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Component Maintenance Strategies and Risk Analysis for Random Shock Effects Considering Maintenance Costs

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Highlights

- Proposing the reliability model under the influence of random shocks.
- The sensitivity analysis is performed on system degradation.
- Two importance indicators are proposed for two types of maintenance strategies.
- Considers the impact of maintenance cost risk on system maintenance frequency.

Abstract

Maintenance can improve a system's reliability in a long operation period or when a component has failed. The reliability modeling method that uses the stochastic process degradation model to describe the system degradation process has been widely used. However, the existing reliability models established using stochastic processes only consider the internal degradation process, and do not fully consider the impact of external random shocks on their reliability modeling. Furthermore, the existing theory of importance does not consider the actual factors of maintenance cost. In this paper, based on the reliability modeling of random processes, the degradation rate under the influence of random shocks is introduced into the time scale function to solve the impact of random shocks on product reliability, and two cost importance measures are proposed to guide the maintenance selection of the components under limited resources in the system. Finally, a subsystem of an aircraft hydraulic system is analyzed to verify the proposed method's performance.

Keywords
System reliability, Preventive maintenance, Importance measure, Maintenance cost, Random shock.

1. Introduction

Performance degradation of a system is inevitable in its life cycle due to environmental influences and wear and tear of the system. When the performance degradation reaches a certain level, the system may malfunction. At this point, maintenance is crucial for system performance recovery. Maintenance differs for various components throughout the life cycle due to the performance differences between components. In addition, due to the interaction between components, the maintenance sequence of different components can have a large impact on reliability. [32]. Therefore, it is particularly important to study the mapping relationship between components and system functions. By making a reasonable decision method of component maintenance, the system paralysis caused by its failure can be reduced and the safety of the whole system running cycle can be improved.

The purpose of the maintenance policy is to use less resources to make the maintenance products have high reliability or availability. Due to the increasing reliability of current products, it is becoming more and more difficult to collect product failure data. Reliability modeling methods for fault data

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are sometimes not feasible. Therefore, degradation-based reliability analysis and maintenance strategies have attracted more and more attention in recent years [13,14]. Common reliability degradation modeling includes based on system performance [23,24,33] and based on system maintenance cost [27,34]. According to the unique structure of the system, a mathematical model will be established based on performance degradation to analyze its performance degradation law. Common mathematical models include Markov model [18,22], Bayesian model [3-5], importance measures model [11,35], etc. Reliability modeling based on maintenance costs is usually linked to preventive maintenance, because preventive maintenance at different time intervals will greatly affect system maintenance costs. For example, Khatab et al. [17] established a comprehensive optimization model to decide the optimal detection period and degradation threshold level. Combining system monitoring status and historical degradation data, Cai et al. [6] proposed a re-prediction method for remaining useful life to improve the accuracy of prediction. Alvarez et al. [1] proposed a stochastic dynamic programming model to minimize the total maintenance cost per unit time. Levitin et al. [19,20] proposed a task success probability evaluation method based on an event transfer, which enabled optimal preventive replacement scheduling.

Since product failure is random, even if preventive maintenance [9,29] or condition-based maintenance [25] has been carried out on the system, the product may still fail unexpectedly, causing downtime. Therefore, in recent years, the resilience analysis of the system has gradually attracted attention. The resilience analysis of the system is mainly about the ability of the system to quickly recover to the pre-disturbance occurs when or after the disturbance occurs. For example, Xing [30,31] considered the system's resilience, proposed a reliability analysis method for cascading fault modeling, and established mitigation strategies for the resilience system to deal with cascading failures. Combining the Markov model with dynamic Bayesian networks (DBNs), Cai et al. [7,8] proposed a resilience evaluation method applicable to various external disasters. Due to the system failure caused by external disturbance, random shock [10] and uncertainty analysis [21] are also considered in the maintenance strategy. Specifically, Bian et al. [2] studied a reliability model for multi-component systems that relies on a competitive failure process to characterize the impact of random shocks on the components of the system. Gao et al.[15] considered the new shock effect modes caused by multiple external shocks, and established a reliability model of the soft and hard competitive failure process of systems or devices with degraded shock correlation.

As far as we know, the existing reliability analysis and maintenance strategies only consider the reliability upgrade of system components and a single maintenance cost strategy. Furthermore, few papers consider the impact of exposed external environmental factors on system maintenance. This paper proposes two new importance indicators that comprehensively consider various maintenance costs and conditions of different maintenance strategies. Specifically, this work has the following contributions.

(1) Proposing the reliability model under the influence of random shocks. In this reliability model, degradation rate and dynamic failure threshold are introduced into the time scale function, which can effectively reflect the changes of system reliability after external shocks.
(2) Proposing two important measures based on maintenance cost (CIIM, CJIIM) and their evaluation methods under different maintenance decisions, including the first type of preventive maintenance strategy and the second type of mixed maintenance strategy.
(3) Conducting a numerical example of an aircraft hydraulic energy subsystem containing a series and parallel configurations. The effectiveness of the proposed cost-based important measure versus the traditional performance-based important measure is investigated.

The rest of this paper is organized as follows. Section 2 analyzes the impact of external shocks on product reliability modeling and failure threshold reduction. Section 3 describes the analysis of system maintenance costs and single and binary measures based on maintenance costs to identify the components or groups of components that require the most maintenance of the system. In section 4, a subsystem of aircraft hydraulic system is taken as an numerical example to verify the effectiveness of the proposed method. Finally, the conclusion is given in Section 5.

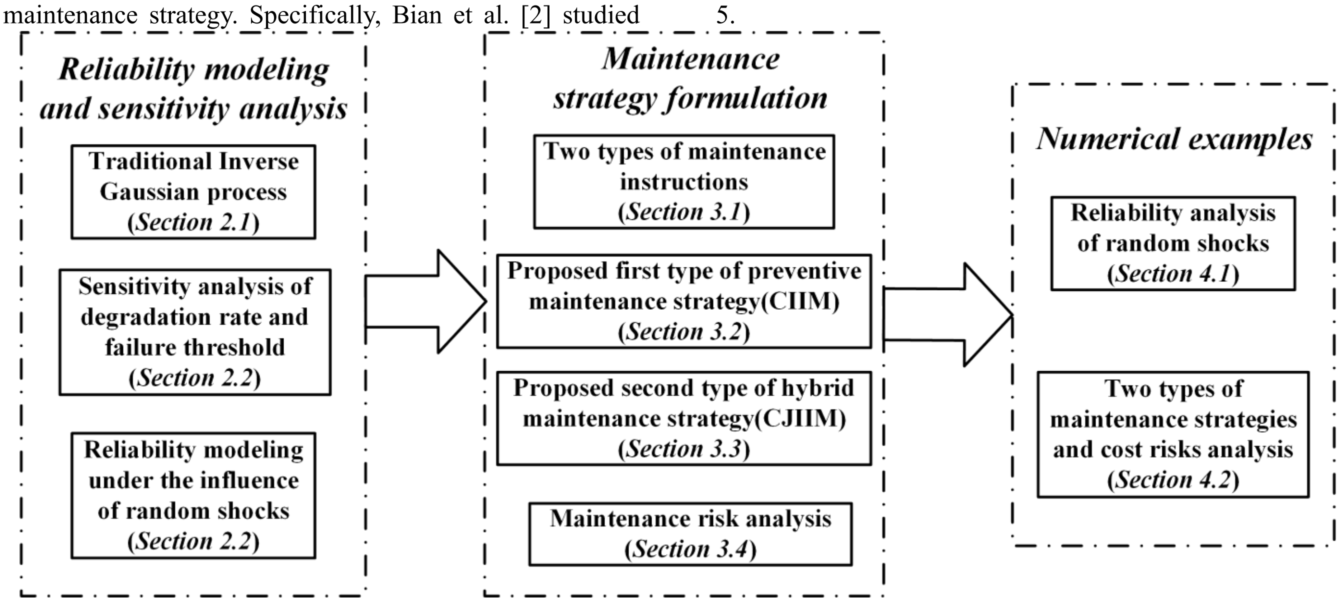

Fig. 1. Framework of maintenance strategy and risk assessment under random shocks.

