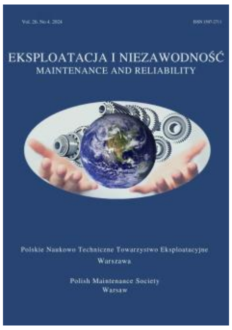

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Bayesian network approach for dynamic fault tree with common cause failures and interval uncertainty parameters

Indexed by: Web of Science Group

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Highlights

- Combine BWM and HFS to avoid the influence of expert subjectivity in β-factor model.
- The interval theory is introduced to deal with the uncertainty parameters.
- A framework for reliability evaluation of dynamic fault tree with CCF based on CTBN is proposed.
- The calculation time of proposed method is shorter than that of DTBN-based method.

Abstract

Traditional fault tree analysis often assumes that the basic events are independent and the failure parameters are known. Therefore, it is powerless to deal with the correlation among basic events and the uncertainty of failure parameters due to the small failure data. Therefore, a framework based on continuous-time Bayesian network is proposed to evaluate the reliability of fault tree with common cause failures (CCF) and uncertainty parameters. Firstly, the best-worst method (BWM) and hesitant fuzzy set (HFS) are introduced to address the issue of β-factor being influenced by experts' subjectivity. Then, the interval theory is introduced to deal with the uncertainty parameters. Based on continuous-time Bayesian network, the conditional probability functions of logic gates (i.e. AND gate, OR gate, spare gate, priority AND gate) with CCF are derived, and the upper and lower bounds of failure probability of top event can be solved. Finally, the fault trees of CPU system and brake signal transmission subsystem are given to verify the effectiveness of the proposed framework.

Keywords

fault tree analysis (FTA), continuous-time Bayesian network (CTBN), common cause failures (CCF), best-worst method (BWM), hesitant fuzzy set (HFS), interval theory

1. Introduction

Fault Tree Analysis (FTA) is a graphical deductive method based on Boolean algebra, which is widely used in industrial fields, such as aerospace [1], automotive [2], and transportation [3]. The fault tree can be divided into static and dynamic fault trees, according to whether it is related to the sequence of events. Logic gates (such as AND gate and OR gate) are commonly employed in static fault trees, and it is generally assumed that the basic events are independent. There are mature theories for achieving qualitative and quantitative analysis of static fault trees [4,5]. Dugan [6] proposed a dynamic fault tree analysis approach by incorporating time-related logic gates, such as priority AND gates and spare gates.

With the increasing complexity of engineering systems, the correlations between failures are also becoming increasing apparent. The correlation of failures may degrade the system reliability in many cases. Additionally, common cause failure (CCF) is an important reason for failure correlation. CCF is defined as the simultaneous failure of several components due to common influencing factors, such as extreme environmental conditions, design defects, or human factors [7]. Over the past

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few decades, some CCF models are conducted [8-10], including the α-factor model, β-factor model, binomial failure rate model, etc. Among them, the β-factor model is the most commonly used model because of its concise failure division. An inherent drawback of the β-factor model is that the β-factor may be affected by experts' subjectivity.

Traditional FTA method could accurately deduce the failure probability of top event but with low-efficiency. In order to accelerate the analysis efficiency, fault trees can be mapped to other models, such as Bayesian network (BN) [11], Markov chain [12], Petri net [13], etc. Bayesian network can estimate and update the probabilities of variables based on failure data, while avoiding the combinatorial explosion problem [14]. Bayesian network encodes the causal relationships among a set of variables by using a graphical model. Jun [15] carried out Bayesian network for fault analysis and developed the corresponding fault identification, fault reasoning, and sensitivity analysis. Guo [16] and Zhang [17] proposed discrete-time Bayesian network to achieve reliability assessment of fault trees with CCF and irrepairable multi-state system respectively. Discrete-time Bayesian network (DTBN) is capable of accurately modelling dynamic failure systems. However, the improving of precision of DTBN is directly accompanied by the refinement of time intervals. As the number of time intervals increases, the size of conditional probability table of BN will significantly increase, resulting in a significant increase in the amount of computation. In addition, DTBN can only calculate the failure probability at integer multiples of the time interval. Therefore, Boudali [18] proposed a continuous-time Bayesian network (CTBN) model to solve the transformation and solution of dynamic fault tree. CTBN does not need to discretize time, so it can calculate the failure probability at any time. Yang [19] conducted reliability analysis for wireless communication networks based on CTBN. Wang [20] conducted reliability modeling and evaluation for rectifier feedback system based on CTBN. Dui [21] conducted reliability and service life analysis of airbag systems based on CTBN. Sturlaugson [22] conducted sensitivity analysis on CTBN as applied specifically to reliability models. Liu [23] proposed hybrid time Bayesian networks by combining CTBN and DTBN, which allows us to more naturally model dynamic systems with regular and irregularly changing variables. CTBN was also used for fault diagnosis and prognostic. Schupbach [24] proposed a health management method for risk prognosis based on CTBN. Perreault [25] constructed a continuous time Bayesian network from a D-matrix, a common matrix representation of diagnostic model. Bai [26] conducted position loss risk analysis of dynamic positioning systems of semi-submersible drilling units based on CTBN. CTBN can achieve reliability evaluation while the failure distribution type and parameters are known. However, due to the complexity of failure and the cognitive limitation of failure mechanisms, as well as the lack of failure data, there is inevitable uncertainty in the failure distribution. Li [27] proposed CTBN combined with triangular fuzzy numbers for dynamic fault tree analysis. Sturlaugson [28] defined and extended inference to reason under uncertainty in CTBN. However, none of the above researches based on CTBN modeled CCF, that would lead to correlation of failures.

In response to the above mentioned issues, the best-worst method (BWM) and hesitant fuzzy set (HFS) are combined to determine the β-factor to mitigate the influence of expert subjectivity in the β-factor model. Binary interval numbers are introduced to account for the uncertainty in failure parameters. Subsequently, the CTBN is constructed for dynamic logic gates, and the conditional probability density functions for the corresponding gate outputs are derived. The reliability evaluation of dynamic fault tree's top event can be achieved finally.

The main contribution of this paper is: a systematic reliability evaluation framework for fault tree with CCF and interval uncertainty parameters based on CTBN is proposed. Compared with DTBN-based reliability evaluation, the advantages of proposed framework are: the proposed framework does not need to discretize the time, enabling the calculation of failure probability at any time; the calculation time of the proposed framework is significantly shorter than that of DTBN-based framework. In addition, the influence of expert subjectivity in the β-factor model is properly reduced and the interval theory is introduced to deal with the uncertainty parameters.

The remainder of this paper is structured as follows: Section 2 presents the framework for reliability evaluation of dynamic fault tree based on CTBN. Section 3 provides two case studies of a CPU system and a brake signal transmission subsystem based on the proposed method. Section 4 summarizes the conclusions of this research.

2. Framework for reliability evaluation of dynamic fault tree

The flowchart of proposed framework is depicted in Fig. 1. Initially, BWM is used to determine the weight of each expert. Subsequently, the β -factor is obtained by fusing the experts' evaluations. After that, the fault tree and the corresponding CTBN model will be obtained. Finally, the cumulative distribution function of top event can be derivated obtained according to CTBN' rules.

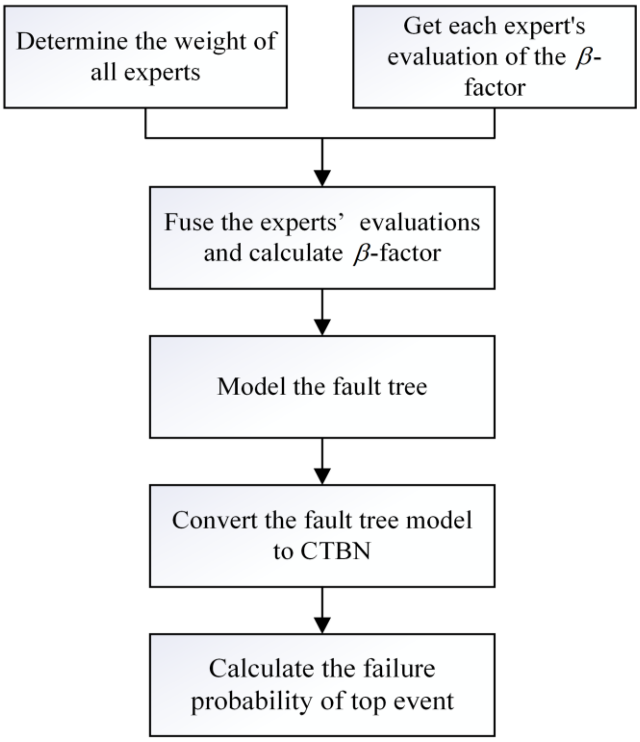


Fig. 1. Flowchart of proposed framework.

2.1. β -factor model based on BWM and HFS

2.1.1. β -factor model

The β -factor model divides the failure of basic event into two independent parts: an individual failure and a failure caused by CCF [29]. The failure probability of the basic event can be divided as follows,

$$P_A^{\mathrm{tot}} = P_A + P_{CC} \tag{1}$$

where P_A^{tot} is the total failure probability of event A, P_A is the individual failure probability, and P_{CC} is the failure probability caused by CCF.

