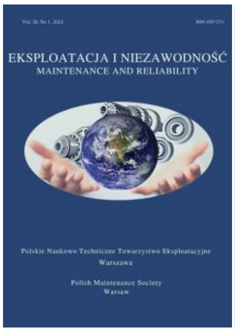

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Remaining useful life prediction of equipment considering dynamic thresholds under the influence of maintenance

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Highlights

- To predict RUL by considering dynamic thresholds.
- A multi-stage maintenance-impact degradation model is established.
- The parameters are estimated using the MLE and Bayesian formula.
- The proposed approach can enhance the precision of RUL prediction.

Abstract

A novel approach for predicting remaining useful life (RUL) is proposed for situations where maintenance threshold and failure threshold exhibit dynamic behavior due to uncertainties in degradation and the influence of detection strategies during maintenance processes. The approach introduces maintenance threshold error to establish a multi-stage maintenance-impact degradation model with dynamic maintenance threshold based on the Wiener process. This model considers the impact of maintenance on degradation rate, amount, and path. Moreover, by using the first hitting time (FHT) and introducing failure threshold error to reflect the dynamic behavior of the failure threshold, the formula for predicting equipment RUL is derived. The model parameters are estimated using both the maximum likelihood estimation (MLE) approach and Bayesian formula. The proposed approach was validated with simulation data and gyroscope degradation data, and the results demonstrate its ability to effectively enhance the precision of equipment RUL prediction.

Keywords
dynamic thresholds, Wiener process, multi-stage degradation model, remaining useful life

1. Introduction

Prognostics and health management (PHM) is an indispensable technology that is often employed to ensure secure operational functionality and augment the monetary effectiveness of equipment [8,23]. Lately, the precise forecast of RUL has become a pivotal element for the successful utilization of PHM technology. Thus, research revolving around RUL forecasting has garnered substantial attention from scholars locally and internationally, materializing as a major area of interest in the reliability domain and producing abundant outcomes [2,6,17].

Currently, among the various research approaches, the random process-based approach based on equipment degradation is more promising and valuable due to the high longevity, stability, and well-developed monitoring techniques of modern equipment [12,21]. The primary goal of this approach is to establish a degradation model that describes the equipment degradation process [28]. The commonly used degradation models include the Wiener process [10,13,31], Gamma process, and others[11,16,22]. The Wiener process, in particular, has been widely applied in research due to its flexibility in describing the degradation process, analytical

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expression for the FHT [32,34], compatibility with other influencing factors, and so on. However, most of the extant RUL prediction research about the Wiener process focuses on discussions regarding the time scale[19], random parameter setting [3,30,33], failure threshold assumption [7], and modeling the influence of vibration shock [5] on the degradation process. The influence of maintenance on the equipment degradation process is often neglected. In engineering practice, the degradation condition of the equipment is improved to some extent due to running maintenance protection. For this situation, some scholars have conducted research on RUL prediction of equipment under the influence of maintenance.

Wang and other scholars [27] stems from Wiener process, viewed the effect of maintenance on equipment degradation process as an effect on the degradation amount and used normal distribution to represent the variation of the degradation amount after maintenance. Reference [26] did not only rely on the normal distribution to depict the change in degradation resulting from maintenance but also delved into exploring the use of an exponential distribution to depict the amount of degradation wreaked by maintenance. Pei H, Si X, Hu C [20] argued that contemplating the effect of maintenance on the degradation rate is equally significant as considering its effect on the degradation amount. They developed a degradation model with a multi-stage diffusion process to predict the RUL. Although this approach can effectively enhance the precision of RUL prediction, it postulates that the degradation amount detected before each maintenance event is equal to the preset maintenance threshold, meaning that the change in performance degradation amount strictly begins from the preset maintenance threshold after each maintenance event. In actuality, due to the vagueness revolving around equipment degradation, detection mechanisms, the degradation amount detected before maintenance may surpass the maintenance threshold. Therefore, using the preset maintenance threshold as the starting point for equipment performance degradation amount may lead to inaccuracies in degradation modeling. Similarly, due to uncertainties in degradation and the influence of detection strategies, the actual failure threshold may exceed its preset value. At this point, the RUL prediction formula calculated based on the FHT and the preset failure threshold may be inaccurate.

Therefore, in an effort to upgrade the precision of equipment RUL prediction, this article addresses the degradation of equipment with maintenance impacts during its lifecycle. Based on the multi-stage degradation modeling theory [14,18,29], a maintenance-impact degradation model according to the Wiener process is established, which takes into account dynamic maintenance threshold. Further research is conducted on the updating of the degradation path after maintenance, and a formula for estimating RUL that considers dynamic failure threshold using the notion of FHT is put forward, based on the aforementioned degradation model. The availability of the proposed approach is verified through simulation and examples, providing a new approach and thinking to upgrade the precision of RUL prediction. The organization of this article is outlined as follows. The maintenance-impact degradation model is introduced in Section 2. Section 3 introduces the derivation process of RUL prediction formula. The estimation of parameters is introduced in Section 4. Section 5 describes the case analysis process, where the proposed approach is validated by simulation data and gyroscope degradation data. Section 6 is the conclusion of the article.

2. Maintenance-impact degradation model

2.1. Problem description and assumptions

To enhance the efficacy of equipment usage, maintenance activities are always present throughout the life cycle of the equipment. Upon each maintenance, the equipment's performance is enhanced while the extent of performance is reduced in contrast to its prior state. The degradation track of the equipment exhibits multi-stage features, as evidenced in Figure 1.

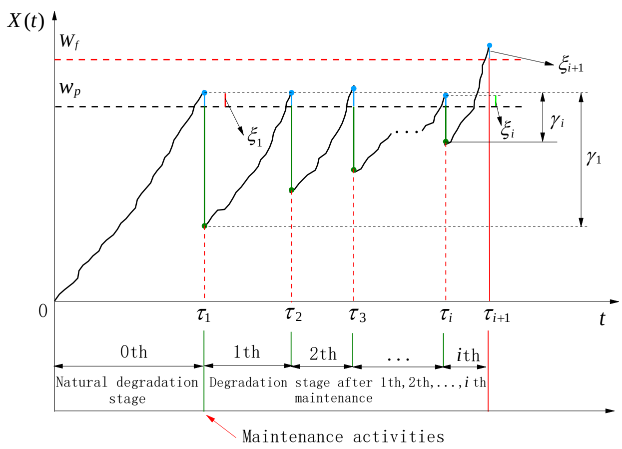


Fig. 1. Equipment degradation track under the influence of maintenance.

In Figure 1, $0 \sim \tau_1$ represents the natural degradation stage, which has no maintenance impact. When the degradation amount $X(t)$ arrives the preset maintenance threshold w_p, maintenance and guarantee activities are initiated for the equipment. After the first maintenance, the equipment's performance is improved, and the reduced degradation amount is defined as the amount of change impacted by maintenance, $\gamma_1$as indicated. However, due to uncertainty in degradation and the influence of detection strategies, the amount of degradation prior to the maintenance may surpass the maintenance threshold, causing errors in the jumping-off point of the next stage of degradation. Thus, the jumping-off point of the next stage of degradation should be demarcated as $w_p + \xi_1 - \gamma_1$, ξ_1 is the maintenance threshold error after the 1th maintenance. $\tau_1 \sim \tau_2$ indicates the equipment degradation stage after the 1th maintenance, and the starting point of the degradation amount in this stage is $w_p + \xi_1 - \gamma_1$; $\tau_i \sim \tau_{i+1}$ indicates the equipment degradation stage after the i th maintenance, and the starting point of the degradation amount in this stage is $w_p + \xi_i - \gamma_i$, which γ_i is the degradation amount of the maintenance impact after the i th maintenance. i is the number of times of maintenance and $i \in Z^+$. ξ_i is the maintenance threshold error after the i th maintenance. Without loss of generality, it is supposed that when the equipment runs to the time τ_{i+1}, the equipment has failed due to the performance degradation which has surpassed the error threshold of failure W_f or $W_f + \xi_{i+1}$, where ξ_{i+1} is the failure threshold error.

