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Belief reliability-based design optimization method with quantile index under epistemic uncertainty

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Highlights

- Epistemic uncertainty brings great risk in product reliability design.
- Uncertainty theory is introduced to quantify epistemic uncertainty.
- A new belief reliability quantile index is put forward for reliability analysis.
- Belief reliability-based design optimization methods are proposed using the quantile index.
- The proposed method shows good accuracy and efficiency.

Abstract

Product reliability design optimization is affected by epistemic uncertainty greatly, which leaves significant risks in product use. In this paper, a new belief reliability-based design optimization (BRBDO) method under epistemic uncertainty is established to handle this problem. First, the belief reliability theory is introduced into the design optimization problem, and a quantile index is proposed to quantify belief reliability level based on uncertainty theory, through which a rapid analysis method called first order belief reliability analysis (FOBRA) method is developed. Then, according to the different trade-off strategies, two types of design optimization models are established, and corresponding FOBRA-based computation methods are also demonstrated. Finally, several case applications are studied to verify the effectiveness of the analysis and design optimization methods proposed in this paper. The results indicate that the BRODO method with the quantile index can save a lot of computational time with acceptable accuracy and can rationally cope with epistemic uncertainty.

Keywords
belief reliability, reliability design optimization, epistemic uncertainty, quantile index, uncertainty theory

1. Introduction

1.1 Background

With the development of industry, product reliability has been paid more and more attention. In this context, product design often requires decision makers to balance reliability with other attributes, such as performance, cost, weight, etc., which is the fundamental goal of carrying out reliability-based design optimization (RBDO).

Generally, since the product reliability is significantly influenced by various uncertainties, the quantification and propagation of uncertainty have become the core issues in reliability design. In existing research and applications, however, the mathematical theories utilized for describing uncertainty (especially epistemic uncertainty) still have various deficiencies such as theoretical inconsistency, which may bring design risks in real cases. Therefore, there is a great urgent to find a better method to characterize uncertainties, thus providing a solid foundation for RBDO. Further, with the uncertainty description and the properties of iterative optimization, we also need to find a reasonable and efficient algorithm to obtain the optimal results. Facing with these demands, in this paper, a new belief reliability-based design

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optimization (BRBDO) method considering epistemic uncertainty will be proposed.

1.2 Literature Review

The research regarding uncertainty in RBDO mainly include two aspects. One of them is the utilization of different mathematical models to describe uncertainty, and the other goes to the efficiency improvement of optimization in the context of corresponding mathematical tools.

(1) Theories for uncertainty in RBDO

According to the source and characteristics of uncertainty, it is usually categorized as two types: aleatory uncertainty and epistemic uncertainty [15]. Among them, the aleatory uncertainty characterizes the inherent randomness of the objective world, such as the fluctuation of material parameters caused by microscopic in-homogeneity. Epistemic uncertainty is the uncertainty caused by the limitation of human knowledge and lack of information, such as physical parameter information obtained from small sample statistics, knowledge limitations caused by complex physical processes, etc.

In existing studies, due to the advantages of probability theory in dealing with aleatory uncertainty, reliability-based design optimization (RBDO) methods based on probability theory have been widely studied and applied, but there has not been consensus for representing epistemic uncertainty. In order to deal with epistemic uncertainty, Bayesian method is applied to reliability analysis and design, which is still based on probability theory [26,30]. In literature, some non-probabilistic mathematical methods are introduced. Based on fuzzy theory, Cai et.al created a fuzzy reliability measurement framework [3]. Cremona and Gao applies possibility measure to the field of structural reliability analysis [5]. Mourelatos and Zhou proposed a possibility-based design optimization (PBDO) model under incomplete information [23]. Bae and Canfield proposed a reliability analysis method based on evidence theory [1]. Mourelatos and Zhou proposed a general evidence-based design optimization (EBDO) model [24]. In addition, Ben-Haim and Elishakoff used interval variables to describe basic structural variables, and proposed a convex model for structural reliability analysis [2]. Jiang et al. further established a reliability design optimization model based on interval theory [12]. Wang et al. used a novel polar transformation to achieve a unified reliability analysis taking both random variables and bounded intervals into account [27]. Recently, researchers have also proposed methods such as reliability-based topology design optimization [7], system-level reliability design optimization [16], and reliability-based tolerance design optimization [10] that consider the effects of uncertainty.

Although these methods can solve some product reliability analysis and design optimization problems affected by epistemic uncertainty, they still have various theoretical defects [14]. For example, possibility metric does not satisfy duality axiom, evidence theory and interval analysis may cause interval extension problems when calculating system reliability. The establishment of uncertainty theory provides another way to deal with epistemic uncertainty in product reliability. Uncertainty theory is a new set of axiomatic mathematical theory created by Liu, which is believed to better handle epistemic uncertainty [18]. Zeng et al. introduced uncertainty theory into reliability analysis, and first proposed the concept of belief reliability, which was defined as the uncertain measure that the system can operate normally [32]. Subsequently, Wen et al. further improved the system reliability measurement and analysis method based on uncertainty theory [29,33]. Kang systematically expounded the framework and research progress of belief reliability theory [13]. In terms of reliability design optimization, Chen proposed a belief reliability-based design optimization (BRBDO) model [4]. Numerous studies have shown that belief reliability theory can effectively deal with epistemic uncertainty in reliability engineering and optimization [11,17,18].

(2) Efficiency improvement method in RBDO

Reliability design optimization is to iteratively find the optimal design point on the basis of multiple reliability assessments, which often requires a lot of computing resources. Therefore, how to improve the optimization efficiency is another key issue in reliability design.

Improving reliability design optimization efficiency can generally be achieved in two ways. The first is to improve reliability assessment efficiency in reliability design optimization. The basic idea of this class of methods is to simplify each reliability assessment calculation in the optimization process using a specific method, thus improving the overall optimization efficiency. Typical methods include first order reliability method in RBDO, the equivalent optimization model of PBDO [23], and the method of approximate calculation of constraints using interval analysis in EBDO [8], etc. The second is to improve the overall optimization strategy. This class of methods tends to transform the double-loop procedure into a single-loop procedure by decoupling the optimization and reliability assessment under a specific reliability measure, thus accelerating the solution of design optimization models. Representative methods include sequential optimization and reliability assessment (SORA) method in RBDO [6], the sequential algorithm in PBDO [34], and the decoupling strategy of reliability design optimization model based on interval theory [22]. It is clear that the practical use of these methods is greatly determined by the mathematical tools used to describe uncertainty in reliability assessment process. As mentioned previously, these methods may have deficiencies in uncertainty quantification. However, the basic ideas of these methods are still worth learning, which is important for us to propose reasonable optimization algorithms. In terms of belief reliability-based design optimization, the research on improving the optimization efficiency is relatively limited. Chen tried to improve the overall optimization efficiency by improving the reliability analysis efficiency, and gave a deterministic equivalent model of BRBDO when the performance margin function is strictly monotonic [4]. However, in the general non-monotonic case, the BRBDO model can only solve the reliability constraints by uncertain simulation algorithm [35], which greatly limits the optimization efficiency.

