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Dynamic grouping maintenance optimization by considering the probabilistic remaining useful life prediction of multiple equipment

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Highlights

- The probabilistic RUL prediction is obtained by using LSTM and VAE resampling.
- A multi-equipment dynamic grouping maintenance model is established.
- The gazelle optimization algorithm is used to solve the optimization model.
- The effectiveness of the proposed method is verified by the numerical case with 6 wind turbines.

Abstract

For multi-equipment maintenance of modern production equipment, the economic correlation and degradation uncertainty may lead to insufficient or excessive maintenance, increasing maintenance costs. This paper proposes a dynamic grouping maintenance method based on probabilistic remaining useful life (RUL) prediction for multiple equipment. Long short term memory (LSTM) is developed to predict the equipment probability RUL by the Variational Auto-Encoder (VAE) resampling. Then, the dynamic grouping maintenance model is constructed to minimize the maintenance cost rate under the known probabilistic RUL information. The gazelle optimization algorithm (GOA) is used to determine the optimal maintenance time for each equipment. To better verify the effectiveness of the proposed method, a numerical case with six wind turbines is introduced to analyse the performance of GOA. Moreover, the advantages of dynamic grouping maintenance is verified by comparing with independent maintenance, whose maintenance cost rate is reduced by 10.01%.

Keywords

long short term memory, probabilistic remaining useful life prediction, Variational Auto-Encoder, Dynamic grouping maintenance, gazelle optimization algorithm.

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1. Introduction

In industrial production, production systems consisting of multiple equipment that perform specific production tasks according to certain rules. However, due to the pursuit of production efficiency by production companies and the prolonged use of multiple production equipment, the problem of equipment failure leading to the increase in equipment maintenance costs is prominent [1]. The equipment maintenance cost can be effectively reduced by reasonably arranging the multi-equipment maintenance plan based on the equipment degradation information. At the same time, the

equipment failure of the production process can be avoided as much as possible.

Currently, researchers have done a lot of research on maintenance. There are many maintenance decision-making methods, including condition-based maintenance [2], maintenance based on degenerate models such as nonlinear Wiener process[3], Markov process [4], gamma process [5]. With the development of artificial intelligence technology, data-based predictive maintenance is gradually maturing [6], which needs to predict the remaining useful life (RUL) of equipment

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based on data processing and information analysis and to determine the maintenance strategy with the lowest cost within the maintenance window of opportunity. Predictive maintenance based on data needs to consider three aspects, remaining life prediction, maintenance model building, and optimization algorithm.

For RUL prediction, most of the researchers only considers point prediction of the remaining useful life of equipment. Wu et al. [7] proposed a deep convolutional migration network with spatial pyramid pooling to improve the prediction accuracy of bearings. Jafari et al. [8] A particle filter-based RUL estimation technique for Li-ion batteries combining Kalman filter (KF) with particle filter (PF) for predicting the remaining useful life of the battery. Cui et al. [9] also all point to the prediction of the remaining life of equipment. Due to some uncertainty in the RUL predictions, some scholars have constructed a probabilistic RUL prediction framework to output an estimate of the probability density of the target RUL in recent years. Wang et al. [10] used least squares, non-informative distributions, and Markov Chain Monte Carlo methods (without dropout sampler) to predict and obtain the uncertainty of the RUL of the bearings. Nguyen et al. [11] proposed to predict the RUL distributions of components through a combination of probabilistic models and deep recurrent neural networks and then derive formulas for RUL uncertainty to enable the quantification of RUL uncertainty for multicomponent systems.

For equipment maintenance modelling, many scholars have also done much research in the past. Many of which are based on physical models [3][5][12], and there have also been studies on data prediction-based maintenance schedules [2][13][14]. However, fewer researchers in the past have considered maintenance schedules that predict information uncertainty.

Nguyen et al. [15] proposed a novel dynamic predictive maintenance (PdM) framework using Long Short-Term Memory (LSTM) networks for fault prediction, which provides the probability of system failure in different time scales to determine the moment of maintenance activities. Chen et al. [16] build on this foundation by proposing multivariate LSTM networks to obtain the degradation prediction distributions and to determine the optimal maintenance moment. Kim et al. [17] proposed a new method for determining future data measurement schedules by scheduling future component

inspections after estimating the RUL uncertainty. So, probabilistic RUL predictions is a way forward for predictive maintenance.

However, none of them considered multi-equipment maintenance. Lee et al. [18] proposed a Deep Reinforcement Learning (DRL) approach to planning predictive maintenance of aircraft engines by integrating data-driven probabilistic RUL predictions into a deep reinforcement learning model and then determining the engine replacement moment based on the trend of the RUL predictions. Mitici M et al. [19] proposed a data-driven predictive maintenance framework for multiple components using Monte Carlo discard methods and convolutional neural networks to obtain probabilistic RUL predictions and develop a multi-component maintenance plan based on the predictions. They considered probabilistic RUL prediction for multi-equipment maintenance but ignored the effect of economic correlation between multi-equipment maintenance. In contrast, for the economic correlation of multi-equipment, some scholars [20][21] developed dynamic grouping maintenance models to reduce the maintenance cost of multi-equipment systems. They all consider dynamic grouping maintenance to solve the problem of the economic correlation of multi-equipment, but they did not consider probabilistic RUL prediction.

For optimization algorithms, some scholars have adopted classical solution algorithms such as using genetic algorithm (GA) [22] and particle swarm algorithm (PSO) [3]. However, it is easy to fall into the local optimal solution [23]. Some other scholars have adopted newly developed algorithms in recent years, such as the hybrid whale swarm algorithm [24] and the peacock algorithm [25]. However, there is less research on maintenance model solving using the gazelle optimization algorithm (GOA), which has strong search capability and can improve the solving accuracy.

According to the existing research status, the research gap is summarized as follows. (1) Many researchers have focused on the RUL prediction, but it is essential to consider probabilistic RUL estimation by considering the prediction uncertainty. So, the probabilistic RUL prediction based on resampling and LSTM is proposed by considering the equipment degradation data. (2) Fewer researchers have considered the economic relevance of multi-equipment maintenance, so it is necessary to

the dynamic grouping maintenance for the system with multiple equipment. (3) Since classical optimization algorithms are prone to fall into local optimal solutions, the gazelle optimization algorithm (GOA) is introduced to strengthen the search ability of the global optimal solution of the model.

Therefore, aiming at the uncertainty of equipment remaining useful life prediction and the economic correlation of multi-equipment maintenance, this paper proposes a multi-equipment dynamic grouping maintenance decision-making method based on probabilistic remaining useful life (RUL) prediction.

The innovations of this paper include the following.

- (1) Probabilistic RUL prediction of equipment is proposed by using the LSTM model after resampling the equipment degradation data using a Variational Auto-Encoder (VAE).
- (2) A dynamic grouping maintenance model for multiple equipment is established based on the probabilistic RUL prediction for determining the optimal maintenance time for maintenance equipment.
- (3) GOA is introduced to solve the maintenance decision model by considering the advantages of the robust search capability.

The rest of this paper is structured as follows. Section 2 establishes the dynamic maintenance model. The GOA procedure is introduced to optimize the model in Section 3. Section 4 implements the numerical experiments to verify the advantages of the proposed method. Finally, Section 5

summarizes the important findings.

2. Establishment of the model

The model in this paper consists of two parts: one is a probabilistic RUL prediction model with resampling and LSTM, and the other is a multi-equipment dynamic grouping maintenance decision model.

2.1. Probabilistic RUL prediction based on resampling and LSTM

The collected data are first processed by the sliding average filtering method to reduce the invalid data and obtain the processed degraded data. Then, the VAE is used to resampling the degradation data of the equipment with the normal distribution as the prior distribution, and K groups of degradation data are obtained by K times resampling. Next, the LSTM prediction model is trained by using the equipment degradation data. Finally, to predict the probability RUL of the equipment, K groups of new data after K resampling are used as the input of the LSTM model to obtain K groups of output. The predicted results on day ct are taken from all K sets of outputs, and the probability distribution is obtained by statistical calculation through the frequency distribution method. This probability distribution represents the probability RUL prediction of the equipment on the ct th day. The framework of probabilistic RUL prediction based on resampling and LSTM is shown below.

