16)$$

$$WC = C_r M(w) = -C_r \ln R\,(w) \quad (17)$$

where, w represents the failure time, $h(\tau)$ is the probability density function of the product failing within time τ, WC is the total warranty loss cost, $R(w)$ is the reliability at the end of the warranty period, C_r is the cost required for repairing each good or item during the warranty period.

Leemis[34] provides a formal definition of reliability, which is stated as: 'The reliability of an item refers to the probability that it can successfully perform its intended function under specified environmental conditions over a specific period of time.' Therefore, based on this definition, it can be inferred that the primary random variable in traditional reliability models is the failure time τ. However, reliability models can also encompass other random variables or parameters, resulting in reliability models that incorporate covariates. The studies by Deleveaux[35], Blue[36], and Hassan[8] attempted to correlate the mean and variance of the main quality characteristics of products with warranty costs by adopting reliability models with covariates. The reliability models for target-is-best $R_N(w)$

and smaller-the-better $R_S(w)$ product characteristics addressed in this paper were proposed by Blue[36] and Hassan[8]:

$$R_N(w) = \frac{1}{\sqrt{1+2b\sigma^2 w^c}} exp\left[-\left(a + \frac{b(\mu-T)^2}{1+2b\sigma^2 w^c}\right) w^c\right] \quad (18)$$

$$R_S(w) = \frac{1}{\sqrt{1+2b\sigma^2 w^c}} exp\left[-\left(a + \frac{b\mu^2}{1+2b\sigma^2 w^c}\right) w^c\right] \quad (19)$$

where, μ and σ^2 are respectively the mean and variance of the product quality characteristics; a, b and c are parameters of the reliability model; T is the target value of the product quality characteristic.

3.2. Constructing total cost optimization model

In summary, from both the manufacturer's and customer's perspectives, the quality assessment of micro-holes is closely related to tolerance costs, rework costs, scrap costs, and warranty costs. The purpose of this paper is to obtain high-quality products at the lowest cost during the micro-manufacturing process. Therefore, the proposed method aims to simultaneously minimize tolerance, rework, scrap, and warranty costs to find the optimal processing parameter settings.

To consider the impact of model parameter estimation errors on the total cost optimization model, this paper adopts interval estimation for model parameters. Based on robust design principles, it constructs an optimization model to enhance solution robustness. Considering economic factors such as manufacturing and warranty costs, an optimization model for the economical design of laser micro-drilling process parameters has been established:

$$\min_{x,\theta_\mu,\theta_\sigma} E(TC) = \lambda * (d_0 + \frac{d_1}{3\hat{\sigma}_y} + C_{rework} P(y \le LSL) + C_{scrap} P(y \ge USL)) + (1-\lambda) * (-Cr\, ln[R(w)]).$$

$$\text{s.t. } f(y) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} exp\left\{-\frac{1}{2}\left[\frac{(y-\hat{\mu})^2}{\hat{\sigma}^2}\right]\right\},$$

$$\hat{\mu}(x) = \boldsymbol{X}\widehat{\boldsymbol{\theta}}_\mu,\ \hat{\sigma}(x) = \boldsymbol{X}\widehat{\boldsymbol{\theta}}_\sigma,$$

$$CI_{\widehat{\boldsymbol{\theta}}_\mu} = \left[\widehat{\boldsymbol{\theta}}_\mu(j) - t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}_1^2(\boldsymbol{X}'\boldsymbol{X})^{-1}},\ \widehat{\boldsymbol{\theta}}_\mu(j) + t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}_1^2(\boldsymbol{X}'\boldsymbol{X})^{-1}}\right],\quad CI_{\widehat{\boldsymbol{\theta}}_\sigma} = \left[\widehat{\boldsymbol{\theta}}_\sigma(j) - t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}_2^2(\boldsymbol{X}'\boldsymbol{X})^{-1}},\ \widehat{\boldsymbol{\theta}}_\sigma(j) + t_{\frac{\alpha}{2},n-k}\sqrt{\hat{\sigma}_2^2(\boldsymbol{X}'\boldsymbol{X})^{-1}}\right],$$

