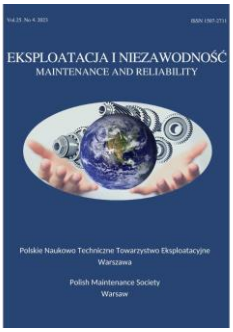

Eksploatacja i Niezawodnosc – Maintenance and Reliability
Volume 25 (2023), Issue 4
journal homepage: http://www.ein.org.pl

Article citation info:
Song M, Zhang Y, Yang F, Wang X, Guo G, Maintenance policy of degradation components based on the two-phase Wiener process, Eksploatacja i Niezawodnosc – Maintenance and Reliability 2023: 25(4) http://doi.org/10.17531/ein/172537

Maintenance policy of degradation components based on the two-phase Wiener process

Indexed by: Web of Science Group

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Highlights

- A maintenance policy with two inspection intervals and a preventive-replacement threshold.
- A fixed change point degradation threshold is adopted in the two-phase Wiener process.
- A reasonable degradation failure time distribution is given.
- The process is explained and the feasibility is proved through a numerical example.
- The impact of the cost parameters is analyzed through sensitivity analysis.

Abstract

This paper proposes a condition-based maintenance policy for the two-phase Wiener degradation process components. The main contribution of this article is to provide the time distribution of degradation failures for the two-phase Wiener process degradation component, as well as the modeling and solving methods for two-phase maintenance. The two-phase maintenance policy includes two-phase inspection and preventive replacement maintenance operations. The established optimization maintenance policy model aims to minimize long-term operation costs. The specific cost calculation equation and the solution method of the maintenance model are given. The feasibility of the maintenance policy model is verified using the two-phase degradation data of the Liquid Coupling Devices. The Particle swarm optimization algorithm can stably solve the described problem, and the results show that the two-phase maintenance policy can be more economical and improve components availability. After that, we also analyzed the impact of the cost parameters on the maintenance policy through sensitivity analysis.

Keywords

condition-based maintenance, two-phase inspection, two-phase Wiener process, Particle Swarm Optimization, sensitivity analysis

1. Introduction

Generally, machines and components go through a series of degradation states before eventual failure occurs. Inspecting machine degradation and repairing it timely could reduce maintenance costs, operational downtime, and safety hazards [21]. In the research, the characteristic quantity can be selected to reflect the degradation of equipment [11]. Over time, the degradation will gradually increase, and the equipment will fail after reaching a certain level. By analyzing the degradation mechanism and the degradation data, mathematical models can be built to obtain the time distribution of degradation failures [14]. Some degradation process models are established by stochastic processes such as the Wiener or Gamma process, and the failure time distribution can be derived according to the given failure threshold [8]. For the Wiener process, the failure time distribution follows the inverse Gaussian (IG) distribution [5]. For the Gamma process, some researchers also give its

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failure time distribution [22]. Both stochastic processes can model linear and nonlinear models, the difference being that the Gamma process is non-decreasing, while the Wiener process can cope with the situation that the amount of degradation fluctuates over time. Another type of degradation model is the general path model [7]. The model assumes that the degradation path is a certain path affected by random effects. After the random parameters are determined, the degradation will conform to a fixed route, and the fault occurrence time will also become a fixed value, so the degradation path model cannot model the time uncertainty. However, in practical situations, it is difficult to fully grasp the randomness of the environment or the equipment itself. It is more reasonable to use stochastic process modeling to take this uncertainty into account. At the same time, random processes such as the Wiener process or Gamma process have good mathematical properties, which can better combine prior distribution and express the influence of the environment through covariates [4].

But sometimes the degradation phenomenon cannot be expressed using a single model. In the study, it is found that some devices showed the characteristics of two-phase degradation during the degradation process. After the degradation reaches a certain turning threshold, the degradation mechanism or form changes. For some equipment, the degradation may be very fast at the beginning, and degradation slows down after the turning threshold is reached. It may also start slowly, but after the transition threshold, the degradation accumulates quickly. The degradation path model became an option when describing the two-phase degradation situation. While research on the two-phase degradation process in recent years has focused on stochastic processes. Some scholars use the same degradation model for the two-phase degradation, such as the Wiener-Wiener model proposed by Yan et al [30]. and the Gamma-Gamma model proposed by Fouladirad and Grall [6]. Some scholars also use different stochastic processes when modeling the two-phase degradation process, such as the Gamma-Wiener model proposed by WANG et al [26]. The research objects include the two-phase degradation process of the Liquid Coupling Devices (LCD) [9], Lithium-ion batteries [25], or bearings [12,23], and the multi-phase degradation process of the high-voltage-pulse capacitor [5]. Research content includes parameter estimation and reliability or remaining life prediction [13], also the accuracy of small sample modeling [27].

In general, the two-phase degradation process has attracted the attention of researchers, and two stochastic process models, the Gamma process and the Wiener process are mainly used in modeling.