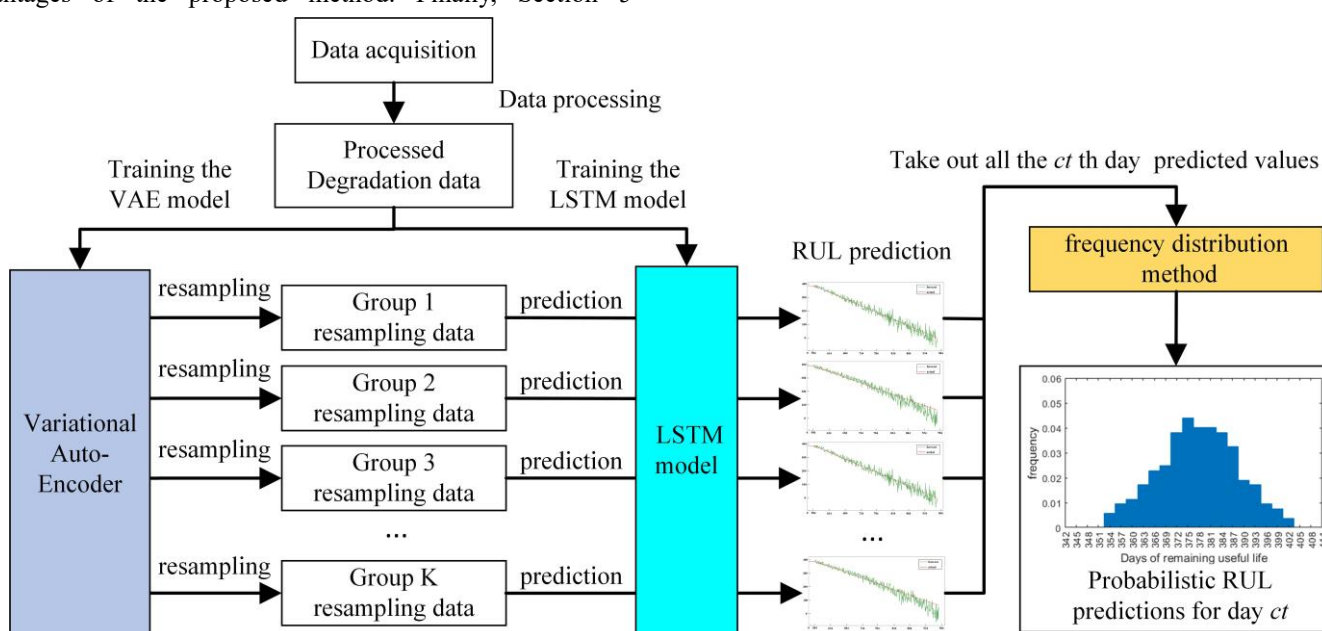


Fig. 1. The framework of probabilistic RUL prediction based on resampling and LSTM.

Meanwhile, in order to ensure the validity of the data, this paper first uses the sliding average filtering method [26] to deal with the outliers of the data before resampling the degraded data by the VAE for improving the quality of data and avoiding the effects of invalid data. Sliding average filtering is a common signal processing technique, which can effectively reduce invalid data and preserve trend information. The core idea is to estimate the current value by the arithmetic mean of a set of recent data points. For each point in the dataset, the sliding average is calculated as follows:

$$ma_d = \frac{x_{d-n+1} + x_{d-n+2} + \dots + x_d}{n} \quad (1)$$

Where the data sequence is $x \in \{x_1, x_2, \dots, x_d\}$, the sliding average sequence is $\{ma_1, ma_2, \dots, ma_d\}$, the window size is n , and the total number of data in the dataset is d .

2.1.1. Resampling of Variational Auto-Encoder

VAE [27] is an unsupervised learning generative model that combines the ideas of auto-encoder and variational inference, aiming to learn the latent distribution of the input data and generate new samples to complete the resampling of the

degraded data. VAE can be implemented to map the input data x to an approximate distribution by establishing the relationship between the input variable x and the latent variable z through neural networks.

The specific steps involved in the resampling process using VAE in this paper are summarized as follows.

- (1) Set the equipment degradation data as the input variable x of the VAE model;
- (2) Train the VAE model. Map it to the latent space (the latent space is assumed to obey a prior distribution, and a normal distribution is chosen in this paper) via the encoder $q_\theta(z|x)$, and compute the mean μ_x and the variance parameter σ_x^2 for the latent space of each input variable;
- (3) Resampling from the latent space randomly to generate the latent variable z_x ;
- (4) The decoder reduces each latent variable to generate a new sample with an approximate prior distribution, which constitutes a set of resampled data;
- (5) Repeat Steps (3) and (4) for K times based on the reparameterization of the latent variable to generate K sets of the resampled data.

The VAE workflow is shown in Fig. 2.

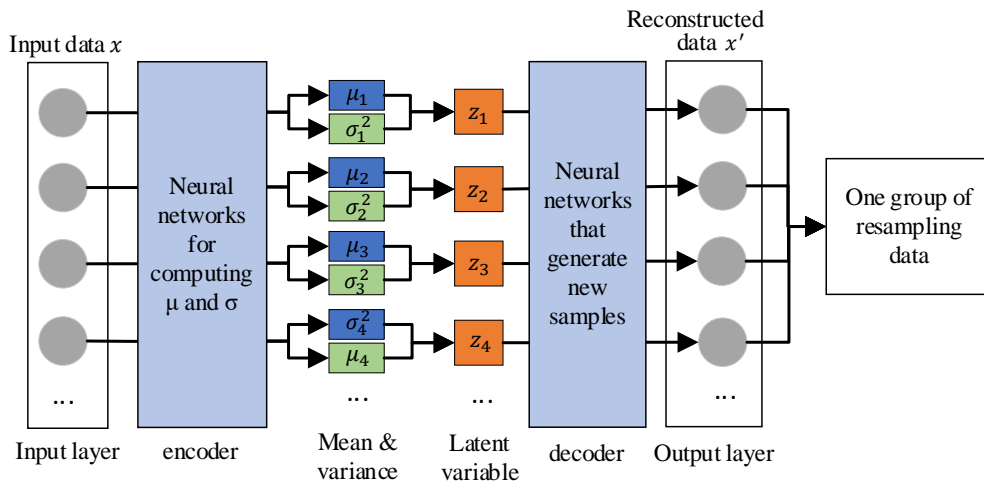


Fig. 2. Working flow chart of VAE.

The optimization objective function formulation [27] for VAE is as follows:

$$L_x(\phi, \theta) = \operatorname{argmax}\{E_{z \sim q}[\log(p_\theta(x|z))] - D_{KL}(q_\theta(z|x)||p_\theta(z))\} \quad (2)$$

ϕ and θ , respectively, the parameters of the encoder and decoder, model considering $p_\theta(z)$ and $q_\theta(z|x)$ obey the normal distribution, namely $p_\theta(z) \in N(0, 1)$, $q_\theta(z|x) \in N(\mu_x, \sigma_x^2)$. μ_x and σ_x^2 represent the mean and variance generated by the

encoder $q_\theta(z|x)$, respectively.

2.1.2. LSTM model construction

LSTM is a particular Recurrent Neural Network (RNN) architecture for processing long sequence data [28] based on memory function. LSTM controls the transmission state through a particular gate structure, including an input gate,

a forgetting gate, and an output gate, which can capture the dependency of the long sequences in a better way. The prediction effect is improved when dealing with the time series prediction problem [29].

In this paper, equipment degradation data is brought into the LSTM model to complete the training and testing of the model. Finally, the LSTM model with completed training is obtained. The cell diagram of the LSTM model is shown in Fig. 3.

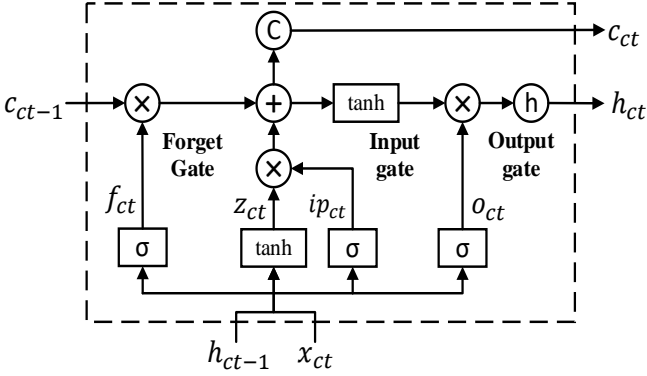


Fig. 3. Cell diagram of the LSTM model.

The expression for the LSTM model is shown as follows.

