



Article citation info:

Wang Z, Shangguan W, Peng C, Cai B, Similarity Based Remaining Useful Life Prediction for Lithium-ion Battery under Small Sample Situation Based on Data Augmentation, *Eksploracja i Niezawodność – Maintenance and Reliability* 2024; 26(1) <http://doi.org/10.17531/ein/175585>

Similarity Based Remaining Useful Life Prediction for Lithium-ion Battery under Small Sample Situation Based on Data Augmentation

Indexed by:



Zongyao Wang^a, Wei Shangguan^{a,b,*}, Cong Peng^a, Baigen Cai^{a,b}

^a School of Electronics and Information Engineering, Beijing jiaotong university, Beijing, 100044, China

^b China State Key Laboratory of Rail Traffic control and safety, Beijing jiaotong university, Beijing, 100044, China

Highlights

- The realistic degradation trajectories are generated based on the single exponential model and Sobol sampling.
- The Pearson distance is used to assess the similarity between the predicted reference trajectory and the actual trajectory.
- Based on similarity measures and kernel density estimation, point estimation and uncertainty estimation of RUL are realized.

Abstract

Lithium-ion batteries find extensive application in transportation, energy storage, and various other fields. However, gathering a significant volume of degradation data for the same type of lithium-ion battery devices becomes challenging in practice due to variations in battery operating conditions and electrochemical properties, among other factors. In this small sample situation, accurately predicting the remaining useful life (RUL) of the battery holds great significance. This paper presents a RUL prediction method that is based on data augmentation and similarity measures. Firstly, by utilizing the single exponential model and Sobol sampling techniques, it is possible to generate realistic degradation trajectories, even with just one complete run-to-failure degradation dataset. Subsequently, the similarity between the generated prediction reference trajectories and actual degradation trajectories is evaluated using the Pearson distance. Following that, the point estimation of RUL is performed through weighted averaging. Then, the uncertainty of the RUL predictions is quantified using kernel density estimation. Finally, the effectiveness of the proposed RUL prediction method is validated using two NASA lithium-ion battery datasets. Results demonstrate the practicality and effectiveness of the proposed method.

Keywords

Lithium-ion batteries, remaining useful life, similarity-based prediction, Sobol sampling, kernel density estimation

This is an open access article under the CC BY license (<https://creativecommons.org/licenses/by/4.0/>)

1. Introduction

The sustainable development of the global transportation industry, energy storage industry, and other fields faces significant challenges due to energy shortages, climate change, and pollutant emissions [4]. As a result, there is an urgent need for researchers to conduct comprehensive studies on innovative forms of energy storage. Lithium-ion batteries, compared to other forms of energy storage, are portable energy sources with high energy density, environmental friendliness, and long cycle

life. As a result, they have found extensive applications in various domains, including new energy vehicles, unmanned aerial vehicles, high-speed trains, and portable electronic devices [13,32]. For example, the development of electric vehicles is inseparable from the continuous improvement of the performance of on-board lithium-ion power batteries [37]. However, as the number of charges and discharges increases, the performance of the battery gradually deteriorates. This

(*) Corresponding author.

E-mail addresses:

Z. Wang (ORCID: 0000-0002-4153-627X) 22110059@bjtu.edu.cn, W. Shangguan wshg@bjtu.edu.cn, C. Peng 19111064@bjtu.edu.cn, B. Cai bgecai@bjtu.edu.cn,

phenomenon is commonly referred to as battery aging 6. Battery aging not only results in economic losses but also poses risks such as fire, explosion, and other catastrophic accidents, seriously impacting the safety of individuals and their property. To prevent these disasters, it is crucial to promptly monitor the health status of lithium-ion batteries and conduct research on predicting their remaining useful life (RUL).

The prediction of RUL for lithium-ion batteries is at the heart of battery health management and operation and maintenance systems. This approach utilizes existing degradation data to estimate a reasonable failure time, which plays a crucial role in ensuring the safe and efficient operation of batteries 16. Presently, RUL prediction algorithms for lithium-ion batteries can be broadly classified into two types: model-based and data-driven approaches 14.

The model-based approach is based on the equivalent circuit model 25 and electrochemical model 17 of lithium-ion batteries for prediction. However, such methods require professional technical support and strict experimental conditions, which makes these RUL prediction methods less popular due to the complexity of the methods 26.

The data-driven approach is the most commonly utilized method for predicting battery RUL. This method does not rely on a specific physical model as it directly employs degradation data to construct a prediction model 8. Data-driven methods can be divided into two main categories: machine learning methods and statistical learning methods 33. Machine learning methods mainly include Support Vector Regression (SVR), Artificial Neural Networks (ANN) 24, Bayesian Networks (BN) 2 and so on. At present, there are many research studies on RUL prediction of lithium-ion batteries based on machine learning. Wang et al. 30 proposed an improved anti-noise adaptive (ANA)-LSTM algorithm which has higher accuracy compared to traditional RNN and LSTM methods. Zhao et al. 48 used a combination of eigenvector selection and SVR on establishing the relationship between lithium-ion battery capacity and health index for more accurate RUL prediction. Ren et al. 19 proposed an Auto-CNN-LSTM model, which is able to achieve accurate RUL prediction by mining deep hidden information in limited data. In practice, many researchers often integrate multiple prediction models to predict the RUL of a battery in order to effectively address the stochastic nature of

the prediction results of a single data-driven model and improve the robustness of the prediction. Zhao et al. 49 proposed a fused neural network framework based on broad learning system (BLS) and LSTM for predicting the RUL of lithium batteries, and validated the effectiveness and superiority of the method on two datasets. In addition, there are some other practical and relevant algorithms, such as the BLS-particle swarm optimization (PSO) prediction framework 43 or the PSO- Gate Recurrent Unit (GRU) 42 prediction method. However, these prediction methods based on machine learning are considered black-box models. They rely on significant amounts of historical data to train the algorithms, with have poor interpretability and stability.

Statistical learning methods mainly include Wiener process and various filtering algorithms such as Kalman Filter (KF), Particle Filter (PF), etc. The modeling process of the Wiener is based on the assumption of the Markov property. While this assumption is valid in many real-world cases, it does not always hold true 12. Xue et al. 36 developed a RUL prediction method for lithium-ion batteries combining Unscented Kalman Filter (UKF) and SVR with better prediction accuracy compared to the traditional KF and UKF. Zhang et al. 41 used variational mode decomposition to decompose the degradation data of lithium batteries into trend and residual terms, and integrated the PF and Gaussian process regression (GPR) to predict the RUL of the batteries. Wang et al. 31 proposed a singular filtering (SF)-GPR-LSTM residual capacity estimation model for lithium batteries, which can effectively evaluate the full life cycle remaining capacity of batteries at extremely low temperatures. Zhang et al. 46 proposed a RUL prediction method based on Unscented Particle Filter (UPF) and utilized the Markov Chain Monte Carlo (MCMC) technique after the resampling operation to solve the sample impoverishment problem, which is better compared to the traditional PF and UPF methods with better performance. However, the stochastic filtering approach has some limitations. The use of Kalman filters is based on the assumption that both process noise and sensor noise are Gaussian distributed 12. PF is a more generalized filtering method, but suffers from particle degeneracy and scarcity. In addition, the prediction accuracy of filter-based methods relies on a priori knowledge of the physical behavior; however, domain knowledge is not always available

or is too expensive to obtain 9.

In recent years, similarity-based prediction (SBP) methods have gained significant attention. SBP is categorized as a type of machine learning method. This prediction method does not rely on any prior knowledge about the degradation model and only requires a small amount of historical data to accurately predict RUL based on similarities between samples. Zhang et al. 45 proposed a RUL estimation method for mechanical systems based on the similarity of phase-space trajectories, which searches for the optimal segment in the reference degradation trajectories compared with the current degradation trajectories, and realizes accurate RUL estimation by calculating the similarity. Xia et al. 34 proposed a similarity-based self-encoder multiscale ensemble prediction method and validated the effectiveness and superiority of the method on the CMAPSS dataset. Although these traditional SBP methods have achieved some results, they also have some shortcomings.

1) Traditional SBP methods do not necessitate a large amount of degradation data, but they do have fundamental requirements regarding the quantity of data. In practice, obtaining degradation data for lithium-ion batteries of the same type is difficult, and available degradation information is limited, posing a significant challenge to accurately predicting the RUL of the battery.

