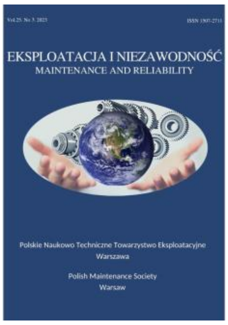

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An adaptive method based on PC-Kriging for system reliability analysis of truss structures

Indexed by: Web of Science Group

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Highlights

- Global metamodel quickly estimates the failure probability of the truss with sufficient accuracy.
- Representative failure modes are obtained to deal with redundant failure members.
- Local metamodels are constructed to calculate the system reliability index of the truss.
- An adaptive method performs reliability analysis for truss structures with high accuracy and efficiency.

Abstract

In practice, a truss consists of a large number of members which makes it a complex system. This leads to difficulties to estimate the system reliability due to computational costs. An adaptive method is thereby proposed to deal with this issue. It constructs a global metamodel to quickly estimate the rough reliability index of a truss. According to the estimated reliability index, the differential evolution algorithm is performed to generate more samples located in an expanded domain so that more representative failure modes can be identified. Combined with AK-SYSi, local metamodels of representative failure mods are built, and updated through active learning. When the convergence criterion is satisfied, the results of system reliability analysis can be obtained. Eventually, two examples of truss structures are studied to illustrate the superiority of the proposed method in balancing accuracy and efficiency. The results indicate that the proposed method makes a good balance between accuracy and efficiency when it is applied to analyze the system reliability of the truss.

Keywords
trade-off, truss structures, system reliability analysis, global and local metamodel, representative failure modes.

1. Introduction

A truss structure is usually a complex system due to its numerous members. Considering its failure modes to perform reliability evaluation is especially of great significance as this has better accuracy than the estimation in terms of component-level reliability. The failure probability of the structure at the component level can be defined as [42]:

$$P_f = Prob(g(X)\leqslant 0) = \int_{g(x)\leqslant 0} f_X(x)dx \qquad (1)$$

where $f_X(x)$ is the joint probability density function (PDF) of the vector

$X = [x_1,x_2,\dots,x_n]^T$ and $g(X)$ denotes the performance function. However, it is difficult to calculate the failure probability directly according to Eq. (1) due to the multi-dimensional integral it has. Thus, numerous approaches have been proposed to deal with this issue. One of the simulation-based methods called Monte Carlo Simulation (MCS) [16; 30] is regarded as the most accurate method to estimate the failure probability.

Although MCS can estimate the failure probability accurately, its large computational costs prompt researchers to make efforts to investigate more efficient methods. Many variance-reduction techniques were put forward such as subset simulation (SS) [36; 44], and importance sampling (IS) [9].

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These techniques can decrease the number of samples to estimate failure probability, but they have a limited degree of enhancement of efficiency. To further improve computational efficiency, numerous metamodels have been developed to approximately replace the real performance function. For example, polynomial chaos expansion (PCE) [2], Kriging [12; 21; 22], deep neural networks(DNN) [18] and support vector machines [7] are commonly implemented in reliability analysis. Besides, many novel metamodels also were proposed in [3; 19; 20; 27; 28; 29] to solve the issues of microwave components such as the nonlinearity of electrical characteristics and the curse of dimensionality. To improve the efficiency of electromagnetic (EM) simulations, different surrogates were proposed such as feature-based regression surrogates [25], response feature surrogates [26], and other novel surrogates [23; 24; 37; 38; 39]. When only metamodels are applied to analyze the practical structures, there are still high computational costs. Therefore, metamodels combined with variance-reduction techniques were proposed continuously in recent years [5; 10; 13; 31; 32; 35; 40; 41; 48; 50; 52]. Echard et al proposed an AK-MCS method that combined the Kriging model and MCS and adopted the U learning function as well as the corresponding convergence criterion to perform an active learning algorithm to update the surrogate model so that the failure probability can be estimated efficiently and accurately. Based on AK-MCS, Sun et al put forward a new learning function called the least improvement function (LIF) to improve the accuracy of the estimated failure probability and developed a new method to deal with more complicated issues such as nonlinear performance function and high dimensional practical problem. Nevertheless, when the failure probability is very small, a large sample population is still needed since AK-MCS and other methods generate samples by MCS. To overcome this defect, Kriging models combined with other variance-reduction methods were proposed [13; 32; 41; 48]. AK-SS combined the Kriging model and SS [13] was developed to estimate the small failure probability. Xu et al developed a novel method named AK-MSS for the estimation of small failure probabilities. This approach made a sequence for the samples based on the distances between the samples and the original in the standard normal space to select new samples in the next level, rather than based on the predicted values of performance function in SS. Both AK-SS and AK-MSS made contributions to decrease the computational efforts of small failure probabilities.

The above methods can estimate the component-level reliability, but they are hard to be applied to estimate the system reliability of a structure directly. Research on system reliability based on sampling-based methods also has been developed [6; 11; 46; 49]. For example, Fauriat and Gayton proposed AK-SYS [11]] to conduct system reliability analysis. For AK-SYS, local metamodels were built for corresponding failure modes, and the failure probability was estimated according to the principle of a series system or parallel system. Three strategies were involved to proceed with the active learning process, and the local metamodel with the greatest influence on the failure will be updated which is determined as the strategy of AK-SYS. This method considers multiple failure modes and has high accuracy and efficiency for the estimation of system failure probability. Meanwhile, an improved method called AK-SYSi based on AK-SYS was proposed by Yun et al [49]. It intends to identify the correct failure mode to be updated. Hence, a refined U learning function was defined and applied to some examples that verify the effectiveness of AK-SYSi.

However, these methods of system reliability analysis are based on the failure modes that have been identified. For a truss, the failure modes as well as their performance functions are usually unknown and the reliability analysis usually takes a large number of computational costs to keep a high accuracy. Therefore, we intend to develop a general framework including the identification of failure modes to estimate the system reliability of the truss with a good trade-off between accuracy and efficiency. Xu [47] proposed a two-stage method to analyze the system reliability, which estimates the failure probability of a structure through the global metamodel and local metamodels. Inspired by this two-stage method, a novel adaptive method is proposed to estimate the system reliability of a truss structure and deal with the computational challenges. It firstly estimates a rough reliability index of a truss by the global metamodel, and according to the geometric meaning of the reliability index, the DE algorithm is conducted to expand the sample domain to identify more failure modes. Then the shortest failure modes are selected as the representative failure modes to represent the longer failure modes that include them. The last stage is constructing the local metamodel for each representative failure mode to estimate the failure probability and reliability index of the truss and its representative failure modes.

This paper is organized as follows. Section 2 provides the AK-SYSi method to illustrate the refined U learning function briefly. Section 3 introduces the proposed method for the system reliability analysis of the truss. It mainly contains the construction of the global metamodel, the identification of representative failure modes, and the construction of the local metamodel for system reliability estimation. Besides, two truss structures are discussed in Section 4 to validate the proposed method. Eventually, the conclusion is summarized in Section 5.