Based on the above statements, the assumptions are made regarding critical issues such as modeling and the derivation of RUL formulas are as follows:

Assumption 1: The improvement in equipment performance after undergoing maintenance is reflected by the change in the amount of post- maintenance equipment degradation.

Assumption 2: When the degradation performance of the equipment reaches or surpasses the predetermined maintenance threshold, maintenance and support activities are promptly carried out to restore the degradation to a certain degree.

Assumption 3: The proportion of maintenance time to the entire life cycle of the equipment is relatively small and can be ignored.

Assumption 4: The degradation process described in this article by applying the Wiener process.

Assumption 5: Based on the above assumptions, after each i th maintenance, the equipment continues to degrade from the start point of the previous degradation $w_p + \xi_i - \gamma_i$, when the detected degradation amount $X(t)$ is exactly equal to the predetermined maintenance threshold w_p, the ξ_i will be zero. Likewise, when the detected degradation amount $X(t)$ reaches the predetermined failure threshold W_f, ξ_{i+1} is set to 0.

Assumption 6: Taking real-world situations into account, it is assumed that the errors in maintenance and failure thresholds follow normal distribution.

2.2. Maintenance-impact degradation model based on wiener process

Given the aforementioned assumption, the maintenance-impact degradation model of the equipment using the Wiener process [4,15,35] after i th maintenance can be established by:

$$X_i(t) = \begin{cases} \mu_0(t-\tau_0) + \sigma_0 B(t-\tau_0)[(0 \le t \le \tau_{i+1})(i=0)] \\ w_p + \xi_i - \gamma_i + \mu_i(t-\tau_i) + \\ \sigma_i B(t-\tau_i)[(\tau_i \le t \le \tau_{i+1})(i=1,2,\ldots n; n \in Z^+)] \end{cases} \quad (1)$$

Where $X_i(t)$ represents the degradation amount of equipment performance at time t; w_p represents a given maintenance threshold; ξ_i represents the error of maintenance threshold, which is used to reflect the dynamic nature of the maintenance threshold and $\xi_i \sim N(\mu_{\xi_i}, \sigma_{\xi_i}^2)$. $\boldsymbol{\gamma_i}$ represents the improvement amount of equipment degradation due to maintenance activities, and will be determined by the degradation amount detected before and after maintenance; $\mu_i (i \ge 0)$ represents the drift parameter, which represents the degradation rate of the equipment after each i th maintenance; $\sigma_i (i \ge 0)$ represents the diffusion parameter, which represents the uncertainty of the degradation path; $B(\cdot)$represents the standard Brownian motion.

3. RUL prediction

Due to the influence of maintenance activities, the complete degradation process of the equipment is separated into several degradation stages. Depending on the maintenance frequencies (MFs), the degradation process is doubly divided into three parts: the natural degradation stage, the degradation stage from the first maintenance until the final maintenance, and the degradation stage from the final maintenance until failure. As a result, the following RUL prediction analysis and formula derivation will revolve around these three parts.

Assuming that the equipment has undergone i th ($1 < i \leq 2 \leq 3 \leq \cdots \leq n, n \in Z^+$) maintenances, the degradation in performance detected during any operational time(OT) $t_{i,j}$ is noted by $X(t_{i,j})$, where $j(j \geq 1, j \in Z^+)$ represents the number of tests. When the performance degradation detected after a period of time $r_{i,j}$ reaches or reaches the predetermined failure threshold, the equipment is supposed to have failed. At this point, the $r_{i,j}$ is the RUL and the residual degradation time (RDT) of the equipment after the last maintenance, based on the FHT [32], which is defined as:

$$R_{i,j} = inf\{r_{i,j}: X(t_{i,j} + r_{i,j}) \geq W_f, X(t_{i,j}) < W_f\} \\ = inf\begin{Bmatrix} r_{i,j}: X(t_{i,j} + r_{i,j}) - X(t_{i,j}) \geq W_f - X(t_{i,j}), \\ X(t_{i,j}) < W_f \end{Bmatrix} \quad (2)$$

Based on Eq. (1) and (2), it can be derived that the PDF of $r_{i,j}$ is [9,36]:

$$f_{R_{i,j}}(r_{i,j}|X(t_{i,j})) = \frac{W_f - X(t_{i,j})}{\sqrt{2\pi\sigma_i^2 r_{i,j}^3}} exp\left[-\frac{\left(W_f - X(t_{i,j}) - \mu_i r_{i,j}\right)^2}{2\sigma_i^2 r_{i,j}}\right] \quad (3)$$

The specific predicted value of $r_{i,j}$ is:

$$r_{i,j} = E(r_{i,j}) = \int_{-\infty}^{+\infty} r_{i,j} f_{R_{i,j}}(r_{i,j}) d(r_{i,j}) \quad (4)$$

It is noticed that Eq. (3) is applicable for RUL prediction with a fixed failure threshold. However, this approach does not take into account the error in failure threshold. Therefore, when considering the error in failure threshold, the Eq. (3) should be modified as:

$$f'_{R_{i,j}}(r_{i,j}|X(t_{i,j})) = \frac{W_f + \xi_{i+1} - X(t_{i,j})}{\sqrt{2\pi\sigma_i^2 r_{i,j}^3}} \cdot exp\left[-\frac{\left(W_f + \xi_{i+1} - X(t_{i,j}) - \mu_i r_{i,j}\right)^2}{2\sigma_i^2 r_{i,j}}\right] (5)$$

Due to the dynamic and stochastic nature of the failure threshold to be represented, Eq. (5) cannot yet be used as the target PDF and doubly derivation is necessary. To this end, Lemma 1 [24] is provided.

Lemma 1: if $Z \sim TN(\mu, \sigma^2), A \in R, B \in R^+$, then:

$$E\left[Z \cdot exp\left(\frac{-(Z-A)^2}{2B}\right)\right] = \frac{\sqrt{B}}{\Phi(\mu/\sigma)(B+\sigma^2)} \cdot \\ \begin{bmatrix} \sqrt{\frac{B\sigma^2}{2\pi}} exp\left(-\frac{A^2\sigma^2 + \mu^2 B}{2B\sigma^2}\right) + \frac{A\sigma^2 + B\mu}{\sqrt{B+\sigma^2}} \cdot \\ exp\left(-\frac{(\mu-A)^2}{2(B+\sigma^2)}\right) \Phi\left(\frac{A\sigma^2 + B\mu}{\sqrt{(B+\sigma^2)B\sigma^2}}\right) \end{bmatrix}$$

According to the Lemma 1, deeming the $W_f + \xi_{i+1} - X(t_{i,j})$ is Z, it can be deduced that the PDFs of the RUL and RDT to any OT during the period from the last maintenance of the equipment can be expressed as:

$$f'_{R_{i,j}}(r_{i,j}|X(t_{i,j})) = \frac{W_f + \xi_{i+1} - X(t_{i,j})}{\sqrt{2\pi\sigma_i^2 r_{i,j}^3}} \cdot exp\left[-\frac{(W_f + \xi_{i+1} - X(t_{i,j}) - \mu_i r_{i,j})^2}{2\sigma_i^2 r_{i,j}}\right] = \\ \frac{1}{\sqrt{2\pi B r_{i,j}^2}} \cdot \frac{\sqrt{B}}{\Phi\left(\frac{\alpha}{\sigma_{\xi_{i+1}}}\right)(B + \sigma_{\xi_{i+1}}^2)} \cdot \begin{bmatrix} \sqrt{\frac{B\sigma_{\xi_{i+1}}^2}{2\pi}} exp\left(-\frac{A^2\sigma_{\xi_{i+1}}^2 + \alpha^2 B}{2B\sigma_{\xi_{i+1}}^2}\right) + \\ \frac{A\sigma_{\xi_{i+1}}^2 + B\alpha}{\sqrt{B + \sigma_{\xi_{i+1}}^2}} exp\left(-\frac{(\alpha - A)^2}{2(B + \sigma_{\xi_{i+1}}^2)}\right) \cdot \\ \Phi\left(\frac{A\sigma_{\xi_{i+1}}^2 + B\alpha}{\sqrt{(B + \sigma_{\xi_{i+1}}^2)B\sigma_{\xi_{i+1}}^2}}\right) \end{bmatrix} \quad (6)$$

where $B = \sigma_i^2 r_{i,j}$; $A = \mu_i r_{i,j}$; $\alpha = W_f + \mu_{\xi_{i+1}} - X(t_{i,j})$.

