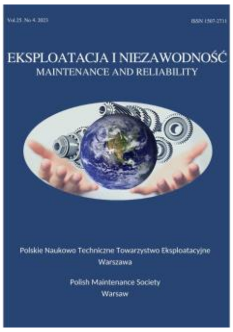

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A Z-number and MABAC method based on reliability analysis and evaluation of product design concept

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Highlights

- Utilizes Z-numbers for detailed customer-centric product design evaluation.
- Proposes MABAC for optimal design choice using expert and attribute weight integration.
- Comparing trapezoidal and triangular fuzzy numbers in customer-centric design decisions.

Abstract

Modular design is a significant method for complicated product development. In the context of modular design, involving users in concept assessment boosts a product's appeal but also introduces decision uncertainty and unreliability. As a solution, this paper proposed a hybrid method by integrating expert consensus modeling, attribute weighting, Z-number, and the Multi-Attribute Border Approximation Area Comparison (MABAC) method. Initially, a consensus model is established using consistency theory to determine expert weights, and attribute priorities are determined through the entropy weighting method. Subsequently, the Z-number-based MABAC method ranks the alternatives, determining the optimal solution among them. Using an automated outdoor cleaning vehicle as an example, the proposed method is compared to other techniques. The sensitivity analysis and the comparisons show that the proposed method improves the reliability and objective of the decision-making process.

Keywords

product design concept evaluation, Z-number; MABAC; reliability; modular design

1. Introduction

In today's competitive market environment, ensuring the reliability of product design and optimization of user experience, as well as reducing product manufacturing costs, are critical to the success of enterprises. New Product Development (NPD) is a comprehensive process that covers the entire process from the concept phase to the actual market launch, with reliability plays a key role throughout. During this process, a deep understanding of user needs, market trends, and technological advances is essential and significant. Reliability is not only about the stability and durability of the product but also about the trust of the user. Therefore, reliability is considered as a core element in NPD [1]. Summers [2] applied reliability analysis from behavioral economics to product development to address uncertainties, unforeseen uncertainties, and complexity in the development process. A new model was established, indicating that the success of frameworks, practices, tools, and decisions depends on reducing the impact of decision errors. Li [3] proposed a Physical Information-based Ensemble Learning (PIEL) method for fatigue reliability analysis of aviation engine blade systems. Through case studies and method comparisons,

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it was demonstrated that the propose has high computational accuracy and efficiency, making it an effective reliability analysis method for blade-disk systems. Chen [4] introduced a user reliability incentive mechanism considering user reliability requirements to address energy reliability resource allocation problems, aiding optimal decision-making from a market perspective. A reliability incentive mechanism-based master-slave dual-layer game model was proposed, innovating reliability models in terms of reliability constraints. Li [5] presented a Multivariate Ensemble Hierarchical Linkage (ME-HL) strategy, decomposing complex evaluation systems through HL strategies and synchronously mapping responses of subsystems using ME models. This construction of a multi-level system reliability framework demonstrated significant advantages in computational accuracy and efficiency for system reliability assessment problems. Mashal [6] based on a comprehensive literature review and expert judgment, established a model identifying the most critical criteria influencing smart grid reliability from a user perspective. Applying the fuzzy analytic hierarchy process, the criteria were analyzed and prioritized using triangular fuzzy numbers and triangular membership functions. The previous researches are related to various phases during the design, manufacture and maintenance process. Integrated strategies have been employed to ensure the reliability of data, components, and the machine throughout the entire product development and serve process. Therefore, the reliability of a new product, and a holistic approach is essential to address potential challenges at various stages of development and product lifecycle.

The abbreviation list is presented in Table 1.

Table 1. Abbreviation list.

Abbreviation	Full title
MABAC	Multi-attribute border approximation area comparison
NPD	New product development
MFD	Modular function deployment
MADM	Multi-attribute decision making
TFSN	Trapezoidal fuzzy numbers
TFN	Triangular fuzzy numbers
TOPSIS	Technique for order preference by similarity to ideal solution
VIKOR	VlseKriterijuska Optimizacija I Komoromisno Resenje

Positioned as a pivotal intermediary in the product development cycle, product design involves numerous crucial elements, including user needs assessment, market analysis, technical evaluation, sustainability and environmental considerations and lifecycle management. These aspects need to be holistically considered to ensure the final product meets user needs, boasts market competitiveness, and aligns with feasibility and economic requirements. As well as a challenge start of the NPD, product design is also a creative endeavor, serves as the phase where new products are conceived, developed, and tested for originality. Improvement methodologies for product development are gaining greater prominence due to global competition in this phase. As an effective methodology, the utilization of Modular Function Deployment (MFD) significantly enhances the product quality, especially for complicated products[7]. Modular design is constructed by several procedures which are shown in Fig. 1, forming a systematic and complicated strategy. The design task is subdivided into numerous requirements, and solutions are generated and aggregated into multiple design schemes by MFD theory, a decision-making process named design concept evaluation need to be done to select the optimal design scheme. During this period, as part of the evaluation data are obtained from the experts, the reliability of information is also crucial for the enterprises. To enhance objectivity in evaluations, multiple experts allocate preferences based on prepared attributes. However, cognitive biases among experts may lead to biases such as seeking or accepting information consistent with their existing views while neglecting or excluding contradictory information, impacting the objectivity of decisions. Collective discussion is one way to obtain the information from the experts. During the discussion[8], groupthink may arise, causing members to avoid expressing viewpoints differing from the mainstream. This lack of diversity may affect the innovation and comprehensiveness of decisions. Experts may exhibit overconfidence in their judgments and predictions, overlooking or underestimating uncertainty factors, leading to inaccurate risk assessments and undesirable decision outcomes. Questionnaire in the form of linguistic information is another way to acquire the information from the experts. Darko [9] proposed a Probability Reliable Language Multi-Attribute Decision Model (PRLMADM) to address uncertainties in decision-making and assess the reliability of information using a Probability Reliable Language Term Set (PRLTS). Pablo [10] analyzed and compared composite material laminated plate designs optimized under different conditions, illustrating the interaction between model parameter uncertainty and design variability. The results indicated the necessity of considering model uncertainty and variability in reliability optimization. Addressing this issue, this paper proposes a method to transform Z-number into trapezoidal fuzzy numbers and analyzes their implications for unreliable information. This approach aims to mitigate the impact of unreliable information on decision outcomes. Additionally, by integrating the MABAC method, the paper combines evaluation information from multiple experts to rank and select the optimal solution among the given alternatives.

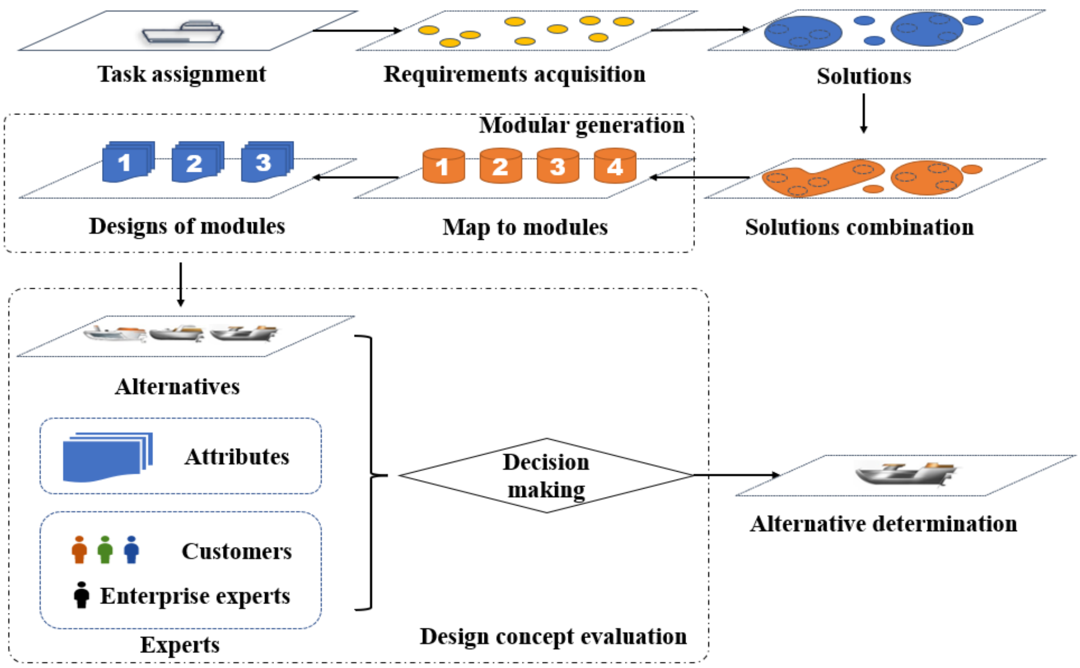


Fig. 1. Process of modular product concept design.

In the field of Multi-Attribute Decision-Making (MADM), balancing different factors and addressing uncertainty is a challenging task. To better tackle this challenge, this research introduces various decision methods, including Z-number and MABAC methods. In some cases, reliability analysis needs to consider fuzziness and uncertainty, and Z-number can effectively handle this situation for fuzzy reliability assessment. The motivation of this paper is to explore the application of Z-number and MABAC in the modular design evaluation phase, analyze their advantages and applicability, and address some of the limitations and challenges of traditional MADM methods.

With the participation of the customers, uncertainty and unreliability are often present in the data and information in the evaluation phase of modular design, adding complexity to the decision-making process. Traditional precise mathematical methods struggle to handle such uncertainty, while fuzzy set theory provides an effective tool for dealing with fuzzy information. Z-number, as a manifestation of fuzzy set theory, holds the potential for addressing issues of uncertainty and unreliability. The MABAC method is a MADM approach that combines fuzzy set theory with the boundary approximation technique. It proves effective in handling fuzzy data and uncertainty. MABAC offers advantages such as adaptability and simplicity in computation, making it widely applicable to decision problems across various domains. The MABAC method provides a comprehensive and accurate assessment of alternatives, with the potential to enhance decision quality and reduce subjective biases in the decision-making process.

The advantage of Z-number and the MABAC ranking method in handling uncertainty and unreliability while also accounting for the trade-offs among multiple attributes for design concept evaluation. A novel MADM approach based on Z-number and the MABAC method to select the optimal product design scheme.

This paper presents three contributions:

1) This paper establishes a decision model for customer-

centric modular product design evaluation. In order to better express customers' uncertain opinions, this paper utilizes the concept of Z-number to represent customer evaluation information and the degree of its reliability, then transforms it into trapezoidal fuzzy numbers for research. This approach provides a more detailed representation of customer requirements compared to classical fuzzy numbers.

2) Regarding the selection of the optimal design solution in the product design process, this paper proposes a method that combines the multi-expert decision evaluation matrix with consistency theory and the entropy weight method. This method effectively integrates expert weights and attribute weights and ultimately ranks and selects the product design solutions through the MABAC method.

3) By comparing the ranking results of trapezoidal fuzzy numbers and triangular fuzzy numbers, it is evident that trapezoidal fuzzy numbers possess a broader representational capacity and stronger capability in handling fuzzy boundaries. When dealing with multiple sources of information, especially in customer-centric design decision processes, they are better equipped to address complex fuzzy information and unreliable data.

The remaining structure of this paper is as follows. Section 2 reviews relevant decision theories and methods, including MADM, Z-number, and the MABAC method. Section 3 presents definitions derived from the relevant theories. In Section 4, an optimal selection method based on multiple schemes is proposed. In Section 5, an example of a waste-cleaning vehicle is applied, and a comparison with other methods is conducted to validate the proposed approach. Section 6 concludes with sensitivity analysis, effectiveness assessment, and key findings.

2. Literature review

This section includes three parts. First, the related MADM theories and their applications in design concept evaluation are reviewed in section 2.1. Afterward, section 2.2 is about Z-number and TFSN. Section 2.3 reviews the MABAC method.

2.1 MADM applications in design concept evaluation

Design concept evaluation plays a critical role in the concept design process[11]. In the early decades of product design, design schemes are usually determined by the project manager, where the users' real requirements are ignored[12]. However, with the improvement of product service systems, user' participation has become a key factor in design concept evaluation[13, 14]. User involvement in new product development is implemented in two ways. One effective way is to directly invite the experienced customer as a supervisor to participate in design quality projects at all stages during the new product development[15]. However, as the specialization and complexity of products increase, it is difficult for users to have a comprehensive and profound understanding of various aspects of product design and production requirements. Another proper way is to participate in the design concept evaluation procedure, their preferences are considered in decision-making[16, 17].