2. Review of basic theory

2.1. AK-SYSi method

System reliability evaluation is a complicated issue as it has to consider many factors. It is effective to consider the structure as a series system or parallel system to estimate the failure probability. Consider E_i to be a failure event with the failure domain $g_i(X) < 0$, where $g_i(\cdot)$ is the performance function of the *i*-th failure event. Thus, for the series system, the failure probabilities can be expressed as Eq. (2), where the I_F^{series} is defined as Eq. (3).

$$P_f^{series} = Prob\left(\bigcup_{i=1}^{n} E_i\right) = Prob\left(\bigcup_{i=1}^{n} g_i(X) < 0\right)$$

$$= E\left(I_F^{series}\right) \approx \frac{1}{N}\sum_{i=1}^{N} I_F^{series}(x_i) \tag{2}$$

$$I_F^{series} = \begin{cases} 1, \min\limits_{i=1,\ldots,n} g_i(X) < 0 \\ 0, \min\limits_{i=1,\ldots,n} g_i(X) \geqslant 0 \end{cases} \tag{3}$$

And for the parallel system, its failure probabilities can be expressed as Eq. (4), where the $I_F^{parallel}$ is defined as Eq. (5).

$$P_f^{paralle} = Prob\left(\bigcap_{i=1}^{n} E_i\right) = Prob\left(\bigcap_{i=1}^{n} g_i < 0\right)$$

$$= E\left(I_F^{parallel}\right) \approx \frac{1}{N}\sum_{i=1}^{N} I_F^{parallel}(x_i) \tag{4}$$

$$I_F^{parallel} = \begin{cases} 1, \max\limits_{i=1,\ldots,n} g_i(X) < 0 \\ 0, \max\limits_{i=1,\ldots,n} g_i(X) \geqslant 0 \end{cases} \tag{5}$$

The AK-SYSi is an active learning method to estimate the failure probability, so the learning function plays a significant role. In AK-SYS, three strategies were proposed to estimate the system probability of failure. The composite criterion approach (#3) was determined to be applied to analyze system reliability due to its efficiency. However, the third strategy has some deficiencies in the identification of the minimum failure mode or the maximum failure mode. They might be misidentified sometimes. Considering improving the defects AK-SYS has, the AK-SYSi refined the learning function adopted in the AK-SYS method. For the series and parallel system, the refined U learning function is expressed as Eq. (6) and Eq. (7) respectively.

$$U(X) = \begin{cases} \frac{|\hat{g}_\Theta(X)|}{\sigma_{\hat{g}_\Theta(X)}} = \min\limits_{i=1,\ldots,k} \frac{|\hat{g}_i(X)|}{\sigma_{\hat{g}_i(X)}} \hat{g}_i(X) > 0 \forall i = 1,\ldots,k \Theta = arg \min\limits_{i=1,\ldots,k} \frac{|\hat{g}_i(X)|}{\sigma_{\hat{g}_i(X)}} \\ \frac{|\hat{g}_\Theta(X)|}{\sigma_{\hat{g}_\Theta(X)}} = \max\limits_{i=\hat{1},\ldots,\hat{k}} \frac{|\hat{g}_i(X)|}{\sigma_{\hat{g}_i(X)}} \hat{g}_i(X) \leqslant 0 \exists i = 1,\ldots,k \Theta = arg \max\limits_{i=\hat{1},\ldots,\hat{k}} \frac{|\hat{g}_i(X)|}{\sigma_{\hat{g}_i(X)}} \end{cases} \tag{6}$$

$$U(X) = \begin{cases} \frac{|\hat{g}_\Theta(X)|}{\sigma_{\hat{g}_\Theta(X)}} = \min\limits_{i=1,\ldots,k} \frac{|\hat{g}_i(X)|}{\sigma_{\hat{g}_i(X)}} \hat{g}_i(X) \leqslant 0 \forall i = 1,\ldots,k \Theta = arg \min\limits_{i=1,\ldots,k} \frac{|\hat{g}_i(X)|}{\sigma_{\hat{g}_i(X)}} \\ \frac{|\hat{g}_\Theta(X)|}{\sigma_{\hat{g}_\Theta(X)}} = \max\limits_{i=\hat{1},\ldots,\hat{k}} \frac{|\hat{g}_i(X)|}{\sigma_{\hat{g}_i(X)}} \hat{g}_i(X) > 0 \exists i = 1,\ldots,k \Theta = arg \max\limits_{i=\hat{1},\ldots,\hat{k}} \frac{|\hat{g}_i(X)|}{\sigma_{\hat{g}_i(X)}} \end{cases} \tag{7}$$

The refined U learning function in AK-SYSi considers the sign of the predicted value of the performance function so that it can decrease the misidentified probability of minimum or maximum failure modes to proceed with active learning. Once there is one mode in the failure domain, it leads to the failure of the system for the series system, while all modes are located in the failure domain, it results in the failure of the parallel system. For the series system, at one sample, if the predicted values of all modes are larger than zero, the mode Θ with minimum learning function value is determined, otherwise, the value of the learning function is determined as the maximum value. For the parallel system, if the predicted values of all modes are smaller than or equal to zero, the mode Θ with minimum learning function value is determined, otherwise, select the maximum value of the learning function. Meanwhile, the indicator Θ of failure modes is also determined.

2.2. PC-Kriging model

The Polynomial-Chaos Kriging (PC-Kriging) model is the combination of polynomial chaos expansions and the Kriging model. It can be regarded as the universal Kriging with a specific form of the trend. The universal Kriging model comprises of a regression model and stochastic process, which is described as:

$$G(x) = F(x,\beta) + z(x) = f^T(x)\beta + \sigma^2 z(x) \tag{8}$$

where the first term is a realization of a regression function, σ^2 is the variance of the Gaussian process, and $z(x)$ denotes a Gaussian stochastic process with zero-mean. The covariance between x_i and x_j is defined as:

$$Cov\left(z(x_i),z(x_j)\right) = \sigma^2 R(x_i,x_j;\theta) \tag{9}$$

where σ^2 is the process variance and $R(x_i,x_j;\theta)$ is the correlation function between samples x_i and x_j with hyperparameter θ.