1.3 Contribution

According to the above analysis, it is believed that the uncertainty theory is more competitive in describing uncertainty in reliability design optimization problems. However, there is still not a complete model and computation framework for BRBDO problems. In this paper, aiming to more effectively perform reliability design optimization for products affected by epistemic uncertainty under belief reliability, we will firstly propose a rapid analysis method of belief reliability, and then establish BRBDO models and computation methods correspondingly. The main contributions of this paper are summarized as follows:

1) This paper proposes a belief reliability quantile index based on uncertainty theory and puts forward an approximate belief reliability analysis method accordingly.
2) This paper establishes two types of BRBDO models and provides corresponding equivalent simplified models on the basis of the developed rapid belief reliability analysis method.

1.4 Organization

The remainder of this paper is organized as follows. Section 2 will briefly introduce the axioms, definitions and theorems of uncertainty theory. Section 3 will propose the first-order belief reliability analysis (FOBRA) method and analyze its mathematical properties. New design optimization models on the basis of FOBRA will be established in Section 4. Finally, several cases will be studied to verify the effectiveness of the method proposed in this paper.

2. Preliminary

In this section, some related concepts in uncertainty theory and belief reliability are introduced as the preliminary of this paper.

Uncertain measure $\mathcal{M}$ is the basis of our research that indicates the belief degree that an event occurs. Mathematically, let $\mathcal{L}$ be a nonempty set, and Γ a σ-algebra over Γ, then each element $\Lambda \in \mathcal{L}$ is an event and the uncertain measure should satisfy the following axioms [18, 19]:

Axiom 1 (Normality Axiom) $\mathcal{M}\{\Lambda\} = 1$ for the universal set Γ.

Axiom 2 (Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 3 (Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have

$$\mathcal{M}\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leqslant \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

Axiom 4 (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1,2,\cdots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\{\prod_{k=1}^{\infty} \Lambda_i\} = \bigwedge_{k=1}^{\infty} \mathcal{M}\{\Lambda_k\},$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1,2,\cdots$, respectively.

Definition 1(Uncertainty space [18]) Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ and $\mathcal{M}$ an uncertain measure. Then the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

Definition 2(Uncertain variable [18]) An uncertain variable ξ is a measurable function from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

Definition 3(Uncertainty distribution [18]) The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \leqslant x\}$$

for any real number x.

Definition 4(Expected value [18]) Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geqslant x\}dx - \int_{-\infty}^{0} \mathcal{M}\{\xi \leqslant x\}dx$$

provided that at least one of the two integrals is finite.

Definition 5(Variance [18]) Let ξ be an uncertain variable with finite expected value e. Then the variance of ξ is

$$V[\xi] = E[(\xi - e)^2].$$

In this paper, the normal uncertain variable is the basic tool for reliability analysis method. Here we introduce its distribution and corresponding properties.

Definition 6 [18] An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, x \in \mathbb{R}$$

denote by $\mathcal{N}(e, \sigma)$ where e and σ are real numbers with $\sigma > 0$. A normal uncertainty distribution is called standard if $e = 0$ and $\sigma = 1$.

Theorem 1 [18] The normal uncertain variable $\xi \sim \mathcal{N}(e, \sigma)$ has an expected value e and a variance σ^2.

Theorem 2 [18] Let ξ_1 and ξ_2 be independent normal uncertain variables $\mathcal{N}(e_1, \sigma_1)$ and $\mathcal{N}(e_2, \sigma_2)$, respectively. Then the sum $\xi_1 + \xi_2$ is also a normal uncertain variable $\mathcal{N}(e_1 + e_2, \sigma_1 + \sigma_2)$. The multiplication of a normal uncertain variable $\mathcal{N}(e, \sigma)$ and a scalar number $k > 0$ is also a normal uncertain variable $\mathcal{N}(ke, k\sigma)$.

Theorem 3 [18]: If ξ is a normal uncertain variable $\mathcal{N}(e, \sigma)$, then ζ is a normal uncertain variable if

$$\zeta = \frac{\xi - e}{\sigma}. \quad (1)$$

As for the application of uncertain measure in reliability engineering, Liu first proposed the reliability index in terms of system life and Boolean states [20]. Later, Zeng et al. named this reliability metric as belief reliability and interpreted the metric as the belief degree of the system to be reliable [32]. Zhang et al. and Kang extended the measurement of reliability and proposed the belief reliability regarding performance margin [13, 33], which is the basis for reliability design optimization of this paper. We hereby give the definition of belief reliability in terms of performance margin when it is mainly affected by epistemic uncertainty.

Definition 7 (Belief reliability [32]) Assume a product contains uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ that are mainly affected by epistemic uncertainty and there is a performance margin function G such that the product is working if and only if $G(\xi_1, \xi_2, \cdots, \xi_n) > 0$. Then the belief reliability is mathematically defined as

$$R_B = \mathcal{M}\{G(\xi_1, \xi_2, \cdots, \xi_n) > 0\}. \quad (2)$$

3. Belief reliability analysis with performance margin function

Based on the definition of belief reliability, it is studied that only the R_B of products with strictly monotonic performance margin functions can be easily calculated via inverse uncertainty distribution [32]. For products with non-monotonic performance margin functions, R_B can only be approximated using an uncertain simulation algorithm [35], which will bring an unbearable computational load in the design optimization problem.

To assist the reliability assessment in design optimization and improve the calculation efficiency, this section will propose a novel belief reliability index called the quantile index based on uncertainty theory and develop a rapid reliability analysis method accordingly. The quantile index has good properties with a linear performance margin function, so the first-order information of the performance margin function is mainly utilized in the reliability analysis method, and we call this method the first-order belief reliability analysis (FOBRA) method. In this paper, the quantile index and FOBRA will be established based on the normality assumption, which means that all variables of the product will be treated as normal uncertain variables with mean and variance sufficient to describe all its characteristics.

3.1 Belief reliability quantile index

The belief reliability quantile index (BRQI) is defined as follows.

Definition 8 (Belief reliability quantile index) Let $\boldsymbol{\tau} = \tau_1, \tau_2, \cdots, \tau_n$ be an uncertain vector and $G(\boldsymbol{\tau})$ be the performance margin function. The belief reliability quantile index γ is defined as

$$\gamma = \frac{E[G]}{\sqrt{V[G]}}, \tag{3}$$

where $E[G]$ and $V[G]$ are the expected value and variance of $G(\boldsymbol{\tau})$, respectively.