Design concept evaluation is a decision-making process considering various information obtained from different fields, and ranking the solutions in a finite set of conflicting and incommensurable solutions. MADM is one of the most proper methods in analytical decision-making theories[18]. As the information is obtained from experts, most of them are unreliable and uncertain.

To solve the problem, a fuzzy set is integrated into MADM methods in design concept evaluation. The fuzzy theory describes the uncertainty of information using fuzzy information and utilizes membership functions to describe the fuzzy boundaries of objects[19]. Fuzzy sets handle fuzzy information in decision-making through membership functions. The fundamental idea is to extend the characteristic function, which can only take values in 0 or 1, to a membership function that can take arbitrary values on the interval [0,1]. Maysa[20] provides a multiple-experts fuzzy-TOPSIS decision-making model to illustrate the need to better address uncertainties in rating. Li[21] presents a dynamic diagnostic strategy based on reliability analysis and distance-based VIKOR with heterogeneous information. It can be seen that MADM methods are very important and useful in reliability analysis.

Based on different membership functions, fuzzy information can be classified into various forms. Interval numbers represent the most basic form of fuzzy information, with membership functions having values of only 0 or 1[22]; fuzzy numbers are a type of convex fuzzy set, characterized by a real-valued membership function; rough numbers approximate the fuzziness of information through upper and lower approximations, resulting in a step-like membership function[23]; binary semantics represent the fuzziness of information by constructing bivariate binary functions[24]; intuitionistic fuzzy sets establish membership functions using both membership and non-membership dimensions[25]; Picture fuzzy sets consider membership, non-membership, and neutrality simultaneously, establishing membership functions based on multiple dimensions[26, 27].

Optimal selection of product design schemes is an important application area of fuzzy theory. Thurston[28] was the first to apply fuzzy theory to the optimization of product design schemes, transitioning from a classical linguistic information environment based on precise numbers to a linguistic information environment based on fuzzy theory. In recent years, multi-attribute decision-making methods combined with fuzzy theory have rapidly developed in the field of optimal product design. Fuzzy numbers, vague sets, interval numbers, rough sets, and soft sets have all been applied to the optimization of product design schemes. Fuzzy numbers [29, 30] and vague sets[31] describe the fuzzy membership relationships between various objectives and different sets using membership functions. However, the determination of membership functions still requires additional data information, and feasible feedback mechanisms are needed when determining fuzzy data [32]. Interval numbers[33, 34] represent the randomness of information distribution within a certain range centered around a precise value. However, in decision-making, they introduce inherent preferences and uncertainties. Moreover, when there is overlap between the intervals representing different scheme decision results, it becomes difficult to differentiate the superiority of one interval over the other.

2.2 Z-number

In design concept evaluation, as the experts are constructed in different fields[35], their acknowledgement and experiences vary due to different individual backgrounds. Hence, the reliability gap between each expert for each aspect is huge. Z-number is an effective way to describe the reliability gaps. The form of Z-number includes two parts, the former part shows the preference obtained from the expert, while the latter part represents the possibility of this preference. Since Z-number was proposed by Zadeh in 2011[36], scholars proposed reasonable information conversion and aggregation methods considering both preference and reliability.

The construction of derivative forms of Z-number and the corresponding algorithm proposals represent important methods for establishing a Z-number fuzzy linguistic information environment. Researchers have introduced various improved forms of Z-number based on different problems, such as Z*-numbers[37], Z-advanced numbers[38], Z^{+}-numbers[39], and multi-dimensional Z-number[40]. These forms have been accompanied by targeted computational methods. Additionally, some studies have explored Z-number through probability-possibility distributions. For instance, they have simplified relevant operations using typical probability distribution processes like Gaussian distributions[41].

Another important method for establishing a Z-number fuzzy linguistic information environment is the transformation of Z-number into other forms of fuzzy numbers, such as crisp numbers or other fuzzy numbers like triangular and trapezoidal fuzzy numbers. Aliyev[42] treated the $\tilde{A}$ and $\tilde{B}$ components of Z-number as a pair of equally weighted fuzzy numbers and established a fuzzy linguistic information environment through information aggregation. However, the $\tilde{A}$ and $\tilde{B}$ components in Z-number represent different content with varying degrees of importance, making a simple merging approach unsuitable. Subsequently, Z-number transformation methods based on different levels of importance emerged[43](Li et al., 2022). Kang[44] proposed conversion algorithms to transform Z-number into triangular and trapezoidal fuzzy numbers. Following this, numerous researchers utilized triangular or trapezoidal fuzzy numbers to establish fuzzy linguistic information environments based on Z-number, referred to in this article as Z-number fuzzy linguistic information environments[45, 46].

In the field of product design and manufacturing, Qi[47] proposed a method for optimal product design scheme selection in the Z-number fuzzy linguistic information environment by comprehensively considering user requirements and design parameters, combining design parameters, user preferences, and reliability. Zhu[48] based on the Z-number fuzzy linguistic information environment, utilized the Analytic Hierarchy Process (AHP) to calculate the weights of decision attributes for optimal product design scheme selection. They then introduced an improved multi-attribute boundary approximation region comparison method to establish a model for optimal product design scheme selection. Considering customer needs and decision experts' confidence, Liu[49] established a model for optimal product design scheme selection in the Z-number fuzzy linguistic information environment. They analyzed customer needs to determine the set of decision attributes, used AHP to determine attribute weights, applied the Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) method for scheme ranking, and established the selection model. In the Z-number fuzzy linguistic information environment, Aydoğan [50] introduced an improved axiomatization design method that incorporates judgments and preferences into the axiomatization design process.

2.3 MABAC method

The MABAC method is used for multi-attribute decision-making. It takes into consideration the interrelationships between different attributes as well as the importance of each attribute. It aims to rank the alternatives based on their performance across multiple attributes by creating a border approximation area for each alternative. The method takes into account the decision maker's subjective preferences and allows for the consideration of uncertainty and imprecision in the decision-making process.

This method categorizes the objects to be evaluated and then performs comparisons and rankings within each category. Ali Ebadi TORKAYESH and others[51] conducted a comprehensive literature review of 117 recent articles on the latest developments and applications of MABAC, considering concepts of sustainability and economic feasibility. Jiang[27] extended the MABAC method using Picture Fuzzy Sets based on Prospect Theory (PT) and utilized fuzzy number models for evaluating different suppliers. Huang[46] integrated a design alternative evaluation model that combines Z-cloud rough numbers, best-worst method, and MABAC method. Mandal[52] proposed a hybrid method based on the entropy method, deviation-based method, and MABAC method with interval-valued spherical fuzzy sets to choose the most effective plastic waste management process in which the decision experts' and attribute weights are completely unknown. Salman[53] integrated AHP and MABAC method to determine the best dimple-roughened plate parameter arrangement in a jet-impinged dimple-roughened solar air collectors. Tao[54] proposed a prospect theory-based MABAC method integrated with novel interactional operations and similarity measures in MADM problems. In design concept evaluation, the MABAC method is also important, Zhu[48] and Chen[16] implemented this method in the heat exchanger design scheme and mortise and tenon joint structure selection.

3. Preliminaries

The concept of Z-number was initially introduced by Zadeh and is associated with the reliability of information, used for computations involving unreliable information. Z-number establish a connection between evaluative information and reliability information and are defined as Z = (A, B). A Z-number comprises two components: Component A restricts the allowable values for the uncertain real-valued variable x; Component B quantifies the reliability of the first component.

Triangular and trapezoidal fuzzy numbers are both commonly used methods for representing fuzzy quantities, each with its characteristics[55]. Triangular fuzzy numbers are characterized by their succinct expression, strong intuitiveness, and convenient mathematical operations, however, their precision is limited. Triangular fuzzy numbers have limited capacity for representing asymmetric, discontinuous, or complex-shaped fuzzy set memberships, and their membership distribution is constrained. When representing the membership distribution of fuzzy sets, triangular fuzzy numbers can only express symmetric, linearly changing distributions. On the other hand, trapezoidal fuzzy numbers, by introducing an additional key value, either the left shoulder or the right shoulder, offer greater flexibility compared to triangular fuzzy numbers. They can represent a wider range of fuzzy sets, accurately describing asymmetric, discontinuous, or complex-shaped memberships[56]. Therefore, this study employs trapezoidal fuzzy numbers for the uncertainty component A in the Z-number due to their enhanced flexibility, accurate representation, and broader applicability. Additionally, triangular fuzzy numbers are used to represent the unreliability component B in the Z-number.

Definition 1[57] A trapezoidal fuzzy number A is defined by a four-dimensional array (a_1, a_2, a_3, a_4), and its membership function $\mu_A(x)$ is shown below, with the corresponding function graph depicted in Fig. 2.

$$\mu_A(x) = \begin{cases} 0, & x\epsilon(-\infty, a_1) \\ \frac{(x-a_1)}{(a_2-a_1)}, & x\epsilon[a_1, a_2] \\ 1, & x\epsilon\ [a_2, a_3] \\ \frac{(a_4-x)}{(a_4-a_3)}, & x\epsilon[a_3, a_4] \\ 0, & x\epsilon(a_4, +\infty) \end{cases} \quad (1)$$

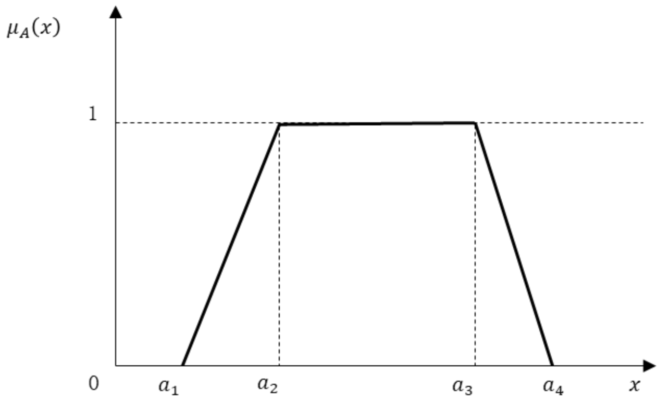


Fig. 2. A trapezoid fuzzy number.

Definition 2[58] A triangular fuzzy number B is defined by a three-dimensional array (b_1, b_2, b_3), and its membership function $\mu_B(x)$ is shown below, with the corresponding function graph depicted in Fig. 3.

$$\mu_B(x) = \begin{cases} 0, & x\epsilon(-\infty, b_1) \\ \frac{(x-b_1)}{(b_2-b_1)}, & x\epsilon[b_1, b_2] \\ \frac{(b_3-x)}{(b_3-b_2)}, & x\epsilon[b_2, b_3] \\ 0, & x\epsilon(b_3, +\infty) \end{cases} \quad (2)$$

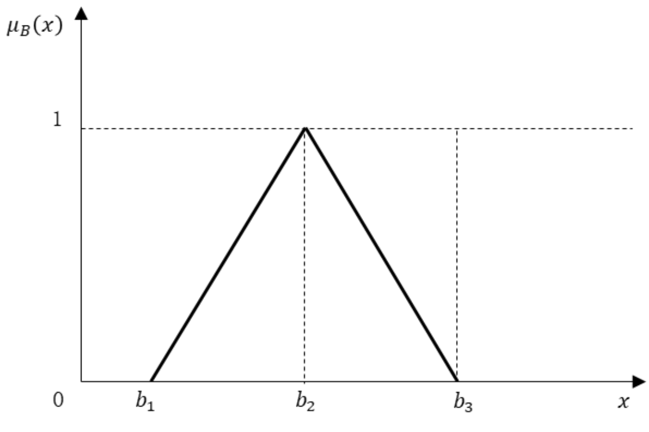


Fig. 3. A triangular fuzzy number.