The β -factor $\beta_A = \frac{P_{CC}}{P_A^{\mathrm{tot}}}$, means the proportion of the failure probability caused by CCF to the total failure probability.

Usually, β -factors are determined by experts. Therefore, β -factors are susceptible to experts' subjectivity, resulting in biased reliability evaluations. In order to obtain more objective and comprehensive reliability evaluations, group decision-making is used instead of individual decision-making. In group decision-making, it is necessary to obtain the weight of each expert to integrate all experts' perspectives on the decision issue. Best-worst method (BWM) is introduced to determine the weight of each expert. Simultaneously, experts may hesitate during evaluations. Hesitant fuzzy set (HFS) is introduced to prevent information loss caused by single-value evaluations.

2.1.2. Best-worst method

BWM is a pairwise comparison-based method for multi-objective decision-making problems [30]. Compared with analytic hierarchy process (AHP), BWM can effectively reduce the number of comparisons. When there are n criteria, AHP requires $\frac{n(n-1)}{2}$ comparisons, whereas BWM requires $2n-3$ comparisons (i.e. all criteria compared with the best criterion and worst criterion respectively) [30].

The process of BWM is as follows:

(1) Determine the best criterion c_B and the worst criterion c_W from the decision criteria $\boldsymbol{c} = \{c_1, c_2, \dots c_n\}$ that affect expert weights.

(2) Determine the preferences of the best criterion over all the other criteria $\boldsymbol{A}_B^c = \{A_{B1}^c, A_{B2}^c, \dots, A_{Bn}^c\}$, where A_{Bi}^c is a number among 1 to 9, and $A_{BB}^c = 1$.

(3) Determine the preferences of all the other criteria over the worst criterion $\boldsymbol{A}_W^c = \{A_{1W}^c, A_{2W}^c, \dots, A_{nW}^c\}$, where A_{jW}^c is a number from 1 to 9, and $A_{WW}^c = 1$.

(4) Obtain the optimal weight of each criterion.

The solution should be found to minimize the maximum absolute difference between $|\frac{\omega_B^c}{\omega_i^c} - A_{Bi}^c|$ and $|\frac{\omega_i^c}{\omega_W^c} - A_{iW}^c|$. The corresponding linear optimization model is

$$min \ \ \xi, s.t. \begin{cases} |\omega_B^c - A_{Bi}^c \cdot \omega_i^c| \le \xi \\ |\omega_i^c - A_{iW}^c \cdot \omega_W^c| \le \xi \\ \sum_{i=1}^n \omega_i^c = 1 \\ \omega_i^c \ge 0, i = 1,2,\dots,n \end{cases} \tag{2}$$

Solving model (2), the optimal weights $\{\omega_1^c, \omega_2^c, \dots, \omega_n^c\}$ and optimal solution ξ^* are obtained. And then it is necessary to do the consistency test of the preferences $\boldsymbol{A}_B^c$ and $\boldsymbol{A}_W^c$. The consistency ratio (CR) is proposed,

$$CR = \frac{\xi^*}{CI} \tag{3}$$

where CI is the consistency index, which can be obtained according to the preference of the best criterion over the worst criterion A_{BW}^c, as shown in Table 1. If $CR < 0.1$, it is considered that the preferences meet the consistency requirement, thus the obtained optimal weights are reliable.

Table 1. Consistency index (CI) table.

A_{BW}^c	1	2	3	4	5	6	7	8	9
CI	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

(5) Determine the best expert e_B and the worst expert e_W among all experts $\boldsymbol{e} = \{e_1, e_2, \dots, e_m\}$.

(6) Determine the preferences of the best expert over all the other experts $\boldsymbol{A}_B^e = \{A_{B1}^e, A_{B2}^e, \dots, A_{Bm}^e\}$, where A_{Bi}^e is a number from 1 to 9, and $A_{BB}^e = 1$.

(7) Determine the preferences of the preference of all the other experts over the worst expert $\boldsymbol{A}_W^e = \{A_{1W}^e, A_{2W}^e, \dots, A_{mW}^e\}$, where A_{jW}^e is a number among 1 to 9, and $A_{WW}^e = 1$.

(8) Obtain the weight of each expert.

An optimization model similar to equation (1) is used to find the optimal weights of each expert $\{\omega_1^e, \omega_2^e, \dots, \omega_m^e\}$.

2.1.3. Hesitant fuzzy set

Sometimes, experts might hesitate while evaluating the β -factor, hesitant fuzzy set might be employed to describe the hesitancy. If the jth expert $e_j (j = 1,2, \dots, m)$ hesitates to evaluate the l_j possible values of β -factor, then his evaluations can be represented by hesitant fuzzy element $\boldsymbol{h}_j = \{h_1(x), h_2(x), \dots, h_{l_j}(x)\}$.

Hesitant fuzzy weighted averaging (HFWA) operator [31] is used to fuse the evaluations of all experts.

$$HFWA(h_1, h_2, \dots, h_m) = \bigoplus_{j=1}^{m} (\omega_j^e \boldsymbol{h}_j) \tag{4}$$

where $\oplus$ represents additive operation among hesitant fuzzy elements [31]. Further, the average of $HWFA(h_1, h_2, \dots, h_m)$ is used to obtain the β -factor,

$$\beta = \frac{\mathrm{sum}(HFWA(h_1, h_2, \dots, h_m))}{\mathrm{length}(HFWA(h_1, h_2, \dots, h_m))} \tag{5}$$

where $sum(HFWA)$ and length $(HFWA)$ represent the summation and length of $HWFA$ respectively.

2.2. Interval theory

In fault tree analysis, it is often assumed that the failure distribution types and parameters of basic events are known. However, small failure data poses a challenge in acquiring precise failure parameters in practice. Interval number is commonly used to describe parameter uncertainty due to its ability to utilise minimal data information.

Interval number $\tilde{\lambda}_A = [\lambda_A^-, \lambda_A^+]$ represents the failure rate of the basic event A. In which, $\tilde{\lambda}_A$ means the failure rate of A, λ_A^+ and λ_A^- represent the upper and lower bounds of failure rate respectively. If $\lambda_A^+ = \lambda_A^-$, $\tilde{\lambda}_A$ will degrade into a deterministic failure rate.

Moreover, when making a comparison between two interval numbers, the possibility degree is frequently used. Assuming $\tilde{A} = [A^-, A^+]$, $\tilde{B} = [B^-, B^+]$, then the possibility degree $P(\tilde{A} \le \tilde{B})$ is

$$P(\tilde{A} \le \tilde{B}) = \begin{cases} 0, & A^- \ge B^+ \\ 0.5 \cdot \frac{B^+ - A^-}{A^+ - A^-} \cdot \frac{B^+ - A^-}{B^+ - B^-}, & B^- \le A^- < B^+ \le A^+ \\ \frac{0.5B^- - A^- + 0.5B^+}{A^+ - A^-}, & A^- < B^- < B^+ \le A^+ \\ \frac{B^- - A^-}{A^+ - A^-} + \frac{A^+ - B^-}{A^+ - A^-} \cdot \frac{B^+ - 0.5A^+ - 0.5B^-}{B^+ - B^-}, & A^- < B^- \le A^+ < B^+ \\ \frac{B^+ - 0.5A^+ - 0.5A^-}{B^+ - B^-}, & B^- \le A^- < A^+ < B^+ \\ 1, & A^+ < B^- \end{cases} \tag{6}$$

There are six position relationships between $\tilde{A}$ and $\tilde{B}$, shown in Fig. 2. It was considered that $\tilde{A}$ is superior to $\tilde{B}$ when $P(\tilde{A} \le \tilde{B}) < 0.5$, denoted by $\tilde{A} \succ \tilde{B}$; it was considered that $\tilde{A}$ is indifferent to $\tilde{B}$ when $P(\tilde{A} \le \tilde{B}) = 0.5$, denoted by $\tilde{A} \sim \tilde{B}$; it was considered that $\tilde{A}$ is inferior to $\tilde{B}$ when $P(\tilde{A} \le \tilde{B}) > 0.5$, denoted by $\tilde{A} \prec \tilde{B}$ [33,34].

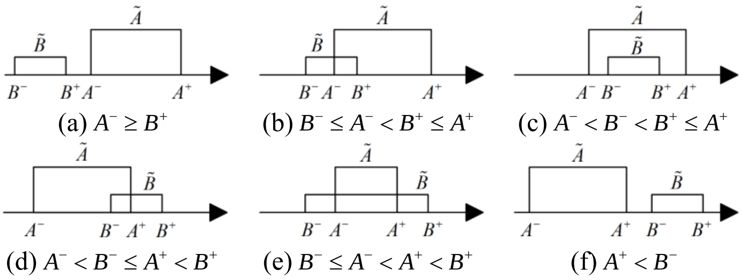


Fig. 2. Six position relationships between interval numbers.