With regards to the correlation function, the Matern-5/2 correlation function is adopted and expressed as:

$$R_{M5/2}(x_i,x_j;\theta) = \prod_{k=1}^{M}\left(1+\sqrt{5}\frac{|x_i-x_j|}{\theta_k}+\frac{5|x_i-x_j|^2}{3\theta_k^2}\right)\exp\left(-\sqrt{5}\frac{|x_i-x_j|}{\theta_k}\right) \tag{10}$$

where $\theta = \{\theta_k, k=1,2,\dots,M\}$. Following, the estimates of β and σ^2 can be expressed as:

$$\hat{\beta} = \beta(\theta) = \left(F^T R_\theta^{-1} F\right)^{-1} F^T R_\theta^{-1} Y \tag{11}$$

$$\hat{\sigma}^2 = \sigma^2(\theta) = \frac{1}{N}(Y-F\beta)^T R_\theta^{-1}(Y-F\beta) \tag{12}$$

where R_θ denotes the correlation matrix with the term $R_{\theta,ij} = R_{M5/2}(x_i,x_j;\theta)$, $i,j = 1,2,\dots,N$. Since $\hat{\beta}$ and $\hat{\sigma}^2$ is related to θ, the θ needs to be estimated. The maximum likelihood estimation is often utilized:

$$\theta = \underset{\theta}{\operatorname{argmin}}(\det \quad R_\theta)^{\frac{1}{N}}\hat{\sigma}^2 \tag{13}$$

Finally, the response value of the sample can be predicted. For an unknown point x with Gaussian distribution, its mean $\hat{\mu}(x)$ and variance $\hat{\sigma}^2(x)$ are respectively given as:

$$\hat{\mu}(x) = f(x)^T\beta + r(x)^T R_\theta^{-1}\left(Y-F\hat{\beta}\right) \tag{14}$$

$$\hat{\sigma}^2(x) = \hat{\sigma}^2\left(1-r(x)^T R_\theta^{-1} r(x) + u(x)^T\left(F^{T^T} R_\theta^{-1} F\right)^{-1} u(x)\right) \tag{15}$$

where $r(x) = \left[R_{m5/2}(x,x_1),\dots,R_{m5/2}(x,x_N)\right]^T$ and $u(x) = F^T R_\theta^{-1} r(x) - f(x)$.

Different from the Kriging model, PC-Kriging replaces the trend of the universal Kriging model with the polynomial chaos expansions. More details of PCE refer to [34]. Hence, the PC-Kriging is given as:

$$G(x) = \sum_{\tau\in\Theta}\psi_\tau(x)\beta_\tau + z(x) \tag{16}$$

where $\psi_\tau(x)$ are the multivariate orthonormal polynomials, β_τ denotes the corresponding coefficients. Two ways, namely the sequential and optimal approaches, are often utilized to determine the set of orthonormal polynomials in PC-Kriging. The former is adopted in this paper. To conveniently construct the PC-Kriging model, the UQLab toolbox [33] is used.

2.3. Subset simulation

Subset simulation (SS) is a variance-reduced technique for the estimation of the small failure probability. It is assumed that F is the final failure event, the failure can be defined as a product of sequential intermediate failure events $F_1 \supset F_2 \supset \cdots \supset F_m = F$. So, the failure probability is defined as:

$$P(F) = P(F_m) = P(F_m|F_{m-1})P(F_{m-1}) = P(F_1)\prod_{i=1}^{m-1}P(F_{i+1}|F_i) \tag{17}$$

During the implementation process of the SS, the modified M-H algorithm with delayed rejection is adopted to generate conditional samples of the next SS level [53]. The procedure of SS is as follows.

(1) Ninitial samples are generated based on the distribution of each random variable.

(2) Obtain the performance function values of Nsamples, and rank them in ascending order.

(3) Set the conditional probability p_0 that is suggested as 0.1 [1], and the first p_0N samples are selected to generate the next $(1-p_0)N$ samples by the modified M-H algorithm with delayed rejection.

(4) Repeat (2)(3) until the value of the p_0N-th sample is smaller than zero.

(5) Calculate the failure probability via Eq. (17).

3. An adaptive method for system reliability analysis of truss structures

Herein, a method that intends to identify the failure modes and estimate the system failure probability for the truss structure is proposed. The global metamodel is constructed to approximately estimate the system reliability index of the truss firstly. Then based on the system reliability index estimated by the global metamodel, the dominant failure domain is expanded by the differential evolution algorithm so that the representative failure modes are determined. Finally, the local metamodel for each representative failure mode is built to estimate the system failure probability in terms of the principle of AK-SYSi.

3.1.Construction of the global metamodel

To quickly estimate the system reliability index of the truss, a global metamodel is first constructed to estimate the rough reliability index so that the rough failure domain can be identified. Herein, a method called PCK-USS is proposed to quickly construct the global metamodel. It is based on the framework of PC-Kriging and SS. Meanwhile, three acceleration strategies are introduced to assure computational efficiency.

3.1.1. Load factor of the performance function

The aim of constructing the global metamodel is to estimate the system reliability roughly. It is of great significance to find a proper index to reflect the global failure of the structure for the construction of performance function. Zhao and Ono [51] proposed a method to establish the performance function via the load factor. And it is appropriate to construct the performance function of truss structures as well. The performance function is defined as:

$$G(X) = \lambda(M,P) - 1 \tag{18}$$

Where λ denotes the load factor that is the function of M and P, M represents the capacities of members of the truss, and P represents the loads. The key to constructing the performance function focuses on the load factor. Once the load factor is determined, the performance function can be built. For a truss, its ultimate load can be obtained through the limit analysis of the truss in ANSYS, and calculate further its load factor. It is assumed that the loads $(P_1,P_2,\ldots,P_n)$ are applied to a truss and the ultimate load is F_{lim}. The loads $(P_2,\ldots,P_n)$ can be given by P_1 shown in Eq. (9).

$$(P_1,P_2,\ldots,P_n) = P_1 \times \left(1,\frac{P_2}{P_1},\ldots,\frac{P_n}{P_1}\right) = P_1 \times (1,\xi_2,\ldots,\xi_n) \tag{19}$$

A very large value of P_1 is denoted as F that can cause the failure of the truss, and the loads to conduct the limit analysis is $(F,F\xi_2,\ldots,F\xi_n)$. Set the iteration time T, and start limit analysis in ANSYS. The ultimate load F_{lim} can be obtained. The time to reach the limit state is t, so Eq. (10) can be built according to the principle of solving the ultimate load in ANSYS.

$$\frac{F_{\text{lim}}}{F} = \frac{t}{T} \tag{20}$$

Then the ultimate load is deduced as:

$$F_{\text{lim}} = \frac{t}{T} \times F \tag{21}$$

After obtaining the ultimate load, the load factor is expressed as:

$$\lambda = \frac{F_{\text{lim}}}{P_1} \tag{22}$$

Therefore, the performance function based on the load factor can be constructed. When λ is smaller than 1, the loads $(P_1,P_2,\ldots,P_n)$ will lead to the failure of the truss as the ultimate load is not greater than P_1. If λ is greater than 1, the truss is safe as the P_1 does not reach the ultimate load.