$$R(w) = \frac{1}{\sqrt{1+2b\sigma^2 w^c}} exp\left[-\left(a + \frac{b(\mu-T)^2}{1+2b\sigma^2 w^c}\right) w^c\right] \quad (20)$$

$$0 \le \lambda \le 1,$$

where, λ represents the manufacturer's emphasis on internal costs, that is, the weight assigned to internal costs. $\lambda \in [0,1]$. When $\lambda=0$, it suggests the manufacturer prioritizes external cost reduction over internal costs, favoring a low-margin, high-volume marketing strategy. When λ is 0.5, it implies that the manufacturer places equal importance on both internal and external costs. When $\lambda=1$, it indicates the manufacturer opts for a high-margin, low-volume strategy, focusing significantly on reducing its internal costs with less emphasis on external costs.

4. Case Study

This paper employs a micro-drilling example to test the effectiveness of the proposed method. The micro-drilling experiment is derived from literature[37], focusing on the parameter design issue of the micro-drilling process. The output response of this process is the radius of the micro-hole y with a target-is-best characteristic, with the upper and lower specification limits set at 39.8μm and 40.2μm, respectively. The closer the value is to the target value T=40μm, the higher the quality of the hole produced. If the radius of the micro-hole exceeds the upper limit, it will be scrapped, incurring scrap costs; if the radius of the micro-hole is less than the lower limit, it will be reworked, incurring rework costs. Assume the unit rework cost is 10, and the scrap cost is 200. Experimental data were collected using a four-axis CNC femtosecond laser micro-machining center and a 3^3 full factorial design. Three processing parameters were identified as significant controllable factors: average power (x_1), Q-switch frequency (x_2), and cutting speed (x_3). The actual values of each parameter and their corresponding coded values are shown in Table 2, and the experimental results are presented in Table 3.

Table 2. Actual and encoded values of parameters.

Parameters	level		
	-1	0	1
x_1: average power/mW	50	100	150
x_2: Q-switch frequency/Hz	500	650	800
x_3: cutting speed /(0.01mm/s)	4	6	8

Table 3. Experimental results.

x_1	x_2	x_3	y	$\bar{y}$	σ
-1	1	1	26.806 22.300 22.454	23.853	2.558
-1	0	0	27.200 22.406 34.043	27.883	5.849
-1	-1	-1	27.462 23.507 44.908	31.959	11.387
-1	0	1	29.698 26.800 26.800	27.766	1.673
-1	0	0	43.460 41.287 44.908	43.218	1.823
-1	0	-1	58.671 60.843 52.946	57.487	4.079
-1	-1	1	23.179 21.730 23.403	22.770	0.908
-1	0	0	36.941 35.492 40.562	37.665	2.611
-1	1	-1	49.530 55.773 52.152	52.485	3.135
0	1	0	36.216 31.319 35.492	34.342	2.643
0	0	0	52.152 61.568 57.222	56.980	4.713
0	-1	0	57.946 59.395 65.189	60.843	3.833
0	0	0	29.698 30.422 26.800	28.973	1.916
0	0	0	42.011 39.838 42.735	41.528	1.508
0	0	0	57.222 53.998 55.049	55.423	1.644
0	-1	0	25.800 24.300 29.698	26.599	2.786
0	0	0	38.389 35.165 38.389	37.314	1.861
0	1	0	52.876 59.395 55.773	56.014	3.266
1	1	-1	33.319 34.043 34.043	33.802	0.418
1	0	0	52.152 55.049 57.946	55.049	2.897
1	-1	1	60.119 57.222 55.773	57.705	2.213
1	0	-1	26.698 27.525 28.973	27.732	1.152
1	0	0	44.908 48.530 46.357	46.598	1.823
1	0	1	62.292 66.638 59.395	62.775	3.646
1	-1	-1	26.800 28.249 27.525	27.524	0.724
1	0	0	36.216 39.838 40.562	38.872	2.328
1	1	1	54.325 49.254 59.395	54.324	5.070

The Kolmogorov-Smirnov test verified the response distribution. MATLAB's lillietest confirmed hypothesis validity with P>0.5. This outcome indicates that the response variable follows a normal distribution. Assume in the warranty cost model, the parameters are w=12, a=0.001, b=0.025, c=0.8, Cr=50. Referring to Hassain[18], the tolerance cost model coefficients are assumed to be d_0=10, d_1=3. Based on the process parameters for micro-drilling in Table 2, along with the sample mean and standard deviation of the micro-hole radius, confidence intervals for the model parameters $\widehat{\boldsymbol{\theta}}_\mu$ and $\widehat{\boldsymbol{\theta}}_\sigma$ for mean and standard deviation responses can be derived from Equations (9) and (10), respectively. Subsequently, the response surface models for the mean and standard deviation are fitted using Equations (7) and (8), respectively.