One issue with interval number operations is that interval extension occurs when an interval number is encountered multiple times during the operation [35]. For example, $\tilde{A} = [0,1]$,$\tilde{B} = \tilde{A}e^{-\tilde{A}}$. According to the rules of interval operation, $\tilde{B} = [0,1] \times \left[\frac{1}{e}, 1\right] = [0,1]$. However, when $\tilde{A} = [0,1]$, the derivative $\frac{\partial \tilde{A}e^{-\tilde{A}}}{\partial \tilde{A}}$ is nonnegative, which means $\tilde{B} = [0, \frac{1}{e}] \subset [0,1]$. To solve this problem, it is necessary to simplify the calculation as much as possible. If interval numbers occur multiple times in the calculation, the calculation can be transformed into an optimization problem.

2.3. Bayesian network for fault tree with CCF

The CTBN can be used to analyse the reliability of dynamic fault trees. The unit step function is used to describe the failure sequence of logic gate's input events. And the impulse function is used to describe the conditional probability density function of logic gate's output.

Take a simple Bayesian network, shown in Fig. 3, as an example. t_A and t_B are the failure times of *A* and *B* respectively. According to the characteristics of the unit step function $u(x)$, $u(t_A - t_B)$ indicates that event *A* fails earlier than *B*. Similarly, $u(t_B - t_A)$ means that *A* fails later than *B*. For the output, according to the sieving property of the pulse function δ, $p\delta(t - \tau)$ indicates that the probability of *C* failing at time τ is p.

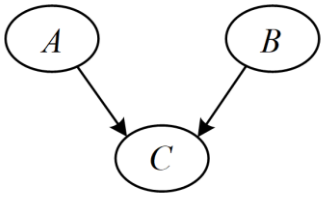
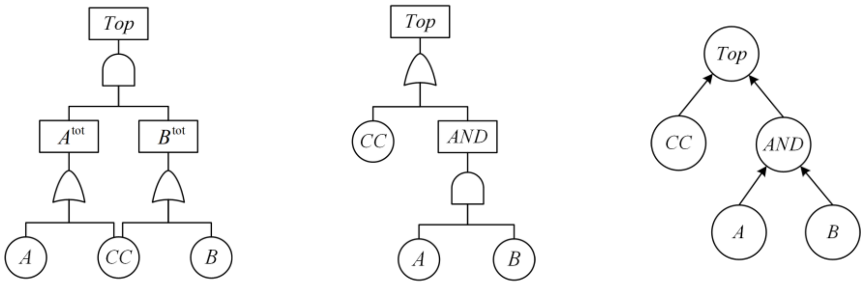


Fig. 3. A simple Bayesian network.

For the fault tree with CCF, it needs to be transformed into Bayesian network, and then the conditional probability density function can be derived. The conditional probability density functions of AND gate, OR gate, spare gates, and priority AND gate with CCF are derived as follows.

2.3.1. AND gate fault tree with CCF.

For AND gate fault tree with CCF, the corresponding BN is shown in Fig. 4.



(a)Fault tree (b) Equivalent fault tree (c)BN

Fig. 4. AND gate fault tree with CCF and the corresponding BN.

The failure of basic event can be divided into two parts: an individual failure and a failure caused by CCF. A^{tot} is divided into *A* and *CC*, B^{tot} is divided into *B* and *CC*. If *CC* fails or both events *A* and *B* fail, the top event will fail. In Fig. 4 (b) and (c), *AND* denotes the top event failure resulting from individual failure of *A* and *B*.

Assuming that failures follow exponential distribution, the probability density function and cumulative distribution function of *A*, *B* and *CC* are as follows,

$$\tilde{f}_A(t) = \tilde{\lambda}_A e^{-\tilde{\lambda}_A t}, \tilde{F}_A(t) = 1 - e^{-\tilde{\lambda}_A t}, (t > 0) \tag{7}$$

$$\tilde{f}_B(t) = \tilde{\lambda}_B e^{-\tilde{\lambda}_B t}, \tilde{F}_B(t) = 1 - e^{-\tilde{\lambda}_B t}, (t > 0) \tag{8}$$

$$\tilde{f}_{CC}(t) = \tilde{\lambda}_{CC} e^{-\tilde{\lambda}_{CC} t}, \tilde{F}_{CC}(t) = 1 - e^{-\tilde{\lambda}_{CC} t}, (t > 0) \tag{9}$$

where $\tilde{\lambda}_A$, $\tilde{\lambda}_B$, $\tilde{\lambda}_{CC}$ represent the failure rates of *A*, *B*, *CC* respectively, and they are expressed by interval numbers. If $\tilde{\lambda}_A^{\text{tot}}$, $\tilde{\lambda}_B^{\text{tot}}$, and β_A (or β_B) are known, $\tilde{\lambda}_A$, $\tilde{\lambda}_B$and $\tilde{\lambda}_{CC}$ will be obtained. Given the value of β_A, the calculation proceeds as follows,

$$\tilde{\lambda}_A = (1 - \beta_A)\tilde{\lambda}_A^{\text{tot}} \tag{10}$$

$$\tilde{\lambda}_{CC} = \beta_A \tilde{\lambda}_A^{\text{tot}} \tag{11}$$

$$\tilde{\lambda}_B = \tilde{\lambda}_B^{\text{tot}} - \tilde{\lambda}_{CC} \tag{12}$$

Assume that *A* and *B* fail at time t_A, t_B respectively. According to the principle of AND gate, the conditional probability density function of AND gate's output is

$$f_{AND|A,B}(t|t_A, t_B) = u(t_B - t_A)\delta(t - t_B) + u(t_A - t_B)\delta(t - t_A) \tag{13}$$

Thus, the joint density function is

$$\tilde{f}_{AND,A,B}(t, t_A, t_B) = f_{AND|A,B}(t|t_A, t_B)\tilde{f}_A(t_A)\tilde{f}_B(t_B) \tag{14}$$

Integrate the joint density function to get the marginal probability density function of *AND*,

$$\tilde{f}_{AND}(t) = \int_0^\infty \int_0^\infty [u(t_B - t_A)\delta(t - t_B) + u(t_A - t_B)\delta(t - t_A)]\tilde{f}_A(t_A)\tilde{f}_B(t_B)\mathrm{d}t_A\mathrm{d}t_B$$
$$= \int_0^\infty \delta(t - t_B)\tilde{f}_B(t_B)\int_0^{t_B} \tilde{f}_A(t_A)\,\mathrm{d}t_A\mathrm{d}t_B + \int_0^\infty \delta(t - t_A)\tilde{f}_A(t_A)\int_0^{t_A} \tilde{f}_B(t_B)\,\mathrm{d}t_B\mathrm{d}t_A$$
$$= \tilde{f}_B(t)\tilde{F}_A(t) + \tilde{f}_A(t)\tilde{F}_B(t) = [\tilde{F}_A(t)\tilde{F}_B(t)]' \tag{15}$$

If either *AND* or *CC* fails, the top event will fail. Therefore, the conditional probability density function of top event is

$$f_{Top|AND,CC}(t|t_{AND}, t_{CC}) = u(t_{AND} - t_{CC})\delta(t - t_{CC}) + u(t_{CC} - t_{AND})\delta(t - t_{AND}) \tag{16}$$

Thus, the joint density function is

$$\tilde{f}_{Top,AND,CC}(t, t_{AND}, t_{CC}) = f_{Top|AND,CC}(t|t_{AND}, t_{CC})\,\tilde{f}_{AND}(t_{AND})\tilde{f}_{CC}(t_{CC}) \tag{17}$$

By integrating the joint density function, the marginal probability density function of top event is

$$\tilde{f}_{Top}(t) = \int_0^\infty \int_0^\infty [u(t_{AND} - t_{CC})\delta(t - t_{CC}) + u(t_{CC} - t_{AND})\delta(t - t_{AND})]\,\tilde{f}_{AND}(t_{AND})\tilde{f}_{CC}(t_{CC})\mathrm{d}t_{AND}\mathrm{d}t_{CC}$$
$$= \tilde{f}_{CC}(t)\left(1 - \tilde{F}_{AND}(t)\right) + \tilde{f}_{AND}(t)\left(1 - \tilde{F}_{CC}(t)\right)$$
$$= \tilde{f}_{AND}(t) + \tilde{f}_{CC}(t) - [\tilde{F}_{AND}(t)\tilde{F}_{CC}(t)]' \tag{18}$$

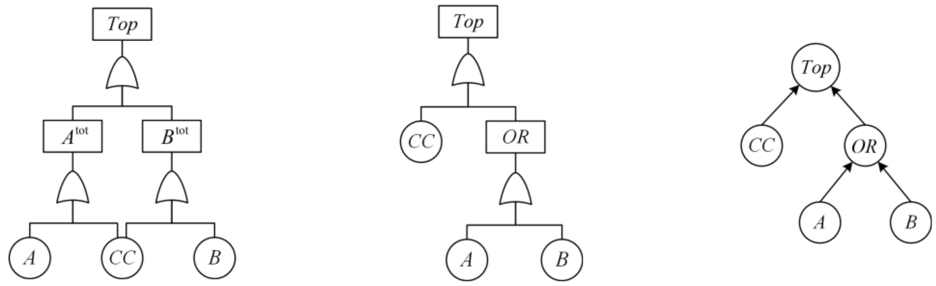
Integrate Eq. (18) to get the cumulative distribution function of top event as

$$\tilde{F}_{Top}(t) = P(\tau < t) = \int_0^t \tilde{f}_{Top}(\tau)\,\mathrm{d}\tau = \tilde{F}_A(t)\tilde{F}_B(t) + \tilde{F}_{CC}(t) - \tilde{F}_A(t)\tilde{F}_B(t)\tilde{F}_{CC}(t) \tag{19}$$

Eq. (19) enables the calculation of failure probability of top event of AND gate fault tree with CCF at any given time.