Input state:

$$z_{ct} = \tanh(W_z x_{ct} + U_z h_{ct-1} + b_z) \quad (3)$$

Forget Gate:

$$f_{ct} = \sigma(W_f x_{ct} + U_f h_{ct-1} + b_f) \quad (4)$$

Input gate:

$$i_{p_{ct}} = \sigma(W_{ip} x_{ct} + U_{ip} h_{ct-1} + b_{ip}) \quad (5)$$

Intermediate state:

$$o_{ct} = \sigma(W_o x_{ct} + U_o h_{ct-1} + b_o) \quad (6)$$

Current state:

$$c_{ct} = f_{ct} \cdot c_{ct-1} + i_{p_{ct}} \cdot z_{ct} \quad (7)$$

Output gate:

$$h_{ct} = \tanh(c_{ct}) \otimes o_{ct} \quad (8)$$

Where W and U denote the weight matrix, σ denotes the sigmoid activation function, and b denotes the model bias. x_{ct} represents the input value of the current moment (the ct day), h_{ct-1} denotes the output value of the previous moment, and c_{ct-1} is the output state of the previous moment. c_{ct} represents the output state of the current moment, and h_{ct} represents the output value of the current moment.

2.1.3. Probabilistic RUL prediction

In this paper, the LSTM model has been trained in Section 2.1.2

to predict based on the resampled data in Section 2.1.1. The specific steps of probabilistic RUL prediction are as follows.

(1) Take the K sets of resampled data of equipment i as the input of the completed trained LSTM model, and get the K sets of outputs (RUL prediction);

(2) Take all the RUL prediction values (h_{ct}) at the current moment (day ct) as the total number of samples from the K sets of outputs of equipment i ;

(3) According to frequency distribution in statistics, a probability distribution can be obtained by the occurrence frequency of each value in the total number of samples. An observation group is obtained by remaining useful life at time point t respectively, and the number of observations in which the value of h_{ct} falls into each observation group is counted and denoted as $n_{i,t}$;

(4) The relative frequency of each observation group is calculated according to Equation (9), and the relative frequencies of all observation groups (all remaining useful life (t)) are formed into a probability distribution. It is used as the probabilistic RUL prediction result for this equipment i at the current moment (day ct).

$$P_{i,t} = \frac{n_{i,t}}{K} \quad (9)$$

$n_{i,t} = \{\text{The } h_{ct} \text{ value of equipment } i \text{ is the number of } t\}$

Where $P_{i,t}$ indicates that the relative frequency at which the RUL predicted value (h_{ct}) of equipment i is equal to t , $n_{i,t}$ denotes the number of samples at which the RUL prediction value (h_{ct}) of equipment i is equal to t , and K is the total samples for equipment i (total number of h_{ct}).

In addition, in order to facilitate the maintenance of the decision model, the probabilistic RUL need to be converted into failure probabilities $\varphi_{i,j}$. The equipment probabilistic RUL can reflect the failure probability of equipment, so the failure probability $\varphi_{i,j}$ is denoted as:

$$\varphi_{i,j} = P_{i,t} \quad (10)$$

Where $\varphi_{i,j}$ denotes the probability that equipment i fails on day j based on the day ct prediction. The relationship between day j and the current moment ct is:

$$j = t + ct \quad (11)$$

2.2. Multi-equipment dynamic grouping maintenance decision model based on probability RUL prediction

This paper arranges the maintenance plan for multi-equipment based on the probabilistic RUL prediction results of multi-

equipment, which completes the maintenance decision-making model with dynamic grouping. Among them, the maintenance method adopts replacement maintenance, and the evaluation index is the maintenance cost rate. Determine the optimal maintenance time for each piece of equipment to minimize the maintenance cost rate of the maintenance decision model.

2.2.1. Dynamic grouping maintenance policy

Given the uncertainty of the remaining useful life prediction of multi-equipment and the economic correlation of multi-equipment maintenance, this paper considers the cost-based dynamic grouping strategy for multi-equipment maintenance decisions. Dynamic grouping maintenance has good compatibility, and equipment can be divided into different groups according to different maintenance horizons [20] so that different maintenance arrangements can be made for different groups of equipment, which can avoid the limitations of equipment grouping by short-term vision and long-term vision.

The multi-equipment dynamic grouping maintenance decision model determines the optimal maintenance time of each piece of equipment according to the evaluation index (maintenance cost rate) in the model. Because the grouping of equipment and the optimal maintenance time of each group of equipment will directly affect the maintenance cost rate for the

maintenance decision model, in the maintenance model, the equipment with the same optimal maintenance time is regarded as a group, and the equipment with the optimal maintenance time alone can be regarded as a group.

2.2.2. Multi-equipment dynamic grouping maintenance decision model

A multi-equipment dynamic grouping maintenance model is established by considering essential maintenance cost, downtime loss cost, sudden failure and emergency maintenance cost, and depreciation loss cost. The symbols used in this model are defined in Table 1. Meanwhile, this paper makes the following assumptions:

- Multi-equipment can be maintained on the same day;
- Only replacement maintenance is considered in the maintenance mode;
- Assume that one spare part cost and one shutdown cost are spent for each equipment maintained, and only one entry and exit cost of maintenance tools is spent for multiple equipment in the same group;
- Describe sudden failure and emergency maintenance costs in the form of mathematical expectations;

Table 1. Symbol definition table.

Symbol	Explain	Symbol	Explain
C_r	Maintenance cost rate for the maintenance decision model	C_i^v	Downtime cost of equipment i
		C_e	Cost of lost production per unit of time
C_w	Total cost of the maintenance decision model	T_s	Maintenance equipment downtime
t_i	Optimal maintenance time for equipment i	C_s	Downtime costs for all equipment
		C_p	Sudden failure and emergency maintenance costs for all equipment
M	The number of equipment	E_i^n	The mathematical expectation of a sudden failure of the equipment i before the optimal maintenance time t_i
N	Maximum maintenance days of the equipment		
C_{sm}	Essential maintenance costs for a single equipment	C_{em}	Emergency maintenance cost of single equipment
C_f	Cost of labor		
C_{ec}	Maintenance tool input and output costs	C_a	The cost of production accidents caused by equipment failure
C_{sp}	Spare parts cost		
C_m	Essential maintenance costs for all equipment	$\varphi_{i,j}$	Probability of sudden failure of equipment i on day j
G	Number of equipment groups	C_i^d	Depreciation expense of equipment i
G_j	Maintenance schedules for day j		
$X_{i,j}$	Decision variable of whether equipment i will be maintained on day j	T_d	Design life of spare parts
		C_o	Depreciation costs incurred by all equipment

In the maintenance method, the objective function (evaluation index) is to minimize the maintenance cost rate C_r for the maintenance decision model. The maintenance cost rate can be obtained by determining the relationship between the total cost C_w of the maintenance model and the sum of the accumulated running time of all equipment ($\sum_{i=1}^M t_i$). It represents the Single equipment maintenance cost per day, and the smaller the maintenance cost rate, the lower the maintenance cost. The maintenance cost rate of the maintenance model is C_r , it can be expressed as:

$$C_r = \frac{C_w}{\sum_{i=1}^M t_i} \quad (12)$$

Among them, the total cost of maintenance model C_w includes essential maintenance cost C_m , downtime loss cost C_s , sudden failure and emergency maintenance cost C_p , and depreciation loss cost C_o , C_w can be expressed as:

$$C_w = C_m + C_s + C_p + C_o \quad (13)$$

(1) Essential maintenance costs

Essential maintenance cost Indicates the essential cost of maintaining the equipment.

