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Analysis of remaining useful life of slope based on nonlinear wiener process

Indexed by:



Ming Huang^{a,b}, Weihai Yu^a, Fan Yang^{a,b,*}

^a School of Civil and Hydraulic Engineering, Hefei University of Technology, China

^b Anhui Province Key Laboratory of Water Conservancy and Water Resources, China

Highlights

- A remaining useful life (RUL) prediction model is proposed to better tackle the life evaluation problem in the slope degradation process.
- The probability density function (PDF) of RUL is deduced by the least squares method (LSM) and the maximum likelihood estimation method (MLEM).
- A linear model (M1) and two nonlinear models (M2 and M3) are estimated and compared using the measured displacement data of the slope.

Abstract

A remaining useful life (RUL) prediction model based on the nonlinear Wiener process is proposed to better tackle the life evaluation problem in the slope degradation process. Taking the displacement of the slope as its performance degradation index, and the nonlinear Wiener process is used to establish the RUL prediction model of the slope. For this model, the least squares method (LSM) is used to estimate the drift coefficients, the maximum likelihood estimation method (MLEM) is used to estimate the diffusion parameters, and then the probability density function (PDF) of the RUL of the slope is deduced and the RUL is predicted. The proposed model is verified by slope engineering examples. The results demonstrated that the RUL of the degradation model based on the nonlinear Wiener process has a greater prediction accuracy than the linear Wiener process. Because the various nonlinear functions have varying slope adaptations, and it can predict the RUL of a slope more accurately, which can provide more reliable preventive maintenance decisions.

Keywords

nonlinear Wiener process, slope, displacement, parameter estimation, RUL prediction

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1. Introduction

As one of the four major global geological hazards (earthquakes, floods, landslides and debris flows), landslides often destroy buildings, block traffic, damage rivers and cause casualties. It poses a serious hazard to human life and engineering construction[1-3]. Slope displacement monitoring is an important means and measure to grasp the status of slopes and ensure their safety[4,5]. As the most significant parameter to characterize slope changes, slope displacement can reflect the changing condition and development trend of slopes. Therefore, if the development trend of slope displacement can be

accurately monitored and predicted[6,7], disaster mitigation and prevention measures can be taken as early as possible.

Since the slope displacement monitoring data directly reflects the overall safety of the slope, the degradation process has received much attention[8,9]. He et al. established a slope displacement vector angle and displacement rate prediction criterion using the data from the slope monitoring point of Xin Tan, and the prediction results matched with the actual slope instability time and pattern[10]. Ma constructed a grey least squares support vector machine prediction model to predict

(*) Corresponding author.
E-mail addresses:

M. Huang 2009800034@hfut.edu.cn, W. Yu yuweihai12345@163.com, F. Yang (ORCID: 0000-0002-6532-8619) fyang@hfut.edu.cn

future slope displacements using the sequence of measured slope displacements[11]. Venkatesan et al. demonstrated the applicability of the improved Bayesian classification technique to landslide sensitivity early warning models on data mining[12]. Considering the displacement monitoring information, Cheng et al. proposed an analysis method to judge the stability of soil slopes, and a non-linear function relationship between the strength reduction coefficient and slope displacement is established[13]. Considering the displacement information fusion, Feng et al. proposed a dynamic early warning method for open pit slopes to carry out real-time zonal safety warnings of slopes and predict the stability of future excavation processes of slopes[14]. Chakraborty et al. used multiple linear regression (MLR) and artificial neural networks (ANN) to predict the slope stability and compared the finite element outputs with the developed prediction models to find the best prediction model[15]. Considering the particle migration and variation, Qi et al. proposed a particle swarm optimization algorithm to predict the slope excavation displacement and stability[16]. In recent years, some new models have been proposed to predict the slopes. Liu et al. proposed a Physics-informed Data Assimilation method for landslide displacement forecasting, which can enhance the prediction ability of the physical forecasting model and solve the problem of ignoring the physical significance of landslides in the mathematical prediction model[17]. Wang et al. proposed a new methodology for analyzing slope stability based on three techniques: interferometric synthetic aperture radar, unmanned aerial vehicle, and ground-based interferometric synthetic aperture radar [18]. Besides, Lin et al. proposed a combined neural network prediction model that combines a temporal convolutional neural network and a bidirectional long short-term memory neural network to address the shortcomings of traditional recurrent neural networks in predicting displacement-fluctuation-type landslides[19]. All in all, the prediction work based on displacement monitoring data has achieved more achievements, but it can only predict the displacement value for a short period, and the remaining useful life prediction of slope can grasp the time of slope damage and can carry out better maintenance work.

