Eksploatacja i Niezawodnosc – Maintenance and Reliability
Volume 25 (2023), Issue 2
journal homepage: http://www.ein.org.pl

Article citation info:
Gao W, Wang Y, Zhang X, Wang Z, Quasi-periodic Inspection and Preventive Maintenance Policy Optimisation for a system with Wiener Process degradation. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2023: 25(2) http://doi.org/10.17531/ein/162433

Quasi-periodic Inspection and Preventive Maintenance Policy Optimisation for a system with Wiener Process degradation

Indexed by: Web of Science Group

Wenke Gao[a,*], Yu Wang[b], Xiwen Zhang[c], Zhuangzhuang Wang[c]

[a] SINO-German college of intelligent manufacturing, Shenzhen technology university, Shenzhen, China, 518118
[b] SANY Heavy Machinery Limited.Kunshan, China
[c] School of energy and power engineering, Lanzhou university of technology, Lanzhou, China,

Highlights

- Implementation of quasi-periodic inspection and maintenance (QI&M) for a degrading system.
- Development of a model to optimize the degradation threshold and maintenance for QI&M.
- Example on degradation of a piston pump and obtained the impact on optimal QI&M.

Abstract

Periodic inspection policy is performed for some degradation systems to check their degradation states, whereas it is usually difficult to implement on time due to impact of some random factors. Inspections and some maintenance actions are implemented in an inspection window with random, and thus how to optimize the inspection windows and the degradation threshold of the system to perform preventive maintenance (PM) are beneficial in practice. To this end, an optimisation of quasi-periodic inspection and PM policy with inspection window is proposed for a degradation system whose degradation followed Wiener process with a linear drift. Assume that PM can change the degradation rate and inspections are randomly performed in each inspection window. After optimisation, the optimal interval of the inspection window, the degradation threshold of PM and PM policy are determined by minimising the long-term running cost rate of the system. Finally, modeling and optimisation are illustrated using the degradation process of an axial piston pump, and the sensitivity analysis of some key parameters is conducted.

Keywords
Preventive maintenance; Wiener process; Quasi-periodic inspection policy; Axial piston pump.

1. Introduction

Degradation modeling has been widely studied to evaluate the reliability in some electronics manufacturing or mechanical systems. The performance index of the degenerate system is usually assumed to be a quality characteristic that the degradation is resulted from a gradual accumulation of damage in a system's life cycle [6, 31, 33]. The failure occurs when the accumulative degradation reaches a specified threshold which is defined in terms of the requirement of running situation for the system. The measurement values are called the degradation data which are random values and contain rich reliability information. These data are used to evaluate the system reliability in some real cases. Stochastic processes are usually applied to describe the system degradation. Wherein the Wiener process is one of these stochastic processes and some maintenance policies are optimised based on the developed model using the measurement data [25, 34, 37]. Measurement errors also considered to enhance the model accuracy in the acceleration degradation process and remaining useful life prediction [10, 26]. As one approach to obtain the degradation data, inspection actions are conducted to identify the real-time degradation of the system, and maintenance decision-making is performed when the degradation value reaches a defined threshold. Periodic and sequential inspections have been widely studied, and used in some tests of electronic system [5, 16, 23, 28]. However, the

(*) Corresponding author.
E-mail addresses: W. Gao (ORCID:0000-0003-3590-1995) email:gaowenke@sztu.edu.cn, Y. Wang, 18393813260@163.com, X. Zhang 974172488@qq.com, Z Wang 863538914@qq.com

inspection action of some systems is unable to be implemented on time due to some random factors, such as transportation cycles, maintenance ability and supply of some spare parts, which makes a periodic inspection to be a random event distributed in each inspection window, and also renders the periodic inspection to be a quasi-periodic event. Therefore, how to obtain the optimal quasi-periodic inspection policy with an inspection window, determine the optimal PM threshold and corresponding policy for the system with degradation performance needs to be discussed deeply.

Inspection policies have been discussed in the past years. Nakagawa et al. [21] reviewed these research before 2010, and summarized them as periodic and non-inspection. Some developments are made in recent years for periodic inspection policy. Munford and Shahani[20] considered a one-parameter policy which has the property of decreasing (increasing) intervals between successive inspection times if the system has an increasing (decreasing) failure rate. Naoto et al., [12] discussed an inspection policy for the modified inspection model considering the system failure due to any inspection and obtained the nearly optimal inspection policy. Levitin et al. [13] modeled non-repairable systems subject to a delay-time failure process involving hidden and fatal failures in two stages, and scheduled inspections is performed to detect the hidden failure. Driessen et al. [7] studied a mono-unit system that is characterized by three distinct deterioration states, and PM is triggered by a given number of inspection which is periodical imperfect. Cavalcante et al. [4] built a model of inspection of a protection system, in which the inspection outcome provides imperfect information of the state of the system, and the preventive replacement is conducted based on the inspection result. Zhang et al.[36] investigated a PM policy for a three-state system considering both imperfect inspections and imperfect repairs, in which cost rate and reliability of the system are derived by a recursive method. Moakedi et al. [17] modeled a block-based inspection policy for a multi-unit system with stochastic dependence, in which some units may develop a hidden failure and others may experience healthy, defective and revealed failures, and block-based inspection policy was considered to detect and fix both defects and hidden failures. Jin et al. [11] considered an integrated inspection policy, which combines perfect and imperfect inspections and is flexible in that the frequencies of imperfect inspections need not be common for all perfect inspections periods. Zhao et al. [39] developed a maintenance model for a mono-unit system with atypical degradation path, of which the pattern can be influenced by inspections. In the model, the system degradation is assumed to decrease by a random value instantaneously, while the degrading rate is elevated after each inspection. Seyedhosseini et al. [24] introduced an optimal periodic inspection policy for a two-unit system, in which the failure of the first component is hidden and the second component has three work states.