For a single equipment essential maintenance cost C_{sm} , it can be expressed as:

$$C_{sm} = C_f + C_{ec} + C_{sp} \quad (14)$$

The essential maintenance cost for all equipment can be expressed as:

$$C_m = (C_f + C_{sp}) \cdot M + C_{ec} \cdot G \quad (15)$$

Where, determine whether there is equipment with a maintenance schedule on the day j . If there is, note G_j equals 1, and vice versa, note G_j equals 0. The total number of equipment groups, denoted as G , is obtained by summing up the values of G_j for all days. The number of equipment groups G can be denoted as:

$$G_j = \begin{cases} \sum_{i=1}^M X_{i,j} \geq 1, G_j = 1 \\ \sum_{i=1}^M X_{i,j} < 1, G_j = 0 \end{cases} \quad (16)$$

$$G = \sum_{j=1}^N G_j \quad (17)$$

The decision variable $X_{i,j}$ indicates whether or not replacement maintenance is performed for equipment i on the day j , which can be expressed as:

$$\begin{cases} X_{i,j} = 1, \text{Replace and maintain equipment } i \text{ on day } j \\ X_{i,j} = 0, \text{No maintenance is performed on equipment } i \text{ on day } j \end{cases} \quad (18)$$

(2) Shutdown loss cost

The downtime cost of a single equipment C_i^v can be

expressed as:

$$C_i^v = C_e \cdot T_s \quad (19)$$

The downtime cost C_s of dynamic grouping maintenance of all equipment can be expressed as:

$$C_s = M \cdot C_e \cdot T_s \quad (20)$$

(3) Sudden failure and emergency maintenance costs

The cost of sudden failures and emergency maintenance considers the losses caused by inadequate equipment maintenance. By considering the probabilistic RUL prediction results $\varphi_{i,j}$ for different equipment, costs of sudden failures and emergency maintenance of all equipment is C_p , C_p can be expressed as:

$$C_p = \sum_{i=1}^M (E_i^n \cdot (C_{em} + C_i^v)) \quad (21)$$

Among them, the emergency maintenance cost of single equipment C_{em} can be expressed as:

$$C_{em} = C_{sm} + C_a \quad (22)$$

In addition, the mathematical expectation E_i^n for a sudden failure of the equipment i before the optimal maintenance time t_i is expressed as:

$$E_i^n = \sum_{j=1}^{t_i} \varphi_{i,j} \quad (23)$$

Where $\varphi_{i,j}$ comes from the probabilistic RUL prediction of equipment i in section 2.1.3, and the total probability value of occurrence of failure for each piece of equipment is 1, i.e., $\sum_{j=1}^N \varphi_{i,j} = 1$.

Therefore, the costs of sudden failures and emergency maintenance of all equipment is C_p , it can be expressed as:

$$\begin{aligned} C_p &= \sum_{i=1}^M (E_i^n \cdot (C_{em} + C_i^v)) \\ &= \sum_{i=1}^M \left(\sum_{j=1}^{t_i} \varphi_{i,j} \cdot (C_f + C_{ec} + C_{sp} + C_a + (C_e \cdot T_s)) \right) \end{aligned} \quad (24)$$

(4) Depreciation loss expense

When determining the optimal time to maintain equipment, consider the cost of depreciation loss per equipment, which represents the loss due to excessive equipment maintenance.

The depreciation cost of equipment i is C_i^d . It represents the wasted value of the replaced parts due to the maintenance of the equipment at the optimal maintenance time t_i , C_i^d is expressed as:

$$C_i^d = \frac{(T_d - t_i)}{T_d} \cdot C_{sp} \quad (25)$$

Where the optimal maintenance time t_i for equipment i is obtained from $X_{i,j}$, and when a decision variable $X_{i,j}$ exists with

a value of 1, then the optimal maintenance time t_i for equipment i is j . The expression is:

$$t_i = \begin{cases} X_{i,j} = 1, t_i = j \\ X_{i,j} \neq 1, \text{nonexecution} \end{cases} \quad (26)$$

Therefore, the depreciation cost of all equipment C_o is expressed as:

$$C_o = \sum_{i=1}^M \frac{(T_d - t_i)}{T_d} \cdot C_{sp} \quad (27)$$

Finally, the total cost of the maintaining model is C_w , it can be expressed as:

$$\begin{aligned} C_w &= C_m + C_s + C_p + C_o \\ &= (C_f + C_{sp} + (C_e \cdot T_s)) \cdot M + G \cdot C_{ec} + \sum_{i=1}^M \left(\sum_{j=1}^{t_i} \varphi_{i,j} \cdot \right. \\ &\quad \left. (C_f + C_{sp} + C_{ec} + C_a + (C_e \cdot T_s)) + \frac{(T_d - t_i)}{T_d} \cdot C_{sp} \right) \end{aligned} \quad (28)$$

The maintenance cost rate of the maintenance model is C_r , it can be expressed as:

$$\begin{aligned} C_r &= \frac{C_w}{\sum_{i=1}^M t_i} \\ &= \frac{\left((C_f + C_{sp} + (C_e \cdot T_s)) \cdot M + G \cdot C_{ec} + \sum_{i=1}^M \left(\sum_{j=1}^{t_i} \varphi_{i,j} \cdot (C_f + C_{sp} + C_{ec} + C_a + (C_e \cdot T_s)) + \frac{(T_d - t_i)}{T_d} \cdot C_{sp} \right) \right)}{\sum_{i=1}^M t_i} \end{aligned} \quad (29)$$

Note: The relationship between the decision variable $X_{i,j}$ and the optimal maintenance time t_i is shown in Equation(26).

In addition, the maintenance decision model takes into account the following main constraints:

$$\begin{cases} \sum_{j=1}^N X_{i,j} = 1; \\ X_{i,j} \in \{0,1\}; \\ 1 \leq i \leq M; 1 \leq j \leq N; \end{cases} \quad (30)$$

- Consider choosing the optimal maintenance time, which means that within the maximum maintenance days N for each piece of equipment i , only one day can be selected as the optimal maintenance time t_i .
- Consider the maintenance mode of each piece of equipment. The maintenance decision variable $X_{i,j}$ is 1 or 0. That is, the maintenance mode of the equipment is replacement maintenance or no maintenance.
- Consider the range of equipment i and day j in the decision variable $X_{i,j}$, where the number of equipment is M and the maximum maintenance

days of equipment is N ;

3. GOA procedures

The Gazelle Optimization Algorithm (from now on referred to as GOA) is a meta-heuristic optimization algorithm proposed by Agushaka and Ezugwu et al. [30] in 2022. This algorithm simulates the behavior of gazelle to avoid predators and solves the problem by simulating the movement, foraging, and escape of antelopes. The gazelle optimization algorithm includes global search, local search, and gazelle escape. Compared with traditional algorithms such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO), which tend to fall into the problem of local optimal solutions [23], the GOA algorithm has robust searchability and can better find the optimal solution of the model so that the solution accuracy is higher. Therefore, this paper applies the Gazelle Optimization Algorithm (GOA) to solve the maintenance decision model.

In this paper, the objective function (maintenance cost rate C_r is used as the fitness function to assess the fitness of gazelles in a particular location. Where the smaller C_r is, the higher the fitness f is, indicating the better solution at that location, and the fitness formula is:

$$f = \frac{1}{C_r} \quad (31)$$

The basic steps of GOA optimization solution are as follows:

(1) Random initialization of populations

$$\begin{aligned} X &= \begin{bmatrix} X_{1,1} & X_{1,2} & \cdots & X_{1,N} \\ X_{2,1} & X_{2,2} & \cdots & X_{2,N} \\ \vdots & \vdots & X_{i,j} & \vdots \\ X_{M,1} & X_{M,2} & \cdots & X_{M,N} \end{bmatrix} \\ &X_{i,j} \in \{0,1\} \end{aligned} \quad (32)$$

X is a matrix of position vectors of a candidate population, and the position vector is the maintenance decision variable $X_{i,j}$ in Section 2.2.2, consisting of 0 or 1.

(2) Construct an elite gazelle matrix

By evaluating the fitness of all the gazelles (X) in this generation, the gazelle with the highest fitness is selected as the top gazelle, and the position vector matrix of this gazelle is used as the elite matrix, which is used for searching and finding the gazelles in the next step. The elite matrix is:

$$Elite = \begin{bmatrix} X'_{1,1} & X'_{1,2} & \dots & X'_{1,N} \\ X'_{2,1} & X'_{2,2} & \dots & X'_{2,N} \\ \vdots & \vdots & X'_{i,j} & \vdots \\ X'_{M,1} & X'_{M,2} & \dots & X'_{M,N} \end{bmatrix} \quad (33)$$

$$X'_{i,j} \in \{0,1\}$$

(3) Global search

At this stage, when gazelles are free to graze without predators or predators, they adopt the Brownian movement. Seemingly random motion when the displacement conforms to a standard (Gaussian) probability distribution function, where the average value and variance are $\mu = 0$ and $\sigma^2 = 1$, respectively. Its location is updated as follows:

$$gazelle_{iter+1} = gazelle_{iter} + S \cdot R \cdot R_B \cdot (Elite_{iter} - R_B \cdot gazelle_{iter}) \quad (34)$$

Where S represents the moving speed of the gazelle, R_B represents a random vector based on Brownian motion, R is a random number between 0 and 1, and $gazelle_{iter+1}$ is the next iteration of the current solution $gazelle_{iter}$.