Definition 3[44] Let $A = \{\langle x, \mu_A(x)\rangle | x\epsilon(0,1)\}$ and $B = \{\langle x, \mu_B(x)\rangle | x\epsilon(0,1)\}$. $A(x)$ represents a trapezoidal membership function, and $B(x)$ represents a triangular membership function. According to the method proposed by Kang, the reliability component B is transformed into a specific number α.

$$\alpha = \frac{\int x\mu_B(x)dx}{\int \mu_B(x)dx} \quad (3)$$

By incorporating the weight α onto the uncertainty component A, the weighted Z-number Z^α is obtained as follows: $Z^\alpha = \left\{\left(x, \mu_z(x) | \mu_z(x) = \mu_A\left(\frac{x}{\sqrt{\alpha}}\right)\right)\right\}$, consequently, the transformed trapezoidal fuzzy number Z^A is given by: $Z^A = (\sqrt{\alpha}a_1, \sqrt{\alpha}a_2, \sqrt{\alpha}a_3, \sqrt{\alpha}a_4)$

Example 1 Let $A = (0.1,0.2,0.3,0.4)$ and $B = (0.5,0.7,0.9)$. $A(x)$ represents a trapezoidal membership function, and $B(x)$ represents a triangular membership function. According to the Definition 4 proposed the operational laws, we obtain as follows:

$$\mu_B(x) = \begin{cases} 0, & x\epsilon(-\infty, 0.5) \\ \dfrac{(x-0.5)}{(0.7-0.5)}, & x\epsilon[0.5,0.7] \\ \dfrac{(0.9-x)}{(0.9-0.7)}, & x\epsilon[0.7,0.9] \\ 0, & x\epsilon(0.9, +\infty) \end{cases}$$

$$\alpha = \frac{\int x\mu_B(x)dx}{\int \mu_B(x)dx}$$

$$= \frac{\int_{0.5}^{0.7} x\frac{(x-0.5)}{(0.7-0.5)}dx + \int_{0.7}^{0.9} x\frac{(0.9-x)}{(0.9-0.7)}dx}{(0.9-0.5)/2} = 0.7$$

$$Z^A = (\sqrt{\alpha}a_1, \sqrt{\alpha}a_2, \sqrt{\alpha}a_3, \sqrt{\alpha}a_4)$$
$$= (\sqrt{0.7}\times 0.1, \sqrt{0.7}\times 0.2, \sqrt{0.7}\times 0.3, \sqrt{0.7}\times 0.4) = (0.084,0.167,0.251,0.335)$$

Definition 4[59] The fundamental operations of trapezoidal fuzzy numbers and triangular fuzzy numbers are similar. In this paper, taking trapezoidal fuzzy numbers as an example, let $A_1 = (a_1, a_2, a_3, a_4)$ and $A_2 = \left(a_1', a_2', a_3', a_4'\right)$, the corresponding rules for basic operations are listed as follows:

1) $A_1 \oplus A_2 = [a_1 + a_1', a_2 + a_2', a_3 + a_3', a_4 + a_4']$
2) $A_1 \times n = [na_1, na_2, na_3, na_4]$
3) $A_1 \otimes A_2 = [a_1 \cdot a_1', a_2 \cdot a_2', a_3 \cdot a_3', a_4 \cdot a_4']$
4) $A_1 / A_2 = [a_1/a_1', a_2/a_2', a_3/a_3', a_4/a_4']$

Definition 5[60] The distance between these two trapezoidal fuzzy numbers $dis(A_1, A_2)$ can be represented as:

$$dis(A_1, A_2) = \sqrt{\frac{1}{4}[(a_1 - a_1')^2 + (a_2 - a_2')^2 + (a_3 - a_3')^2 + (a_4 - a_4')^2]} \quad (4)$$

Example 2 Let $A_1 = (1,2,3,4)$ and $A_2 = (4,3,2,1)$ be two trapezoidal fuzzy numbers. According to the Definition 5 proposed the operational laws, we obtain as follows:

$$dis(A_1, A_2) = \sqrt{\frac{1}{4}[(1-4)^2 + (2-3)^2 + (3-2)^2 + (4-1)^2]}$$
$$= 2.236$$

4. Proposed method

To select the optimal design among a large number of product design schemes, this paper introduces a comprehensive approach. Firstly, the weights of various experts are calculated using consistency theory to establish an expert consensus decision model, which is then used to determine the weights of individual attributes. Subsequently, a scheme prioritization and ranking method is proposed, utilizing the Z-number based MABAC method to obtain the best possible solution. In summary, the proposed approach combines expert consensus modeling, attribute weighting, and the Z-number based MABAC method to effectively and systematically identify the optimal product design scheme from a multitude of alternatives. The evaluation and selection process of the optimal design concept is shown in Fig. 4

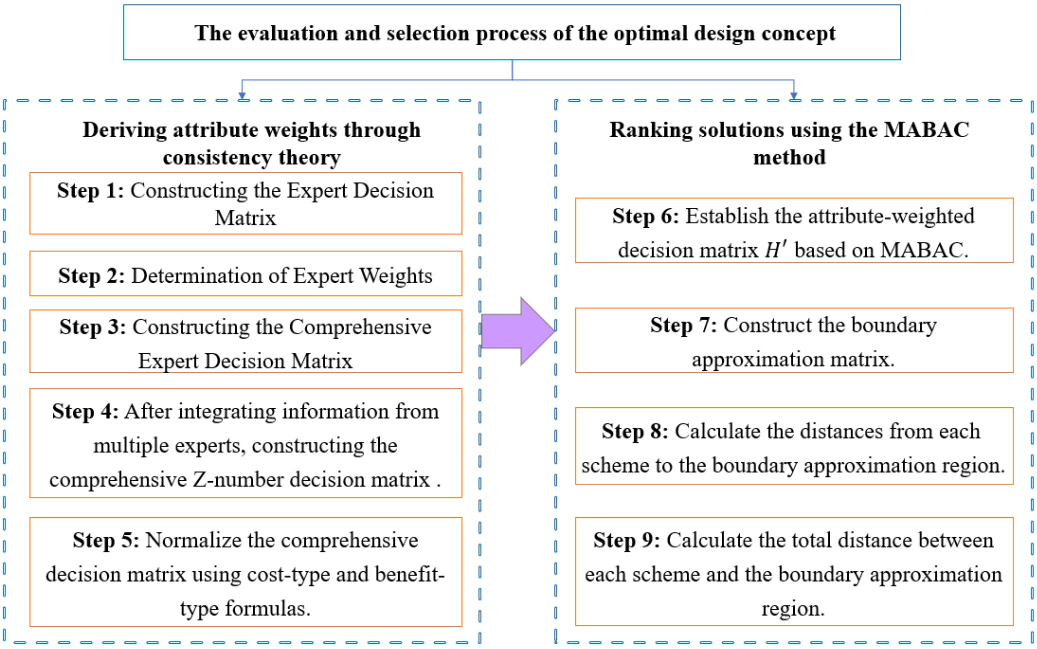


Fig. 4. The product design optimization process.

Step 1: Constructing the expert decision matrix

The decision experts simultaneously provide evaluation information and reliability information in a multi-level scale format during the assessment. The decision assessment is conducted by the experts considering the provided "evaluation information" comprehensively. This process establishes two sets of variables: the evaluation information set and the reliability information set. Using Z-number $Z_{ij}^{(k)} = (A_{ij}, B_{ij})$, the k decision expert's evaluation of design scheme i in dimension j is represented, where A_{ij} and B_{ij} represent the evaluation information set and the reliability information set, respectively.

The decision matrix for expert k is as follows:

$$D^{(k)} = \begin{bmatrix} Z_{11}^{(k)} & Z_{12}^{(k)} & \cdots & Z_{1n}^{(k)} \\ Z_{21}^{(k)} & Z_{22}^{(k)} & \cdots & Z_{2n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{m1}^{(k)} & Z_{m2}^{(k)} & \cdots & Z_{mn}^{(k)} \end{bmatrix}, k = 1,2,\dots,K; i = 1,2,\dots,m; j = 1,2,\dots,n \tag{5}$$

Converting it into trapezoidal fuzzy number representation, $E^{(k)}$ represents the transformed matrix

$$E^{(k)} = \begin{bmatrix} (a_{11}^{(k)}, b_{11}^{(k)}, c_{11}^{(k)}, d_{11}^{(k)}) & \cdots & (a_{1n}^{(k)}, b_{1n}^{(k)}, c_{1n}^{(k)}, d_{1n}^{(k)}) \\ (a_{21}^{(k)}, b_{21}^{(k)}, c_{21}^{(k)}, d_{21}^{(k)}) & \cdots & (a_{2n}^{(k)}, b_{2n}^{(k)}, c_{2n}^{(k)}, d_{2n}^{(k)}) \\ \vdots & \ddots & \vdots \\ (a_{m1}^{(k)}, b_{m1}^{(k)}, c_{m1}^{(k)}, d_{m1}^{(k)}) & \cdots & (a_{mn}^{(k)}, b_{mn}^{(k)}, c_{mn}^{(k)}, d_{mn}^{(k)}) \end{bmatrix} \tag{6}$$

Step 2: Determination of expert weights

The distance between two trapezoidal fuzzy numbers is expressed as formula (4). The group consistency of expert k evaluation information can be represented by con_k.

$$con_k = \frac{1}{\sum_h^K \sum_{i=1}^m \sum_{j=1}^n dis\left(Z_{ij}^h, Z_{ij}^k\right)} \tag{7}$$

Where $dis\left(Z_{ij}^{(h)}, Z_{ij}^{(k)}\right)$ represents the distance between the $Z_{ij}^{(h)}$ and , $Z_{ij}^{(k)}$, $h, k \epsilon (1,2, \ldots, K)$ and $h \neq k$.

Therefore, each expert's weight ω_k based on consistency is:

$$\omega_k = \frac{con_k}{\sum_{k=1}^K con_k} \tag{8}$$

Step 3: Constructing the comprehensive expert decision matrix

The comprehensive weighted decision matrix E, considering expert weights, can be obtained using the following formula:

$$E = \left(\omega_1 D^{(1)} + \omega_2 D^{(2)} + \cdots + \omega_k D^{(k)}\right), k = 1,2 \ldots, K \tag{9}$$

Converting it into a comprehensive decision matrix E_{ij} with trapezoidal fuzzy numbers:

$$E_{ij} = \left(\sum_{k=1}^K a_{ij}^{(k)}\, \omega^{(k)}, \sum_{k=1}^K b_{ij}^{(k)}\, \omega^{(k)}, \sum_{k=1}^K c_{ij}^{(k)}\, \omega^{(k)}, \sum_{k=1}^K d_{ij}^{(k)}\, \omega^{(k)}\right) \tag{10}$$

Applying the following formula to normalize the comprehensive decision matrix, P_{ij} represents the normalized matrix:

$$P_{ij} = \left(P(a_{ij}), P(b_{ij}), P(c_{ij}), P(d_{ij})\right) = \left(\frac{a'_{ij}}{\sum_{i=1}^m a'_{ij}}, \frac{b'_{ij}}{\sum_{i=1}^m b'_{ij}}, \frac{c'_{ij}}{\sum_{i=1}^m c'_{ij}}, \frac{d'_{ij}}{\sum_{i=1}^m d'_{ij}}\right) \tag{11}$$

Where, $i = 1,2, \ldots, m$ and $j = 1,2, \ldots, n$.

Applying the entropy weight formula, the values e_j for each attribute are calculated as follows:

$$e_j = -\frac{1}{\ln(m)} \sum_{i=1}^m P_{ij} \ln\left(P_{ij}\right) \tag{12}$$

The weight λ_j for attribute c_j is:

$$\lambda_j = \frac{|1-e_j|}{\sum_{j=1}^n |1-e_j|} \tag{13}$$

Step 4: After integrating information from multiple experts, construct the comprehensive Z-number decision matrix H.

