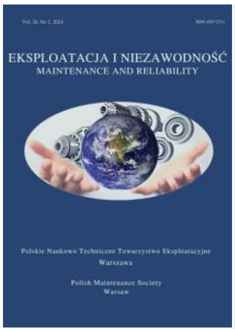

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Dynamic reliability calculation of random structures by conditional probability method

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Highlights

- The Conditional probability method for dynamic reliability.
- The Kriging model for numerical sampling method.
- The Kriging sampling process.

Abstract

Reliability is sometimes computed as the likelihood of achieving an intended function in the presence of uncertainties, and this is known as dynamic reliability by the conditional probability approach. These techniques can produce incredibly accurate reliability estimates. This work uses the dynamic response spanning action Markov hypothesis for the composite random reliability problem. Two steps are needed to describe conditional probability: first, the Taylor expansion approach is used to derive a 2nd-order approximate formula for determining the dynamic reliability of the random structure. The second step is to come up with a mathematical sampling strategy based on the statistical analysis's Kriging model. The Kriging interpolation model's sampling process satisfies the nonlinear association between structural random boundaries and dynamic reliability. Consequently, the finite element results can be used immediately to anatomize the impact of random structural parameters on dynamic reliability, bypassing the arduous and time-consuming theoretical derivation. The numerical example results show that the sampling method based on the Kriging model is unconcerned about the ratio used to represent dispersion and provides extra benefits in computational verisimilitude and calculation productivity.

Keywords

dynamic reliability, stochastic structure, Taylor expansion method, response surface method, Kriging model.

1. Introduction

Uncertainties are inherent in several aspects of engineering systems, including but not limited to material qualities, loads, and geometrical parameters. Reliability analysis offers a suitable framework for addressing uncertainties and assessing the probability of failure. This strategy is becoming more prevalent in other disciplines. A large body of literature and applications under the title "Reliability Engineering" has existed in engineering for many years. In practical engineering, the responses of structures often follow probability distributions with arbitrary shapes due to the propagation of uncertainties

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such as external loads, material properties, and geometric dimensions [1,2]. Reliability or risk management is one of the main concerns in the design phase, as the occurrence of faults in technical systems, e.g., in an aircraft, spacecraft, or nuclear power plant, can have catastrophic consequences. Therefore, the expectation of higher reliability and lower environmental impact has become essential. Reliability-Based Design (RBD) is a design method that has gained popularity in practice as it allows this expectation to be considered at the design stage. Reliability analyses are widely used in industry, ranging from structural engineering [3-5] to mechanical engineering [6, 7] and material design [8-10]. Various reliability analysis strategies have been developed to estimate the probability of technical structure failure effectively. Sampling-based methods include Monte Carlo simulation (MCS) and its advanced variants [11–13]. Although these methods can be used for problems of varying complexity, the computational effort required for practical engineering cases is generally high. Analytical approximation methods, such as the first-order reliability method (FORM) [14, 15] and the second-order reliability method (SORM) [16, 17]. These methods have proven efficient in estimating the probability of failure but can lead to incorrect results for problems with high nonlinearity or multiple design points.

Engineering structures contain several uncertain factors in many aspects, such as the randomness of material parameters, geometric dimensions, boundary conditions, damping, etc. These random factors will negatively impact the dynamic response analysis results. Under certain conditions, it may also become a dominant factor, so it is very necessary to consider the randomness of structural parameters in dynamic reliability analysis. In recent years, some dynamic reliability studies have been carried out considering the dual randomness of tailoring and structural parameters, but they have failed to achieve systematically effective results. The difficulty lies in 1) The dynamic reliability analysis centered on the randomness of the load is based on the random vibration theory, while the reliability analysis considering the randomness of the structural parameters is based on the random variable pattern. The coupled solution of the two is difficult to express in a unified model: 2) Some existing analysis methods are mainly based on stochastic finite element and Monte Carlo sampling methods. However, the stochastic finite element method involves intricate formulas and cumbersome calculations. Meanwhile, in the Monte Carlo method, the amount of calculation is too large to be applied in practical engineering. There arises a necessity to find a simple, convenient, practical, and efficient reliability analysis method. Random vibration on viscoelastic materials extends across various domains such as structural engineering [18,19], transportation [20,21], civil engineering [22,23], aeronautical engineering [24,25], and complex recovery system assessment [26], simulation approaches for complex systems [27], large-scale load-carrying structures [28].

