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A SIMULATION APPROACH ON RELIABILITY ASSESSMENT OF COMPLEX SYSTEM SUBJECT TO STOCHASTIC DEGRADATION AND RANDOM SHOCK

METODA SYMULACYJNA OCENY NIEZAWODNOŚCI ZŁOŻONEGO SYSTEMU PODLEGAJĄCEGO PROCESOM DEGRADACJI STOCHASTYCZNEJ I NARAŻONEGO NA OBCIĄŻENIA LOSOWE

Many systems are affected by different random factors and stochastic processes, significantly complicating their reliability analysis. In general, the performance of complicated systems may gradually, suddenly, or continuously be downgraded over times from perfect functioning to breakdown states or may be affected by unexpected shocks. In the literature, analytic reliability assessment examined for special cases is restricted to applying the Exponential, Gamma, compound Poisson, and Wiener degradation processes. Consideration of the effect of non-fatal soft shock makes such assessment more challenging which has remained a research gap for general degraded stochastic processes. Through the current article, for preventing complexity of analytic calculations, we have focused on applying a simulating approach for generalization. The proposed model embeds both the stochastic degradation process as well randomly occurred shocks for two states, multi-state, and continuous degradation. Here, the user can arbitrarily set the time to failure distribution, stochastic degradation, and time to occurrence shock density function as well its severity. In order to present the validity and applicability, two case studies in a sugar plant alongside an example derived from the literature are examined. In the first case study, the simulation overestimated the system reliability by less than 5%. Also, the comparison revealed no significant difference between the analytic and the simulation result in an example taken from an article. Finally, the reliability of a complicated crystallizer system embedding both degradation and soft shock occurrence was examined in a three-component standby system.

Keywords: system reliability; multi-state system; competing failures; stochastic degradation; random shocks; discrete event simulation.

Prawidłowe działanie wielu systemów zależy od różnych czynników losowych i procesów stochastycznych, co znacznie komplikuje analizę niezawodności tych układów. Parametry pracy skomplikowanych systemów mogą ulegać stopniowemu, nagłemu lub stałemu obniżeniu ze stanu doskonałego funkcjonowania do stanu awaryjnego. Wpływ na nie mogą też mieć niespodziewane obciążenia. W literaturze przedmiotu, analityczną ocenę niezawodności stosuje się do badania przypadków szczególnych i ogranicza do badania degradacji w oparciu o proces wykładniczy, proces gamma, złożony proces Poissona i proces Wienera. Ocena niezawodności z uwzględnieniem wpływu obciążeń miękkich, nieprowadzących do całkowitej awarii, stanowi większe wyzwanie tworząc lukę w badaniach nad ogólnymi stochastycznymi procesami degradacji. Aby uniknąć złożonych obliczeń analitycznych, w niniejszej pracy skupiliśmy się na zastosowaniu podejścia symulacyjnego w celu uzyskania generalizacji. Proponowany model obejmuje zarówno stochastyczny proces degradacji, jak i losowo występujące obciążenia i uwzględnia przypadki degradacji systemów dwustanowych, wielostanowych oraz degradacji ciągłej. Posługując się tym modelem, użytkownik może dowolnie ustawiać rozkład czasu do uszkodzenia, degradację stochastyczną, czas do wystąpienia obciążenia, funkcję gęstości prawdopodobieństwa wystąpienia obciążenia, a także jego nasilenie. Trafność oraz możliwości zastosowania przedstawionego modelu zilustrowano na podstawie dwóch studiów przypadków dotyczących cukrowni oraz przykładu zaczerpniętego z literatury. W pierwszym studium przypadku, poziom niezawodności systemu obliczony na podstawie symulacji różnił się o mniej niż 5% od wyniku otrzymanego na drodze analitycznej. Porównanie nie ujawniło również żadnej istotnej różnicy między wynikiem analitycznym a symulacyjnym w przykładzie pochodzącym z literatury. Artykuł wieńczy analiza niezawodności złożonego układu krystalizatora, obejmująca zarówno degradację, jak i występowanie miękkich obciążeń w trójelementowym systemie krystalizatora z rezerwą.

Słowa kluczowe: niezawodność systemu; system wielostanowy; uszkodzenia konkurujące; degradacja stochastyczna; obciążenia losowe; symulacja zdarzeń dyskretnych.