4.1. Optimization results for different methods

Manufacturers with different marketing strategies place varying levels of importance on internal and external costs. The structure of differential loss costs can somewhat influence the determination of laser micro-hole processing parameter levels. Therefore, by choosing the value of λ appropriately according to their own type, manufacturers can obtain the optimal processing parameter levels that match their type.

This section compares the proposed method with other approaches for analyzing the total lifecycle costs of researched products. Table 4 presents the response means, standard deviations, and various component costs associated with these methods. Hassan[18]developed a comprehensive cost model that included tolerance costs, rework costs, scrap costs, and manufacturing costs. However, the entire lifecycle cost model developed in Hassan's work uses constant parameters for the mean and variance models, thus failing to consider the uncertainty of model parameters. Lv et al.[38] utilized the multi-objective optimization algorithm "gamultiobj" to minimize the product lifecycle costs and achieve the Pareto frontier. Subsequently, the optimal solution was selected using the grey relational analysis technique based on criteria importance through intercriteria correlation approach. Although this method does not consider the uncertainty in model parameters, it accounts for the uncertainty in predicted response losses caused by fluctuations in predicted values. To demonstrate the superiority of the method proposed in this paper, under the condition of λ=0.5, a comparative analysis of the optimization results is conducted.

Based on the conditions and objective function given by Equation (20), the Fmincon function in MATLAB optimization tools is utilized to find a feasible solution. Given the need to set an initial point in the function, Fmincon is susceptible to being trapped in local optima. To obtain a global optimum, this study selected 1000 initial sample points within the feasible region, which tend to be stable points or boundary points. The convergence criterion is to ensure the ratio of change in response values is less than 10^{-6}. The optimization results are shown in Table 4.

Table 4. Comparison table of optimization results for different methods.

Method	μ	σ	Tolerance	Rework	Scrap	Warranty	Expected total cost
Proposed method	**40.1393**	1.6750	10.5970	4.8554	**83.9490**	18.0810	**58.7412**
Hassain(2012)	40.2010	2.4548	**10.4074**	5.0016	87.0251	29.5556	65.9948
Lv et al.(2023)	39.8344	**1.4416**	10.6937	**3.9990**	98.0958	**14.6202**	63.7043

As shown in Table 3, for the proposed method, the level of the controllable factor is $x^* = (-0.0034, 0.0002, -0.0001)$, resulting in a total cost of 58.7412. The optimal level of processing parameters $x^* = (-0.0820, 0.1144, -1.0000)$ and a total cost of 65.9948 of Hassain' (2012) approach. The optimal level of processing parameters and total cost for Lv et al.'(2023) method are $x^* = (0.0999, 0.0267, 0.0984)$ and 63.7043, respectively. It is evident that the proposed method incurs the lowest cost (58.7412), while the cost associated with Hassan's (2012) method is the highest (65.9948), with Lv et al.'s (2023) cost falling in between (63.7043). The expected total cost, a composite indicator that considers various cost factors, reflects the economic efficiency of the entire production process. From this perspective, the proposed method demonstrates superior performance in cost control.

The mean of the proposed method is very close to the target value of 40. This indicates that the process parameters are set with high precision, allowing for stability and consistency in outputs despite uncertainties. In micro-manufacturing, precise setting and execution of these parameters are critical for achieving high-quality outputs. The standard deviation measures the dispersion of data. The proposed method has a standard deviation of 1.6750, compared to 2.4548 for Hassan (2012) and 1.4416 for Lv et al.(2023). The smallest standard deviation, seen in Lv et al.'s (2023) method, results from effectively managing fluctuations in predicted responses by dynamically adjusting production parameters to minimize output variability. Hassan's (2012) process exhibits the greatest variability, and due to its association with tolerance costs related to fluctuations in product quality characteristics, it incurs the lowest tolerance costs.

