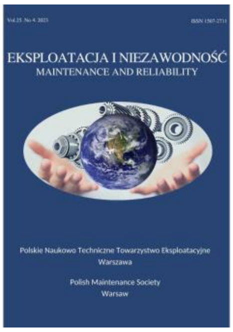

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An integrated approach to estimate storage reliability with masked data from series system

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Highlights

- The masked data of components from the storage system is investigated.
- The initial reliability of the storage products are introduced.
- The improved EM-like algorithm is used to update the testing data.
- An LS-based EM-like algorithm is proposed for estimating the storage reliability with masked data.

Abstract

Storage reliability is of importance for the products that largely stay in storage in their total life-cycle such as warning systems for harmful radiation detection, and many kinds of defense systems, etc. Usually, the field-testing data can be available, but the failure causes for a series system cannot be always known because of the masked information. In this paper, the storage reliability model with possibly initial failures is studied on the statistical analysis method when the masked data are considered. To optimize the use of the masked survival data from storage systems, a technique based on the least squares (LS) method with an EM-like algorithm, is proposed for the series system. The parametric estimation procedure based on the LS method is developed by applying the algorithm to update the testing data, and then the LS estimation for the initial reliability and failure rate of the components constituting the series system are investigated. In the case of exponentially distributed storage lifetime, a numerical example is provided to illustrate the method and procedure. The results should be useful for accurately evaluating the production reliability, identifying the production quality, and planning a storage environment.

Keywords
storage reliability, masked data, EM-LS algorithm, series system, LS method

1. Introduction

Some systems, such as warning systems for harmful radiation detection, rescue systems, spare parts for aircrafts, etc., may spend most of their time in storage, but once needed, must be fully functional [18]. Especially for some one-shot products, such as missiles, that can be used for only one time, and destroyed or extensively rebuilt after the use [19]. These types of products are being in a dormant state in the life cycle, and the operation time is usually very short compared with the time in storage. Consider the performance-based logistics in real life, storage reliability dominates the efforts in achieving the mission reliability goal. Nevertheless, the failure mechanisms of the products involved are completely different from those in active use, which imply that the storage reliability model should be considered by taking the operation reliability into account, and the reliability analysis for this type of products in storage is therefore important in practice.

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Based on different assumptions, there have been some storage models proposed [6, 14, 20, 34, 35,]. In the existing literatures, the storage reliability models are commonly to assume that the products are perfect at the beginning of the storage. Theoretically, the operational reliability of this type of product is not always 100% at the beginning of storage, and the initial probability of reliability should be considered in the evaluation model. Factually, some one-shot products may have only 96% operational reliability when they are newly produced. For example, to assess the storage reliability with initial failures, Zhao and Xie [37] proposed a generalized storage reliability model and estimated the initial reliability of storage product based on the least square method. Based on the simple non-parametric estimate of the current reliability, they [38] also studied the problem of parametric estimation based on a simple Weibull distribution assumption. Without consideration of the initial probability of failures, an excessively optimistic prediction on the storage reliability would be obtained. Therefore, the initial reliability embedded in the storage reliability models is necessary and has an important practical significance.

Obviously, operation reliability degrades with the storage time of product, and may become very low after the products are stored for a certain time period. To ensure that the products can complete the specified mission when required, condition monitoring, replacement or maintenance has to be carried out periodically [3, 10, 17, 27, 10, 17, 9, 22, 12], meanwhile, some field-observed data is recorded. In order to combine all available data to obtain a more accurate estimate, Zhang et al. [34] proposed an integrated approach to estimate the storage reliability based on the combinational estimation of the failure numbers and field-testing data. Consider the case of very few or even no failures, the reliability of the product cannot be very low [5], Zhang et al. [35] applied the E-Bayesian estimate of the failure probabilities into the approach proposed in document [34]. However, all the survival data from components constituting a parallel system is not always observed directly. The field-testing data can be available, but the failure causes cannot be always known due to various reasons such as high cost, difficulty to diagnosis, lack of enough information, etc. This is the case where the lifetimes of components are masked for the causes of some failures are unknown or the components resulting in the system failures cannot be completely identified [29, 30]. As far as a parallel system is concerned, it fails if and only if all components constituting the system fail, and the success data from the surviving system may contain some failed components, and the exact successes from the components are masked. Generally, the cause may only be isolated to some subset of the system's components, and the equivalent data of components from the system is usually called masked data [28]. Therefore, the field data from storage components integrating with masked data has great significance in theory for evaluating the storage life of component.

In 1988, Usher and Hodgson [30] analyzed the component reliability based on the masked system life data using maximum likelihood (ML) technique. For a general series system, the expression of the likelihood function was derived by Usher & Hodgson [28, 29], and the ML estimation was considered in the cases of two and three-component systems, and it was shown that the closed-form ML estimators are algebraically intractable. The masked data analysis was also studied by Hutto, Mazzuchi, & Sarkani [8] for a superimposed renewal process. Lin et al. [15] extended the results of Usher and Hodgson (1988) by deriving exact ML estimates for the general case of a series system, and developed a Bayesian approach [16] which considers prior information on the component reliabilities. Consider a parallel system with non-identical exponential lifetime components, Sarhan et al. [24, 25, 26] investigated the Bayes estimates of component reliabilities using masked-system life data. For the series-parallel and parallel-series hybrid systems with masked data, wang et al. [31] investigated the ML and interval estimation of parameters. Using the masked data from accelerated life test, Cai et al. [2] estimated the reliability of a log-normal series system based on the EM algorithm, and then derived the approximate confidence intervals of failure rate and investigated the ML estimations of the failure rate using the EM measure [1]. Assuming that the components have Weibull life distributions, Misaei et al. [21] estimated the parameters of the competing risks model with masked failure data. In software reliability, the ML estimation of software reliability were also considered for superimposed nonhomogeneous Poisson processes in the case of the masked data [23, 32, 33, 36]. Based on the exponential life distribution of storage component and system, Zhao et al. [39] investigated the masked failure data from series system being in storage based on the LS method, but not considered the masked success data about the storage system with parallel connection.

Life data of components from storage products are often used to estimate the storage reliability of the individual components. These estimates are useful since they reflect the storage reliability of the components under actual storage conditions. In this paper, the masked storage life data from the series system are applied into the storage reliability model with the possible initial failures, where the masked data are the groups of binomial-type system failure data. The masked data of failed components is excavated from the failed series systems. Consider closed-form ML estimates are algebraically intractable, a general method of the parametric estimates is developed by applying a modified EM algorithm to LS estimation, and the EM-like algorithm is applied for updating the testing data. In the case of exponential storage lifetime of components constituting series system, the method and procedure are formulized in detail. Finally, a numerical example is also provided to illustrate the method and procedure. The results should be useful for planning a storage environment, decision-making concerning the maximum length of storage, maintenance strategy optimization and identifying the production quality.