The reliability analysis of structures under random loads falls into the category of dynamic reliability. Among them, the dynamic reliability research based on the first transcendence problem has made great progress, forming theoretical methods based on the spanning process and theoretical methods based on the diffusion process. Three analytical techniques, as an example of Multiple Stripe Analysis (MSA) [29-30], cloud analysis [31], and Incremental Dynamic Analysis (IDA) [32]. A multi-state manufacturing system's dynamic reliability analysis under a non-homogeneous continuous-time Markov process (NHCTMP) [33]. In the literature, four key types of techniques can be found for seismic fragility analysis of structures, i.e. the empirical, hybrid, experimental, and analytical methods [34–37]. However, in these studies, all structure parameters are assumed to be deterministic. To evaluate the dynamic reliability, response, and possible failure of buried corroded pipeline under rockfall impact [38]. In recent years, certain developments have been made in the reliability research of composite stochastic systems that consider structural and excitation randomness.

The analysis methods can be summarized as conditional random response method [39-40] and conditional reliability method [41-43]; there are three types of conditional iteration methods [44]. Among them, the conditional reliability method extends the deterministic structural dynamic reliability results to unconditional dynamic reliability considering the random parameters of the structure. Methods to improve the shearer reliability, stability, and vibration reduction in the coal mining process [45]. Spencer and Elishakoff [41] calculated the reliability of single-degree-of-freedom linear and nonlinear systems based on the Kolmogorov equation of dynamic reliability and the relationship between conditional probability and total probability; Zhao et al. [42] used the point estimation method and response surface method to calculate the unconditional reliability of multi-degree-of-freedom hysteresis structures are achieved. Abhijit and Subrata [43] used the Taylor expansion method to solve the unconditional probability and analyze the reliability of concrete dams under earthquake action. The conditional reliability method has clear mathematical concepts and clear analysis processes and can estimate the impact of random parameters on reliability from multiple angles. However, there are also problems, such as the tedious derivation process, and the computational complexity will increase with the larger dimension of the random variable. It isn't easy to apply it to complex structures.

In view of this, this research considers the impact of structural random parameters based on deterministic structural dynamic reliability analysis. It proposes to fit the nonlinear correlation among dynamic reliability and structural random boundaries through the Kriging interpolation model. Then, it establishes a mathematical sampling method based on the Kriging model. At the same time, a conditional reliability calculation method based on the second-order Taylor expansion is derived. Finally, the application characteristics of the two methods are compared and illustrated through numerical examples, and the effectiveness of the methods is analyzed. Parallel to these studies, this study investigates a wide range of dynamic reliability analyses, including the Monte Carlo method of dynamic reliability analysis, the 2nd-order Taylor expansion method of dynamic reliability analysis, and the Kriging sampling method of dynamic reliability analysis. Therefore, the researchers assumed that this objective was attainable. Although the different methods' effects on the structure performance may differ from the dynamic reliability analysis, the different methods influencing the dynamic reliability analysis are presented in Table 1.

Table 1. The different methods influencing the dynamic reliability analysis are presented.

Reference	Dynamic reliability	Monte Carlo method	2nd order Taylor expansion method	Kriging sampling method	Methodology
Zhian L et al. (2023) [46]	√	√	×	√	Semi-Parallel Active learning method based on Kriging (SPAK)
B. Echard et al. (2013) [47]	√	√	×	√	Active learning Kriging-based and Monte Carlo (AK-MCS) simulation
B. Echardet, al. (2011) [48]	√	√	×	√	Active learning reliability method combining Kriging and Monte Carlo Simulation
Nicolas Lelièvre et, al. (2018) [49]	√	√	×	√	AK-MCSi
Qing Guo et al. (2019) [50]	√	×	×	√	Active learning Kriging model-Directional importance sampling (ALK-DIS) method
Yan Shi et, al. (2019) [51]	√	√	×	√	Double-loop optimization algorithm combined with Monte Carlo Simulation-Active learning Kriging method
Xiaoping Du et, al. (2005) [52]	√	×	×	×	Reliability-Based Design (RBD) method
Zhangli Hu et, al. (2021) [53]	√	√	√	×	Second-order reliability methods
This Work	√	√	√	√	

2. Conditional probability method for dynamic reliability analysis of random structures:

The dynamic reliability theory assumes of random response crossing over the first failure. When the structural response is a stationary random process, its dynamic reliability can be expressed as a Poisson process. For high limits, the probability of crossing the limit is very small, and the event is rare, so the Poisson process is acceptable. However, the assumption that crossover events are independent is difficult to accept for narrow-band processes. The semi-analytical approximation method can be used to correct the shortcomings of the Poisson process method. Among them, the approximation of Vanmarke and Corotis, which obey the Markov process based on the number of crossovers, is considered the most accurate. The dynamic reliability formula corresponding to the bilateral limit is:

$$R(x_0)=\exp\left[\frac{T\sigma_{\dot{x}}}{2\pi\sigma_x}\exp\left(-\frac{r^2}{2}\right)\frac{1-\exp\left(-\sqrt{\frac{\pi}{2}}qr\right)}{1-\exp\left(-\frac{r^2}{2}\right)}\right] \quad (1)$$