The Lv et al. (2023) method minimizes rework through stringent quality control and the application of predictive models. However, a potential side effect of this method is an elevated scrap rate, particularly notable in industries where extremely high product quality is pursued. The proposed method balances economic benefits with resource efficiency by optimizing costs throughout the product lifecycle, creating a more cost-effective production model. Warranty costs refer to the additional expenses incurred due to product quality issues. Hassan (2012) had the highest warranty costs at 29.5556, followed by the proposed method at 18.0810, while Lv et al. (2023) had the lowest at 14.6202. This indicates that Lv et al.'s (2023) method effectively maintains product quality, resulting in lower warranty costs.

The analysis above indicates that the proposed method offers distinct advantages in robustness and cost-efficiency, particularly from a cost-benefit perspective. For production environments aiming to optimize costs while maintaining product quality, the proposed method is a superior choice. Economically, the proposed method reduces total costs by 10.99% and 7.79% compared to two other methods, offering significant economic advantages.

When the confidence level 1-α is 0, Equation (20)'s model parameters degenerate to constants, resembling traditional robust optimization models that overlook parameter uncertainty. Thus, these traditional methods are a specific case of our proposed approach.

Based on the above results, two interesting findings can be observed:

1) The optimization method that incorporates model parameter uncertainty yields processing parameters whose response mean is closest to the 40μm target and results in a smaller standard deviation, thereby minimizing fluctuations in predicted response values. Consequently, accounting for model parameter uncertainty not only enhances predictive accuracy but also leads to improved optimization outcomes.

2) When estimating model parameters amid uncertainty, the proposed method ensures optimal processing parameters maintain quality and control costs, even in the worst-case scenarios. Therefore, considering model parameter uncertainty makes the economic efficiency and robustness of the optimization results more balanced and reasonable, ultimately resulting in the lowest internal and external total costs.

4.2. Sensitivity analysis

When λ =0.5, in order to understand the impact of some model assumptions on the optimal solution, sensitivity analysis is conducted on different warranty periods and unit scrap costs.

4.2.1. Sensitivity analysis for warranty period *w*

When conducting sensitivity analysis on the warranty period w, the values of unit scrap cost, rework cost, and other model assumptions are held constant. The value of w is varied from 6 to 12, with an increment of 6, and the optimization results are shown in Table 5.

Table 5. Effect of different values of w on optimization results.

w	x_1 x_2 x_3	Tolerance	Rework	Scrap	Warranty	Expected total cost
6	0.0006 -0.0033 0.0136	10.6024	4.7867	85.1305	11.6517	56.0856
12	-0.0034 0.0002 -0.0001	10.5970	4.8554	83.9490	18.0810	58.7412
18	0.0167 0.0001 0.0190	10.6110	4.8404	83.8066	21.9894	60.6236
24	0.0920 -0.0050 0.0020	10.6417	4.8318	83.0156	24.0872	61.2882
30	0.0751 0.0081 0.0226	10.6423	4.7950	83.7169	26.9649	63.0596

From Table 6, it can be observed that both warranty costs and expected total costs increase with the lengthening of the warranty period. Since tolerance, rework, and scrap costs relate to specification limits and fluctuations in micro-hole quality, their changes are not significant as the warranty period lengthens. Therefore, as the warranty period for micro-manufacturing products lengthens, the total cost of the product increases. This is because extending the warranty period typically involves more after-sales service, maintenance costs, and potential warranty liabilities. For merchants, trade-offs and considerations should be made in order to develop an appropriate warranty strategy. In developing warranty strategies, businesses must consider market demand, cost factors, and product quality to ensure the warranty period meets customer expectations and promotes corporate sustainability.

Table 6. Effect of different values of C_{scrap} on optimization results.