1. Introduction

Reliability is common scientific characteristic of a system with commutability, operability, or usability upon any request to accomplish the relevant nominated tasks over time to finally evaluate the system potential or performance. In this regard, their assessment is a

crucial analytic task given the huge complexity and solving the many states equation especially in the presence of stochastic degradation process and random arrival shock with unknown severity. This context has remained a research gap, which has attracted much attention in the literature by Patelli et al. [17]

In general, the system reliability is analyzed in three ways: binary or two states, multi-state, and degradation process which present system state continuously over time. Commonly, to avoid heavy calculations, reliability assessment is carried out for a few states. Such an approach employs an oversimplification in many real-life situations where the system is accomplished based on assuming a comprehensive range of states, varying from perfect functioning to complete breakdown.

Conventionally, system reliability assessment should follow tedious computations for simultaneously solving a large number of differential equations to calculate the probability of the system being in each state. Then system reliability is computed based on summation of probability of all states where the system functions well. In many real cases, a system may become degraded when being subject to random shocks with different degrees of severity. This may predispose the system to fail suddenly or accelerate their degradation process. Hence, applying analytic methods for reliability assessment is complicated especially for multi-state systems. Accordingly, in the present study, we have developed an appropriate and efficient simulation approach for this issue to develop the professional capabilities required by analysts.

Degradation refers to either performance degradation (e.g. power output of a generator) or some measure of actual degradation (e.g., toxic concentration in a chemical process or fatigue crack in a gear). Commonly, the degradation process reveals a continuous alteration of the system state over time. Once a proper degradation variable is selected, degradation data, when properly measured, could provide substantial information as there are quantitative measurements (not just at discrete points of time or their failure). Indeed, it is possible to make powerful reliability inferences from degradation data even when there are no failures.

The degradation process could be modeled using the experimental data through degradation path modeling method. Stochastic degradation process tends to model the degradation variable over time while considering the measurement error. This kind of modeling consists of two terms to present the deterministic behavior of the variable level through a linear, quadratic, exponential and other terms alongside the error term which is described by a given random term; e.g. Gamma, Logistic or Weibull distribution (Nikulin et al. [16]).

Additionally, most engineering systems suffer from catastrophic events occurring randomly and they could cause sudden breakdown or initiate other mechanisms which accelerate the failure process. Hence, the time of shock occurrences and their severity are presented by two random variables. A sophisticated review on shock modeling methods in reliability engineering has been presented by Finkelstein Maxim and Cha Ji Hwan [7].

Through current research, we focused on answering to the following research questions.

1. How to estimate reliability of a multi-component multi-state system on the based on simulation modeling?
2. What is the consequence of randomly occurred shocks?
3. How to estimate reliability of a gradually stochastic degraded system?

The rest of this paper is organized as follows. Section 2 reviews the literature on reliability assessments of binary, multi-state, and degraded processes using analytic methods and simulation models. Due to complex calculations of the analytic method, essential basis as well the proposed simulation model are presented in section 3. The validation method for the proposed simulation model presented on section 4. Section 5 discusses a real case study to clarify the proposed method in details. Finally, section 6 closes the paper with concluding remarks, advantages, and drawbacks to be covered by future research.

2. Literature review

Many research efforts have been made to assess reliability of systems. So far, a great deal of attention has been paid to system reliability analysis which deals with a binary state system describing system states using functioning or failure states via a specified random variable. Over the past few decades, reliability practitioners have been working on analyzing system reliability using more data collected during the system life time. In this way, the system state has been evaluated over time discretely or continuously through multi-state or degraded level. Some reliability experts have also focused on other sources accelerating failure process such as shock or hazardous events.

An analytic model to evaluate a degrading binary system during a fatal shock has been presented in Riascos-Ochoa et al. [19]. They fitted a phase-type distribution to inter-arrival time in case of shock occurrence. This approach helps users evaluate one single component reliability. Another research Caballé and Castro [3] proposed a model with internal degradation under a gamma process and random shocks with non-homogeneous Poisson process. In addition, they analyzed the robustness of the solution by changing the input parameters. Also, binomial shock process was evaluated in Eryilmaz [5]. They extended their model to the presence of shock dependent processes using the Markov chain. An analytic model for a single component on the presence of hard failure (shocks) and soft failures (degradation) for fault-tolerant systems was probed in Liu et al. [14]. They also implemented a proposed model on an example to show the applicability to many systems via a model with cumulative shocks based on batch Markovian arrival process Montoro-Cazorla and Pérez-Ocón [15]. In their model, shock processes are interdependent and the system failure occurs when the number of cumulative shocks exceeds the defined threshold. A Stress-Strength model was developed in Hao et al. [9] for soft and hard failures and their interactions. The results revealed a positive correlation between shock process and degradation performance and the mutually dependent processes had direct effects on the system reliability. In Rafiee et al. [18], a generalized mixed shock model involving fatal and non-fatal shocks was presented analytically. In this paper, three types of shock patterns were taken into account. A sensitivity analysis was applied on an example of micro-electro-mechanical system to show the application of the proposed model.