When $j = 0$, $X(t_{i,0}) = w_p + \xi_i - \gamma_i$, the PDFs of the final RUL and the total RDT of the equipment after the last maintenance is:

$$f_{R_{i,0}}\left(r_{i,0}\middle|W_f + \Delta\xi_{i+1} - w_p + \gamma_i\right) = \frac{W_f + \Delta\xi_{i+1} - w_p + \gamma_i}{\sqrt{2\pi\sigma_i^2 r_{i,0}^3}} \cdot \\ exp\left[-\frac{\left(W_f + \Delta\xi_{i+1} - w_p + \gamma_i - \mu_i r_{i,0}\right)^2}{2\sigma_i^2 r_{i,0}}\right] \quad (7)$$

where $\Delta\xi_{i+1} = \xi_{i+1} - \xi_i$ and $\Delta\xi_{i+1} \sim N(\mu_{\Delta\xi_{i+1}}, \sigma_{\Delta\xi_{i+1}}^2)$, therefore, according to the Lemma 1, the target PDF for the RUL of the equipment after the last maintenance, which is the total RDT after the last maintenance, can be derived as:

$$f'_{R_{i,0}}\left(r_{i,0}\middle|W_f + \Delta\xi_{i+1} - w_p + \gamma_i\right) = \frac{1}{\sqrt{2\pi B r_{i,0}^2}} \cdot \frac{\sqrt{B}}{\Phi\left(\frac{\alpha}{\sigma_{\Delta\xi_{i+1}}}\right)\left(B + \sigma_{\Delta\xi_{i+1}}^2\right)} \cdot \\ \begin{bmatrix} \sqrt{\frac{B\sigma_{\Delta\xi_{i+1}}^2}{2\pi}} exp\left(-\frac{A^2\sigma_{\Delta\xi_{i+1}}^2 + \alpha^2 B}{2B\sigma_{\Delta\xi_{i+1}}^2}\right) + \\ \frac{A\sigma_{\Delta\xi_{i+1}}^2 + B\alpha}{\sqrt{B + \sigma_{\Delta\xi_{i+1}}^2}} \\ exp\left(-\frac{(\alpha - A)^2}{2\left(B + \sigma_{\Delta\xi_{i+1}}^2\right)}\right) \Phi\left(\frac{A\sigma_{\Delta\xi_{i+1}}^2 + B\alpha}{\sqrt{\left(B + \sigma_{\Delta\xi_{i+1}}^2\right)B\sigma_{\Delta\xi_{i+1}}^2}}\right) \end{bmatrix} \quad (8)$$

where $B = \sigma_i^2 r_{i,0}$; $A = \mu_i r_{i,0}$; $\alpha = W_f + \mu_{\Delta\xi_{i+1}} - w_p + \gamma_i$.

Furthermore, the PDF of the RDT regarding to the degradation stage prior to the last maintenance is:

$$f_{R_{i-1,j}}\left(r_{i-1,j}\middle|w_p + \xi_i - X(t_{i-1,j})\right) = \frac{1}{\sqrt{2\pi r_{i-1,j}^2} \cdot \Phi\left(\frac{\alpha}{\sigma_{\xi_i}}\right)\left(B + \sigma_{\xi_i}^2\right)} \cdot \\ \begin{bmatrix} \sqrt{\frac{B\sigma_{\xi_i}^2}{2\pi}} exp\left(-\frac{A^2\sigma_{\xi_i}^2 + \alpha^2 B}{2B\sigma_{\xi_i}^2}\right) + \\ \frac{A\sigma_{\xi_i}^2 + B\alpha}{\sqrt{B + \sigma_{\xi_i}^2}} exp\left(-\frac{(\alpha - A)^2}{2(B + \sigma_{\xi_i}^2)}\right) \Phi\left(\frac{A\sigma_{\xi_i}^2 + B\alpha}{\sqrt{(B + \sigma_{\xi_i}^2)B\sigma_{\xi_i}^2}}\right) \end{bmatrix} \quad (9)$$

where $B = \sigma_{i-1}^2 r_{i-1,j}$; $A = \mu_{i-1} r_{i-1,j}$; $\alpha = w_p + \mu_{\xi_i} - X(t_{i-1,j})$.

Similarly, during the degradation period from the $i-1$ th maintenance to the i th maintenance, the PDF of the total RDT can be obtained as:

$$f_{R_{i-1,0}}\left(r_{i-1,0}\middle|\gamma_{i-1} + \Delta\xi_i\right) = \frac{\gamma_{i-1} + \Delta\xi_i}{\sqrt{2\pi\sigma_{i-1}^2 r_{i-1,0}^3}} \cdot$$

$$exp\left[-\frac{\left(\gamma_{i-1}+\Delta\xi_i-\mu_{i-1}r_{i-1,0}\right)^2}{2\sigma_{i-1}^2 r_{i-1,0}}\right]=\frac{1}{\sqrt{2\pi B r_{i-1,0}^2}}\cdot\frac{\sqrt{B}}{\Phi\left(\frac{\alpha}{\sigma_{\Delta\xi_i}}\right)\left(B+\sigma_{\Delta\xi_i}^2\right)}\cdot$$

$$\left[\begin{array}{l}\sqrt{\frac{B\sigma_{\Delta\xi_i}^2}{2\pi}}exp\left(-\frac{A^2\sigma_{\Delta\xi_i}^2+\alpha^2 B}{2B\sigma_{\Delta\xi_i}^2}\right)+\\ \frac{A\sigma_{\Delta\xi_i}^2+B\alpha}{\sqrt{B+\sigma_{\Delta\xi_i}^2}}exp\left(-\frac{(\alpha-A)^2}{2(B+\sigma_{\Delta\xi_i}^2)}\right)\cdot\\ \Phi\left(\frac{A\sigma_{\Delta\xi_i}^2+B\alpha}{\sqrt{(B+\sigma_{\Delta\xi_i}^2)B\sigma_{\Delta\xi_i}^2}}\right)\end{array}\right] \quad (10)$$

where $B=\sigma_{i-1}^2 r_{i-1,0}$; $A=\mu_{i-1}r_{i-1,0}$; $\alpha=\gamma_{i-1}+\mu_{\Delta\xi_i}$.