3.1.2. Acceleration strategies of active learning

The first acceleration strategy is the uniform design (UD). It is an experimental design method that generates samples as uniformly as possible. The UD includes a uniform design table and a usage table. $U_n(q^t)$ denotes the uniform design table, where "U" denotes the UD, "n" represents the number of experiments, "q" denotes the number of factor levels, and "t" is defined as the maximum number of columns of the table [14]. For the usage table, it is provided to determine samples that have the minimum error. To obtain the UD samples conveniently, programming of UD has been written via MATLAB, and the deviation uniformity measure of these UD samples is according to the centered L2-discrepancy (CD2) [32]. In this paper, UD is used to generate the initial DoE to construct the initial metamodel.

The K-means++ clustering is the second acceleration strategy that is considered as an improvement of the K-means clustering. K-means++ clustering is different from K-means in the selection of the initial clustering centers. K-means++ clustering only selects the one initial clustering center randomly rather than selects the initial k clustering centers randomly like K-means clustering. Then it determines the next clustering centers which are farther from the present centers. The procedure of this algorithm can be concluded as follows.

(1) Select one center u_1 randomly from the whole sample points;

(2) Calculate the distance from each rest sample point to the center;

(3) Determine a new center in terms of a weighted probability distribution;

(4) Repeat (2)(3) until obtaining k initial centers;

(5) Search for k clustering centers according to the K-means algorithm.

More details of the K-means clustering algorithm are in [43].

The last acceleration strategy is a convergence criterion based on the failure probability. As for the U learning function, it can rapidly converge to an accurate failure probability in terms of the convergence criterion $\min(U(x)) > 2$. However, the value of $\min(U(x))$ is hard to reach the threshold for more complex performance function or more random variables even though the estimated failure probability is already close to the actual value. The convergence criterion $\min(U(x)) > 2$ of U learning function may not be appropriate at this time. Hence, Cui and Ghosn [6] proposed a more efficient and robust convergence criterion based on the estimated failure probability. When the failure probabilities of the last three iterations reach the convergence criterion, the active learning process will be terminated. In each SS level, the convergence criterion is given as:

$$\left|\frac{P_f^s - P_f^{s-1}}{P_f^s}\right| \& \left|\frac{P_f^s - P_f^{s-2}}{P_f^s}\right| < \varepsilon \tag{23}$$

where the P_f^s, P_f^{s-1} and P_f^{s-2} represent the estimated failure probabilities of three successive iterations respectively. The error ε is determined as 0.01.

3.1.3. Global metamodel construction

To estimate the global failure probability, a method consisting of the PC-Kriging model, subset simulation, and the acceleration strategies of convergence is proposed. Combined with the performance function of the load factor λ, the process to construct the global metamodel is concluded as six steps, as shown in Fig. 1.

Step 1: Generate N samples via MCS in the first SS level and the initial design of experiments (DoE) by UD in the standard normal space. Here, the initial DoE to construct the initial PC-Kriging consists of sample points generated by UD. For samples generated by UD, they are located in the domain $[\mu - k\sigma, \mu + k\sigma]$. The number of sample points of the initial DoE is chosen as 31 in this paper.

Step 2: Construct or update the PC-Kriging model. Input the initial DoE or updated DoE that includes newly added samples. The values of the performance function of DoE can be obtained based on the limit analysis as well as the corresponding computation introduced in subsection 3.1.1, and the global PC-Kriging model can be constructed by the UQlab toolbox.

Step 3: Compute the predicted values of the performance functions of N samples through the constructed PC-Kriging model, and estimate the failure probability in this SS level.

Step 4: Judge whether the estimated failure probability meets the convergence criterion. If it does not reach the convergence criterion, enter step 5. Otherwise, determine the value of the $p_0 N$-th sample, if it is smaller than 0, compute the final failure probability, i.e. step 6, if not, enter the next SS level, and repeat step 3 to step 4.

Step 5: Determine the initial sample point with minimum learning function value for K-means++ clustering, and further obtain K samples to be added to DoE. The clustering number K is determined as 2. Following consider the sample which has minimum learning function value in each subdomain respectively to be added to the DoE. If the performance function values of K samples are half positive and half negative, the K samples are determined to be added to the DoE, otherwise, determine one sample with the minimum value of learning function in the whole sample domain to be added to the DoE. According to the above operation, the design of experiments would be updated. When the updated DoE is determined, return to step 2 to build a new PC-Kriging model.

Step 6: Compute the global failure probability.

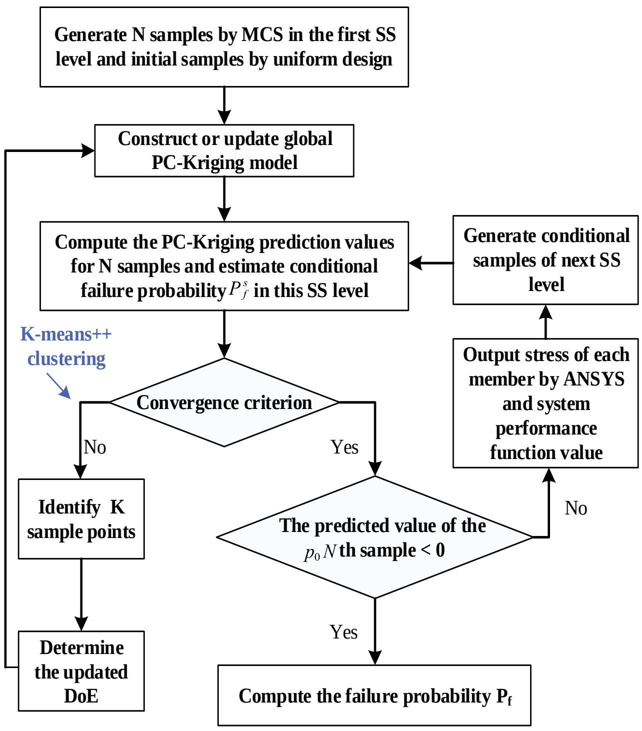

Fig. 1. The flowchart of the method to construct the global metamodel.

3.2. Identification of representative failure modes

The reliability index estimated by the global metamodel helps to identify the rough failure domain of the truss. Considering the identified failure domain may not contain the real failure domain. DE (Differential Evolution) algorithm is applied to expand the identified failure domain to make it closer to the real failure domain so that more representative failure modes may be identified.

3.2.1. New samples for identifying more failure modes

Since the number of samples to construct the global metamodel is small, fewer failure modes might be identified. In order to guarantee the accuracy of identified failure modes, the differential evolution (DE) algorithm [45] is applied to increase samples for searching more failure modes. The new samples generated by DE are named DE DoE in this research.

In the standard normal space, the reliability index β can be illustrated as the shortest distance from the original to the limit state surface [4]. And the reliability index can be obtained by the transformation $\beta = -\Phi^{-1}(P_f)$, where P_f is calculated by the global metamodel. The range of DE DoE is limited in $[\beta-\Delta,\beta+\Delta]$ where Δ is a constant to determine the range of the failure domain. Note that the generation of DE DoE is based on the identified failure samples. The identified failure samples may not be on the surface of β, so it is essential to perform the unification of β for the identified failure samples.