RUL prediction, as the basis for real-time mastering of system operation status and making predictive maintenance

plans, has already gained interest in several domains[20-22], including slope engineering. RUL prediction methods are mainly divided into mechanistic model-based RUL prediction and degradation data-based RUL prediction. The former is accurate but very difficult to model the degradation process. The latter does not require prior knowledge of the product degradation mechanism and only requires real-time measurement of the product output parameters to reveal the internal and external influences on the bases of the measured parameters. The prediction of RUL based on degradation data has become a research hotspot in recent years. Wang et al. applied a one-dimensional linear Wiener process with drift to model and used the maximum likelihood estimation method (MLEM) to estimate the initial parameters, and the effectiveness of the Wiener process-based method for predicting the RUL of an aeronautical hydraulic axial piston pump is verified by the final experimental results[23]. Freitas et al. considered a linear degradation model with only the slope and assumed that the inverse of the slope obeys the Weibull distribution, and a Bayesian estimation method for the parameters as well as the product reliability is proposed[24]. Oliveira & Colosimo compared three methods (the pseudo-life approximation method, the analytical method and the simulation method) to estimate the failure distribution on the bases of the linear degradation model[25]. Zhu et al. established a Wiener process model on the bases of the performance degradation through battery degradation data, and its reliability function analysis can provide a scientific and accurate assessment of lithium batteries[26]. Based on a multi-stage Wiener process degradation model, Liu et al. proposed a method to predict the remaining life of an aero-engine and provide a basis for the formulation of aero-engine maintenance plans[27]. Based on a one-dimensional linear Wiener process, Li et al. established a bank slope degradation model to verify the remaining life prediction for bank slopes, which can provide technical support for early warning of bank slopes[28]. On this basis, Feng et al. constructed a random-effects Wiener process model for the slopes of water diversion projects and used Bayesian methods to achieve updating of parameters[29]. For upcoming measured displacements, a precise estimation of the slope's RUL may be made. However, current research on the RUL of slopes only stays in the linear degradation model. Due

to the complex and variable operating environment of slopes, only considering linear degradation does not adequately reflect the health state of slopes and may cause errors in the life prediction results, so it is necessary to study the feasibility of the non-linear degradation model for the health state assessment of slopes.

In light of the aforementioned issues, based on a nonlinear Wiener process, a model is proposed to estimate the slopes' RUL. The slope displacement is used as its degradation index, the nonlinear Wiener process model is used to describe the displacement degradation of the slope, and the unknown parameters are obtained through two-step estimation. The proposed model can realize the prediction of the remaining useful life of the slope and provide a guiding basis for the later maintenance decision of the slope.

2. Modeling

2.1 Wiener process modeling

The Wiener process can describe the non-monotonic performance degradation process and individual variability and has good computational power, so it is one of the widely used performance degradation models in the field of reliability[30]. If the Wiener process $\{B(t), t > 0\}$ satisfies the following three properties.

(1) $B(0) = 0$ is certain to hold.

(2) $B(t)$ is a smooth, independent increment.

(3) The increment $\Delta B(t) = B(t + \Delta t) - B(t); \Delta t > 0$, follows a normal distribution with mean and variance of 0 and respectively.