According to the Eq. (9), when $i=1$, the PDF of the RDT in the natural degradation stage is:

$$f_{R_{0,j}}\left(r_{0,j}\middle|w_p+\xi_1-X(t_{0,j})\right)=\frac{1}{\sqrt{2\pi r_{0,j}^2}\cdot\Phi\left(\frac{\alpha}{\sigma_{\xi_1}}\right)\left(B+\sigma_{\xi_1}^2\right)}\cdot$$

$$\left[\begin{array}{l}\sqrt{\frac{B\sigma_{\xi_1}^2}{2\pi}}exp\left(-\frac{A^2\sigma_{\xi_1}^2+\alpha^2 B}{2B\sigma_{\xi_1}^2}\right)+\frac{A\sigma_{\xi_1}^2+B\alpha}{\sqrt{B+\sigma_{\xi_1}^2}}exp\left(-\frac{(\alpha-A)^2}{2(B+\sigma_{\xi_1}^2)}\right)\cdot\\ \Phi\left(\frac{A\sigma_{\xi_1}^2+B\alpha}{\sqrt{(B+\sigma_{\xi_1}^2)B\sigma_{\xi_1}^2}}\right)\end{array}\right] (11)$$

where $B=\sigma_0^2 r_{0,j}$; $A=\mu_0 r_{0,j}$; $\alpha=w_p+\mu_{\xi_1}-X(t_{0,j})$.

When $j=0$, $X(t_{0,0})=0$in Eq. (11), the Eq. (11) represents the PDF of the total RDT in the natural degradation stage.

According to the above inference, the RUL of the OT in the natural degradation stage is:

$$r_{0,j}=\left\{E(r_{0,j})+\sum_{i=2}^{n}E(r_{i-1,0})+E(r_{i,0})\right\}$$

$$\{n>1,n\in Z^+;j\geq 0,j\in Z^+\} \quad (12)$$

where $E(r_{0,j})=\int_{-\infty}^{+\infty}r_{0,j}\,f_{R_{0,j}}(r_{0,j})d(r_{0,j})$;

$$E(r_{i-1,0})=\int_{-\infty}^{+\infty}r_{i-1,0}\,f_{R_{i-1,0}}(r_{i-1,0})d(r_{i-1,0});$$

$$E(r_{i,0})=\int_{-\infty}^{+\infty}r_{i,0}f'_{R_{i,0}}(r_{i,0})d(r_{i,0}).$$

The RUL of the OT in any stage of degradation between the first and last maintenance is:

$$r_{i-1,j}=\{\textstyle\sum_{i=k}^{1}E(r_{i-1,j})+\sum_{i=k+1}^{n}E\left(r_{i-1,0}\right)+E\left(r_{i,0}\right)\}\{k\geq 2,k\in Z^+;n>1,n\in Z^+;j\geq 0,j\in Z^+\} \quad (13)$$

where $E(r_{i-1,j})=\int_{-\infty}^{+\infty}r_{i-1,j}\,f_{R_{i-1,j}}(r_{i-1,j})d(r_{i-1,j})$;

$$E(r_{i-1,0})=\int_{-\infty}^{+\infty}r_{i-1,0}\,f_{R_{i-1,0}}(r_{i-1,0})d(r_{i-1,0});$$

$$E(r_{i,0})=\int_{-\infty}^{+\infty}r_{i,0}f'_{R_{i,0}}(r_{i,0})d(r_{i,0}).$$

The RUL of the OT in the degradation period since its last maintenance is:

$$r_{i,j}=E(r_{i,j})=\int_{-\infty}^{+\infty}r_{i,j}f'_{R_{i,j}}(r_{i,j})d(r_{i,j}) \quad (14)$$

In conclusion, it can be stated that the RUL of the equipment at any OT t_i during its lifespan, after undergoing i th maintenance, can be expressed as：

$$r_i=\begin{cases}r_{0,j},\tau_0\leq t_i\leq\tau_1\\ r_{i-1,j},\tau_{i-1}\leq t_i\leq\tau_i\\ r_{i,j},\tau_i\leq t_i\leq\tau_{i+1}\end{cases} \quad (15)$$

4. Estimation of parameters

The RUL formula of the equipment has unknown parameters is $\theta=(\mu_{\xi_i},\sigma_{\xi_i}^2,\mu_{\Delta\xi_i},\sigma_{\Delta\xi_i}^2,\mu_{\xi_{i+1}},\sigma_{\xi_{i+1}}^2,\mu_{\Delta\xi_{i+1}},\sigma_{\Delta\xi_{i+1}}^2,\mu_i,\sigma_i^2)$, where $\mu_{\xi_i},\sigma_{\xi_i}^2$ and $\mu_{\Delta\xi_i},\sigma_{\Delta\xi_i}^2$ belong to the distribution parameters of the maintenance threshold error ξ_i and the maintenance error interval $\Delta\xi_i$, respectively, while $\mu_{\xi_{i+1}},\sigma_{\xi_{i+1}}^2$ and $\mu_{\Delta\xi_{i+1}},\sigma_{\Delta\xi_{i+1}}^2$ belong to the distribution parameters of the failure threshold error ξ_{i+1} and the failure-maintenance threshold error interval $\Delta\xi_{i+1}$. The parameter μ_i,σ_i^2 belongs to the degradation model and all parameters are independent of each other. In light of this situation, this paper first utilizes the MLE [1] approach to estimate the parameters of the maintenance threshold error, the maintenance threshold error interval, the failure threshold error, and the failure threshold error interval in the model. Subsequently, the MLE approach and Bayesian formula [1,10] are applied to estimate the degradation parameters in the degradation model.

4.1. Estimation of parameters for failure and maintenance threshold error

Assuming that the amount of performance degradation of the equipment detected in the last inspection before the i th $i(i=1,2,\ldots.n)$ maintenance was X_{τ_i}; when $i=n+1$, the equipment failed with the last detected performance degradation of $X_{\tau_{i+1}}$ before the failure. we can analyze upon the previous assumption, the maintenance threshold error and the failure threshold error of the equipment after the i th maintenance may be expressed as:

$$\xi_i=X_{\tau_i}-w_p \quad (16)$$

$$\xi_{i+1}=X_{\tau_{i+1}}-W_f \quad (17)$$

the logarithm of the likelihood function of the $\mu_{\xi_i},\sigma_{\xi_i}^2$ is:

$$\ln L\left(\mu_{\xi_i},\sigma_{\xi_i}^2\right)=-\frac{n+1}{2}\ln 2\pi-\frac{n+1}{2}\ln\sigma_{\xi_i}^2-\frac{\sum_{i=1}^{n+1}(\xi_i-\mu_{\xi_i})^2}{2\sigma_{\xi_i}^2} \quad (18)$$

By finding partial derivatives of Eq. (18) for $\mu_{\xi_i},\sigma_{\xi_i}^2$ and letting them to zero, we can acquire:

$$\begin{cases}\widehat{\mu_{\xi_i}}=\frac{\sum_{i=1}^{n+1}\xi_i}{n+1}\\ \widehat{\sigma_{\xi_i}}^2=\frac{\sum_{i=1}^{n+1}(\xi_i-\widehat{\mu_{\xi_i}})^2}{n+1}\end{cases} \quad (19)$$

Upon doubly consideration, due to the random characteristic of equipment degradation and the uncertainty of measurement techniques, the maintenance and failure thresholds at different MFs are not entirely equal in practicality. Therefore, the expressions of μ_{ξ_i}, $\sigma_{\xi_i}^2$, and $\mu_{\xi_{i+1}}$, $\sigma_{\xi_{i+1}}^2$ have been respectively adjusted as:

$$\begin{cases}\mu_{\xi_i}=\xi_i\\ \widehat{\sigma_{\xi_i}}^2=\frac{\sum_{i=1}^{n+1}(\xi_i-\frac{\sum_{i=1}^{n+1}\xi_i}{n+1})^2}{n+1}\end{cases} \quad (20)$$

$$\begin{cases}\mu_{\xi_{i+1}}=\xi_{i+1}\\ \widehat{\sigma_{\xi_{i+1}}}^2=\frac{\sum_{i=1}^{n+1}(\xi_i-\frac{\sum_{i=1}^{n+1}\xi_i}{n+1})^2}{n+1}\end{cases} \quad (21)$$