(4) Local search

In this stage, the escape behavior of gazelles after finding predators is divided into two stages, and each stage constantly adopts two opposite directions of movement according to the parity of the number of iterations [30]. Stage 1: Take Levi's flight when spotting predators. The second stage: After the predator is spotted, the predator pursues the gazelle, it taking a Brownian motion and later in Levi's flight. The local search phase is as follows:

Stage 1: Gazelle When spotting a predator, the gazelle takes a Levi flight to escape:

$$gazelle_{iter+1} = gazelle_{iter} + S \cdot \mu \cdot R \cdot R_L \cdot (Elite_{iter} - R_L \cdot gazelle_{iter}) \quad (35)$$

Where μ is -1 or 1, indicating two directions of motion. R_L represents a random number vector based on a Levy distribution.

Stage 2: The behavior of a predator chasing a gazelle after being spotted:

$$gazelle_{iter+1} = gazelle_{iter} + S \cdot \mu \cdot CF \cdot R_B \cdot (Elite_{iter} - R_L \cdot gazelle_{iter}) \quad (36)$$

Where $CF = \left(1 - \frac{iter}{Maxiter}\right)^{\left(2 \frac{iter}{Maxiter}\right)}$, CF represents the cumulative effect of predators.

(5) Updating the elite gazelle matrix

After completing the global and local searches, the top gazelle and elite gazelle matrices are updated if a higher fitness gazelle emerges.

(6) Gazelle escape

The gazelle has a specific survival rate when facing a

predator, and the hunting success rate of the predator is represented by PSRs, which affects the gazelle's escape ability and prevents the algorithm from being trapped in a local minimum. The process can be expressed as follows:

$$gazelle_{iter+1} = \begin{cases} gazelle_{iter} + CF[LB + R \cdot (UB - LB)] \cdot U, r \leq PSR_S \\ gazelle_{iter} + [PSR_S(1 - r) + r](gazelle_{r1} - gazelle_{r2}), else \end{cases} \quad (37)$$

r is a random number between 0 and 1.

(7) Return the optimal value

When the maximum number of iterations is reached, the gazelle position with the highest fitness is returned as the optimal solution of the optimized solution.

The GOA algorithm's steps to solve this paper's model are shown in Fig. 4.

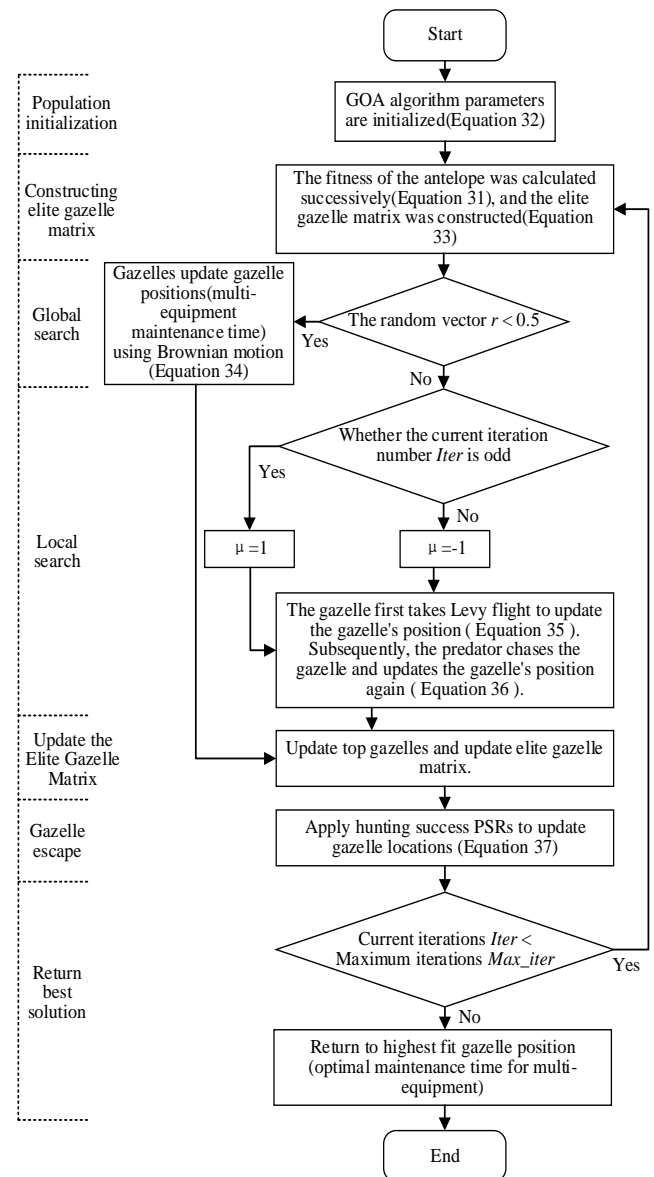


Fig. 4. Flowchart of GOA algorithm.

4. Numerical case

This paper considers six wind turbines of the same type as case objects for validation. Wind turbines are usually installed in remote and harsh environments, which is a major obstacle for equipment maintenance, resulting in relatively high maintenance costs for wind turbines. At the same time, since multiple wind motors are often installed in the same area, the economic relevance should be considered when scheduling maintenance work. Multiple equipment are repaired as much as possible in a single maintenance, and the maintenance strategy of dynamic grouping is consistent with this maintenance scenario.



Fig. 5. Main bearing diagram of wind turbine.

At present, the composition of the wind turbine includes eight significant systems, such as wind wheel, spindle, gearbox, and generator. Due to the critical and unique role of the main bearing [31], the damage to the main bearing can be regarded as damage to the whole wind turbine. Therefore, when assessing the health of a wind turbine, it can be simplified to assess the operational degradation of the main bearing [32]. The maintenance method in this paper only considers replacement maintenance and each maintenance piece of equipment consumes one spare part. According to the maintenance cost of a wind farm in northern China, this paper uses a multi-equipment dynamic grouping maintenance decision model based on probabilistic remaining useful life (RUL) prediction to obtain the minimum maintenance cost rate. The main bearing of the wind turbine is shown in Fig.5.

4.1. Data set description

The Weibull distribution is a flexible distribution model, which accommodates a wide range of life distribution shapes. In the operation of electrical and mechanical equipment, the Weibull distribution can fully reflect the effects of operating time and conditions on the equipment's life [33]. The Weibull distribution is a powerful tool for describing various lifetime distributions [34]. It is very suitable for a life degradation model.

This paper uses a simulation dataset to model the degradation of six main bearings for wind turbines (hereafter referred to as equipment). A degradation process with 8 Weibull distributions is used for each piece of equipment to simulate the eight degradation features of the equipment. Only one degradation value exists as an eigenvalue for each feature for each day of each equipment, and the number of samples for each feature represents the total number of days of life of that equipment. The labels of eight eigenvalues at the ct moment are the equipment's remaining useful life (RUL) at the ct moment. The parameters of different Weibull distributions are adjusted to describe the degradation of the six pieces of equipment through six diversified experiments to obtain the simulation dataset of the six pieces of equipment. The formula for the Weibull degradation distribution is as follows:

$$f(ct, m_s, \eta_s) = \frac{m_s}{\eta_s} \left(\frac{ct}{\eta_s}\right)^{m_s-1} \cdot e^{-\left(\frac{ct}{\eta_s}\right)^{m_s}} \quad (38)$$

Where ct denotes the current moment, m_s is the shape parameter of feature s , η_s is the scale parameter of feature s , $f(ct, m_s, \eta_s)$ is the degenerate value of the feature s . Where $s \in \{0,8\}$ and $m_s > 0, \eta_s > 0, t > 0$.

The degradation simulation parameters for each device have a range of m_s : 534.38~1234.38, a range of η_s : 1.8173~2.8173, and a range of ct (a range of number of samples): 947~989. The degradation process of the six equipment of the same type is shown in Fig. 6. The total life days of the six equipment are shown in Table 2.