In the formula: $r=\frac{x_0}{\sigma_x}$, x_0 is the given bilateral limit value; q is the shape parameter of the spectral density $S_{yy}(\omega)$, $\bar{q}=\sqrt{1-\frac{\bar{\lambda}_1}{\bar{\lambda}_0\bar{\lambda}_2}}$ where λ_j is the response auto-power spectral density function, and the calculation formula is:

$$\lambda_j=\int_{-\infty}^{\infty}\omega^k S_{yy}(\omega)d\omega \quad (2)$$

Equation (1) is the conditional dynamic reliability when the structural parameters are considered as definite values. To account for the randomness of the structural parameters, the calculation formula suitable for the unconditional dynamic reliability is derived below. The random variable vector of structural parameters is symbolized by X, which normally indicates the randomness of structure mass and rigidity. For a given X, the reliability at this time can be obtained according to equation (1) $R_c(t|X)$, so the unconditional dynamic reliability formula can be written as:

$$R_{uc}(t)=\int_x R_c(t|X)f_x(X)\,dX \quad (3)$$

In the formula: $f_x(X)$ is the joint probability density of X. Equation (3) is equivalent to solving multiple integrals, which usually can only rely on numerical methods. In this article, two methods for solving equation (3) are presented: one is to derive the second-order conditional probability calculation formula based on Taylor expansion; the other is to establish a Kriging model-based numerical sampling method and illustrate the effectiveness and applicability of the method is illustrated by numerical examples.

2.1. 2nd order Taylor expansion method:

Expand the conditional reliability. $R_c(t|X)$

$$R_c(t|X)=\bar{R}_c(X_0)+\sum_{k=1}^{N}(X_k-X_{k0})\,R_{c,k}^{1}(X_0)+\frac{1}{2}\sum_{l=1}^{N}\sum_{k=1}^{N}(X_k-X_{k0})(X_l-X_{l0})R_{c,kl}^{II}(X_0) \quad (4)$$

According to the definition of mathematical expectation of multidimensional random variables, unconditional reliability is the mathematical expectation of conditional dynamic reliability. $R_c(t|X)$

$$R_{uc}=R_c(t|X)=\bar{R}_c(X_0)+\sum_{k=1}^{N}(X_k-X_{k0})\,R_{c,k}^{1}(X_0)+\frac{1}{2}\sum_{l=1}^{N}\sum_{k=1}^{N}(X_k-X_{k0})(X_l-X_{l0})R_{c,kl}^{II}(X_0) \quad (5)$$

The second term of equation (5) (X_k-X_{k0}) is equal to 0, so equation (5) can be simplified and written as:

$$R_{uc}=\bar{R}_c(X_0)+\frac{1}{2}\sum_{l=1}^{N}\sum_{k=1}^{N}(X_k-X_{k0})(X_l-X_{l0})R_{c,kl}^{II}(X_0) \quad (6)$$

If the random variable X is transformed into the standard normal space U, then equation (6) can be further simplified as:

$$R_{uc}(U)=\bar{R}_c(U_0)+\frac{1}{2}\sum_{l=1}^{N}\sum_{k=1}^{N}((X_k-X_{k0})^2)R_{c.kl}^{II}(U_0) \quad (7)$$

$\bar{R}_c(U_0)$ is the reliability of the random variable when taking the mean value, and the expression is shown in equation (1). Let $\bar{R}_c=\exp(-T\bar{\alpha})$, and then we have $R_{c,k}^{I}=\alpha_k^{I}\bar{R}_c$, $R_{c,kl}^{II}=\alpha_k^{I}\alpha_l^{I}\bar{R}_c-\alpha_k^{II}\bar{R}_c$, where α, α_k^{I} and α_{kl}^{II} are the crossing rate, the first-order derivative, and the second-order derivative of the crossing rate with random variables, respectively. The conditional expected crossing rate is written as follows:

$$\alpha=\frac{ab}{c} \quad (8)$$

Then there are:

$$a=\frac{1}{2\pi}\frac{\sigma_{\dot{x}}}{\sigma_x};\qquad A=1-\exp\left(-\sqrt{\frac{\pi}{2}}\frac{x_0}{\sigma_x}q\right) \quad (9)$$

$$B=\exp\left(-\frac{x_0^2}{2\sigma_x^2}\right),\qquad b=A.B; \quad (10)$$

$$c=\exp\left(\frac{1}{2}\left(\frac{x_0}{\sigma_x}\right)^2\right)-1 \quad (11)$$