Similarly, we can obtain the expressions for $\mu_{\Delta\xi_i}$, $\sigma_{\Delta\xi_i}^2$ and $\mu_{\Delta\xi_{i+1}}$, $\sigma_{\Delta\xi_{i+1}}^2$ as follows:

$$\begin{cases}\mu_{\Delta\xi_i}=\Delta\xi_i=\xi_i-\xi_{i-1}\\ \widehat{\sigma_{\Delta\xi_i}}^2=\frac{\sum_{i=2}^{n+1}(\mu_{\Delta\xi_i}-\frac{\sum_{i=2}^{n+1}\mu_{\Delta\xi_i}}{n+1})^2}{n+1}\end{cases} \quad (22)$$

$$\begin{cases}\mu_{\Delta\xi_{i+1}}=\Delta\xi_{i+1}=\xi_{i+1}-\xi_i\\ \widehat{\sigma_{\Delta\xi_{i+1}}}^2=\frac{\sum_{i=2}^{n+1}(\mu_{\Delta\xi_i}-\frac{\sum_{i=2}^{n+1}\mu_{\Delta\xi_i}}{n+1})^2}{n+1}\end{cases} \quad (23)$$

4.2. Estimation of parameters for degradation parameters

Assuming that the equipment's amount of performance degradation detected by the time $t_{i,j}$ is $x_{i,j}$, where $i(0\le i\le 1\le 2\le\cdots\le n, n\in Z^+)$ is the MFs and $j(0\le j\le 1\le 2\le\cdots\le h, h\in Z^+)$ is the time of degradation sample detections. Therefore, during the degradation stage from the i th maintenance movement to the next maintenance movement, the logarithmic likelihood function based on the sample data is:

$$l(\mu_i,\sigma_i^2|\Delta x_{i,j})=-\frac{h}{2}\ln 2\pi+\sum_{j=1}^{h}\ln\Delta t_j+h\ln\sigma_i^2+\sum_{j=1}^{h}\frac{(\Delta x_{i,j}-\mu_i\Delta t_{i,j})^2}{\sigma_i^2\Delta t_{i,j}} \quad (24)$$

By finding partial derivatives of Eq. (24) for μ_i,σ_i^2 and letting them to zero, we can acquire:

$$\begin{cases}\hat{\mu}_i=\frac{1}{h}\sum_{j=1}^{h}\frac{\Delta x_{i,j}}{\Delta t_{i,j}}\\ \hat{\sigma}_i^{\,2}=\left[\frac{1}{h}\sum_{j=1}^{h}\frac{(\Delta x_{i,j}-\hat{\mu}_i\Delta t_{i,j})^2}{\Delta t_{i,j}}\right]\end{cases} \quad (25)$$

In Eq. (25), under the assumption of other known conditions, the parameter σ_i^2 that characterizes the degradation path of the equipment is decided by the parameter μ_i that characterizes the degradation rate of the equipment. This suggests that once the equipment's degradation rate is determined, its degradation path is correspondingly determined and fixed. However, during the degradation course of the equipment, due to the randomness of the degradation course, the degradation path exhibits uncertainty. Even if the calculated degradation rate of equipment is the same, its degradation path is not unique. Therefore, in the parameter estimation of degradation parameters, it is necessary to consider the issue of parameter updating.

Assuming that the degradation parameters of the equipment after maintenance and update are $\mu_i,\tilde{\sigma}_i^{\,2}(i\ge 1)$, which are updated from the degradation parameters $\Theta=(\mu_{i-1},\sigma_{i-1}^2)$ before maintenance, we can acquire from the Bayesian formula:

$$L(\mu_i,\tilde{\sigma}_i^{\,2}|\Delta x_{i,j},\Theta)\propto L(\Delta x_{i,j}|\mu_i,\sigma_i^2)\cdot L(\mu_i,\sigma_i^2|\Theta)\propto$$
$$\exp\left[-\sum_{j=1}^{h}\frac{(\Delta x_{i,j}-\mu_i\Delta t_j)^2}{2\sigma_i^2\Delta t_j}\right]\cdot\exp\left[-\frac{(\mu_i-\mu_{i-1})^2}{2\sigma_{i-1}^2}\right]\propto$$
$$\exp\left[-\sum_{j=1}^{h}\left(\mu_i^2\left(\frac{\Delta t_j}{2\sigma_i^2}+\frac{1}{2\sigma_{i-1}^2}\right)-\left(\frac{\Delta x_{i,j}}{\sigma_i^2}+\frac{\mu_{i-1}}{\sigma_{i-1}^2}\right)\mu_i\right)\right]\propto$$
$$\exp\left[-\frac{(\mu_i-\frac{\sum_{j=1}^{h}\left(\Delta x_{i,j}\sigma_{i-1}^2+\mu_{i-1}\sigma_i^2\right)}{\sum_{j=1}^{h}\sigma_{i-1}^2\Delta t_j+\sigma_i^2})^2}{2\cdot\sum_{j=1}^{h}\frac{\sigma_i^2\sigma_{i-1}^2}{\sigma_{i-1}^2\Delta t_j+\sigma_i^2}}\right] \quad (26)$$

According to Eq. (26), the parameter expression of the updated degradation path is given by:

$$\tilde{\sigma}_i^{\,2}=\sum_{j=1}^{h}\frac{\sigma_i^2\sigma_{i-1}^2}{\sigma_{i-1}^2\Delta t_j+\sigma_i^2} \quad (27)$$

The parameters σ_i^2 and σ_{i-1}^2 can be calculated using Eq. (25). Similarly, when $i=0$, Eq. (27) became:

$$\tilde{\sigma}_0^{\,2}=\sum_{j=1}^{h}\frac{\sigma_0^2\sigma_0^2}{\sigma_0^2\Delta t_j+\sigma_0^2} \quad (28)$$

5. Case analysis

5.1. Simulation validation

Utilizing the independent increment property of the Wiener process [10,24], this article first randomly generates degenerate increment data using Matlab software. Secondly, given the parameter values: $W_f=0.67$, $w_p=0.52$, $\Delta t=1$, $i=3$, $X_{1,0}=0.12004$, $X_{2,0}=0.21523$, $X_{3,0}=0.27916$, a set of degenerate data is simulated to prove the availability of the approach mentioned in this article, as shown in Figure 2.

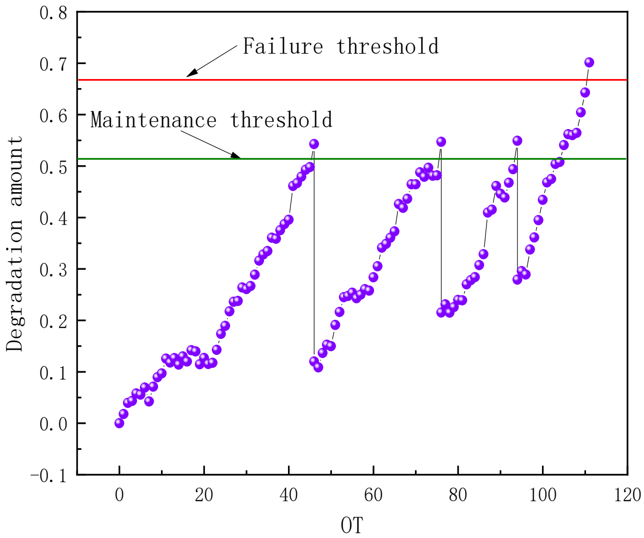


Fig.2. Simulated degradation data under maintenance influence.