The decision matrix H can be denoted as

$$H = \frac{1}{K}\left(E^{(1)} + E^{(2)} + \cdots + E^{(K)}\right) \tag{14}$$

Step 5: Normalize the comprehensive decision matrix H using cost-type and benefit-type formulas, $Z(x_{ij})$ represents the normalized matrix.

benefit-type:

$$Z(x_{ij}) = \left(\left[\frac{a_{ij} - \min_j a_{ij}}{\max_j a_{ij} - \min_j a_{ij}}\right]\left[\frac{b_{ij} - \min_j b_{ij}}{\max_j b_{ij} - \min_j b_{ij}}\right]\left[\frac{c_{ij} - \min_j c_{ij}}{\max_j c_{ij} - \min_j c_{ij}}\right]\left[\frac{d_{ij} - \min_j d_{ij}}{\max_j d_{ij} - \min_j d_{ij}}\right]\right) \tag{15}$$

cost-type:

$$Z(x_{ij}) = \left(\left[\frac{\max_j a_{ij} - a_{ij}}{\max_j a_{ij} - \min_j a_{ij}}\right]\left[\frac{\max_j b_{ij} - b_{ij}}{\max_j b_{ij} - \min_j b_{ij}}\right]\left[\frac{\max_j c_{ij} - c_{ij}}{\max_j c_{ij} - \min_j c_{ij}}\right]\left[\frac{\max_j d_{ij} - d_{ij}}{\max_j d_{ij} - \min_j d_{ij}}\right]\right) \tag{16}$$

Step 6: Establish the attribute-weighted decision matrix H' based on MABAC.

The decision matrix H' can be denoted as

$$H' = \lambda_j(1 + H) \tag{17}$$

Step 7: Construct the boundary approximation matrix G_j.

The boundary region of attribute c_j can be represented as:

$$G_j = \left(\prod_{i=1}^m H'\right)^{\frac{1}{m}} \tag{18}$$

Step 8: Calculate the distances from each scheme to the boundary approximation region.

The distances Q_{ij} between each scheme and the boundary approximation region can be calculated using the following formula:

$$Q_{ij} = H' - G_j \tag{19}$$

Step 9: Calculate the total distance between each scheme and the boundary approximation region.

The total distance Q_i of scheme A_i for each attribute can be obtained using the arithmetic mean aggregation function.

$$Q_i = \sum_{j=1}^n Q_{ij} \tag{20}$$

Rank the design schemes based on the distances and select the optimal scheme. If $Q_1 > Q_2$, it indicates that scheme A_1 is preferred over A_2. The design schemes can be sorted based on the values of Q_i, thus selecting the optimal design scheme.

5. Case study

The requirements of the users are significant for high-tech intelligent product development. In the design concept evaluation stage of the customer-oriented product, both technical and aesthetic are critical for improving the quality of the product. An automatic guided outdoor cleaning vehicle is an intelligent driverless transport system that can be used for cleaning purposes. It is an automated guided vehicle that can be used in various applications such as cleaning squares. This process presents a design for an automated guided cleaning vehicle as an example of how the proposed hybrid MADM methodology can be used to optimize the design schemes. In order to ensure superior performance of the system during the design process, we have employed a comprehensive approach, the process of which is illustrated in Fig. 5. The following will provide a detailed overview of this process.

1) User requirements and product functionality analysis

The starting point of the design is to deeply understand user requirements and the functionalities the product should possess. We divide this process into two key modules: user requirement analysis and product functionality analysis. These modules are connected through the flow of information to ensure alignment between product functionality and user expectations.

2) Attribute identification and selection

To define the critical attributes required for the product, we introduce the attribute identification and selection module. This module identifies and selects attributes from both user requirements and product functionalities, determining the attributes that need attention in the design process. This aids in sieving out the most important attributes from a multitude of possibilities. Given that each attribute holds different significance for the product, we need to determine and allocate attribute weights to quantitatively measure their importance. This approach allows for a more accurate evaluation of the impact of each attribute on the design proposal, avoiding excessive focus on certain minor attributes.

3) Comprehensive assessment and selection

Taking into account the weights of different attributes, we use the comprehensive assessment and Selection module to evaluate the merits of various design proposals. This aids in selecting the design proposal that performs best across multiple attribute trade-offs, thereby maximizing user satisfaction and product performance.

4) Optimal attribute scheme and modular design

Following the comprehensive assessment, we obtain the optimal attribute scheme. However, a system typically comprises multiple modules. To facilitate better design and implementation, we introduce the attribute assessment and selection module as well as the modular design module. The former ensures that each module thoroughly considers critical attributes, while the latter guarantees coherence and efficiency at the module level across the entire system.

Through the aforementioned series of modules and processes, we successfully break down the intricate design process into relatively independent modules, with each module responsible for dealing with specific attributes or factors. The black-box diagram clearly illustrates the flow of information and relationships between different modules, enabling a more systematic approach to design and optimization. This method has played a pivotal role in the design of automated guided outdoor cleaning vehicles providing a solid foundation for the ultimate product performance and user satisfaction.

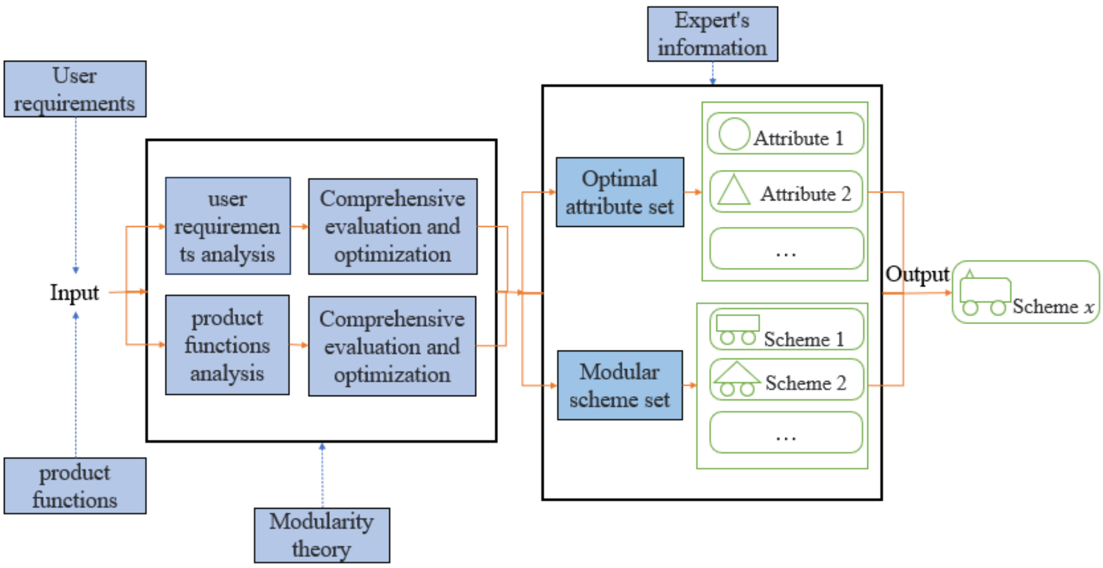


Fig. 5. Early stage of customer-oriented product development process.

At the same time, we use this case as a basis for further research. We establish a functional structure model based on information flow, create product module divisions based on user requirements, and apply a set partitioning approach to handle the composition and structure of the product[61]. The automatic guided outdoor cleaning vehicle is a complex electromechanical product that operates within a defined area, combining sweeping, and vacuuming functions. There are multiple product functional components, which are illustrated in Fig. 6 and summarized in Table 2.

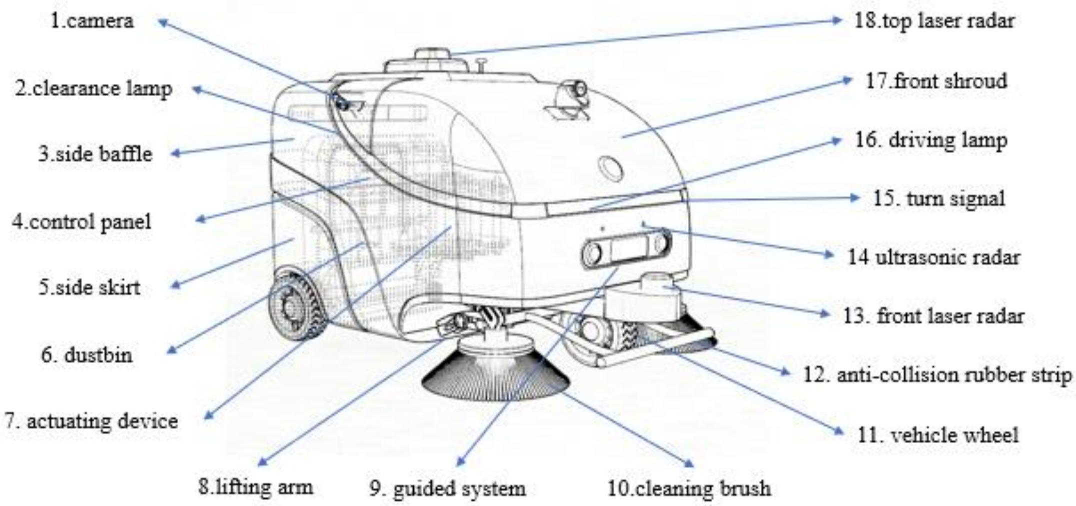


Fig. 6. Components of the intelligent unmanned cleaning vehicle.

Table 2 Component list of the intelligent unmanned cleaning vehicle

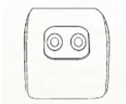
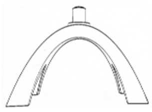
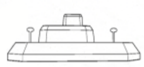
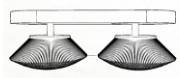
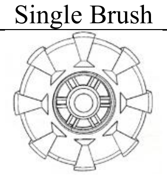
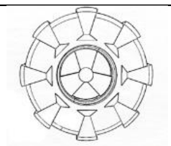
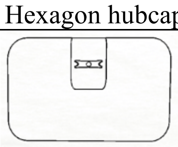
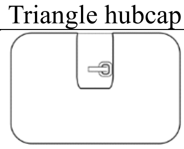
Serial number	Name	Serial number	Name
1	camera	2	clearance lamp
3	side baffle	4	control panel
5	side skirt	6	dustbin
7	actuating device	8	lifting arm
9	guided system	10	cleaning brush
11	vehicle wheel	12	bumper rubber strip
13	front laser radar	14	ultrasonic radar
15	turn signal	16	driving lamp
17	front shroud	18	top laser radar

Through cluster relationship calculations[62], the automatic guided cleaning vehicle modular division of the automated guided cleaning vehicle is shown in Table 3. For different modules, reasonable design solutions are established, and a total of 32 product design solutions are obtained through aggregation. Based on the division of different modules, we analyzed the corresponding sub-solutions related to each module, as shown in Table 4, and selected 8 feasible solutions from them.

Table 3. The modules of the intelligent unmanned cleaning vehicle.

Serial number	Module	Components	Number of schemes
P1	Sensor module	1,9,13,14,18	4
P2	Cleaning module	8,10	2
P3	Power module	4,7,11	2
P4	Vehicle body module	2,3,5,6,12,15,16,17	2

Table 4. Each module of the sub-solutions.

Serial number	A (label)	B (label)	C (label)	D (label)
P1	 Single camera	 Dual camera	 Laser radar	 Infrared radar
P2	 Single Brush	 Double Brush		
P3	 Hexagon hubcap	 Triangle hubcap		
P4	 Sliding-Type Trash Bin Door	 Handle-Type Trash Bin Door		

By comprehensively considering product functionality and user requirements, establish a set of preferred attributes for the solution. Based on user feedback, establish the decision attribute table as shown in Table 5.

Table 5. Decision attribute.

Attributes	Detailed information	Type
1	Usability and maintainability	Benefit
2	Work efficiency	Benefit
3	Work quality	Benefit
4	Working endurance	Benefit
5	Aesthetic appeal	Benefit
6	Manufacturing cost	Cost

Simultaneously, invite two experts from the design field and two experts from the production workshop, along with two customers with extensive consumer experience, for a total of six experts to provide evaluation information for the above case in the form of a 7-level scale and reliability information in the form of a 5-level scale, with the corresponding relationships are shown in Table 6.

Table 6. Expert decision information and corresponding fuzzy numbers.