2. Maintenance strategy analysis based on degradation process

2.1 Traditional Inverse Gaussian process

In practice the factors which affect the degradation process such as the inhomogeneity of the material and the uncertainty of the external load all have certain randomness. Stochastic process methods can describe the time correlation effectively, and they are commonly used in the study of performance degradation. Commonly used stochastic process methods include Wiener process, inverse Gaussian process and gamma process. In this paper, we use inverse Gaussian (IG) process to model degradation because it is suitable for monotonically increasing degradation processes such as wear and fatigue. In addition, the IG process is flexible, allowing the examination of different characteristics of the deterioration process [16].

Let the product degradation process $\{y(t), t > 0\}$ obey the IG process with parameters μ, σ and time scale function $\Lambda(t)$, then the IG degradation process applies as follows

$$y(t) \sim IG(\mu\Lambda(t), \frac{\mu^3}{\sigma^2}\Lambda^2(t)) \tag{1}$$

where $\Lambda(t)$ is a non-negative increasing time scale function, which is used to represent the properties of a random degradation process. The probability density function (PDF) of $y(t)$ is

$$f(y(t)|\mu\Lambda(t), \frac{\mu^3}{\sigma^2}\Lambda^2(t)) = \sqrt{\frac{\mu^3\Lambda^2(t)}{2\pi\sigma^2 y^3(t)}} exp\left[-\frac{\mu(y(t)-\mu\Lambda(t))^2}{2\sigma^2 y(t)}\right] \tag{2}$$

Let $\lambda = \frac{\mu^3}{\sigma^2}$, so the independent increment $\Delta y(t) = y(t+\Delta t) - y(t)$ follows the IG distribution:

$$\Delta y(t) \sim IG(\mu\Delta\Lambda(t), \lambda\Delta\Lambda^2(t)) \tag{3}$$

Thus, the PDF of $\Delta y(t)$ is

$$f(\Delta y(t)) = \sqrt{\frac{\lambda\Delta\Lambda^2(t)}{2\pi\Delta y^3(t)}} exp\left[-\frac{\lambda(\Delta y(t)-\mu\Delta\Lambda(t))^2}{2\mu^2\Delta y(t)}\right] \tag{4}$$

The cumulative distribution function (CDF) of $\Delta y(t)$ can be expressed as

$$F(\Delta y(t)) = \Phi\left[\sqrt{\frac{\lambda\Delta\Lambda^2(t)}{\Delta y(t)}}\left(\frac{\Delta y(t)}{\mu\Delta\Lambda(t)} - 1\right)\right] + exp\left(\frac{2\lambda}{\mu}\Delta\Lambda(t)\right)\Phi\left[-\sqrt{\frac{\lambda\Delta\Lambda^2(t)}{\Delta y(t)}}\left(\frac{\Delta y(t)}{\mu\Delta\Lambda(t)} + 1\right)\right] \tag{5}$$

where $\Phi(\cdot)$is the CDF of the standard Gaussian distribution. Let D denote the failure threshold of the product, then the reliability function based on IG process can be expressed as:

$$R(t) = P(T \le t) = P(y(t) > D)$$
$$= \Phi\left[\sqrt{\frac{\lambda}{D}}\left(\frac{D}{\mu} - \Lambda(t)\right)\right] + exp\left(\frac{2\lambda\Lambda(t)}{\mu}\right)\Phi\left[-\sqrt{\frac{\lambda}{D}}\left(\frac{D}{\mu} + \Lambda(t)\right)\right] \tag{6}$$

2.2 Proposed degradation rate and failure threshold variation

In the actual operation process, in addition to the natural degradation, the product will also be affected by external factors such as corrosion and impact to different degrees. These factors can be thought of as a generalized random shock with additional effects on the level of product degradation. However, reliability modeling based on stochastic processes generally only assumes the degradation trajectory as a linear degradation, that is, $q = 1$ in the time scale function $\Lambda(t) = kt^q$, which is inconsistent with the actual degradation process, thus affecting the accuracy of reliability modeling.

In addition, for the failure threshold, it is generally assumed that the failure threshold is fixed. However, under the influence of external shock, the frequency and magnitude of shock will affect the change of failure threshold. Therefore, the conventional fixed failure threshold affects the accuracy of stochastic process-based reliability modeling and estimation of remaining useful life, which will bring safety risks for subsequent maintenance. Therefore, it is necessary to model the degradation rate and failure threshold variation under the influence of generalized random shock to improve the accuracy of reliability estimation.

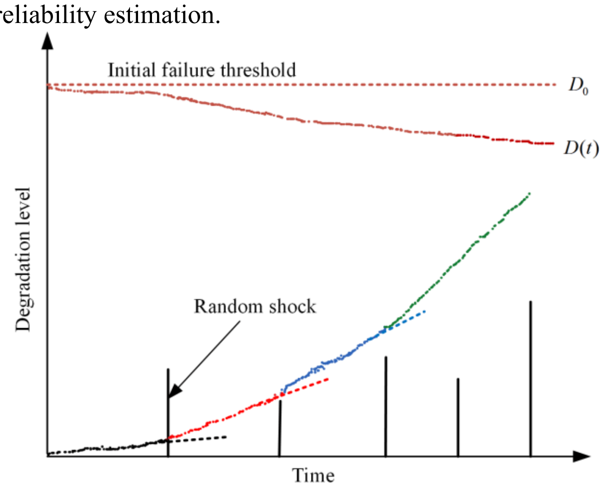

Fig. 2. Degradation trajectory and failure threshold variation with external shock.

It is assumed that there are natural degradation and random shocks in the system. Therefore, the overall degradation of the system is equal to the combined effect of natural degradation and shocks degradation. The random shocks can be divided into fatal shocks and non-fatal shocks. For fatal shocks, it can cause instant system failure. For nonfatal shocks, it has an effect on natural degradation processes. Poisson process and their variants are widely used to simulate random shocks to systems[15]. In this paper, the arrival of random shocks is assumed according to the composite Poisson process, $\{N(t), t \ge 0\}$, with arrival rate v. Here $N(t)$ represents the number of nonfatal shocks.

Consider system with random shocks and natural degradation. The overall degradation is represented by $M(t)$. It is assumed that natural degradation processes and random shocks are interdependent. The effect of non-fatal shocks are mainly manifested as degradation increment and degradation rate, which is represented as an exponential function. Then the total degradation can be expressed as[28]

$$M(t) = \beta \cdot \eta(te^{\psi_1(M(t))N(t)+\psi_2(M(t))\sum_{j=1}^{N(t)} X_j}) + \psi_2(M(t))\sum_{j=1}^{N(t)} X_j \tag{7}$$

where β is a random variable. $\eta()$is the natural degradation process. $\psi_1(M(t))$ and $\psi_2(M(t))$ are parameters representing the effects of non-fatal shocks, depending on the current degradation level $M(t)$. X_j is the jth nonfatal shock.