To prove the availability of our approach, we conduct comparative analysis with approaches that do not consider degradation path updating, those which consider degradation path updating but not dynamic maintenance thresholds, those which consider both degradation path updating and dynamic maintenance thresholds but ignore dynamic failure thresholds. For ease of comparison, we label our approach as A3, while the other approaches are labeled as A0, A1, and A2. The estimation outcomes of the unknown parameters for each approaches are presented in Table 1.

Table 1. Parameter estimation outcomes.

parameters	MFs (i)				
	$i=0$	$i=1$	$i=2$	$i=3$	
				i	$i+1$
μ_i	0.011797	0.014231	0.018500	0.024900	-
σ_i^2	5.6979E-06	1.1479E-05	1.6860E-05	1.5575E-05	-
μ_{ξ_i}	-	0.02266	0.02700	0.02908	0.03014
$\sigma_{\xi_i}^2$	-	0.8027E-05	0.8027E-05	0.8027E-05	0.8027E-05
$\mu_{\Delta\xi_i}$	-	-	0.00434	0.00208	0.00106
$\sigma_{\Delta\xi_i}^2$	-	-	0.1879E-05	0.1879E-05	0.1879E-05

It can be tented from Figure 2 that the degradation trajectory of the equipment after three repairs is divided into four stages of degradation. In order to highlight the verification effect and increase rationality, this article selects four OTs arbitrarily in each degradation stage for the prediction of RUL. According to the derivation of the RUL prediction formula for the four approaches, the RDT corresponding to each OT in each degradation stage should be calculated first. The concrete calculation outcomes can be calculated in Table 2.

In Table 2, considering the updated degradation path and dynamic maintenance threshold, A2 and A3 approaches resulted in a predicted RDT closer to the actual remaining degradation time (ARDT) compared to the A0 and A1 approaches. This indicates that an approach considers both the updated degradation path and dynamic maintenance threshold can effectively upgrade the precision of predicting the RDT, thus enhancing the precision of predicting the RUL. In particular, in the degradation stage after three maintenance cycles, the A3 approach predicted the RDT closer to the practical value than the A2 approach. This is because the A3 approach doubly considers the dynamic failure threshold based on the A2 approach, demonstrating the importance of considering the dynamic failure threshold in improving the precision of prediction. To doubly illustrate the differences among the four approaches, the RUL and relative errors (REs) outcomes calculated by four approaches and Actual remaining useful life (ARUL) are shown in Table 3.

Table 2. RDT calculated using four approaches corresponding degradation stage at different MFs

MFs (i)	OT	A0	A1	A2	A3	ARDT
$i=0$	0	41.36	44.03	45.91	45.91	46
	1	39.79	42.51	44.36	44.36	45
	3	37.59	40.35	42.26	42.26	43
	11	30.69	33.37	35.26	35.26	35
$i=1$	46	25.64	28.02	29.89	29.89	30
	51	20.60	23.04	24.92	24.92	25
	55	16.22	18.62	20.52	20.52	21
	67	4.94	7.04	8.93	8.93	9
$i=2$	76	15.18	16.40	17.98	17.98	18
	77	14.28	15.61	17.17	17.17	17
	88	4.44	5.59	6.17	6.17	6
	89	2.14	3.17	4.74	4.74	5
$i=3$	94	15.04	15.66	15.66	16.92	17
	95	14.39	14.99	14.99	15.68	16
	99	10.20	10.81	10.81	11.67	12
	100	8.58	9.20	9.20	10.46	11

Table 3. RUL and REs calculated by four Approaches.

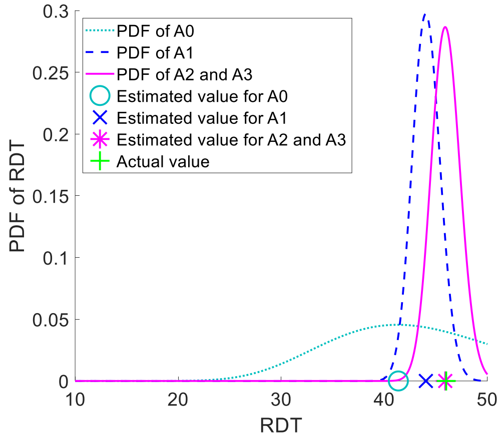
OT	A0	A1	A2	A3	ARUL
0	97.22 (-12.41%)	104.11 (-6.21%)	109.44 (1.41%)	110.70 (-0.27%)	111
1	95.65 (-13.05%)	102.59 (-6.74%)	107.89 (-1.92%)	109.15 (-0.77%)	110
3	93.45 (-13.47%)	100.43 (-7.01%)	105.79 (-2.05%)	107.05 (-0.88%)	108
11	86.55 (-13.45%)	93.45 (-6.55%)	98.79 (-1.21%)	100.05 (0.05%)	100
46	55.86 (-14.06%)	60.08 (-7.57%)	63.53 (-2.26%)	64.79 (-0.32%)	65
51	50.82 (-15.30%)	55.10 (-8.17%)	58.56 (-2.40%)	59.82 (0.30%)	60

OT	A0	A1	A2	A3	ARUL
55	46.44 (-17.07%)	50.68 (-9.50%)	54.16 (-3.29%)	55.42 (-1.04%)	56
67	35.16 (-20.09%)	39.10 (-11.14%)	42.57 (-3.25%)	43.83 (-0.39%)	44
76	30.22 (-13.66%)	32.06 (-8.40%)	33.64 (-3.89%)	34.90 (-0.29%)	35
77	29.32 (-13.76%)	31.27 (-8.03%)	32.83 (-3.44%)	34.09 (0.26%)	34
88	19.48 (-15.30%)	21.25 (-7.61%)	21.83 (-5.09%)	23.09 (0.39%)	23
89	17.18 (-21.91%)	18.83 (-14.41%)	20.40 (-7.27%)	21.66 (-1.55%)	22
94	15.04 (-11.53%)	15.66 (-7.88%)	15.66 (-7.88%)	16.92 (-0.47%)	17
95	14.39 (-10.06%)	14.99 (-6.31%)	14.99 (-6.31%)	15.68 (-2.00%)	16
99	10.20 (-15.00%)	10.81 (-9.92%)	10.81 (-9.92%)	11.67 (-2.75%)	12

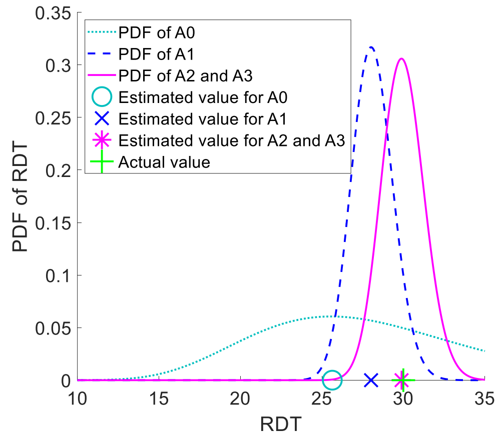
In table 3, it is evident that the prediction deviation for the A0 and A1 approaches surpasses that of the other two approaches. This indicates that ignoring dynamic maintenance thresholds can lead to a decrease in the precision of RUL, while taking them into account can effectively enhance the precision of the forecast. Furthermore, on the basis of considering dynamic maintenance thresholds, the prediction deviation of the A3 approach is lower than that of the A2 approach. That is because the A3 approach, in addition to considering dynamic maintenance thresholds like the A2 approach, takes into account dynamic failure thresholds - illustrating that, in addition to considering dynamic maintenance thresholds to improve the precision of prediction outcomes, dynamic failure thresholds are also a critical factor that cannot be ignored.

To visually illustrate the differences between the four approaches and doubly substantiate the above conclusions, the PDF of RDT predicted by the four approaches, using different OTs (0, 46, 76, and 94), was created following different MFs, as depicted in Figure 3.