Restriction	Symbol	TRFNs
Extremely poor/low	EP/EL	(0, 0, 1, 2)
Very poor/low	VP/VL	(1, 2, 3, 4)
Poor/low	P/L	(2, 3, 4, 6)
Fair	F	(3, 4, 6, 7)
Good/high	G/H	(4, 6, 7, 8)
Very good/high	VG/VH	(6, 7, 8, 9)
Extremely good/high	EG/EH	(8, 9, 10, 10)
Very uncertain	VU	(0, 0.1, 0.3)
Uncertain	U	(0.1, 0.3, 0.5)
Moderate	M	(0.3, 0.5, 0.7)
Certain	C	(0.5, 0.7, 0.9)
Very certain	VC	(0.7, 0.9, 1)

Based on the expert evaluation results, construct the corresponding decision matrix. Taking the evaluation information from Expert 1 as an example, their decision matrix is shown below. Using the evaluation information, similar decision matrix models can be established for the other experts.

$$D^{(1)}=\begin{bmatrix} (VG,C) & (EG,M) & (VG,VC) & (G,M) & (VG,C) & (H,VC) \\ (EG,C) & (EG,C) & (VG,VC) & (G,M) & (VG,C) & (H,M) \\ (EG,VC) & (VG,C) & (EG,VC) & (G,C) & (G,C) & (H,C) \\ (EG,VC) & (EG,VC) & (EG,VC) & (G,C) & (VG,VC) & (H,C) \\ (VG,C) & (EG,VC) & (EG,VC) & (G,C) & (VG,VC) & (VH,VC) \\ (EG,VC) & (VG,C) & (VG,C) & (VG,VC) & (EG,VC) & (VH,VC) \\ (EG,VC) & (VG,C) & (VG,C) & (EG,VC) & (VG,VC) & (H,C) \\ (EG,VC) & (EG,C) & (EG,VC) & (EG,VC) & (VG,VC) & (H,C) \end{bmatrix}$$

According to the correspondence between linguistic information in the table and trapezoidal fuzzy numbers and triangular fuzzy numbers, the decision matrix $D^{(1)}$ can be transformed into:

$$E^{(1)}=\begin{bmatrix} ((6,7,8,9),(0.5,0.7,0.9)) & ((8,9,10,10),(0.3,0.5,0.7)) & \dots & ((4,6,7,8),(0.7,0.9,1)) \\ ((8,9,10,10),(0.5,0.7,0.9)) & ((8,9,10,10),(0.5,0.7,0.9)) & \dots & ((4,6,7,8),(0.3,0.5,0.7)) \\ ((8,9,10,10),(0.7,0.9,1)) & ((6,7,8,9),(0.5,0.7,0.9)) & \dots & ((4,6,7,8),(0.5,0.7,0.9)) \\ ((8,9,10,10),(0.7,0.9,1)) & ((8,9,10,10),(0.7,0.9,1)) & \dots & ((4,6,7,8),(0.5,0.7,0.9)) \\ ((6,7,8,9),(0.5,0.7,0.9)) & ((8,9,10,10),(0.7,0.9,1)) & \dots & ((6,7,8,9),(0.7,0.9,1)) \\ ((8,9,10,10),(0.7,0.9,1)) & ((6,7,8,9),(0.5,0.7,0.9)) & \dots & ((6,7,8,9),(0.7,0.9,1)) \\ ((8,9,10,10),(0.7,0.9,1)) & ((6,7,8,9),(0.5,0.7,0.9)) & \dots & ((4,6,7,8),(0.5,0.7,0.9)) \\ ((8,9,10,10),(0.7,0.9,1)) & ((8,9,10,10),(0.5,0.7,0.9)) & \dots & ((4,6,7,8),(0.5,0.7,0.9)) \end{bmatrix}$$

To compare the linguistic information provided by experts, according to formula (3), the Z-number matrix is transformed into a trapezoidal fuzzy number matrix. Taking Expert 1's decision information as an example, their decision matrix is transformed into:

$$E^{(1)}=\begin{bmatrix} (5.020,5.857,6.693,7.530) & (5.657,6.364,7.071,7.071) & \dots & (3.724,5.586,6.517,7.448) \\ (6.693,7.530,8.367,8.367) & (6.693,7.530,8.367,8.367) & \dots & (2.828,4.243,4.950,5.657) \\ (7.448,8.379,9.309,9.309) & (5.020,5.857,6.693,7.530) & \dots & (3.347,5.020,5.857,6.693) \\ (7.448,8.379,9.309,9.309) & (7.448,8.379,9.309,9.309) & \dots & (3.347,5.020,5.857,6.693) \\ (5.020,5.857,6.693,7.530) & (7.448,8.379,9.309,9.309) & \dots & (5.586,6.517,7.448,8.379) \\ (7.448,8.379,9.309,9.309) & (5.020,5.857,6.693,7.530) & \dots & (5.586,6.517,7.448,8.379) \\ (7.448,8.379,9.309,9.309) & (5.020,5.857,6.693,7.530) & \dots & (3.347,5.020,5.857,6.693) \\ (7.448,8.379,9.309,9.309) & (6.693,7.530,8.367,8.367) & \dots & (3.347,5.020,5.857,6.693) \end{bmatrix}$$

Using the consistency theory and formula (4) and (7), the total distance between expert evaluation information can be determined, which reveals the consistency of each decision expert's evaluations. The expert weights are determined by formula (8). The correlation coefficients and weights of each expert are shown in Table 7.

Table 7. Correlation coefficients and expert weights based on the consistency theory.

Decision experts (k)	Total distance (dis(k))	Consistency (con(k))	Weights ($w^{(k)}$)
Expert 1	286.538	0.00349	0.163
Expert 2	262.492	0.00381	0.177
Expert 3	289.875	0.00345	0.161
Expert 4	304.165	0.00329	0.153
Expert 5	255.579	0.00391	0.182
Expert 6	283.963	0.00352	0.164

Determined by the weights of each expert, the comprehensive decision matrix E_{ij} is as follows for Expert 1's weighted matrix:

$$E_{ij}=\begin{bmatrix} (4.913,5.922,6.788,7.654) & (5.638,6.625,7.474,7.915) & \dots & (4.326,5.632,6.499,7.366) \\ (5.646,6.651,7.503,7.936) & (4.420,5.669,6.500,7.194) & \dots & (3.961,5.367,6.214,7.060) \\ (5.871,6.906,7.787,8.227) & (4.517,5.663,6.514,7.365) & \dots & (4.215,5.708,6.608,7.508) \\ (5.619,6.684,7.578,8.179) & (5.898,6.780,7.661,8.238) & \dots & (4.359,5.813,6.662,7.267) \\ (5.268,6.322,7.206,7.941) & (5.839,6.872,7.752,8.202) & \dots & (5.210,6.268,7.184,8.100) \\ (5.037,6.120,7.343,7.958) & (5.217,6.226,7.092,7.806) & \dots & (4.905,6.119,7.035,7.952) \\ (5.356,6.368,7.253,7.986) & (4.699,5.886,6.769,7.654) & \dots & (3.817,5.440,6.324,7.206) \\ (5.724,6.776,7.675,8.258) & (4.776,5.798,6.801,7.534) & \dots & (3.601,5.401,6.301,7.201) \end{bmatrix}$$

By applying formula (11), the comprehensive decision matrix for this expert is normalized. Through the entropy weighting method, the weights for corresponding attributes can be calculated from formula (12) and (13) in the normalized matrix. The weights for each attribute are shown in Table 8.

Table 8. The weight for each attribute.

Attribute	C1	C2	C3
Weight	[0.065,0.091,0.017,0.100]	[0.158,0.124,0.271,0.125]	[0.237,0.339,0.203,0.075]
Attribute	C4	C5	C6
Weight	[0.172,0.198,0.322,0.375]	[0.242,0.190,0.085,0.200]	[0.126,0.058,0.102,0.125]

Summarizing the information from various experts and performing information fusion, the comprehensive Z-number decision matrix H is constructed using formula (14) as shown below:

$$H=\begin{bmatrix}(4.898,5.922,6.789,7.658) & (5.625,6.611,7.458,7.892) & \dots & (4.309,5.627,6.495,7.363)\\ (5.640,6.648,7.501,7.935) & (4.439,5.682,6.513,7.204) & \dots & (3.944,5.358,6.204,7.051)\\ (5.892,6.931,7.815,8.249) & (4.525,5.672,6.524,7.377) & \dots & (4.219,5.708,6.607,7.507)\\ (5.672,6.721,7.615,8.198) & (5.889,6.766,7.644,8.212) & \dots & (4.415,5.851,6.698,7.287)\\ (5.303,6.342,7.226,7.954) & (5.814,6.847,7.724,8.174) & \dots & (5.181,6.252,7.167,8.082)\\ (4.965,6.052,7.292,7.913) & (5.177,6.201,7.068,7.797) & \dots & (4.903,6.112,7.028,7.943)\\ (5.334,6.358,7.241,7.970) & (4.682,5.877,6.760,7.644) & \dots & (3.846,5.458,6.342,7.226)\\ (5.708,6.762,7.662,8.251) & (4.759,5.782,6.790,7.518) & \dots & (3.598,5.397,6.297,7.196)\end{bmatrix}$$

By applying formula (15) and formula (16), the matrix is normalized, resulting in the matrix $\widehat{H}$:

$$\widehat{H}=\begin{bmatrix}(0.000,0.000,0.000,0.000) & (0.818,0.799,0.780,0.683) & \dots & (0.551,0.699,0.698,0.697)\\ (0.746,0.720,0.694,0.467) & (0.000,0.000,0.000,0.000) & \dots & (0.781,1.000,1.000,1.000)\\ (1.000,1.000,1.000,0.997) & (0.059,0.000,0.009,0.172) & \dots & (0.608,0.609,0.582,0.558)\\ (0.779,0.792,0.805,0.911) & (1.000,0.931,0.934,1.000) & \dots & (0.484,0.449,0.487,0.771)\\ (0.407,0.416,0.426,0.499) & (0.948,1.000,1.000,0.962) & \dots & (0.000,0.000,0.000,0.000)\\ (0.067,0.129,0.490,0.430) & (0.509,0.450,0.458,0.588) & \dots & (0.176,0.157,0.144,0.135)\\ (0.439,0.432,0.441,0.526) & (0.168,0.174,0.204,0.437) & \dots & (0.843,0.888,0.857,0.830)\\ (0.815,0.833,0.851,1.000) & (0.221,0.094,0.229,0.312) & \dots & (1.000,0.956,0.903,0.859)\end{bmatrix}$$

The weighted Z-number decision matrix H' is determined using formula (17) as follows:

$$H'=\begin{bmatrix}(0.065,0.091,0.017,0.100) & (0.287,0.223,0.482,0.210) & \dots & (0.195,0.099,0.173,0.212)\\ (0.113,0.157,0.029,0.147) & (0.158,0.125,0.271,0.125) & \dots & (0.224,0.116,0.204,0.250)\\ (0.130,0.182,0.034,0.200) & (0.167,0.124,0.273,0.147) & \dots & (0.203,0.093,0.161,0.195)\\ (0.116,0.163,0.031,0.191) & (0.316,0.239,0.524,0.250) & \dots & (0.187,0.084,0.152,0.221)\\ (0.091,0.129,0.024,0.150) & (0.308,0.248,0.542,0.245) & \dots & (0.126,0.058,0.102,0.125)\\ (0.069,0.103,0.025,0.143) & (0.238,0.180,0.395,0.199) & \dots & (0.148,0.067,0.117,0.142)\\ (0.094,0.130,0.024,0.153) & (0.185,0.146,0.326,0.180) & \dots & (0.232,0.110,0.189,0.229)\\ (0.118,0.167,0.031,0.200) & (0.193,0.136,0.333,0.164) & \dots & (0.252,0.113,0.194,0.232)\end{bmatrix}$$

The boundary approximation region decision matrix is obtained using formula (18) and (19), based on this, the distance matrix Q_{ij} for each alternative to the boundary approximation region is constructed:

$$Q_{ij}=\begin{bmatrix}(-0.032,-0.046,-0.009,-0.057) & (0.063,0.052,0.102,0.025) & \dots & (0.004,0.009,0.016,0.016)\\ (0.016,0.02,0.003,-0.01) & (-0.066,-0.046,-0.109,-0.06) & \dots & (0.033,0.026,0.047,0.054)\\ (0.033,0.045,0.008,0.043) & (-0.057,-0.047,-0.107,-0.038) & \dots & (0.012,0.003,0.004,-0.001)\\ (0.019,0.026,0.005,0.034) & (0.092,0.068,0.144,0.065) & \dots & (-0.004,-0.006,-0.005,0.025)\\ (-0.006,-0.008,-0.002,-0.007) & (0.084,0.077,0.162,0.06) & \dots & (-0.065,-0.032,-0.055,-0.071)\\ (-0.028,-0.034,-0.001,-0.014) & (0.014,0.009,0.015,0.014) & \dots & (-0.043,-0.023,-0.04,-0.054)\\ (-0.003,-0.007,-0.002,-0.004) & (-0.039,-0.025,-0.054,-0.005) & \dots & (0.041,0.02,0.032,0.033)\\ (0.021,0.03,0.005\ \ 0.043) & (-0.031,-0.035,-0.047,-0.021) & \dots & (0.061,0.023,0.037,0.036)\end{bmatrix}$$

Finally, the total distance between each alternative and the boundary approximation region matrix is calculated using formula (20), as shown in Table 9. The average value is taken as the representative of the corresponding fuzzy number. By comparing the order of the alternatives based on this, it is found that Alternative 8 is the optimal choice.