C_{scrap}	μ	σ	Tolerance	Rework	Scrap	Warranty	Expected total cost
80	40.1297	1.6656	10.6004	4.8317	33.7230	17.9274	33.5413
90	40.0921	1.5038	10.6649	4.7162	38.0509	15.4557	34.4439
100	40.1192	1.6566	10.6036	4.8054	42.3615	17.7799	37.7752
110	40.1053	1.5459	10.6469	4.7557	46.3896	16.0987	38.9455
120	40.1178	1.5351	10.6514	4.7868	50.1558	15.9499	40.7720

C_{scrap}	μ	σ	Tolerance	Rework	Scrap	Warranty	Expected total cost
130	40.1126	1.5524	10.6441	4.7755	54.6270	16.2047	43.1257
140	40.1420	1.5614	10.6405	4.8547	57.8250	16.3770	44.8486
150	40.0660	1.5226	10.6568	4.6494	64.5982	15.7147	47.8095
160	40.0671	1.5093	10.6626	4.6491	68.7646	15.5161	49.7962
170	40.1313	1.5475	10.6462	4.8242	70.5703	16.1528	51.0968
180	40.1727	1.6051	10.6230	4.9336	73.4498	17.0788	53.0426
190	40.1446	1.5423	10.6484	4.8567	78.2042	16.0917	54.9005
200	40.1393	1.6750	10.5970	4.8554	83.9490	18.0810	58.7412
210	40.1027	1.5608	10.6407	4.7515	88.8528	16.3203	60.2826
220	40.1015	1.5483	10.6459	4.7476	92.9909	16.1319	62.2581

Table 6 reflects the overall impact of changes in unit scrap cost on various costs, as well as the optimal process mean and variance. When other unit costs and warranty model parameters remain constant, an increase in unit scrap cost leads to higher scrap costs and expected total costs. Meanwhile, there are slight variations in the mean and standard deviation under different rework cost coefficients, indicating some fluctuations in the production process, but overall differences are not significant. Variations are minimal, likely due to modeling that accounted for model parameter uncertainty, leading to optimized processing parameters and notably stable production, with fluctuations limited to a narrow range.

When the unit scrap costs are 210 and 220 respectively, an increase in unit scrap cost leads to stricter control over micro-holes quality, resulting in higher tolerance costs to ensure compliance with better quality standards. Higher tolerance costs indicate a more stable micro-hole manufacturing process, requiring less additional work for repair or adjustment. This can reduce subsequent maintenance and warranty costs associated with micro-holes.

4.2.2. Sensitivity analysis for different cost weights λ

Due to the varying marketing strategies that manufacturers may employ at different stages, there is also a differential emphasis on internal and external costs. Therefore, based on equation (20), an analysis of the values of internal costs, external costs, and total costs under different cost weights is conducted. Fifteen points are uniformly selected from 0 to 1 for optimization, and the optimization results are presented in Figure 1.

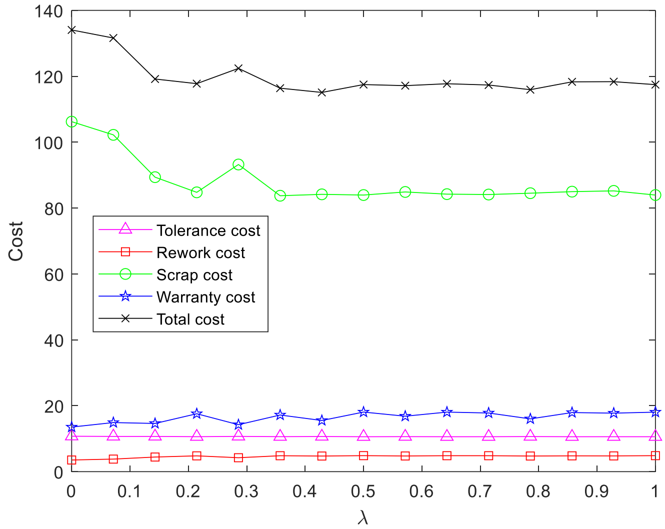

Fig.1. The impact of different values of λ on the optimization results.