2) The similarity measure based on Euclidean distance is commonly utilized in SBP. However, this metric is not applicable when the predicted reference sample and the actual sample only exhibit identical major degradation trends.

3) Traditional SBP methods typically offer only point estimates of RUL, failing to provide any uncertainty intervals. However, uncertainty intervals are crucial for decision-makers in real operating conditions, as the information provided by point estimates alone is insufficient 35.

To address the aforementioned challenges, this paper presents a novel RUL prediction method for lithium-ion batteries. This method leverages data augmentation and similarity measures to handle small sample problems. The contribution of this paper can be summarized as follows:

1) By utilizing the single exponential model and Sobol sampling, data augmentation can be achieved even under conditions of limited samples. The generated degradation trajectories can provide data support for achieving accurate

RUL predictions.

2) To assess the correlation between various degradation trajectories in terms of their overall trend, the Pearson distance was selected as the similarity measure between the predicted reference trajectory and the actual trajectory.

3) In this paper, kernel density estimation (KDE) is employed to quantify the uncertainty in RUL predictions, aiming to minimize the impact of prediction uncertainty on decision-making in engineering practice.

The rest of this paper is organized as follows. Section 2 provides a concise overview of the battery degradation data augmentation method, which is based on an exponential model and Sobol sampling. In Section 3, we outline the proposed similarity-based RUL prediction method. The experimental validation of this method on the NASA lithium-ion battery dataset is presented and discussed in Section 4. Lastly, Section 5 offers concluding remarks for the entire paper.

2. Data augmentation for similarity based RUL prediction

A. Problem description

In real-world working conditions, acquiring a substantial volume of condition monitoring (CM) data for the same type of Li-ion battery proves challenging due to numerous external interferences and internal factors. For example, technological advancements have gradually enhanced the reliability of batteries, resulting in significantly extended cycle times from their operation commencement to failure. Consequently, procuring a sizable dataset depicting the complete life cycle degradation of identical battery types becomes prohibitively expensive 47. In RUL prediction, the problem of lack of sufficient run-to-failure degradation data is known as the small sample problem.

In recent years, with the rapid development of artificial intelligence (AI), more and more researchers have utilized deep learning prediction methods to predict the RUL of lithium-ion batteries. However, these methods rely on large amounts of degradation data and labeled RULs to train the model and are no longer suitable for RUL prediction in small sample situations [7,22,40].

In this case, the SBP method is a better choice to directly measure the similarity between different degradation trajectories and achieve accurate RUL prediction by weighted

averaging [10,12].

The first step in utilizing traditional similarity-based RUL prediction methods is to collect diverse CM data to guarantee the accuracy of RUL prediction [35]. When there is only one predicted reference trajectory, the prediction results for RUL become highly unstable and uncertain, as they heavily rely on the quality of that single trajectory. Even if the reference trajectory is divided into multiple segments using a sliding window, the instability and uncertainty persist.

However, in some situations, only one complete run-to-failure degradation data is even available. For instance, there is not a significant amount of historical degradation data for lithium-ion batteries that are newly put into the system. As batteries generally undergo a complete life-cycle degradation experiment prior to being deployed, we have and have only one complete run-to-failure degradation data. The aim of this paper is to achieve accurate SBP under small sample conditions with only one complete run-to-failure degradation data. The availability of only one degradation trajectory and the lack of multiple reference trajectories for prediction pose a significant challenge to the SBP method.

To address this issue, this paper presents a data augmentation method that utilizes a single exponential model and Sobol sampling. The proposed method for data augmentation only necessitates one complete set of run-to-failure CM data and the degradation information available for the battery under test.

B. The degradation model of the battery

The capacity of a lithium-ion battery gradually diminishes as the battery ages. When the capacity drops below a certain percentage of the standard capacity value, it can be considered as battery failure. As a result, researchers typically use capacity as the degradation or condition indicator for lithium-ion batteries. Essentially, predicting the RUL of a battery entails predicting when the battery's capacity will reach the failure threshold.

In practice, researchers often employ the double exponential model to establish an empirical degradation model for capacity [11,15], which is represented by Eq (1).

$$C(k) = a_1 \cdot \exp(a_2 \cdot k) + a_3 \cdot \exp(a_4 \cdot k) \quad (1)$$

where $C(k)$ denotes the capacity of the k -th charging cycle and

a_1, a_2, a_3, a_4 are the parameters of the double-exponential model.

However, when fitting incomplete run-to-failure capacity data using a double-exponential model, the range of values for the model parameters fluctuates significantly as the available degradation information continues to change. This variability is not conducive to data augmentation.

In order to solve this problem, the single exponential model is chosen as the empirical degradation model of battery capacity in this paper. This model has only two unknown parameters and demonstrates high stability even as the available degradation information changes. Moreover, the single exponential model can also accurately estimate the change rule of battery capacity.

The expression for the single exponential empirical degradation model of battery capacity is shown in Eq (2).

$$C(k) = a \cdot \exp(b \cdot k) \quad (2)$$

Assume there is a complete run-to-failure capacity data $C = (C_1, C_2, \dots, C_N)$, and not run-to-failure capacity data $Q = (Q_1, Q_2, \dots, Q_{ts})$, $ts < N$. The Levenberg-Marquardt algorithm was used to estimate the unknown parameters of the empirical degradation models of C and Q . The 95% confidence intervals for the four sets of unknown parameters were obtained as shown below.

$$A = \{(a_c^{min_c}, a_c^{max_c}, a_q^{min_q}, a_q^{max_q})\} \quad (3)$$

where a_c^{min} denotes the lower bound of parameter a in the empirical model of degradation data C , a_c^{max} denotes the upper bound of parameter a in the empirical model of degradation data Q , and so on.

C. data augmentation based on Sobol sampling

In recent years, Generative Adversarial Networks (GAN) have become the most commonly used technique for augmenting degradation data. GANs aim to enrich the data in a data-driven manner and generate realistic degradation trajectories, thereby enhancing the performance of RUL prediction methods [5,46].

However, to ensure the credibility of the generated data, GAN methods typically require more than two historical degradation trajectories as references [5,46]. In practice, it is possible that only one complete run-to-failure degradation trajectory can be collected. In such cases, the GAN method is

no longer applicable.

Random sampling algorithms are quasi-random and limited to one cycle. However, once this period is exceeded, they are repeated and cease to be mutually independent random numbers. In contrast to random sampling methods, Sobol sequence sampling methods focus on generating uniform distributions in probability space, by which local clustering can be avoided [29]. Sobol samples are created by rapidly converging to a uniformly distributed (with a low bias) sequence of deterministic numbers, which outperforms sampling such as Latin hypercube sampling (LHS), which is a method of sampling other low biased sequences [20,28]. The necessary computational procedure for Sobol's method of generating quasi-random sequences in one dimension is shown below 1.

We aim to generate a sequence $\{x^1, x^2, \dots\}, 0 < x^i < 1$ with small differences in the unit interval. At first, we need a set of direction numbers v_i , and each v_i is a binary fraction.

To obtain v_i , we need to choose a polynomial with coefficients chosen from $\{0,1\}$, which is a primitive polynomial in the domain Z_2 . A primitive polynomial of degree d in Z_2 can be constructed

$$P \equiv x^d + m_1 x^{d-1} + \dots + m_{d-1} x + 1 \quad (4)$$

where (m_1, m_2, \dots) is the coefficients of P , and each m_i is 0 or 1. We use the coefficients of P to define a recursive equation to compute v_i as follows.

$$v_1 = m_1 v_{i-1} \oplus m_2 v_{i-2} \oplus \dots \oplus m_{d-1} v_{i-d-1} \oplus v_{i-d} \oplus \left[\frac{v_{i-d}}{2^d} \right] \quad (5)$$

where \oplus denotes a bit-by-bit exclusive-or operation, and the last term is v_{i-d} shifted right d places.

Finally, we can generate the sequence x^1, x^2, \dots by the following equation.

$$x^n = b_1 v_1 \oplus b_2 v_2 \oplus b_3 v_3 \oplus \dots \quad (6)$$

where $[b_1, b_2, b_3, \dots]$ is the binary representation of n .

The steps of the proposed capacity degradation data augmentation technique based on single exponential model and Sobol sampling are shown in Algorithm 1.

Algorithm 1 Proposed Data Augmentation Approach

Input: The number of generated degradation trajectories n , parameter set A of the single exponential model.

Output: The simulation degradation trajectories.