Non-periodic inspection policy, such as sequential or random inspection, is performed for some systems which often work for a job with random working times [21]. Cao et al. [2] proposed a PM model which subjects to sequential inspection for a three-stage failure process, and two-level sequential inspections, postponed maintenance and opportunistic maintenance are considered in the PM model. Babishin et al., [1] introduced a k-out-of-n:G system whose components subject to soft and hard failures, both failures are inspected non-periodically and maintained by replacement and minimal repair. Xiao et al., [32] considered a mono-unit system that may fail due to either hard failures or soft failures, and the wait time of the system was utilized to conduct inspections and maintenance. Yue and Gao [35] considered a deteriorating operating system which executes a job with random working times, of which the occurrence of the failure follows Geometric process and is detected by random inspection action. Castro and Landesa [3] introduced of a new model to characterize the dependence between degradation processes and implementation of a condition-based maintenance strategy considering non-periodic inspection times. Zhao and Nakagawa [40] optimised a random inspection policy considering random procedure times, compared it with periodic inspection and computed a modified checking cost for random inspection, and defined first, last and overtime policies for inspections in a decision-making. Raza and Ulansky [22] described a mathematical model of predictive maintenance based on prognostics and health management, proposed a new method to determine the optimal periodicity of predictive inspection, and showed that the predictive maintenance is unconditionally more efficient than corrective maintenance by numerical examples. It can be found that most research considered the inspection interval as a fixed value or as a random time according with the production period. To convenience production management, maintenance window is performed to implement maintenance actions, whereas the influence factors are complex while not single[9]. To consider these influences, maintenance window is viewed as a feasible policy to obtain the performance state of some degradation systems. Mosheiov et al., [19] studied scheduling problems for two-machine flow shop and open shop with a maintenance start window to minimize the makespan. Mahdi et al., [18] focused on specifying maintenance opportunity window (MOW) in job-shop production systems, and developed mathematical models and formulae to determine the MOW. Zhang and Yang [38] proposed a state-based maintenance policy with multifunctional maintenance windows to handle failure mechanisms of a multistate industrial asset with environmental disturbance. In the maintenance window, the inspection can be scheduled according to the short-term requirements of production.

It can be found from above studies that most inspection policies are implemented based on the system running time or its job cycle. The job cycle-based inspection policy considers the production process, though the detection result of the inspection is the same as time-based policy. For running time-based inspection policy, the time interval of the inspection is a fixed or a sequential decrease value. Time-based inspection policy, such as periodic or sequential inspection, is convenience for the management of the maintenance and it yet can find the degradation state of the system. The similarity of these two policies is to inspect the degradation state by the inspection, and the difference is that the inspection interval is fixed or random. Some research studied inspection policy in multi-state system for condition-base maintenance policy, few of them considered inspection policy for continuous degradation system and inspection window. Thus, how to determine the optimal period of some inspection windows and the degradation threshold for

PM are helpful to some real cases. Aiming at the performance degradation system, this study proposes a quasi-periodic inspection and PM policy for a continuous degradation system with inspection window. Assume that the system degradation process is based on the Wiener process with a linear drift, and inspections are implemented in the inspection windows with random. The PM and replacement threshold, PM number and the period of the inspection window are implemented are determined by the maintenance policy optimisation. Finally, the feasibility of the model is illustrated through a real case study on the degradation process of an axial piston pump.

The remaining parts of this study are organized as follows. In Section 2, the system degradation is described by a Wiener stochastic process model. Section 3 is devoted to describe the maintenance policy modelling and optimization. Section 4 provides a real case study about the degradation process of an axial piston pump to illustrate the proposed maintenance policy. Conclusions are drawn in Section 5.

For ease of reference, some notations are stated as follows:

a_j	the PM factor of the jth PM interval
$B(t)$	the standard Brownian motion
C	the total maintenance cost of the replacement cycle
C_i	the inspection cost
C_f	the failure replacement cost.
C_r	the preventive replacement cost
$f(t)$	the probability density function
$g(t)$	the system long-running cost rate function
l	the failure threshold of the system performance
N	the number of PM
$R(t)$	the reliability function
τ_i	the time of the i-th inspection of PM
T	the period of the inspection window
w	The PM threshold of the system performance
W	the length of inspection window
$X(t)$	the system degradation performance
Y_i	The PM interval of the i-th PM
Y	the total length of the replacement cycle
μ	the drift parameter
σ	the diffusion parameter

2. Degradation model

Wiener process is an independent incremental process, and is widely used in the degradation modelling for some systems, of which some performance indexes can be detected by inspections. It can describe the non-monotonic performance degradation process and has good computational analysis ability in the field of engineering [27, 29].

Let the stochastic process $\{X(t): t \in R^+\}$ denote the underlying degradation process of the system over the running time t. Herein, we consider a general linear underlying degradation process, where the degradation state $X(t)$ with a linear drift at t is expressed as:

$$X(t) = \mu t + \sigma B(t) \tag{1}$$

where μ denotes the drift coefficient, σ represents the diffusion coefficient, and $\{B(t): t \geq 0\}$ is the standard Brownian motion process with $\sigma B(t) \sim N(0, \sigma^2 t)$ for $t>0$, which is used to describe time-correlated structure. Wiener process is not monotonously increasing but the mean degradation is linearly increasing in t, i.e., $E(X(t)|\mu)=\mu t$.

The standard Wiener process $\{B(t): t \geq 0\}$ is characterised by the following properties:

(1) $B(0)=0$;

(2) $B(t)$ has independent increments: for every $t>0$, the future increments $B(t+u)-B(t)$, $u \geq 0$, are independent of the past values $B(s)$, $s<t$.

(3) $B(t)$ has Gaussian increments: $B(t+u)-B(t)$ is normally distributed with mean 0 and variance u, $B(t) \sim N(0, t)$;

(4) $B(t)$ has continuous paths: $B(t)$ is continuous in t.

Let $l(l>0)$ be the degradation threshold of the system performance. Assume that the degradation process of the system performance is subject to the above linear Wiener motion with a drift. The system fails and then a replacement is to be performed when the first hitting time of Wiener process $\{X(t), t \geq 0\}$ reaches or exceeds the failure the threshold l, the corresponding lifetime is $\mathscr{L}$, which is defined as

$$\mathscr{L} = inf\{t: X(t) \geq l | X(0) < l\}$$

Assume that the degradation process $\{X(t), t \geq 0\}$ starts at $\hbar$ with $X(\hbar)=x_\hbar$. If the degradation process $\{X(t), t \geq 0\}$ is first hitting the threshold at a certain time $t(t>\hbar)$ exactly, then the probability that such a process crossed the threshold level before time t is assumed to be negligible.

For $\{X(t), t \geq 0\}$, μt is a continuous function of t in $[0,\infty)$, the probability density function(PDF) of $\mathscr{L}$ can be formulated under above assumption as [8, 14, 27],

$$f(t) = \frac{l}{\sqrt{2\pi\sigma^2 t^3}} exp\left(-\frac{(l-\mu t)^2}{2\sigma^2 t}\right) \tag{2}$$

According to the PDF, the reliability function $R(t)$ can be obtained as bellow,

$$R(t) = P(\mathcal{L} > t) = \int_t^\infty f(u)du$$

$$= \Phi\left(-\frac{\mu t - l}{\sigma\sqrt{t}}\right) - exp\left(\frac{2\mu l}{\sigma^2}\right)\Phi\left(\frac{-\mu t - l}{\sigma\sqrt{t}}\right) \tag{3}$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of a standard normal distribution.