According to equation (8), the first derivative of α concerning the random variable can be directly calculated:

$$\alpha_k^{I}=\left(\frac{\alpha_k^{I}}{\bar{a}}+\frac{b_k^{I}}{\bar{b}}-\frac{c_k^{I}}{\bar{c}}\right)\bar{\alpha} \quad (12)$$

in:

$$\alpha_k^I=\bar{a}\left(\frac{\sigma_{\dot{x},k}^I}{\bar{\sigma}_{\dot{x}}}-\frac{\sigma_{x,k}^I}{\bar{\sigma}_x}\right) \quad (13)$$

$$b_k^I=\sqrt{\frac{\pi}{2}}x_0\left(\frac{q_{x,k}^I}{\bar{\sigma}_{\dot{x}}}-\frac{q\sigma_{x,k}^I}{\bar{\sigma}_x^{\ 2}}\right)(1-\bar{A})\bar{B}+x_0^2\frac{\sigma_{x,k}^I}{\bar{\sigma}_x^{\ 3}}\bar{A}\,\bar{B} \quad (14)$$

$$c_k^I=-\frac{x_0^2}{(\bar{\sigma}_x)^3}\sigma_{x,k}^I(1+\bar{c}) \quad (15)$$

The second-order moment is derived according to equation (12), and the expression is as follows:

$$\alpha_k^{II}=\left(\frac{\alpha_{kl}^{II}\bar{a}-\alpha_k^I\alpha_l^I}{\bar{a}^2}+\frac{b_{kl}^{II}\bar{b}-b_k^Ib_l^I}{\bar{b}^2}-\frac{c_{kl}^{II}\bar{c}-c_k^Ic_l^I}{\bar{c}^2}\right)\bar{a}+\frac{\alpha_k^I\alpha_l^I}{\bar{a}} \quad (16)$$

In the formula:

$$\alpha_{kl}^{II}=\left(\frac{2\sigma_{\dot{x},k}^I\sigma_{\dot{x},l}^I}{\bar{\sigma}_{\dot{x}}^2}-\frac{2\sigma_{x,k}^I\sigma_{x,l}^I-\sigma_{x,kl}^{II}\bar{\sigma}_x+\sigma_{\dot{x},kl}^{II}\bar{\sigma}_{\dot{x}}}{\bar{\sigma}_x\bar{\sigma}_{\dot{x}}}\right)\bar{a} \quad (17)$$

$$b_{kl}^{II}=A^{II}B+2A^IB^I+AB^{II} \quad (18)$$

$$A^I=\sqrt{\frac{\pi}{2}}x_0\left[\frac{q_k^I}{\bar{\sigma}_x}-\frac{q\sigma_{xk}^I}{\bar{\sigma}_x^2}\right](1-\bar{A});\quad B^I=x_0^2\frac{\sigma_{x,k}^I}{\bar{\sigma}_x^{\ 3}}\bar{B} \quad (19)$$

$$A^{II}=\sqrt{\frac{\pi}{2}}x_0\left[\frac{q_{kL}^{II}\bar{\sigma}_x^2-2q_k^I\sigma_{x,k}^I\bar{\sigma}_x+2q\sigma_{x,k}^I-q\sigma_{xkl}^{II}\bar{\sigma}_x}{\bar{\sigma}_x^3}.(1-\bar{A})+\frac{q_k^I\bar{\sigma}_x+q\sigma_{x,k}^I}{\bar{\sigma}_x^2}A_k^I\right] \quad (20)$$

$$B^{II}=x_0^2\left[\frac{\sigma_{x,kl}^{II}}{\bar{\sigma}_x^3}\bar{B}-\frac{3\sigma_{x,k}^I\sigma_{x,l}^I}{\bar{\sigma}_x^4}\bar{B}+\frac{\sigma_{x,k}^I}{\bar{\sigma}_x^3}B_k^I\right] \quad (21)$$

$$c_{kl}^{II}=x_0^4\left(\frac{3\sigma_{x,k}^I\sigma_{x,l}^I-\sigma_{x,kl}^{II}\bar{\sigma}_x}{x_0^2\bar{\sigma}_x^4}+\frac{(\sigma_{x,k}^I)^2}{\bar{\sigma}_x^6}\right)(1+\bar{c}) \quad (22)$$