Theorem 4 If a product has a linear performance margin function $G(\boldsymbol{\tau})$ with respect to $\boldsymbol{\tau}$, the product belief reliability R_B and the quantile index γ satisfies

$$R_B = 1 - \left(1 + exp\left(\frac{\pi\gamma}{\sqrt{3}}\right)\right)^{-1}. \tag{4}$$

Proof Without loss of generosity, set the linear performance margin function to be

$$G(\boldsymbol{\tau}) = a_0 + \sum_{i=1}^{n} a_i\, \tau_i, \tag{5}$$

where $\tau_i \sim \mathcal{N}(e_i, \sigma_i)$. Then, the BRQI can be calculated according to Theorem 2, that is

$$\gamma = \frac{a_0 + \sum_{i=1}^{n} a_i e_i}{\sum_{i=1}^{n} |a_i| \sigma_i}. \tag{6}$$

Meanwhile, we have the $G(\boldsymbol{\tau})$ follows a normal uncertainty distribution with a mean of $a_0 + \sum_{i=1}^{n} a_i e_i$ and a standard deviation of $\sum_{i=1}^{n} |a_i| \sigma_i$. Then the distribution function of $G(\boldsymbol{\tau})$ is

$$\Psi(x) = \left(1 + exp\left(\frac{\pi(a_0 + \sum_{i=1}^{n} a_i e_i - x)}{\sqrt{3}\sum_{i=1}^{n} |a_i| \sigma_i}\right)\right)^{-1}, \quad x \in \mathbb{R} \tag{7}$$

Therefore, the belief reliability can be calculated as

$$R_B = 1 - \Psi(0) = 1 - \left(1 + exp\left(\frac{\pi(a_0 + \sum_{i=1}^{n} a_i\, e_i)}{\sqrt{3}\sum_{i=1}^{n} |a_i|\, \sigma_i}\right)\right)^{-1}$$
$$= 1 - \left(1 + exp\left(\frac{\pi\gamma}{\sqrt{3}}\right)\right)^{-1}. \tag{8}$$

The theorem is thus proved.

Remark 1 The γ is called a quantile index because its value can be regarded as a quantile of the standard normal distribution in terms of R_B when G is a linear function. Specifically, $\Phi_{st}(-\gamma) = 1 - R_B$, where Φ_{st} represents the distribution function of a standard normal uncertain variable.

Remark 2 The belief reliability R_B of a product with a linear performance margin function is positively correlated with its belief reliability quantile index γ.

3.2 FOBRA method

In this section, we develop a calculation method for BRQI to estimate the product belief reliability with performance margin function. Generally, when the performance margin function G is a linear function with respect to $\boldsymbol{\tau}$, the BRQI can be calculated directly according to Theorem 2, and the belief reliability can be easily calculated by Theorem 4. For the nonlinear case, the calculation of BRQI involves solving complex multiple integrals, which is extremely difficult in many cases. To handle this, we tend to use the first-order Taylor expansion of the function G to acquire an approximate BRQI.

Because the uncertainty distributions of elements in $\boldsymbol{\tau}$ may be in different scales, in this paper, the normal uncertain variables $\tau_i (i = 1,2 \cdots, n)$ are first unified to a standard uncertain space using Equation (1). Then, an expansion point on the G can be suggested with the standardized performance margin function.

Definition 9 (Standardized performance margin function) Let $G(\boldsymbol{\tau})$ be the performance margin function with respect to a normal uncertain vector $\boldsymbol{\tau} = (\tau_1, \tau_2, \cdots, \tau_n)$. Then, the equivalent performance margin function $\overline{G}(\overline{\boldsymbol{\tau}})$ with respect to a standard normal uncertain vector $\overline{\boldsymbol{\tau}} = (\overline{\tau_1}, \overline{\tau_2}, \cdots, \overline{\tau_n})$ is called the standardized performance margin function.

Theoretically, all the points on the surface of $\overline{G}(\overline{\boldsymbol{\tau}}) = 0$ can be the Taylor expansion point of $\overline{G}(\overline{\boldsymbol{\tau}})$, but not all the results with an arbitrary expansion points can cover the linear character of the function $\overline{G}$. Therefore, we make the following suggestion.

Let $\overline{G}(\overline{\boldsymbol{\tau}})$ be a standardized performance margin function of a product, then the point on surface $\overline{G}(\overline{\boldsymbol{\tau}}) = 0$ with the closest distance to the origin point is suggested to be the expansion point for $\overline{G}$. Coincidentally, the above expansion point has a same geometric meaning in the Cartesian coordinate system with Most Probable Point (MPP) [21]. In this regard, the search of this point can be implemented using the HLRF algorithm [9, 25]. Since the algorithm is to acquire the point that can check the belief degree of being reliable, we call this point as the belief degree checking point (BDCP).

Proposition 1 Let $G(\boldsymbol{\tau})$ be the performance margin function of a product with a standardized performance margin function $\overline{G}(\overline{\boldsymbol{\tau}})$. If $\overline{G_L}(\overline{\boldsymbol{\tau}^*}) = b_0 + \sum_{i=1}^{n} b_i \overline{\tau_i^*}$ is the first order Taylor expansion of $\overline{G}(\overline{\boldsymbol{\tau}})$ at BDCP, then the BRQI can be approximated by

$$\gamma = \frac{b_0}{\sum_{i=1}^{n}|b_i|}. \quad (9)$$

Proof Let $G_L(\boldsymbol{\tau}^*)$ be the inverse transformed function of $\overline{G_L}(\overline{\boldsymbol{\tau}^*})$ in the original uncertainty space, where $\boldsymbol{\tau}^* = (\tau_1^*, \tau_2^*, \cdots, \tau_n^*)$ with $\tau_i^* \sim \mathcal{N}(e_i^*, \sigma_i^*)$. Then, $G_L(\boldsymbol{\tau}^*)$ is just a first order Taylor expansion of $G(\boldsymbol{\tau})$ at $\boldsymbol{\tau}^*$, which can be regarded as the approximation of $G(\boldsymbol{\tau})$. In other words, the BRQI of $G_L(\boldsymbol{\tau}^*)$ can be used to approximate the BRQI of $G(\boldsymbol{\tau})$.

If we write $G_L(\boldsymbol{\tau}^*)$ as

$$G_L(\boldsymbol{\tau}^*) = a_0 + \sum_{i=1}^{n} a_i\, \tau_i^*, \quad (10)$$

then according to Equation (1), we have

$$b_0 = a_0 + \sum_{i=1}^{n} a_i\, e_i^*, \quad (11)$$

$$b_i = a_i \sigma^*. \quad (12)$$

According to Definition 8, the BRQI in terms of $G_L(\boldsymbol{\tau}^*)$ can be calculated as

$$\gamma_L = \frac{E[G_L(\boldsymbol{\tau}^*)]}{\sqrt{V[G_L(\boldsymbol{\tau}^*)]}} = \frac{a_0 + \sum_{i=1}^{n} a_i e_i^*}{\sum_{i=1}^{n}|a_i|\sigma_i^*} = \frac{b_0}{\sum_{i=1}^{n}|b_i|}, \quad (13)$$

which is the approximation of γ for $G(\boldsymbol{\tau})$.