Let a degradation test be performed on n systems, and the test degradation data for the i-th equipment at the jth time be recorded as $x_{i,j}$. Then, the increment of measured degradation data for the i-th equipment between the jth and (j-1)th is $\Delta x_{i,j} = x_{i,j} - x_{i,j-1}$, and $\Delta t_{i,j} = t_{i,j} - t_{i,j-1}$. Then, the parameters of Wiener process can be estimated by the maximum likelihood method, and the likelihood function can be written as,

$$L(\mu, \sigma^2) = \prod_{i=1}^{n}\prod_{j=1}^{m_i} \frac{1}{\sqrt{2\sigma^2\pi\Delta t_{ij}}} exp\left(-\frac{(\Delta x_{ij} - \mu\Delta t_{ij})^2}{2\sigma^2\Delta t_{ij}}\right) \tag{4}$$

The maximum likelihood estimates for the drift coefficient μ and the diffusion parameter σ can be obtained as follows,

$$\hat{\mu} = \frac{\sum_{i=1}^{n} x_{im_i}}{\sum_{i=1}^{n} t_{im_i}} \tag{5}$$

$$\hat{\sigma}^2 = \frac{1}{\sum_{i=1}^{n} m_i}\left(\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{(\Delta x_{ij})^2}{\Delta t_{ij}} - \frac{\left(\sum_{i=1}^{n} x_{im_i}\right)^2}{\sum_{i=1}^{n} t_{im_i}}\right) \tag{6}$$

According to above formulas, the reliability model can be obtained by the tested degradation data of the system. Based on the reliability model, some maintenance policies can be developed for the system.

3. Maintenance Policy Modelling and Optimisation

3.1 Introduction and assumptions of maintenance process

For degraded equipment, the degenerative status of a system can be detected by an auto-detection system or an inspection policy. About auto-detection system, we will discuss in our following study. Herein, we discuss inspection policy. As mentioned in the Section 1, a periodic inspection action usually is influenced by some random factors and is implemented in an inspection window. Thus, the periodic inspection plans are randomly distributed in the inspection windows and exhibited as a quasi-periodic inspection. Consequently, the *j*th inspected time is marked as τ_i, the length of the inspection window is W which is a constant, and the length of the inspection interval is T, which starts from t=0 or $t=\tau_i$. A PM is performed when the inspected degradation index reaches the threshold w(w<l). A preventive replacement is performed after (N-1)th PM when the inspected degradation index firstly reaches or exceeds the threshold w. A failure replacement is conducted when the inspected degradation index firstly reaches or exceeds the failure threshold l. The inspection processes of the *i*-th PM interval is shown on Figure 1, where Y_i is the *i*-th PM interval.

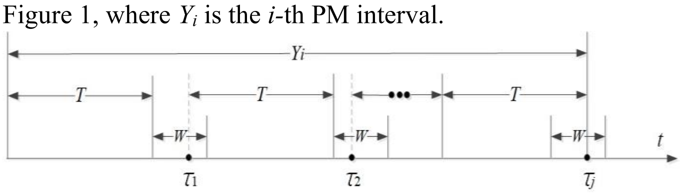

Fig. 1 The *i*-th PM interval.

According to the maintenance process, there are four cases in the maintenance process which are shown as Figure 2-5. Figure 2 exhibits that a failure replacement is performed before the *i*-th (0≤i≤N-1) PM and some quasi-periodic inspections are conducted within each PM interval. It can be found that the degradation index of the (j-1)th inspection $X(\tau_{j-1})$<w and $X(\tau_j)\geq l$ within the *i*-th PM interval. Then, a failure replacement is conducted at τ_R in the *i*-th PM interval.

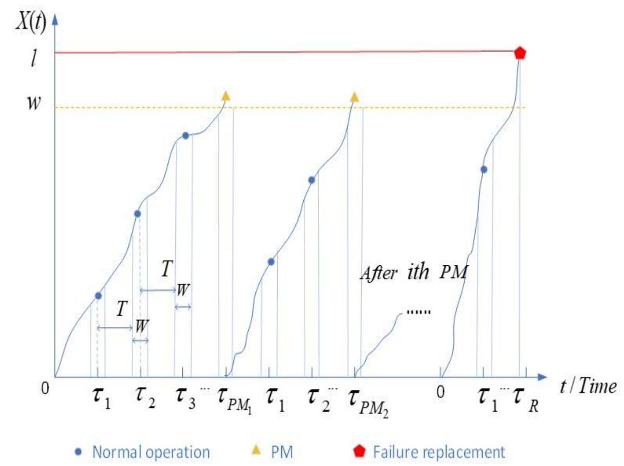

Fig.2 Failure Replacement.

Figure 3 displays that the degradation index within [w, l) can be detected by the last inspection in each PM interval, which triggers i(i=N-1) times PM.

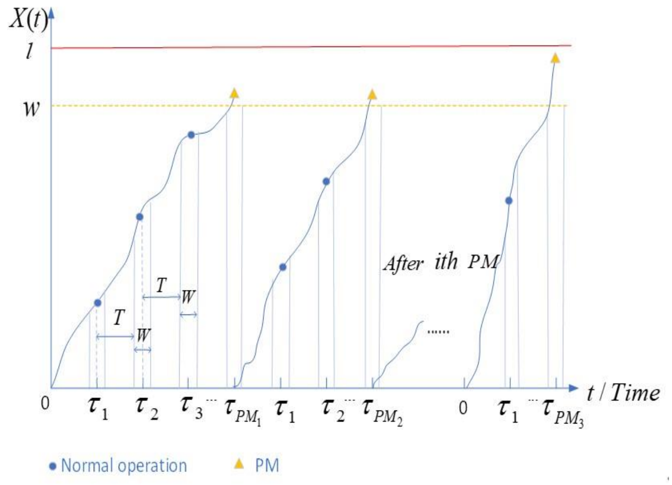

Fig.3 Preventive Maintenance.

After N-1 times PM, two cases may arise: a failure replacement or a preventive replacement. Figure 4 shows a failure replacement is triggered if the degradation index of the *j*th inspection is $X(\tau_j)$>l within the (N-1)th PM interval.