The narrowband process q shown in equation (1) is also random, and the mean value and its derivative can be obtained according to the moments of each order of the power spectrum function:

$$\bar{q}=\sqrt{1-\frac{\bar{\lambda}_1}{\bar{\lambda}_0\bar{\lambda}_2}},\qquad q_k^I=\frac{1-\bar{q}^2}{2\bar{q}}\left(\frac{\lambda_{0,k}^I}{\bar{\lambda}_0}+\frac{\lambda_{2,k}^I}{\bar{\lambda}_2}-\frac{2\lambda_{1,k}^I}{\bar{\lambda}_1}\right) \quad (23)$$

$$q_k^{II}=\frac{1-\bar{q}^2}{2\bar{q}}\left(\frac{\lambda_{0,kl}^{II}\bar{\lambda}_0-\lambda_{0,k}^I\lambda_{0,k}^I}{\bar{\lambda}_0^{\ 2}}+\frac{\lambda_{2,kl}^{II}\bar{\lambda}_2-\lambda_{2,k}^I\lambda_{2,l}^I}{\bar{\lambda}_2^{\ 2}}-\frac{2\lambda_{1,kl}^{II}\bar{\lambda}_1-\lambda_{1,k}^I\lambda_{1,l}^I}{\bar{\lambda}_1^{\ 2}}\right)-\frac{1+\bar{q}^2}{\bar{q}(1-\bar{q}^2)}q_k^Iq_l^I \quad (24)$$

It can be seen from the above results that as long as the first-order derivative and the second-order derivative of the variance of each response quantity concerning the random parameters are obtained, unconditional reliability can be obtained according to the aforementioned method.

2.2. Numerical sampling method based on the Kriging model:

Concerning the problem shown in equation (3), a discernible correlation exists between conditional dynamic reliability and structural random parameters. However, there is no clear analytical expression for this relationship due to nonlinear factors such as structure, response, and load. This article aims to introduce the Kriging method for unconditional dynamic reliability calculation, building upon the Kriging approach. The main work includes fitting conditional dynamic reliability and numerical simulation of unconditional reliability, which are explained separately in the subsequent sections.

1) Kriging method fitting of conditional dynamic reliability:

A certain response surface form is used to fit the relationship between conditional dynamic reliability $R(t|x_1,x_2,\dots,x_n,)$ and structural random parameters $x_1,x_2,\dots,x_n$ including the selection of response surface form and sample test design method. Based on the Kriging [54] method, formula (3) can be expressed as:

$$\tilde{R}(t|X)=f^T(X)\beta+z(X) \quad (25)$$

Among them: β is the regression coefficient; *f(X)* is the polynomial coefficient, which is usually taken as a fixed constant and does not affect the approximation accuracy; $z(X)$ is a random function with a mean value of 0, a variance of σ^2 and a covariance matrix of:

$$\text{cov}[z(x_i),z(x_j)]=\sigma^2R(x_i,x_j),\quad i,j=1,2,\dots,n \quad (26)$$

$R(x_i,x_j)$ is the spatial correlation equation of any two sample points in the space, which plays a decisive role in the accuracy of the simulation, is generally chosen to be.

Gaussian form:

$$R(x_i,x_j)=\exp\left|-\sum_{k=1}^p\theta_k\left|x_i^k-x_j^k\right|^2\right| \quad (27)$$

The parameter θ_k can be given by the highest probability evaluation of the function, that is, the construction problem of the optimal Kriging model adapted into a nonlinear unconstrained optimization problem, whose form is:

$$\max_{\theta_k>\theta}-\frac{\left[n\ln\hat{\sigma}^2+\ln(\det R)\right]}{2} \quad (28)$$

2) Numerical simulation of unconditional dynamic reliability equation (25) obtains the fitting form of conditional dynamic reliability. Substitute it into equation (3) to obtain the unconditional probabilistic dynamic reliability:

$$R(t)=\int_x R(t|X)f_x(X)\,dX=\int_x\left(f^T(X)\beta+z(X)\right)f_x(X)\,dX \quad (29)$$

According to the definition of the expected value of a multidimensional random variable, equation (29) is synonymous with the mathematical expectation of calculating *R(t|X)*, and can be calculated by numerical simulation. First, according to the statistical characteristics of the random variables, N groups of samples are extracted from the sample space, and N conditional dynamic reliability values are calculated. Subsequently, the Kriging response surface of the conditional reliability is obtained using the method of step 1), and finally, the average value of the conditional dynamic reliability is used to estimate unconditional dynamic reliability. Among them, the Kriging response surface's fitting accuracy and calculation efficiency are pivotal factors for determining the effectiveness of this approach.