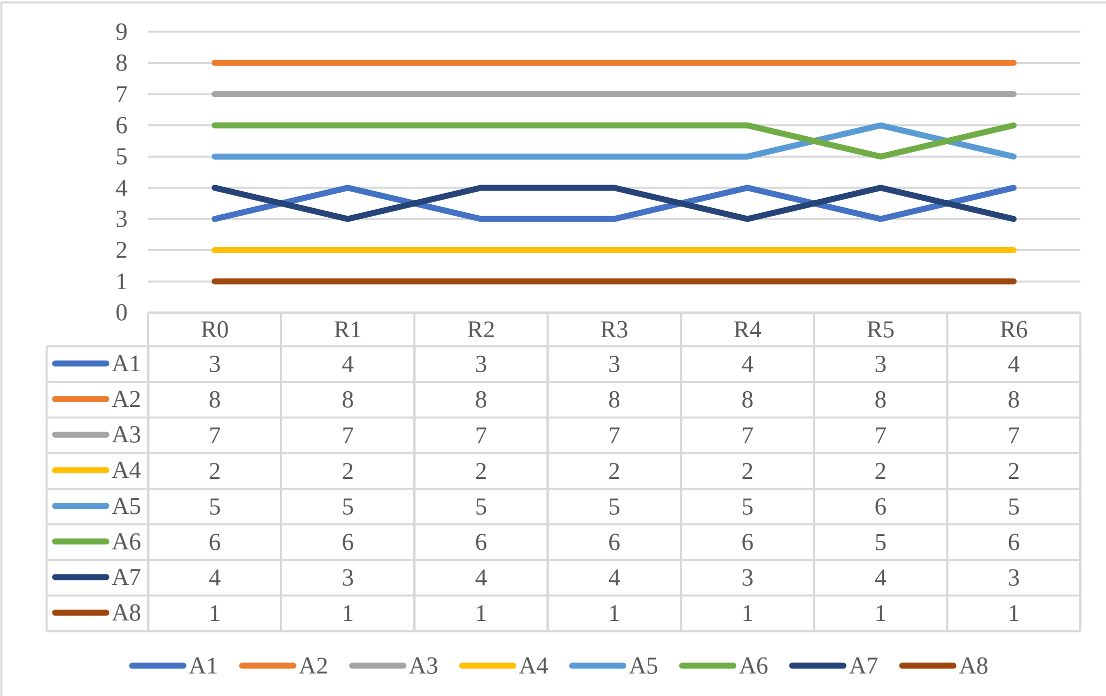
Table 9. The rankings of the 8 alternatives according to MABAC.

Alternative	Values of the attribute function	Average	Ranking
A1	$[0.108, 0.18, 0.228, 0.405]$	0.230	3
A2	$[-0.244, -0.213, -0.235, 0.053]$	-0.160	7
A3	$[-0.182, -0.015, -0.123, 0.197]$	-0.031	8
A4	$[0.271, 0.318, 0.309, 0.481]$	0.345	2
A5	$[-0.009, 0.145, 0.121, 0.271]$	0.132	5
A6	$[-0.018, 0.038, -0.027, 0.308]$	0.075	6
A7	$[0.045, 0.141, 0.159, 0.572]$	0.229	4
A8	$[0.375, 0.488, 0.411, 0.753]$	0.507	1

6. Discussion

6.1 Sensitivity analysis

In the initial phase, sensitivity analysis was conducted to assess the stability of the proposed method. The analysis results are shown in Fig. 7. To begin with, this paper performed sensitivity validation on the Z-number MABAC method through an experiment involving variations in attribute weights. The diagram below illustrates the changes in selected crucial attributes at different incremental levels. In round R0, all attributes maintained their initial weights. In round R1, the weight of attribute C1 was increased by 80%, while the weights of other attributes and variables remained unchanged. Similarly, in rounds R2 to R6, the weights of corresponding attributes C2 to C6 were increased to 1.8 times their original values, while the weights of other attributes remained constant. For instance, we calculated the ranking of alternative solutions in Round R1, and rankings were computed for the other rounds as well. As depicted in the graph below, there is a high degree of consistency in the rankings of alternative solutions. Across the six rounds of assessment, option A8 secured the first rank in all instances. Among the options, A2, A3, A4, A5, and A6 retained their rankings consistently throughout the experiment, with only A1 and A7 showing some fluctuations in comparison. The results of the sensitivity analysis underscore the method's substantial stability. The ranking of alternative solutions displayed minimal divergence, maintaining a predominantly stable state.



	R0	R1	R2	R3	R4	R5	R6
A1	3	4	3	3	4	3	4
A2	8	8	8	8	8	8	8
A3	7	7	7	7	7	7	7
A4	2	2	2	2	2	2	2
A5	5	5	5	5	5	6	5
A6	6	6	6	6	6	5	6
A7	4	3	4	4	3	4	3
A8	1	1	1	1	1	1	1

Fig. 7. Results of sensitivity analysis.

6.2 Comparative analysis and discussion

In order to reduce the subjective and improve the reliable and precise of our method, besides the fuzzy Z-number theory, several strategies are integrated in this method, as is shown in Fig. 8. Firstly, the modular design method can clarify the responsibilities and functions of each module, making each module more independent. This division can reduce the overall complexity of the system and reduce the influence of subjectivity on the entire system. Secondly, considering the knowledge and background of the experts are not even, the expert weight is processed based on the consistency theory by their information without any additional input, so as to reduce the affect generated by the experts' background. Thirdly, the entropy method is an excellent objective weight determination way. In our method, the attribute weight is obtained by combining the entropy weight method to further reduce the subjective influence in the decision-making process. By aggregating Z-number, the MABAC method and the strategies above, the reliability and precision of this process have been enhanced.

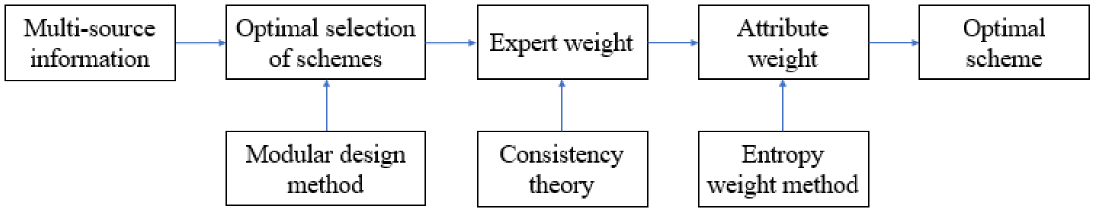


Fig. 8. The process of determining the optimal solution.

To better illustrate the advantages of this method, we conducted an effectiveness analysis and compared our proposed method with other existing approaches to ascertain its validity and reasonableness. Therefore, this paper proposes the trapezoidal fuzzy number- TOPSIS method and trapezoidal fuzzy number- VIKOR method based on the trapezoidal fuzzy number- VIKOR method and proposes the trigonometric fuzzy number-MABAC method based on the MABAC method. The following is the process of the proposed method:

1) Experiment I: compared with the Trapezoidal Fuzzy Number-TOPSIS method, the frameworks are shown in Fig. 9. The TOPSIS method is straightforward to comprehend, with a relatively intuitive calculation process. It is suitable for cases where positive and negative ideal solutions are known, particularly well-suited for handling simple decision problems. Simultaneously, considering the distances between alternative solutions and the positive and negative ideal solutions allows it to depict the proximity of alternative solutions. However, it tends to overlook the differences among alternative solutions and does not account for weight distribution and inter-attribute correlations, potentially struggling with complex multi-expert group decision-making problems. The core of the MABAC method is boundary approximation, which considers the positions of alternative solutions in a multi-dimensional attribute space, beyond merely focusing on the distance between positive and negative ideal solutions. Moreover, it can handle cases where weights are unevenly distributed across different attributes, permitting adjustments to attribute weights based on specific circumstances to better reflect decision-makers' preferences.

2) Experiment II: compared with the Trapezoidal Fuzzy Number-VIKOR method, the frameworks are shown in Fig. 10. The VIKOR method takes into account the distances between positive and negative ideal solutions and the worst alternative solution. In contrast to the TOPSIS method, it comprehensively reflects the differences among alternative solutions. It can handle cases of unevenly distributed attribute weights and is well-suited for situations with uncertain weight distributions. However, in some simple cases, it might become overly complex, with a relatively intricate calculation process. When dealing with a large number of alternative solutions, the computational workload could be substantial. On the other hand, the calculation process of the MABAC method is relatively straightforward. Additionally, since the MABAC method comprehensively considers factors such as boundary approximation, weight distribution, and range limits, it may exhibit greater robustness and adaptability when dealing with complex real-world decision-making problems. It has the potential to handle multi-alternative, multi-attribute decision problems more effectively.

3) Experiment III: compared with the Triangle Fuzzy Number-MABAC method, the frameworks are shown in Fig. 11. The Trapezoidal Fuzzy Number approach offers greater flexibility when compared to triangle fuzzy numbers. It can more accurately describe complex uncertainties by considering varying degrees of fuzziness and variability. Trapezoidal fuzzy numbers excel in specific application scenarios, such as risk assessment and control systems, as they can better adapt to the complexity of real-world problems and provide a more precise reflection of uncertainties. While Triangle Fuzzy Numbers are computationally simpler and widely applicable, their expressive power is relatively limited. They may struggle to describe certain complex uncertainty situations, especially when multiple sources of fuzziness are present in attribute indicators. In the context of multi-expert group decision-making, Triangle Fuzzy Numbers might not capture the true characteristics of actual uncertainties accurately, leading to less precise predictive or decision outcomes.

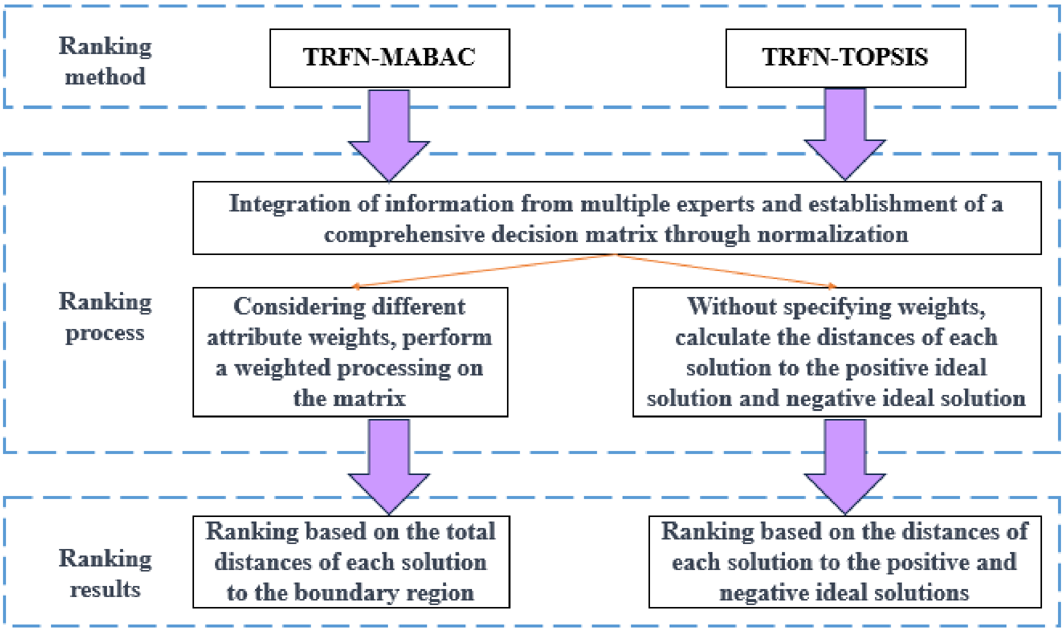


Fig. 9. The frameworks of experiment I.