- 1: Using Eqs. (4)-(6), two-dimensional sobol sequence samples sp are generated.
- 2: From the parameter set A , find the global maximum values \widetilde{a}_{max} and \widetilde{b}_{max} , and the global minimum values \widetilde{a}_{min} and \widetilde{b}_{min} of the parameters a and b .
- 3: The generated Sobol sequence is mapped to the specified range by the following equation.

$$a_{new} = \widetilde{a}_{min} + sp(:, 1) * (\widetilde{a}_{max} - \widetilde{a}_{min})$$

$$b_{new} = \widetilde{b}_{min} + sp(:, 2) * (\widetilde{b}_{max} - \widetilde{b}_{min})$$

where $sp(:, i)$ refers to the i -th dimension of data in the Sobol sequence.

- 4: The generated new parameters a_{new} and b_{new} are brought into Eq. (2) to simulate n degenerate trajectories.
 - 5: **for** $j = 1 : n$ **do**
 - 6: If $min(C_j) \leq$ failure threshold, then this degradation trajectory will be retained, otherwise this trajectory will be discarded, where C_j is the j th degenerate trajectory.
 - 7: **end for**
 - 8: Ultimately, the degradation trajectory that reaches the failure threshold is used as the predicted reference trajectory for similarity-based prediction.
-

3. III. Similarity based RUL prediction and uncertainty management

In this section, this paper presents a framework for predicting RUL of lithium-ion batteries. The framework is based on data augmentation and similarity measures specifically designed for small-sample cases. Fig. 1 illustrates the flowchart of the framework. The proposed prediction framework is divided into two main parts.

The left side of the dashed line represents the data preparation stage, which addresses the issue of limited predicted reference trajectories in small samples through the use of data augmentation techniques.

On the right side of the dashed line lies the RUL prediction stage. Here, the process begins by calculating the similarity measure between the generated degradation trajectories and the actual degradation trajectories using the Pearson distance. Subsequently, utilizing this similarity information, point estimates for RUL are computed. Finally, to overcome the limitation of traditional SBP methods in providing uncertainty results, the KDE method is employed to effectively manage the uncertainty associated with RUL prediction.

The rest of this section discusses in detail the specific process of predicting RUL based on SBP.

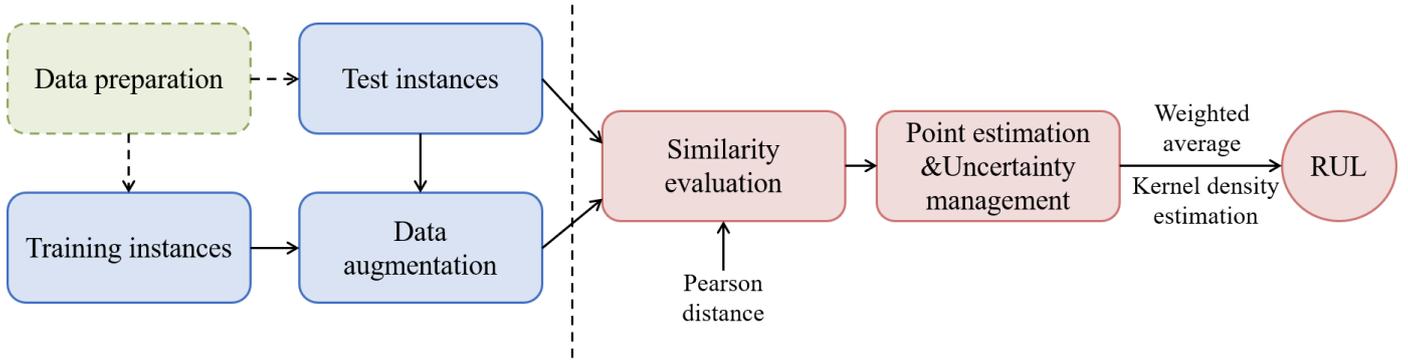


Fig. 1. The proposed RUL prediction based on data augmentation and similarity measure.

A. Similarity evaluation

Evaluating the similarity between different degradation trajectories is a crucial process for SBP, as it plays a pivotal role in inferring RUL and providing predictive reference examples.

In traditional similarity evaluation methods, many researchers usually use different distance metrics to measure the similarity between degradation trajectories. The basic distance metrics include Euclidean distance 32, Manhattan distance 12, Cosine distance 40, and so on. Meanwhile, to simultaneously account for the impacts of distance and correlation on similarity, Wang et al. 28 utilized complex invariant distance (CID) as an alternative to Euclidean distance-based similarity calculation.

In addition to distance-based similarity representations, there are many statistical techniques to compute the similarity between degradation trajectories, such as dynamic time warping (DTW) 18. DTW is suitable for calculating the similarity between trajectories of different lengths and is not applicable in the prediction scenario discussed in this paper.

In this paper, the degradation trajectory of the simulation augmentation in the small-sample case provides an approximate estimation of the capacity's degradation trend. However, it does not capture the noise and capacity regeneration phenomena that occur during the capacity change process. To effectively measure the correlation between these degradation trends, the Pearson distance was chosen as a means to calculate the similarity between degradation trajectories using the expression shown below.

$$d_p = 1 - \rho_{X,Y} = 1 - \frac{E[(X-\bar{X})(Y-\bar{Y})]}{\sigma_X \sigma_Y} \quad (7)$$

where X and Y represent two different degradation trajectories, \bar{X} and \bar{Y} denote the mean of X and Y , and σ_X, σ_Y denote the standard deviation of X and Y .

B. RUL point estimation

Based on the similarity measures between the degradation trajectories, we can predict the RUL of the battery by point estimation. Assuming that the current moment is t_s and the failure moment of the battery is t_f , then the RUL can be defined as the following equation.

$$RUL = t_f - t_s \quad (8)$$

It is assumed that there are M predicted reference degradation trajectories for the same type of battery. The estimated RUL (ERL) for different reference trajectories is defined as $ERL_i, i = 1, 2, \dots, M$. The point estimate of the battery RUL can be obtained by weighting the predictions based on similarity measures, as defined in Eq (9).

$$\widehat{rul} = \sum_{k=1}^M weight_k \cdot ERL_k \quad (9)$$

where $weight_k$ represents the weight of different predicted reference degradation trajectories. The equation for calculating the weight based on Pearson distance is shown below.

$$weight_k = \frac{s_{p,k}}{\sum_{i=1}^M s_{p,i}} = \frac{\frac{1}{d_{p,k}}}{\sum_{i=1}^M \frac{1}{d_{p,i}}} \quad (10)$$

where $\frac{1}{d_{p,k}}$ is the reciprocal of the Pearson distance for the k -th reference trajectory.

C. Uncertainty management

Traditional similarity-based prediction methods solely provide a deterministic RUL without assessing the uncertainty of the prediction. However, in practical applications, it is crucial for decision-makers to assess the uncertainty of RUL predictions (uncertainty management). Decision-makers rely on this information to understand the accuracy and reliability of the predictions [10]. Additionally, uncertainty management serves as the foundation for implementing proactive risk defense and

intelligent operations and maintenance strategies for devices.

This paper utilizes the KDE technique to merge the weights and ERLs of individual reference degradation trajectories. This approach enables the management of uncertainty in battery RUL prediction results. KDE is a nonparametric estimation method that can estimate the RUL distribution without making any assumptions [33].

It is assumed that $x = (x_1, x_2, \dots, x_n)$ are independent identically distributed samples from a specific distribution with an unknown probability density function $f(\cdot)$. The kernel density method was used to estimate the density as follows.

$$\hat{f}(x) = \frac{1}{Nh} \sum_{i=1}^n K\left(\frac{x-x_i}{h}\right) \quad (11)$$

where $K(\cdot)$ is the kernel function, h is the bandwidth, and n is the size of sample.

In this paper, we choose the most widely used Gaussian kernel function as the kernel function of KDE, whose expression is shown in Eq (12).

$$K\left(\frac{x-x_i}{h}\right) = \frac{1}{\sqrt{2\pi}h} \exp\left(-\frac{(x-x_i)^2}{2h^2}\right) \quad (12)$$

For a more concise representation, $weight_k$ is denoted as w_k and ERL_k is denoted as r_k . Taking w_k and r_k of different degradation trajectories into the KDE, the density of the RUL of the tested trajectory is estimated as follows.

$$\hat{f}(r_u) = \sum_{i=1}^M \frac{w_i}{\sqrt{2\pi}h} \exp\left(-\frac{(r_u-r_k)^2}{2h^2}\right) \quad (13)$$

With the probability density function, uncertainty management of SBP methods can be realized by calculating the cumulative distribution function of RUL, determining the confidence interval boundaries, and then determining the confidence intervals of the predicted RUL at a given confidence level.