Suppose a sample $X(x_1,x_2,...,x_n)$ is a failure sample. It is necessary to convert it to β. Like the normalization of X:

$$X' = \left(\frac{x_1}{\sqrt{x_1^2+...+x_n^2}},\frac{x_2}{\sqrt{x_1^2+...+x_n^2}},...,\frac{x_n}{\sqrt{x_1^2+...+x_n^2}}\right) \tag{24}$$

the unification of β for X can be expressed as:

$$\hat{X} = \left(\frac{\beta x_1}{\sqrt{x_1^2+...+x_n^2}},\frac{\beta x_2}{\sqrt{x_1^2+...+x_n^2}},...,\frac{\beta x_n}{\sqrt{x_1^2+...+x_n^2}}\right) \tag{25}$$

where $\hat{X}$ is the sample converted to β.

When the DE DoE is obtained, transform samples among it into the space of the identified failure samples so that the real DE DoE can be obtained. The process of generating DE DoE where $\Delta= 1$ is shown in Fig. 2, where the red five-pointed stars denote the initial samples converted to β, and the blue circles represent the DE DoE.

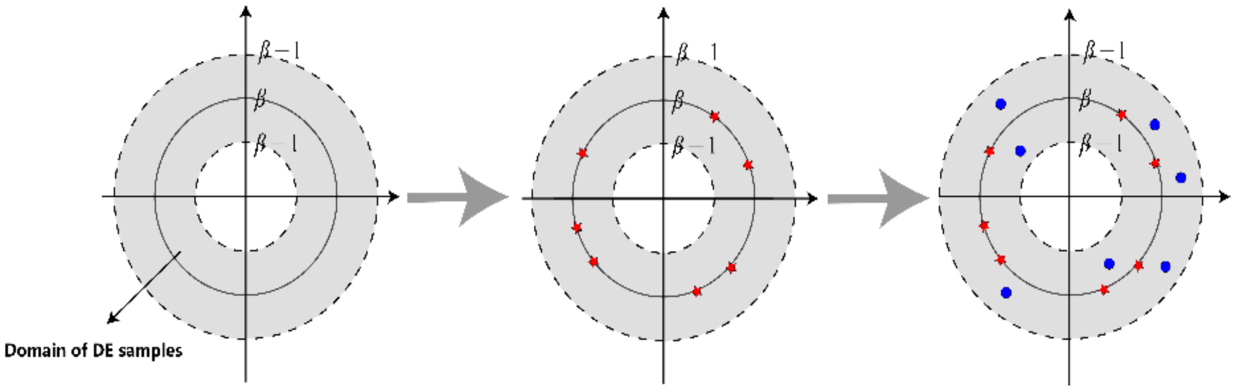

Fig. 2. Illustration of the generation of DE DoE.

Representative failure modes

Once the DE DoE is obtained, the global metamodel is used to select the failure samples from the DE DoE. Then, the limit analysis is initiated for these failure samples to identify new failure modes. If there are no new failure modes, add them to corresponding samples of failure modes so as to improve the accuracy of the initial local metamodels. There might be some redundant failure members identified by stress in the failure modes. To deal with this issue, we select representative failure modes that are the shortest failure modes to be on behalf of the failure modes with redundant members. As shown in Fig. 3, if the shortest failure modes are included in the longer failure modes, the longer failure modes are classified into the shortest failure modes. The representative failure modes of the truss are determined thereby.

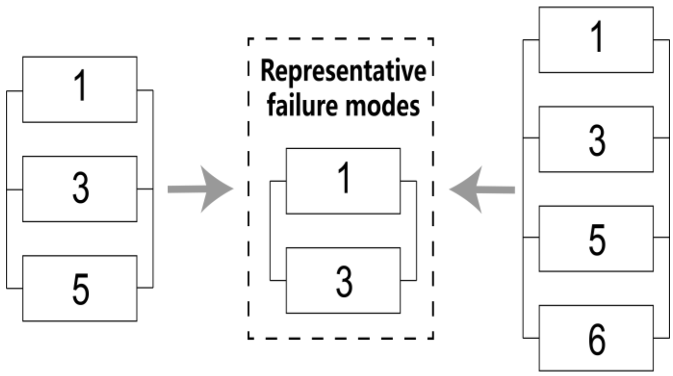

Fig. 3. Determination of the representative failure modes.

3.3. Local metamodel to estimate the system failure probability

Once the representative failure modes have been identified, the system reliability analysis based on them initiates. According to AK-SYSi, local metamodels for all representative failure modes are built and updated via active learning to accurately estimate the system failure probability.

3.3.1. Construction of the local metamodel

When the representative failure modes as well as their corresponding samples have been identified, constructing local metamodels of failure modes becomes the key step to estimating the system failure probability. The stress-strength interference theory is usually performed to build the performance functions of failure modes. Suppose one failure mode has $k$failure components, and the performance function of each component is expressed as:

$$Z_i = R_i - S_i (i{=}1{,}2{,}\ldots{,}k) \quad (26)$$

where R_i denotes the yield strength of a component and S_i is its corresponding response. A failure mode of the truss can be considered as a parallel system, and its performance function can be defined as:

$$G_l = \max\{Z_i{,}i{=}1{,}2{,}\ldots{,}k\} \quad (27)$$

Since the failure of a truss structure consists of a series of failure modes, it is regarded as a series system of multiple failure modes. Therefore, the performance function of the truss system is indicated as Eq. (17), where l denotes the number of failure modes.

$$G_s = \min\{G_l{,}l{=}1{,}2{,}\ldots{,}j\} \quad (28)$$

There are two strategies for estimating the failure probability of each failure mode. One is building a metamodel for each member, and calculating the failure probability through multiple metamodels. The other is constructing one metamodel directly for each failure mode according to its samples as well as the corresponding value of the performance function. The latter was demonstrated to be efficient and accurate in [47], so it is adopted to construct the local metamodels of failure modes in this research.

It is found that the stresses of members are needed to construct Eq. (15). The nonlinear analysis is conducted to determine the failure sample. For the samples that lead to no failure of the truss, the stress of each member does not reach its yield strength. The stress of each failure member can be obtained by elastic analysis so that the performance function of a failure mode is built directly according to Eq. (16). However, for the samples that cause the failure of the truss, they will lead to the failure of the truss structure when the load reaches the ultimate load of the truss, and the stresses of failure members equal to their yield strength. Hence, the performance functions of failure members cannot be built as the stress is no more than the yield stress. To solve this issue, another elastic analysis of the finite element model is performed, which adopts the incremental load between the ultimate load and the real load applied to simulate the stress that is after the yield of a member. Then the real stress is defined as the sum of the stresses of nonlinear and elastic analysis. The local metamodel is thereby constructed.