Based on the above analysis, the first-order belief reliability analysis (FOBRA) method for an arbitrary performance margin function can be summarized in Fig. 1. For the linear case, it is straightforward to calculate the BRQI as well as the belief reliability. For the nonlinear case, the BDCP is first obtained by the iterative operation of HLRF algorithm, then the performance margin function is linearized by first order Taylor expansion on BDCP, and thus the approximate BRQI as well as the belief reliability can be obtained.

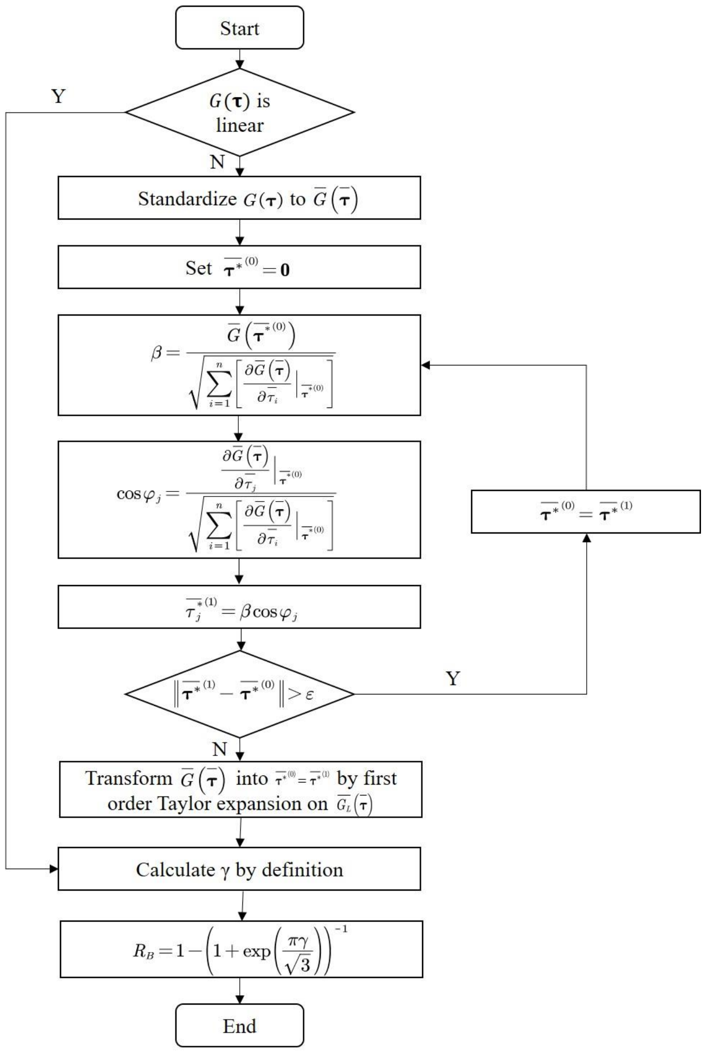

Fig. 1. Flow chart of the FOBRA method.

Obviously, FOBRA method avoids multiple integration operations as well as large-scale sampling operations, thereby ensuring that it can achieve a rapid analysis of belief reliability for products affected by epistemic uncertainty. In addition, FOBRA method guarantees a certain evaluation accuracy, especially when the performance margin function is weakly nonlinear.

In addition, the equivalent normalization method is introduced for cases where the basic variables do not follow normal uncertainty distributions. Equivalent normalization means replacing the original distribution with a normal distribution so that the original distribution has the same cumulative distribution function value and the same derivative value at the BDCP as that normal distribution. Assume that $\Psi(x)$ is the original non-normal distribution function of a basic variable, τ^* is the belief degree checking point, $\Phi(x)$ is the equivalent normal distribution function with unknown e and σ, $\Psi'(x)$ and $\Phi'(x)$ are the derived functions of $\Psi(x)$ and $\Phi(x)$ respectively. Then, e and σ can be obtained by solving the following system of equations:

$$\begin{cases} \Psi(\tau^*) = \Phi(\tau^*) = \left(1 + exp\left(\frac{\pi(e-\tau^*)}{\sqrt{3}\sigma}\right)\right)^{-1}, \\ \Psi'(\tau^*) = \Phi'(\tau^*) = \frac{\pi\, exp\left(\frac{\pi(e-\tau^*)}{\sqrt{3}\sigma}\right)}{\sqrt{3}\sigma\left(1+exp\left(\frac{\pi(e-\tau^*)}{\sqrt{3}\sigma}\right)\right)^2}. \end{cases} \tag{14}$$

After acquiring the parameters, the obtained normal distribution function $\Phi(x)$ can be used to replace the original distribution $\Psi(x)$ for FOBRA.

4. Belief reliability-based design optimization models

This section will propose belief reliability-based design optimization (BRBDO) models as well as solving methods. According to different trade-off strategies between input resources and reliability, two types of BRBDO models are proposed. The notations used in this section are as follows.

C: cost function indicating the resource investment of the product,

$\boldsymbol{X}$: vectors of design-related uncertain variables,

$\boldsymbol{P}$: vectors of design-independent uncertain variables,

$\boldsymbol{d}$: design vector (the vector of expected values of $\boldsymbol{X}$),

G_k: k-th performance margin function of the product,

C^t: specified input resource limit,

R_k^t: specified minimum belief reliability for the k-th performance,

γ_k^t: specified minimum quantile index corresponding to the belief reliability for the k-th performance,

$\boldsymbol{d}^L$: lower tolerance limit of the design vector,

$\boldsymbol{d}^U$: upper tolerance limit of the design vector,

m: total number of product performance margin functions,

n_X: dimension of $\boldsymbol{X}$,

n_P: dimension of $\boldsymbol{P}$,

Φ_i: uncertainty distribution of X_i,

Ψ_i: uncertainty distribution of P_i,

Φ_i^{-1}: inverse uncertainty distribution of X_i,

Ψ_i^{-1}: inverse uncertainty distribution of P_i.

4.1 Resource minimum BRBDO model

In general, designers want to invest as few resources as possible for a product with specified reliability design goals. The design optimization model corresponding to this optimization strategy is called the resource minimum design optimization model in this paper. The model can be expressed as **Model 1** under the uncertain measure.