4. Experiment and Discussion

A. Data description

The lithium-ion battery dataset provided by the NASA Ames Prediction Center of Excellence (PCoE) was used to validate the effectiveness of the proposed RUL prediction method [3]. The battery operates at an ambient temperature of 24°C [21]. Repeated charge/discharge cycles can accelerate the aging of lithium-ion batteries. The capacity resulting from these cycles serves as a suitable indicator to describe the degradation process of lithium-ion batteries.

In this paper, the capacity degradation data of B0005 and B0006 batteries are selected to validate the similarity-based RUL prediction method proposed in this paper. A lithium-ion battery is considered to have failed when 30% of its rated capacity has decayed, and the failure threshold of the capacity is 1.4Ah. The capacity degradation data of B0005 and B0006 batteries are shown in Fig. 2.

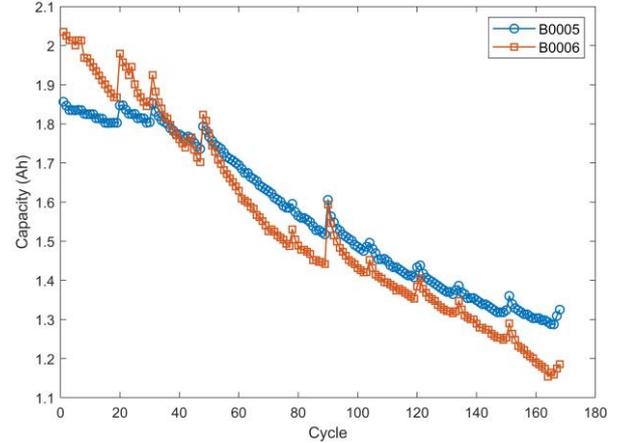


Fig. 2. Degradation data of B0005 and B0006 batteries.

In the experimental session, B0005 and B0006 lithium-ion batteries are used as the objects to be predicted, respectively, and it is assumed that the complete run-to-failure capacity data of the other battery is available.

Furthermore, experiments were conducted on the RW-09 and RW-10 Li-ion battery datasets, also obtained from the PCoE, in order to further validate the proposed methodology. The failure thresholds for both RW-09 and RW-10 batteries are also set at 1.4 Ah [27]. The capacity degradation data of RW-09 and RW-10 batteries can be seen in Fig. 3.

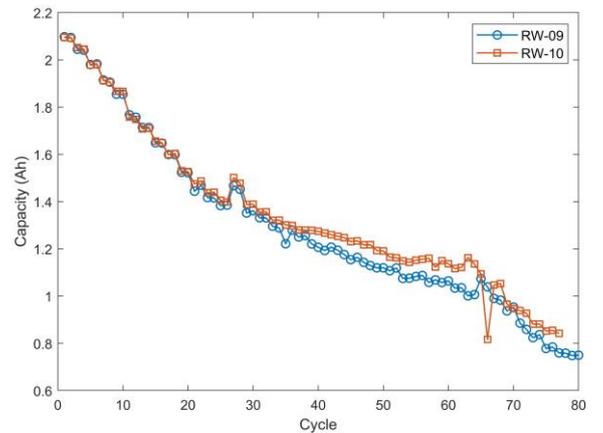


Fig. 3. Degradation data of RW-09 and RW-10 batteries.

We utilize the proposed method for RUL prediction and verify the validity. To further illustrate the effectiveness of the

proposed method, we compare it with four prediction methods: Euclidean similarity-based method (ESB), cosine similarity-based method (CSB), Manhattan similarity-based method (MSB), and complex invariant similarity-based method (CISB). These methods replace the Pearson distance with other distance measures when calculating the similarity, and the rest of the parameters are consistent with the methods proposed in this paper.

In addition, we compare the performance of the proposed method with the sliding window-based similarity prediction method (denoted as SW). The basic idea of the SW method to realize RUL prediction is to divide the degradation trajectory into many segments using a sliding window and find the trajectory segment that is most similar to the actual degradation trajectory as the ERL of that trajectory, and the rest of the operation is similar to that of the traditional similarity-based RUL prediction. In this paper, the sliding window size of SW is set to 20 and the Pearson distance is used to calculate the similarity between trajectories.

Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), and Mean Absolute Error (MAE) were chosen as the evaluation metrics to evaluate the performance of various RUL prediction methods, and the formulas for RMSE, MAPE, and MAE are shown below.

$$RMSE_k = \sqrt{\frac{1}{R} \sum_{i=1}^R (rul_i - \widehat{rul}_i^k)^2} \quad (14)$$

$$MAPE_k = \frac{1}{R} \sum_{i=1}^R \left| \frac{rul_i - \widehat{rul}_i^k}{rul_i} \right| \quad (15)$$

$$MAE_k = \frac{1}{R} \sum_{i=1}^R |rul_i - \widehat{rul}_i^k| \quad (16)$$

where rul_i denotes the true RUL under the i -th prediction starting point (SP), \widehat{rul}_i^k denotes the result of the RUL predicted by the k -th model at this moment, and R is the total number of predicted points. The smaller the values of RMSE, MAE, and MAPE, the closer the predicted RUL is to the true RUL, indicating better prediction performance.

B. RUL prediction for B0006

In this case, the RUL of the battery is predicted from the nine moments of [40:5:80] for the B0006 lithium-ion battery to verify the effectiveness of the proposed method. It is important to note that the available degradation information of the battery being tested varies depending on the starting point for prediction.

First, due to the insufficiency of capacity degradation sample data, we utilize the proposed Sobol sampling method to realize the augmentation of degradation data.

Based on the B0005 battery capacity degradation data, the Levenberg-Marquardt algorithm was utilized to estimate the upper and lower bounds of the corresponding single exponential model parameters a_c and b_c as (1.931, 1.954) and (-0.002662, -0.002529), respectively.

When the prediction starting point is different, the available degradation information of the battery to be tested is different, leading to differences in the results of the degradation model parameter estimation. Therefore, the single exponential degradation model of available capacity data is established at each prediction starting point. The parameters of the degradation model under different prediction starting points are shown in Table 1.

Table 1. - Parameter estimation results for B0006 available degradation data under different prediction starting points.

SP	a_Q	b_Q
40	(2.010, 2.055)	(-0.003670, -0.00269)
45	(2.013, 2.055)	(-0.003751, -0.002928)
50	(2.011, 2.056)	(-0.003648, -0.002871)
55	(2.013, 2.054)	(-0.003620, -0.002964)
60	(2.019, 2.059)	(-0.003738, -0.003154)
65	(2.023, 2.062)	(-0.003801, -0.003266)
70	(2.023, 2.063)	(-0.003900, -0.003389)
75	(2.024, 2.063)	(-0.003966, -0.003489)
80	(2.044, 2.082)	(-0.004213, -0.003784)

The upper and lower limits of the parameters of the single exponential degradation model are determined under different prediction starting points, the number of simulation samples n is set to 20, and Algorithm 1 is utilized to generate reference degradation trajectories to provide data support for similarity-based RUL prediction. As an example, the capacity degradation trajectories generated using Algorithm 1 when the prediction starting point is 80 cycles are shown in Fig. 4.

In Fig. 4, the solid blue line represents the real B0006 degradation trajectory and the dashed lines represent the degradation trajectories generated by Algorithm 1. As can be seen in Fig. 4, the generated degradation trajectories are able to roughly encompass the overall degradation trend of the B0006 battery, but lack accurate estimates of noise or capacity regeneration phenomena. However, the focus of the similarity-based prediction method is on predicting the approximate range

of RUL for different prediction starting points, rather than accurately predicting the details of capacity degradation.

The degradation trajectories generated by using Sobol sampling are regarded as training trajectories, and their Pearson distances to the tested trajectories are calculated at different prediction starting points, and then the estimated RUL is calculated by using Eqs. (9)-(10). In addition, the RULs of the B0005 battery are simultaneously estimated by the prediction methods of ESB, CSB, MSB, CISB, and the sliding window to obtain the comparison results between the RULs estimated by the multiple methods and the true RUL comparison results are shown in Fig. 5. As depicted in Fig. 5, the RUL prediction results obtained from the proposed method show a closer alignment to the true RUL across various prediction starting points. Additionally, the RUL prediction curves exhibit minimal fluctuation, thus substantiating the robustness of the proposed approach.