For analyzing system reliability in shock-degradation models, some methodologies have been presented. For example, Huang et al. [10] offered an analytic method for reliability assessment for a system affected by smooth degradation with the gamma process and traumatic failure caused by Poisson shock process. In their proposed method, given the two processes in that, with increase in the degradation level, the probability of traumatic failure caused by a random shock increased. A degradation model for in civil structures was presented by Wang et al. [22]. This model had two main components: non-increasing stochastics parameters and existing correlation between load processes and degradation. They also developed this model on a numerical example to analyze sensitivity of reliability on degradation and shock processes as well as the load-deterioration correlation. They found that the system reliability was very sensitive to variations of cumulative deterioration and shock. Further, two different types of dependent competing failure processes were modeled in An and Sun [1].

At first, a shock and a degradation process were considered simultaneously along with the existing interdependency between them. Secondly, multiple degradation processes with their correlation were added to the model. Finally, by extending a numerical example, sensitivity analysis was made to evaluate the effects of parameter models on the system reliability. The reliability of a system based on the presence of fatigue degradation and shock processes was evaluated by Zhang et al. [26]. They considered retardation event in their model, i.e. fatigue procedures were retarded when the shock occurred in the

system. They evaluated the reliability in two case studies: in the first one, they considered shock processes as a fixed time period, while in the second, shock occurred at various time periods. The reliability of load-sharing systems with dependent shock and degradation processes was investigated by Che et al. [4]. In their assumed system, the failure time of components, the time between arrival of shocks, and their interaction were stochastic, so they used an analytical method to analyze the reliability of the system. Experimental results indicated that the reliability in load-sharing systems was lower than in simple parallel systems.

Some authors used a simulation model to model and analyze the reliability of systems. Monte Carlo simulation was applied to analyze the system reliability of a degradation-shock process model in Fan et al. [6]. In their simulation model, the shock process was influenced by the soft failures (degradation) and random shocks were categorized into three zones based on their magnitudes. In Warrington and Jones [23], a discrete event simulation with path-sets methodology was presented to generate a dynamic model for analyzing the reliability of system. Another simulation model for reliability of a self-healing network for scalable and fault-tolerant, parallel runtime environments was evaluated by Angskun et al. [2]. They used a simulation method to calculate the system under failure conditions. Gola [8] focused on the way to estimate system reliability with changing machine due to maintain the production process stability using Enterprise Dynamics software.

In Vaisi et al. [21], an availability reliability model for a two-machine robotic cell was presented for different sources of uncertainty. They implemented this structure on a multi-state transmission system. Further Juan et al. [12] presented a simulation methodology in a time-dependent building for civil engineering structures. They discovered that the simulation method could offer more advantages over other approaches, since it could measure details such as multi-state systems and discover critical components in a structure. Many researchers such as Kosicka et al. [13], Jasiulewicz-Kaczmarek and Gola [11], Zaim et al. [25] and Sobaszek et al. [20] focused on the way to increase system reliability in different aspects. Interested readers may follow some beneficial methods in

In the case of reliability assessment for a multi-state system, analytic calculation is highly complex. Most cases have focused on the exponential time to failure and time to repair distribution. Wenjie et al. [24] proposed a reliability index for a repairable multi-state component using homogenous continuous time Markov chain process. Their method was limited to a short time repair process. Further, they tried to balance the maintenance cost and lifetime of multi-state components in an illustrated example. The complexity of computation of analytic methods encourages researchers to use efficient simulation techniques especially for reliability assessment of multi-component multi-state systems.

As the literature suggests, almost all existing methods have been conducted based on restricted assumptions. The reliability assessment of a multi-state system which degrades randomly and suffers from competing random shock effects when all random variables could not be modeled in any free given process has been research gap so far. This paper proposes an efficient computer simulation approach in reliability assessment of a multi-state system subject to stochastic degradation process and randomly occurring non-fatal shocks. The proposed method has no restriction on applying Markov or semi-Markov process. It has also the capability to be applied for any time of occurrence of shocks, and for their relevant consequences and any random degradation process.