Model 1:

$$\begin{cases} \min\limits_{\boldsymbol{d}} \ C(\mathbf{d}) \\ \text{subject to} \\ \quad \mathcal{M}\{G_k(\boldsymbol{X},\boldsymbol{P}) \geq 0\} \geq R_k^t, k = 1,2,\cdots,m \\ \quad \boldsymbol{d}^L \leq \boldsymbol{d} \leq \boldsymbol{d}^U. \end{cases} \tag{15}$$

(1) Monotonic case

When the performance margin functions in the model are strictly monotonic with respect to $\boldsymbol{X}$ and $\boldsymbol{P}$, where $X_1, X_2, \cdots, X_{n_X}, P_1, P_2, \cdots, P_{n_P}$ are independent uncertain variables, the reliability constraints of **Model 1** can be transformed into an equivalent expression according to uncertainty theory. Specifically, if the performance margin function $G_k(\boldsymbol{X},\boldsymbol{P})$ is continuous, and strictly increasing with respect to $X_1, X_2, \cdots, X_r, P_1, P_2, \cdots, P_s$ and strictly decreasing with respect to $X_{r+1}, X_{r+2}, \cdots, X_{n_X}, P_{s+1}, P_{s+2}, \cdots, P_{n_P}$, then **Model 1** can be equivalently expressed as **Model 2**.

Model 2:

$$\begin{cases} \min\limits_{\boldsymbol{d}} \ C(\mathbf{d}) \\ \text{subject to} \\ \quad G_k(\Phi_1^{-1}(R_k^t), \cdots, \Phi_r^{-1}(R_k^t), \Phi_{r+1}^{-1}(1-R_k^t), \cdots \\ \quad \quad \Phi_{n_X}^{-1}(1-R_k^t), \Psi_1^{-1}(R_k^t), \cdots, \Psi_s^{-1}(R_k^t), \Psi_{s+1}^{-1}(1-R_k^t), \cdots, \\ \quad \quad \Psi_{n_P}^{-1}(1-R_k^t)\big) \leq 0, \quad k = 1,2,\cdots,m \\ \quad \boldsymbol{d}^L \leq \boldsymbol{d} \leq \boldsymbol{d}^U. \end{cases} \tag{16}$$

Obviously, **Model 2** can be solved by various non-linear optimization algorithms as it is a deterministic optimization model.

(2) Non-monotonic case

In the situation where the performance margin functions are non-monotonic (or the property of monotony is difficult to identify), solving the belief reliability constraint in **Model 1** would be significantly hard. Therefore, the FOBRA method proposed in Section 3 is utilized to generate a new belief reliability constraint. Since the BRQI is strictly positively correlated with the estimated belief reliability in the FOBRA method, the BRQI can be used to replace the belief reliability constraint in **Model 1**, which is written as

$$\gamma_k(\boldsymbol{X},\boldsymbol{P}) \geq \gamma_k^t, \quad k = 1,2,\cdots,m, \tag{17}$$

where γ_k^t can be directly transformed from R_k^t as

$$\gamma_k^t = \frac{\sqrt{3}}{\pi} ln((1-R_k^t)^{-1}-1). \tag{18}$$

Based on the above replacement, solving **Model 1** requires executing FOBRA during each iteration to guarantee the satisfaction of the constraints, and thus the optimal result can be obtained through performing overall optimization in the outer loop. A typical solving method is to utilize the classical genetic algorithm by imposing FOBRA-based reliability constraints in the selection operator.

4.2 Reliability maximum BRBDO model

In practical engineering, designers also face another optimization problem, that is, how to reasonably allocate resources for the product design to maximize reliability under given resource constraints. The design optimization model corresponding to this optimization strategy is called the reliability maximum BRBDO model in this paper. The model can be expressed as **Model 3** under the uncertain measure.

Model 3:

$$\begin{cases} \max\limits_{\boldsymbol{d}} \ \ \wedge_{k=1}^m \mathcal{M}\{G_k(\boldsymbol{X},\boldsymbol{P}) \geq 0\} \\ \text{subject to} \\ \quad C(\boldsymbol{d}) \leq C^t \\ \quad \boldsymbol{d}^L \leq \boldsymbol{d} \leq \boldsymbol{d}^U. \end{cases} \tag{19}$$

(1) Monotonic case

When the performance margin functions in the model are strictly monotonic with respect to $\boldsymbol{X}$ and $\boldsymbol{P}$, where $X_1,X_2,\cdots,X_{n_X},P_1,P_2,\cdots,P_{n_P}$ are independent uncertain variables, the objective function of **Model 3** can be transformed into an equivalent expression according to uncertainty theory. Specifically, if the performance margin function $G_k(\boldsymbol{X},\boldsymbol{P})$ is continuous, and strictly increasing with respect to $X_1,X_2,\cdots,X_r,P_1,P_2,\cdots,P_s$ and strictly decreasing with respect to $X_{r+1},X_{r+2},\cdots,X_{n_X},P_{s+1},P_{s+2},\cdots,P_{n_P}$, then **Model 3** can be equivalently expressed as **Model 4**.

Model 4:

$$\begin{cases} \max\limits_{\boldsymbol{d}} \ \ \wedge_{k=1}^m \{\alpha_k \mid G_k(\Phi_1^{-1}(1-\alpha_k),\cdots,\Phi_r^{-1}(1-\alpha_k), \\ \qquad \Phi_{r+1}^{-1}(\alpha_k),\cdots,\Phi_{n_X}^{-1}(\alpha_k),\Psi_1^{-1}(1-\alpha_k),\cdots \\ \qquad \Psi_s^{-1}(1-\alpha_k),\Psi_{s+1}^{-1}(\alpha_k),\cdots,\Psi_{n_P}^{-1}(\alpha_k)\Big)=0\} \\ \text{subject to} \\ \quad C(\boldsymbol{d}) \leq C^t \\ \quad \boldsymbol{d}^L \leq \boldsymbol{d} \leq \boldsymbol{d}^U. \end{cases} \tag{20}$$

Apparently, **Model 4** is also a deterministic optimization model that can be solved by commonly used optimization algorithms.

(2) Non-monotonic case

Similarly, in non-monotonic cases, FOBRA method is utilized to analyze the belief reliability in each iteration. Considering the positive correlation between the BRQI and belief reliability, the BRQI is used to replace the belief reliability objective function in **Model 3**, i.e., the objective function is replaced by

$$\wedge_{k=1}^m \gamma_k. \tag{21}$$

Then, a typical solving method of the new **Model 3** could be the genetic algorithm with a fitness function established using FOBRA method.

5. Case study

5.1 Cantilever beam structure

In this case study, a cantilever beam structure (as shown in Fig. 2) with a single monotonic performance margin function is studied. The reliability analysis of the structure was first performed by the FOBRA method in Section 5.1.1, which verified the accuracy of the FOBRA method. Then, resource minimum design optimization and reliability maximum design optimization of the structure are studied respectively in Section 5.1.2 and 5.1.3 to show the accuracy and efficiency of the FOBRA-based design optimization method.

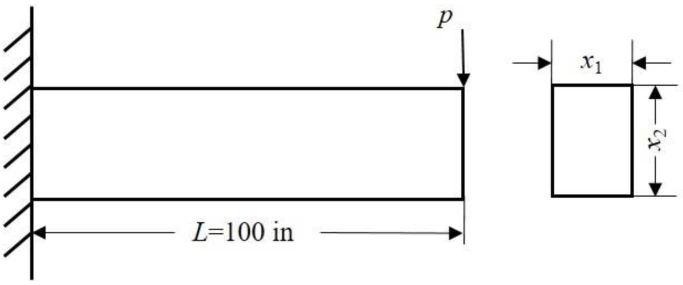

Fig. 2. Cantilever beam structure.