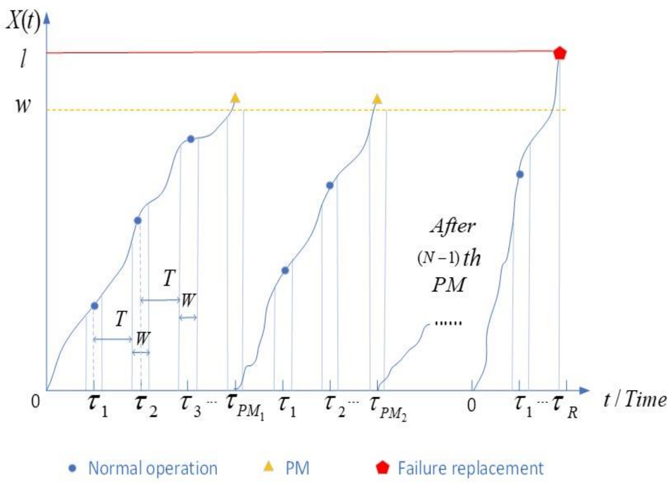

Fig.4 Failure Replacement.

Figure 5 displays that a preventive replacement is conducted if the degradation of the jth inspection is $w\leq X(\tau_j)$<l within the (N-1)th PM interval.

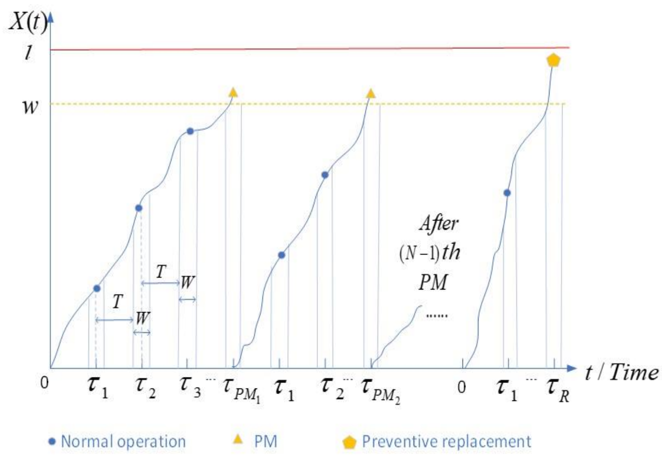

Fig.5 Preventive Replacement.

According to the above description and specifications, the maintenance model for the system is based on the following five assumptions:

1) A quasi-periodic inspection is implemented to obtain the degradation index of the system in each PM interval. The i-th inspection time τ_i is to obey the uniform distribution within the i-th inspection window which has a given length W. The inspection is perfect and can truly reflect the system's degradation state, and the inspection cost is a constant C_i.

2) The system is running normally and maintenance is not necessary if $0 \leq X(t) \leq w$. Additionally, the system fails and is triggered a failure replacement when the degradation index is $X(t) \geq l$. A failure replacement is conducted with a constant cost C_f.

3) A PM is performed if the degradation index of the system is $w \leq X(t) < l$, where w is the threshold of PM. The PM is imperfect and can decrease the degradation index to zero, whereas it can enlarge the degradation rate. The degradation rate of the system before and after a PM meets $\mu_j = a_j \mu$, where μ_j is the drift parameter of the jth PM interval, and a_j is the PM factor of the jth PM interval. The corresponding cost is a constant C_p.

4) A preventive replacement is performed when the system has undergone $N-1$ times PM and firstly reaches the threshold for preventive replacement, and the corresponding cost is a constant C_r.

5) The time for inspection, PM, preventive and failure replacement can be negligible.

3.2 Maintenance Policy Modelling

According to the maintenance process and assumptions, the quasi-periodic inspection and PM policy is considered for the mono-unit repairable system. One replacement for the system can be viewed as one renewing with the related cost. The process can be regarded as a renewal reward process, and we can get the system long-running cost rate according to the renewal reward theory.

$$g(T, w, N) = \lim_{t \to +\infty} \frac{C(t)}{t} = \frac{E(C)}{E(Y)} \tag{7}$$

where $g(\bullet)$ is the system long-running cost rate function. $E(C)$ and $E(Y)$ are the expectation of the maintenance cost and the length of the replacement cycle in the replacement interval. T is the period of the inspection window, w is the threshold value of the system degradation for the PM, and N is the times of PM.

On the basis of the maintenance process and model assumptions, the maintenance cost during the renewal cycle can be expressed as follows:

$$\begin{aligned} C = &\sum_{j=1}^{N-1} \left\{ \prod_{k=1}^{j-1} \begin{pmatrix} P(w < X_k(\tau_1) < l) \\ +P(X_k(V) < w \cap w < X_k(V+\tau_N) < l) \end{pmatrix} \begin{pmatrix} I_{(l<X_j(\tau_1))}(C_f) + I_{(w<X_j(\tau_1)<l)}(C_p) + I_{(X_j(V)<w \cap X_j(V+\tau_N)>l)}\,(C_f + MC_i) \\ +I_{(X_j(V)<w \cap w<X_j(V+\tau_N)<l)}\,(C_P + MC_i) \end{pmatrix} \right\} \\ &+ \prod_{j=1}^{N-1} \begin{pmatrix} P\big(w < X_j(\tau_1) < l\big) \\ +P\big(X_j(V) < w \cap w < X_j(V+\tau_N) < l\big) \end{pmatrix} \begin{pmatrix} I_{(l<X_N(\tau_1))}(C_f) + I_{(w<X_N(\tau_1)<l)}(C_r) + I_{(X_N(V)<w \cap X_N(V+\tau_N)>l)}\,(C_f + MC_i) \\ +I_{(X_N(V)<w \cap w<X_N(V+\tau_N)<l)}\,(C_r + MC_i) \end{pmatrix} \end{aligned} \tag{8}$$

Similarly, the renewal cycle length Y can be denoted in the following:

$$\begin{aligned} Y = &\sum_{j=1}^{N-1} \left\{ \prod_{k=1}^{j-1} \begin{pmatrix} P(w < X_k(\tau_1) < l) \\ +P(X_k(V) < w \cap w < X_k(V+\tau_N) < l) \end{pmatrix} \begin{pmatrix} I_{(l<X_j(\tau_1))}(\tau_1) + I_{(w<X_j(\tau_1)<l)}(\tau_1) + I_{(X_j(V)<w \cap w<X_j(V+\tau_N)<l)}\,(V+\tau_N) \\ +I_{(X_j(V)<w \cap X_j(V+\tau_N)>l)}\,(V+\tau_N) \end{pmatrix} \right\} \\ &+ \prod_{j=1}^{N-1} \begin{pmatrix} P\big(w < X_j(\tau_1) < l\big) \\ +P\big(X_j(V) < w \cap w < X_j(V+\tau_N) < l\big) \end{pmatrix} \begin{pmatrix} I_{(l<X_N(\tau_1))}(\tau_1) + I_{(w<X_N(\tau_1)<l)}(\tau_1) + I_{(X_N(V)<w \cap w<X_N(V+\tau_N)<l)}\,(V+\tau_N) \\ +I_{(X_N(V)<w \cap X_N(V+\tau_N)>l)}\,(V+\tau_N) \end{pmatrix} \\ = &\sum_{j=1}^{N-1} \left\{ \prod_{k=1}^{j-1} \begin{pmatrix} P(w < X_k(\tau_1) < l) \\ +P(X_k(V) < w \cap w < X_k(V+\tau_N) < l) \end{pmatrix} \Big(I_{(w<X_j(\tau_1))}(\tau_1) + I_{(X_j(V)<w \cap w<X_j(V+\tau_N))}(V+\tau_N) \Big) \right\} \\ &+ \prod_{j=1}^{N-1} \begin{pmatrix} P\big(w < X_j(\tau_1) < l\big) \\ +P\big(X_j(V) < w \cap w < X_j(V+\tau_N) < l\big) \end{pmatrix} \Big(I_{(w<X_N(\tau_1))}(\tau_1) + I_{(X_N(V)<w \cap w<X_N(V+\tau_N))}\,(V+\tau_N) \Big) \end{aligned} \tag{9}$$

where $I_Q(Z)$ is an indicator function of the set Q, that is,

$$I_Q(Z) = \begin{cases} 1, & if\ Z \in Q \\ 0, & otherwise \end{cases}$$

τ_i (i=1, 2, ..., M) represents time of the i-th inspection in one inspection window [T, T+W]. Thus, $V = \sum_{i=1}^{M-1} \tau_i$. To compute $g(T,w,N)$, it needs to get the expectation $E(C)$ and $E(Y)$.

$$E(C)=\sum_{j=1}^{N-1}\left\{\begin{array}{l}\prod_{k=1}^{j-1}\left(\begin{array}{l}\frac{1}{W}\int_{T}^{T+W}P\left(w<X_k(\tau_1)<l\right)d\tau_1\\+\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}f_V(V)\left(P\left(X_k(V)<w\cap X_k(V+\tau_N)>l\right)\right)dVd\tau_N\end{array}\right)\\ \left(\begin{array}{l}C_f\frac{1}{W}\int_{T}^{T+W}P\left(l<X_j(\tau_1)\right)d\tau_1+C_p\frac{1}{W}\int_{T}^{T+W}P\left(w<X_j(\tau_1)<l\right)d\tau_1\\+\sum_{M=2}^{+\infty}\left(\left(C_f+MC_i\right)\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}f_V(V)\left(P\left(X_j(V)<w\cap X_j(V+\tau_N)>l\right)\right)dVd\tau_N\right)\\+\sum_{M=2}^{+\infty}\left(\left(C_p+MC_i\right)\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}f_V(V)\left(P\left(X_j(V)<w\cap w<X_j(V+\tau_N)<l\right)\right)dVd\tau_N\right)\end{array}\right)\end{array}\right\}$$

$$+\left(\prod_{j=1}^{N-1}\left(\begin{array}{l}\frac{1}{W}\int_{T}^{T+W}P\left(w<X_j(\tau_1)<l\right)d\tau_1\\+\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}f_V(V)\left(P\left(X_k(V)<w\cap X_k(V+\tau_N)>l\right)\right)dVd\tau_N\end{array}\right)\right)$$

$$\left\{\begin{array}{l}C_f\frac{1}{W}\int_{T}^{T+W}P\left(l<X_N(\tau_1)\right)d\tau_1+C_p\frac{1}{W}\int_{T}^{T+W}P\left(w<X_N(\tau_1)<l\right)d\tau_1\\+\sum_{M=2}^{+\infty}\left(\left(C_f+MC_i\right)\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}f_V(V)\left(P\left(X_N(V)<w\cap X_N(V+\tau_N)>l\right)\right)dVd\tau_N\right)\\+\sum_{M=2}^{+\infty}\left(\left(C_p+MC_i\right)\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}f_V(V)\left(P\left(X_N(V)<w\cap w<X_N(V+\tau_N)<l\right)\right)dVd\tau_N\right)\end{array}\right\}\qquad(10)$$

The calculation of $E(Y)$ is stated as follow:

$$E(Y)=\sum_{j=1}^{N-1}\left\{\begin{array}{l}\prod_{k=1}^{j-1}\left(\begin{array}{l}\frac{1}{W}\int_{T}^{T+W}P\left(w<X_k(\tau_1)<l\right)d\tau_1\\+\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}f_V(V)\left(P\left(X_k(V)<w\cap X_k(V+\tau_N)>l\right)\right)dVd\tau_N\end{array}\right)\\ \left(\begin{array}{l}\frac{1}{W}\int_{T}^{T+W}\tau_1\,P\left(w<X_j(\tau_1)\right)d\tau_1\\+\sum_{M=2}^{+\infty}\left(\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}(V+\tau_N)f_V(V)\,P\left(X_j(V)<w\cap w<X_j(V+\tau_N)\right)dVd\tau_N\right)\end{array}\right)\end{array}\right\}$$

$$+\left(\prod_{j=1}^{N-1}\left(\begin{array}{l}\frac{1}{W}\int_{T}^{T+W}P\left(w<X_j(\tau_1)<l\right)d\tau_1\\+\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}f_V(V)\left(P\left(X_k(V)<w\cap X_k(V+\tau_N)>l\right)\right)dVd\tau_N\end{array}\right)\right)$$

$$\left(\begin{array}{l}\frac{1}{W}\int_{T}^{T+W}\tau_1\,P\left(w<X_N(\tau_1)\right)d\tau_1\\+\sum_{M=2}^{+\infty}\left(\frac{1}{W}\int_{T}^{T+W}\int_{(M-1)T}^{(M-1)(T+W)}(V+\tau_N)f_V(V)\,P\left(X_N(V)<w\cap w<X_N(V+\tau_N)\right)dVd\tau_N\right)\end{array}\right)\qquad(11)$$

According to the model assumptions, $V=\sum_{i=1}^{M-1}\tau_i$ is the sum of $M-1$ independent random variables subjected to the uniform distribution, and thus the PDF of V can be expressed as follows:

$$f_V=\begin{cases}\frac{1}{W^M(M-1)!}\\ \sum_{r=0}^{k}(-1)^r\binom{M}{r}(V-(M-1)T-rW)^{M-1},\\ kW+MT\le V\le(k+1)W+MT,\ k=0,1,\cdots,M-1\\ 0,\qquad \text{Others}\end{cases}$$

(12)