Table 2. The total life days of the 6 equipment.

equipment	Total life days
A	974
B	989
C	968
D	965
E	965
F	986

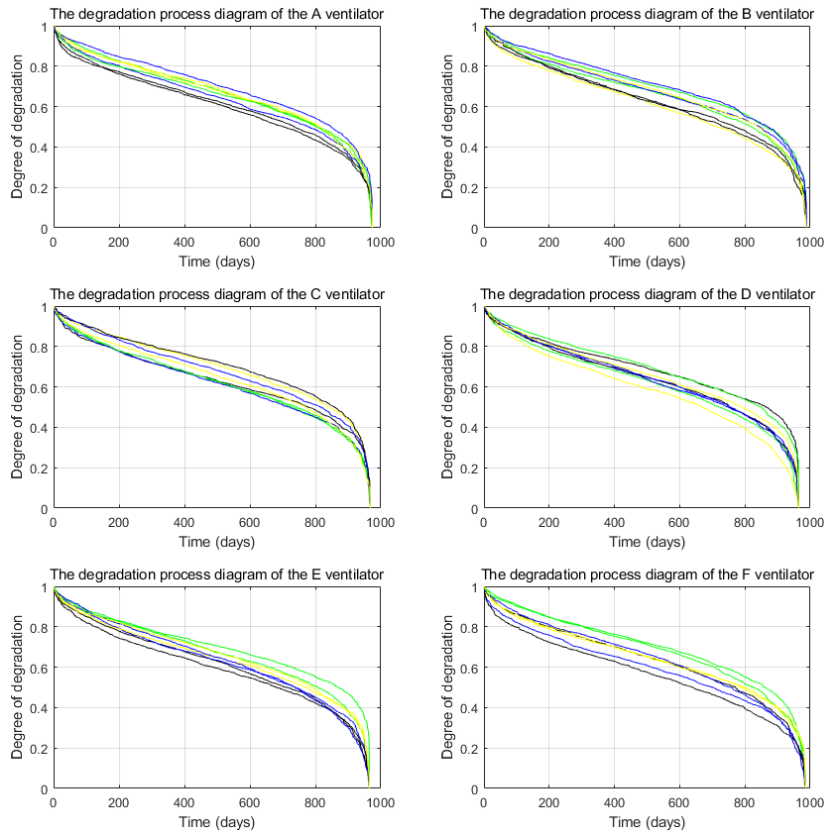


Fig. 6. Degradation process of 6 equipment.

4.2. Probabilistic RUL prediction of multi-equipment

The following is the probabilistic RUL prediction process of the wind turbine (after this, referred to as the equipment). Firstly, outliers in the data are dealt with using a sliding average filtering method to reduce invalid data. Then, the LSTM prediction model was trained and tested on degraded data from one equipment (equipment A) using the LSTM. The first 60 % of the data set is used as the training set (only this part of the data is selected for training). In order to evaluate the performance of the prediction model on unknown data, the remaining 40 % is used as the test set to test the prediction effect. The results are shown in Fig. 7. Finally, this equipment's trained LSTM prediction model is obtained.

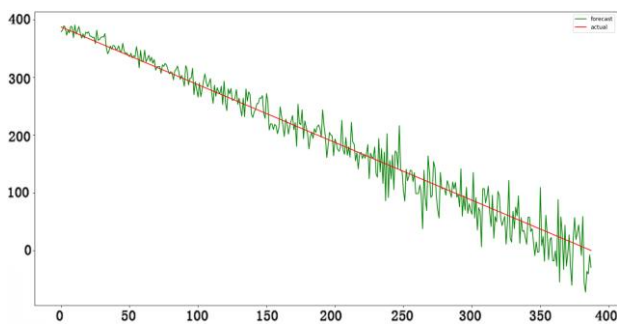


Fig. 7. Continuous RUL prediction diagram for one piece of equipment.

Then, VAE is used to resample the degradation data of this equipment 200 times, and 200 groups of reconstructed data of this equipment are obtained. At the current moment (day 584), we need to evaluate the probabilistic RUL prediction of equipment, so these reconstructed data are taken as the first 60% (first 583 days) of the data as the known data to be brought into the trained LSTM prediction model respectively, to get the 200 sets of outputs for this equipment on day 584. All the predicted values on the 584th day were taken from 200 sets of outputs for statistical calculation to obtain the probability distribution. This probability distribution indicates uncertainty in the RUL prediction of this equipment on the 584th day, and the probability distribution diagram is shown in Fig. 8.

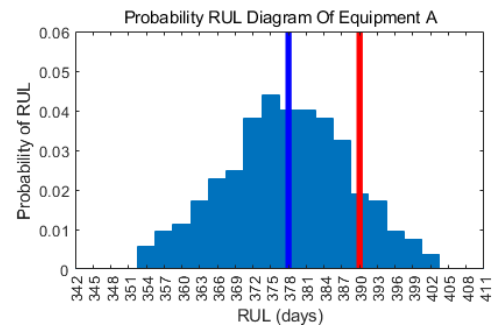


Fig. 8. Probability distribution diagram of RUL prediction for one piece of equipment on the 584th day.

For the equipment's training and test results in Fig. 7, the root mean square error (RMSE) value is 23.903. Among them, the remaining life at the current time (day 584) is predicted to be 382 days, and the actual remaining life is 390 days.

In Fig. 8, the blue line represents the average predicted RUL value, and the red line represents the actual RUL value.

To illustrate the accuracy of the prediction results, we compare the model prediction results (probabilistic RUL distribution) with the real data (real RUL values). Based on the probabilistic RUL prediction results in Fig. 8, this paper uses mean square error (MSE) and percentage error (PE) to evaluate the prediction accuracy between the probabilistic RUL distribution and the real RUL values. In this case, MSE is

a measure of prediction accuracy by calculating the mean of the sum of squares of the differences between the predicted and real values, and PE is a measure of prediction accuracy by calculating the differences between the predicted and real values, which is expressed in the form of a percentage. According to the calculation, the MSE value of its 200 predicted values is 26.44 and the average PE value is 3.53 %. The smaller the RMSE, MSE, and PE values, the higher the prediction accuracy, indicating that the equipment probability RUL prediction has a certain reliability.

Similarly, this paper considers probability RUL prediction for six equipment of the same type in a wind farm, and the prediction results are shown in Fig. 9:

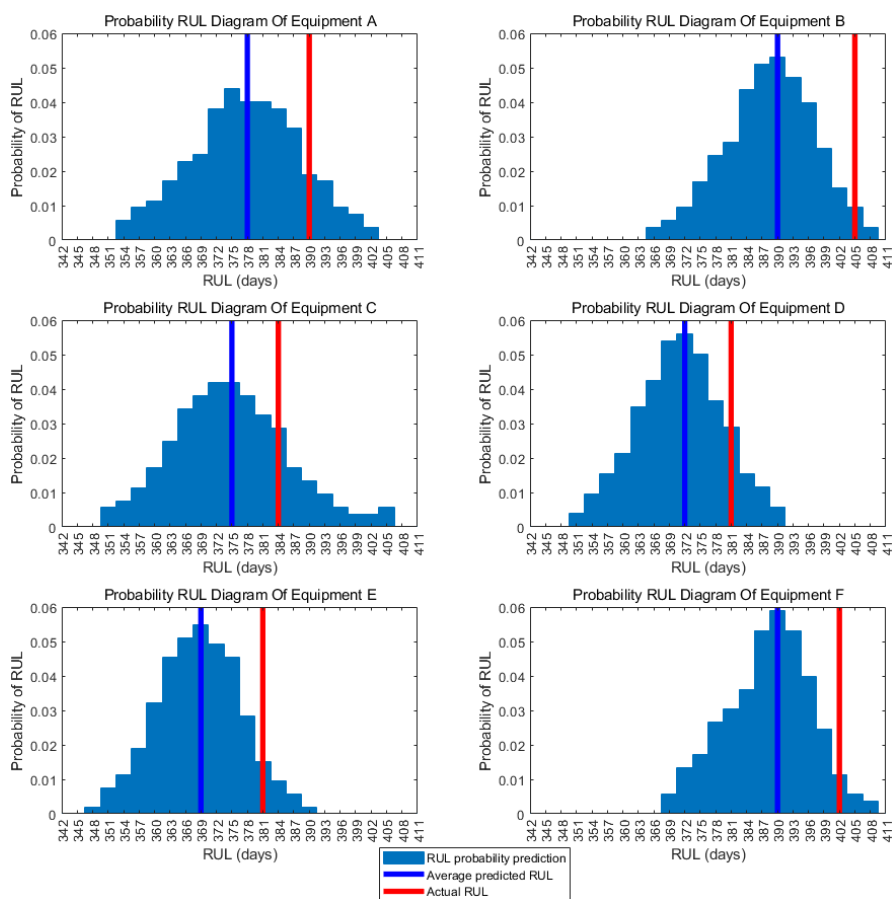


Fig. 9. RUL probability distribution for six equipment.