Then $B(t)$ satisfying the above conditions is called the standard Wiener process and is used for modeling. Let $X(t)$ denote the amount of degradation of the slope at t . The equation for modeling the degradation of the slope is:

$$X(t) = x(0) + at + \sigma_B B(t) \quad (1)$$

Let $x(0)$ denotes the displacement of the slope at the initial moment. It is usually considered that $x(0) = 0$ [31] in engineering applications, define $X(t) = X(t) - x(0)$, then the constructed linear Wiener process can be expressed as:

$$X(t) = at + \sigma_B B(t) \quad (2)$$

where a is the drift coefficient, and σ_B is the diffusion coefficient.

The drift coefficient of the Wiener process in Eq. (2) is a linear function of time. However, with the change of external

environment, the displacement of slope in the degradation process usually has non-linear characteristics, it is more applicable to build a displacement degradation model for nonlinear Wiener processes, which can be expressed as [32]:

$$X(t) = x(0) + a \int_0^t \mu(b; t) dt + \sigma_B B(t) \quad (3)$$

where $\mu(b; t)$ and $B(t)$ are mutually independent, and $\int_0^t \mu(b; t) dt$ is a non-linear function parameterized by b , which is used to characterize the non-linear nature of slope displacement degradation.

2.2 RUL prediction

The RUL of a slope is the time at which the failure threshold is first reached by a random degradation process. If the time when the equipment first reaches the failure threshold is predicted at t_i based on historical data, the RUL of the slope is obtained, the basic principle of which is shown in Fig. 1.

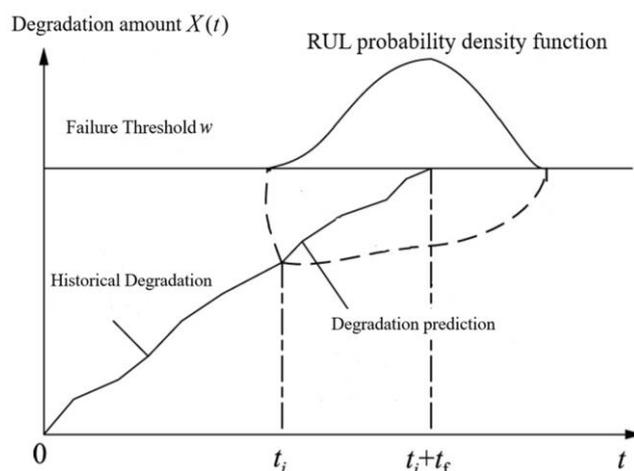


Fig. 1. Sketch of the principle of RUL prediction.

In Fig. 1, w is the set failure threshold and the RUL t_f usually occurs at the maximum value of the PDF of RUL. During operation, the slope degrades continuously and its RUL decreases with time, when the quantity of degradation reaches the pre-set failure threshold w for the first time, the slope will suffer instability damage, at which point the RUL of the slope is zero. Therefore, the end of slope life is the time when the random degradation process $X(t)$ first crosses the failure threshold w , the RUL of the slope t_f can be defined as:

$$t_f = \inf\{t_f: X(t_f + t_i) \geq w | X(t_i) < w\} \quad (4)$$

where t_f is the RUL, t_i is the current moment of the slope, and $X(t_i)$ is the current moment of the slope displacement.

2.3 Life prediction models

RUL prediction by linear Wiener process has been widely used in the fields of machinery, lithium batteries, etc., and the application results tend to be mature, among which the literature [28,29] has achieved good results in slope RUL prediction. However, different systems often exhibit different nonlinear characteristics during degradation, so it is necessary to choose a nonlinear function that is compatible with the degradation characteristics of the system, which helps to improve the prediction accuracy. In the field of slope engineering, there is less research on the selection of non-linear functions, the degradation model based on the nonlinear Wiener process has achieved good results in the fields of machinery and battery, etc. So this article builds on their degradation laws study experience with nonlinear function selection[6,33,34], and focuses on the analysis of two forms of nonlinear functions, $b\exp(bt)$ and b' in slope engineering.