2.3.1. OR gate fault tree with CCF

The OR gate fault tree with CCF and the corresponding BN are shown in Fig. 5, where *OR* in Fig. 5(b) and (c) denotes the top event failure resulting from individual failure of *A* and *B*.



(a) Fault tree (b) Equivalent fault tree (c) BN

Fig. 5. OR gate fault tree with CCF and the corresponding BN.

The probability density functions of *A*, *B*, and *CC* are the same as those in Section 2.3.1. The marginal probability density function of *OR* is

$$\tilde{f}_{OR}(t) = \tilde{f}_A(t) + \tilde{f}_B(t) - [\tilde{F}_A(t)\tilde{F}_B(t)]' \tag{20}$$

If either the *OR* or *CC* fails, the top event will fail. Therefore, the marginal probability density function of top event is

$$\tilde{f}_{Top}(t) = \tilde{f}_{Or}(t) + \tilde{f}_{CC}(t) - [\tilde{F}_{Or}(t)\tilde{F}_{CC}(t)]' \tag{21}$$

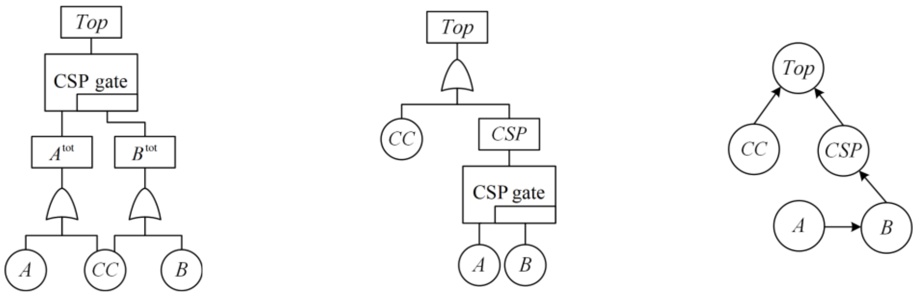
Integrate Eq. (21) to get the cumulative distribution function of top event as

$$\tilde{F}_{Top}(t) = P(\tau < t) = \int_0^t \tilde{f}_{Top}(\tau)\,\mathrm{d}\tau = [\tilde{F}_A(t) + \tilde{F}_B(t) - \tilde{F}_A(t)\tilde{F}_B(t)]\left(1 - \tilde{F}_{CC}(t)\right) + \tilde{F}_{CC}(t) \tag{22}$$

Eq. (22) enables the calculation of the failure probability of top event of OR gate fault tree with CCF at any given time.

2.3.3 Cold spare (CSP) gate fault tree with CCF

The CSP gate fault tree with CCF and the corresponding BN are shown in Fig. 6, where *OR* in Fig. 6 (b) and (c) denotes the top event failure resulting from individual failure of *A* and *B*.



(a) Fault tree (b) Equivalent fault tree (c) BN

Fig. 6. CSP gate fault tree with CCF and the corresponding BN.

The probability density functions of *A*, *B*, and *CC* are the same as those in Section 2.3.1. According to the principle of CSP gate, *B* will only work after *A* fails. Therefore, the conditional failure rate of *B* in the CSP gate is

$$\tilde{\lambda}_{B\_CSP|A}(t|t_A) = u(t - t_A)\tilde{\lambda}_B \tag{23}$$

Hence, the conditional probability density function of *B* is

$$\begin{aligned}\tilde{f}_{B\_CSP|A}(t|t_A) &= \tilde{\lambda}_{B\_CSP|A}(t|t_A)e^{-\int_0^t \tilde{\lambda}_{B\_CSP|A}(\tau|t_A)\mathrm{d}\tau} \\ &= u(t - t_A)\tilde{f}_B(t - t_A)\end{aligned} \tag{24}$$

Integrate Eq. (24) to get the marginal probability density function of *B* in CSP gate as

$$\begin{aligned}\tilde{f}_{B_{CSP}}(t) &= \int_0^\infty \tilde{f}_{B\_CSP|A}(t|t_A)\tilde{f}_A(t_A)\,\mathrm{d}t_A \\ &= \int_0^t \tilde{\lambda}_B e^{-\tilde{\lambda}_B(t-t_A)}\lambda_A e^{-\tilde{\lambda}_A t_A}\,\mathrm{d}t_A \\ &= \begin{cases}\dfrac{\tilde{\lambda}_A\tilde{\lambda}_B}{\tilde{\lambda}_B - \tilde{\lambda}_A}(e^{-\tilde{\lambda}_A t} - e^{-\tilde{\lambda}_B t}), \tilde{\lambda}_A \neq \tilde{\lambda}_B \\ \tilde{\lambda}_A{}^2 e^{-\tilde{\lambda}_A t}t, \qquad \tilde{\lambda}_A = \tilde{\lambda}_B\end{cases}\end{aligned} \tag{25}$$

Integrate Eq. (25) to get the cumulative distribution function of *B* in CSP gate as

$$\tilde{F}_{B\_CSP}(t) = \begin{cases}1 - \frac{\tilde{\lambda}_A e^{-\tilde{\lambda}_B t} - \tilde{\lambda}_B e^{-\tilde{\lambda}_A t}}{\tilde{\lambda}_A - \tilde{\lambda}_B}, & \tilde{\lambda}_A \neq \tilde{\lambda}_B \\ 1 - e^{-\tilde{\lambda}_A t} - \tilde{\lambda}_A t e^{-\tilde{\lambda}_A t}, \tilde{\lambda}_A = \tilde{\lambda}_B\end{cases} \tag{26}$$

If *B* fails, then *CSP* will fail immediately. Therefore, the conditional probability density function of *CSP* is

$$f_{CSP|B\_CSP}(t|t_B) = \delta(t - t_B) \tag{27}$$

Thus, the joint density function is

$$\tilde{f}_{CSP,B\_CSP}(t, t_B) = f_{CSP|B\_CSP}(t|t_B)\tilde{f}_{B\_CSP}(t_B) \tag{28}$$

By integrating the joint density function, the marginal probability density function of *CSP* is

$$\begin{aligned}\tilde{f}_{CSP}(t) &= \int_0^\infty f_{CSP|B\_CSP}(t|t_B)\tilde{f}_{B\_CSP}(t_B)\,\mathrm{d}t_B \\ &= \int_0^\infty \delta(t - t_B)\tilde{f}_{B\_CSP}(t_B)\,\mathrm{d}t_B = \tilde{f}_{B\_CSP}(t)\end{aligned} \tag{29}$$

So it is clear that $\tilde{F}_{CSP}(t) = \tilde{F}_{B\_CSP}(t)$

If either *CSP* or *CC* fails, the top event will fail. Therefore, the marginal probability density function of top event is

$$\tilde{f}_{Top}(t) = \tilde{f}_{CSP}(t) + \tilde{f}_{CC}(t) - [\tilde{F}_{CSP}(t)\tilde{F}_{CC}(t)]' \tag{30}$$

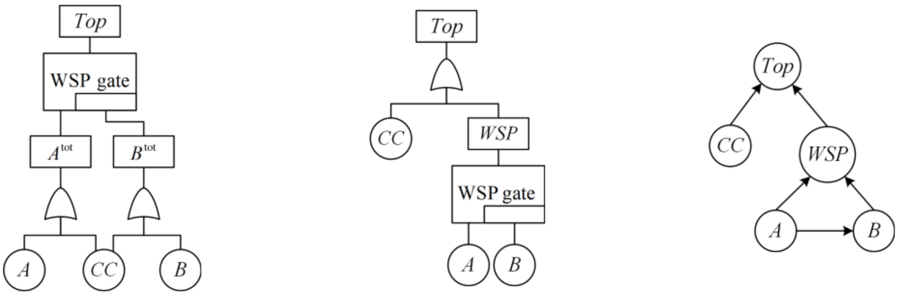
Integrate Eq. (30) to get the cumulative distribution function of top event as

$$\begin{aligned}\tilde{F}_{Top}(t) &= P(\tau < t) = \int_0^t \tilde{f}_T(\tau)\,\mathrm{d}\tau \\ &= \begin{cases}1 - \dfrac{\tilde{\lambda}_A e^{-\tilde{\lambda}_B t} - \tilde{\lambda}_B e^{-\tilde{\lambda}_A t}}{\tilde{\lambda}_A - \tilde{\lambda}_B} e^{-\tilde{\lambda}_{CC} t}, \tilde{\lambda}_A \neq \tilde{\lambda}_B \\ 1 - (1 + \tilde{\lambda}_A t)e^{-(\tilde{\lambda}_A + \tilde{\lambda}_{CC})t}, \qquad \tilde{\lambda}_A = \tilde{\lambda}_B\end{cases}\end{aligned} \tag{31}$$

Eq. (31) enables the calculation of the failure probability of top event of CSP gate fault tree with CCF at any given time.