3.3.2. Active learning of the local metamodel

Once the initial local metamodels of representative failure modes have been constructed, active learning initiates. In this stage, the refined U learning function of AK-SYSi is applied to determine which local metamodel corresponds to the sample among the population N_s to be updated. Then the sample with a minimum value of learning function is selected as the best sample to update the local metamodel. Different from AK-SYSi, the convergence criterion is the same as the global metamodel, which is in terms of the estimated system failure probability to terminate iteration. When the active learning reaches the convergence criterion, the iteration ends. Meanwhile, the system failure probability is also obtained according to Eq. (2). Then, the failure probabilities of representative failure modes are computed through their local metamodels.

3.4. Procedure of the proposed method

Based on the introduction above, the procedure of the proposed method is summarized in Fig. 4. This method mainly includes the construction of the global metamodel, identification of representative failure modes, and local metamodel to estimate the system failure probability. The stage of construction of the global metamodel aims to provide a rough reliability index of the truss. Then samples to identify more failure modes are expanded by the DE algorithm according to the rough reliability index for determining the representative failure modes.

Eventually, the local metamodels for representative failure modes are constructed via active learning to estimate the system failure probability and reliability index.

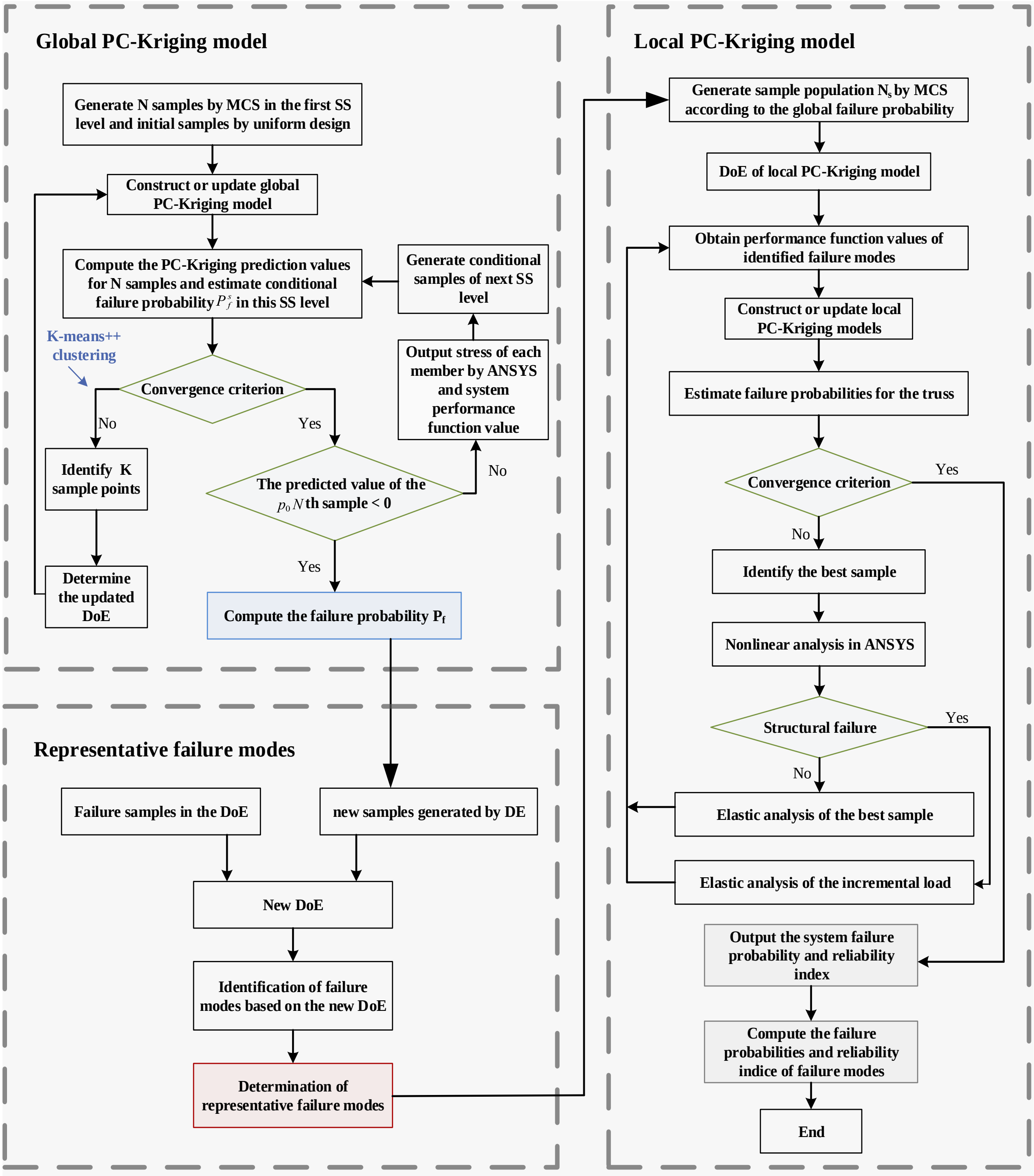

Fig. 4. Flowchart of the proposed method.

4. Validation of the proposed method

A plane truss and a space truss are discussed in this section to illustrate the applicability of the proposed method. Since this method is simulation-based, the results for the two examples are both performed for fifteen runs. Table 1 lists the parameters adopted for two examples. Besides, to make a quantitative evaluation of the efficiency of the proposed method, the number of calls to the performance function of the truss N_{call} is considered as the evaluation standard.

Table 1. Parameters of the proposed method for two examples.

Example	N	P_0	k	K	ε	Δ
1	1000	0.1	5	2	0.01	1
2	1000	0.1	4	2	0.01	1

4.1. Example 1: truss bridge structure

This example is a plane truss comprised of 25 members with ideal elastic-plastic behavior [17]. As shown in Fig. 5, at the 9th and 10th nodes, loads P_1 and P_2 are applied respectively. The parameters of members are listed in Table 2, and twenty-seven random variables are considered, including the yield strength of 25 members as well as two loads. The correlation of random variables is not considered, and the corresponding parameters about random variables are listed in Table 3.

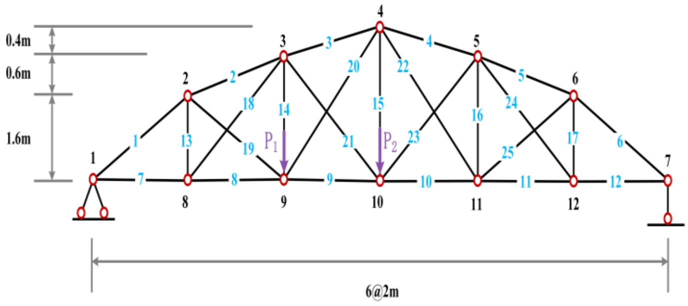

Fig. 5. Truss bridge of example 1.