The nonlinear degenerate models derived by inserting a and b into Eq. (3) for the integration process are shown in Eqs. (5) and (6) and labeled M2 and M3, respectively, whereas the linear degenerate model described in Eq. (2) is labeled M1.

$$M2: X(t) = at^b + \sigma_B B(t) \quad (5)$$

$$M3: X(t) = a(e^{bt} - 1) + \sigma_B B(t) \quad (6)$$

The constructed model demonstrates that M2 and M3 add nonlinear functions $\mu(b, t)$ compared to M1, M1 is a special case of M2 when $b=1$. M2 and M3 are two different forms of nonlinear construction. M2 is the power function form and M3 is the exponential form.

In the process of calculating the RUL of a slope, the key is to calculate the PDF of the slope RUL. By the definition of the RUL, the first reach time in the Wiener process obeys the inverse Gaussian distribution. Since the PDF of RUL is explicit for linear Wiener processes and implicit for nonlinear Wiener processes, in order to make the PDF explicit, it is represented by the approximate expression. The PDFs[35] for each degradation model were obtained separately as follows:

$$f_{M1}(t) = \frac{w}{\sqrt{2\pi\sigma_B^2 t^3}} \exp\left(-\frac{(w-at)^2}{2\sigma_B^2 t}\right) \quad (7)$$

$$f_{M2}(t) \cong \frac{w-at^b(1-b)}{\sigma_B \sqrt{2\pi t^3}} \exp\left(-\frac{(w-at^b)^2}{2\sigma_B^2 t}\right) \quad (8)$$

$$f_{M3}(t) \cong \frac{w-a\beta(t)}{\sigma_B \sqrt{2\pi t^3}} \exp\left(-\frac{(w-a\gamma(t))^2}{2\sigma_B^2 t}\right) \quad (9)$$

$\gamma(t)$ and $\beta(t)$ in Eq. (9) can be presented as Eq. (10).

$$\begin{cases} \gamma(t) = \exp(bt) - 1 \\ \beta(t) = (1 - bt)\exp(bt) - 1 \end{cases} \quad (10)$$

3. Parameter estimation

The parameters in each model constructed were determined based on the characteristics of the monitoring data. The slope life PDF obtained in Eqs. (7)-(10), the M1 uses the MLE to estimate the parameters. Let $0 = t_1 < t_2 < t_3 < \dots < t_n = t$ denotes the monitoring moment of the slope during the period $[0, t]$, corresponding to the amount of slope degradation $X = \{x_1, x_2, x_3, \dots, x_{n1}\}$. The likelihood function for the degradation of the slope displacement is:

$$L(a, \sigma_B) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma_B^2 \Delta t_i}} \exp\left(-\frac{(\Delta x_i - a\Delta t_i)^2}{2\sigma_B^2 \Delta t_i}\right) \quad (11)$$

$$\begin{aligned} \ln L(a, \sigma_B) &= -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(\sigma_B^2) - \frac{1}{2\sigma_B^2} \sum_{i=1}^n \frac{(\Delta x_i - a\Delta t_i)^2}{\Delta t_i} + \\ &\ln\left(\prod_{i=1}^n \frac{1}{\sqrt{\Delta t_i}}\right) \end{aligned} \quad (12)$$

Find the partial derivatives of a and σ_B^2 respectively, and make them equal to 0, i.e.:

$$a = \frac{\sum_{i=1}^n \Delta x_i}{\sum_{i=1}^n \Delta t_i} \quad (13)$$

$$\sigma_B^2 = \frac{1}{n} \sum_{i=1}^n \frac{(\Delta x_i - a\Delta t_i)^2}{\Delta t_i} \quad (14)$$

Among others, $\Delta x_i = x_i - x_{i-1}$, $\Delta t_i = t_i - t_{i-1}$.