2.3.4. Warm spare (WSP) gate fault tree with CCF

The WSP gate fault tree with CCF and the corresponding BN are shown in Fig. 7, where *WSP* in Fig. 7 (b) and (c) denotes the top event failure resulting from individual failure of *A* and *B*.



(a) Fault tree (b) Equivalent fault tree (c) BN

Fig. 7. WSP gate fault tree with CCF and the corresponding BN.

The probability density functions of A, B, and CC are the same as those in Section 2.3.1. According to the principle of WSP gate, when A is working, the failure rate of B is α times the failure rate of normal operation. Hot spare (HSP) gates can be considered as special WSP, where the value of α is 1. when A fails, the failure rate of B is the failure rate of normal operation. Therefore, the conditional failure rate of B in the WSP gate is

$$\tilde{\lambda}_{B\_WSP|A}(t|t_A) = u(t_A - t)\alpha\tilde{\lambda}_B + u(t - t_A)\tilde{\lambda}_B \quad (32)$$

Hence, the conditional probability density function of B is

$$\begin{aligned}\tilde{f}_{B\_WSP|A}(t|t_A) &= \tilde{\lambda}_{B\_WSP|A}(t|t_A)e^{-\int_0^t \tilde{\lambda}_{B\_WSP|A}(\tau|t_A)\mathrm{d}\tau} \\ &= u(t_A - t)\alpha\tilde{f}_B(t)[1 - F_B(t)]^{\alpha-1} \\ &\quad + u(t - t_A)\tilde{f}_B(t)[1 - F_B(t_A)]^{\alpha-1}\end{aligned} \quad (33)$$

According to the principle of WSP gate, if both A and B fail, WSP will fail, so the conditional probability density function of WSP is

$$\begin{aligned}f_{WSP|A,B\_WSP}(t|t_A, t_B) &= u(t_A - t_B)\delta(t - t_A) \\ &\quad + u(t_B - t_A)\delta(t - t_B)\end{aligned} \quad (34)$$

Thus, the joint density function is

$$\begin{aligned}\tilde{f}_{WSP,A,B\_WSP}(t, t_A, t_B) &= f_{WSP|A,B\_WSP}(t|t_A, t_B) \\ &\quad \tilde{f}_{B\_WSP|A}(t_B|t_A)\tilde{f}_A(t_A)\end{aligned} \quad (35)$$

By integrating the joint density function, the marginal probability density function of WSP is

$$\tilde{f}_{WSP}(t) = \int_0^\infty \int_0^\infty f_{WSP|A,B\_WSP}(t|t_A, t_B)\tilde{f}_{B\_WSP|A}(t_B|t_A)\tilde{f}_A(t_A)\mathrm{d}t_A\mathrm{d}t_B$$
$$= \begin{cases} (1 - e^{-\tilde{\lambda}_B\alpha t})\alpha\tilde{\lambda}_A e^{-\tilde{\lambda}_A t} + \dfrac{(1 - e^{-[\tilde{\lambda}_A + (\alpha-1)\tilde{\lambda}_B]t})\tilde{\lambda}_A\tilde{\lambda}_B e^{-\tilde{\lambda}_B t}}{\tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B}, & \tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B \neq 0 \\ (1 - e^{-\tilde{\lambda}_B\alpha t})\alpha\tilde{\lambda}_A e^{-\tilde{\lambda}_A t} + \tilde{\lambda}_A\tilde{\lambda}_B t e^{-\tilde{\lambda}_B t}, & \tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B = 0 \end{cases} \quad (36)$$

Integrate Eq. (36) to get the cumulative distribution function of WSP as

$$\tilde{F}_{WSP}(t) =$$
$$\begin{cases} \alpha(1 - e^{-\tilde{\lambda}_A t}) + (e^{-(\tilde{\lambda}_A + \alpha\tilde{\lambda}_B)t} - 1)(\dfrac{\alpha\tilde{\lambda}_A}{\tilde{\lambda}_A + \alpha\tilde{\lambda}_B} + \dfrac{\tilde{\lambda}_A\tilde{\lambda}_B}{(\tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B)(\tilde{\lambda}_A + \alpha\tilde{\lambda}_B)}) + \dfrac{\tilde{\lambda}_A(1 - e^{-\tilde{\lambda}_B t})}{\tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B}, & \tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B \neq 0 \\ \alpha(1 - e^{-\tilde{\lambda}_A t}) + (e^{-(\tilde{\lambda}_A + \alpha\tilde{\lambda}_B)t} - 1)\dfrac{\alpha\tilde{\lambda}_A}{\tilde{\lambda}_A + \alpha\tilde{\lambda}_B} + \dfrac{\tilde{\lambda}_A}{\tilde{\lambda}_B} - \dfrac{\tilde{\lambda}_A}{\tilde{\lambda}_B}e^{-\tilde{\lambda}_B t} - \tilde{\lambda}_A t e^{-\tilde{\lambda}_B t}, & \tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B = 0 \end{cases} \quad (37)$$

If either WSP or CC fails, the top event will fail. Therefore, the marginal probability density function of top event is

$$\tilde{f}_{Top}(t) = \tilde{f}_{WSP}(t) + \tilde{f}_{CC}(t) - [\tilde{F}_{WSP}(t)\tilde{F}_{CC}(t)]' \quad (38)$$

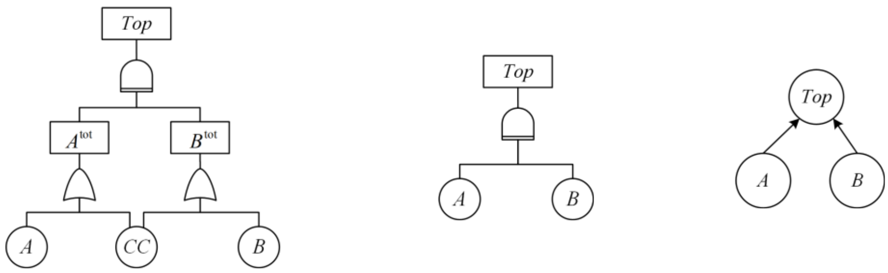
Integrate Eq. (38) to get the cumulative distribution function of top event as

$$\tilde{F}_{Top}(t) = P(\tau < t) = \int_0^t \tilde{f}_{Top}(\tau)\,d\tau$$
$$= \begin{cases} 1 + e^{-\tilde{\lambda}_{CC}t}[\alpha(1 - e^{-\tilde{\lambda}_A t}) + (e^{-(\tilde{\lambda}_A + \alpha\tilde{\lambda}_B)t} - 1)(\dfrac{\alpha\tilde{\lambda}_A}{\tilde{\lambda}_A + \alpha\tilde{\lambda}_B} + \dfrac{\tilde{\lambda}_A\tilde{\lambda}_B}{(\tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B)(\tilde{\lambda}_A + \alpha\tilde{\lambda}_B)}) + \dfrac{\tilde{\lambda}_A(1 - e^{-\tilde{\lambda}_B t})}{\tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B} - 1], & \tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B \neq 0 \\ 1 + e^{-\tilde{\lambda}_{CC}t}[\alpha(1 - e^{-\tilde{\lambda}_A t}) + (e^{-(\tilde{\lambda}_A + \alpha\tilde{\lambda}_B)t} - 1)\dfrac{\alpha\tilde{\lambda}_A}{\tilde{\lambda}_A + \alpha\tilde{\lambda}_B} + \dfrac{\tilde{\lambda}_A}{\tilde{\lambda}_B} - \dfrac{\tilde{\lambda}_A}{\tilde{\lambda}_B}e^{-\tilde{\lambda}_B t} - \tilde{\lambda}_A t e^{-\tilde{\lambda}_B t} - 1], & \tilde{\lambda}_A + (\alpha - 1)\tilde{\lambda}_B = 0 \end{cases} \quad (39)$$

Eq. (39) enables the calculation of the failure probability of top event of CSP gate fault tree with CCF at any given time.

2.3.5. Priority AND (PAND) gate fault tree with CCF

The PAND gate fault tree with CCF and the corresponding BN are shown in Fig. 8.



(a) Fault tree (b) Equivalent fault tree (c) BN

Fig. 8. PAND gate fault tree with CCF and the corresponding BN.