This article attempts to find suitable maintenance policies for the two-phase degradation phenomenon, as the changes in two-stage degradation may affect the arrangement of maintenance policies in different phases. The utilization of preventive maintenance can reduce the occurrence of unexpected failures and reduce losses [20]. Preventive maintenance can be roughly divided into time-based maintenance (TBM) and condition-based maintenance (CBM) [1]. Condition-based maintenance can incorporate the inspected equipment status data into the judgment basis for maintenance decisions so that the decisions made are more flexible and accurate. Some research examples show that adopting condition-based maintenance is more economical than time-based maintenance [2,10,19]. A common CBM is to set a preventive maintenance threshold. Once it is inspected that the degradation amount reaches the threshold, maintenance measures are taken. The inspection is also divided into continuous monitoring and interval inspection. When the inspection cost is high and cannot be ignored when compared with the maintenance cost, or it is difficult to implement continuous monitoring, interval inspection is generally adopted. And the inspection interval sometimes becomes a decision variable of maintenance policy. At the same time, it is worth noting that some researchers have adopted different intervals in different phases when conducting condition-based maintenance. Some literature also concluded that different intervals can achieve better results than fixed detection intervals with cost savings [3,15-16,18].

From the above, it can be seen that CBM provides a more flexible model for maintenance, and has proved its superiority in saving costs, so it has attracted many researchers into this field so far. At the same time, two-phase maintenance policies are also emerging. In the case of two-phase degradation, rational use of two-phase condition maintenance may produce good results.

Therefore, this paper will propose a two-phase maintenance policy for which the degradation can be seen as the two-phase Wiener process. Because the Wiener process can deal with general situations, whether the degradation is monotonous or not. The model-solving process taking the Wiener process as an example can also be applied to other stochastic processes, such as the gamma process. This paper's contribution is provided a clear calculation equation for the maintenance costs and the proper optimization method under the above maintenance framework. The two-phase Wiener process mentioned in this paper is under the condition of a fixed change point degradation threshold while the change point of time will be variable, which means it will enter the next phase if the degradation amount exceeds this threshold. A reasonable degradation failure time distribution different from those in the existing literature is given. The method proposed in this paper can guide the maintenance decision of two-stage degraded components. The rest of this paper is organized as follows. The literature review will be introduced in Section 2 comparing the differences between this paper and existing papers. The two-phase Wiener process is introduced in Section 3. The maintenance policy is described and established in Section 4. The formulation and optimization are shown in Section 5. The feasibility and superiority of the model are illustrated with numerical examples in Section 6, followed by sensitivity analysis. The conclusion and prospect are made in Section 7.

2. Literature review

2.1. Two (Multi) -phase stochastic process

Feng et al. [5] proposed a three-phase degradation model based on capacitance degradation of high-voltage pulse capacitors in 2012 and gave a cumulative distribution function of a lifetime. The authors consider that the change point between different phases is fixed. At the same time, since the life distribution is not calculated as a whole, the cumulative distribution function of life is discontinuous or even not monotonically increasing. Yan et al. [30] conducted a similar two-phase study, which has the same shortcomings as the above literature. Besides the above literature, Wang et al. [23] and Wang et al. [24] all consider that the fault will occur in the first phase. Kong et al. [12] study the two-phase Wiener degradation process considering the sharp increase at change points, the expression of degradation quantity includes indicator function. The model is also extended to a multi-phase degradation process. When calculating the reliability function, the change point is considered a random variable. Wen et al. [28] proposed a multiple change-point Wiener degradation process considering unit heterogeneity. Updating the parameters of the posterior function through Bayesian methods. Because the residual life is calculated by phase iteration, the Monte Carlo simulation method is used while the uncertainty of parameters is considered.

In this paper, a fixed phase-change threshold is considered that divides a degradation path into two phases. When the degradation amount exceeds the threshold, the degradation enters the second phase. Thus the time distribution of the phase-change point can be obtained according to the fixed phase-change threshold distinguish from the case above where the change point is fixed, or the change point is subject to a specific distribution independent of the degradation amount. Because the degradation can reflect the state of the components, it is more reasonable to distinguish the degradation phases by the degradation amount. As for the cumulative distribution function of life, the function curve is continuous in this paper and is not in the form of a piecewise function.

2.2. Two-phase maintenance

Some researchers have adopted a multi-phase maintenance strategy in their research on condition-based maintenance. Ni et al. [17] adopted the adaptive maintenance policy and time-dependent maintenance policy, making maintenance strategies more economical by adopting different maintenance thresholds and inspection intervals. Naderkhani et al. [15] regard the detected state information as the covariate of the proportional risk model. Two different inspection intervals are used in the maintenance framework, and the results show that it has better performance than the fixed inspection interval and the age-based strategy. Naderkhani and Makis [16] modeled the degradation process as a three-state continuous time homogeneous hidden Markov process with two unobservable operating states and one observable fault state. The decision variable includes two sampling intervals and two control thresholds. The optimization objective is to minimize the long-term average maintenance cost per unit time of partially observable degraded systems. Under the framework of this article, the sojourn time in each state follows an exponential distribution. Ponchet et al. [18] assessed a maintenance model for a multi-degradation mode system, the degradation process is considered to be the Gamma process. The degradation mode changes after a certain time point and can be inspected. The adapted rule sets different inspection intervals and preventive maintenance thresholds after the time point and is compared with the global rule. The Numerical examples show that better results can be achieved with the adaptive rule. Yan et al. [29] proposed a non-fixed periodic inspection strategy. Under the multi-phase degradation model proposed in the article, each phase is set with an inspection interval, and in the last phase, two inspection intervals are added by setting a threshold. It is believed that failure will be observed when the failure threshold is exceeded. In addition, a penalty cost is set for not being found in time during state transition. It is believed that failure will occur during all phases before the last phase with little probability.