The nonlinear degenerate models M2 and M3 have parameters $\theta(a, b, \sigma_b)$, and the nonlinear least squares approach is used to estimate the unknown parameters $\theta(a, b)$ so that A in Eq. (15) is minimized.

$$A = \min \sum_{i=1}^n \left(X(t) - a \int_0^1 \mu(t; b) dt \right)^2 \quad (15)$$

Let $Z(t) = X(t) - a \int_0^1 \mu(t; b) dt = \sigma_B B(t)$, $Z(t)$ can be viewed as a special case of a linear Wiener process with a drift coefficient equal to 0, and estimating parameters using the MLE. $\Delta Z(t) = Z(t+1) - Z(t)$, ($t = 1, \dots, n$) represents the degradation increase at Δt , the maximum likelihood function $L(\sigma_B^2)$ is:

$$L(\sigma_B^2) = \prod_{i=1}^n \frac{1}{\sigma_B \sqrt{2\pi \Delta t}} \exp\left(-\frac{(\Delta Z(t))^2}{2\sigma_B^2 \Delta t}\right) \quad (16)$$

Derivation of Eq. (16):

$$\frac{\partial \ln L(\sigma_B^2)}{\partial \sigma_B^2} = -\frac{n}{2\sigma_B^2} + \sum_{i=1}^n \left(\frac{(\Delta Z(t))^2}{2\Delta t \sigma_B^4} \right) = 0 \quad (17)$$

Then the maximum likelihood estimates for σ_B^2 is:

$$\sigma_B^2 = \frac{1}{n} \sum_{j=1}^n \frac{(\Delta Z(t))^2}{\Delta t} \quad (18)$$

4. Example analysis

Using the amount of slope displacement degradation as the characteristic value, two slopes in China are used as examples for calculation using the model in the article to verify the adaptability of each model.

4.1 Brief description of the case

Case 1: The Xin Tan landslide, which occurred in June 1985, is located in Hubei Province, China, down 27km from the Three Gorges Dam Project. The elevation of the leading and trailing edges of the landslide is 70~900m, with a relative height difference of more than 800m. The total volume of the landslide is about 30 million m³. The back edge of the landslide and the western boundary is a steep wall of bedrock composed of Devonian Permian sandstone and limestone, while the eastern

boundary is a fissured surface cut into the avalanche accumulation; the accumulation is generally 30~40m thick, dominated by gravel and clay; the underlying bedrock surface is Silurian sand and shale, and the form is relatively complex.

Case 2: A water diversion channel project is located in Xinjiang Province, China. The geological conditions of the channel slope are complex, with the upper part of the slide being loess and the underlying bedrock of the sand and gravel being Tertiary mudstone. The slope has a history of landslides and is still in a creep-slip state.

4.2 Degradation data and failure thresholds

According to the slope displacement monitoring information, Case 1 takes the measured displacement data from May 1978 to September 1984 at the monitoring site, and Case 2 takes the measured displacement data from July 18th, 2011 to July 23th, 2017 for Wiener process degradation modeling, and their performance degradation trends are shown in Fig. 2.