The probability density functions of A, B, and CC are the same as those in Section 2.3.1. According to the principle of PAND gate, top event will fail if and only if A fails earlier than B. CCF will cause A^{tot} and B^{tot} simultaneously failure and the top event will not fail, so the conditional probability density function of top event is

$$f_{Top|A,B}(t|t_A,t_B) = u(t_B - t_A)\delta(t - t_B) + u(t_A - t_B)\delta(t - \infty) \quad (40)$$

Thus, the joint density function is

$$\tilde{f}_{Top,A,B}(t,t_A,t_B) = f_{Top|A,B}(t|t_A,t_B)\tilde{f}_A(t_A)\tilde{f}_B(t_B) \quad (41)$$

By integrating the joint density function, the marginal probability density function of top event is

$$\tilde{f}_{Top}(t) = \tilde{F}_A(t)\tilde{f}_B(t) \quad (42)$$

Integrate Eq. (42) to get the cumulative distribution function of top event as

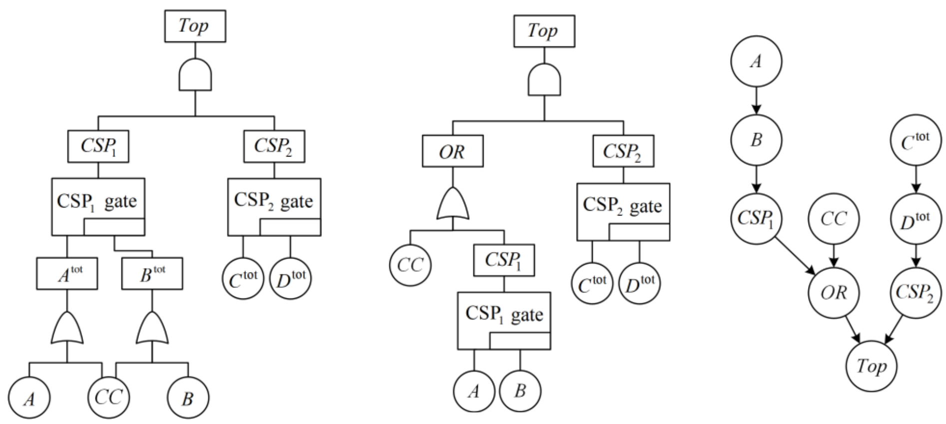
$$\tilde{F}_{Top}(t) = \int_0^t \tilde{f}_{Top}(\tau)\mathrm{d}\tau = \frac{\tilde{\lambda}_A - (\tilde{\lambda}_A + \tilde{\lambda}_B)e^{-\tilde{\lambda}_B t} + \lambda_B e^{-(\tilde{\lambda}_A + \tilde{\lambda}_B)t}}{\tilde{\lambda}_A + \tilde{\lambda}_B} \quad (43)$$

The Eq. (43) enables the calculation of the failure probability of top event of PAND gate fault tree with CCF at any given time.

3. Dynamic fault tree case

Case 1:

This case is specific case studies adapted from the reference [16]. The digital safety-level distributed control system is the control mechanism for nuclear reactor devices, which plays an important role in ensuring the stable operation of nuclear power plants. As an important part of the distributed control system, the emergency shutdown system is generally designed to be multi-channel redundant to increase its reliability. The structure of the emergency shutdown system is a 2oo3 structure (two CPU systems out of three are required for the system to work). The CPU system is modelled to analyse reliability. A CPU system consists of two components A and C, and both A and C have a spare part, named B and D respectively. When A fails, B starts to work; when C fails, D starts to work. If A, B, C and D fail, the system will fail. A and B have a common cause CC. When CC fails, A and B fail immediately. According to failure statistic data, the total failure rate of A and B is set as $\tilde{\lambda}_A^{\text{tot}} = \tilde{\lambda}_B^{\text{tot}} = [5 \times 10^{-5}, 6 \times 10^{-5}]\text{h}^{-1}$, and the total failure rate of C and D is set as $\tilde{\lambda}_C^{\text{tot}} = \tilde{\lambda}_D^{\text{tot}} = [5 \times 10^{-5}, 5.2 \times 10^{-5}]\text{h}^{-1}$. The evaluations of β_A evaluated by three experts are: $h_1 = \{0.1, 0.15\}, h_2 = 0.1, h_3 = \{0.1, 0.2\}$ respectively. The CPU system fault tree and corresponding Bayesian network with CCF are shown in Fig. 9.



(a) Fault tree (b) Equivalent fault tree (c) BN

Fig. 9. CPU system fault tree with CCF and the corresponding BN.

The steps of dynamic FTA based on CTBN are as follows:

Step 1: Determine the weight of all criteria.

Assume that there are three criteria that affect the weights of experts: education level c_1, length of service c_2 and professional title c_3. The obtained optimal weights are shown in Table 2.

Table 2. The weights of the criteria table.

	c_1	c_2	c_3
A_B^c (c_3)	2	6	1
A_W^c (c_2)	3	1	6
ω^c	0.3	0.1	0.6

The result of consistency test:

$$CR = \frac{\xi^*}{CI} = \frac{0.0046}{3} = 0.01533 < 0.1 \quad (44)$$

Therefore, the preferences meet the consistency requirement and the obtained weights are reliable.

Step 2: Determine the weights of all experts.

Determine the best expert, the worst expert and preferences. The obtained optimal weights of three experts e_1, e_2, e_3 are shown in Table 3.

Table 3. The weights of experts table.

	e_1	e_2	e_3
A_B^e (e_1)	1	2	4
A_W^e (e_3)	4	2	1
ω^e	0.5715	0.2856	0.1429

The result of consistency test:

$$CR = \frac{\xi^*}{CI} = \frac{0.0044}{1.63} = 0.0027 < 0.1 \quad (45)$$

Therefore, the preferences meet the consistency requirement and the obtained weights are reliable.

Step 3: Fuse the experts' evaluation results and calculate β_A.

According to the information provided in Table 3, the weights of the three experts are $\boldsymbol{\omega}^e = \{0.5715, 0.2856, 0.1429\}$. The weighted average is calculated,

$$HFWA(h_1, h_2, h_3) = (0.1000, 0.1150, 0.1289, 0.1435) \quad (46)$$

The estimated $\beta_A = 0.12185$.

Step 4: Derive the cumulative distribution function of top event.

$\tilde{\lambda}_A$, $\tilde{\lambda}_B$, $\tilde{\lambda}_{CC}$ can be deduced from Eqs. (10)-(12), $\tilde{\lambda}_A = [4.39075 \times 10^{-5}, 5.2689 \times 10^{-5}]\mathrm{h}^{-1}$, $\tilde{\lambda}_B = [4.39075 \times 10^{-5}, 5.2689 \times 10^{-5}]\mathrm{h}^{-1}$, $\tilde{\lambda}_{CC} = [6.0925 \times 10^{-6}, 7.311 \times 10^{-6}]\mathrm{h}^{-1}$ respectively.

The cumulative distribution function of OR is obtained as

$$\tilde{F}_{OR}(t) = \tilde{F}_{CSP_1}(t) + \tilde{F}_{CC}(t) - \tilde{F}_{CSP_1}(t)\tilde{F}_{CC}(t) = 1 - (1 + \tilde{\lambda}_A t)e^{-(\tilde{\lambda}_A + \tilde{\lambda}_{CC})t} \quad (47)$$

The cumulative distribution function of CSP_2 is obtained as

$$\tilde{F}_{CSP_2}(t) = 1 - e^{-\tilde{\lambda}_C^{\mathrm{tot}} t} - \tilde{\lambda}_C^{\mathrm{tot}} t e^{-\tilde{\lambda}_C^{\mathrm{tot}} t} \quad (48)$$

Then the cumulative distribution function of top event is

$$\tilde{F}_{Top}(t) = [1 - (1 + \tilde{\lambda}_A t)e^{-(\tilde{\lambda}_A + \tilde{\lambda}_{CC})t}](1 - e^{-\tilde{\lambda}_C^{\mathrm{tot}} t} - \tilde{\lambda}_C^{\mathrm{tot}} t e^{-\tilde{\lambda}_C^{\mathrm{tot}} t}) \quad (49)$$

It can be found that $\tilde{\lambda}_A, \tilde{\lambda}_C^{\mathrm{tot}}$ occurs more than once, resulting in interval extension. Thinking $\tilde{\lambda}_A, \tilde{\lambda}_C^{\mathrm{tot}}, \tilde{\lambda}_{CC}$ are variables, the derivative $\frac{\partial \tilde{F}_{Top}(\tilde{\lambda}_A, \tilde{\lambda}_C^{\mathrm{tot}}, \tilde{\lambda}_{CC}, t)}{\partial \tilde{\lambda}_A}$, $\frac{\partial \tilde{F}_{Top}(\tilde{\lambda}_A, \tilde{\lambda}_C^{\mathrm{tot}}, \tilde{\lambda}_{CC}, t)}{\partial \tilde{\lambda}_{CC}}$, $\frac{\partial \tilde{F}_{Top}(\tilde{\lambda}_A, \tilde{\lambda}_C^{\mathrm{tot}}, \tilde{\lambda}_{CC}, t)}{\partial \tilde{\lambda}_C^{\mathrm{tot}}}$ are nonnegative. Therefore, the upper bound and the lower bound of $\tilde{F}_{Top}(t)$ are obtained when $\tilde{\lambda}_A, \tilde{\lambda}_C^{tot}, \tilde{\lambda}_{CC}$ take the upper bounds and the lower bounds respectively.