Figure 1 illustrates the variation trends of expected loss costs under 15 different values of λ. When λ=0, there is a significant gap between the sum of internal costs and external costs. The sum of internal costs (comprising tolerance and scrap costs) amounts to 120.5276, while the warranty cost reaches its minimum at 13.5308. This is because, at λ=0, the company completely disregards internal costs and focuses on external costs. When the gap between internal and external costs reaches its maximum, the company can consider a low-margin, high-volume marketing strategy. It can adjust the tolerance levels within a certain range to achieve cost optimization. As λ changes from 0 to 0.5, the manufacturer's marketing strategy begins to shift towards a low-margin, high-volume approach. At this stage, to enhance brand recognition, establish market credibility, and prestige, the company pays significant attention to losses caused by customer dissatisfaction after product use. As λ gradually increases from 0.5 to 1, manufacturers may gradually move away from the low-margin, high-volume strategy and start to pay more attention to internal costs, with scrap costs showing a downward trend. When λ=1, the company completely disregards external costs and focuses on internal costs. Tolerance and scrap costs are minimized, and warranty costs increase, suggesting that the company may need to strengthen the monitoring and management of external quality of micro-holes to reduce warranty costs. With lower internal costs for micro-holes and higher per-unit profits, the company can opt for a high-margin, low-volume marketing strategy.

The aforementioned results are merely the outcomes of sensitivity analysis on a subset of model assumptions and parameters. Other parameters can be investigated in a similar manner as described above. This paper aims to demonstrate sensitivity analysis methods for selected parameters and their notable findings, thus not encompassing all parameter results.

4.3. Numerical Case Study: Revisiting the Micro-Drilling Process

To verify the economic and robustness of the proposed modeling technique, Monte Carlo simulations were used to study process variations of different degrees in the micro-drilling process.

In the Monte Carlo simulation, the original micro-drilling process data forms the basis for generating a full factorial design with three repetitions at each design point for every simulation run, facilitating data collection from simulated experiments. The first scenario involves high variability design, where the mean and standard deviation of the original samples at each design point are used to generate normal random samples. The second scenario involves low variability design, which is achieved by multiplying the original sample standard deviation by 0.25 to generate normal random variables with reduced variability by 75%, while other methods are created in a similar manner[39]. Subsequently, three samples are randomly drawn from the normal random samples to form the data for simulated experiments, with the specific experimental structure detailed in Table 7. This process is repeated 500 times to generate 500 sets of experimental data.

Table 7. Experimental framework for numerical cases

x_1	x_2	x_3	$\bar{y}$	σ	Simulation data
-1	1	1	23.853	2.558	$y_{1,1}$ $y_{1,2}$ $y_{1,3}$
-1	0	0	27.883	5.849	$y_{2,1}$ $y_{2,2}$ $y_{2,3}$
-1	-1	-1	31.959	11.387	$y_{3,1}$ $y_{3,2}$ $y_{3,3}$
-1	0	1	27.766	1.673	$y_{4,1}$ $y_{4,2}$ $y_{4,3}$
-1	0	0	43.218	1.823	$y_{5,1}$ $y_{5,2}$ $y_{5,3}$
-1	0	-1	57.487	4.079	$y_{6,1}$ $y_{6,2}$ $y_{6,3}$
-1	-1	1	22.770	0.908	$y_{7,1}$ $y_{7,2}$ $y_{7,3}$
-1	0	0	37.665	2.611	$y_{8,1}$ $y_{8,2}$ $y_{8,3}$
-1	1	-1	52.485	3.135	$y_{9,1}$ $y_{9,2}$ $y_{9,3}$
0	1	0	34.342	2.643	$y_{10,1}$ $y_{10,2}$ $y_{10,3}$
0	0	0	56.980	4.713	$y_{11,1}$ $y_{11,2}$ $y_{11,3}$
0	-1	0	60.843	3.833	$y_{12,1}$ $y_{12,2}$ $y_{12,3}$
0	0	0	28.973	1.916	$y_{13,1}$ $y_{13,2}$ $y_{13,3}$
0	0	0	41.528	1.508	$y_{14,1}$ $y_{14,2}$ $y_{14,3}$
0	0	0	55.423	1.644	$y_{15,1}$ $y_{15,2}$ $y_{15,3}$
0	-1	0	26.599	2.786	$y_{16,1}$ $y_{16,2}$ $y_{16,3}$
0	0	0	37.314	1.861	$y_{17,1}$ $y_{17,2}$ $y_{17,3}$
0	1	0	56.014	3.266	$y_{18,1}$ $y_{18,2}$ $y_{18,3}$
1	1	-1	33.802	0.418	$y_{19,1}$ $y_{19,2}$, $y_{19,3}$
1	0	0	55.049	2.897	$y_{20,1}$ $y_{20,2}$ $y_{20,3}$
1	-1	1	57.705	2.213	$y_{21,1}$ $y_{21,2}$ $y_{21,3}$
1	0	-1	27.732	1.152	$y_{22,1}$ $y_{22,2}$ $y_{22,3}$
1	0	0	46.598	1.823	$y_{23,1}$ $y_{23,2}$ $y_{23,3}$
1	0	1	62.775	3.646	$y_{24,1}$ $y_{24,2}$ $y_{24,3}$
1	-1	-1	27.524	0.724	$y_{25,1}$ $y_{25,2}$ $y_{25,3}$
1	0	0	38.872	2.328	$y_{26,1}$ $y_{26,2}$ $y_{26,3}$
1	1	1	54.324	5.070	$y_{27,1}$ $y_{27,2}$ $y_{27,3}$