The remainder of this paper is organized as follows. For predicting the storage reliability, Section 2 gives a generalized storage reliability model with initial reliability. Section 3, an EM-like approach based on the masked data is presented for estimating the parameters of proposed models in the case of exponential distribution. For illustrating the EM-like method, a numerical case is provided to demonstrate the method and procedure in Section 4. Lastly, Section 5 gives the conclusions of the paper.

2. Storage reliability model with initial failures

Generally, storage reliability is considered to be the ability of a product that can still be able to perform its required functions after it has been in the storage state for a certain period of time under specific storage environment [2]. For illustrating the EM measure which estimates storage reliability based on LS estimates, in this section, a generalized models for storage component and system are developed and some assumptions from the engineering practice are presented.

2.1 A generalized storage reliability model

Usually, the storage reliability of products can be defined as the probability that the product can perform its specific function for a period of specific storage time [39]. So, the storage reliability at time t can be represented as

$$R(t) = P\{T > t\} \tag{1}$$

where, the storage lifetime T of a product is a real stochastic variable, and the event "$T > t$" means that the product can perform its specific function after a period of storage time t. In practice, the real storage lifetime T of a product is hard to observe, but it is commonly truncated in the manner that the value of T can be known to be between two specific time points, greater or less than a given value of the time at which it is tested. If the storage lifetime T is assumed as an exponential distribution with parameter λ, then the storage reliability at time t based on Eq. (1) can be given by

$$R(t) = exp(-\lambda t), t \geq 0.$$

In this case, the reliability is equal to 1 at the beginning $t = 0$, which means that the products or systems are perfect before they arrive at the storage stage. Furthermore, if let Ω represent certainty that 'the product is perfect at time zero', then $P(\Omega) = 1$ and Eq. (1) can be rewritten as

$$R(t) = P\{(T > t) \cap \Omega\} = P\{T > t, \Omega\}. \tag{2}$$

Obviously, Eq. (2) means that the storage reliability is equal to the probability of joint event, where the joint event is comprised of a basic event $'(T > t)'$ and a certain event $'\Omega'$.

However, this is not always true in engineering that the products are perfect at the beginning of storage, and the initial reliability of the product is an essential part that should be taken into consideration [37]. Denoted by A the basic event that 'the initial state of the product', then Eq. (2) can be represented as $R(t) = P\{T > t, A\}$. Using the law of the probability multiplication, the storage reliability of the product can be deduced as

$$R(t) = P(A) \cdot P\{T > t \mid A\} = R_0 R_S(t), t \geq 0. \tag{3}$$

Where, R_0 is the probability which equals to $P(A)$, and $R_S(t)$ is the conditional probability $P\{T > t \mid A\}$.

2.2 Initial reliability and inherent reliability

Theoretically, R_0 in Eq. (3) can be interpreted as the initial reliability of the product and takes value in interval $[0,1]$, and may not be completely known for the products in storage. The meaning of initial reliability R_0 can be different for different types of products. For one-shot devices, R_0 should be the probability that the devices without storage can finish its required function successfully. Obviously for this kind of products in storage, R_0 can never be completely known. However, for electronic components, R_0 may be interpreted as the proportion of non-defective components. If all the products are inspected before the storage starts, the initial reliability R_0 could be assumed to be known. However, 100% testing is not practical because of the extra high costs for inspection, or even impossible for many types of products, such as explosive products.

The conditional probability $R_S(t)$ is usually called as the inherent storage reliability [37], which reveals entirely the effect of the storage on the products and the failure process of the products due to the material deterioration in storage. In engineering, some common lifetime distributions, such as exponential, Weibull or lognormal distributions, can be applied to model the inherent storage reliability. Obviously, R_0 and $R(0)$ are all the storage reliability at time zero and have $R(0) = R_0$, so the inherent storage reliability is equal to 1 at time $t = 0$ in Eq. (3). Therefore, the expression $1 - R_S(t)$ can be deduced as a cumulative failure distribution function, and have the distribution family $\{F(t,\theta), \theta \in \Theta\}$ with an unknown parameter θ, i.e.

$$R(t,\theta) = R_0 \cdot (1 - F(t,\theta)).$$

2.3 Modeling assumptions

To evaluate the storage reliabilities of components constituting a series system, it is necessary to develop some measures for estimating the unknown parameters using the field and masked data. Based on the engineering practice and periodic testing data, some assumptions are listed as follows:

(a). Experiment environment for the storage systems and components keep unchanged.

(b). Systems or components are not always perfect at the beginning of storage, i.e., the initial reliability of storage satisfies $0 \leq R_0 \leq 1$.

(c). The series system composed of m components denoted by $1, 2, \cdots, m$. At the beginning of experiment, the same type of N systems, and N_{i0} components i $(i = 1, 2, \cdots, m)$ are put into storage.

(d). The product systems or components are sampled randomly at time $t_j (j = 1,2, \cdots, v)$, and not returned storage experiment.

(e). Number of tested samples is much less than the total size of the population at each testing time.

(f). The outcome of the testing is s-independent, and all the failed or survived samples among tested ones can be detected.

(g). The masked data of components is obtained from the failed series systems.

Apparently, the assumptions (a) – (g) stem from the case of storage practice, where assumption (a) is taken to ensure the effect of stable storage conditon on the reliability of products and the homology of testing data. Assumption (b) indicates the initial reliability of storage samples, and (c) – (g) determine the activity of testing.

2.4 Reliability model of storage system with series connection

In practice, the exponential life distribution can simulate the degradation process of material in the storage state and also has the simple form [18], the lifetime of systems and components are usually assumed as this type distribution. To simplify the method presented in this paper, the exponential distribution is applied for the inherent storage reliability although the method can be valid for a general lifetime distribution. The reliability expressed in Eq. (1) for the i-th components is therefore rewritten as

$$R_i(t) = R_{0i}\, exp(-\lambda_i t), i = 1,2, \cdots, m, \quad (4)$$

where, R_{0i} is the initial reliability of i-th storge component, and λ_i is the failure ratio.