In Eq. (7), $\psi_1(M(t))$ represents the effect of nonfatal impact on the degradation rate and $\psi_2(M(t))$ represents the effect of nonfatal impact on the degradation increment. As the degradation trajectory $\eta(\cdot)$is the linear, $\psi_1(M(t))$ is a linear logarithmic function of $M(t)$, and $\psi_2(M(t))$ is proportional to $M(t)$, the closed form of the model can be obtained expression.

$$\psi_1(M(t)) = a_1 \, ln(M(t)) + b_1 \tag{8}$$

$$\psi_2(M(t)) = a_2 M(t) \tag{9}$$

$M(t) = \beta \cdot te^{(a_1 \ln M(t) + b_1)N(t) + (a_2 M(t)) \sum_{j=1}^{N(t)} X_j} + a_2 M(t) \sum_{j=1}^{N(t)} X_j = exp\{[ln(\beta \cdot t) + b_1 N(t) - ln(1 - a_2 \sum_{j=1}^{N(t)} X_j)]/(1 - a_1 N(t))\}$ (10)

where a_1,a_2,a_3 are non-negative coefficients and b_1,b_2 are real values. The total degradation $M(t)$ includes natural degradation and random shocks degradation. Therefore, the degradation rate q in the time scale function should be the partial derivative of the total degradation increment $M(t)$ with respect to time *t*.

$$q = \frac{\partial M(t)}{\partial t} \quad (11)$$

Therefore, the degradation rate on the time scale function is related to the degradation amount under the joint influence of natural degradation and random shocks. When the frequency and magnitude of random shocks per unit time increase, the degradation rate on the time scale will increase, and the degradation trajectory will become steeper. In addition, under the influence of external shock, the frequency and magnitude of the shock will affect the change of failure threshold. The conventional fixed failure threshold affects the accuracy of reliability modeling based on stochastic processes, which will bring safety risks to subsequent maintenance. Therefore, it is necessary to analyze the dynamic failure threshold under the influence of generalized random shock.

For the shock process, the impact of the cumulative degradation process on the shock failure process is reflected in the gradual decrease of the shock failure threshold. When the total amount of degradation caused by natural and random shocks continues to increase, the product's ability to withstand external shocks becomes weaker. At this time, the product will be more sensitive to random shocks, and the shock failure threshold will gradually decrease. Assuming that the initial failure threshold of the system is D_0, the failure threshold is reduced due to continuous random shock degradation during operation. The amount of degradation in $D(t)$ is positively related to the cumulative degradation level of the shock. Therefore, the dynamic failure threshold at time *t* can be expressed as

$$D(t) = D_0 - \kappa(\psi_2(M(t)) \sum_{j=1}^{N(t)} X_j)$$

$$= D_0 - \kappa a_2 \{exp\{[ln(\beta \cdot t) + b_1 N(t) - ln(1 - a_2 \sum_{j=1}^{N(t)} X_j)]$$

$$/(1 - a_1 N(t))\}\} \sum_{j=1}^{N(t)} X_j \quad (12)$$

Considering that the actual degradation trajectory may be affected by external shocks during product operation, the degradation rate should not be a fixed value. In addition, the impact of external shocks can change the product failure threshold. Therefore, the system reliability function considering the degradation rate and dynamic failure threshold can be expressed as

$R(t) = P(T \le t) = P(Y(t) > D(t))$

$$=\Phi\left[\sqrt{\frac{\lambda}{D(t)}}\left(\frac{D(t)}{\mu} - kt^{\frac{\partial M(t)}{\partial t}}\right)\right] + exp\left(\frac{2\lambda kt^{\frac{\partial M(t)}{\partial t}}}{\mu}\right)$$

$$\Phi\left[-\sqrt{\frac{\lambda}{D(t)}}\left(\frac{D(t)}{\mu} + kt^{\frac{\partial M(t)}{\partial t}}\right)\right] \quad (13)$$

Due to the complexity of the components and their interactions, the downtime hazards and economic losses caused by the faults of the components are more serious than ever before. Hence, it is an important topic to study the interaction between components and system functions, properly maintain the different components in the system, reduce the risks caused by component faults, and improve system reliability. So the next section presents guidance for system maintenance based on cost importance measures.

3. Cost-based maintenance strategy and risk analysis

Due to the limitation of maintenance resources, it is impractical to maintain all components of the system. Therefore, it is necessary to determine the impact of each component on system function, so as to provide more effective guidance in system maintenance activities. Component importance measure is the important way to identify some key components that have an impact on system function. In this section, we propose cost-based importance measure to recognize some weaknesses in a system, providing some guidance for the formulation of maintenance strategies.

3.1 Maintenance cost analysis

We consider the following three maintenance costs during system maintenance [27].

a) Cost of improving the component reliability. In the system, the cost of improving the reliability of different components varies, although the levels of improvement might be the same.

b) Cost due to the component failure. If a critical system component fails, system needs to be shutdown for repair or replacement. Repair costs depend on the complexity of the component and the time required to repair or replace it.

c) Cost of the system downtime. A system can usually perform a specific function. When the system has a malfunction and needs to be maintained, this will cause the system to shut down. When the system is shut down, there will be downtime loss, which is also a cost to consider.

In the actual maintenance process, there are two types of situations:

1) **The first type of maintenance.** When the system is running to the point where regular maintenance is required, but the components in the system have not yet failed, it is essential to recognize the weak links of a system and perform preventive maintenance.
2) **The second type of maintenance.** A sudden failure of the system during operation requires maintenance. In order to reduce system downtime, in addition to repairing faulty components, it is also necessary to recognize weak links in a system and perform maintenance ahead of time.

The following assumptions are be made in the maintenance cost analysis:

- The system consists of *n* components, each of which is in perfect condition in the initial stage.
- Each of the *n* components has two states, namely, the running state and the failure state.

Maintenance costs for the type 1 of maintenance include downtime losses due to system downtime during maintenance and the cost of improving component reliability. Its total maintenance cost can be expressed as:

$$C^{T1}(t) = \sum_{i=1}^{n}(c_i(t) \cdot d_i) + c_{s,i}(t)d_i \qquad (14)$$

where $c_i(t)$ is the cost of component i to improve reliability at time t. d_i is the variable that determines whether component i requires maintenance. $c_{s,i}(t)$ is the cost of the system downtime loss due to maintenance of component i.

Maintenance costs for the type 2 maintenance increases the repair cost of faulty components, and its total maintenance cost can be expressed as

$$C^{T2}(t) = c_k(t) + c_{s,k}(t) + \sum_{i=1,i\neq k}^{n}(c_i(t) \cdot d_i) \qquad (15)$$

where $c_k(t)$ is the repair cost of the failure component k. $c_{s,k}(t)$ is the cost of the system shutdown loss caused by the fault component k. $c_i(t)$ is the cost of choosing component i for preventive maintenance when repairing the faulty component k.

For the first type of maintenance, none of the components failed during maintenance. At this time, it is essential to find the impact of each component failure on the system function based on the importance measure. This type of maintenance is the effect of component on system. For the second type of maintenance, when a component fails and needs to be repaired, the faulty component will affect the maintenance options for other normal components. Therefore, this type of maintenance is the combined effect of faulty and normal components on the system.