Accordingly, the performance metrics of each prediction method are shown in Table 2. To further illustrate the validity of the proposed method, this paper also employs two kinds of generalized data-driven methods - GPR 16 and SVR 39- to predict the RUL of lithium-ion batteries.

For SVR, the kernel function is set as linear. And the kernel function of GPR is squared-exponential. In this case, a data-driven model is utilized to establish a mapping between the degradation data of B0005 and the available degradation information of B0006, which ultimately leads to the prediction of RUL. The prediction results of GPR and SVR are also recorded in Table 2.

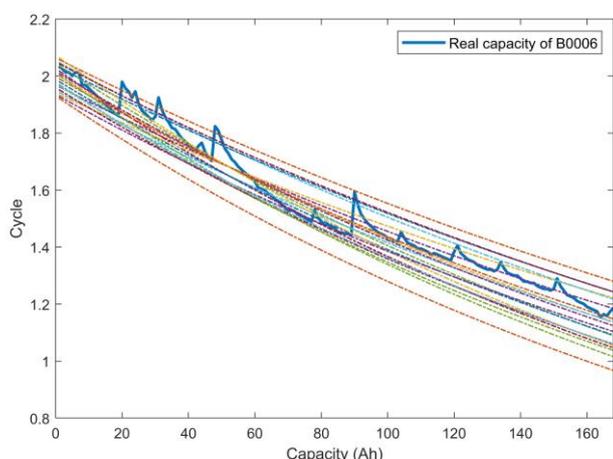


Fig. 4. The generated data by the single exponential model when the starting point is 80 cycles.

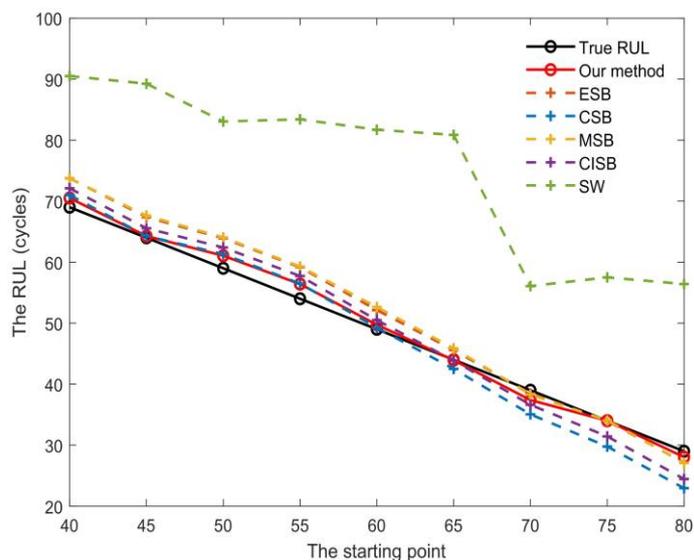


Fig. 5. RUL prediction results for B0006.

Table 2. - RUL prediction results of multiple methods with different prediction starting points for B0006. Lowest values are presented in bold.

Method	RMSE	MAPE	MAE
Our method	1.3501	0.0218	1.0688
ESB	3.3457	0.0533	2.8177
CSB	3.1256	0.0658	2.5610
MSB	3.4925	0.0569	2.9873
CISB	2.8436	0.0582	2.5479
SW	27.0108	0.5820	26.4174
GPR	35.0619	0.7062	34.2222
SVR	34.0539	0.6974	33.4444

As can be seen from Fig. 5, the RUL predicted by our method is closer to the true RUL compared to all other methods under different prediction starting points. In addition, as shown in Table 2, our method has the minimum value under all three evaluation metrics, indicating the validity and superiority of the method. Compared with ESB, CSB, MSB, CISB, SW, GPR and SVR, the MAPE of our method was reduced by 59.10%, 66.87%, 61.69%, 62.54%, 96.25%, 96.91% and 96.87%, respectively.

After obtaining the weights and estimated RUL values of each degradation trajectory, we utilize KDE to manage the uncertainty of the prediction results, providing a reliable basis for implementing maintenance decisions and risk management. As an example, the probability density distribution of the RUL obtained from the KDE method estimation with a prediction starting point of 40 cycle is shown in Fig. 6.

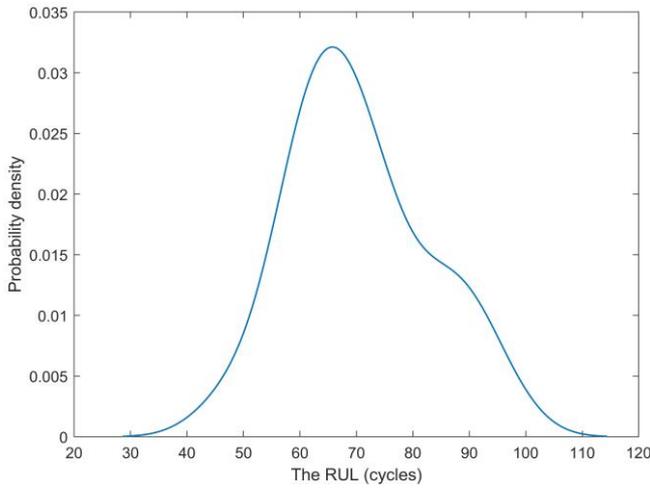


Fig. 6. Probability density function of the estimated RUL for B0006 when the starting point is 40 cycles.

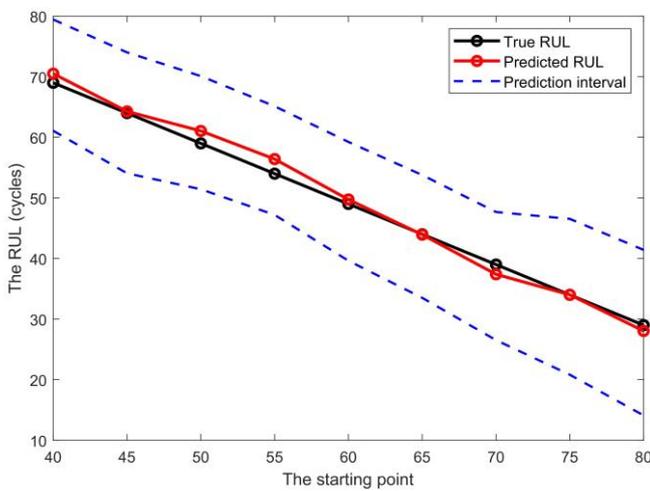


Fig. 7. The predicted RUL and confidence interval of our method for B0006.

The 50% (i.e., 25%-75%) confidence intervals for the RUL prediction results of the proposed method are shown in Fig. 7.

As can be seen from Fig. 7, the actual RULs are located in the predicted confidence intervals, which proves the better reliability and robustness of the proposed method compared to the traditional method of predicting the RULs using only point estimation.

C. RUL prediction for B0005

In this case, we assume that the degradation trajectory of the B0006 Li-ion battery is available, and predict the RUL of the B0005 battery by the proposed RUL prediction framework at different prediction starting points.

Before RUL prediction, it is necessary to estimate the range of values for the parameters of the capacity degradation model for data augmentation. 95% confidence intervals for the values

of parameters a_c and b_c for the B0006 battery capacity are (2.003, 2.030) and (-0.00334, -0.003181). The range of parameter values for the degradation model of the available capacity of the B0005 battery under different prediction starting points is shown in Table 3.

Table 3. - Parameter estimation results for B0005 available degradation data under different prediction starting points.

SP	a_q	b_q
40	(1.829, 1.850)	(-0.0008647, -0.0003694)
45	(1.831, 1.853)	(-0.001026, -0.0005686)
50	(1.831, 1.853)	(-0.001036, -0.0006242)
55	(1.833, 1.855)	(-0.001129, -0.0007586)
60	(1.836, 1.859)	(-0.001248, -0.0008874)
65	(1.837, 1.863)	(-0.001330, -0.0009539)
70	(1.837, 1.866)	(-0.001470, -0.001077)
75	(1.844, 1.875)	(-0.001654, -0.001262)
80	(1.879, 1.910)	(-0.002045, -0.001678)

When the prediction starting point is 60 cycles, the degradation trajectory generated using the data augmentation method proposed in this paper is shown in Fig. 8.