The probability can be calculated as Eq.(13) if the degradation index of the system exceeds the failure threshold at the Mth inspection τ_M and less than the PM threshold w at the (M-1)th inspection τ_{M-1}.

$$P\left(X(V)<w\cap X(V+\tau_M)>l\right)=\left(1-F_{T_2}(V)\right)\left(\int_0^w F_{l-x}(\tau_M)f_\phi(x;V)dx\right)\qquad(13)$$

where,

$$F_{l-x}(\theta)=\Phi\left(\frac{a_j\mu\theta-(l-x)}{\sigma\sqrt{\theta}}\right)+exp\left(\frac{2a_j\mu(l-x)}{\sigma^2}\right)\Phi\left(\frac{-a_j\mu\theta-(l-x)}{\sigma\sqrt{\theta}}\right)$$

(14)

$$f_\phi(x;t)=\frac{1}{\sqrt{2\pi\sigma^2t}}exp\left(-\frac{\left(x-a_j\mu t\right)^2}{2\sigma^2t}\right)\qquad(15)$$

If the degradation index of the system is between the PM threshold w and the failure replacement threshold l at the Mth inspection. Then, the probability can be calculated as

$$P\left(X(V)<w\cap w<X(V+\tau_M)<l\right)=\left(1-F_{T_2}(V)\right)\int_0^w\left(F_{w-x}(\tau_M)-F_{l-x}(\tau_M)\right)f_\phi(x;V)dx\qquad(16)$$

where

$$F_{w-x}(\theta)=\Phi\left(\frac{a_j\mu\theta-(w-x)}{\sigma\sqrt{\theta}}\right)+exp\left(\frac{2a_j\mu(w-x)}{\sigma^2}\right)\Phi\left(\frac{-a_j\mu\theta-(w-x)}{\sigma\sqrt{\theta}}\right)\qquad(17)$$

Considering that $g(T,w,N)$ is complicated function, the optimal

T^*, w^* and N^* is difficult to determine via an analytical method. To avoid the derivation and differential operation for $g(N, T, w)$, Hooke–Jeeves algorithm is used here to determine the optimal T^*, and w^*. Details of the algorithm readers can refer references [15].

The solving process is shown in the following briefly.

Step 1: Give a known N beginning from 2 to a limited number (N_{max}), and use the Hooke–Jeeves algorithm to find the optimal solution T^*, w^* and $g_{min}(T^*, w^*; N)$.
Step 2: Set N=N+1, and use the Hooke–Jeeves algorithm to determine the optimal T^*, w^* and $g_{min}(T^*, w^*; N)$.
Step 3: Go to step 2 if N<N_{max}.
Finally, Fine the smallest $g_{min}(N,T^*,w^*)$ for different N, and determine the optimal N^*.

4. Case Study and Analysis

4.1 Case Study

In this case, we take the performance degradation of a hydraulic axial piston pump as an example to illustrate the proposed model. As an important power element in a hydraulic system, the hydraulic axial piston pump commonly experiences wear in its three main friction pairs. The oil film becomes unstable with the wear increases, thereby increasing the internal leakage and reducing the volume efficiency of the axial piston pump, which fails once the oil leakage reaches the threshold. The wear process is a degradation process and is difficult to measure directly, whereas the total leakage oil can be characterized by the return oil of the axial piston pump. Measurement of the return oil shows that it is random values and independently incremented on the basis of running time. Wang et al.[30] tested the degradation process of the axial piston pump by return oil, and predicted its remaining useful life. Using the tested degradation data, the degradation model is developed by Wiener process with a linear drift, where μ=0.0024,σ^2=1.2067e-04.

Assume that the running state of the axial piston pump can be obtained via some quasi-periodic inspections in the inspection windows, in which inspection randomly distributes and followed the uniform distribution. A PM is performed when the return oil is between the PM threshold w and the failure threshold l. A failure replacement is performed when the return oil firstly reaches or exceeds the failure threshold, and a preventive replacement is performed after the axial piston pump experiences $N-1$ times PM. Therefore, the inspection and maintenance policy is followed the proposed model above.

Tab.1 Model parameters.

Items	Parameters	remarks
C_r	900¥	/
C_f	2600¥	/
C_i	30¥	/
C_p	300¥	/
W	20h	/
l	2.8l/min	[30]
a_j	1+0.3j (j=1,…,)	Assumption

Some parameters used in the proposed model are listed in Table 1. Then, the three-dimension of (g,w,T) is shown as Fig.6, from which the existence of g_{min} can be found directly.

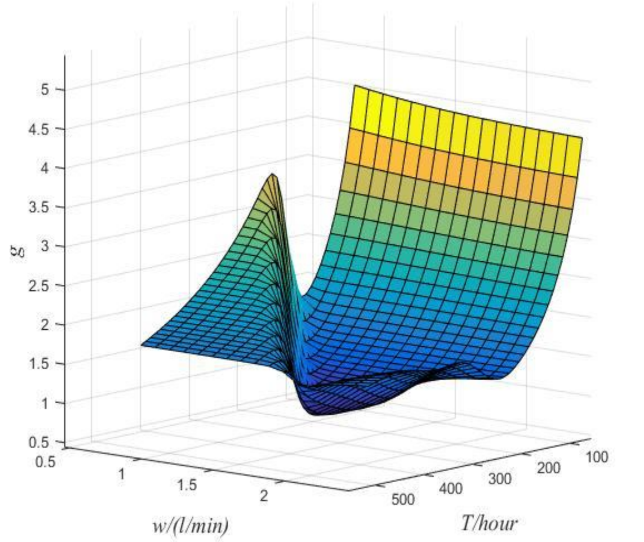

Fig. 6 Plot of g-w-T.

The optimal T^*, w^* and N^* can be obtained using the search algorithm mentioned above. The optimal result is shown in Table 2. The optimal period of the inspection window is T^*=310h, the PM threshold is w^*=1.8L/min, and the PM number is N^*=4. The minimum cost rate is g_{min}=0.9231. The results mean that the axial piston pump can be inspected in each inspection window within the length W=20h randomly, the period of inspection window is 310h, the axial piston pump is preventively maintained if the return oil reaches 1.8L/min and less than 2.8L/min, and it is replaced when PM number reaches 4 or return oil reaches or exceeds 2.8L/min firstly.

Tab.2 Optimisation results.

Items	g_{min}	T^*(h)	w^*(L/min)
N=2	1.0672	360	1.6
N=3	0.9928	340	1.7
N=4	0.9231	310	1.8
N=5	1.1124	240	2
N=6	1.1144	240	2

4.2 Sensitivity analysis

In this case, W a_j, and l are three key parameters in the model, and thus we mainly discuss the sensitivity analysis for the optimal results. Results showed that the minimum cost rate g_{min}, the optimal PM number N^*, the optimal PM threshold w^* and the optimal inspection window period T^* can be influenced by the change of W and a_j, and l within a certain range.