3.2 Proposed cost importance measure-based single maintenance measure

For the first type of maintenance, it is necessary to find out the degree of influence of different component failure to the system function according to the importance measure. The integrated importance measure (IIM) describes the affect of component performance degradation on system performance degradation, which could be expressed as

$$I_i^{IIM}(t) = R_i(t) \cdot \lambda_i(t) \cdot \frac{\partial R_S(t)}{\partial R_i(t)} \qquad (16)$$

where $R_i(t)$ is the reliability of component i, $\lambda_i(t)$ is the component failure rate. The component's failure rate can be expressed as $\lambda_i(t) = -\frac{dR_i(t)/dt}{R_i(t)}$. Therefore, the IIM of component i is

$$I_i^{IIM}(t) = -\frac{dR_i(t)}{dt} \cdot \frac{\partial R_S(t)}{\partial R_i(t)} \qquad (17)$$

The IIM of n components represent the decrease in the system reliability per unit time. The reliability of the system composed of n components can be expressed as $R_S(t) = f(R_1(t), R_2(t), \dots, R_n(t))$.

$$\frac{dR_S(t)}{dt} = \frac{df(R_1(t), R_2(t), \dots, R_n(t))}{dt} = \sum_{i=1}^{n}\frac{dR_i(t)}{dt} \cdot \frac{\partial R(t)}{\partial R_i(t)} = -\sum_{i=1}^{n} I_i^{IIM}(t) \qquad (18)$$

According to Eq. (18), the reliability change per unit time is the sum of the IIM of the n components. Thus, the IIM of component i is the contribution of the reliability change of component i to the system reliability change at time t. The higher the IIM value of a system component, the higher the importance of that component. The cost IIM based maintenance measure is

$$I_i^{CIIM}(t) = \frac{C^{T1}(t)}{I_i^{IIM}(t)} \qquad (19)$$

where $I_i^{CIIM}(t)$ describes the priority of each component in the system during the first type of maintenance. When $I_i^{CIIM}(t)$ is small, component i should be maintained first. Therefore, for the first type of maintenance, according to the size of $I_i^{CIIM}(t)$, the change of the system maintenance cost due to the failure of each component can be found.

3.3 Proposed maintenance cost-based binary measure

For the second type of maintenance, when a component fails and needs to be repaired, the faulty component will affect the maintenance options of other normal components. Such maintenance is the combined effect of faulty and normal components on the system. Thus, a new binary importance measure is needed to assess the relationship between the components and the cost of system maintenance.

The joint integrated importance measure (JIIM) describes the degree to which one component of the system is repaired and affects the reliability of other components. JIIM is a binary importance measure as follows

$$I_i^{JIIM}(t)_{X_k(t)} = I_i^{IIM}(t)_{X_k(t)=0} - I_i^{IIM}(t)_{X_k(t)=1} \qquad (20)$$

where $I_i^{IIM}(t)_{X_k(t)=0}$ is the contribution of component i to the change in system reliability when the component k is in a failure state. $I_i^{IIM}(t)_{X_k(t)=1}$ is the contribution of component i to the change in system reliability when component k is in a working state. The difference $I_i^{JIIM}(t)_{X_k(t)}$ is defined as the JIIM of component i when component k is repaired. Eq.(20) can be expanded and written as

$$I_i^{JIIM}(t)_{X_k(t)} = I_i^{IIM}(t)_{X_k(t)=0} - I_i^{IIM}(t)_{X_k(t)=1}$$
$$= R_i(t) \cdot \lambda_i(t) \cdot \left[\frac{\partial f(R_1(t), R_2(t), \dots, R_n(t)|R_k(t) = 0)}{\partial R_i(t)} - \frac{\partial f(R_1(t), R_2(t), \dots, R_n(t)|R_k(t) = 1)}{\partial R_i(t)}\right]$$
$$= R_i(t) \cdot \lambda_i(t) \cdot \left(\frac{\partial R_S(t)_{R_k(t)=0}}{\partial R_i(t)} - \frac{\partial R_S(t)_{R_k(t)=1}}{\partial R_i(t)}\right) \qquad (21)$$

The failure rate of component i can be expressed as $\lambda_i(t) = -\frac{dR_i(t)/dt}{R_i(t)}$. Therefore, the JIIM can be expressed as

$$I_i^{JIIM}(t)_{X_k(t)} = -\frac{dR_i(t)}{dt} \cdot \left(\frac{\partial R_S(t)_{R_k(t)=0}}{\partial R_i(t)} - \frac{\partial R_S(t)_{R_k(t)=1}}{\partial R_i(t)}\right) \qquad (22)$$

The IIM describes the contribution of the failure of component i to changes in system reliability, while JIIM describes the contribution of component i to changes in system reliability after the component k is repaired. JIIM describes the degree to which the maintenance of each component improves the reliability of the system when the second type of maintenance occurs in the system.

$$\frac{d(R_S(t)_{R_k(t)=0} - R_S(t)_{R_k(t)=1})}{dt} = \frac{dR_S(t)_{R_k(t)=0}}{dt} - \frac{dR_S(t)_{R_k(t)=1}}{dt}$$
$$= \sum_{i=1,i\neq k}^{n} \frac{dR_i(t)}{dt} \cdot \frac{\partial R_S(t)_{R_k(t)=0}}{\partial R_i(t)} - \sum_{i=1,i\neq k}^{n} \frac{dR_i(t)}{dt} \cdot \frac{\partial R_S(t)_{R_k(t)=1}}{\partial R_i(t)}$$
$$= \sum_{i=1,i\neq k}^{n} \frac{dR_i(t)}{dt} \cdot \left[\frac{\partial R_S(t)_{R_k(t)=0}}{\partial R_i(t)} - \frac{\partial R_S(t)_{R_k(t)=1}}{\partial R_i(t)}\right]$$
$$= -\sum_{i=1,i\neq k}^{n} I_i^{JIIM}(t)_{X_k(t)} \qquad (23)$$

According to Eq. (23), when the component k is repaired, the reliability change per unit time of the system is the sum of the JIIM values of the n-1 components except the faulty component

k. So, the JIIM of component i is the contribution of the reliability variation of component i to the system reliability change while component k is repaired. The cost JIIM based maintenance measure is

$$I_i^{CJIIM}(t) = \frac{C^{T2}(t)}{I_i^{JIIM}(t)x_{k(t)}} \quad (24)$$

The physical meaning of Eq. (24) is that when the component k is repaired, when the value of $I_i^{CJIIM}(t)$ is large, the maintenance cost of the component i to the whole system changes greatly. Therefore, for the second type of maintenance, according to the size of $I_i^{CJIIM}(t)$, the change in the system maintenance cost due to each component being repaired can be found.