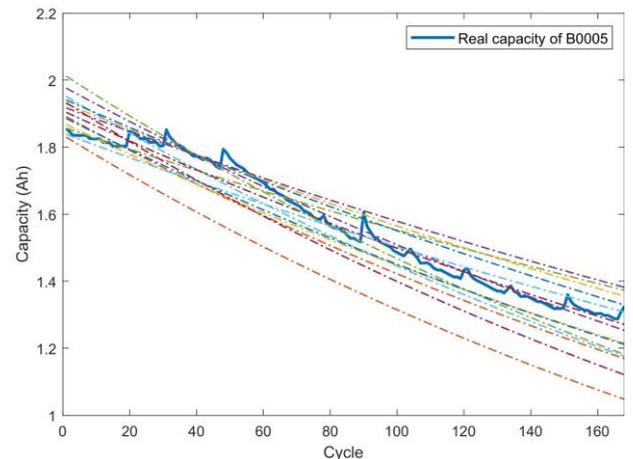


Fig. 8. The generated data by the single exponential model when the starting point is 60 cycles.

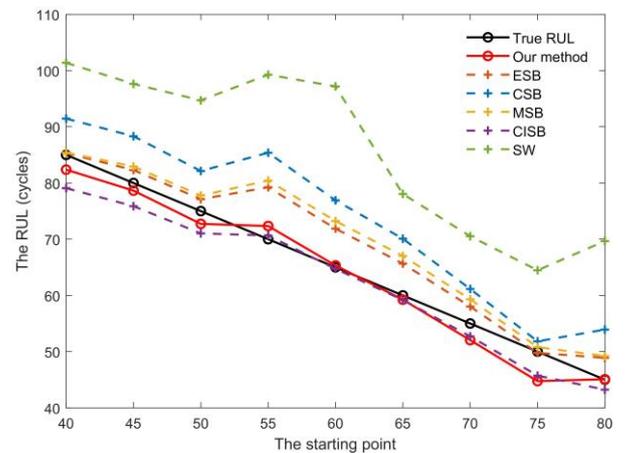


Fig. 9. RUL prediction results for B0005.

Table 4. - RUL prediction results of multiple methods with different prediction starting points for B0005. Lowest values are presented in bold.

Method	RMSE	MAPE	MAE
Our method	2.5030	0.0322	1.9973
ESB	4.7024	0.0599	3.7394
CSB	9.2069	0.1325	8.4659
MSB	5.5732	0.0733	4.5712
CISB	3.2573	0.0408	2.6751
SW	21.7186	0.3344	20.8777
GPR	18.2513	0.2375	16
SVR	31.0658	0.5708	30.5108

In Fig. 8, the solid blue line is the real capacity degradation trajectory of the B0005 battery, and the dashed lines are the generated degradation trajectories. From the figure, it can be seen that the generated degradation trajectory cannot simulate the actual degradation of capacity finely, but it can cover the actual capacity degradation trend well.

The generated data is used as the prediction reference trajectories to predict the RUL of the B0005 battery utilizing the similarity-based prediction method. The RUL results predicted by each model when the prediction starting point is different are shown in Fig. 9. Accordingly, the performance metrics of each prediction method are shown in Table 4.

In Fig. 9, the black curve represents the true RUL and the red curve represents the RUL predicted by the proposed method. As can be seen from Fig. 9, the RUL predicted by the method proposed in this paper is closer to the real RUL than the comparison method, and the trend is more stable with insignificant fluctuations. In addition, as shown in Table 6, the prediction of RUL using the proposed method has the lowest RMSE, MAPE and MAE, which proves the validity and superiority of the method. Taking RMSE as an example, the proposed method reduces ESB, CSB, MSB, CISB, SW, GPR and SVR by 46.77%, 72.81%, 55.09%, 23.18%, 88.47%, 86.29% and 91.94% respectively.

After obtaining the weights and ERLs of each predicted reference trajectory under different prediction starting points, the KDE method was utilized to quantify the uncertainty of the RUL prediction. As an example, the probability density function of the RUL prediction obtained using KDE estimation with a prediction starting point of 60 cycles is shown in Fig. 10.

The 50% confidence intervals for the RUL predictions are shown in Fig. 11. From Fig. 11, it can be seen that the prediction

intervals of the proposed method can cover the true RUL for any prediction starting point, which proves the reliability and robustness of the proposed method.

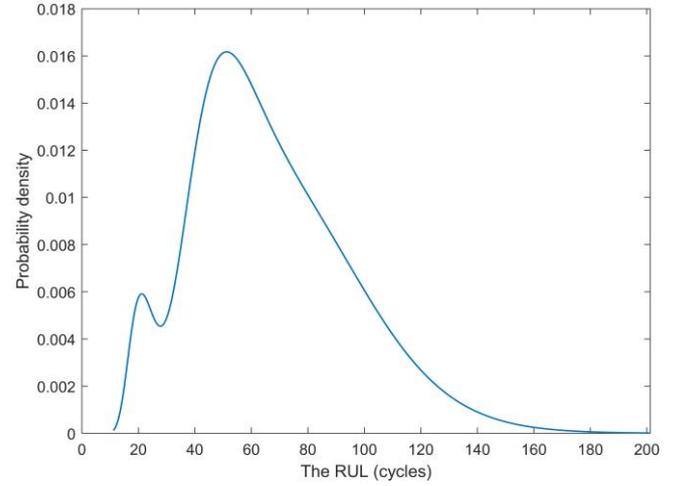


Fig. 10. Probability density function of the estimated RUL for B0005 when the starting point is 60 cycles.

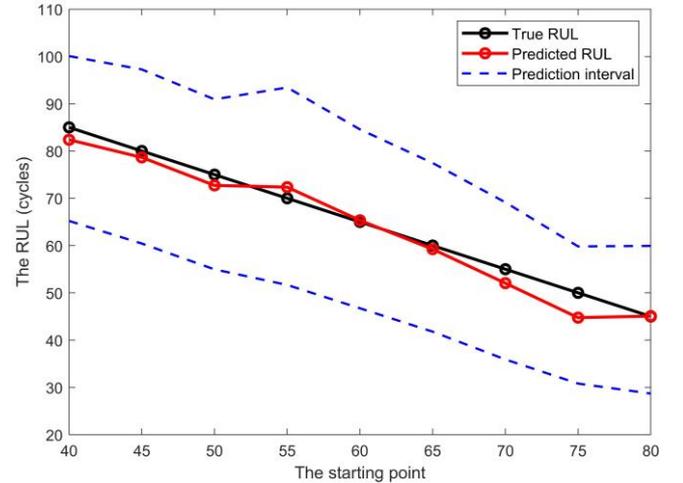


Fig. 11. The predicted RUL and confidence interval of our method for B0005.

D. RUL prediction for RW-10

In this case, we utilize the capacity data of RW-09 to predict the RUL of RW-10.

Assuming a starting point of 20 cycles, we obtained confidence intervals for the degradation data parameters of RW-09 and a portion of RW-10 using Eq. (2), respectively:

$$A = \{(1.99, 2.016), (-0.01252, -0.01153), (2.146, 2.193), (-0.01848, -0.01652)\} \quad (17)$$

In this case, we utilize Er to describe the prediction error, which is defined as shown below.

$$Er_k = |rul_i - \widehat{rul}_i^k| \quad (18)$$

According to Eq. (17) and Algorithm 1, 20 degradation

trajectories are generated as shown in Fig. 12.

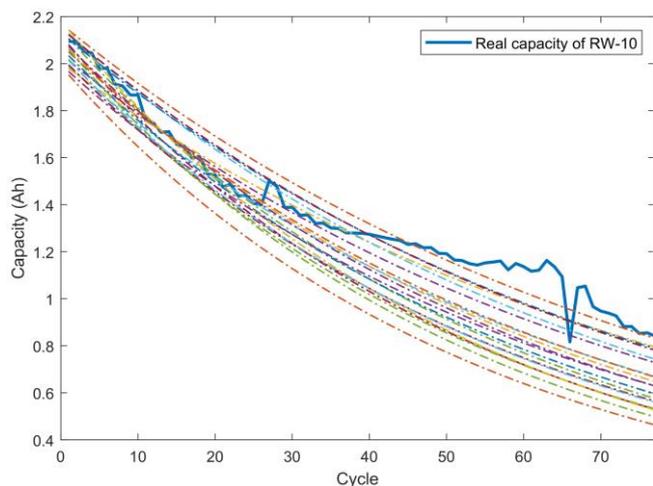


Fig. 12. The generated data by the single exponential model when the starting point is 20 cycles.