W is the one of the parameters that can be changed by user, a reasonable W should be determined based on the above analysis and requirement of users. The optimal results with different W are computed in Table 3, we can find that g_{min} is increasing and T^* is decreasing with the increase of W clearly, w^* and N^* are stable with the change of W. That is to say, the increase of W narrows the period of the inspection window, whereas it enlarges the failure risk and indirectly causes the increment of the system long running cost rate. Thus, users should reduce the length of W as much as possible.

Tab.3 Relationship between W and T*/w*/N*/gmin.

W	T^*	w^*	N^*	g_{min}
10	322	1.8	4	0.9214
20	310	1.8	4	0.9231
40	302	1.8	4	0.9285
60	288	1.8	4	0.9371

a_j describes the effect of PM and is an assumed parameter. From Table 4, it can be found that g_{min} and w^* are increasing, N^* and T^* are decreasing rapidly with the increase rate of a_j. Decreasing of N^* means that the whole running time of the axial piston pump is shorten. Reduction of T^* represents that the frequency of the inspection is increased. These results display that improving the PM effect is undoubtedly beneficial for users.

Tab.4 Optimisation results with different a_j.

a_j	g_{min}	N^*	w^*	T^*
1+0.20j	0.9158	5	1.7	342
1+0.25j	0.9322	4	1.7	323
1+0.30j	0.9231	4	1.8	310
1+0.50j	0.9654	3	2.0	244

The failure threshold l is defined by user according to the requirements of the system. In this work, l is the flow of the return oil in the axial piston pump, which is in inverse proportionto the volumetric efficiency of the axial piston pump. l usually is relatively small for aviation field and is relatively large in the civilian equipment. The influence of l on optimal results is shown as Table 5. It can be found that g_{min} is decreasing, w^* and T^* are increasing with the rise of l, whereas N^* remains stable. According these results, a small l need pay a high long running cost rate of the system.

Tab.5 Optimisation results with different l.

l	g_{min}	w^*	N^*	T^*
2.4	1.1080	1.5	4	262
2.6	1.0072	1.7	4	293
2.8	0.9231	1.8	4	310
3.0	0.8532	1.9	4	338

5. Conclusion

In this study, considering a wide fact that inspections for some degradation systems are unable to perform on time and usually performed in an inspection window, a maintenance policy of a quasi-periodic inspection and PM with inspection windows is proposed for a repairable system with a performance degradation process that is subject to the Wiener process. By minimising the system long-running cost rate, the optimal period $T^*$of inspection windows, the degradation threshold w^* for PM and PM number N^* are obtained. The developed model is beneficial for making a maintenance schedule, although each inspection is a random event. Case study shows that g_{min} keeps increase and T^* is getting hitched with the increase of W and a_j, while keeps opposite trend with the increase of the failure threshold l. N^* is stable with the increase of W and l, and yet decrease with the increase of a_j. w^* is rising with the increase of a_j and l, nevertheless it is stable with the change of W. Therefore, the length of the inspection window should be reduced as much as possible to decrease the system long-term cost rate and increase the average working time if the production and maintenance ability are allowable. However, considering that a certain inspection window length can become more flexible for maintenance and production scheduling, users should consider W in a comprehensive and multifaceted manner.

The model developed in this study is also suitable for other degradation processes, such as Gaussian Process and Gamma processes. The length of inspection window usually limited by the production capacity, such as repairman, we will discuss it in our future work.

Reference

1. Babishin, V., Y. Hajipour, and S. Taghipour, Optimisation of Non-Periodic Inspection and Maintenance for Multicomponent Systems. Eksploatacja i Niezawodność- Maintenance and Reliability, 2018. 20(2): 327-342. https://doi.org/10.17531/ein.2018.2.20
2. Cao, X., et al., A preventive maintenance model subject to sequential inspection for a three-stage failure process. International Journal of Performability Engineering, 2019. 15(1). https://doi.org/10.23940/ijpe.19.01.p8.7687
3. Castro, I.T. and L. Landesa, A dependent complex degrading system with non-periodic inspection times. Computers & Industrial Engineering, 2019. 133: 241-252. https://doi.org/10.1016/j.cie.2019.04.053
4. Cavalcante, C.A., P.A. Scarf, and M. Berrade, Imperfect inspection of a system with unrevealed failure and an unrevealed defective state. IEEE Transactions on Reliability, 2019. 68(2): 764-775. https://doi.org/10.1109/TR.2019.2897048
5. de Jonge, B. and P.A. Scarf, A review on maintenance optimization. European Journal of Operational Research, 2019.
6. Dinh, D.-H., P. Do, and B. Iung, Degradation modeling and reliability assessment for a multi-component system with structural dependence. Computers & Industrial Engineering, 2020. 144: 106443. https://doi.org/10.1016/j.cie.2020.106443
7. Driessen, J., H. Peng, and G. Van Houtum, Maintenance optimization under non-constant probabilities of imperfect inspections. Reliability Engineering & System Safety, 2017. 165: 115-123. https://doi.org/10.1016/j.ress.2017.03.020
8. Duan, F.J., G.J. Wang, and W.M. Wei, Remaining Useful Life Estimation Based on Wiener Degradation Process With Mixed Random Effects, in *Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling*. 2020. 1-6. https://doi.org/10.1109/APARM49247.2020.9209431