3.4 Cost-based risk analysis

Cost is a crucial factor in the maintenance process. During regular system maintenance, improper maintenance intervals may result in system over-maintenance or under-maintenance. Therefore, a cost-based risk analysis of the system is essential. During system maintenance, the expected loss due to the failure of the system with n components can be expressed as:

$$\bar{C}_f = \sum_{k=1}^{n} \frac{\lambda_k}{\lambda_S}\bar{C}_k = \frac{\lambda_1(t)}{\lambda_S(t)}\bar{C}_1 + \frac{\lambda_2(t)}{\lambda_S(t)}\bar{C}_2 + \cdots + \frac{\lambda_n(t)}{\lambda_S(t)}\bar{C}_n \quad (25)$$

where $\bar{C}_k$ is the expected loss when system component k fails, and $\lambda_s(t)$ is the system failure rate, which is related to the connections between the components. Therefore, the failure risk of a repairable system is

$$K = (1 - R_S(t)) \cdot \bar{C}_f = (1 - R_S(t)) \cdot \sum_{k=1}^{n} \frac{\lambda_k}{\lambda_S}\bar{C}_k \quad (26)$$

The relationship between the failure risk and the maintenance cost in a certain period can be determined by comparing the risk of failure K with the system maintenance cost. When the failure risk in a given period is less than the system maintenance cost, the failure risk is relatively small in this period. Thus, the components' maintenance frequency can be reduced during this period, reducing unnecessary system maintenance costs. When the failure risk in a given period is greater than the system maintenance cost, the failure risk is relatively large in this period. Thus, the maintenance frequency of the system should be increased to reduce the probability of system failure. The system maintenance frequency is determined by cost-based risk analysis to ensure that the system maintenance is in the optimal state throughout the entire life cycle.

4. Numerical examples

In this section, the CIIM and CJIIM methods are applied to a subsystem of the aircraft hydraulic system(AHS), and the application effect and effectiveness of the method are further verified. Based on references Dui et al.[12] Gao et al.[15] and Tanner et al.[26], some coefficients are extended to illustrate the model proposed in this paper. At the same time, the simulation and sensitivity analysis of the model are carried out. Firstly, the main components of a subsystem of the AHS are selected as maintenance objects, and the composition of the system and the functions of different components in the system are introduced. Then, combined with the key components of the system, a component maintenance sequence strategy considering external shocks and threshold changes is introduced, and CIIM and CJIIM are used to evaluate the impact of maintenance costs of different maintenance decisions. Finally, based on the numerical results, the CIIM and CJIIM of important components in the whole system are analyzed, and the optimal maintenance decisions for the components in the system are formulated to achieve the effect of improving the system reliability and the lowest maintenance cost in the whole maintenance process.

An AHS is crucial for aircraft reliability since it provides hydraulic power for aircraft rudder deflection and other important tasks. Therefore, the AHS typically has a redundant structure to ensure high reliability of the aircraft. Fig. 3 shows a subsystem of one of the hydraulic systems.

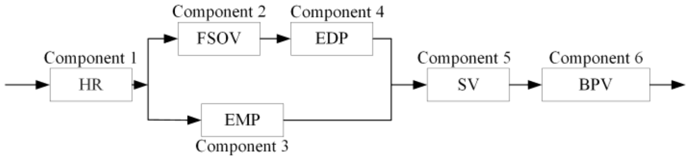

Fig. 3. A subsystem of an AHS.

The hydraulic oil from the hydraulic reservoirs (HR) is converted to high-pressure hydraulic oil after passing through the fire shut-off valve (FSOV) and engine-driven pump (EDP) or electric motor pump (EMP). Adjustable hydraulic power is obtained after the high-pressure hydraulic oil passes through the servo valve (SV) and back-pressure valve (BPV), providing hydraulic power to the hydraulic system, such as the rudder, brake, and landing gear of the aircraft.

4.1 System reliability analysis under random shock and dynamic failure threshold

We use the proposed model to simulate the degradation process under random shock, which consists of the natural degradation process that occurs during system operation and the cumulative damage magnitude caused by random shocks. Assuming that the load of a random shock is less than the failure threshold $D(t)$, possibility of different shock arrival is nonfatal. The drift parameters, shape parameters and initial failure threshold are shown in Table 1.

Table 1. Parameter values for reliability analysis of AHS.

	HR	FSOV	EMP	EDP	SV	BPV
μ	6.63×10^{-3}	4.51×10^{-3}	2.33×10^{-3}	9.07×10^{-3}	7.42×10^{-3}	3.95×10^{-3}
λ	3.41×10^{-5}	1.37×10^{-5}	7.15×10^{-5}	9.63×10^{-6}	6.45×10^{-6}	6.43×10^{-5}
D_0	0.45×10^{-2}	0.25×10^{-2}	0.15×10^{-2}	0.55×10^{-2}	0.30×10^{-2}	0.20×10^{-2}

In this model, the arrival of the shock follows a compound Poisson process with a rate v. Suppose $\psi_1 = 0.01 \cdot ln(M(t))$, and $\psi_2 = 0.05 \cdot M(t)$; i.e. $a_1 = 0.01$, $a_2 = 0.05$, $b_1 = 0$. According to the subsystem structure diagram, the reliability expression of the subsystem is determined as

$$R_S(t) = R_1(t) \cdot R_6(t) \cdot (R_2(t) \cdot R_4(t) + R_3(t) - R_2(t) \cdot R_3(t) \cdot R_4(t)) \cdot R_5(t)$$

In the case, we get the reliability curve shown in Fig. 4.

Fig. 4 shows that the reliability under the proposed random shock and dynamic failure thresholds degrades faster than the reliability without the shock effect. After the system is affected by random shocks, the performance degradation caused by random shocks accumulates in each component of the system, and the performance degradation of each component is then accumulated in the system, resulting in faster system reliability degradation under random shocks than without shocks. The proposed model is basically consistent with our assumptions, and the impact of random shocks on the health of the system is relatively serious.

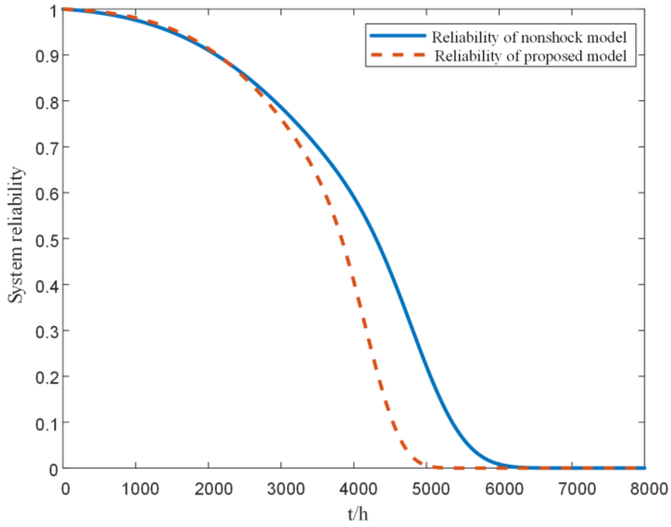

Fig. 4. Comparison of two types of models.

Therefore, the proposed model has good engineering and rationality in the case of both external impact and natural degradation. The proposed degradation of the combined effects of random shocks and dynamic failure thresholds provides a new perspective for reliability modeling.