5.1.1 Reliability analysis of the cantilever beam structure

In this case, we only consider the maximum displacement of the cantilever beam structure under the load P as the key performance. Then, the performance margin function can be written as:

$$G = D - \left(\frac{4L^3}{Ex_1x_2}\right) \times \sqrt{\left(\frac{p_1}{x_2^2}\right)^2 + \left(\frac{p_2}{x_1^2}\right)^2}, \qquad (22)$$

where the length of the beam L=100 *in*, the allowable maximum displacement D=2.5 *in*, and the rest of the parameters are regarded as uncertain variables because of epistemic uncertainty, whose meaning and distribution information are shown in Table 1. As an additional note, the mean and standard deviation of the uncertain variables can be obtained by the graduation formula [36].

Table 1 Uncertain variable information of the cantilever beam structure

Uncertain variable	Distribution type	Mean	Std. dev
Load lateral p_1 (*lb*)	Normal	1000	100
Load vertical p_2 (*lb*)	Normal	500	100
Young's modulus E (*psi*)	Normal	29×10^6	1.45×10^6
Width x_1 (*in*)	Normal	3	0.3
Thickness x_2 (*in*)	Normal	4	0.4

According to the FOBRA algorithm, BRQI of the cantilever beam structure is $\gamma = 1.5499$. According to Equation (4), the structural belief reliability estimated by BRQI can be obtained as

$$R_{\text{estimated}} = 1 - \left(1 + exp\left(\frac{\pi\gamma}{\sqrt{3}}\right)\right)^{-1} = 0.9433.$$

Further, since the performance margin function is strictly monotonic with respect to all uncertain variables, the theoretical solution of its belief reliability can also be calculated by the operational law of uncertainty theory as

$$\begin{aligned} R_{\text{theory}} &= \mathcal{M}\{G \geq 0\} \\ &= \alpha \Big| \left\{ D - \left(\frac{4L^3}{\Phi_E^{-1}(1-\alpha)\cdot\Phi_{x_1}^{-1}(1-\alpha)\cdot\Phi_{x_2}^{-1}(1-\alpha)}\right) \sqrt{\left(\frac{\Phi_{p_1}^{-1}(\alpha)}{\Phi_{x_2}^{-1}(1-\alpha)^2}\right)^2 + \left(\frac{\Phi_{p_2}^{-1}(\alpha)}{\Phi_{x_1}^{-1}(1-\alpha)^2}\right)^2} = 0 \right\} \\ &= 0.9430. \end{aligned}$$

Among them, Φ_E^{-1}, $\Phi_{x_1}^{-1}$, $\Phi_{x_2}^{-1}$, $\Phi_{p_1}^{-1}$ and $\Phi_{p_2}^{-1}$ are the inverse distribution functions of E, x_1, x_2, p_1 and p_2, respectively. The relative error between the approximate solution obtained by FOBRA and the theoretical analytical solution is 0.0318%.

In terms of computation time, the FOBRA algorithm takes 0.05 milliseconds and the analytical solution takes 0.04 seconds under the same hardware configuration. In addition, the uncertain simulation algorithm proposed in [35] takes 13.52 seconds at a sampling size of 1e5, which is much higher than the previous two. Obviously, FORBA algorithm is much more efficient than uncertain simulation algorithm. This gap will be even larger in design optimization due to the large number of calls to the reliability analysis process.

In order to further verify the accuracy and robustness of FOBRA, the reliability assessment results of the structure are investigated when the parameters vary within a certain range. The calculation results of BRQI, the theoretical value of reliability, and the estimated value of reliability are shown in Fig. 3. Among them, d_1 and d_2 are the mean values of x_1 and x_2 , the theoretical value is calculated according to the operational law of uncertainty theory, and the estimated value is the approximate belief reliability calculated by Equation (4). It can be found that the FOBRA method is very accurate at low levels of structural reliability. As the reliability level of the structure becomes higher, the FOBRA method slightly overestimates the belief reliability of the structure. Overall, the accuracy and robustness of FOBRA is satisfactory.

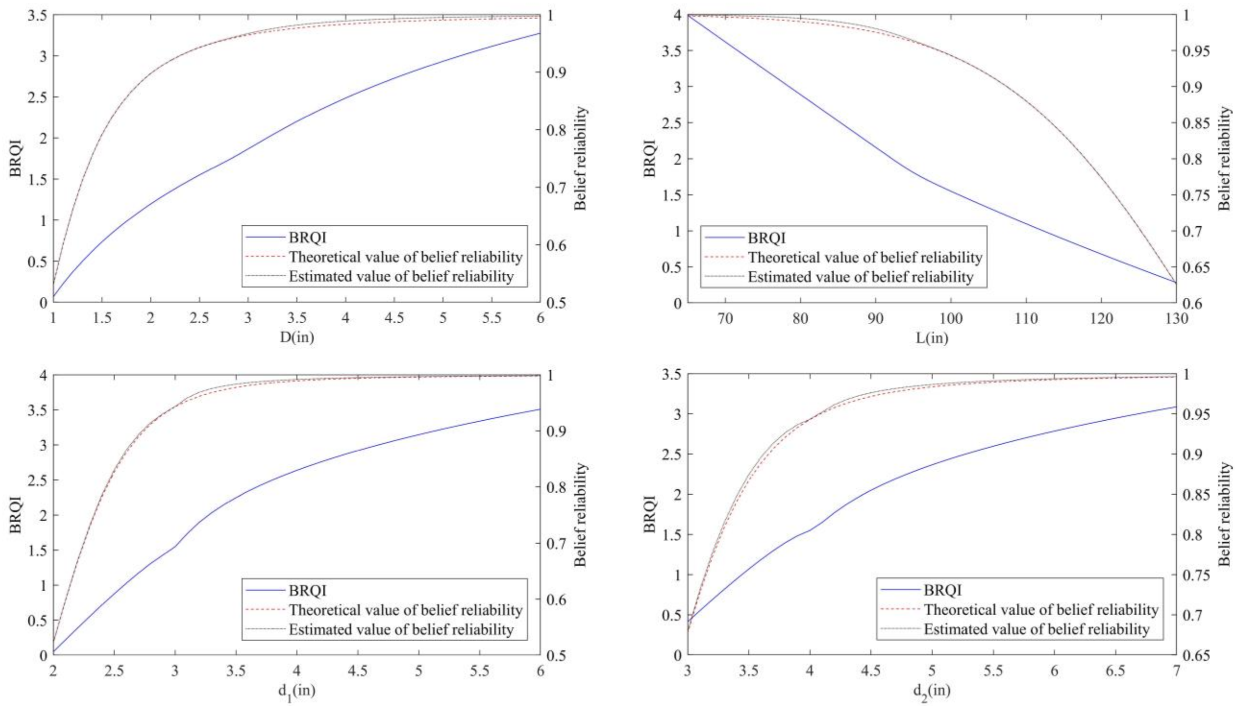

Fig. 3. Belief reliability analysis results of the cantilever beam.