Table 3 shows the MSE and PE values of the six equipment. The MSE values of the six equipment range from 18.32 to 26.44, with an average MSE value of 23.84. The PE values of the six equipment range from 2.92% to 4.14%, with an average PE value of 3.42%. According to the analysis in Table 3, it can be seen that the prediction accuracy of the six equipment is relatively good, and the probabilistic RUL prediction has some reliability.

Table 3. MSE and PE values for six equipment.

equipment	MSE	PE
A	26.44	3.53%
B	25.53	4.14%
C	25.39	3.11%
D	21.53	2.92%
E	18.32	3.36%
F	25.81	3.47%

4.3. Multi-equipment maintenance experiment design

This is to prove the superiority of the multi-equipment dynamic grouping maintenance model in reducing maintenance costs and that the GOA algorithm has the advantage of solving accuracy when solving the model. This paper's two simulation experiments are designed: the algorithm comparison experiment and the maintenance strategy comparison experiment.

Both simulation software uses MATLAB R2021b, and the hardware configuration is Intel (R) Core (TM) i5-13400F 2.50GHz; 32GB.

(1) Algorithm performance analysis

To demonstrate the advantages of the Gazelle Optimization Algorithm (GOA) in terms of solution accuracy, the algorithm comparison experiment compares the GOA, GA, and PSO algorithms. All three algorithms have the following initial settings. The population size is 200, the number of iterations is 700, and the number of repeated experiments is 30. All three algorithms use the probabilistic RUL prediction results in Fig. 9 and the maintenance model parameters in Table 4 to solve the multi-equipment dynamic grouping maintenance decision model. Finally, the box plot is used to show the solution results of the three algorithms, and the optimal value, average value, and median are used to analyze the algorithm's simulation results.

Table 4. Table of maintenance model parameters.

Parameter	Value (units)
M	6 (sets)
N	1000 (days)
C_f	6000 (RMB/set)
C_{sp}	150,000 (RMB/set)
C_{ec}	35,000 (RMB/time)
C_e	400 (RMB/hour)
T_s	72 (hours)
T_d	1000 (days)
C_a	50000 (RMB/time)

(2) Maintenance strategy comparison

After proving the advantages of the GOA algorithm in solving the dynamic grouping maintenance model in terms of accuracy, in order to prove that the maintenance decision of a multi-equipment dynamic grouping has certain advantages in

reducing maintenance costs, the maintenance strategy comparison experiment compares the dynamic grouping maintenance of multi-equipment with the independent maintenance of multi-equipment without considering the dynamic grouping. The maintenance strategy comparison simulation results are analyzed using three indicators: the maintenance cost rate, the total maintenance cost, and the number of days used by all equipment. The maintenance models for both strategies are solved using the GOA algorithm, both using the probabilistic RUL prediction results in 4.2 and the maintenance model parameters in Table 4.

Independent maintenance of multiple equipment s means that the maintenance strategy is based on each equipment's probability RUL prediction results, the minimum maintenance cost rate of each independent equipment is the target, and the optimal maintenance time corresponding to each independent equipment is obtained. Finally, the optimal maintenance time of all independent equipment constitutes a multi-equipment independent maintenance decision.

4.4. Multi-equipment maintenance experimental results

(1) Results of algorithm performance

The GOA, GA, and POS algorithms run the multi-equipment dynamic grouping maintenance model 30 times, and the solution result box line diagram is shown in Fig. 10. The comparison of the three indicators of optimal value, average value, and median is shown in Table 5.

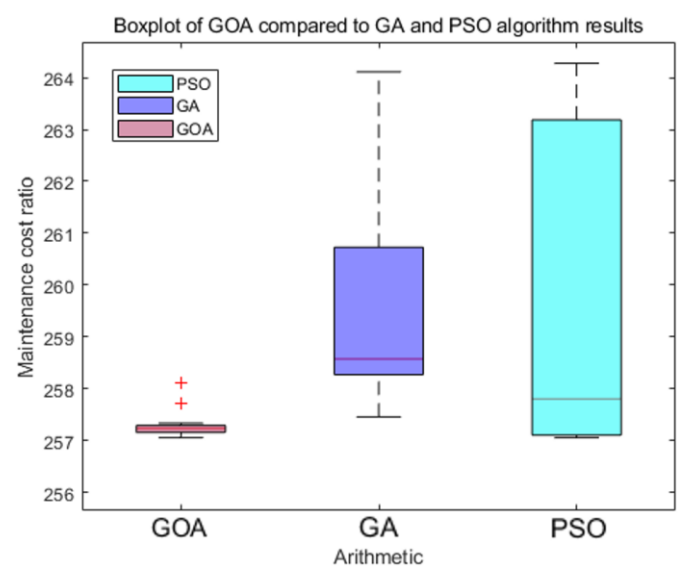


Fig. 10. Boxplot comparison of GOA, GA, and PSO algorithm results.

Table 5 Index comparison of GOA, GA, and PSO algorithm results

(Unit: yuan)	GOA	GA	PSO
Optimum (Min)	257.05	257.46	257.05
Mean	257.26	259.62	259.29
Median	257.23	258.57	257.80

According to Fig. 10, it can be seen that the GOA algorithm is significantly higher than the GA and PSO algorithms in terms of solution accuracy. According to Table 5, the results of the GOA algorithm are better than those of the GA and PSO algorithms in terms of optimal value, average value, and median, indicating that the GOA algorithm has certain advantages over the classical algorithms GA and PSO in solving the dynamic grouping maintenance model. It is easier to obtain a lower maintenance cost rate.

(2) Comparison results of two maintenance strategies

We compare the results of multi-equipment dynamic grouping maintenance decisions with those of multi-equipment independent maintenance decisions. In order to show more intuitively, Fig.11 considers the first 583 days based on Fig. 9, shows the probability distribution of the total life of 6 equipment, and shows the decision results of the two maintenance strategies. Fig.12 shows the maintenance plan of the two maintenance strategies.

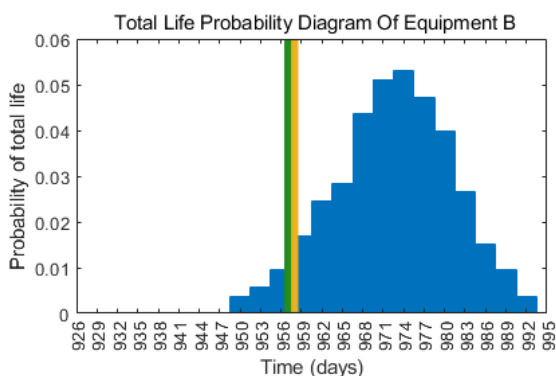
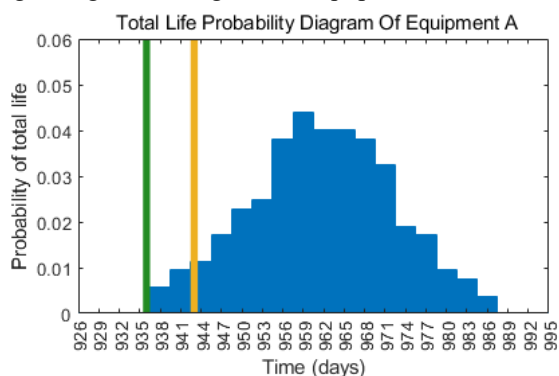
In Fig.11, the green line is the optimal maintenance time for each equipment of the dynamic grouping maintenance strategy, and the orange line is the optimal maintenance time for each equipment of the independent maintenance strategy. In Fig.12, the green 'stars' constitute the maintenance plan of the dynamic grouping strategy, and the orange 'stars' constitute the maintenance plan of the independent maintenance strategy.

According to Fig.11 and Fig.12, six equipment are divided

into two groups according to the dynamic grouping maintenance plan. Among them, the optimal maintenance time of wind power equipment A, C, D, and E is 936 days, and they are regarded as a group. The optimal maintenance time for wind power equipment B and F is 957 days for both, and they are regarded as a group. The essential maintenance cost of the maintenance plan (including cost of labor, spare parts cost, maintenance tool input and output costs) is 1006000 yuan, the cost of downtime loss is 172800 yuan, the mathematical expectation of sudden failure and emergency maintenance cost is 224600 yuan, and the cost of equipment depreciation loss is 51300 yuan. Therefore, the total cost of the dynamic grouping maintenance plan is 1454700 yuan, the average running days of the six pieces of equipment is 943 days, and the maintenance cost rate (the average daily maintenance cost of each equipment) is 257.1 yuan.