From the above literature, using different inspection intervals can achieve better results. However, it can also be seen that the maintenance policy is established under various degradation model assumptions, and the maintenance and inspection arrangements will also be slightly different. In this paper, a condition-based maintenance policy with two different inspection intervals for the two-phase Wiener degradation process is proposed.

3. Two-phase Wiener process

In many practical situations, the degradation of components has two different phases. This paper considers the two-phase Wiener process, the degradation rate of the two phases is different, and the degradation path is continuous. Let $X = \{X(t): t \geq 0\}$ be the degradation process. Fig. 1 presents a degradation path following a two-phase Wiener process. A fixed phase-change threshold X_P divides a degradation path into two phases. The system is in the first phase when it is brand new. The degradation increases stochastically and will hit the threshold X_P. After that, the degradation goes into the second phase. The degradation rate in the second phase is different from that of the first phase. Once the degradation value exceeds a critical value D, the system is regarded to fail.

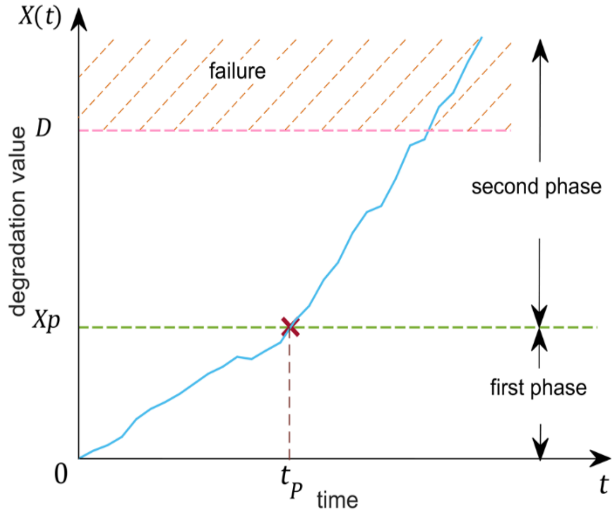

Fig. 1. Two-phase degradation process.

Assume $X(0) = 0$ for any system. The two-phase Wiener process is given as follows:

$$X(t) = \begin{cases} \mu_1 t + \sigma_1 B(t), & 0 < t \leq t_P \\ X_P + \mu_2 (t - t_P) + \sigma_2 B(t - t_P), & t > t_P \end{cases} \quad (1)$$

where t_P is the hitting time for the threshold X_P; μ_1, μ_2 are the drift parameters for the first phase and the second phase, σ_1, σ_2 are the diffusion parameters for the first phase and the second phase; $B(t) \sim N(0, t)$ is the standard Brownian motion.

In each phase, the first passage time to a fixed increment Δx follows the inverse Gaussian distribution, i.e.

$$f_i(\Delta t|\Delta x) = \frac{\Delta x}{\sigma_i \sqrt{\Delta t^3}} \varphi\left(\frac{\Delta x - \mu_i \Delta t}{\sigma_i \sqrt{\Delta t}}\right), i = 1,2 \quad (2)$$

where $\varphi(x)$ is the probability density function of the standard normal distribution.

Then the probability density function of t_P is given by

$$f_1(t_P|X_P) = \frac{X_P}{\sigma_1 \sqrt{t_P{}^3}} \varphi\left(\frac{X_P - \mu_1 t_P}{\sigma_1 \sqrt{t_P}}\right) \quad (3)$$

When the degradation value exceeds D, a failure occurs. The reliability function of the system can be calculated as follows:

$$\begin{aligned} R(t) &= \Pr(T > t) = \Pr(X(t) < D) \\ &= \Pr(X(t) \leq X_P) + \Pr(X_P < X(t) < D) \\ &= \int_t^\infty f_1(u|X_P) du \\ &+ \int_0^t f_1(u|X_P) du \int_{t-u}^\infty f_2(v|D - X_P)\, dv \end{aligned} \quad (4)$$

where T is the failure time. The reliability of the component is the sum of the probability of degradation in phase 1 and the probability of degradation in stage 2 but not exceeding the degradation failure threshold.

The reliability function is the basis of the calculation of the maintenance costs.

4. Policy description

If the degradation path follows the two-phase Wiener process, failure will only appear in the second phase. So at the beginning of the degradation, a less frequent inspection can be adopted to save costs. We consider a two-phase inspection scheme. Inspect the component at time $t_i(i=1,2,...)$. It is assumed the inspection is perfect and non-destructive. If the degradation is in the first phase, the time interval to the next inspection is I_1. If the degradation is in the second phase and a decision is made to leave the system operating, the time interval to the next inspection is I_2. The inspection cost is C_I. A preventive replacement threshold $\tau \in (X_P, D)$ is set in the policy. At an inspection epoch, if the degradation value is less than τ, no replacement is taken; if the degradation value exceeds τ but is less than D ($\tau \le X(t) < D$), preventive replacement is performed; If the degradation value is larger than D, corrective replacement is performed. The replacement time is negligible. So degradation value will change to 0 instantly after replacement. The costs of the preventive replacement and corrective replacement are C_P and C_C, respectively.

The maintenance policy based on the degradation amount is concisely illustrated in Fig 2.