5.1.2 Resource minimum design optimization

The resource minimum design optimization problem of the cantilever beam structure is first studied. The FOBRA-based resource minimum design optimization model can be established as follows:

$$\begin{cases} \min \quad A = d_1 \cdot d_2 \\ \text{subject to} \\ \quad \gamma(x_1, x_2, p_1, p_2, E, D, L) \geq \gamma^t \\ \quad \gamma^t = \frac{\sqrt{3}}{\pi} ln((1-R^t)^{-1}-1) \\ \quad 0 \leq d_1 \leq 100 \\ \quad 0 \leq d_2 \leq 100. \end{cases} \tag{23}$$

In this problem, the design variables are the mean values of x_1 and x_2 represented as d_1 and d_2, which are required to be not greater than 100 *in*, the constraint is the specified quantile index acquired from the specified belief reliability R^t, and the objective function is the cross-sectional area of the cantilever beam (denoted as *A*).

Table 2 shows the optimization results of the above model under different R^t. Because the performance margin function of the product is strictly monotonic, it can be solved by the equivalent deterministic model (refer to **Model 2**). In Table 2, the optimal solution obtained by this method is used as the reference optimal solution to evaluate the relative error.

Table 2. Results of the resource minimum design optimization of the cantilever beam.

R^t	Optimal solution via FOBRA			Optimal solution via equivalent model			Relative error
	d_1(*in*)	d_2(*in*)	A(in^2)	d_1(*in*)	d_2(*in*)	A(in^2)	
0.999	3.814	4.199	16.02	3.993	5.140	20.52	21.93%
0.99	3.372	4.327	14.59	3.415	4.524	15.45	5.57%
0.98	3.285	4.109	13.50	3.206	4.385	14.06	3.98%
0.97	3.230	3.979	12.85	3.071	4.325	13.28	3.24%
0.95	3.133	3.838	12.02	2.952	4.161	12.28	2.12%
0.9	2.955	3.673	10.85	2.865	3.812	10.92	0.64%

The results also indicate that the resource minimum optimal solutions solved via FOBRA are comparatively accurate when the specified belief reliability is low, and there may be a larger error when the specified belief reliability becomes higher. This is mainly because the optimization process with highly specified belief reliability may enlarge the error caused by nonlinearity in FOBRA, thus affecting the precision.

In terms of the time cost, under the same hardware configuration and genetic algorithm parameter setting, the average solution time via FOBRA is 0.12 seconds, while the average solution time via the equivalent deterministic model is 83.03 seconds. Results show that the FOBRA-based design optimization model has significantly higher solution efficiency. The main reason for the efficiency gap is that the reliability assessment in the equivalent deterministic model solution requires solving transcendental equations, which is much more computationally intensive than FOBRA.

5.1.3 Reliability maximum design optimization

The reliability maximum design optimization problem of the cantilever beam structure is also studied. The FOBRA-based reliability maximum design optimization model can be established as follows:

$$\begin{cases} \max \quad \gamma(x_1, x_2, p_1, p_2, E, D, L) \\ \text{subject to} \\ \qquad d_1 \cdot d_2 \leq A^t \\ \qquad 0 \leq d_1 \leq 100 \\ \qquad 0 \leq d_2 \leq 100. \end{cases} \tag{24}$$

In this problem, the design variables are the mean values of x_1 and x_2 represented as d_1 and d_2, which are required to be not greater than 100 *in*, the constraint is the specified cross-sectional area denoted as A^t, and the objective function is the BRQI of the cantilever beam.

Table 3 shows the optimization results of the above model under different A^t. Similar to the previous problem, the results of the equivalent deterministic model are also used as the reference optimal solution. Table 3 indicates that the optimal solutions solved via FOBRA are very accurate, with relative errors of less than 1%.

Table 3. Results of the reliability maximum design optimization of the cantilever.

A^t (in^2)	Optimal solution via FOBRA				Optimal solution via equivalent model			Relative error
	d_1(*in*)	d_2(*in*)	γ	R	d_1(*in*)	d_2(*in*)	R	
11	2.747	4.004	1.243	0.9050	2.896	3.798	0.9038	0.13%
12	3.114	3.854	1.616	0.9494	2.981	4.025	0.9429	0.69%
13	3.220	4.038	1.970	0.9727	3.070	4.235	0.9657	0.72%
14	3.320	4.217	2.320	0.9853	3.258	4.297	0.9796	0.58%
15	3.391	4.423	2.682	0.9923	3.496	4.290	0.9873	0.51%
16	3.792	4.217	3.669	0.9987	3.501	4.570	0.9923	0.64%

In addition, under the same hardware configuration and genetic algorithm parameter setting, the average solution time via FOBRA is 0.20 seconds, while the average solution time via the equivalent deterministic model is 38.40 seconds. The efficiency of optimization based on the FORBA algorithm still has a significant advantage.

5.2 Reliability design optimization of vehicle side impact

In order to verify that the proposed method is applicable to complex non-monotonic situations, a multi-dimensional vehicle side impact design optimization problem is studied in this case [31].

The design objective is to minimize the vehicle weight (denoted as W) under given safety constraints of the side impact. The information of all variables and several deterministic parameters (denoted as $x_1 \sim x_{11}$) are listed in Table 4. The design variables are the mean values of 7 variables (denoted as $d_1 \sim d_7$) that are related to weight and safety. And they all have corresponding design upper and lower bounds. Because of the limitation of data and knowledge of these variables and parameters, they are regarded as normal uncertain variables in this paper.

Table 4. Parameters of the vehicle side impact design optimization problem.

Variable	Std. dev	Lower limit	Mean value	Upper limit
B-Pillar inner x_1	0.03	0.5	d_1	3
B-Pillar reinforcement x_2	0.03	0.45	d_2	2.7
Floor side inner x_3	0.03	0.5	d_3	3
Cross members x_4	0.03	0.5	d_4	3
Door beam x_5	0.05	0.875	d_5	5
Door belt line reinforcement x_6	0.03	0.4	d_6	3

Variable	Std. dev	Lower limit	Mean value	Upper limit
Roof rail x_7	0.03	0.4	d_7	3
Parameter	Std. dev	Mean value		
Material of B-Pillar inner x_8	0.006	0.345		
Material of floor side inner x_9	0.006	0.192		
Barrier height x_{10}	10	0		
Barrier hitting position x_{11}	10	0		

The finite element model shown in Fig.4 is used to simulate the side impact, where the vehicle was impacted by an obstacle with an initial velocity of 33 mph.