3. Computer simulation model basis

Basically, the reliability analysis of a multi-state system depends its component features, states and the system RBD. Suppose a multi-state system consisting of N components where each component j could have k_j different states corresponding to its performance rates, represented by Eq. 1:

$$g_j = \{g_{j1}, g_{j2}, \dots, g_{jl}, \dots, g_{jk_j}\} \quad (1)$$

where, g_{ji} is the performance rate of component j in the state $i \in \{1, 2, \dots, k_j\}$. Suppose that the performance rates are arranged in a descending order at different states of each component. For example, in state g_{j1} , the performance rate of component j is perfect and complete. When the state transfers from g_{j1} to g_{j2} , the performance rate will decrease for example to 90%. This descending status will continue until the state of the component reaches g_{jl} . In the remaining states $\in \{l+1, l+2, \dots, k\}$, the system does not have an adequate performance level which could be called complete breakdown. So, the element j may be functioning well in the state $q \in \{1, 2, \dots, l\}$. Figure 1 displays typical performance rates for a system at 12 different states which could be considered during the last four states.

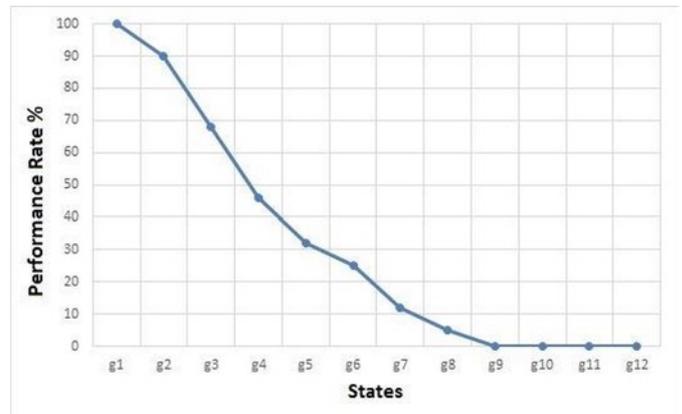


Fig. 1. A typical degrading system performance over time

The probabilities related to the different states of the component j at any instant time t can be displayed by the Eq. 2:

$$p_j(t) = \{p_{j1}(t), p_{j2}(t), \dots, p_{jl}(t), \dots, p_{jk_j}(t)\} \quad (2)$$

where $p_{ji}(t) = \Pr\{G_j(t) = g_{ji}\}$ and $\sum_{i=1}^{k_j} p_{ji}(t) = 1$.

So the reliability of each component presents by Eq. 3:

$$R_j(t) = p_{j1}(t) + p_{j2}(t) + \dots + p_{jl}(t) = \sum_{i=1}^l p_{ji}(t) \quad (3)$$

Finally, in a multi-state system with N series components, the reliability is equal to Eq. 4:

$$R_s(t) = \prod_{j=1}^N R_j(t) \quad (4)$$

Also, for a multi-state system with N parallel component, the system reliability is calculated by Eq. 5:

$$R_s(t) = 1 - \prod_{j=1}^N (1 - R_j(t)) \quad (5)$$

Degradation processes act as a failure mode and are often defined by a smoothing continuous damage accumulated over time. In addition, temporal variability should be taken into account during the system degradation process. It usually may be modeled by a given deterministic curve beside an error terms follows from a given statistically density function such as Exponential, Gamma, Logistic, Weibull, or etc. In a multi-state system with N elements, the performance rate of component j in the state i may be reduced or updated due to its relevant deterioration process or shock occurrence as Eq. 6:

$$\dot{g}_{ji}(t) = \alpha_{ji}(t) \cdot g_{ji}(t) \quad (6)$$

where, $0 < \alpha_{ji}(t) < 1$ is the degradation rate of performance of element j in the state i at time t , and the prime accent on g denotes the updated values for the performance rate.

Suppose $X_j(t)$ represents the degradation level of component j at time t . For a continuous or discrete degraded process, each component fails if the relevant level exceeds its threshold l_j . Let T_{dj} be the failure time of degradation process; thus, the reliability of the system while only considering with degradation process can be then estimated by the fraction of the time when the system performance level is greater than the threshold level l . Equivalently, in mathematical terms for each component, the reliability is given by Eq. (7):

$$R_j(t) = \Pr(t < T_{dj}) = \Pr(X_j(t) > l_j) = \Pr(g_j(t) > l_j) \quad (7)$$

Consequently, the simulation model extracts the component reliability output through the counting ratio of the desired condition over the total runs. Note that after running the simulation model over given period of time, this equation is applied for individual estimation of only single component reliability not the system reliability. Formerly, system reliability could be calculated using reliability block diagram indicating how component reliability contributes to the success or failure of a complex system. After a few repetitions of the process for different simulated observation periods, reliability curve can be illustrated.