For a series system with m components, the storage lifetime T can be written as $min\,(T_1, T_2, \cdots, T_m)$, and the storage reliability at time t should be

$$\begin{aligned} R(t) &= p\{min(T_1, T_2, \cdots, T_m) > t\} \\ &= p\{T_1 > t, \cdots, T_i > t, \cdots, T_m > t\} \\ &= \textstyle\prod_{i=1}^{m} \Pr\{T_i > t\} = \prod_{i=1}^{m} R_i(t) \\ &= R_{01}R_{02} \cdots R_{0m} \cdot exp[-(\lambda_1 + \lambda_2 + \cdots + \lambda_m)t]. \end{aligned} \quad (5)$$

Therefore, based on the components $1, 2, \cdots, m$ of the series system, the inherent storage reliability of system using Eq. (3) and Eq. (5) can be expressed as

$$R_s(t) = \frac{R_{01}R_{02} \cdots R_{0m}}{R_0} \cdot exp[-(\lambda_1 + \lambda_2 + \cdots + \lambda_m)t].$$

Using Eq. (3), the storage reliability of system at time t can be expressed as

$$R(t) = R_0\, exp(-\lambda t), \quad (6)$$

where, λ is the failure ratio and R_0 the initial reliability. Then the initial reliability for the system, using Eq. (5) and Eq. (6), can be deduced as

$$R_0 = R_{01}R_{02}\cdots R_{0m}\cdot exp[(\lambda-\lambda_1-\lambda_2-\cdots-\lambda_m)t] \quad (7)$$

Especially, Eq. (7) can be simplified as $R_0 = R_{01}R_{02}\cdots R_{0m}$ as $t = 0$, and which indicates the initial reliability of system can be determined by its components. Furthermore, according to Eq. (5), the mean time to failure (MTTF) of the system can be deduced as

$$ET|_n = \int_0^\infty R(t)dt = \frac{1}{\lambda_1+\lambda_2+\cdots+\lambda_n} \quad (8)$$

3. EM-like approach for estimating parameter based on masked data

For the components of storage system, the field failure data can be collected at testing time. In fact, we only can observe that the storage system is failed or survived at testing time, and it is difficult to determine which one or ones have been failed or survived. As far as a surviving series system is concerned, all the components of the system must be survived. Apparently, all the failed components may be unknown or masked for a failed series system. Therefore, how to dig the masked data of components in a storage system is very important for properly evaluating the reliabilities of components constituting the system.

3.1 Masked failure data

Suppose that the product systems with series connection in storage have the binomial-type failure data at sequential observation times $t_1 < t_2 < \cdots < t_v$. For obtaining the masked data, some entire products are in the storage states, meanwhile, some components constituting the product are also put into storage. At testing time $t_j\,(j = 1,2,\cdots,v)$, $n_j\,(n_j \ll N)$ products (i.e., systems) are randomly sampled from N storage products, where f_j products failed and $s_j\,(j = 1,2,\cdots,v)$ ones survived. In the meantime, n_{ij} of the $N_{i0}\,(n_{ij} \ll N_{i0})$ components, for the component i in storage, are randomly sampled, where f_{ij} failed components are detected and $s_{ij}\,(i = 1,2,\cdots,m)$ components are surviving. The detailed data from product systems and components are listed in Table 1.

Table 1. Binomial-type failure data at sequential observation times $t_j\,(j = 1,2,\cdots,v)$.

Causes of failures	Testing times			
	t_1	t_2	$\cdots$	t_v
System	(n_1, f_1, s_1)	(n_2, f_2, s_2)	$\cdots$	(n_v, f_v, s_v)
Component 1	(n_{11}, f_{11}, s_{11})	(n_{12}, f_{12}, s_{12})	$\cdots$	(n_{1v}, f_{1v}, s_{1v})
Component 2	(n_{21}, f_{21}, s_{21})	(n_{22}, f_{22}, s_{22})	$\cdots$	(n_{2v}, f_{2v}, s_{2v})
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Component m	$(n_{m1}, f_{m1}, s_{m1}$	$(n_{m2}, f_{m2}, s_{m2}$	$\cdots$	$(n_{mv}, f_{mv}, s_{mv}$

Note that the data in Table 1 satisfies $n_j = f_j + s_j$, $n_{ij} = f_{ij} + s_{ij}$, $i = 1,2,\cdots,m$, $j = 1,2,\cdots,v$.

The failure of the product (system) is not identified on which component's failures, and then called to be masked. For a series connection system, the system would fail as long as one component in the system fails, and the cause may only be isolated to some subset F of the system's components for a series system, where nonempty subset $F \subseteq \{1, 2, \cdots, m\}$. Especially, the surviving component set for a series system is $\{1, 2, \cdots, m\}$. For demonstrating the masked data in series system clearly, the general form of data displays in Table 2 and represents the life length and true cause of failure, where the true cause of system failure was found by simply observing the minimum life length of the three components A, B and C. Apparently, there are four types of masked data for a two-component system and six types for a three-component one. Assuming that only one component in the system fails at some time points, Table 2 lists the six types of masked data for a random sample of systems.

Table 2. Different types data for a three-component system with masking.

System	Time to failure	Component causing failure	Masking set S_i
S_1	T_1	A	$\{A\}$
S_2	T_2	B	$\{B\}$
S_3	T_3	C	$\{C\}$
S_4, S_5, S_6	T_4	A^*	$\{A,B\}\{A,C\}\{A,B,C\}$
S_4, S_7, S_6	T_5	B^*	$\{A,B\}\{B,C\}\{A,B,C\}$
S_5, S_7, S_6	T_6	C^*	$\{A,C\}\{B,C\}\{A,B,C\}$

Note that the notations A, B and C suggest that the true cause of system failure can be found, and A^*, B^*, C^* indicate that the cause of system failure is only isolated to some subset $\{A, B, C\}$ of the system's components.

3.2 Parameter estimation without masked data

Note that if the masked data is not considered, then the storage reliability of each component at testing times using ML method can be simply estimated as

$$\hat{R}_i(t_j) = \frac{n_{ij} - f_{ij}}{n_{ij}} = \frac{s_{ij}}{n_{ij}}, i = 1,2,\cdots,m\,;\ j = 1,2,\cdots,v.$$

To avoid the inverted data of the point estimation, Zhang et al. [35] obtained the following E-Bayesian estimates of storage reliability at testing time.

$$\hat{R}_i(t_j) = 1 - \frac{1}{c-1}\iint_{0<a<1,1<b<c} \frac{a + f_{ij}}{a + b + s_{ij}} dadb, \quad (9)$$

where the hyper-parameters a, b should satisfy $0 < a < 1, b > 1$ and it is suitable for c to take 2~8 [7].