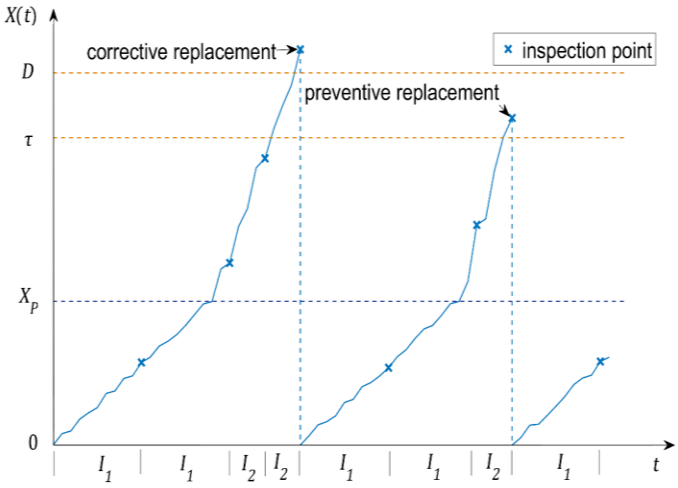

Fig. 2. Description of the proposed maintenance policy.

The objective is to minimize the long-run average cost per unit time by jointly optimizing I_1, I_2, and τ.

5. Formulation and optimization

5.1. Formulation

According to the renewal theory, the long-run cost rate can be expressed by the expected cost rate in one cycle. Cost items include maintenance costs and inspection costs. Then the long-run cost rate can be expressed as

$$\lim_{t\to\infty}\frac{C(t)}{t}=\frac{E(C_T)}{E(T_L)} =\frac{P_1C_P+P_2C_c+E(N)C_I}{E(T_L)} \tag{5}$$

where $C(t)$ is the cost at time t, C_T is the cost in one cycle, T_L is the length of a cycle. P_1 and P_2 are the probability that the renewal cycle ends with preventive replacement or corrective replacement, respectively. N is the inspection times in one cycle.

When $\tau \le X(t) < D$, a preventive replacement will be carried out at inspection point t. Accumulate the probability of implementing preventive maintenance at each inspection point, the probability that the renewal cycle ends with preventive replacement P_1 can be calculated as follows:

$$P_1 = \int_0^\infty f_1(u|X_P)du \iint\limits_{\substack{u+y\le N_1I_1\\ u+y+z>N_1I_1}} f_2(y|\tau-X_P)f_2(z|D-\tau)dydz + \sum_{n_2=1}^{\infty}\int_0^\infty f_1(u|X_P)du\iint\limits_{\varphi_1} f_2(y|\tau-X_P)f_2(z|D-\tau)dydz \tag{6}$$

where the first term in Equation (6) represents the probability of preventive replacement at the time N_1I_1, and the second term represents the probability of preventive replacement at the time $N_1I_1+n_2I_2(n_2=1,2,...)$; N_1 indicates the number of times of inspection with I_1 as the inspection interval, its calculation method will be explained later; φ_1: $\{N_1I_1+(n_2-1)I_2<u+y\le N_1I_1+n_2I_2, u+y+z>N_1I_1+n_2I_2\}$, $u+y$ indicates the time when the degradation amount reach the preventive replacement threshold τ, while $u+y+z$ indicates the time when the degradation amount reach the failure threshold D, φ_1 is equivalent to the condition that $\{X(N_1I_1+(n_2-1)I_2)<\tau$, $\tau\le X(N_1I_1+n_2I_2)<D\}$, representing that no maintenance action was carried out at the previous inspection point, and preventive maintenance was carried out at this inspection point.

When preventive maintenance was not performed at the previous inspection point t_{i-1}, at the same time the degradation amount exceeds the failure threshold D ($X(t_i)\ge D$), then a corrective replacement will be implemented at inspection point t_i. The probability that the renewal cycle ends with corrective replacement P_2 can be calculated as follows:

$$P_2 = \iint\limits_{u+v\leq N_1 I_1} f_1(u|X_P) f_2(v|D-X_P)dudv + \sum_{n_2=1}^{\infty} \int_0^{\infty} f_1(u|X_P)du \iint\limits_{\varphi_2} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz \quad (7)$$

where the first term in Equation (7) represents the probability of corrective replacement at the time $N_1 I_1$, and the second term represents the probability of corrective replacement at the time $N_1 I_1 + n_2 I_2 (n_2 = 1,2,\dots)$; $\varphi_2: \{u + y > N_1 I_1 + (n_2 - 1)I_2, u + y + z \leq N_1 I_1 + n_2 I_2\}$ is equivalent to the condition that $\{X(N_1 I_1 + (n_2 - 1)I_2) < \tau,\ X(N_1 I_1 + n_2 I_2) \geq D\}$.

According to the probability that the cycle ends at a specific time, the expected number of inspections and the length of the renewal cycle can be calculated as follows:

The expected number of inspections in one cycle is

$$E(N) = \int_0^{\infty} N_1 f_1(u|X_P)du \iint\limits_{\substack{u+y\leq N_1 I_1 \\ u+y+z>N_1 I_1}} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz + \iint\limits_{u+v\leq N_1 I_1} N_1 f_1(u|X_P) f_2(v|D-X_P)dudv + \sum_{n_2=1}^{\infty} \int_0^{\infty} (N_1 + n_2) f_1(u|X_P)du \iint\limits_{\varphi_1} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz + \sum_{n_2=1}^{\infty} \int_0^{\infty} (N_1 + n_2) f_1(u|X_P)du \iint\limits_{\varphi_2} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz \quad (8)$$

when replacement is implemented at time $N_1 I_1$, the number of inspections that have been implemented is N_1. The number of inspections when replacement is implemented at time $N_1 I_1 + n_2 I_2$ is $N_1 + n_2$. Equation (8) evolved from Equation (6) -(7) to obtain the expected number of inspections within a cycle.