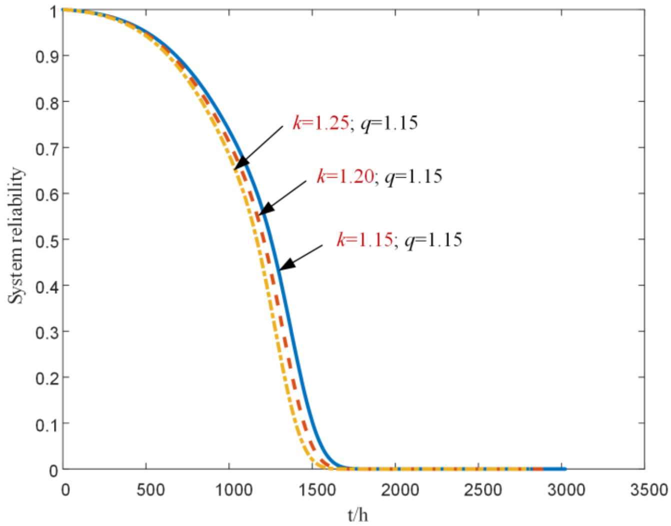

Fig. 5. Analysis of the proposed model for sensitivity k.

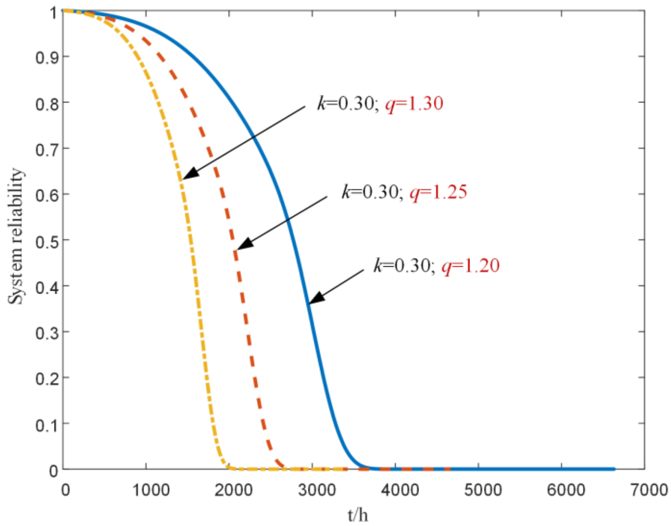

Fig. 6. Analysis of the proposed model for sensitivity q.

Fig. 5 and 6 show the sensitivity of degradation coefficient k and degradation rate q to changes in reliability under the influence of random shocks. As can be seen from the figure, as the parameter q increases, all reliability distributions have been shifted to the left. This indicates that the rate of system or component degradation is faster as the number of shocks increases. The effect of degradation proportional coefficient k on system reliability is similar to that of degradation rate q. When the degradation proportional coefficient k increases, the time scale function of performance degradation increases, resulting in a larger increment of performance degradation within the same time period, which causes the system reliability distribution to move to the left.

Due to the limitation of maintenance resources and cost, it is not practical to carry out preventive maintenance for all components of the system after the system reaches the inspection and maintenance cycle. Therefore, it it important to select some key components for maintenance to improve system performance and reduce system failure risks with limited maintenance resources.

4.2 Maintenance cost-based measure strategy

For the first type of maintenance, when the system is operating to the extent that regular maintenance is required, but the components in the system have not yet failed, it is necessary to identify the weak links of the system and carry out preventive maintenance.

According to Eq. (16), the six components IIM are

$$I_1^{IIM}(t) = R_1(t)\lambda_1(t)\frac{\partial R_S(t)}{\partial R_1(t)} = R_1(t)\lambda_1(t)(R_2(t)R_4(t)(1 - R_3(t)) + R_3(t))R_5(t)R_6(t)$$

$$I_2^{IIM}(t) = R_2(t)\lambda_2(t)\frac{\partial R_S(t)}{\partial R_2(t)} = R_2(t)\lambda_2(t)(R_1(t)R_4(t)(-R_3(t)))R_5(t)R_6(t)$$

$$I_3^{IIM}(t) = R_3(t)\lambda_3(t)\frac{\partial R_S(t)}{\partial R_3(t)} = R_3(t)\lambda_3(t)(R_1(t)(-R_2(t)R_4(t)))R_5(t)R_6(t)$$

$$I_4^{IIM}(t) = R_4(t)\lambda_4(t)\frac{\partial R_S(t)}{\partial R_4(t)} = R_4(t)\lambda_4(t)(R_1(t)R_2(t)(-R_3(t)))R_5(t)R_6(t)$$

$$I_5^{IIM}(t) = R_5(t)\lambda_5(t)\frac{\partial R_S(t)}{\partial R_5(t)} = R_5(t)\lambda_5(t)R_1(t)(R_2(t)R_4(t)(1 - R_3(t)) + R_3(t))R_6(t)$$

$$I_6^{IIM}(t) = R_6(t)\lambda_6(t)\frac{\partial R_S(t)}{\partial R_6(t)} = R_6(t)\lambda_6(t)R_1(t)(R_2(t)R_4(t)(1 - R_3(t)) + R_3(t))R_5(t)$$

Fig. 7 shows the IIM of the components. IIM of the component first increases and then decreases with time. This is because the reliability of each component is high at the beginning, so the importance is relatively low at the beginning. As time runs, the failure rate of each component increases, and the importance becomes greater. But components 1, 5, and 6 have a high rate of increase, indicating their importance. Components 2, 4, and 3 increase slowly because they are redundant with each other.

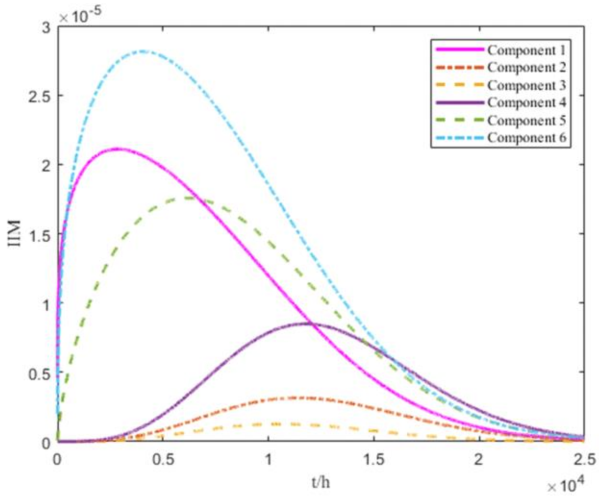

Fig. 7. IIM of components.

This indicates that the components without redundancy in the system should be prioritized during system maintenance. Since IIM does not consider maintenance costs, however, in maintenance practice, different components have different importance and maintenance costs, so they should be handled with different priorities. It is therefore crucial to include maintenance costs in the materiality analysis.

We assume that the system downtime cost is around US \$10,000 per day. Based on the data of reference [12], this paper determined the cost parameters of different components, as shown in Table 2.

Table 2. Maintenance costs and repair costs of the components in USD.

No.	Component	Maintenance cost	Repair cost
1	HR	6,000	10,000
2	FSOV	9,000	15,000
3	EMP	12,000	20,000
4	EDP	10,000	16,000
5	SV	8,000	13000
6	BPV	7,000	11000

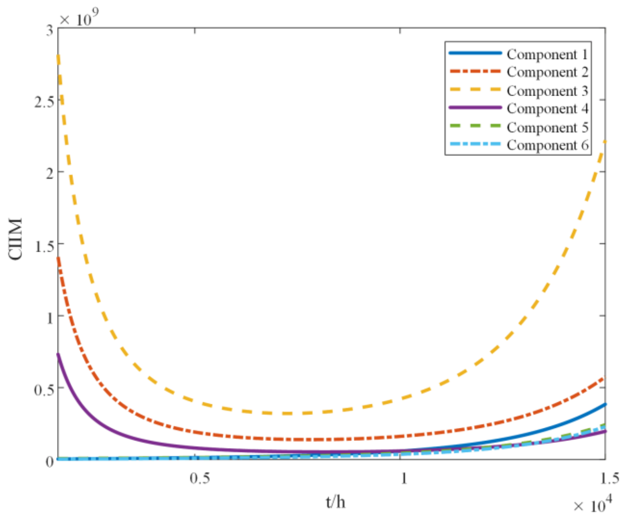

Fig. 8. Maintenance Sequence for Cost-Based IIMs.