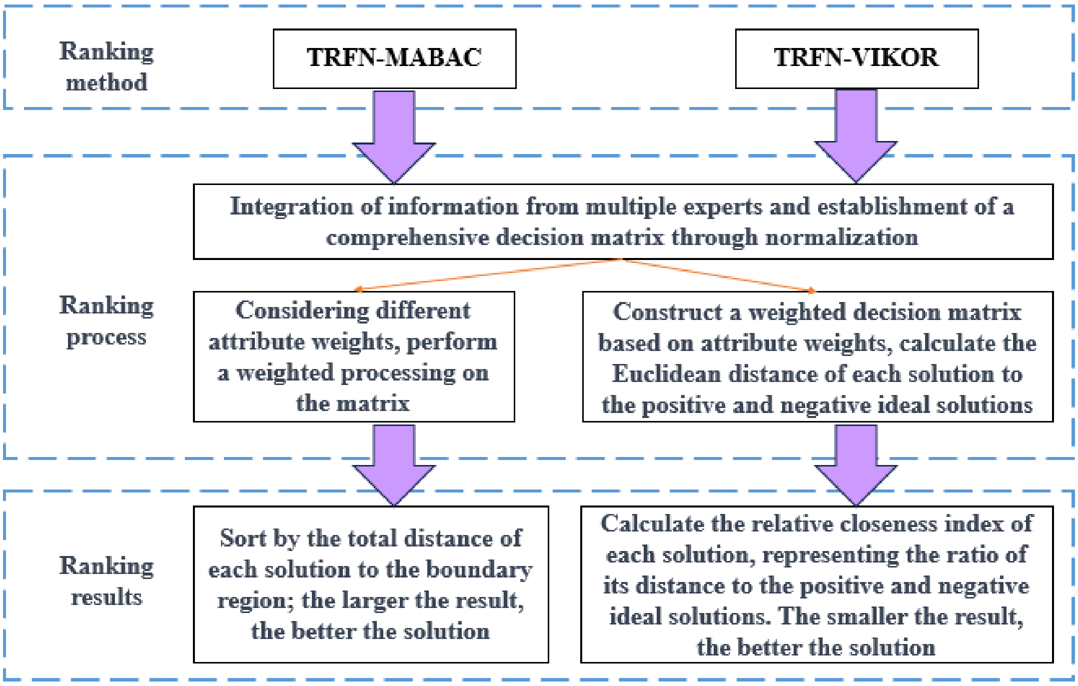


Fig. 10. The frameworks of experiment II.

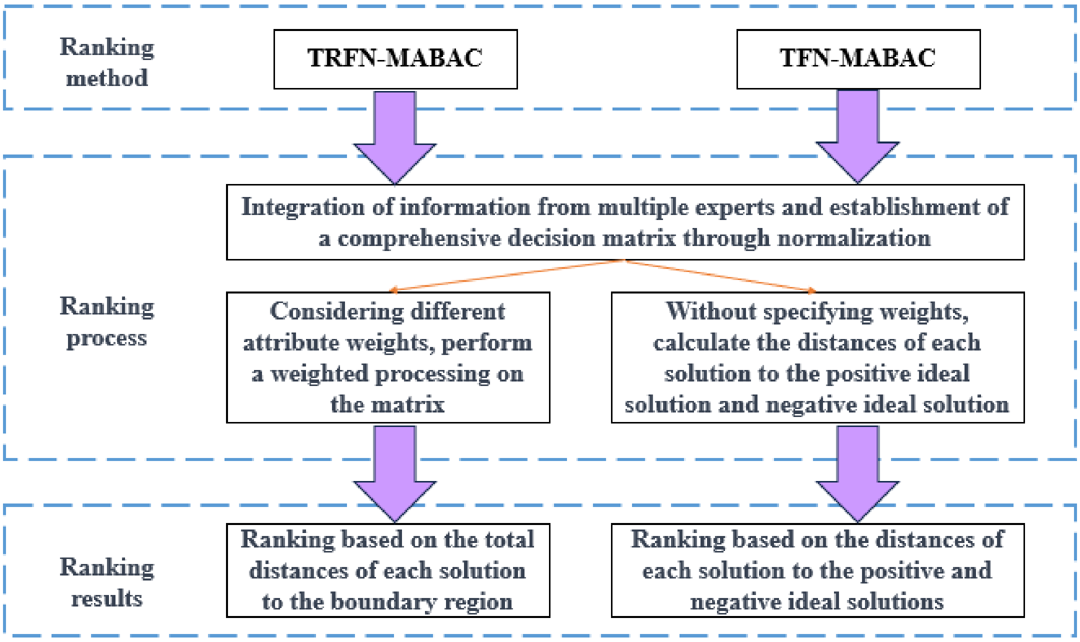


Fig. 11. The frameworks of experiment III.

By conducting the method comparison, the rankings for each alternative solution are computed as shown in Fig. 12. In the TOPSIS method, the ranking order is A8 ≻ A4 ≻ A1 ≻ A7 ≻ A5 ≻ A6 ≻ A3 ≻ A2, indicating that A8 is the optimal solution. In the VIKOR method, the ranking order is A4 ≻ A1 ≻ A7 ≻ A8 ≻ A5 ≻ A6 ≻ A3 ≻ A2, indicating that A4 is the optimal solution. In contrast, in the Triangular Fuzzy Number—MABAC method, the ranking order is A8 ≻ A4 ≻ A7 ≻ A1 ≻ A5 ≻ A6 ≻ A3 ≻ A2, indicating that A8 is the optimal solution.

The Spearman's rank correlation coefficient graphs for the sorting results of each method are shown in Fig. 13. From the graphs, it can be observed that the TRFN-TOPSIS method and the TRFN-MABAC method exhibit the highest correlation. However, whether it is TFN or TRFN, the correlation between the VIKOR method and other methods is not high, especially the correlation between TRFN-VIKOR method and TFN-MABAC method, which is only 0.833.

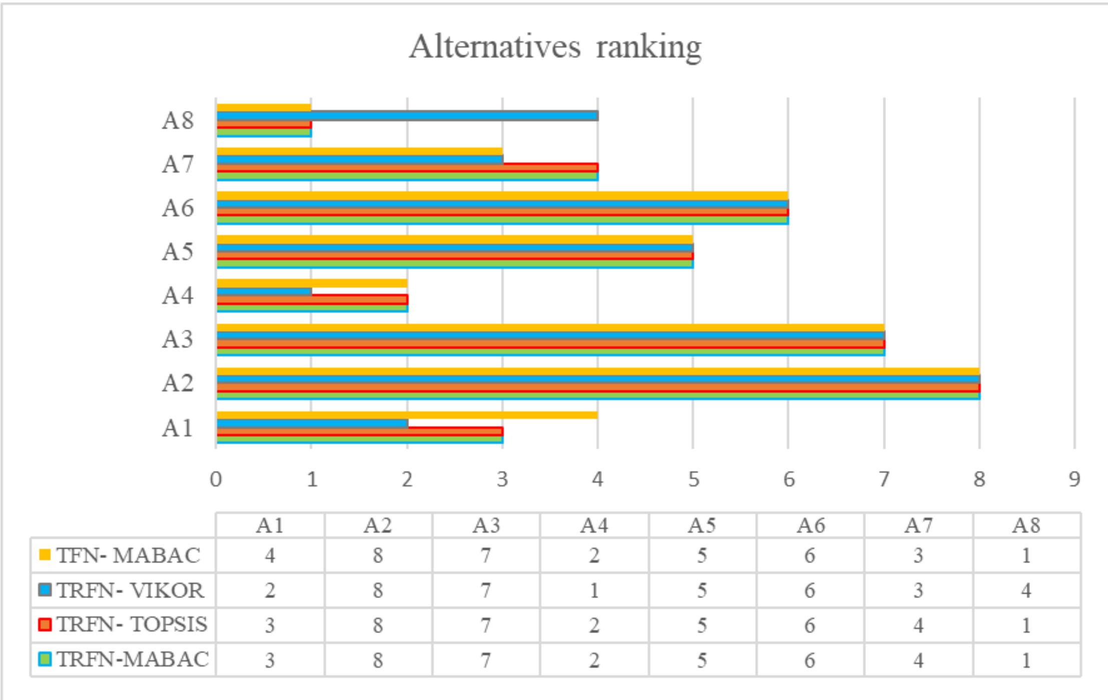


Fig. 12. The rankings of the alternatives determined by different methods.

	TRFN-MABAC	TRFN-TOPSIS	TRFN-VIKOR	TFN-MABAC
TRFN-MABAC	1.000	1.000	0.857	0.976
TRFN-TOPSIS	1.000	1.000	0.857	0.976
TRFN-VIKOR	0.857	0.857	1.000	0.833
TFN-MABAC	0.976	0.976	0.833	1.000

Fig. 13. The Spearman correlation coefficients between the results of the methods.

7. Conclusions

Addressing the reliability of the expert information, this paper presents a MADM method that integrated Z-number and MABAC in the product conceptual design evaluation process. In this paper, a design concept evaluation selection model based on a modular design framework for complicated products is proposed. In response to the uncertainty and unreliability of expert assessments, we have introduced an innovative method for information processing and transformation, combining Z-number with trapezoidal fuzzy numbers. This integrated approach not only effectively addresses the fuzziness in expert assessments but also enhances the overall reliability of the evaluations. Sensitivity analysis and comparisons show the effectiveness and efficiency of the proposed method. This model provides a viable solution for the rational utilization of expert knowledge in new product research and development, offering a practical approach to dealing with the challenges posed by expert unreliability.

However, there still exists a certain degree of subjectivity in determining the transformation parameters of fuzzy numbers and experts weight allocation, which could potentially influence the outcomes. Future research could leverage artificial intelligence-based big data models and employ deep learning algorithms to establish more precise decision models, thereby enhancing the accuracy of fuzzy number transformation.

Data available statement

The data that supports the findings of this study are available athttps://data.mendeley.com/datasets/7vrpfhyvjw/1.

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References

1. Rogalewicz M, Kujawińska A, Feledziak A. Ensuring the reliability and reduction of quality control costs by minimizing process variability. Maintenance & Reliability/Eksploatacja i Niezawodnosc. 2023;25(2). https://doi.org/10.17531/ein/162626
2. Summers GJ, Scherpereel CM. Flawed decision models and flexibility in product development. Journal of Engineering and Technology Management. 2023;67:101728.
3. Li X-Q, Song L-K, Choy Y-S, Bai G-C. Fatigue reliability analysis of aeroengine blade-disc systems using physics-informed ensemble learning. Philosophical Transactions of the Royal Society A. 2023;381(2260):20220384.
4. Chen B, Chen Y, Zhou H, Bai X, Li B, Guo X. A Bi-level gaming programming for regional integrated energy system considering the users' reliability incentive. Reliability Engineering & System Safety. 2023;229:108839.
5. Li X-Q, Song L-K, Choy Y-S, Bai G-C. Multivariate ensembles-based hierarchical linkage strategy for system reliability evaluation of aeroengine cooling blades. Aerospace Science and Technology. 2023;138:108325.
6. Mashal I. Evaluation and assessment of smart grid reliability using fuzzy multi-criteria decision-making. Applications of fuzzy logic in planning and operation of smart grids: Springer; 2021. p. 67-104. https://doi.org/10.1007/978-3-030-64627-1_3
7. Lima MB, Kubota FI. A modular product design framework for the home appliance industry. The International Journal of Advanced Manufacturing Technology. 2022;120(3):2311-30. https://doi.org/10.1007/s00170-022-08896-9
8. Ma Q, Sun H, Chen Z, Tan Y. A novel MCDM approach for design concept evaluation based on interval-valued picture fuzzy sets. PLOS

ONE. 2023;18(11):e0294596.