Multiplied the time and the probability that the cycle ends at a certain time, the expected length of the renewal cycle can be gotten as

$$E(T) = \int_0^{\infty} N_1 I_1 f_1(u|X_P)du \iint\limits_{\substack{u+y\leq N_1 I_1 \\ u+y+z>N_1 I_1}} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz + \iint\limits_{u+v\leq N_1 I_1} N_1 I_1 f_1(u|X_P) f_2(v|D-X_P)dudv + \sum_{n_2=1}^{\infty} \int_0^{\infty} (N_1 I_1 + n_2 I_2) f_1(u|X_P)du \iint\limits_{\varphi_1} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz + \sum_{n_2=1}^{\infty} \int_0^{\infty} (N_1 I_1 + n_2 I_2) f_1(u|X_P)du \iint\limits_{\varphi_2} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz \quad (9)$$

Substituting Equations (6) - (9) into Equation (5) can obtain the expression of the long-run average cost per unit time.

Availability is defined as the proportion of the normal working time of the system's total working time. In this paper, when the degradation amount is less than the failure threshold D, it is considered normal operation; when the degradation amount is greater than the failure threshold D, it is considered abnormal operation time. Therefore, the calculation equation of availability is

$$A_C = \frac{E(T_A)}{E(T)} \quad (10)$$

where

$$E(T_A) = \int_0^{\infty} N_1 I_1 f_1(u|X_P)du \iint\limits_{\substack{u+y\leq N_1 I_1 \\ u+y+z>N_1 I_1}} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz + \iint\limits_{u+v\leq N_1 I_1} (u+v) f_1(u|X_P) f_2(v|D-X_P)dudv + \sum_{n_2=1}^{\infty} \int_0^{\infty} (N_1 I_1 + n_2 I_2) f_1(u|X_P)du \iint\limits_{\varphi_1} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz + \sum_{n_2=1}^{\infty} \int_0^{\infty} (x+y+z) f_1(u|X_P)du \iint\limits_{\varphi_2} f_2(y|\tau - X_P) f_2(z|D-\tau)dydz \quad (11)$$

Replace the time of the inspection point in the case of corrective replacement in Equation (9) with the time of reaching the failure threshold D to obtain Equation (11).

5.2. Optimization

While calculating the long-run average cost per unit of time, there are several problems we have to face. First of all, the integration domain is discrete. It is needed to add up the integral value of multiple domains. At the same time, we need to define the value of N_1. Therefore, in this section, we will first discuss how to calculate the cost equation given above. After that, we need to find a proper optimization solution method to get the optimal solution.

The approximate calculation method proposed in this paper is converting the change point t_P to a constant value, then the calculation of N_1 will be easier. Calculate the cost and cycle when t_P takes different values, and then add them up according to the corresponding probability. Since there are slight differences between the calculation equation in 4.1, we put the approximate calculation method in Appendix A.

As for the maintenance policy we considered, we wish to find a policy that is economical enough. And it is acceptable when a solution is easily obtained and close to the optimal solution. So adopting the intelligence algorithm to find an approximate optimal solution is a good idea. When finding the proper optimization method, Particle Swarm Optimization (PSO) attracts our attention. Particle Swarm Optimization is a global optimization swarm intelligence algorithm inspired by bird flock foraging behavior, first proposed by Kennedy and Eberhart in 1995 [31]. PSO encodes the solution with real numbers, which has high solution efficiency and fast convergence speed. Through the continuous movement of the particles, the approximate optimal solution is finally obtained. Each particle's moving direction and speed are decided by both the optimal solution of its own and the global optimal solution so far. The two optimal solutions represent individual information and group information respectively. The approximate optimal solution is finally found through the generational search.

The process of PSO is shown as follows:

(1)Set the particle swarm optimization algorithm parameters. Parameters include search space, that is, the lower and upper limits of each variable L and U; population size z; learning factors c_1, c_2; stall generations H; iteration accuracy ξ.

(2)Randomly obtain the position x and velocity v of the initial search point, Calculate the fitness value of the initial search point to initialize the optimal position of the individual and the optimal position of the group.

(3) Particle migration

Update the positions of the particles according to Equation (12). The velocity of the particles at the next iteration is related to the original velocity, individual optimal value, and global optimal value.

$$\begin{gathered} x_i^{t+1} = x_i^t + v_i^{t+1} \\ v_i^{t+1} = wv_i^t + c_1 r_1 (p_i^t - x_i^t) + c_2 r_2 (p_g^t - x_i^t) \end{gathered} \tag{12}$$

where x_i^t is the position of the i th particle

v_i^t is the velocity of the i th particle

w is the inertia weight

c_1, c_2 are the learning factors

r_1, r_2 are random numbers uniformly distributed in the interval [0, 1]

p_i^t is the current best position of the ith particle at the tth iteration

p_g^t is the current global best position at the tth iteration

(4) Calculate the fitness value of the updated particles, and update the individual best position and the global best position

(5) Determine whether the stop condition is reached. If the current iteration number reaches the preset maximum number, or the fitness value in H consecutive generations is less than the predetermined convergence accuracy ξ, the iteration is stopped and the optimal solution is output, otherwise, go to step (2).