The failure probabilities of top event obtained based on proposed method and DTBN-based method are shown in Fig. 10, where the number of time intervals of DTBN-based method is set as 100.

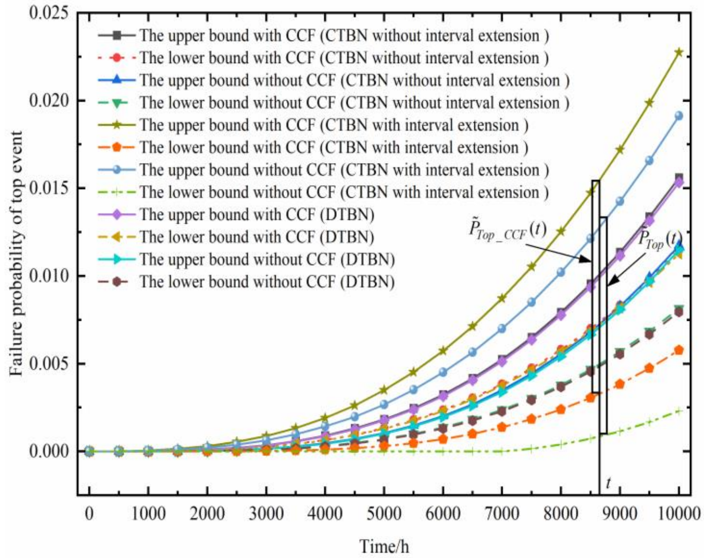


Fig. 10. Failure probability of top event.

At fixed time *t*, the failure probability of top event is an interval number. The probability $\tilde{P}_{Top\_CCF}$ of fault tree with CCF and $\tilde{P}_{Top}$ of fault tree without CCF are shown in Fig. 10. Comparing $\tilde{P}_{Top\_CCF}(t)$ and $\tilde{P}_{Top}(t)$, the possibility degrees are shown in Fig. 11.

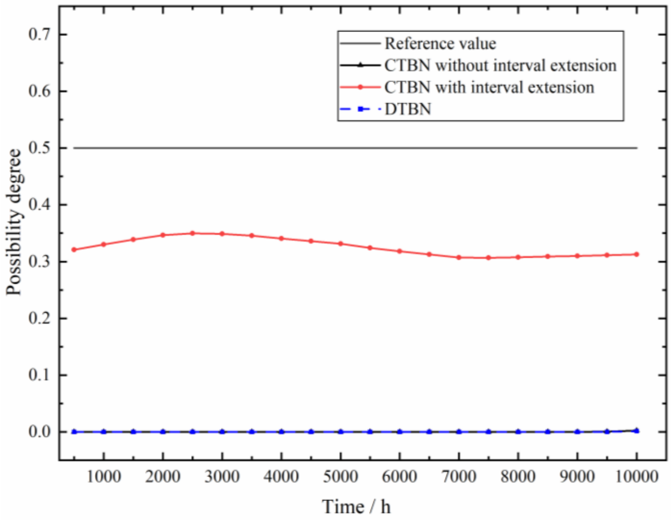


Fig. 11. The possibility degree of $\tilde{P}_{Top_{CCF}}(t) \leq \tilde{P}_{Top}(t)$.

The calculation times of the methods based on CTBN and DTBN is shown in Table 4. The simulation environment: AMD Ryzen 7 6800H CPU @ 3.20 GHz and NVIDIA GeForce RTX 3060 Laptop GPU.

Table 4. The calculation time of the methods.

Method	Calculation time (s)
CTBN-based	8.4690×10^{-5}
DTBN-based	0.6982

From Fig. 10, it can be found that the failure probability of the top event gradually rises in both scenarios with CCF and without CCF. In addition, it can be found that the failure probability of the top event is in a wider range when interval extensions are not be addressed. So, it is important to address the interval extensions to avoid getting the result in a wider range. It can be found that the possibility degree $P(\tilde{P}_{Top\_CCF}(t) \leq \tilde{P}_{Top}(t))$ is significantly lower than 0.5 at any time, shown from Fig. 11. That is, $\tilde{P}_{Top\_CCF} \succ \tilde{P}_{Top}$, which illustrates the necessity of considering CCF in reliability evaluation of dynamic fault tree. From Table 4, it can be found that the calculation time of CTBN-based method is significantly shorter than that of DTBN-based method.

Case 2:

The working principle of Electric Multiple Units brake control system is as follows: The electrical signal generated by the brake control device is transmitted to the brake control device of other vehicles through the information control device; The electronic brake control unit sends out the brake command, analyzes the braking force to be provided, and then transmits it to the brake control unit by electrical signal; After receiving the electrical signal from the electronic brake control unit, the brake control unit converts it into the corresponding pre-controlled pressure signal to the relay valve, and then uses the air braking force to complete the braking operation.

The brake control system consists of three subsystems: brake control subsystem, brake signal transmission subsystem and brake signal generation subsystem. The brake signal transmission subsystem is analyzed below. The brake signal transmission subsystem fault tree and corresponding Bayesian network with CCF are shown in Fig. 12. The failure rates of the basic events are from Reference [36]. The meanings and failure rates of the basic events and intermediate event are shown in Table 5. x_1 and x_2 have a common cause CC. The evaluations of β_{x_1} evaluated by four experts are: $h_1 = \{0.05, 0.08\}, h_2 = \{0.05, 0.1\}, h_3 = 0.1, h_4 = \{0.08, 0.1\}$ respectively.

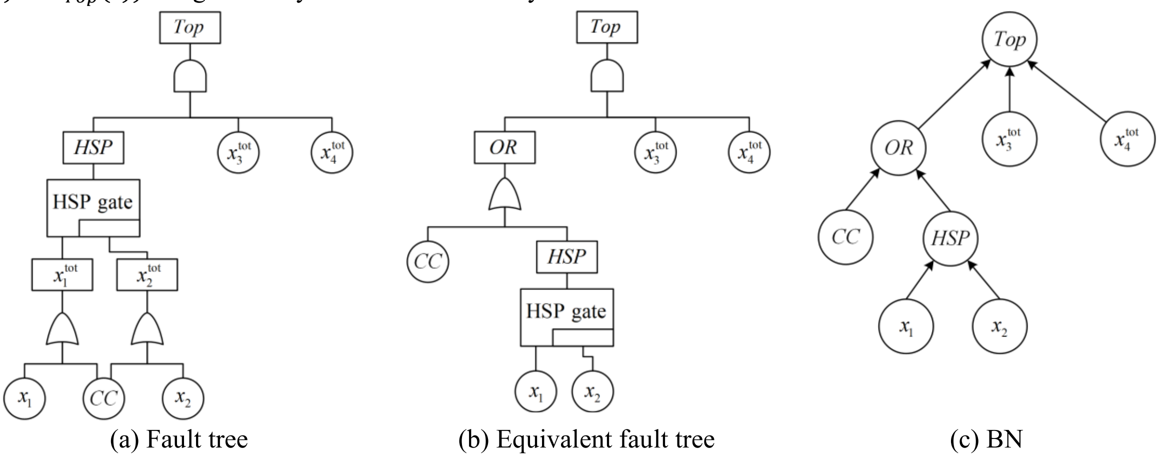


Fig. 12. Brake signal transmission subsystem fault tree with CCF and the corresponding BN.

Table 5. The meanings and failure rates of the basic events.

Event	Meaning	Failure rate ($10^{-6}\mathrm{h}^{-1}$)
x_1	Transmission fault of optical fiber	[2.187,2.673]
x_2	Transmission fault of standby optical fiber	[1.458,1.782]
x_3	Fault of the optical connector	[2.187,2.673]
x_4	Fault of the terminal interface board	[4.365,5.335]
HSP	Transmission fault	

The steps of dynamic FTA based on CTBN are as follows:

Step 1: Determine the weight of all criteria.

Assume that there are three criteria that affect the weights of experts: education level c_1, length of service c_2 and professional title c_3. The obtained optimal weights are shown in Table 6.

Table 6. The weights of the criteria table.

	c_1	c_2	c_3
A_B^c (c_3)	2	6	1
A_W^c (c_2)	3	1	6
ω^c	0.3	0.1	0.6

The result of consistency test:

$$CR = \frac{\xi^*}{CI} = \frac{0.0046}{3} = 0.01533 < 0.1 \tag{50}$$

Therefore, the preferences meet the consistency requirement and the obtained weights are reliable.

Step 2: Determine the weights of all experts.

Determine the best expert, the worst expert and preferences. The obtained optimal weights of four experts e_1, e_2, e_3, e_4 are shown in Table 7.

Table 7. The weights of experts table.

	e_1	e_2	e_3	e_4
A_B^e (e_1)	1	2	3	6
A_W^e (e_4)	6	3	2	1
ω^e	0.5000	0.2500	0.1667	0.0833

The result of consistency test:

$$CR = \frac{\xi^*}{CI} = \frac{0.1380}{3} = 0.046 < 0.1 \tag{51}$$

Therefore, the preferences meet the consistency requirement and the obtained weights are reliable.