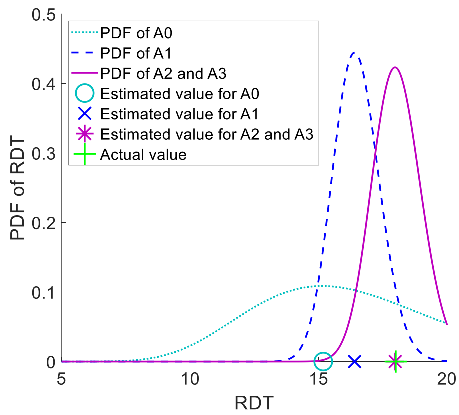
In Figure 3, it is evident that in each degradation phase after MFs is 0, 1, and 2, the predicted values of A2 and A3 approaches are notably closer to the actual values in comparison to A0 and A1 approaches. This illustrates that the approach of considering dynamic maintenance thresholds on the basis of degradation path is necessary and effective in enhancing the precision of prediction outcomes. For the degradation phase after MFs is 3, the introduction of dynamic failure thresholds has led to a significant decrease in the difference between predicted values and actual values of the A3 approach, which once again proves the importance of considering dynamic failure thresholds in improving prediction precision. Through the aforementioned validation, our approach is proven to be valid in upgrading the prediction precision of RUL. To doubly highlight the superiority of our mentioned approach, in addition to considering degradation path updates, a comparison and analysis of A1, A2, and A3 approaches will be conducted through examples.



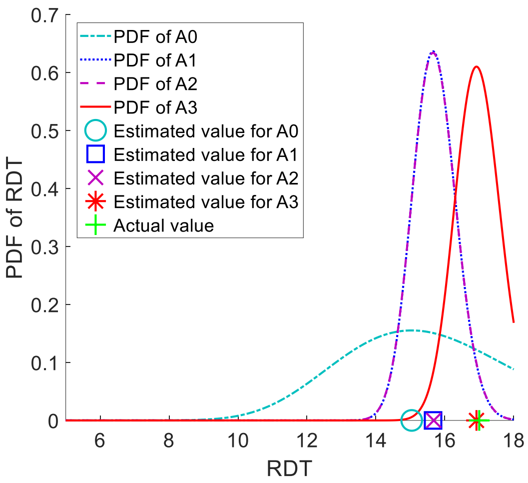
(a) $i = 0$



(b) $i = 1$



(c) $i = 2$



(d) $i = 3$

Fig.3. PDF of RDT predicted by four approaches during the corresponding degradation stage under different MFs.

5.2. Example analysis

Using the drift coefficient of a certain model gyroscope 1 after 3 MFs as performance degradation data from literature [1], the availability of the approach mentioned in this article is validated. The gyroscope is powered on every 2.5 hours, which means the degradation data is recorded by each 2.5 hours. The predetermined maintenance threshold and failure threshold for the gyroscope are 0.3°/h and 0.37°/h, respectively, with a lifespan of 277.5h. The estimation outcomes of the unknown parameters calculated upon the degradation data are shown in Table 4.

Table 4. Estimation outcomes of the unknown parameters.

parameters	MFs（i）				
	$i = 0$	$i = 1$	$i = 2$	$i = 3$	
				i	$i+1$
μ_i	2.42E-03	3.11E-03	4.16E-03	8.53E-03	-
σ_i^2	1.9151E-07	4.8204E-07	6.0420E-10	3.2749E-06	-
μ_{ξ_i}	-	0.0020	0.0020	0.0020	0.0150
$\sigma_{\xi_i}^2$	-	3169E-05	3169E-05	3169E-05	3169E-05
$\mu_{\Delta\xi_i}$	-	-	0.0000	0.0000	0.0130
$\sigma_{\Delta\xi_i}^2$	-	-	3756E-05	3756E-05	3756E-05

According to Table 4 and Eq. (15), RUL prediction values can be acquired for any OT. Similar to the verification process in the simulation, four OTs were selected randomly for RUL prediction in each degradation stage. The calculated RDT, RUL and REs using the three approaches are shown in Table 5 and Table 6, respectively.

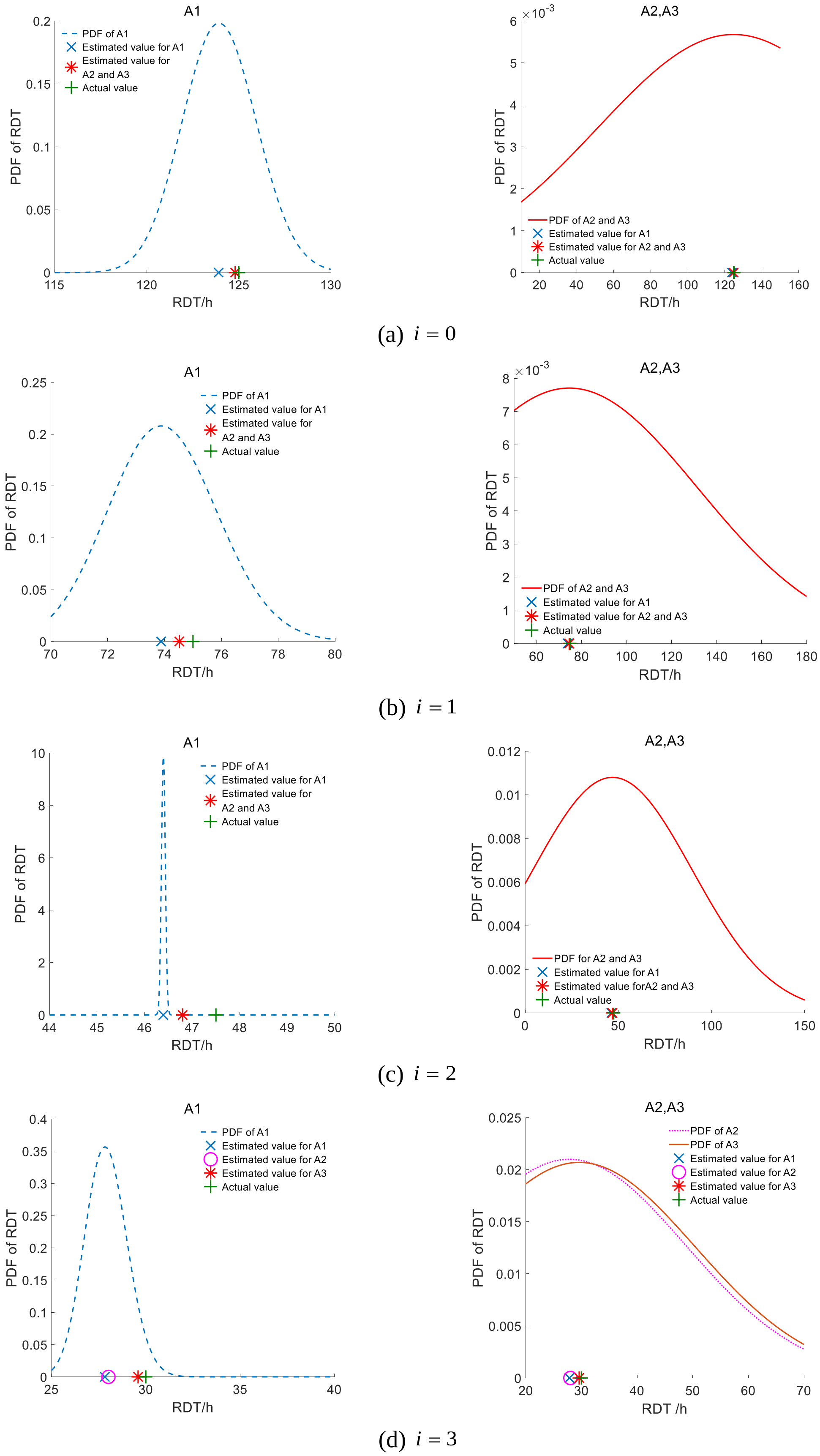
Table 5. RDT calculated using three approaches corresponding degradation stage at different MFs.