For the parameter estimation without masked data, Zhang et al [34, 35] have developed the likelihood function of initial reliability and failure rate, where the parameters of the component reliability can be estimated by ordinary methods separately. When the exponential distributions are applied for the inherent storage lifetimes of components, the ML estimates of the parameters can be obtained by solving the ML equations numerically. According to the function of storage reliability of component in Eq. (4), the set of nonlinear likelihood equations can be given by

$$\begin{cases} \sum_{j=1}^{v} \frac{(n_{ij} - s_{ij})\, exp(-\lambda_i t_j)}{1 - R_{0i}\, exp(-\lambda_i t_j)} = \frac{1}{R_{0i}} \sum_{j=1}^{v} s_{ij} \\ \sum_{j=1}^{v} \frac{t_j (n_{ij} - s_{ij}) R_{0i}\, exp(-\lambda_i t_j)}{1 - R_{0i}\, exp(-\lambda_i t_j)} = \sum_{j=1}^{v} s_{ij} t_j \end{cases},$$

$$i = 1,2,\cdots,m. \quad (10)$$

Note that the set of equations include $2m$ equations, and the closed-form ML estimates are algebraically intractable using the data in Table 1 except under certain tight restrictions.

On the other hand, using the testing data of storage components in Table 1, the reliability of each component at time t_j can be written as

$$R_i(t_j) = R_{0i}\, exp(-\lambda_i t_j), i = 1,2,\cdots,m,\ j = 1,2,\cdots,v.$$

Taking logarithm of the reliability function mentioned above, the linear equations can be given as follows:

$$Y_{ij} = a_i + b_i t_j, \quad (11)$$

where $Y_{ij} = ln[R_i(t_j)], a_i = \ln(R_{0i}), b_i = -\lambda_i, i = 1,2,\cdots,m$, $j = 1,2,\cdots,v$.

Usually, the LS estimates are given as follows since the analytic formulas can be written. For a fixed component i, applying the estimates given in Eq. (9) to replace $R_i(t_j), j = 1,2,\cdots,v$, the LS estimates of parameters a_i and b_i can be written as

$$\hat{b}_i = -\hat{\lambda}_i = \frac{\sum_{j=1}^{v}(t_j - \bar{t})\{\ln[\hat{R}_i(t_j)] - \bar{R}_{iln}\}}{\sum_{j=1}^{v}(t_j - \bar{t})^2},$$

$$\hat{a}_i = \bar{Y}_i - \hat{b}_i \bar{t} = \bar{R}_{iln} + \hat{\lambda}_i \bar{t} = ln(\hat{R}_{0i}).$$

Then the estimates of initial reliability R_{0i} and failure rate λ_i for component i can be expressed as

$$\hat{\lambda}_i = -\frac{\sum_{j=1}^{v}(t_j - \bar{t})\{\ln[\hat{R}_i(t_j)] - \bar{R}_{iln}\}}{\sum_{j=1}^{v}(t_j - \bar{t})^2}, \quad (12)$$

$$\hat{R}_{0i} = exp(\bar{R}_{iln} + \lambda_i \bar{t}), \quad (13)$$

where $\bar{Y}_i = \frac{1}{v}\sum_{j=1}^{v} Y_{ij}$, $\bar{R}_{iln} = \frac{1}{v}\sum_{j=1}^{v} ln[\hat{R}_i(t_j)]$, $\bar{t} = \frac{1}{v}\sum_{j=1}^{v} t_j$, $i = 1,2,\cdots,v$.

Similarly, for the storage system, the LS estimates of parameters λ and R_0 can be represented as

$$\hat{\lambda} = \frac{\sum_{j=1}^{v}(t_j - \bar{t})\{\ln[\hat{R}(t_j)] - \bar{R}_{ln}\}]}{\sum_{j=1}^{v}(t_j - \bar{t})^2}, \quad (14)$$

$$\hat{R}_0 = \exp(\bar{R}_{ln} + \lambda \bar{t}), \quad (15)$$

where $\bar{R}_{ln} = \frac{1}{v}\sum_{j=1}^{v} ln(\hat{R}(t_j)), \bar{t} = \frac{1}{v}\sum_{j=1}^{v} t_j$.

Obviously, the storage reliabilities of components using Eqs. (12) and (13), and systems using Eqs. (14) and (15), can be evaluated and predicted based on the field information without masked data. However, if there exists the masked data as displayed in Table 1, then the likelihood function of all parameters will become a multivariable function with a very high dimension, and therefore the ML estimates of the parameters are difficult to be found [13].

3.3 Parameter estimation with masked data

In some cases, the reliability models can contain both observable variables and hidden or latent variables. If the variables in the reliability model are all the observable variable, then one can directly estimate the parameters in the model using ML or Bayesian estimation technique based on the given data. However, when there exist the hidden variables in the reliability models, the previous mentioned measures cannot estimate these parameters. In 1977, Dempster et al. [4] propose the EM algorithm which can be applied to estimate the parameters of probability models with hidden variables. Recently, the EM algorithm has been applied to solve the ML parametric estimation in software reliability when the masked data are presented [21, 23, 32].

3.3.1 EM algorithm and its modification

For the parametric estimation based on the storage information with masked data, the general ideal of the EM algorithm is to apply the initial value θ^0 of the unknown parameter to calculate the expectation of the hidden data. If the complete data of the storage products are available, then the ML estimate θ^1 in the reliability model can be obtained, and then repeat the procedure by taking θ^1 as the new initial values, until the stable values are found under the specified convergence condition. However, the ML estimates for the parameters in the storage reliability model of components do not exist analytic forms, and indicates that the traditional EM algorithm is very complicated for finding these parameters in the models with masked data. Note that the LS estimates of the parameters in the components' reliability models exist close analytic form as given in formulas (12) and (13). For taking full advantage of the LS technique's advantages, therefore a modified EM-like measure called EM-LS algorithm, is developed and presented as follows:

E-step: Give the initial values of the parameters θ^0, find out the expectations of the failure data of components from the failed system;

M-LS-step: Using the estimated failure data and the non-masked failure data from the series system, to update the field data from components and then estimate the parameter θ^1 based on the LS estimate. Whereafter, the parameter θ^1 will be taken as the new initial values of the E-step, and k times iterations are carried out until the difference of the two neighboring parameters less than a given small positive value ε, i.e.

$$\left|\theta^{(k)} - \theta^{(k-1)}\right| < \varepsilon, \varepsilon \to 0.$$

Up to now, there are no similar studies on the EM-LS algorithm found in the existing researches. Theoretically, if the LS estimates can be easily obtained, then the extended LS algorithm can be used for any model, such as the storage lifetimes follow the exponential, Weibull or log-normal distributions [11]. To verify the effectiveness of the EM-LS algorithm, the case of exponential lifetime for storage component and system is applied in this section.