In Fig. 12, the solid blue line represents the actual capacity degradation trajectory of the RW-10 battery, while the dashed lines represent the generated degradation trajectories. When the predicted starting point is 20 cycles, the true RUL is 9 cycles 27. The RUL of the RW-10 battery was predicted using various methods, and the corresponding predicted results are presented in Table 5.

Table 5. - RUL prediction results of multiple methods for RW-10.

Method	Predicted RUL	Er
Our method	6.9385	2.0165
ESB	6.7756	2.2244
CSB	4.8463	4.1537
MSB	6.9040	2.0960
CISB	6.6566	2.4345
SW	17.8528	8.8528
GPR	5	4
SVR	14	5

Table 5 reveals that the proposed method demonstrates the highest prediction accuracy. When compared to ESB, CSB, MSB, CISB, SW, GPR, and SVR, the proposed method showcases reduced Er values of 9.35%, 51.45%, 3.79%, 17.17%, 77.22%, 49.59%, and 59.67%, respectively.

The KDE algorithm is used to estimate the probability density function of the RUL for RW-10 when the starting point of 20 cycles. The resulting probability density function is shown in Fig.13. Additionally, the proposed method predicts a confidence interval for the RUL of [3.4424, 10.174] at a confidence level of 0.5. These predicted intervals for RUL

enable the development of maintenance strategies to ensure the reliability of lithium-ion batteries.

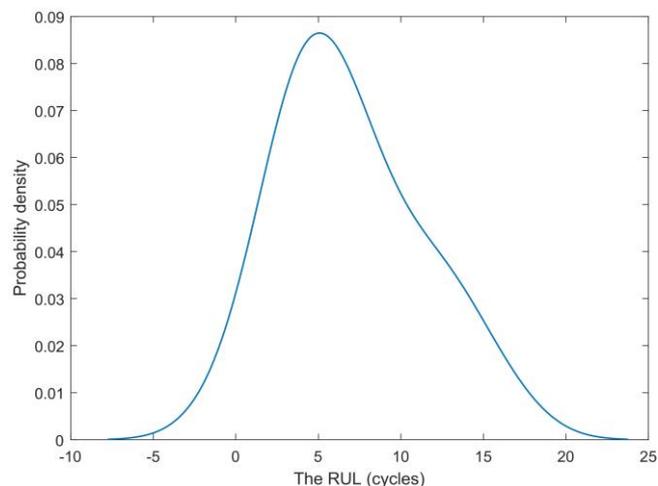


Fig. 13. Probability density function of the estimated RUL for RW-10 when the starting point is 20 cycles.

5. Conclusion

In this paper, we propose a method for predicting RUL of lithium-ion batteries using data augmentation and similarity measures. The proposed method integrates the techniques of Sobol sampling, Pearson distance and kernel density estimation, which ultimately achieves accurate RUL prediction. Our method addresses the challenges posed by inaccurate predictions due to a shortage of run-to-failure degradation data in traditional SBP models under small sample conditions. Moreover, the proposed RUL prediction method is validated on two datasets, proving the effectiveness of the method.

The experimental results show that the method proposed in this paper has higher accuracy compared with ESB, CSB, MSB, CISB, SW, GPR and SVR, and is able to realize accurate RUL prediction under small sample conditions.

There are some limitations of our proposed method:

1) The data augmentation method proposed in this paper lacks the ability to accurately capture the noise and capacity regeneration phenomenon that occurs during the degradation of lithium-ion batteries.

2) The prediction interval of the RUL prediction is wide.

Considering these limitations, future research should prioritize the generation of more realistic degradation trajectories and the development of a more robust approach to managing uncertainty in SBP.

Acknowledgment

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article. This work was funded by the Beijing Natural Science Foundation (No. L211022) and Fundamental Research Funds for the Central Universities (2023YJS004).

References

1. Bratley P, Fox B L. Algorithm 659: Implementing Sobol's quasirandom sequence generator. *ACM Transactions on Mathematical Software (TOMS)*, 1988, 14(1): 88-100, <https://doi.org/10.1145/42288.214372>
2. Dong G, Han W, Wang Y. Dynamic Bayesian network-based lithium-ion battery health prognosis for electric vehicles. *IEEE Transactions on Industrial Electronics*, 2020, 68(11): 10949-10958, <https://doi.org/10.1109/TIE.2020.3034855>
3. Goebel K, Saha B, Saxena A, et al. Prognostics in battery health management. *IEEE instrumentation & measurement magazine*, 2008, 11(4): 33-40, <https://doi.org/10.1109/MIM.2008.4579269>
4. He H, Sun F, Wang Z, et al. China's battery electric vehicles lead the world: Achievements in technology system architecture and technological breakthroughs. *Green Energy and Intelligent Transportation*, 2022: 100020, <https://doi.org/10.1016/j.geits.2022.100020>
5. He R, Tian Z, Zuo M J. A semi-supervised GAN method for RUL prediction using failure and suspension histories. *Mechanical Systems and Signal Processing*, 2022, 168: 108657, <https://doi.org/10.1016/j.ymsp.2021.108657>
6. He W, Williard N, Chen C, et al. State of charge estimation for Li-ion batteries using neural network modeling and unscented Kalman filter-based error cancellation. *International Journal of Electrical Power & Energy Systems*, 2014, 62: 783-791, <https://doi.org/10.1016/j.ijepes.2014.04.059>
7. Kong Z, Jin X, Xu Z, et al. A contrastive learning framework enhanced by unlabeled samples for remaining useful life prediction. *Reliability Engineering & System Safety*, 2023, 234: 109163, <https://doi.org/10.1016/j.res.2023.109163>
8. Li X, Zhang L, Wang Z, et al. Remaining useful life prediction for lithium-ion batteries based on a hybrid model combining the long short-term memory and Elman neural networks. *Journal of Energy Storage*, 2019, 21: 510-518, <https://doi.org/10.1016/j.est.2018.12.011>
9. Li Z, Wu D, Hu C, et al. An ensemble learning-based prognostic approach with degradation-dependent weights for remaining useful life prediction. *Reliability Engineering & System Safety*, 2019, 184: 110-122, <https://doi.org/10.1016/j.res.2017.12.016>
10. Lin Y H, Ding Z Q, Li Y F. Similarity based remaining useful life prediction based on Gaussian Process with active learning. *Reliability Engineering & System Safety*, 2023: 109461, <https://doi.org/10.1016/j.res.2023.109461>
11. Lin Y H, Tian L L, Ding Z Q. Ensemble Remaining Useful Life Prediction for Lithium-ion Batteries with the Fusion of Historical and Real-Time Degradation Data. *IEEE Transactions on Vehicular Technology*, 2023, <https://doi.org/10.1109/TVT.2023.3234159>
12. Liu Y, Hu X, Zhang W. Remaining useful life prediction based on health index similarity. *Reliability Engineering & System Safety*, 2019, 185: 502-510, <https://doi.org/10.1016/j.res.2019.02.002>
13. Lyu G, Zhang H, Zhang Y J, et al. An interpretable remaining useful life prediction scheme of lithium-ion battery considering capacity regeneration. *Microelectronics Reliability*, 2022, 138: 114625, <https://doi.org/10.1016/j.microrel.2022.114625>
14. Meng F, Yang F, Yang J, et al. A power model considering initial battery state for remaining useful life prediction of lithium-ion batteries. *Reliability Engineering & System Safety*, 2023, 237: 109361, <https://doi.org/10.1016/j.res.2023.109361>
15. Miao Q, Xie L, Cui H, et al. Remaining useful life prediction of lithium-ion battery with unscented particle filter technique. *Microelectronics Reliability*, 2013, 53(6): 805-810, <https://doi.org/10.1016/j.microrel.2012.12.004>
16. Pang X, Liu X, Jia J, et al. A lithium-ion battery remaining useful life prediction method based on the incremental capacity analysis and Gaussian process regression. *Microelectronics Reliability*, 2021, 127: 114405, <https://doi.org/10.1016/j.microrel.2021.114405>
17. Prasad G K, Rahn C D. Model based identification of aging parameters in lithium ion batteries. *Journal of power sources*, 2013, 232: 79-85, <https://doi.org/10.1016/j.jpowsour.2013.01.041>
18. Que Z, Xu Z. A data-driven health prognostics approach for steam turbines based on xgboost and dtw. *IEEE Access*, 2019, 7: 93131-93138, <https://doi.org/10.1109/ACCESS.2019.2927488>
19. Ren L, Dong J, Wang X, et al. A data-driven auto-CNN-LSTM prediction model for lithium-ion battery remaining useful life. *IEEE Transactions on Industrial Informatics*, 2020, 17(5): 3478-3487, <https://doi.org/10.1109/TII.2020.3008223>