9. Darko AP, Antwi CO, Asamoah KO, Opoku-Mensah E, Ren J. A probabilistic reliable linguistic PROBID method for selecting electronic mental health platforms considering users' bounded rationality. Engineering Applications of Artificial Intelligence. 2023;125:106716.
10. Salas P, Venkataraman S. Laminate optimization incorporating analysis and model parameter uncertainties for predictable failure. Structural and Multidisciplinary Optimization. 2009;37:541-55. https://doi.org/10.1007/s00158-008-0259-y
11. Chen Z, Zhong P, Liu M, Sun H, Shang K. A novel hybrid approach for product concept evaluation based on rough numbers, shannon entropy and TOPSIS-PSI. Journal of Intelligent & Fuzzy Systems 2021;40:12087-99. https://doi.org/10.3233/JIFS-210184
12. Kimita K, Shimomura Y, Arai T. Evaluation of customer satisfaction for PSS design. Journal of Manufacturing Technology Management. 2009;20(5):654-73. https://doi.org/10.1108/17410380910961046
13. Kwok CP, Tang YM. A fuzzy MCDM approach to support customer-centric innovation in virtual reality (VR) metaverse headset design. Advanced Engineering Informatics. 2023;56:101910.
14. Mirtalaie MA, Hussain OK, Chang E, Hussain FK. A decision support framework for identifying novel ideas in new product development from cross-domain analysis. Information Systems. 2017;69:59-80. https://doi.org/10.1016/j.is.2017.04.003
15. Cristiano JJ, Liker JK, White CCI. Key factors in the successful application of quality function deployment (QFD). IEEE Transactions on Engineering Management. 2001;48(1):81-95. https://doi.org/10.1109/17.913168
16. Chen Z, Zhong P, Liu M, Ma Q, Si G. A novel integrated MADM method for design concept evaluation. Scientific Reports. 2022;12(1):15885. https://doi.org/10.1038/s41598-022-20044-7
17. Qi J, Hu J, Peng Y. New design concept evaluation method involving customer preferences based on rough distance to redefined ideal solution. Computers & Industrial Engineering. 2020;147:106677.
18. Zoghi M, Rostami G, Khoshand A, Motalleb F. Material selection in design for deconstruction using Kano model, fuzzy-AHP and TOPSIS methodology. Waste Management & Research. 2021;40(4):410-9. https://doi.org/10.1177/0734242X211013904
19. Zadeh LA. Fuzzy sets. Information and Control. 1965;8(3):338-53. https://doi.org/10.1016/S0019-9958(65)90241-X
20. Alshraideh M, Ababneh S, Gunay EE, Al-Araidah O. A fuzzy-TOPSIS model for maintenance outsourcing considering the quality of submitted tender documents. Eksploatacja i Niezawodność. 2021;23(3). https://doi.org/10.17531/ein.2021.3.5
21. Li J, DuAn R. Dynamic diagnostic strategy based on reliability analysis and distance-based VIKOR with heterogeneous information. Eksploatacja i Niezawodność. 2018;20(4). https://doi.org/10.17531/ein.2018.4.12
22. Jiang J, Ren M, Wang J. Interval number multi-attribute decision-making method based on TOPSIS. Alexandria Engineering Journal. 2022;61(7):5059-64. https://doi.org/10.1016/j.aej.2021.09.031
23. Zhu G-N, Hu J, Ren H. A fuzzy rough number-based AHP-TOPSIS for design concept evaluation under uncertain environments. Applied Soft Computing. 2020;91:106228.
24. Zhou M, Zheng Y-Q, Chen Y-W, Cheng B-Y, Herrera-Viedma E, Wu J. A large-scale group consensus reaching approach considering self-confidence with two-tuple linguistic trust/distrust relationship and its application in life cycle sustainability assessment. Information Fusion. 2023;94:181-99. https://doi.org/10.1016/j.inffus.2023.01.026
25. Liu S, Yu W, Chan FTS, Niu B. A variable weight-based hybrid approach for multi-attribute group decision making under interval-valued intuitionistic fuzzy sets. International Journal of Intelligent Systems. 2021;36(2):1015-52. https://doi.org/10.1002/int.22329
26. Singh S, Ganie AH. On a new picture fuzzy correlation coefficient with its applications to pattern recognition and identification of an investment sector. Computational and Applied Mathematics. 2021;41(1):8. https://doi.org/10.1007/s40314-021-01699-w
27. Jiang Z, Wei G, Guo Y. Picture fuzzy MABAC method based on prospect theory for multiple attribute group decision making and its application to suppliers selection. Journal of Intelligent & Fuzzy Systems. 2022;42:3405-15. https://doi.org/10.3233/JIFS-211359
28. Thurston DL, Carnahan JV. Fuzzy Ratings and Utility Analysis in Preliminary Design Evaluation of Multiple Attributes. Journal of Mechanical Design. 1992;114(4):648-58. https://doi.org/10.1115/1.2917056
29. Alekperov R, editor Using Fuzzy Z - Numbers When Processing Flexible Queries. Intelligent and Fuzzy Systems; 2022 2022//; Cham: Springer International Publishing. https://doi.org/10.1007/978-3-031-09173-5_52
30. Shidpour H, Da Cunha C, Bernard A. Group multi-criteria design concept evaluation using combined rough set theory and fuzzy set theory. Expert Systems with Applications. 2016;64:633-44. https://doi.org/10.1016/j.eswa.2016.08.022

31. Geng X, Chu X, Zhang Z. A new integrated design concept evaluation approach based on vague sets. Expert Systems with Applications. 2010;37(9):6629-38. https://doi.org/10.1016/j.eswa.2010.03.058
32. Jing L, Yao J, Gao F, Li J, Peng X, Jiang S. A rough set-based interval-valued intuitionistic fuzzy conceptual design decision approach with considering diverse customer preference distribution. Advanced Engineering Informatics. 2021;48:101284.
33. Jabeen K, Ullah K, Akram M, Haleemzai I. Interval Valued Picture Fuzzy Aczel–Alsina Aggregation Operators and Their Application by Using the Multiattribute Decision Making Problem. Journal of Mathematics. 2023;2023:1707867.
34. Xu W, Shang X, Wang J, Xu Y. Multi-Attribute Decision-Making Based on Interval-Valued q-Rung Dual Hesitant Uncertain Linguistic Sets. IEEE Access. 2020;8:26792-813. https://doi.org/10.1109/ACCESS.2020.2968381
35. Chen Z, Zhong P, Liu M, Ma Q, Si G. An integrated expert weight determination method for design concept evaluation. Scientific Reports. 2022;12(1):6358. https://doi.org/10.1038/s41598-022-10333-6
36. Zadeh LA. A Note on Z-numbers. Information Sciences. 2011;181(14):2923-32. https://doi.org/10.1016/j.ins.2011.02.022
37. Banerjee R, Pal SK. Z*-numbers: Augmented Z-numbers for machine-subjectivity representation. Information Sciences. 2015;323:143-78. https://doi.org/10.1016/j.ins.2015.06.026
38. Allahviranloo T, Ezadi S. Z-Advanced numbers processes. Information Sciences. 2019;480:130-43. https://doi.org/10.1016/j.ins.2018.12.012
39. Wu M-C, Mao J-J, Yao A-T, Wu T. The novel entropy measurements of Z +-numbers and their application on multi-attribute decision making problem. Journal of Intelligent & Fuzzy Systems. 2021;40:131-48. https://doi.org/10.3233/JIFS-190300
40. Shen K-W, Wang J-Q, Wang T-L. The arithmetic of multidimensional Z-number. Journal of Intelligent & Fuzzy Systems. 2019;36:1647-61. https://doi.org/10.3233/JIFS-18927
41. Banerjee R, Pal SK, Pal JK. A Decade of the Z-Numbers. IEEE Transactions on Fuzzy Systems. 2022;30(8):2800-12. https://doi.org/10.1109/TFUZZ.2021.3094657
42. Aliev RA, Guirimov BG, Huseynov OH, Aliyev RR. Z-relation equation-based decision making. Expert Systems with Applications. 2021;184:115387.
43. Li Y, Rao C, Goh M, Xiao X. Novel multi-attribute decision-making method based on Z-number grey relational degree. Soft Computing. 2022;26(24):13333-47. https://doi.org/10.1007/s00500-022-07487-w
44. Kang B, Wei D, Li Y, Deng Y. A method of converting Z-number to classical fuzzy number. Journal of Information and Computational Science. 2012;9:703-9.
45. Kang B, Deng Y, Sadiq R. Total utility of Z-number. Applied Intelligence. 2018;48(3):703-29. https://doi.org/10.1007/s10489-017-1001-5
46. Huang G, Xiao L, Pedrycz W, Pamucar D, Zhang G, Martínez L. Design alternative assessment and selection: A novel Z-cloud rough number-based BWM-MABAC model. Information Sciences. 2022;603:149-89. https://doi.org/10.1016/j.ins.2022.04.040
47. Qi J, Hu J, Huang H, Peng Y. New customer-oriented design concept evaluation by using improved Z-number-based multi-criteria decision-making method. Advanced Engineering Informatics. 2022;53:101683.
48. Zhu G-N. Design concept evaluation considering information reliability, uncertainty, and subjectivity: An integrated rough-Z-number-enhanced MCGDM methodology. Advanced Engineering Informatics. 2022;54:101796.
49. Liu Q, Chen J, Wang W, Qin Q. Conceptual Design Evaluation Considering Confidence Based on Z-AHP-TOPSIS Method. Applied Sciences [Internet]. 2021; 11(16). https://doi.org/10.3390/app11167400
50. Aydoğan S, Günay EE, Akay D, Okudan Kremer GE. Concept design evaluation by using Z-axiomatic design. Computers in Industry. 2020;122:103278.
51. Torkayesh AE, Tirkolaee EB, Bahrini A, Pamucar D, Khakbaz A. A Systematic Literature Review of MABAC Method and Applications: An Outlook for Sustainability and Circularity. Informatica. 2023;34:415-48. https://doi.org/10.15388/23-INFOR511
52. Mandal U, Seikh MR. Interval-valued spherical fuzzy MABAC method based on Dombi aggregation operators with unknown attribute weights to select plastic waste management process. Applied Soft Computing. 2023;145:110516.
53. Salman M, Chauhan R, Singh T, Prabakaran R, Kim SC. Experimental investigation and optimization of dimple-roughened impinging jet solar air collector using a novel AHP-MABAC approach. Environmental Science and Pollution Research. 2023;30(13):36259-75.

https://doi.org/10.1007/s11356-022-24765-0

54. Wang T, Wu X, Garg H, Liu Q, Chen G. A prospect theory-based MABAC algorithm with novel similarity measures and interactional operations for picture fuzzy sets and its applications. Engineering Applications of Artificial Intelligence. 2023;126:106787.
55. Giachetti RE, Young RE. Analysis of the error in the standard approximation used for multiplication of triangular and trapezoidal fuzzy numbers and the development of a new approximation. Fuzzy Sets and Systems. 1997;91(1):1-13. https://doi.org/10.1016/S0165-0114(96)00118-2
56. Halidini D, Xhafaj E, Xhafaj A, Halidini E. Ranking the Most Important Attributes of using Google Classroom in online Teaching for Albanian Universities: A Fuzzy AHP Method with Triangular Fuzzy Numbers and Trapezoidal Fuzzy Numbers. 2021. https://doi.org/10.25046/aj060134
57. Dombi J, Jónás T. Ranking trapezoidal fuzzy numbers using a parametric relation pair. Fuzzy Sets and Systems. 2020;399:20-43. https://doi.org/10.1016/j.fss.2020.04.014
58. Anand MC, Bharatraj J. Theory of Triangular Fuzzy Number2017.
59. Banerjee S, Kumar T. Arithmetic Operations on Generalized Trapezoidal Fuzzy Number and its Applications. Turkish Journal of Fuzzy Systems. 2012;3:1309-190.
60. Hu Z, Lin J. An integrated multicriteria group decision making methodology for property concealment risk assessment under Z-number environment. Expert Systems with Applications. 2022;205:117369.
61. Wang S, Li Z, He C, Liu D, Zou G. An Integrated Method for Modular Design Based on Auto-Generated Multi-Attribute DSM and Improved Genetic Algorithm. Symmetry [Internet]. 2022; 14(1). https://doi.org/10.3390/sym14010048
62. Sun J, Chai N, Pi G, Zhang Z, Fan B. Modularization of Product Service System Based on Functional Requirement. Procedia CIRP. 2017;64:301-5. https://doi.org/10.1016/j.procir.2017.03.038