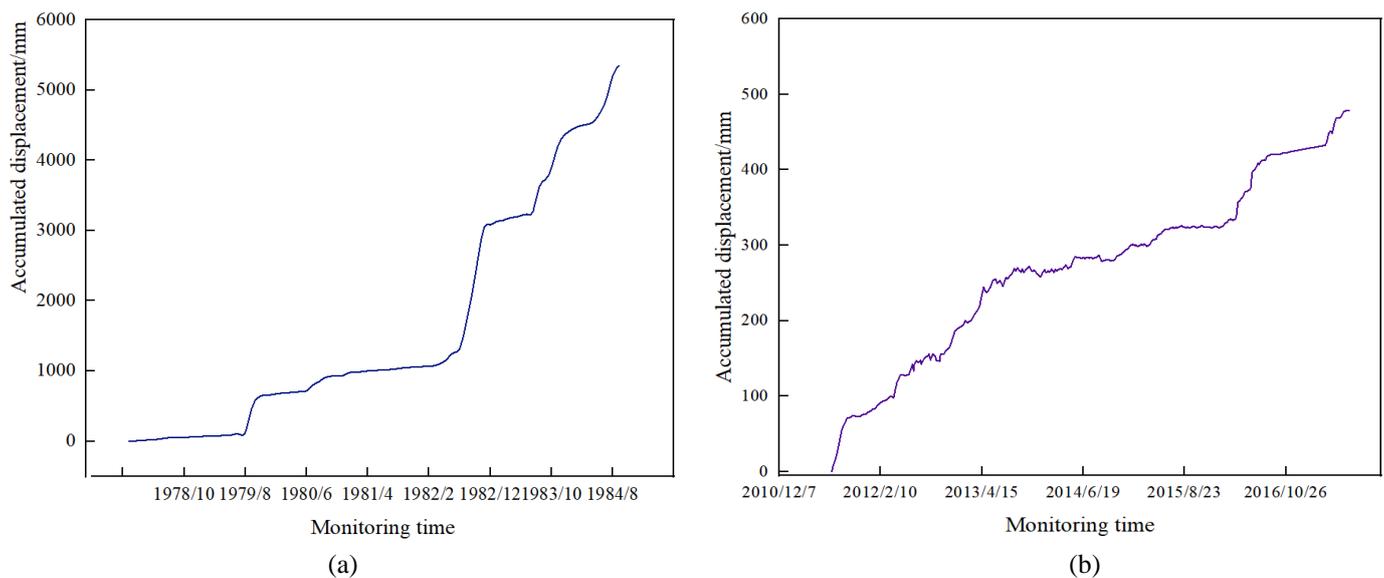


Fig. 2. Displacement degradation curve graph:(a) Case 1, (b) Case 2.

To determine the slope failure threshold, many scholars such as Lin et al.[36], Miao et al.[37], and Qian et al.[38] have made some progress in this field. To consider the important role of displacement monitoring information in slope prediction, He et al. used the monitoring information of slope creep-slip displacement and the correlation between damage variables and stability coefficients, in this way, a displacement warning criterion is constructed based on the safety factor and the slope's initial elastic deformation[39]. According to the literature[39], the slope threshold can be obtained from Eq. (19).

$$w = \frac{F_{cr} S_0}{F_{cr} - 1} \quad (19)$$

Where F_{cr} is the critical safety factor for the slope, and S_0 is the initial elastic deformation of the slope.

According to Eq. (19) and the specification DLT5353-2006[40], the displacement threshold of case 1 can be determined as 7150mm, and the displacement threshold of case 2 is 814mm.

4.3 RUL prediction results and analysis

For the M1 model, the likelihood function is constructed using MLEM and displacement monitoring data as in Eq. (11), and the parameter values can be obtained separately by taking partial derivatives of the parameters through Eq. (12). For the M2 and M3 models, the drift coefficients in the Wiener process are first determined using nonlinear LSM, so that A is the minimum value in Eq. (15), and then the diffusion parameters are determined using MLEM in the same way as above, and the parameter values can be obtained as in Eq. (18). Then each parameter of M1-M3 model was estimated using the measured displacement data of the slope in case 1 of Fig. 2(a), and the estimated values of each model parameter are shown in Table 1 below.

Table 1. Estimated values of different model parameters.

Models	a	b	σ_B
M1	66.854	/	8.64*10 ³
M2	1.256	1.913	652.915
M3	365.951	0.036	6.43*10 ³

Based on the parameters of each model in Table 1, the PDFs of RUL under different degradation models are obtained, as shown in Fig. 3.