3. Example analysis:

The motion equation of a single-degree-of-freedom system subject to stationary random excitation can be expressed as $m\ddot{y}+c\dot{y}+ky=f(t)$, where m is mass, c is damping, and k is stiffness; the auto-power spectral density of stationary random process $f(t)$ is Constant value, $S_{ff}(\omega)=S_0=1$. System parameters m, c, and k are independent normally distributed random variables, with average values of 15, 0.05, and 40, respectively. Assuming that the three random variables have the same variation coefficient v, analyze the dynamic reliability R of the displacement response for time $t = 100$.

The second-order Taylor expansion method and the Kriging model sampling method are used to calculate the dynamic reliability of the random structure. The Monte Carlo method was used to analyze the reliability obtained 10^6 times as the accurate result. To study the impact of system parameter variability on dynamic reliability, the coefficient of variation of random variables changes from 0 to 0.3. Tables 2, 3, and Figures 1 and 2 list the reliability results when the corresponding bilateral displacement response limit values are 3σ and 3.5σ, respectively, where σ is the standard deviation of the displacement steady-state response. When the coefficient of variation is 0, the system structure is deterministic. When the limit values are 3σ and 3.5σ, the corresponding dynamic reliability results are 0.99809 and 0.99879, respectively.

It can be seen from the results in Tables 2, 3, and Figures 1 and 2 that when considering structural variability, the dynamic reliability values obtained by the second-order Taylor expansion method and the Kriging response surface sampling method are both lower than the dynamic reliability value of the deterministic structure. This result is consistent with objective reality. It's attached. It can also be concluded that the results and error trends obtained by the two methods are consistent. The structural dynamic reliability value decreases with the increase of the coefficient of variation. The higher the limit value, the smaller the difference will be.

Table 2. The juxtaposition of the Monte Carlo method against the outcomes of this investigation (with a margin of 3σ).

Coefficient of variation	Monte Carlo method	2nd order Taylor expansion method		Kriging sampling method	
	Reliability	Reliability	Error (%)	Reliability	Error (%)
$v = 0.05$	0.9460765	0.9459862	0.00908	0.9461312	0.00558
$v = 0.10$	0.941336	0.9399672	0.13698	0.9419264	0.07069
$v = 0.15$	0.9268815	0.9329606	0.68953	0.9268926	0.00611
$v = 0.20$	0.915876	0.9501382	3.75422	0.915976	0.02097
$v = 0.25$	0.880352	0.9271094	5.92798	0.8769696	0.48098
$v = 0.30$	0.9361075	0.9132836	8.98796	0.9316416	0.74618

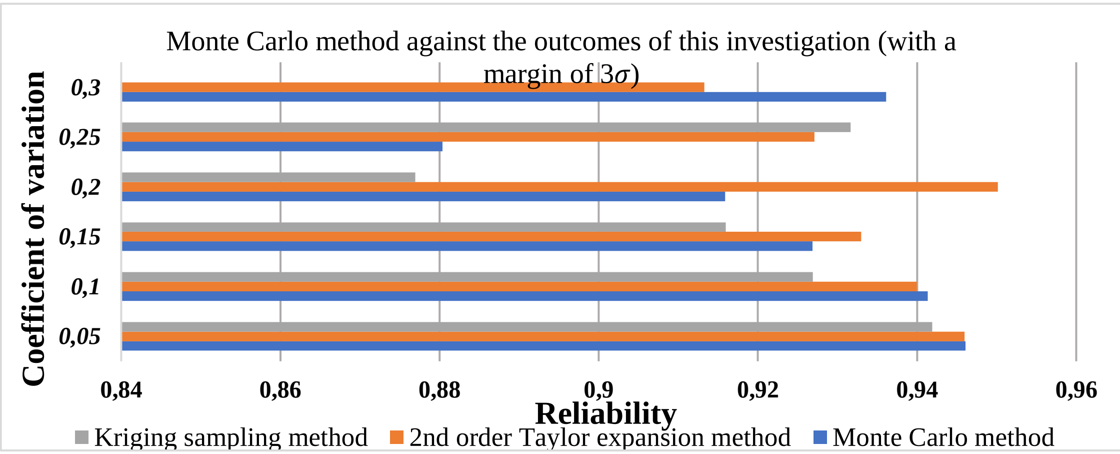

Figure 1. The Comparison of the Monte Carlo method concerning the results of this research (margin 3 σ).

Table 3. The juxtaposition of the Monte Carlo method against the findings of this research (with a margin of 3.5σ).