20. Renardy M, Joslyn L R, Millar J A, et al. To Sobol or not to Sobol? The effects of sampling schemes in systems biology applications. *Mathematical biosciences*, 2021, 337: 108593, <https://doi.org/10.1016/j.mbs.2021.108593>
21. Su C, Chen H, Wen Z. Prediction of remaining useful life for lithium-ion battery with multiple health indicators. *Eksploatacja i Niezawodność*, 2021, 23(1): 176-183, <https://doi.org/10.17531/ein.2021.1.18>
22. Sun C, Ma M, Zhao Z, et al. Deep transfer learning based on sparse autoencoder for remaining useful life prediction of tool in manufacturing. *IEEE transactions on industrial informatics*, 2018, 15(4): 2416-2425, <https://doi.org/10.1109/TII.2018.2881543>
23. Sun F. Green Energy and Intelligent Transportation—promoting green and intelligent mobility. *Green Energy and Intelligent Transportation*, 2022, 1(1): 100017, <https://doi.org/10.1016/j.geits.2022.100017>
24. Tang T, Yuan H. An indirect remaining useful life prognosis for Li-ion batteries based on health indicator and novel artificial neural network. *Journal of Energy Storage*, 2022, 52: 104701, <https://doi.org/10.1016/j.est.2022.104701>
25. Tian H, Qin P, Li K, et al. A review of the state of health for lithium-ion batteries: Research status and suggestions. *Journal of Cleaner Production*, 2020, 261: 120813, <https://doi.org/10.1016/j.jclepro.2020.120813>
26. Wang H, Song W, Zio E, et al. Remaining useful life prediction for Lithium-ion batteries using fractional Brownian motion and Fruit-fly Optimization Algorithm. *Measurement*, 2020, 161: 107904, <https://doi.org/10.1016/j.measurement.2020.107904>
27. Wang J, Zhang S, Li C, et al. A data-driven method with mode decomposition mechanism for remaining useful life prediction of lithium-ion batteries. *IEEE Transactions on Power Electronics*, 2022, 37(11): 13684-13695, <https://doi.org/10.1109/TPEL.2022.3183886>
28. Wang M, Li Y, Zhao H, et al. Combining autoencoder with similarity measurement for aircraft engine remaining useful life estimation. *Proceedings of the International Conference on Aerospace System Science and Engineering 2019*. Springer Singapore, 2020: 197-208, https://doi.org/10.1007/978-981-15-1773-0_14
29. Wang R, Yan F, Shi R, et al. Uncertainty-Controlled Remaining Useful Life Prediction of Bearings with a New Data-Augmentation Strategy. *Applied Sciences*, 2022, 12(21): 11086, <https://doi.org/10.3390/app122111086>
30. Wang S, Fan Y, Jin S, et al. Improved anti-noise adaptive long short-term memory neural network modeling for the robust remaining useful life prediction of lithium-ion batteries. *Reliability Engineering & System Safety*, 2023, 230: 108920, <https://doi.org/10.1016/j.ress.2022.108920>
31. Wang S, Wu F, Takyi-Aninakwa P, et al. Improved singular filtering-Gaussian process regression-long short-term memory model for whole-life-cycle remaining capacity estimation of lithium-ion batteries adaptive to fast aging and multi-current variations. *Energy*, 2023, 284: 128677, <https://doi.org/10.1016/j.energy.2023.128677>
32. Wang T. Trajectory similarity based prediction for remaining useful life estimation. University of Cincinnati, 2010.
33. Wei Y, Wu D. Prediction of state of health and remaining useful life of lithium-ion battery using graph convolutional network with dual attention mechanisms. *Reliability Engineering & System Safety*, 2023, 230: 108947, <https://doi.org/10.1016/j.ress.2022.108947>
34. Xia T, Shu J, Xu Y, et al. Multiscale similarity ensemble framework for remaining useful life prediction. *Measurement*, 2022, 188: 110565, <https://doi.org/10.1016/j.measurement.2021.110565>
35. Xue B, Xu H, Huang X, et al. Similarity-based prediction method for machinery remaining useful life: A review. *The International Journal of Advanced Manufacturing Technology*, 2022, 121(3-4): 1501-1531, <https://doi.org/10.1007/s00170-022-09280-3>
36. Xue Z, Zhang Y, Cheng C, et al. Remaining useful life prediction of lithium-ion batteries with adaptive unscented kalman filter and optimized support vector regression. *Neurocomputing*, 2020, 376: 95-102, <https://doi.org/10.1016/j.neucom.2019.09.074>
37. Xiong R, Kim J, Shen W, Lv C, Li H, Zhu X, Zhao W, Gao B, Guo H, Zhang C, Sun F, Key technologies for electric vehicles, *Green Energy Intelligent Transport 1 (2) (2022) 100041*, <https://doi.org/10.1016/j.geits.2022.100041>
38. Yang F, Yue Z. Kernel density estimation of three-parameter Weibull distribution with neural network and genetic algorithm. *Applied Mathematics and Computation*, 2014, 247: 803-814, <https://doi.org/10.1016/j.amc.2014.09.065>
39. Yang Z, Wang Y, Kong C. Remaining useful life prediction of lithium-ion batteries based on a mixture of ensemble empirical mode decomposition and GWO-SVR model. *IEEE Transactions on Instrumentation and Measurement*, 2021, 70: 1-11, <https://doi.org/10.1109/TIM.2021.3125108>
40. Zeming L, Jianmin G, Hongquan J. A maintenance support framework based on dynamic reliability and remaining useful life. *Measurement*, 2019, 147: 106835, <https://doi.org/10.1016/j.measurement.2019.07.063>

41. Zhang C, Zhao S, He Y. An integrated method of the future capacity and RUL prediction for lithium-ion battery pack. *IEEE Transactions on Vehicular Technology*, 2021, 71(3): 2601-2613, <https://doi.org/10.1109/TVT.2021.3138959>
42. Zhang C, Luo L, Yang Z, et al. Battery SOH estimation method based on gradual decreasing current, double correlation analysis and GRU. *Green Energy and Intelligent Transportation*, 2023, 2(5): 100108, <https://doi.org/10.1016/j.geits.2023.100108>
43. Zhang C, Zhao S, Yang Z, et al. A reliable data-driven state-of-health estimation model for lithium-ion batteries in electric vehicles. *Frontiers in Energy Research*, 2022, 10: 1013800, <https://doi.org/10.3389/fenrg.2022.1013800>
44. Zhang J, Jiang Y, Li X, et al. An adaptive remaining useful life prediction approach for single battery with unlabeled small sample data and parameter uncertainty. *Reliability Engineering & System Safety*, 2022, 222: 108357, <https://doi.org/10.1016/j.res.2022.108357>
45. Zhang Q, Tse P W T, Wan X, et al. Remaining useful life estimation for mechanical systems based on similarity of phase space trajectory. *Expert Systems with Applications*, 2015, 42(5): 2353-2360, <https://doi.org/10.1016/j.eswa.2014.10.041>
46. Zhang X, Miao Q, Liu Z. Remaining useful life prediction of lithium-ion battery using an improved UPF method based on MCMC. *Microelectronics Reliability*, 2017, 75: 288-295, <https://doi.org/10.1016/j.microrel.2017.02.012>
47. Zhang X, Qin Y, Yuen C, et al. Time-series regeneration with convolutional recurrent generative adversarial network for remaining useful life estimation. *IEEE Transactions on Industrial Informatics*, 2020, 17(10): 6820-6831, <https://doi.org/10.1109/TII.2020.3046036>
48. Zhao Q, Qin X, Zhao H, et al. A novel prediction method based on the support vector regression for the remaining useful life of lithium-ion batteries. *Microelectronics Reliability*, 2018, 85: 99-108, <https://doi.org/10.1016/j.microrel.2018.04.007>
49. Zhao S, Zhang C, Wang Y. Lithium-ion battery capacity and remaining useful life prediction using board learning system and long short-term memory neural network. *Journal of Energy Storage*, 2022, 52: 104901, <https://doi.org/10.1016/j.est.2022.104901>