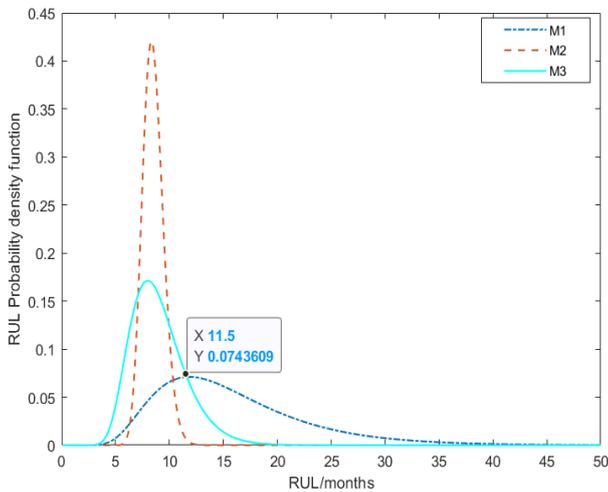


Fig. 3. PDFs of RUL for each degradation model.

Based on the modeling time of September 30, 1984 and the time of large-scale sliding of the Xin Tan landslide on June 12, 1985, it is known that the real RUL of the slope is 8.3 months. It can be observed from Fig. 3 that the RUL predictions for M1-M3 models are 11.5 months, 8.4 months, and 7.8 months, respectively, and the error rates for the M1-M3 models' RUL predictions are 38.5%, 1.22%, and 6.02%. This demonstrates

that:

(a) The nonlinear models M2 and M3 are clearly superior to the linear model M1.

(b) M1 has a larger error in slopes with nonlinear degradation characteristics, whereas M2 and M3 better capture the nonlinear degradation characteristics of slopes.

(c) The applicability of different nonlinear functions to different degradation data is different, resulting in different RUL prediction results. In case 1, model M2 is more applicable and has a stronger prediction impact than model M3.

The predicted life of the M2 model at different moments was further calculated. Using October 1983 as the starting point for prediction, the remaining life was predicted every month for the displacement values at the next monitoring moment, and the model parameters were updated. The results of the predicted values of the remaining life of the slope for each monitoring moment of the M2 model and the comparison of the errors are shown in Fig. 4.

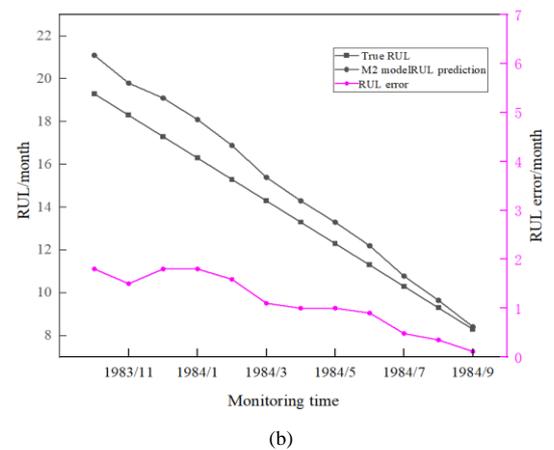
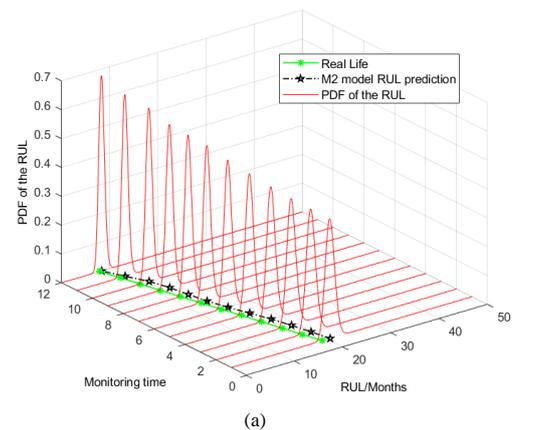


Fig. 4. Comparison of the error between the predicted and real values of the M2 model: (a) M2 prediction results for different time, (b) M2 model RUL error.