3.3.2 EM-LS algorithm for the binominal-type data

Denoted by n the number of tested systems, f the failure number, and s the survive number. If the storage reliability of the ith component is R_i $(i = 1, 2, \cdots, m)$, then the series system has the reliability $R = R_1 R_2 \cdots R_m$. Assuming that these systems' failures are not identified due to which component for a series structure, it can simply be proved that conditional on (n, f). Using the law of large numbers, for $\forall \varepsilon > 0$, one has

$$\lim_{n\to\infty} Pr\left\{\left|\frac{f}{n} - (1 - R)\right| < \varepsilon\right\} = 1.$$

Let f_i^E be the expected number of failures of ith component, then one has the probability

$$\lim_{n\to\infty} Pr\left\{\left|\frac{f_i^E}{n} - (1 - R_i)\right| < \varepsilon\right\} = 1.$$

Therefore, the ratio of f_i^E/n to f/n can be approximated to $(1 - R_i)/(1 - R)$, and the expected number f_i^E can be deduced as

$$f_i^E = \frac{1 - R_i}{1 - R} f, i = 1,2, \cdots, m. \tag{16}$$

Note that a series system failed if and only if one component in the system failed, so the sum satisfies $f_1^E + f_2^E + \cdots + f_m^E \geq f$ since one system failure can be caused by more components.

Concerning on the EM-LS algorithm, the formula (16) will be used to convert the system binomial data (n, f) into component binomial data (n, f_i^E) $(i = 1,2, \cdots, m)$. For the masked data in Table 1, the improved EM-LS algorithm can be presented as follows:

E-step: Given the initial values of the component initial reliabilities $(R_{01}^0, R_{02}^0, \cdots, R_{0m}^0)$ and failure rates $(\lambda_1^0, \lambda_2^0, \cdots, \lambda_m^0)$, calculate as the following steps:

Step 1. At testing times $t_j (j = 1,2, \cdots, v)$, the reliabilities of components are given by

$$R_i(t_j) = R_{0i}^0 \exp\left(-\lambda_i^0 t_j\right), i = 1,2, \cdots, m, \tag{17}$$

and the system reliability is

$$R(t_j) = R_1(t_j) R_2(t_j) \cdots R_m(t_j), \tag{18}$$

where $j = 1,2, \cdots, v$.

Step 2. For the masked system data (n_j, f_j) at testing time t_j, the expected number of failures in the system, using Eq. (16), is given by

$$f_{ij}^E = \frac{1 - R_i(t_j)}{1 - R(t_j)} f_j, i = 1,2, \cdots, m;\ j = 1,2, \cdots, v. \tag{19}$$

Step 3. Update the component binomial data in Table 1 as

$$\begin{cases} n'_{ij} = n_{ij} + n_j \\ f'_{ij} = f_{ij} + f^E_{ij} \end{cases}, \tag{20}$$

where, $i = 1,2,\cdots,m$; $j = 1,2,\cdots,v$.

M-LS-step: Apply the estimated component binomial data calculated by formula (20) to Eq. (19), the reliability of component i at time t_j can be estimated as

$$\hat{R}'_i(t_j) = 1 - \frac{f_{ij} + \theta_{ij} f_j}{n_{ij} + n_j}, \tag{21}$$

where, the parameter $\theta_{ij}(\, i = 1, 2, \cdots, m, j = 1, 2, \cdots, v)$ are computed by

$$\theta_{ij} = \frac{1 - R^0_{0i} \exp(-\lambda^0_i t_j)}{1 - \prod_{i=1}^m [R^0_{0i} \exp(-\lambda^0_i t_j)]}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, v.$$

Furthermore, the LS estimates for the failure rates and initial reliabilities of the components are represented as

$$\begin{cases} \hat{\lambda}^1_i = -\frac{\sum_{i=1}^v (t_j - \bar{t}\,)[ln \hat{R}'_i(t_j) - \bar{R}'_{iln}]}{\sum_{j=1}^v (t_j - \bar{t}\,)^2} \\ \hat{R}^1_{0i} = exp(\bar{R}'_{iln} + \hat{\lambda}^1_i \bar{t}) \end{cases}, \tag{22}$$

where, $\bar{R}'_{iln} = \frac{1}{v}\sum_{j=1}^v ln[\hat{R}'_i(t_j)], \bar{t} = \frac{1}{v}\sum_{j=1}^v t_j\,, i = 1,2,\cdots,m.$

When the LS-step is finished, the E-step will be repeated, but the initial values $(\hat{R}^0_{01}, \hat{R}^0_{02}, \cdots, \hat{R}^0_{0m})$ and $(\hat{\lambda}^0_1, \hat{\lambda}^0_2, \cdots, \hat{\lambda}^0_m)$ will be replaced by the LS estimates $(\hat{R}^1_{01}, \hat{R}^1_{02}, \cdots, \hat{R}^1_{0m})$ and $(\hat{\lambda}^1_1, \hat{\lambda}^1_2, \cdots, \hat{\lambda}^1_m)$. The iteration of the E-step and M-LS-step can be terminated when the stable LS estimates are obtained. Usually, a small positive number ε is fist appointed, and the iteration continues until the difference of two adjacent estimates is lower than the given value ε, i.e.

$$\Delta R_{0i} = |\hat{R}^k_{0i} - \hat{R}^{k-1}_{0i}| < \varepsilon, \qquad \Delta\lambda_i = |\hat{\lambda}^k_i - \hat{\lambda}^{k-1}_i| < \varepsilon.$$

Theoretically speaking, the estimate of the storage reliability more and more approaches to the true value as the number of tested products increases. Usually, the number of storage products for testing, in reality, may be small because of the high expenses such as Missiles, mines, and sonobuoys etc. [18]. However, in the early 1980s, the Reliability Department of Sandia National Laboratories collected and analyzed a relatively large amount of data which depicted the performance of certain electronic parts after long periods of dormant storage, for example, Sandia accumulated the performance data about 261974 resistors and 85976 capacitors from weapons which had stockpiled for 10 years or 20 years old [19]. However, it is rare that the number of storage components and systems for experiment is large in reality. In order to illustrate the practicability and the validity of the proposed EM-LS approach, the data-based examples will be illustrated in Section 4.

4. 4. Illustration

To illustrate the EM-LS algorithm to estimate the storage reliability, a simple series system comprised of two components is considered in this section. For the purpose of illustration, the simulated binomial-type masked data (n_j, f_j, s_j) and (n_{ij}, f_{ij}, s_{ij}) $(i = 1,2,\cdots,m,\ j = 1,2,\cdots,v)$ is listed in Table 3, where the data from the failed systems concludes the masked failure information of components. According to the storage reliability models (5), the initial reliability R_{01}, R_{02} and failure rates λ_1, λ_2 of component 1 and 2, need to be estimated respectively.