9. Gao, W., Optimal sequential preventive maintenance policy for a repairable system with maintenance windows. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 2019. 234(4): 963-977. https://doi.org/10.1177/0954406219886341
10. J, L., et al., Accelerated degradation analysis based on a random-effect Wiener process with one-order autoregressive errors. Eksploatacja i Niezawodnosc-Maintenance and Reliability, 2019. 21(2): 246–255. https://doi.org/10.17531/ein.2019.2.8
11. Jin, L., N. Koyama, and W. Yamamoto, Optimal policy for periodic inspections with flexible imperfect inspection frequencies. Total Quality Science, 2019. 4(3): 128-137. https://doi.org/10.17929/tqs.4.128
12. Kaio, N., T. Dohi, and S. Osaki, Inspection policy with failure due to inspection. Microelectronics Reliability, 1994. 34(4): 599-602. https://doi.org/10.1016/0026-2714(94)90025-6
13. Levitin, G., L. Xing, and H.-Z. Huang, Cost effective scheduling of imperfect inspections in systems with hidden failures and rescue possibility. Applied Mathematical Modelling, 2019. 68: 662-674. https://doi.org/10.1016/j.apm.2018.12.001
14. Lipton, A. and V. Kaushansky, On the first hitting time density for a reducible diffusion process. Quantitative Finance, 2020. 20(5): 723-743. https://doi.org/10.1080/14697688.2020.1713394
15. Long, Q. and C. Wu, A hybrid method combining genetic algorithm and Hooke-Jeeves method for constrained global optimization. Journal of Industrial and Management Optimization, 2014. 10(4): 1279-1296.
16. MF, M. and C.l.n. CAV, Random preventive maintenance policy based on inspection for a multicomponent system using simulation. Eksploatacja i Niezawodnosc-Maintenance and Reliability 2017. 19(4): 552–559. https://doi.org/10.17531/ein.2017.4.8
17. Moakedi, H., M.S. Seyedhosseini, and K. Shahanaghi, A block-based inspection policy for a multi-component system subject to two failure modes with stochastic dependence. Journal of Quality in Maintenance Engineering, 2019. 25(2): 314-339. https://doi.org/10.1108/JQME-05-2018-0044
18. Mokhtarzadeh, M., et al., Determining Maintenance Opportunity Window (MOW) in Job-Shop Systems by Considering Manpower of Maintenance. Journal of Quality Engineering and Production Optimization, 2019. 41(1): 41-54.
19. Mosheiov, G., et al., Two-machine flow shop and open shop scheduling problems with a single maintenance window. European Journal of Operational Research, 2018. 271(2): 388-400. https://doi.org/10.1016/j.ejor.2018.04.019
20. Munford, A.G. and A.K. Shahani, A Nearly Optimal Inspection Policy. Journal of the Operational Research Society, 1972. 23(3): 373-379. https://doi.org/10.1057/jors.1972.56
21. Nakagawa, T., S. Mizutani, and M. Chen, A summary of periodic and random inspection policies. Reliability Engineering & System Safety, 2010. 95(8): 906-911. https://doi.org/10.1016/j.ress.2010.03.012
22. Raza, A. and V. Ulansky, Modelling of predictive maintenance for a periodically inspected system. Procedia CIRP, 2017. 59: 95-101. https://doi.org/10.3390/math8101815
23. Sett, B.K., B.K. Dey, and B. Sarkar, Autonomated Inspection Policy for Smart Factory—An Improved Approach. 2020. 8(10): 1815. https://doi.org/10.3390/math8101815
24. Seyedhosseini, S.M., H. Moakedi, and K. Shahanaghi, Imperfect inspection optimization for a two-component system subject to hidden and two-stage revealed failures over a finite time horizon. Reliability Engineering & System Safety, 2018. 174: 141-156. https://doi.org/10.1016/j.ress.2018.02.024
25. Shahraki, A.F., O.P. Yadav, and H. Liao, A review on degradation modelling and its engineering applications. International Journal of Performability Engineering, 2017. 13(3): 299-314. https://doi.org/10.23940/ijpe.17.03.p6.299314
26. Shahraki, A.F., O.P. Yadav, and H. Liao, A Review on Degradation Modelling and Its Engineering Applications Int J Performability Eng, 2017. 13(3): 299-314. https://doi.org/10.23940/ijpe.17.03.p6.299314
27. Si, X., T. Li, and Q. Zhang, A General Stochastic Degradation Modeling Approach for Prognostics of Degrading Systems With Surviving and Uncertain Measurements. IEEE Transactions on Reliability, 2019. 68(3): 1080-1100. https://doi.org/10.1109/TR.2019.2908492
28. Sun, Q., Z. Ye, and N. Chen, Optimal inspection and replacement policies for multi-unit systems subject to degradation. IEEE Transactions on Reliability, 2018. 67(1): 401-413. https://doi.org/10.1109/TR.2017.2778283
29. Wang, D., et al., Nonlinear-drifted Brownian motion with multiple hidden states for remaining useful life prediction of rechargeable batteries. Mechanical Systems and Signal Processing, 2017. 93(01): 531-544. https://doi.org/10.1016/j.ymssp.2017.02.027
30. Wang, X.J., et al., Remaining useful life prediction based on the Wiener process for an aviation axial piston pump. Chinese Journal of Aeronautics, 2016. 29(3): 779-788. https://doi.org/10.1016/j.cja.2015.12.020
31. Wen, Y., et al., Recent advances and trends of predictive maintenance from data-driven machine prognostics perspective. Measurement, 2022. 187. https://doi.org/10.1016/j.measurement.2021.110276
32. Xiao, H., et al., Optimal inspection policy for a single-unit system considering two failure modes and production wait time. IEEE Transactions on Reliability, 2021: 1-13.
33. Yousefi, N., D.W. Coit, and S. Song, Reliability analysis of systems considering clusters of dependent degrading components. Reliability Engineering and System Safety, 2020. 202: 1-14. https://doi.org/10.1016/j.ress.2020.107005
34. Yu, W., et al., A nonlinear-drift-driven Wiener process model for remaining useful life estimation considering three sources of variability. Reliability Engineering & System Safety, 2021. 212(107631): 1-13. https://doi.org/10.1016/j.ress.2021.107631
35. Yue, D. and Q. Gao, A geometric process model for deteriorating operating system with periodic and random inspections. Quality Technology & Quantitative Management, 2017. 14(3): 269-279. https://doi.org/10.1080/16843703.2016.1208933
36. Zhang, F., J. Shen, and Y. Ma, Optimal maintenance policy considering imperfect repairs and non-constant probabilities of inspection errors. Reliability Engineering & System Safety, 2020. 193: 106615. https://doi.org/10.1016/j.ress.2019.106615
37. Zhang, Z., et al., Degradation data analysis and remaining useful life estimation: A review on Wiener-process-based methods.

European Journal of Operational Research, 2018. 271(3): 775-796. https://doi.org/10.1016/j.ejor.2018.02.033
38. Zhang, Z. and L. Yang, State-Based Opportunistic Maintenance With Multifunctional Maintenance Windows. IEEE Transactions on Reliability, 2021. 70(4): 1481-1494. https://doi.org/10.1109/TR.2020.2995277
39. Zhao, X., et al., Optimal inspection and replacement policy based on experimental degradation data with covariates. IISE Transactions, 2019. 51(3): 322-336. https://doi.org/10.1080/24725854.2018.1488308
40. Zhao, X. and T. Nakagawa, Optimal periodic and random inspections with first, last and overtime policies. International Journal of Systems Science, 2015. 46(9): 1648-1660. https://doi.org/10.1080/00207721.2013.827263