Based on the mean and variance of each set from the initial micro-drilling experiment, 500 sets of experimental data are generated through Monte Carlo simulation. Subsequently, the mean and standard deviation of each set of experimental data are calculated. Different modeling techniques were employed to establish dual-response surface models, considering model parameter uncertainty ($E(TC)$) and not considering model parameter uncertainty ($E(NTC)$). Through equation (20), optimal processing parameters for Monte Carlo simulation (N=500) were obtained. Based on the optimal processing parameters, the mean, standard deviation, internal and external costs, as well as total costs can be calculated, with the results presented in Table 8.

Table 8. Optimization results for different variation scenarios.

Variability level	Model	μ	σ	Tolerance	Rework	Scrap	Warranty	Expected total cost
Low variability	$E(TC)$	40.1695	0.4211	12.3749	4.7113	38.0208	2.1790	28.6430
	$E(NTC)$	40.2000	0.5952	11.6802	5.0000	50.1543	3.7283	35.2814
High variability	$E(TC)$	40.1485	1.5257	10.6554	4.8653	81.9343	15.8495	56.6522
	$E(NTC)$	40.2000	2.3922	10.4180	5.0000	86.7204	28.6780	65.4082

From Table 7 simulation results, it can be observed that the optimization results of the proposed modeling method $E(TC)$, are comparatively superior to those of the method $E(NTC)$. In the low volatility scenario, the optimization results of the proposed method are better in terms of mean, standard deviation, rework costs, scrap costs, warranty costs, and total costs compared to the optimization method that does not consider model parameter uncertainty. The optimal processing parameters are set to (0.0385, -0.0022, -0.0015). Compared to the $E(NTC)$ method, which does not consider model parameter uncertainty and has optimal processing parameters set at (-0.0943, 0.1334, -1.0000), improvements of 5.77%, 24.19%, 41.56%, and 18.82% are achieved in rework costs, scrap costs, warranty costs, and total costs, respectively. In the same way, in the high volatility scenario, the optimal processing parameters obtained under the $E(TC)$ method are set to (0.1033, 0.0003, 0.0101), which correspond to a closer match to the target mean and a smaller standard deviation. In the pursuit of minimizing production fluctuations, the implementation of strict quality control measures and advanced manufacturing technologies will lead to an increase in tolerance costs. Ultimately, the proposed method reduces the total cost while maintaining product quality by balancing internal and external costs. Compared to the $E(NTC)$ method with optimal processing parameters set at (-0.1060, 0.1328, -1.0000), improvements of 5.52%, 44.73%, and 13.39% are realized in scrap costs, warranty costs, and total costs, respectively. This indicates that the presence of uncertainty in model parameter estimation affects the optimization solutions. The optimization method that considers model parameter uncertainty can ensure that the optimization results remain reliable even in the worst-case scenarios. The optimization methods that do not consider uncertainty factors result in optimized solutions that are sensitive to these uncertainties. With greater uncertainty, the differences between the worst-case scenarios of the two methods also become more pronounced. In conclusion, the proposed method outperforms models that neglect model parameter uncertainty. It achieves optimal processing parameters by balancing manufacturing and quality assurance costs.