The flow chart of the Particle Swarm Optimization Algorithm is shown in Fig 3.

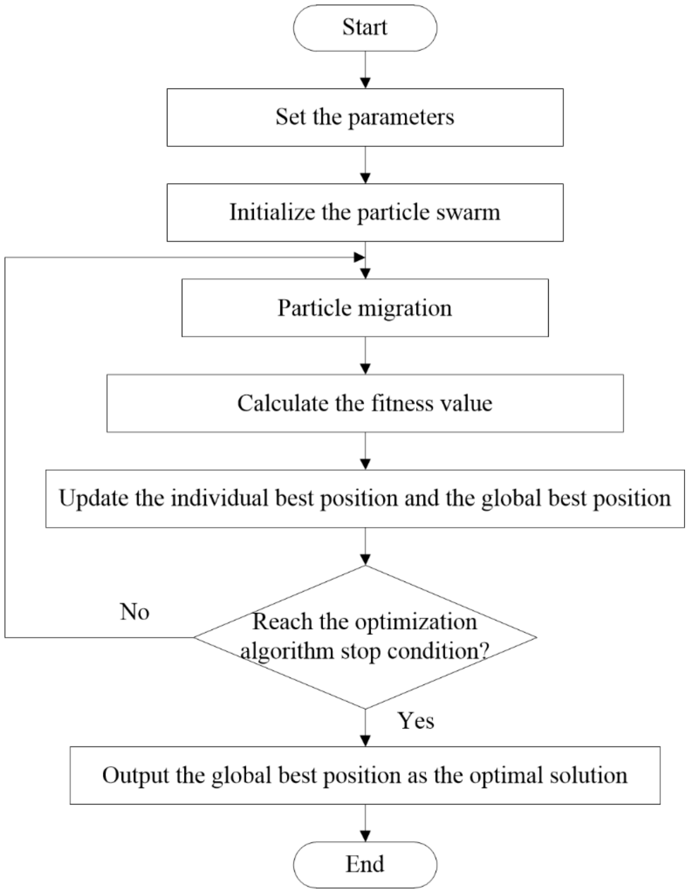

Fig. 3. Particle Swarm Optimization flow chart.

6. Numerical example

6.1. Results and comparison

The feasibility of this two-phase maintenance model is demonstrated based on the degradation data of the fluid coupling mentioned in the literature. Fluid couplings are coupling devices with liquid being the medium. The common failure of the fluid coupling is that the passage is blocked or the oil temperature is too high. The failure of the hydraulic coupling will affect the normal operation of the equipment and result in a loss of efficiency. Therefore, inspection and timely maintenance can avoid system failures which will cause further economic losses. As in the research literature[30], its degradation path follows a two-phase Wiener process, and the parameters are $\mu_1 = 0.2112$, ${\sigma_1}^2 = 0.2084$, $\mu_2 = 0.009$, ${\sigma_2}^2 = 0.0009$, $D = 29.5$mm, $X_P = 15.3$mm. In this part, maintenance cost parameters are set as $C_I = 10$, $C_P = 400$, $C_c = 1000$.

Particleswarm function of MATLAB software is adopted to accomplish the optimization. Related parameters according to the pre-runs are set as particle value's lower limit $L=$ [0.01,0.01,15.3]; particle value's upper limit $U=$[2500,500,29.5]; population size $z=$20. The stop condition is that fitness value improvement is less than $\xi=5\times10^{-5}$ in $H=$15 iterations. Particleswarm function runs 5 times. Each operation obtains the best value of 0.2825. The convergence curve of the optimal fitness value within multiple operations is shown in Fig 4, and the final solution obtained by each operation is shown in Table 1.

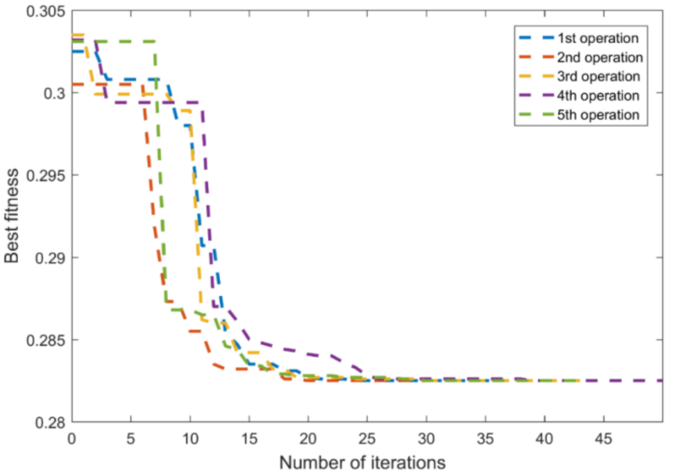

Fig. 4. Convergence curve of operations.

Table 1. The results of each operation.

Operation	I_1	I_2	τ	Iterations	Time/hours	Best Value	Availability
1	1342.5	118.1	27.9	37	2.3	0.2825	0.9998
2	1343.8	117.8	27.9	34	2.1	0.2825	0.9998
3	1343.5	118.4	27.9	43	2.6	0.2825	0.9998
4	1343.6	117.9	27.9	50	3.1	0.2825	0.9998
5	1343.5	118.1	27.9	43	2.6	0.2825	0.9998
average	1343.4	118.1	27.9	41.4	2.5	0.2825	0.9998

According to the multiple operations, it can be presumed that the particle swarm optimization algorithm can be used to obtain the approximate optimal solution stably. As for the first operation, the long-run average cost per unit time is 0.2825, availability is 0.9998, time intervals to the next inspection in the first and the second phase are 1342.5,118.1 respectively, and the preventive-replacement threshold τ is 27.9mm. The average number of iterations over 5 runs is 41.4. It takes nearly two and a half hours when using the Intel Core i5-9400F CPU @2.9GHz. The final solution and the needed time are satisfactory.