Table 3. Failure and success data of system and components.

Testing times/months	Causes of failures		
	System	Component 1	Component 2
6	(50,1,49)	(50,0,50)	(50,1,49)
12	(50,1,49)	(50,1,49)	(50,2,48)
18	(40,1,39)	(40,1,39)	(40,2,38)
24	(40,2,38)	(40,2,38)	(40,3,37)
30	(30,1,19)	(40,2,38)	(40,3,37)
36	(30,3,28)	(30,1,29)	(30,2,28)
42	(20,2,18)	(30,2,28)	(30,2,28)
48	(20,2,18)	(30,2,28)	(30,2,28)
54	(15,2,13)	(20,1,19)	(15,2,13)
60	(15,3,12)	(20,2,18)	(15,3,12)

The cases without and with masked data for a series connection systems are presented as follows:

4.1 Masked failure data from series connection systems

If the masked data from the series systems is not considered in the case, then the failure rate and initial reliability of the components constituting the system, based on Eqs. (9), (12), (13) and using the corresponding data in Table 3, can be estimated as 0.001607 and 0.992667 for component 1, and 0.002962 and 0.998549 for component 2. According to the failure data of system, based on Eqs. (14) and (15), the parameters of the storage reliability function of the system can be estimated as 0.003113 and 0.986241. However, using Newton iterative method and the likelihood equations in Eq. (10), the ML estimates of failure rate and initial reliability can be obtained as

0.002103 and 0.996637 for component 1, and 0.003121 and 0.999788 for component 2, and 0.004989 and 0.991325 for the system.

If the masked data from the storage system is considered in this case, then the proposed EM-LS algorithm would be used to find the known parameters. According to Eqs. (21) and (22), the initial values of component reliability $(\hat{R}_{01}^{0}, \hat{R}_{02}^{0})$ and failure rate $(\hat{\lambda}_{1}^{0}, \hat{\lambda}_{2}^{0})$ can be updated as $(\hat{R}_{01}^{1}, \hat{R}_{02}^{1})$ and $(\hat{\lambda}_{1}^{1}, \hat{\lambda}_{2}^{1})$. The iteration of the E-step and LS-step can be terminated when the stable LS estimates are obtained. The initial reliability and initial failure rate of component 1 and 2 varying with the number of iterations are shown in Fig. 1.

From Fig. 1, one can see that the parameter estimates based on the EM-LS algorithm have steady convergence values. Through about 200 iterations, the convergence values of the initial reliability and failure rate for component 1 are about 0.996017 and 0.001394, and for component 2 are about 0.999031 and 0.001748. Furthermore, using the estimates of parameters for component 1 and 2, the initial reliability of the system based on Eq. (7) at zero time point is 0.995052 and the failure rate based on Eq. (8) is 0.002142. However, applying Eq. (20) to update the data n_{ij} and s_{ij} in Eq. (10), the ML estimates of initial reliabilities and failure rates can be approximated as 0.993704 and 0.001992 for component 1, and 0.995011 and 0.002643 for component 2. Using the estimates of parameters for component 1 and 2, the initial reliability and failure rate of the system can be computed as 0.988746 and 0.003632.

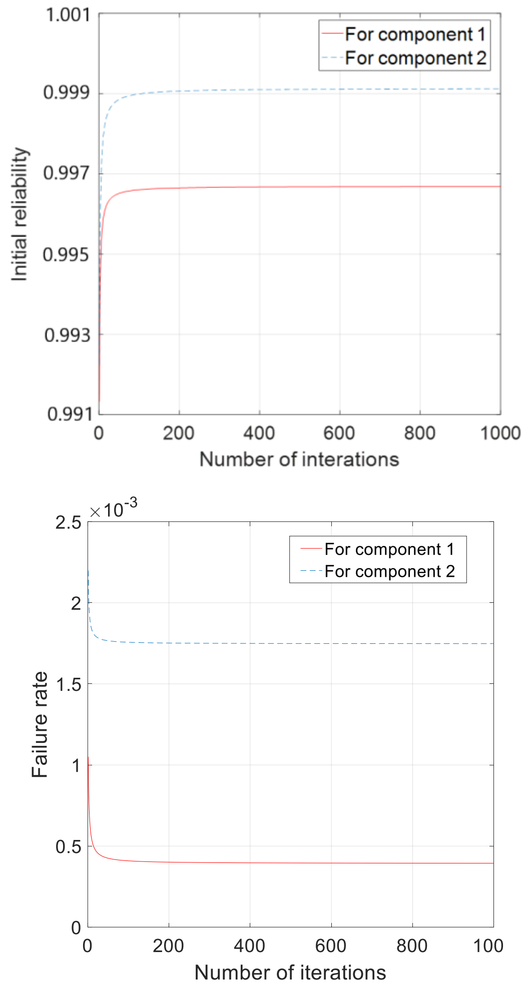

Fig. 1. Initial reliability and failure rate of component 1 and 2.

Clearly, the convergence values, when the data in Table 2 from storage system with and without masked data using LS and ML techniques, the associated estimates are listed in Table 4.

Table 4. Estimates for the parameters of components 1 and 2.

Data	Methods	Component 1		Component 2		System	
		R_{01}	λ_1	R_{02}	λ_2	R_0	λ
Without masked data	LS/N	0.992667	0.001607	0.998549	0.002962	0.986241	0.003113
	ML/N	0.996637	0.002103	0.999788	0.003121	0.991325	0.004989
With masked data	EM-LS/Y	0.996017	0.001394	0.999031	0.001748	0.995052	0.002124
	ML/Y	0.993704	0.001992	0.995011	0.002643	0.988746	0.003632

Note that the acronym "LS/N" in Table 4 denotes the traditional LS method and "ML/N" the ML method without masked data from system, and "EM-LS/Y" is the improved EM-LS method and the traditional ML method which consider the masked data of components from the storage system.

Obviously, the estimates of the failure rates and initial reliabilities with and without applying masked system data differ to each other. The differences between the estimated parameters in the models indicate that the failure data from storage system should be used when the component reliability is evaluated. Using the failure data of components with and without masked data, the estimated reliability by the traditional LS and ML measure without masked data and the EM-LS and ML algorithm with masked data, respectively, are displayed for component 1 in Fig. 2, and for component 2 in Fig. 3, and for the series system in Fig. 4.