According to the independent maintenance plan, the optimal maintenance time from equipment A to equipment F is 943, 958, 941, 940, 937, and 960 days, respectively. The essential maintenance cost of the maintenance plan is 1146000 yuan, the cost of downtime loss is 172800 yuan, the mathematical expectation of sudden failure and emergency maintenance cost is 255450 yuan, and the cost of equipment depreciation loss is 48150 yuan. The total cost of the independent maintenance plan is 1622400 yuan, the average running days of the six pieces of equipment is 946.5 days, and the maintenance cost rate is 285.7 yuan.

The maintenance cost rate of the two maintenance methods, the total maintenance cost, and the average running days of the six equipment are shown in Table 6.



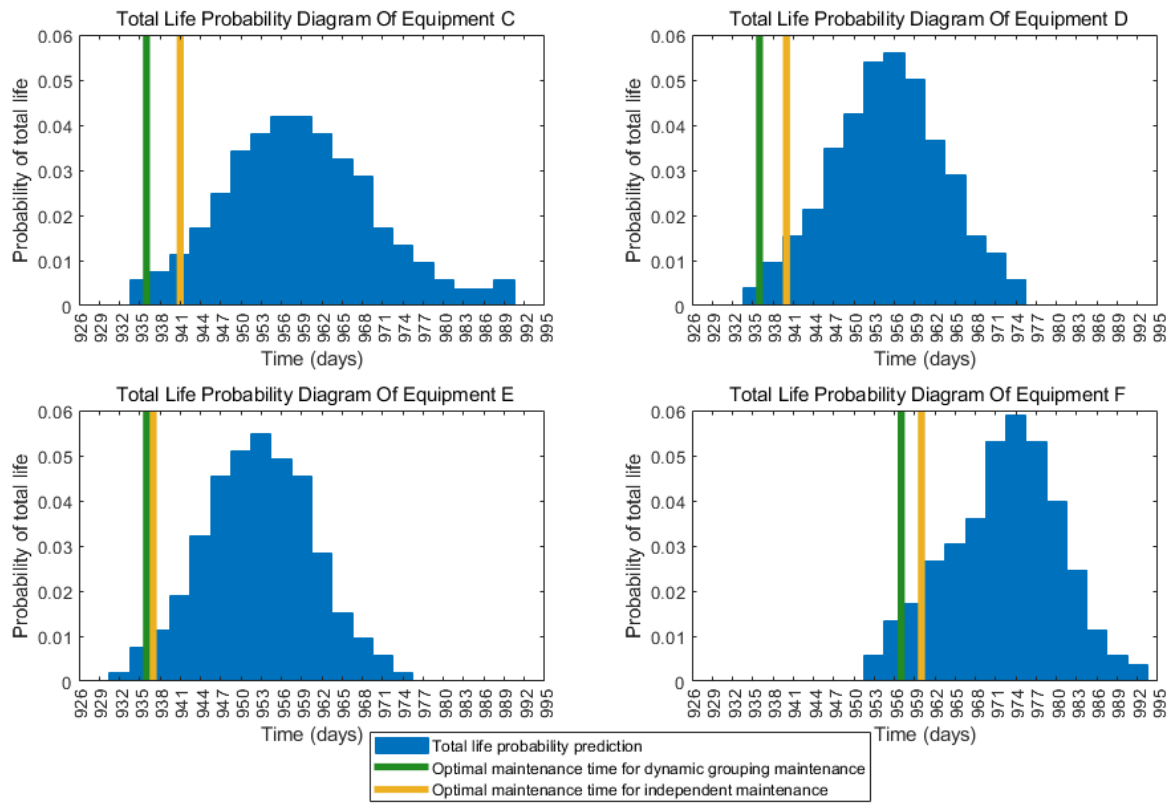


Fig.11. Decision results of dynamic grouping maintenance and independent maintenance.

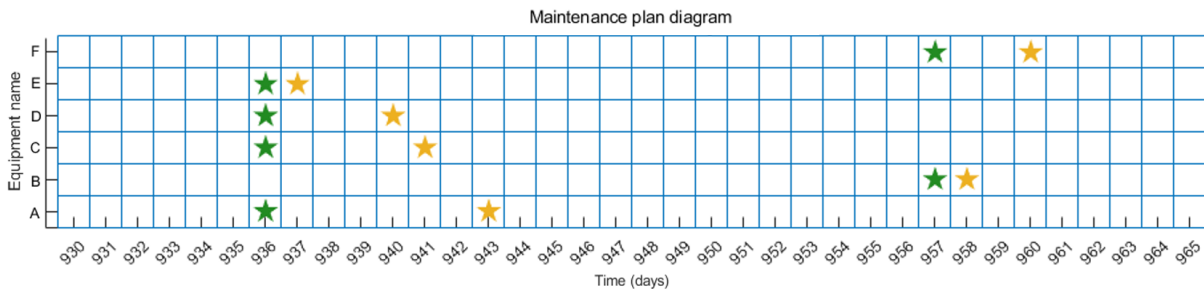


Fig.12. Dynamic grouping maintenance plan and independent maintenance plan.

Table 6. Comparison of results of dynamic grouping maintenance and independent maintenance.

(Unit: yuan)	Maintenance of dynamic grouping	Independent maintenance	difference between the two
Maintenance cost rate	257.1	285.7	28.6
Total maintenance cost	1454700	1622400	167700
Average running days of 6 equipment	943	946.5	3.5

According to Table 6, compared with independent maintenance, the average running days of dynamic grouping maintenance are 3.5 days less than those of independent maintenance. However, in terms of total cost, the former is 167,700 yuan less than the latter, a decrease of 10.34%. Finally, in terms of maintenance cost rate, the former is 28.6 yuan less than the latter, which is reduced by 10.01%.

4.5. Discussion

According to the multi-equipment probability RUL prediction results, multiple new samples can be obtained due to the Variational Auto-Encoder's resampling of the equipment degradation data. LSTM is suitable for processing long sequence data. It can predict the degradation trend more accurately. Therefore, reliable equipment probability RUL prediction results can be obtained by resampling and LSTM.

According to the comparison results of the algorithm, since GOA can continuously jump out of the local optimal solution when solving the dynamic grouping maintenance model, GOA algorithm has certain advantages in solving accuracy compared with the classical algorithms GA and PSO. It is easier to obtain a lower maintenance cost rate.

According to the comparison results of maintenance strategies, it can be seen that the independent maintenance of multi-equipment only determines the optimal maintenance time of each independent equipment based on the probability RUL prediction results of each equipment, which does not consider the cost reduction caused by the dynamic grouping maintenance of multi-equipment (such as the maintenance of the same group of equipment only costs the entry and exit costs of maintenance tools once). So, the maintenance cost rate and total maintenance cost are relatively high. Multi-equipment dynamic grouping maintenance reduces maintenance costs, reduces unnecessary waste, and improves the competitiveness of enterprises compared with multi-equipment independent maintenance.

5. Conclusion

This paper proposes a multi-equipment dynamic grouping maintenance decision-making method based on probabilistic RUL prediction considering the uncertainty of equipment remaining useful life prediction and the economic correlation in the multi-equipment maintenance process. Six wind turbines

with the same type are used as case objects for experimental verification. The experiment summarizes some important findings as follows.

(1) After resampling the degradation data of six devices, the average MSE value between the 200 predicted RUL values and the real RUL values at a specific time is 23.84, and the average PE value is 3.42 %, which can prove that the probability RUL prediction of the equipment has a certain reliability.

(2) By comparing the algorithms, the Gazelle Optimization Algorithm (GOA) is better than the GA and POS algorithms, the optimal value of the solution is relatively better, and the solution accuracy is relatively higher.

(3) Compared with multi-equipment independent maintenance, the maintenance cost rate of multi-equipment dynamic grouping maintenance is reduced by 10.01 %, and the total maintenance cost is reduced by 10.34 %. So, the multi-equipment dynamic grouping maintenance in this paper considers the economic correlation and can effectively reduce the maintenance cost.

In the future, we will consider more components to determine the remaining useful life of the equipment. At the same time, it is more than determining a maintenance plan only once. Over time, it can change the maintenance plan to adapt to new predictive information in time to reduce the maintenance cost.

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