The two-phase maintenance policy is compared with the maintenance policy of a single inspection cycle. The optimization results are shown in Table 2. The long-run average cost per unit time is 0.3135 when a fixed inspection interval in the whole cycle is adopted, and the availability is 0.9996. In this situation, the time interval to the next inspection is 476.6 and the preventive-replacement threshold is 24.7. It can be seen that the two-phase maintenance policy carries out infrequent inspections in the initial phase, while frequent inspections in the later phase can save costs. Under the cost parameters set in this manuscript, the cost of two-phase inspection condition-based maintenance can be reduced by 10% and the availability increased by 0.02% compared with the periodic inspection condition-based maintenance.

Table 2. The results of fixed inspection interval.

I	τ	Best Value	Availability
476.6	24.7	0.3135	0.9996

Next, we will compare the results obtained with the traditional periodic maintenance model which was usually called time-based maintenance(TBM). In the case of time-based maintenance, components will be replaced periodically. The average maintenance cost per unit time under TBM is as follows:

$$\lim_{t\to\infty}\frac{C(t)}{t}=\frac{E(C_T)}{E(T_r)}=\frac{R(T_r)C_P+\overline{R(T_r)}C_c+C_I}{T_r} \tag{13}$$

where T_r is the replacement interval under the time-based maintenance. $\overline{R(t)}=F(t)=1-R(t)$ is the CDF of the failure time. An inspection will be performed at the replacement point T_r to determine whether a preventive or corrective replacement is required.

The availability under TBM is as follows:

$$A_C=\frac{\int_0^{T_r}tF'(t)dt}{T_r\int_0^{T_r}F'(t)dt} \tag{14}$$

Similarly, the maintenance cost per unit time is taken as the fitness value, and the replacement cycle is taken as the decision variable, using the PSO algorithm to solve this problem. The results are shown in Table 3. The optimal replacement interval is 1401.4, the average cost per unit time is 0.303, and the availability is 0.9871. The two-phase maintenance in this paper can save 7% on cost and increase by 1.3% availability compared with time-based maintenance.

As can be seen from the above, adopting the two-phase maintenance policy described in this article has advantages in cost saving and improving availability. The cost and availability comparison of the three policies is shown in Table 4. The two-phase maintenance policy reduces costs by reducing inspection frequency in the first phase and timely grasping the equipment degradation state in the second phase to arrange reasonable maintenance to avoid equipment failure and generate more costs.

Table 3. The results of TBM.

T_r	Best Value	Availability
1401.4	0.303	0.9871

Table 4. Comparison between different policies.

Policy	Cost	Availability
Two-phase maintenance	0.2825	0.9998
Periodic CBM	0.3135	0.9996
TBM	0.303	0.9871

6.2 Sensitivity analysis

To further verify the correctness of the model and explore the impact of each cost parameter on the maintenance decision, a sensitivity analysis is conducted in this section.

Set the preventive replacement costs C_P and the corrective replacement costs C_C as 400, 1000 respectively, and the inspection cost C_I as 2, 10, and 50 in turn. Then run the optimizer to see the influence on the policy. The final result is shown in Table 5 as follows.

Table 5. Optimization results of different inspection costs.

C_I	C_P	C_C	I_1	I_2	τ	Average cost
2	400	1000	1307.7	51.8	28.6	0.2638
10	400	1000	1342.5	118.1	27.9	0.2825
50	400	1000	1390.1	242	26.6	0.3272

As the inspection cost increases, the time intervals to the next inspection in the first phase will slowly increase, while the time intervals to the next inspection in the second phase will significantly increase and the preventive-replacement threshold will decrease.

Then let the inspection cost C_I remain the same, and change the ratio of C_C/C_P. The final result is shown in Table 6 as follows.

Table 6. Optimization results of different C_C/C_P.

C_I	C_P	C_C	I_1	I_2	τ	Average cost
10	400	1000	1342.5	118.1	27.9	0.2825
10	400	4000	1271.9	116.4	27.7	0.2892
10	400	10000	1240.1	117	27.5	0.2923

When the ratio of C_C/C_P increases, it can be seen that the time intervals to the next inspection in the first phase will slowly decrease, the time intervals to the next inspection in the second phase will slightly fluctuate, and the preventive-replacement threshold will decrease.

From the above analysis, it can be seen that when the cost of inspection increases, the frequency of the inspection in the second phase will become lower, and the preventive-replacement threshold will decrease, which means equipment will be replaced in advance to accommodate less frequent inspection. Likewise, when the ratio of C_C/C_P increases, it also causes equipment to be replaced earlier.