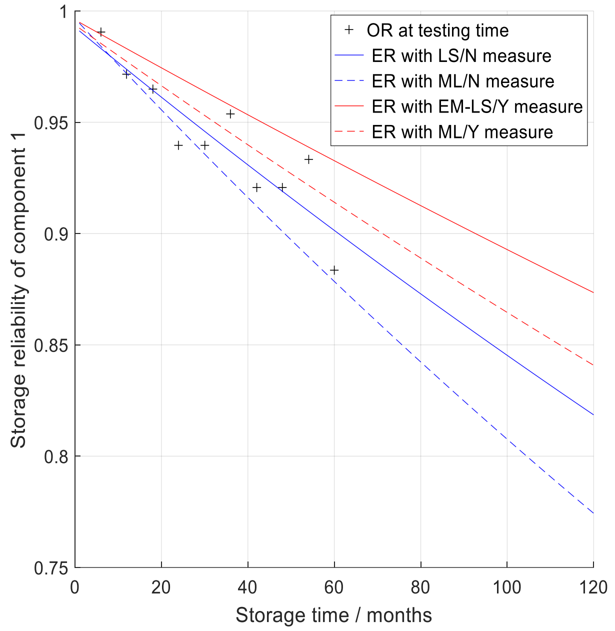

Fig. 2. Observed and estimated reliability for component 1.

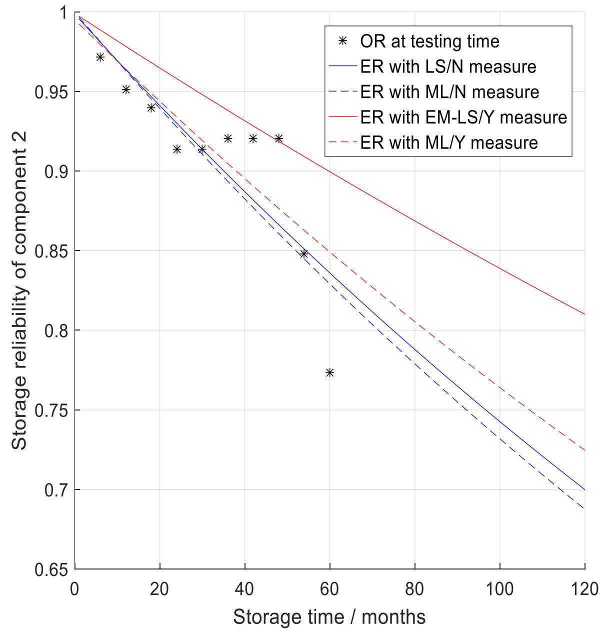

Fig. 3. Observed and estimated reliability for component 2.

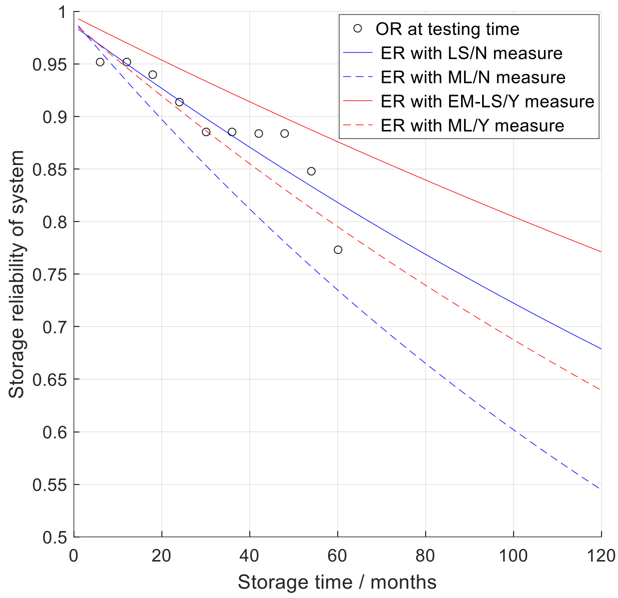

Fig. 4. Observed and estimated reliability for the parallel system.

Note that the acronym "OR" in Fig. 3, Fig. 4 and Fig. 5 denotes the observed reliability, and "ER" denotes the estimated reliability.

From Fig. 2 and Fig. 3, one can apparently observe that the reliability degrades with storage time under the testing condition, and the storage reliabilities with masked data are slightly higher than that without masked data for any components in the system, and the same results for the system in Fig 4. The fittings of storage reliability for the components and system behave as a more optimistic prediction with the masked data. Actually, the latent failure information in the storage system does not fully used when the parameters in the models are only estimated by the observed data without masked data. Furthermore, the predictions of storage reliability are of interest for some engineers based on the estimated parameters, and some estimated results for component 1, component 2 and the series system, are presented in Table 5, Table 6 and Table 7 respectively.

Table 5. Reliability estimates with and without masked data from storage system for component 1.

Storage age t_i (months)	Observed reliability	Estimated reliability			
	E-BE	LSE/N	MLE/N	EM-LSE/Y	MLE/Y
12	0.9714	0.9737	0.9718	0.9795	0.9702
24	0.9398	0.9551	0.9476	0.9632	0.9473
36	0.9539	0.9369	0.9240	0.9473	0.9249
48	0.9206	0.9190	0.9009	0.9316	0.9031

Table 6. Reliability estimates with and without masked data from storage system for component 2.

Storage age t_i (months)	Observed reliability	Estimated reliability			
	E-BE	LSE/N	MLE/N	EM-LSE/Y	MLE/Y
12	0.9515	0.9637	0.9630	0.9783	0.9639
24	0.9136	0.9300	0.9276	0.9580	0.9339
36	0.9206	0.8976	0.8935	0.9381	0.9047
48	0.9206	0.8662	0.8607	0.9186	0.8765

Table 7. Reliability estimates with and without masked data for the storage system

Storage age t_i (months)	Observed reliability	Estimated reliability			
	E-BE	LSE/N	MLE/N	EM-LSE/Y	MLE/Y
12	0.9515	0.9501	0.9337	0.9700	0.9466
24	0.9398	0.9152	0.8795	0.9456	0.9062
36	0.8852	0.8817	0.8284	0.9218	0.8676
48	0.8835	0.8494	0.7802	0.8986	0.8306

where, the acronym “E-BE” denotes the E-Bayesian estimates at the fixed testing point in time, “LSE/N” represents the predicted reliability based on the traditional LS method and “MLE/N” is the predicted reliability based on the traditional ML method without masked data. The acronym “EM-LSE/Y” is the predicted reliability based on the improved EM-LS method and “MLE/Y” is that based on the traditional ML method with masked data.

From the estimated results in Table 5, Table 6 and Table 7, one can find these estimated reliabilities based on the masked data are more optimistic than those without masked data at the determined time, and the results provide an important reference for the engineers which evaluate the reliability of the storage products. Furthermore, if it is required that the reliability of the storage products is above 80%, the allowable storage period under the testing condition can be predicted by the estimated parameters. Specifically, the predicted results, by doing decimals to round up and round down numbers, are presented in Table 8.