Fig. 8 reflects the trend of CIIM based on maintenance cost for six components. As can be seen in Fig. 8, the cost importance measures of different system components vary greatly over time. The CIIMs of components 2, 3 and 4 start at higher levels, then drop to very low levels, and then start to rise. The CIIMs for components 1, 5 and 6 slowly increase with time. Due to a low maintenance cost of components 1 and 6, the overall importance of components 1 and 6 is the least in the first half of the maintenance cost, and their CIIMs slowly increase over time. Compared with the IBMP method proposed in [34], it can be found that the value of the EMP in the two methods is higher, which may be due to its lower maintenance cost and the higher failure rate of the EMP during operation. In addition, the EMP is in an important position in the system structure, and its importance is relatively high. Therefore, compared with the existing IIMs that do not consider maintenance costs, the proposed cost measurement strategy can reflect the relationship between the maintenance of different types of components in the system and the system reliability, which can better and more effectively guide the actual maintenance.

For the second type of maintenance, when a component fails and needs to be repaired, the faulty component affects the maintenance options for other normal components. This maintenance is the combined effect of faulty and normal components on the system.

Fig. 9 shows the JIIM change curves of other components when component k fails. As can be seen in Fig. 9, the joint importance measures of different system components vary greatly over time. Generally speaking, when a component fails, the JIIM of other components increases. When component 1 fails, the JIIM value of component 6 is the highest. It shows that when the faulty component 1 is repaired, the preventive maintenance of the component 6 has the effective impact on the improvement of the system reliability. But it can also be seen that when component 2 fails, component 3 will have a negative value. It can also be shown from the system structure diagram in Fig. 3 that after component 2 fails, component 3 can continue to play a substitute role, so its relative importance is negative. At this time, component 4 and component 2 are in the same channel, so its importance is the highest. The analysis of other components is similar. The joint importance measure shows that the failure of different components in the system will affect the maintenance of other components in the later period, and the sources of these influences are related to their positions in the system structure and their own parameters. However, since JIIM does not consider the maintenance cost of components, in maintenance practice, different maintenance costs should have different priorities. Therefore, the proposed maintenance cost-based binary measure is next simulated.

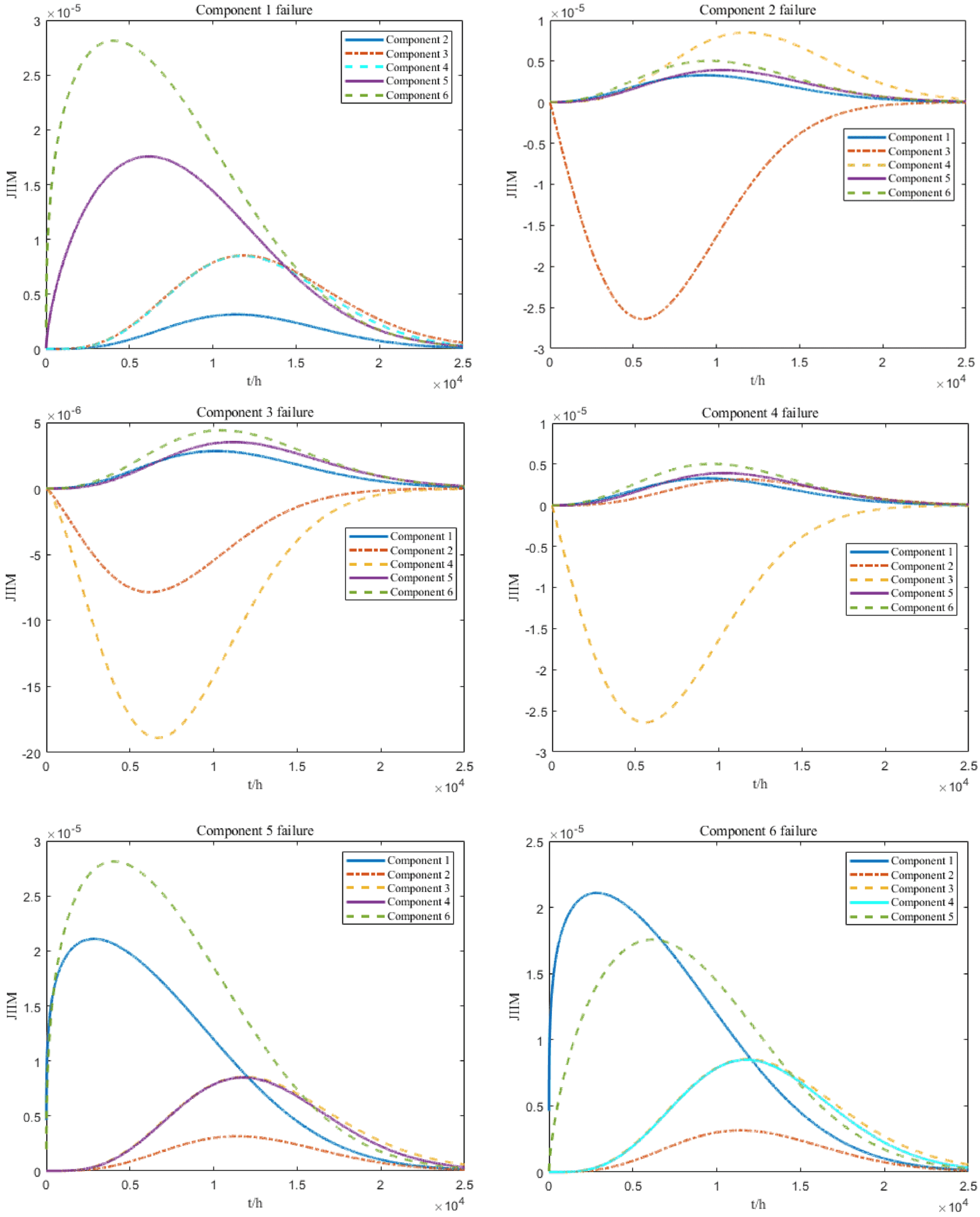

Fig. 9. JIIM of components.

The maintenance costs based on JIIM importance values of the six components are shown in Fig. 10. When component 1 fails and is repaired, the important of components 2, 3, and 4 decrease from larger values to lower values over time, and then start to go up. The importance value of component 2 exceeds that of components 3 and 4, which indicates that the maintenance cost in the beginning has a greater impact on importance, but from the perspective of the whole system cycle, this impact gradually becomes smaller. Also, when component 2 failed, the CJIIM of component 3 showed a negative value. This shows that when component 2 fails, component 3 has a higher maintenance cost, and because of its lower importance level, its cost-based joint importance value changes negatively over time. The analysis of other components is similar. This is similar to the trend in Fig. 8 when using a maintenance cost-based IIM. Comparing with the JIBMP method proposed in [34], it can be found that some components in the two methods have negative values, and these situations all appear in components with redundant channels. This may be because redundant components can function as faulty components to maintain system operation, so components in redundant channels are prone to negative values. In conclusion, compared with the existing IIM and JIIM, which do not consider maintenance cost as a key factor in actual maintenance practice, the importance measure method based on maintenance cost proposed can better reflect the impact of different types of costs on the maintenance sequence of components, so as to guide system maintenance more engineering and rationally.