Coefficient of variation	Monte Carlo method	2nd order Taylor expansion method		Kriging sampling method	
	Reliability	Reliability	Error (%)	Reliability	Error (%)
***v* = 0.05**	0.9592128	0.9592112	0.00021	0.9592104	0.00028
***v* = 0.10**	0.957552	0.958242	0.07128	0.957987	0.04803
***v* = 0.15**	0.9513504	0.9568512	0.61029	0.9527248	0.20394
***v* = 0.20**	0.9363936	0.9547968	1.98403	0.9365761	0.09548
***v* = 0.25**	0.9102336	0.9521472	4.79351	0.9101728	0.64265
***v* = 0.30**	0.905232	0.9393024	8.00139	0.8999536	1.07834

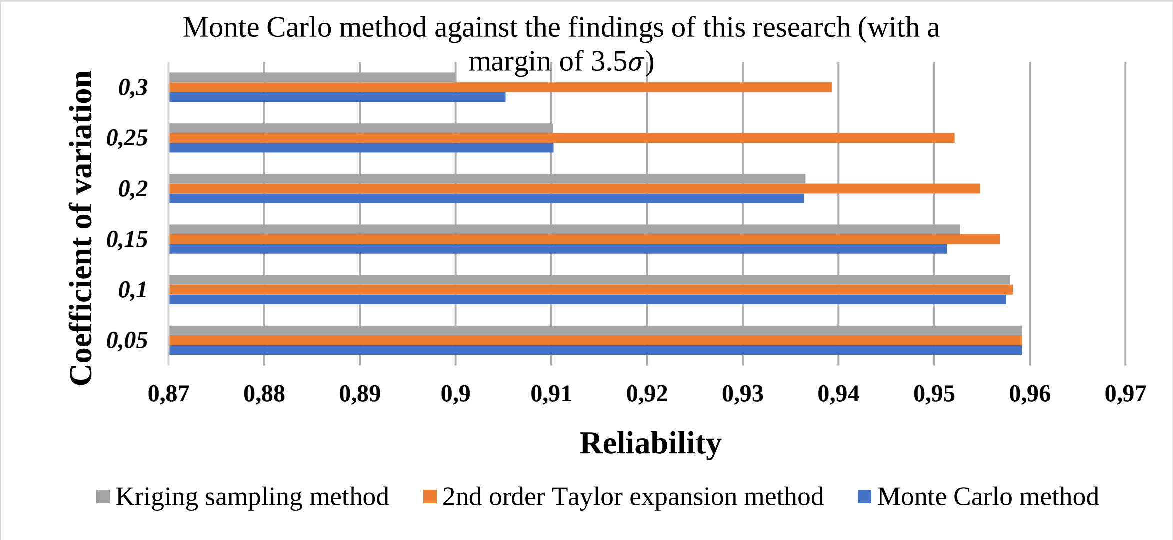

Figure 2. The comparison of the Monte Carlo method concerning the results of this research (margin 3.5 σ).

When deriving based on the Taylor expansion method, only the first two-order terms are taken to simplify the calculation of the conditional probability expansion. It can be observed from the outcomes in Table 3 and Figure 2 that although the second-order Taylor expansion method can obtain the response statistical moments and corresponding reliability with less calculation, the resulting error has a great relationship with the value of the coefficient of variation. Controlled by the two limits, when the structural variation coefficient is lower than 0.2, the computation precision is favorable, and the highest error is 1.98%. Conversely, as the variation coefficient is higher than 0.2, the error escalates swiftly with the rise of the variation coefficient. Specifically, when the variable anomaly is 0.3, the error approaches 8%.