7. Conclusions

This paper proposes a two-phase degradation component maintenance policy for the two-phase degradation phenomenon. For the two-phase Wiener process, the established maintenance policy model optimizes the two-phase inspection interval and the preventive-replacement threshold at the same time. The optimization goal is to minimize the long-run average cost per unit time. The particle swarm optimization algorithm is used to solve the maintenance policy model. The two-phase degradation data of a hydraulic coupler demonstrates the feasibility of the maintenance policy model. Compared with the single inspection interval policy and TBM policy, it can save costs and increase availability. Under the parameter settings in this article, compared with a single fixed cycle inspection condition-based maintenance policy, it can reduce costs by 10% and improve availability by 0.02%. Compared with the time-based maintenance policy, it can save 7% in cost and improve availability by 1.3%. Therefore, in practice, for situations where the degradation process involves two phases and the component state can be expressed using a single feature quantity, the two-phase maintenance policy described in this article can be adopted to reduce costs and improve availability. The Particle Swarm Optimization algorithm can effectively solve the maintenance model. And sensitive analysis shows when the cost of inspection increases, the frequency of the inspection will become lower, and the equipment will be replaced in advance. Likewise, when the ratio of C_C/C_P increases, it also causes equipment to be replaced earlier.

Subsequent research can be extended based on this article. For condition-based maintenance, it is possible in further to collect more data information and analyze the data through intelligent methods such as artificial neural networks. In terms of maintenance policy, more flexible arrangements can also be attempted. For example, a more flexible inspection policy or taking into account the impact of different maintenance actions on equipment state.

Appendix

$$\lim_{t\to\infty}\frac{C(t)}{t}\approx\frac{P_1'C_P+P_2'C_c+E(N)'C_I}{E(T_L)'} \tag{15}$$

where P_1', P_2', $E(N)'$, $E(T_L)'$ can be expressed as follows:

$$\begin{aligned}P_1'=&\sum_{i=1}^{M}f_1(u_i|X_P)\,\Delta u\iint\limits_{\substack{u_i+y\le N_1^iI_1\\u_i+y+z>N_1^iI_1}}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\\&+\sum_{i=1}^{\mathrm{M}}f_1(u_i|X_P)\Delta u\sum_{n_2=1}^{\infty}\iint\limits_{\varphi_1'}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\end{aligned} \tag{16}$$

M is a number that is large enough for the approximate calculation.$u_1=\delta, u_{i+1}-u_i=\Delta u, \delta, \Delta u$ are small numbers selected according to the calculation accuracy.$N_1^i=\left\lceil\frac{u_i}{I_1}\right\rceil$,.$\lceil a\rceil$ means rounding up a, φ_1': $\{N_1^iI_1+(n_2-1)I_2<u_i+y\le N_1^iI_1+n_2I_2, u_i+y+z>N_1^iI_1+n_2I_2\}$

$$\begin{aligned}P_2'=&\sum_{i=1}^{\mathrm{M}}f_1(u_i|X_P)\,\Delta u\int_0^{N_1^iI_1-u_i}f_2(v|D-X_P)\,dv\\&+\sum_{i=1}^{\mathrm{M}}f_1(u_i|X_P)\Delta u\sum_{n_2=1}^{\infty}\iint\limits_{\varphi_2'}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\end{aligned} \tag{17}$$

where φ_2': $\{u_i+y>N_1^iI_1+(n_2-1)I_2, u_i+y+z\le N_1^iI_1+n_2I_2\}$

$$\begin{aligned}E(N)'=&\sum_{i=1}^{\mathrm{M}}N_1^if_1(u_i|X_P)\,\Delta u\iint\limits_{\substack{u_i+y\le N_1^iI_1\\u_i+y+z>N_1^iI_1}}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\\&+\sum_{i=1}^{\mathrm{M}}N_1^if_1(u_i|X_P)\,\Delta u\int_0^{N_1^iI_1-u_i}f_2(v|D-X_P)\,dv\\&+\sum_{i=1}^{\mathrm{M}}f_1(u_i|X_P)\Delta u\sum_{n_2=1}^{\infty}\left(N_1^i+n_2\right)\iint\limits_{\varphi_1'}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\\&+\sum_{i=1}^{\mathrm{M}}f_1(u_i|X_P)\Delta u\sum_{n_2=1}^{\infty}\left(N_1^i+n_2\right)\iint\limits_{\varphi_2'}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\end{aligned} \tag{18}$$

$$\begin{aligned}E(T)'=&\sum_{i=1}^{\mathrm{M}}N_1^iI_1f_1(u_i|X_P)\,\Delta u\iint\limits_{\substack{u_i+y\le N_1'I_1\\u_i+y+z>N_1'I_1}}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\\&+\sum_{i=1}^{\mathrm{M}}N_1^iI_1f_1(u_i|X_P)\,\Delta x\int_0^{N_1^iI_1-u_i}f_2(v|D-X_P)\,dv\\&+\sum_{i=1}^{\mathrm{M}}f_1(u_i|X_P)\Delta u\sum_{n_2=1}^{\infty}\left(N_1^iI_1+n_2I_2\right)\iint\limits_{\varphi_1'}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\\&+\sum_{i=1}^{\mathrm{M}}f_1(u_i|X_P)\Delta u\sum_{n_2=1}^{\infty}\left(N_1^iI_1+n_2I_2\right)\iint\limits_{\varphi_2'}f_2(y|\tau-X_P)f_2(z|D-\tau)dydz\end{aligned} \tag{19}$$

Acknowledgements

This work was supported by the National Science and Technology Major Project of the Ministry of Science and Technology of China, Grant/Award Number: 2015ZX04003002; Jilin Province Major Science and Technology Projects, Grant/Award Number: 20220301015GX.

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