Step 3: Fuse the experts' evaluation results and calculate β_{x_1}.

According to the information provided in Table 6, the weights of the four experts are $\boldsymbol{\omega}^e = \{0.5000, 0.2500, 0.1667, 0.0833\}$. The weighted average is calculated,

$$\begin{aligned} HFWA(h_1, h_2, h_3, h_4) = (&0.0610, 0.0628, 0.0736, 0.0753, \\ &0.0760, 0.0777, 0.0884, 0.0901) \end{aligned} \tag{52}$$

The estimated $\beta_{x_1} = 0.0756$.

Step 4: Derive the cumulative distribution function of top event.

$\tilde{\lambda}_{x_1}$, $\tilde{\lambda}_{x_2}$, $\tilde{\lambda}_{CC}$ can be deduced from Eqs. (10)-(12), $\tilde{\lambda}_{x_1} = [2.0216628 \times 10^{-6}, 2.4709212 \times 10^{-6}]\mathrm{h}^{-1}$, $\tilde{\lambda}_{x_2} = [1.2926628 \times 10^{-6}, 1.5799212 \times 10^{-6}]\mathrm{h}^{-1}$, $\tilde{\lambda}_{CC} = [1.653372 \times 10^{-7}, 2.020788 \times 10^{-7}]\mathrm{h}^{-1}$ respectively.

The cumulative distribution function of *OR* is obtained as

$$\begin{aligned} \tilde{F}_{OR}(t) &= \tilde{F}_{HSP_1}(t) + \tilde{F}_{CC}(t) - \tilde{F}_{HSP_1}(t)\tilde{F}_{CC}(t) \\ &= \tilde{F}_{x_1}(t)\tilde{F}_{x_2}(t) + \tilde{F}_{CC}(t) - \tilde{F}_{x_1}(t)\tilde{F}_{x_2}(t)\tilde{F}_{CC}(t) \end{aligned} \tag{53}$$

Then the cumulative distribution function of top event is

$$\begin{aligned} \tilde{F}_{Top}(t) &= 1 - \left(1 - \tilde{F}_{x_1}(t)\tilde{F}_{x_2}(t)\right)\left(1 - \tilde{F}_{CC}(t)\right)\left(1 - \tilde{F}_{x_3}(t)\right)\left(1 - \tilde{F}_{x_4}(t)\right) \\ &= 1 - \left(1 - (1 - e^{-\tilde{\lambda}_{x_1}t})(1 - e^{-\tilde{\lambda}_{x_2}t})\right)e^{-(\tilde{\lambda}_{CC} + \tilde{\lambda}_{x_3}^{\mathrm{tot}} + \tilde{\lambda}_{x_4}^{\mathrm{tot}})t} \end{aligned} \tag{54}$$

It is clear that $\tilde{\lambda}_{x_1}, \tilde{\lambda}_{x_2}, \tilde{\lambda}_{CC}, \tilde{\lambda}_{x_3}^{\mathrm{tot}}, \tilde{\lambda}_{x_4}^{\mathrm{tot}}$ appear only once in the Eq. (54). Therefore, the interval extension will not happen. The failure probabilities of top event obtained based on proposed method and DTBN-based method are shown in Fig. 13, where the number of time intervals of DTBN-based method is set as 20.

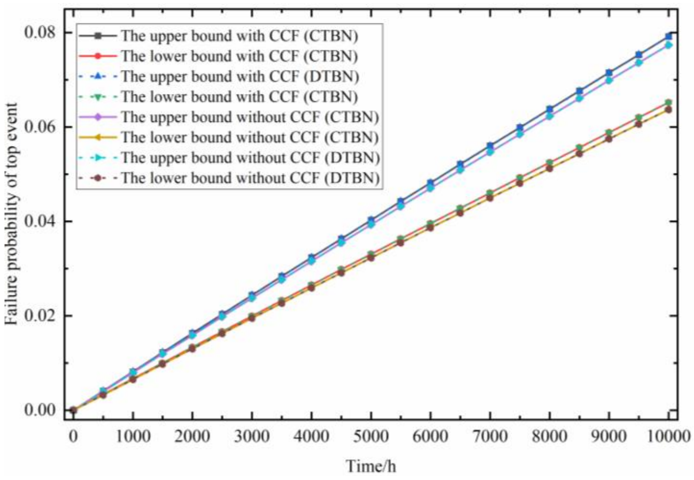


Fig. 13. Failure probability of top event.

At fixed time t, the failure probability of top event is an interval number. Comparing $\tilde{P}_{Top\_CCF}(t)$ and $\tilde{P}_{Top}(t)$, the possibility degrees are shown in Fig. 14.

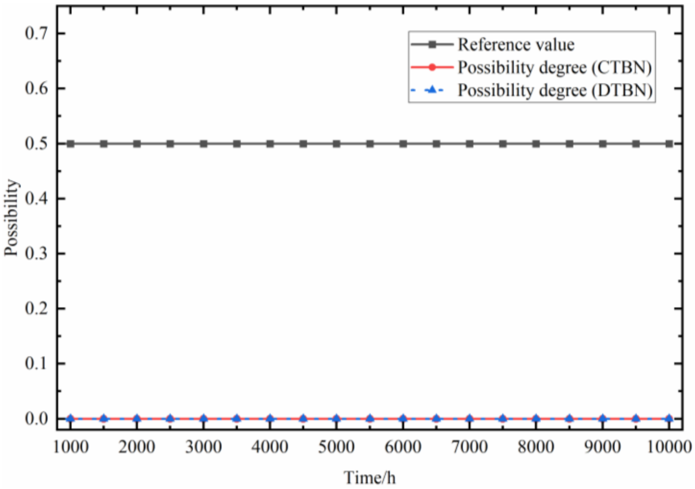


Fig. 14. The possibility degree of $\tilde{P}_{Top\_CCF}(t) \leq \tilde{P}_{Top}(t)$.

The calculation times of the methods based on CTBN and DTBN is shown in Table 8. The simulation environment is the same as that in case 1.

Table 8. The calculation time of the methods.

Method	Calculation time (s)
CTBN-based	4.698×10^{-5}
DTBN-based	0.1582

From Fig. 13, it can be found that the failure probability of the top event gradually rises in both scenarios with CCF and without CCF. It can be found that the possibility degree $P(\tilde{P}_{Top\_CCF}(t) \leq \tilde{P}_{Top}(t))$ is significantly lower than 0.5 at any time, shown from Fig. 14. That is, $\tilde{P}_{Top\_CCF} \succ \tilde{P}_{Top}$, which illustrates the necessity of considering CCF in reliability evaluation of dynamic fault tree. From Table 8, it can be found that the calculation time of CTBN-based method is significantly shorter than that of DTBN-based method.

4. Conclusions

The framework based on CTBN is constructed to evaluate the reliability of dynamic fault tree with CCF and interval uncertainty parameters. BWM is employed to determine the weights of all criteria and experts. β-factors should be evaluated by using hesitant fuzzy set. CTBN is used to derived the probability density functions of dynamic logic gates. The result of two cases verified that CCF will result in a higher failure probability of top event. Therefore, it is necessary to consider CCF in the system. Compared with DTBN-based reliability evaluation framework, the proposed framework does not need to discretize the time, so the failure probability at any time can be calculated. In addition, the calculation time of proposed framework is significantly shorter than that of DTBN-based framework.

In the reliability evaluation of complex systems, the derivation of the conditional probability density function of top event will become extremely complicated. It's my future research on developing simplified or approximated methods. Furthermore, all the failure distributions are assumed to be exponential in this work. It is needed to research on the application of the presented framework for other failure distributions.

Acronyms

AHP	analytic hierarchy process
BN	Bayesian network
BWM	best-worst method
CCF	common cause failure
CI	consistency index
CR	consistency ratio
CSP	cold spare
CTBN	continuous-time Bayesian network
DTBN	discrete-time Bayesian network
FTA	fault tree analysis
HFS	hesitant fuzzy set
HFWA	hesitant fuzzy weighted averaging
HSP	hot spare
PAND	priority AND
WSP	warm spare

Notation

β	β -factor
c_B	best criterion
c_W	worst criterion
$\boldsymbol{A}_B^c$	preferences of the best criterion over all the other criteria
$\boldsymbol{A}_W^c$	preferences of all the other criteria over the worst criterion
A_{BW}^c	preference of the best criterion over the worst criterion
e_B	best expert
e_W	worst expert
$\boldsymbol{A}_B^e$	preferences of the best expert over all the other experts
$\boldsymbol{A}_W^e$	preferences of all the other experts over the worst expert
A_{BW}^e	preference of the best expert over the worst expert
$\tilde{A}$,$\tilde{B}$	interval numbers
$\succ$	superior
$\sim$	indifferent
$\prec$	inferior
$u(x)$	unit step function
δ	pulse function
λ	failure rate
$f(t)$	probability density function
$F(t)$	cumulative distribution function

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