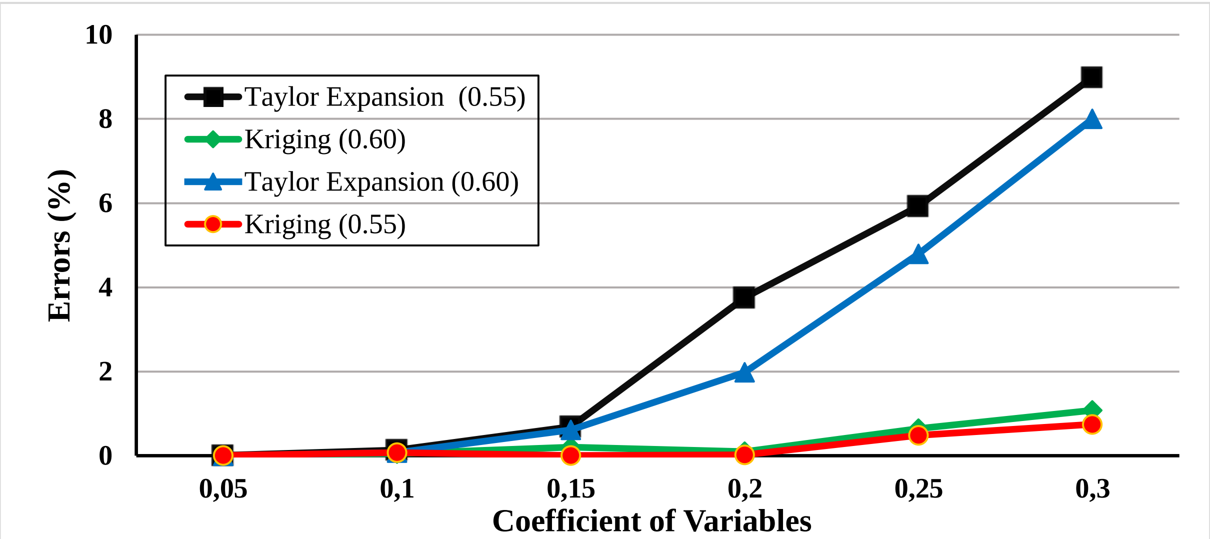

Figure 3. Accuracy of the different limits and different coefficients of variation error.

Table 4. Adjusting the preciseness of the Kriging response surface method.

Coefficient of variation	Sample number	Average relative error (%)	
		Limit 3.5σ	Limit 3.0σ
0.05	100	0.93403×10^{-3}	4.00247×10^{-3}
	70	1.51496×10^{-3}	4.21586×10^{-3}
	50	6.42056×10^{-3}	1.56311×10^{-2}
0.10	100	1.00977×10^{-2}	3.46207×10^{-2}
	70	3.96330×10^{-2}	5.80372×10^{-2}
	50	4.41826×10^{-2}	1.25496×10^{-1}
0.15	100	3.97513×10^{-2}	1.01858×10^{-1}
	70	8.97992×10^{-2}	2.60395×10^{-1}
	50	1.33621×10^{-1}	4.61991×10^{-1}
0.20	100	2.83267×10^{-1}	4.47643×10^{-1}
	70	5.07296×10^{-1}	4.92779×10^{-1}
	50	3.80032	6.96043
0.25	100	0.59284	4.04291
	70	1.09856	5.12987
	50	2.93289	7.50032
0.30	100	4.79305	2.1938
	70	5.62168	11.201
	50	5. 82904	8.78599

When calculating the Kriging sampling method, different sample numbers are first selected to construct the Kriging response surface, and then another 100 groups of samples are selected as error test data. The Kriging simulation accuracy when the number of samples is 50, 70, and 100 under different coefficients of variation is shown in Table 4. It can be seen from the average relative error results that when analyzed by the Kriging sampling method, the response surface model constructed using 50 sample points has achieved good fitting accuracy. The results in Tables 2 and 3 and Figures 1 and 2 show that compared with the Taylor expansion method, the Kriging sampling method has higher calculation accuracy when calculating the conditional probability shown in equation (3), and under the two limits, when the coefficient of variation increases from 0.05 to 0.3, the change in calculation error is not obvious. The maximum error within the value of the coefficient of variation is within one percent. A comparison of the accuracy errors of the two methods at other different limit values and different coefficients of variation can be seen in Figure 3.

4. Conclusion:

This paper proposes a Taylor expansion method and the Kriging model derived from statistical analysis to perform structural reliability analysis. The proposed strategy is found to be economic in the number of calls to the expensive performance function and its results are very accurate for the probability of failure. This approach combines the advantages of two methods: The Kriging interpolation model and the second-order Taylor expansion method.

- According to the different methods for dealing with structural random parameters, two conditional reliability methods are introduced in this paper to solve the problem of composite random reliability: the second-order Taylor expansion method and the numerical sampling method based on the Kriging model. The Kriging sampling process uses the Kriging interpolation model to fit the nonlinear connection among dynamic reliability and structural random parameters. It can be easily calculated directly utilizing the finite element program, avoiding the tedious and arduous theoretical derivation.
- The results of the calculation example show that the Taylor expansion method can determine the statistical moments of the response and the corresponding reliability with less computational effort, but the resulting error has a great relationship with the size of the variation coefficient. In general, the accuracy is lower when the variation coefficient is less than 0.2 high.
- In comparison, the Kriging sampling process is insensitive to the size of the coefficient of variation, and its computational accuracy and efficiency are relatively high. In addition, since the Taylor expansion method requires the creation of specific analysis programs based

on the mean and coefficient of variation of random variables, it is usually impossible to use commercial software directly for reliability analysis. Therefore, the Kriging sampling method is more suitable for dynamic reliability analysis of complex structures.

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