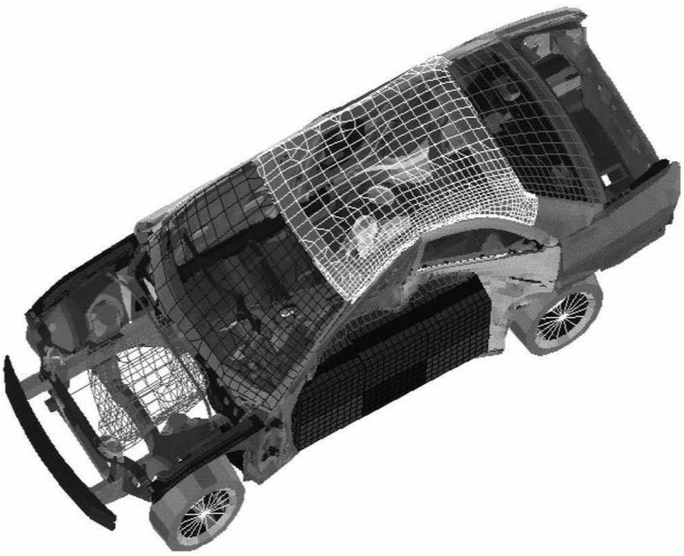

Fig.4. Finite element model of vehicle side impact. [31]

Using a series of response surface functions obtained by finite element analysis as the performance margin functions of the vehicle side impact structure, the design optimization model can be established as follows:

$$\begin{cases} \min \quad W = 1.98 + 4.9d_1 + 6.67d_2 + 6.98d_3 + \\ \qquad\qquad 4.01d_4 + 1.78d_5 + 2.73d_7 \\ \text{subject to} \\ \quad \mathcal{M}\{F_{\text{AL}} \le 1\text{kN}\} \ge R^t \\ \quad \mathcal{M}\{D_{\text{up}} \le 32\text{cm}\} \ge R^t \\ \quad \mathcal{M}\{D_{\text{mid}} \le 32\text{cm}\} \ge R^t \\ \quad \mathcal{M}\{D_{\text{low}} \le 32\text{cm}\} \ge R^t \\ \quad \mathcal{M}\{VC_{\text{up}} \le 0.32\text{cm}\} \ge R^t \\ \quad \mathcal{M}\{VC_{\text{mid}} \le 0.32\text{cm}\} \ge R^t \\ \quad \mathcal{M}\{VC_{\text{low}} \le 0.32\text{cm}\} \ge R^t \\ \quad \mathcal{M}\{F_{\text{ps}} \le 4.0\text{kN}\} \ge R^t \\ \quad \mathcal{M}\{V_{B-\text{Pillar}} \le 9.9\text{m/s}\} \ge R^t \\ \quad d_i^L \le d_i \le d_i^U, i = 1,2,\cdots,7, \end{cases} \tag{25}$$

where

$$\begin{aligned} F_{\text{AL}} &= 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 \\ &\quad + 0.01343x_6x_{10} \\ D_{\text{up}} &= 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} \\ &\quad + 6.63x_6x_9 - 7.77x_7x_8 + 0.32x_9x_{10} \\ D_{\text{mid}} &= 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11x_2x_8 \\ &\quad - 0.0215x_5x_{10} - 9.98x_7x_8 + 22x_8x_9 \\ D_{\text{low}} &= 46.36 - 9.9x_2 - 12.9x_1x_8 + 0.1107x_3x_{10} \\ VC_{\text{up}} &= 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 \\ &\quad + 0.0144x_3x_5 \\ VC_{\text{mid}} &= 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 \\ &\quad + 0.03099x_2x_6 + 0.018x_2x_7 + 0.0208x_3x_8 \\ &\quad + 0.121x_3x_9 - 0.00364x_5x_6 \\ &\quad + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} \\ &\quad + 0.00121x_8x_{11} + 0.00184x_9x_{10} \\ &\quad - 0.02x_2x_2 \\ VC_{\text{low}} &= 0.74 - 0.61x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} \\ &\quad - 0.166x_7x_9 + 0.227x_2x_2 \\ F_{\text{ps}} &= 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} \\ &\quad + 0.009325x_6x_{10} + 0.000191x_{11}x_{11} \\ V_{\text{B-Pillar}} &= 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} \\ &\quad - 0.0198x_4x_{10} + 0.028x_6x_{10}. \end{aligned} \tag{26}$$

Since the performance margin functions in the optimization model are all non-monotonic functions, the model cannot be transformed into an equivalent deterministic model. In addition, the computational cost of using an uncertain simulation algorithm to calculate complex multi-dimensional constraints would be unbearable. Therefore, the FOBRA-based design optimization method proposed in this paper would be a more suitable option.

According to the resource minimum BRBDO model in non-monotonic case, the reliability constraints in the model are transformed into BRQI constraints calculated by FOBRA with $R^t = 0.99$. The optimal design solution acquired by the genetic algorithm is shown in Table 5.

Table 5. Optimization result of the vehicle side impact structure.

Method	Design solution ***d***	*W*
Belief reliability-based design optimization	(0.839,1.434,0.5,1.368,0.875,1.589,0.4)	27.2790
Probabilistic analysis-based design optimization	(0.592,1.337,0.5,1.273,0.875,1.665,0.4)	25.0374

Furthermore, in order to show the effect of the category of uncertainty on design optimization, the optimal solution of the design optimization problem based on probabilistic analysis is also computed under the same parameter setting with normal probability distributions. The result is obtained by using FORM to calculate reliability constraints and genetic algorithm for outer optimization, as shown in Table 5. The results indicate that under the same parameter condition, the optimal solution of BRBDO requires more resources (minimum weight of 27.2790) than probabilistic analysis-based design optimization (minimum weight of 25.0374). In other words, more design margin is required when the product design is influenced by epistemic uncertainty. This is mainly because the consideration of epistemic uncertainty usually makes us inclined to make more conservative decisions, thus allowing the design solution to sacrifice some weight to ensure the level of reliability.

6. Conclusion

Considering the advantages of uncertainty theory in dealing with epistemic uncertainty, this paper studies the reliability design optimization problem for products affected by epistemic uncertainty on the basis of belief reliability theory. The main conclusions are as follows:

1) Based on the definition of belief reliability quantile index, a method for rapid reliability approximate assessment of products affected by epistemic uncertainty, FOBRA, is proposed. It has been proved that the method can effectively assess the belief reliability of products with high accuracy.
2) Considering different trade-off strategies, FOBRA-based resource minimum design optimization model and reliability maximum design optimization model are proposed respectively. The cases show that FOBRA-based reliability maximum design optimization model has very high accuracy, while FOBRA-based resource minimum design optimization model has high accuracy only at low reliability levels.
3) Both FOBRA and the corresponding design optimization methods have very high computational efficiency, representing a huge saving in computational time over other methods.

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