Table 2. Cross section areas of members of example 1.

Members	Cross section areas(m^2)
1-6	15×10^{-4}
7-12	14×10^{-4}
13-17	12×10^{-4}
18-25	13×10^{-4}

Table 3. Distribution types and statistical parameters of random variables.

Random variables	Distribution	Mean	c.o.v
P_1/(kN)	Lognormal	160	0.1
P_2/(kN)	Lognormal	160	0.1
$\sigma_{yi}, i = 1,2,\ldots,25$/(MPa)	Normal	276	0.05

During the process of system reliability analysis for this 25-bar truss, the global metamodel has been completely constructed through averagely 103 iterations. Further, the rough global system reliability index β is calculated, and the domain to obtain DE DoE is determined as $[\beta-1,\beta+1]$. The failure samples among DE DoE and the failure samples included in the samples to construct the global metamodel are performed to identify the representative modes. For one of the runs, the failure modes 1, 1→3, 2→8, 3→9, 2→3→9, 3→4→9 have been identified. As the failure modes 1 and 3→9 are the shortest failure modes, they are selected as the representative failure modes of 1→3, and 2→3→9, 3→4→9 respectively shown in Fig. 6. Herein, three representative failure modes are identified and are denoted as 1, 2→8, and 3→9 respectively. Therefore, three local metamodels corresponding to the representative failure modes are built directly according to their samples.

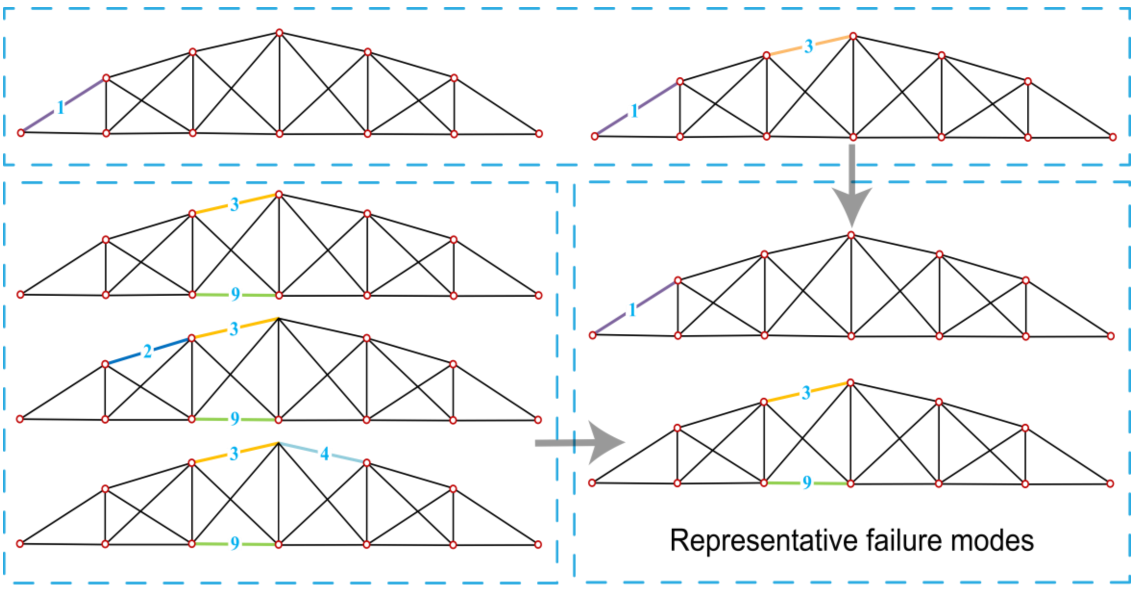

Fig. 6. Representative failure modes of example 1.

The average results of reliability analysis for this example are summarized in Table 4 after the active learning of local metamodels. As shown in Table 4, the reliability index of this truss calculated by the proposed method is 2.6488 that only has a 2.93% relative error compared with MCS. Besides, it also has close results with the method of Xu [47] and the method of Kim et al [17]. This indicates that the proposed method has high accuracy in dealing with the plane truss. Meanwhile, the reliability index of each representative failure mode is computed as well.

In addition, the efficiency of this proposed method is another aspect concentrated on. We compare its efficiency with other methods from the perspective of the number of calls to the performance function. The results of the efficiency investigated are listed in Table 5. The method in [47] is an efficient approach to estimating the failure probability, which needs 271 calls to the performance function to obtain the final results of this example. Nevertheless, the proposed method in this research only needs 141 calls on average to reach convergence, which improves efficiency by 47.97%. The number of calls to the performance function of the proposed method is far less than the method in [17] and MCS. Compared with other methods, the proposed method has a higher computational efficiency. When the truss becomes more complex, this improved efficiency would be more significant. This indicates that the proposed method is efficient in estimating the failure probability of the truss structure.

Table 4. Results of reliability indices analysis of example 1.

Representative failure modes & system	Proposed	Method in [47]	Method in [17]	MCS	Error (%)
1	3.8497	3.6588	3.6334	3.6666	4.99
2→8	4.5958	/	/	/	/
3→9	2.8527	2.5658	2.5920(-3→9)	2.5865(-3→9)	10.29
System	2.6488	2.5614	2.5478	2.5733	2.93

Table 5. Comparisons of the number of calls to the performance function for example 1.

Method	Proposed	Method in [47]	Method in [17]	MCS
Ncall	103+38	271	51,344	460,330

4.2. Example 2: a 25-bar space truss

In this subsection, a 25-bar space truss [8; 15] is considered to investigate the applicability of the proposed method, as shown in Fig. 7. The horizontal load F_1 and vertical load F_2 that are random variables are applied at corresponding nodes respectively. Furthermore, the yield strength of each member is also regarded as a random variable. The section information of members and statistical parameters of random variables are listed in Table 6 and Table 7 respectively. It is assumed that the material of the truss is ideal elastic-plastic with an elastic modulus 2.06×10^5MPa.

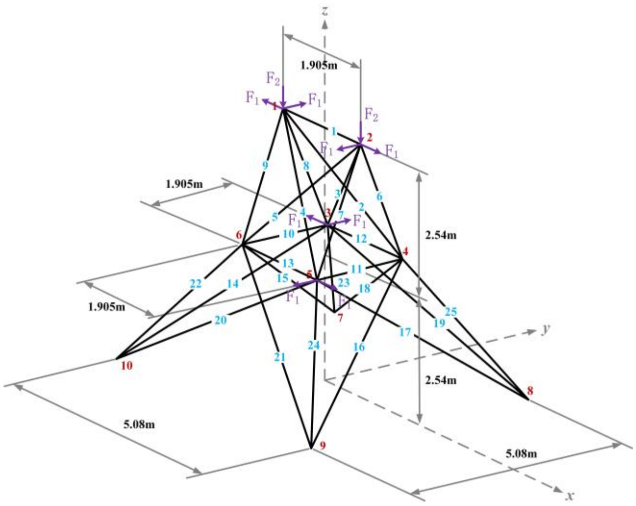

Fig. 7. A 25-bar space truss of example 2.