From case studies and simulation analyses, it has been found that the selection of processing parameters significantly impacts the quality, variability, and total cost of micro-holes. Compared to the $E(NTC)$ method, the $E(TC)$ method suggests that higher average power and cutting speed, along with a lower Q-switch frequency, can improve micro-hole quality while reducing variability and total costs. In terms of average power, a higher average power provides more energy, which is crucial for material removal and micro-hole formation in micro-manufacturing processes. Simultaneously, higher power usually means stronger cutting capabilities, allowing for more effective overcoming of material strength and hardness, thus more easily achieving the desired micro-hole shapes and sizes. Regarding cutting speed, a higher cutting speed helps reduce the heat generated during cutting, which aids in lowering material deformation and surface roughness. At the same time, high cutting speeds can shorten the machining cycle, enhance production efficiency, and to some extent, reduce machining costs. As for the Q-switch frequency, a lower Q-switch frequency implies longer pulse duration and lower average power density. This can diminish local thermal effects and thermal damage to the material. Additionally, a lower Q-switch frequency typically results in a smoother machining surface, as longer pulse durations provide more time for the material surface to cool. Lowering the Q-switch frequency can lengthen processing time, but also reduce damage rates and enhance micro-hole precision, thus diminishing scrap rates and processing variability.

5. Conclusions

Given that businesses adopt differentiated marketing strategies, their levels of concern for internal and external costs vary. Furthermore, engineering systems face many uncertainties, like environmental variations and measurement errors. Under cost constraints, using limited data for response surface models may lead to parameter deviations, compromising optimization robustness. Based on this, this paper proposes a cost-quality design model that considers the internal and external costs of micro- manufacturing. From the perspective of interval estimation of model parameters prediction, this paper combines the idea of robust optimization design to address the economic quality design problem amid model parameter uncertainty. The analysis results indicate:

(1) Firms with different marketing strategies assign varying weights to internal and external costs, leading to varied outcomes. Manufacturers can choose the appropriate λ value for their type to identify the optimal processing parameters.

(2) Considering the uncertainty of model parameters during modeling can reduce variation in the micro-manufacturing process, significantly enhancing the robustness and reliability of the optimal processing parameters.

(3) The proposed method helps engineers make an optimal trade-off between manufacturing cost and warranty cost in micro-manufacturing, thereby reducing the total lifecycle cost of complex products.

The contribution of this paper lies in the proposed method, which integrates product quality characteristics and reliability from an economic perspective. It explores how to conduct economical parameter design amid model parameter uncertainty, constrained by limited experimental resources and manufacturing costs. Additionally, the paper expands the parameter design model by constructing a joint optimization model that encompasses manufacturing costs, warranty costs, and sales strategies, considering the product's entire lifecycle. Furthermore, it offers different marketing strategies based on the weights of internal and external costs.

It should be highlighted that these results may be influenced by experimental conditions and specific material characteristics. Therefore, in practice, adjusting design variables based on specific scenarios and validating paper conclusions with experimental and simulation data is essential. Moreover, to ensure micro-hole quality and processing stability, quality management experts should pay close attention to various parameters during the machining process and promptly adjust and optimize processing conditions.

It is important to note that external costs affecting the product also include factors such as pricing models, market structure types, customer demand, and competitor behavior. How to link these factors to product quality requires further in-depth research. Additionally, in the development and design of products, there are usually many processing parameters that affect the quality characteristics of the product. When experimental resources are limited (many factors to test, few tests possible), researchers prefer to choose fractional factorial designs to reduce the number of experiments and lower the costs. When the number of processing parameters reaches 100, they face the "curse of dimensionality." Techniques such as factor screening[40,41], principal component analysis (PCA)[42], and t-distributed stochastic neighbor embedding (t-SNE)[43] are used to reduce the dimensionality of processing parameters, focusing on key parameters and minimizing information loss. Therefore, further study is required to determine how to conduct cost-effective quality design in micro-manufacturing with high-dimensional processing parameters.

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