Fig. 4(a) depicts the RUL prediction results of each slope monitoring period, demonstrating that the M2 has a stronger prediction impact in Case 1. The RUL prediction errors for each period of the computational model are shown in Fig. 4(b), the results show that the average error of the M2 model is 0.59 months, and as the monitoring period increases and more displacement monitoring data are obtained, the prediction error of the M2 model has a tendency to gradually decrease. The prediction results of M1 and M3 were also calculated for each period, and the average error of model prediction was 3.35 months and 0.91 months respectively. The M1 model notably differs from the M2 model's prediction results, and the M3 model's prediction accuracy is lower than that of the M2 model, which shows that the M2 has better applicability in this case.

Identical calculations as in example 2 above, the degradation modeling of the Wiener process was performed with the measured displacement data from July 18, 2011 to January 29, 2017, and the PDF of its RUL under the M1-M3 model was calculated, as shown in Fig. 5. The damage of the slope occurred in June 2019, and it is known that the RUL is 122 weeks, and the RUL prediction results of M1-M3 model are 112.2 weeks, 125.2 weeks and 116.0 weeks, and the model prediction accuracy is $M2 > M3 > M1$, which shows that the M2 model is more consistent with the degradation characteristics of this slope.

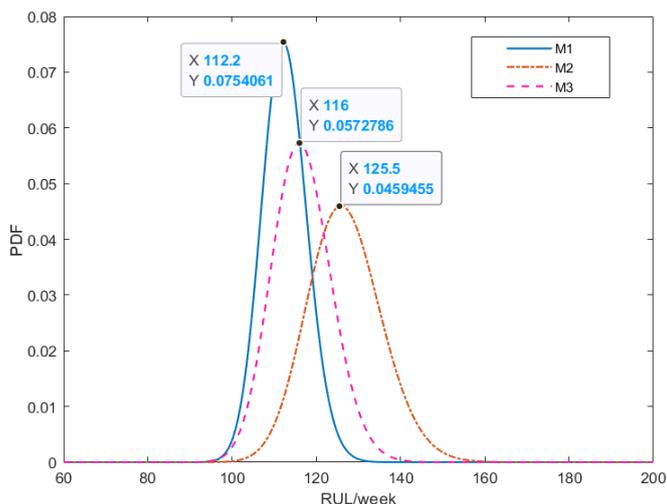


Fig. 5. PDFs of RUL for each degradation model.

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To further verify whether the M2 has better applicability compared with the other two models, using January 29th, 2017 as the starting point to predict the RUL for the subsequent 20 monitoring periods of displacement and updating the model parameters, the average error of prediction of M1-M3 is 9.45 weeks, 2.98 weeks and 7.46 weeks respectively, and the average relative error is 8.59%, 2.71% and 6.78%. It can be seen that the M2 of Case 2 slope is better than M1 and M3, it can predict the RUL of the slope more effectively, and the M2 model in Case 2 can predict the remaining service life of the slope more effectively and provide guidance for maintenance.

5. Conclusion

Concentrating on the RUL prediction issue in slope degradation, the displacement value is used to reflect the slope degradation characteristics as its performance degradation index. Considering that the slope displacement degradation usually presents nonlinear characteristics, the nonlinear Wiener process model is proposed to describe the slope displacement degradation based on monitoring data. The model parameters are determined by MLEM and LSM, and its PDF is derived and the RUL is predicted. According to the analysis of the examples, for Case 1, it is clear that the nonlinear degradation models M2 and M3 are better than the linear model M1, and the M2 has better applicability and higher accuracy of RUL prediction. For Case 2, M2 is also better suited to the slopes' nonlinear degradation characteristics, as can be seen, M2 has a higher prediction accuracy in the engineering example. It is concluded that the nonlinear Wiener process RUL prediction model outperforms the linear Wiener process RUL prediction model in terms of prediction accuracy. Different nonlinear functions adapt to various slopes, and the model with strong adaptability can better predict slope RUL. The results can be used as the basis for the subsequent study of slope health management and provide technical guidance for slope de-risking and strengthening.

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