Table 8. Predictions for the allowable storage period at the reliability 80%

Method		Predicted storage period (months)		
		Component 1	Component 2	System
Without masked data	LS/N	134	75	67
	ML/N	105	72	43
With masked data	EM-LS/Y	157	127	103
	ML/Y	108	82	58

The results in Table 8 indicate that there is a little difference between the predicted storage periods using masked data and using observed data. When the masked data is considered, the estimated storage periods is upper than that of non-masked data (i.e., observed data) under the corresponding methods. As far as the actual cases concerned, one can believe that the estimation with masked data is superior to that without masked data. Moreover, the estimated parameters can be also used to make the prediction of storage reliability. In order to compare the predicted reliabilities with and without masked data, some results are presented in Table 9, Table 10 and Table 11.

Table 9. Predicted reliabilities with and without masked data for component 1.

Storage age t_i (months)	Predicted reliability			
	LSE/N	MLE/N	EM-LSE/Y	MLE/Y
66	0.8928	0.8675	0.9085	0.8713
72	0.8842	0.8566	0.9009	0.8609
78	0.8757	0.8459	0.8934	0.8507
84	0.8673	0.8353	0.8860	0.8406

Table 10. Predicted reliabilities with and without masked data for component 2.

Storage age t_i (months)	Predicted reliability			
	LSE/N	MLE/N	EM-LSE/Y	MLE/Y
66	0.8212	0.8137	0.8902	0.8357
72	0.8068	0.7986	0.8809	0.8226
78	0.7926	0.7838	0.8717	0.8096
84	0.7786	0.7692	0.8626	0.7969

Table 11. Predicted reliabilities with and without masked data for the parallel system.

Storage age t_i (months)	Predicted reliability			
	LSE/N	MLE/N	EM-LSE/Y	MLE/Y
66	0.8031	0.7132	0.8649	0.7780
72	0.7882	0.6922	0.8539	0.7612
78	0.7736	0.6718	0.8431	0.7448
84	0.7593	0.6519	0.8325	0.7288

It can be seen that the predictions of storage reliability with and without masked data have a little difference, and the predicted values with masked data are slightly upper than that without masked data. Maybe one believes that, all the masked data from the storage systems are combined to the observed data from the storage components, can obtain a more accurate estimate. As far as the engineering practice concerned, these results in the Tables 9, 10, and 11 present a more reasonable prediction when the masked data is considered, and it seems that the combined prediction is reasonable measure to evaluate the reliability of products being in storage.

According to the principle of consistent estimator associated with sample size, the masked data from the storage system which imbedded into the observed data can improve the estimated accuracy, because the testing data from the storage component and the masked data from the storage system are all used for estimating the parameters in the proposed models.

5. Conclusion

Consider the storage system with series connection and exponential life distribution, the storage reliability model with initial failures is studied in this paper. If the survival components for a series storage system can not be detected and the cause of system failures is hidden, then the evaluation of storage reliability for a component or system would become difficult.

For making full use of the masked success data from a storage system, an LS measure with an EM-like algorithm is proposed for the series system based on the binomial-type failure data. By applying the EM-LS algorithm to update the testing data, the estimates of initial reliability and failure rate of the components constituting the series system are presented, and a modified EM-like algorithm procedure is developed. Finally, a numerical example is provided to illustrate the EM-LS method, and the proposed measure can make use of the survival components information in the failed series system. The estimates for the storage reliability of the components and system are different with and without masked data, where the storage reliability which considers the masked data in storage system is slightly upper than that without masked data. The proposed EM-LS method has greatly simplified the parametric estimation in the case of the masked data.

In practice, a storage decision can usually be made based on simple engineering judgment and practical circumstance, and the model which consider the masked data is generally worth studying because of making use of the masked success data in the series system. To more accurately evaluate the reliability of the storage system, the data from the existing and the similar products perhaps can be integrated into the field-testing data. This is something that is very much well worth exploring through special processing for obtaining accurate estimates in our future work.

Nomenclature

Acronyms

ML	maximum likelihood	LS	least squares
EM	Expectation and Maximization	MTTF	mean time to failure
MLE	ML estimate	LSE	LS estimate
EM-LS	EM-like measure based on the LS method.		
LS/N	Traditional LS method without masked data from system		
LSE/N	Predicted reliability based on the LS/N measure		
ML/N	Traditional ML method without masked data from system		
MLE/N	Predicted reliability based on the ML/N measure		

EM-LS/Y Proposed EM-LS method with masked data from system

EM-LSE/Y Predicted reliability based on the EM-LS measure

ML/Y Traditional ML method with masked data from system

MLE/Y Predicted reliability based on the ML/Y measure

Notations

T random variable of the storage lifetime

$R_s(t)$ probability that $T > t$ given that the item is perfect at time zero

m number of components constituting the series system

N total number of systems put into storage at the beginning

N_{i0} total number of storage components i ($i = 1, 2, \cdots, m$)

v total number of testing times

λ failure ratio of the storage system

t_j predetermined time points for testing, $j = 1,2, \cdots, v$

λ_i failure ratio of the i-th storage component, $i = 1, 2, \cdots, m$

R_i storage reliability of the i-th component

$R_i(t)$ reliability of the i-th components, $i = 1, 2, \cdots, m$

$\hat{R}_i(t_j)$ MLE of component i at time t_j

$\hat{R}_i'(t_j)$ MLE of component i at time t_j with masked data

$ET|_n$ MTTF of the system which composed of n components.

n_j number of tested systems at time t_j

f_j number of failed systems at time t_j

s_j number of surviving systems at time t_j

N_{i0} number of storage components i

n_{ij} number of tested components i at time t_j

f_{ij} number of failed components i at time t_j

s_{ij} number of surviving components i at time t_j

$\hat{\lambda}$ estimate of failure rate for the series system

$\hat{\lambda}_i$ estimate of failure rate λ_i for component i

λ_i^0 initial value of initial reliability for component i

$\hat{\lambda}_m^k$ failure rate after the k-th iterations based on EM-LS measure

R_0 initial reliability of the storage system

R_{0i} initial reliability of the i-th storage component, $i = 1, 2, \cdots, m$

$\hat{R}_0$ estimate of initial reliability for the series system

$\hat{R}_{0i}$ estimate of initial reliability R_{0i}

R_{0i}^0 initial value of initial reliability for component i

$\hat{R}_{0i}^k$ initial reliability after the k-th iterations based on EM-LS measure

s_i^E expected number of failed components i

n'_{ij} updated number of components i at time t_j with masked data

s'_{ij} updated number of failed components i at time t_j with masked data

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