MFs (i)	OT/h	A1	A2	A3	ARDT
$i = 0$	0	123.90	124.80	124.80	125.00
	2.5	119.8	121.5	121.5	122.50
	95	25.98	27.44	27.44	30.00
	122.5	1.60	2.43	2.43	2.50
$i = 1$	125	73.88	74.52	74.52	75.00
	177.5	19.86	21.91	21.91	22.50
	180	17.61	18.73	18.73	20.00
	190	8.93	9.57	9.57	10.00
$i = 2$	200	46.39	46.80	46.80	47.50
	212.5	34.13	34.47	34.47	35.00
	230	15.38	16.43	16.43	17.50
	245	0.96	1.45	1.45	2.50
$i = 3$	247.5	27.83	28.02	29.59	30.00
	250	24.55	24.55	26.65	27.50
	257.5	18.10	18.10	19.79	20.00
	267.5	7.90	7.90	9.84	10.00

Table.6. RUL and REs calculated by three Approaches.

OT/h	A1	A2	A3	ARUL/h
0	272.00(1.98%)	274.14(1.21%)	275.71(0.65%)	277.50
2.5	267.90(2.58%)	270.84(1.51%)	272.41(0.94%)	275.00
95	174.08(4.61%)	176.78(3.13%)	178.35(2.27%)	182.50
122.5	149.70(3.42%)	151.77(2.08%)	153.34(1.07%)	155.00
125	148.10(2.895%)	149.34(2.07%)	150.91(1.04%)	152.50
177.5	94.08(-5.92%)	96.73(-3.27%)	98.30(-1.70%)	100.00
180	91.83(-5.82%)	93.55(-4.05%)	95.12(-2.44%)	97.50
190	83.15(4.97%)	84.39(-3.55%)	85.96(-1.76%)	87.50
200	74.22(-4.23%)	74.82(-3.46%)	76.39(-1.43%)	77.50
212.5	61.96(-4.68%)	62.49(-3.86%)	64.06(-1.45%)	65.00
230	43.21(-9.03%)	44.45(-6.42%)	46.02(-3.12%)	47.50
245	28.79(-11.42%)	29.47(-9.32%)	31.04(-4.49%)	32.50
247.5	27.83(-7.23%)	28.02(-6.60%)	29.59(-1.37%)	30.00
250	24.55(-10.73%)	24.55(10.73%)	26.65(-3.09%)	27.50
257.5	18.10(-9.50%)	18.10(-9.50%)	19.79(-1.05%)	20.00
267.5	7.90(-21.00%)	7.90(-21.00%)	9.84(-1.60%)	10.00

According to Table 5 and Table 6, it is evident that the prediction values of approach A3 are much closer to the practical values, with a smaller prediction deviation, compared to approaches A1 and A2. In practical engineering, using approaches A1 and A2 may result in excessive replacement of equipment, thereby affecting equipment efficiency. Therefore, in addition to considering the updating of degradation paths and dynamic maintenance thresholds, dynamic failure thresholds should also be valued when predicting RUL. To highlight the differences in the prediction of the three approaches, the total RDT for each degradation stage after MFs is 0, 1, 2, and 3 is plotted as a PDF, corresponding to run times of 0h, 125h, 200h, and 247.5h, respectively. This is shown in Figure 4.



(a) $i = 0$

(b) $i = 1$

(c) $i = 2$

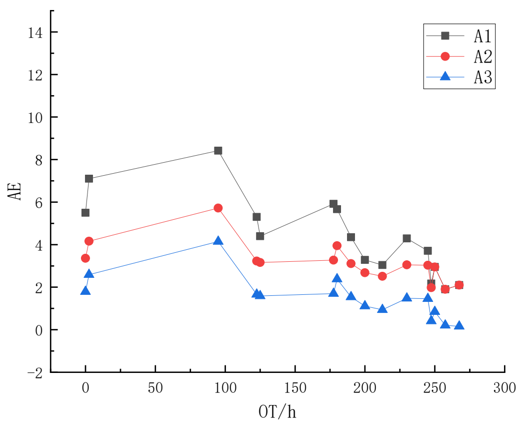
(d) $i = 3$

Fig.4. PDF of RDT predicted by three approaches corresponding degradation stage under different MFs.

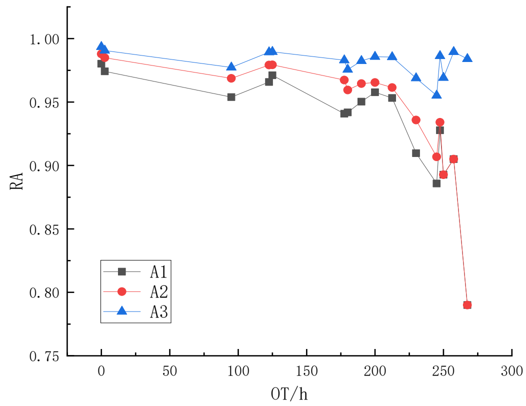
For the various stages of degradation prior to the third repair in Figure 4, the difference between the predicted RDT of A2 and

A3 approaches and the actual values is significantly smaller than that of the A1 approach, demonstrating the efficacy of considering both updated degradation paths and dynamically updated failure thresholds in prediction approaches. However, for the degradation stage after the third repair, while A2 approach shows an improvement in prediction precision compared to the A1 approach, the degree of improvement is limited due to the neglect of dynamic failure thresholds. In contrast, the A3 approach, by doubly considering dynamic failure thresholds, produces predicted values closer to the practical values, which doubly confirms the high precision and availability of prediction approaches that consider both updated degradation paths and dynamic thresholds.

To doubly demonstrate and justify the rationality and high precision of the A3 approach, two common indicators are used to analyze the precision of the predicted RUL: Absolute Error (AE) and Relative Accuracy (RA) [25]. The judgment basis of AE is that the smaller the AE value calculated by the approach, the higher the precision of the approach. As shown in Figure 5(a), the AE value counted by the A3 approach is smaller than the other two approaches, indicating that the A3 approach can effectively upgrade the precision of RUL prediction. RA is judged exactly opposite to AE and is shown in Figure 5(b), where it can again be concluded that A3 approach can effectively upgrade the precision of RUL prediction.



(a)



(b)

Fig.5. AE and RA of RUL predicted by three approaches.

After conducting the aforementioned comparative analysis, it can be inferred that the RUL prediction approach incorporating consideration of degradation path updates, dynamic maintenance thresholds, and dynamic failure thresholds can be valid to enhance the precision of RUL prediction while remaining practical.

6. Conclusion

In this article, a RUL prediction approach that considers dynamic thresholds is proposed to address the problem of traditional maintenance-impact-based prediction approaches treating maintenance and failure thresholds as fixed values, which is inconsistent with actual practice.

1) By combining multi-stage degradation modeling theory, thinking about the impact of maintenance on degradation amount and rate, doubly reflecting on the update of maintenance impact on degradation paths and introducing maintenance threshold errors to reflect the dynamic nature of maintenance thresholds, a maintenance impact-degradation model upon the Wiener process is set up.
2) According to the degradation model, the dynamic nature of failure thresholds is reflected by introducing failure threshold errors, and the RUL prediction model is acquired under the framework of the FHT. The unknown parameters in the model are estimated by combining the MLE approach and the Bayesian formula.
3) The mentioned approach is verified and analyzed by

simulation data and gyroscope degradation data. The outcomes show that under maintenance impact, the approach considering dynamic maintenance thresholds and failure thresholds can be valid to upgrade the precision of RUL prediction.

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