Table 6. Cross section areas of members of example 2.

Type	1	2	3	4	5	6	7	8	9	10	11	12	13
No.	1	2 5	3 4	6 9	7 8	10 11	12 13	14 17	15 16	18 21	19 20	22 25	23 24
Area/ cm^2	4.36	4.56	7.47	2.39	7.52	1.51	1.77	4.88	1.89	1.78	2.63	4.89	7.66

Table 7. Distribution types and statistical parameters of random variables.

Random variables	Distribution	Mean	c.o.v
F_1/(kN)	Normal	88.9	0.2
F_2/(kN)	Normal	22.6	0.2
$\sigma_{yi}, i = 1,2,\dots,25$/(MPa)	Normal	276	0.05

In order to analyze the reliability of this truss, the global metamodel is constructed at first, and the accurate global metamodel is obtained averagely through 156 iterations. Then the reliability index calculated by the global metamodel is used to expand the failure domain so that the DE DoE for identifying new failure modes is generated. It is observed that 3 → 6 and 4 → 9 are the representative failure modes during 15 runs. Fig. 8 shows the process of some failure modes denoted by the representative failure modes. Further, two local metamodels of the representative failure modes are built to perform active learning for system reliability analysis. The results of the analysis for this example are summarized in Table 8. The system reliability computed by the proposed method is 4.6386 which is very close to the benchmark calculated by MCS and the relative error is only 1.76%. Meanwhile, this result is close to the method in [15]. Thus, it can be concluded that the proposed method can analyze the reliability of the truss with high precision.

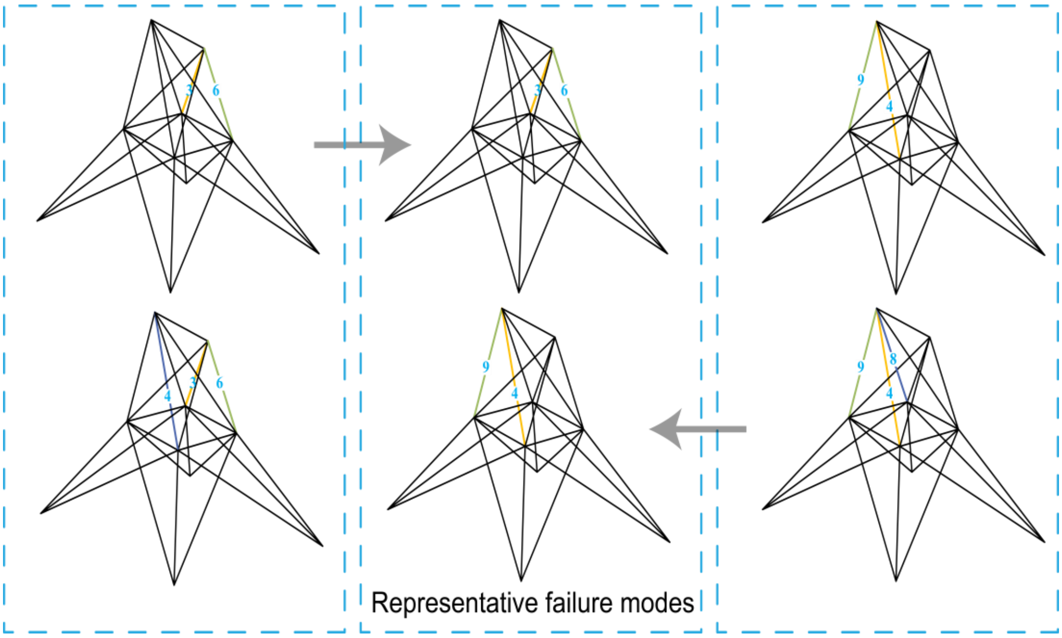

Fig. 8. Representative failure modes of example 2.

Furthermore, Table 8 shows that the failure probability of this space truss is very small as it has a large value of reliability index, so the efficiency of computation becomes an essential aspect to be considered. Once the failure probability is small, more samples are needed to obtain an accurate result, which will result in a large number of computational costs. Table 9 lists the number of calls to the performance function of different methods to investigate the performance of the proposed method in efficiency. It is a huge computational cost for MCS as it needs 2×10^7 calls to the performance function to obtain the accurate reliability index. Nevertheless, this proposed method only needs 172 calls on average to reach convergence. It is found that the method of Jiang et al has higher accuracy than the proposed approach. But there is no big gap between these two methods, and the proposed method might have higher accuracy when the analyzed truss is more complex due to the excellent performance of the PC-Kriging model. Besides, the proposed method has a smaller number of calls to the performance function than the method in [15], which indicates that it has a good trade-off between accuracy and efficiency.

Table 8. Results of reliability indices analysis of example 2.

Representative failure modes & system	Proposed	Method in [15]	MCS	Error (%)
3→6	4.7315	/	/	/
4→9	4.7790	/	/	/
System	4.6386	4.6806	4.7216	1.76

Table 9. Comparisons of the number of calls to the performance function for example 2.

Method	Proposed	Method in [15]	MCS
Ncall	164+8	180	$2{\times}10^7$

5. Conclusion

In this paper, we propose an adaptive method that makes a good trade-off between accuracy and efficiency to perform the system reliability analysis for truss structures. The proposed approach is an adaption of the method of Xu [47]. It adopts the PC-Kriging model as the metamodel since the PC-Kriging merges the merits of Kriging and polynomial chaos expansions and constructs the global metamodel to estimate the global reliability index of the truss roughly and rapidly at first. Then, the DE algorithm is applied to generate more new samples located in the domain $[\beta-\Delta,\beta+\Delta]$ so that more failure modes can be identified. But the determination of Δ in this research is a little arbitrary, and it needs more research to determine the proper value in the future. This operation is beneficial to select more representative failure modes to improve the accuracy of estimation for the system reliability. Based on the identified representative failure modes as well as their corresponding samples, local metamodels are constructed. The active learning process of local metamodels is further initiated to compute the system reliability index of the truss and reliability indices of local failure modes.

This proposed method provides a general framework to analyze the system reliability of a truss structure and can accurately perform reliability analysis with efficiency. Two examples of trusses are investigated to illustrate the trade-off that the proposed method makes. MCS is considered as the most accurate method to generate the benchmark value of the reliability index. Compared with MCS, the relative errors of the two examples are 2.93% and 1.76% respectively. Moreover, the efficiency has been improved greatly. The analysis results verify that the proposed method makes a good trade-off between accuracy and efficiency and is applicative to analyzing the system reliability of the truss structure.

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