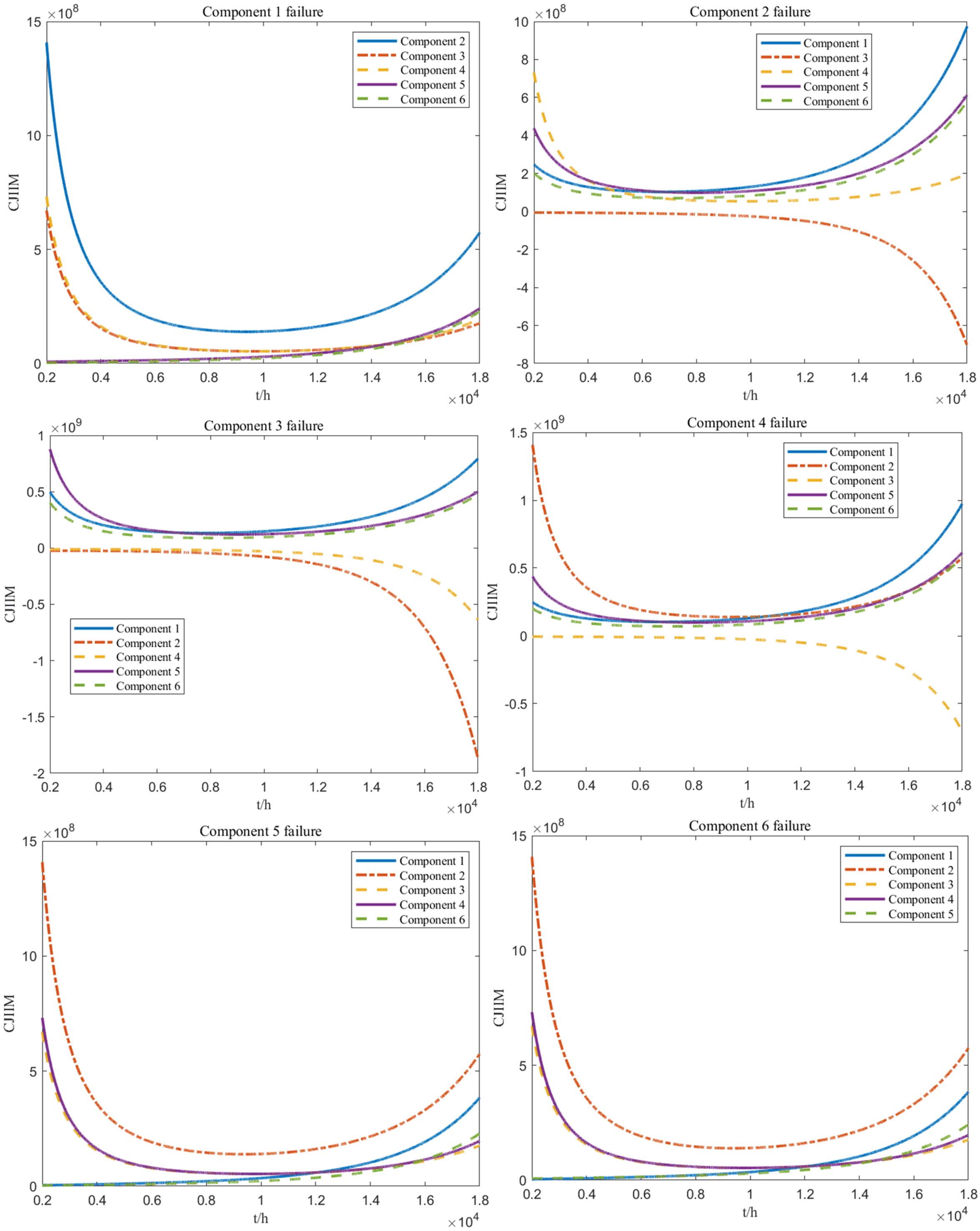

Fig. 10. JIIM based on maintenance cost.

Fig. 11 shows the relationship between the system's failure risk and the maintenance costs over time. It can be seen from Eq. (26) that in the initial operation stage of the system, the failure probability of the system components is low, and the risk of failure is low. Thus, the components' maintenance frequency can be reduced during this period, reducing unnecessary system maintenance costs. Then wear and aging of components will gradually occur, leading to increases in the system's failure rate and failure risk. Therefore, the maintenance frequency of the system should be increased to reduce the probability of system failure. Cost-based risk analysis ensures that the system is at an appropriate maintenance frequency at each stage of its life cycle, so as to reduce the state of over-maintenance or under-maintenance of the system.

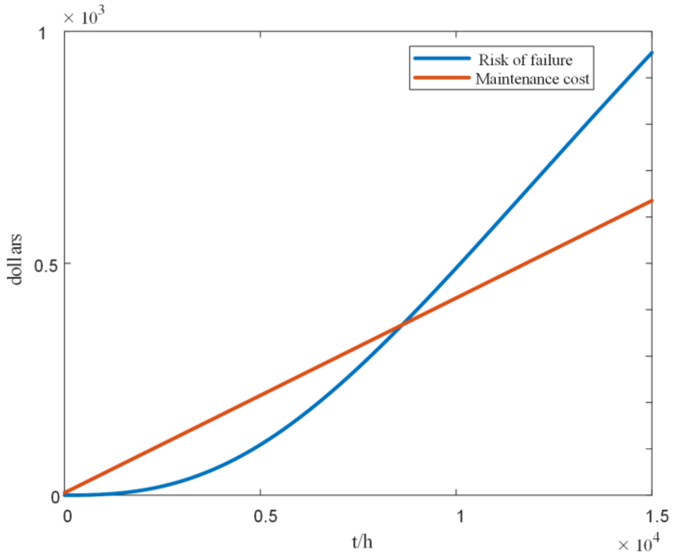

Fig. 11. The relationship between the risk of failure and the maintenance costs.

5. Conclusions and future work

Considering the actual degradation of the product, in addition to natural degradation, it will also be affected to varying degrees by external factors such as corrosion and impact. In addition, under the influence of external factors, the failure threshold of the product will also change. On the basis of traditional reliability modeling, this paper introduces random shock and dynamic failure threshold to jointly conduct reliability modeling. Furthermore, considering the impact of maintenance decisions and system component costs on component importance, this paper proposes a new CIIM and CJIIM measures to evaluate the maintenance strategies for different components in two types of maintenance, respectively. Different from traditional materiality measures, the proposed importance measure takes into account the joint impact of external shocks and maintenance costs on system maintenance, and provides a more reasonable and effective solution for actual system maintenance activities. We found that: 1) There are some ways to improve system reliability, that is, increasing the ability to resist internal deterioration and degradation caused by impact damage. 2) When the component contributes more to the system's lifetime , the cost to restore its performance is higher. 3) The component maintenance cost has a large impact on their importance, whereas this impact tends to decrease over time.

In this work, we consider only two states per component, that is, the state of the system is determined by the combination of failure states for each of its components. In future research, we will consider the impact of the component recovery efficiency on the maintenance strategy and conduct a more comprehensive evaluation.

Acknowledgments

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