Shock is another common competing cause in system failures which accelerates the component degradation rate or random failure processes. The literature has pointed to two kinds of shock; fatal and non-fatal shock. The first type causes rapid disruption while the second accelerates the degradation process. Thus, the transition rate between consecutive states grows progressively.

The proposed simulation method need to following inputs.

1. The system configuration in terms of System Reliability Block Diagram (RBD). For each component, time to failure density function needed to defined necessary. Also specify the component's repair time density function if needed to calculate availability.
2. State transition matrix for each component just for multi-state system. This modulus has no need to be defined for continuous degraded systems.
3. Degradation modulus. For each component just one time dependent degradation model should be described as well a time to failure density function.

4. Random shock occurrence modulus which embeds time to shock occurrence density function and its random effects in terms of a constant or random density function.
5. System reliability/availability estimation modulus.

The first four moduli need some inferences on the system and which are commonly used for reliability analyzers and should be set as simulation model inputs. These moduli are designed individually and some of them are not necessary in all problems. Figure 2 illustrates the proposed simulation method schematically.

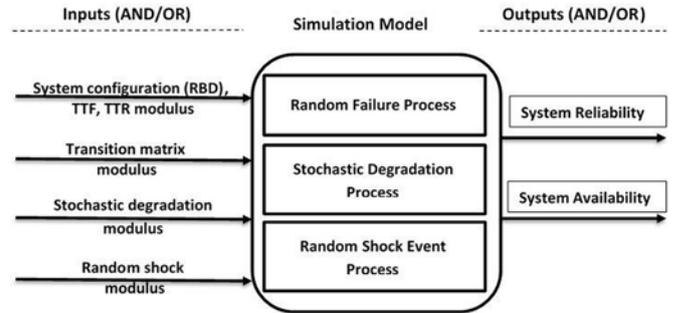


Fig. 2. A schematic view of the proposed computer simulation method

The basic output modulus accounts the system reliability through summation of the probability of existence of a system in a given set of desired states after running the system for a given replications. Here, the system configuration should be defined via the system RBD and breakdown features of some components. They are: 1) failure patterns; hence time to failure (TTF) and time to repair (TTR) density function for each component should be acknowledged experimentally. Degradation process which may be presented by a deterministic curve and randomly distributed residuals/error terms (ET). Random shocks through defining the time between occurrence/arrival (TBA) alongside its severity statistical distribution (S). Here, S represents the substitution of extremely high hazard rate models of a fatal shock; otherwise it is a repeated non-fatal shock which accelerates the degradation process. An alternative approach to apply the soft shock consequences surveyed in the 2nd case study report.

In order to simulate such a system, we applied an object-oriented approach where all components are represented by an individual object using the Enterprise Dynamics Incontrol simulation software; EDTM. In this simulation, the package of almost all well-known density functions is ready to use at its library. So, TTF, TTR, TBA, ET and S could be easily interpreted via a well-known statistical density function. We applied a "Server" to model each component. Accordingly, the number of atoms in the model should be related to the number of components. Here, their relevant cycle time could be adjusted to any constant or random value. So, we set them to zero for immediate processing. In order to estimate the system reliability, it is necessary to eliminate TTR effects. In other words, the repair time should be considered as a very large value to prevent completion of the repairing process. Each simulation run takes a long period of time. When the simulation model is run, a continuous flow of "Product" entity is created. The entity flow simulates the desired component performance and may change the system state over time until one of the failure modes occurs.

At the beginning of simulation, all components are set on their first state with its maximum performance until any significant discrete event occurs over time where the entrance of any entity affects them through degradation process or shock occurrence modulus. These two individual moduli have their own network. In the basic network, the entity is allowed to go to the next Server based on the system RBD. This sequence will continue to meet the last server. Figure 2 typically presents a sample layout for the simulation of only one component

which has four states. Note that this sample network does not reveal the degradation process and shock occurrence and it could cover only random breakdowns. So, no triggers from other modulus affect the basic network. Consequently, the probability of the system being on each state could be calculated by counting the number of entities transitioning into the respective states. For example, consider after a two-hour simulation, the number of entities reaching states 1, 2 and 3 is equal to 200, 150 and 80 respectively.

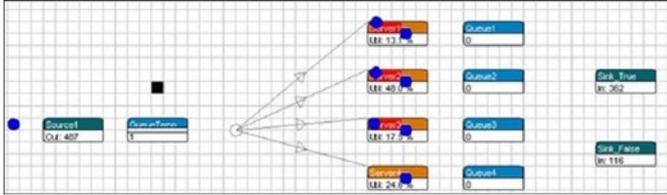


Fig. 3. An Enterprise Dynamic layout for a four-states system

Suppose the component is working both in the first two states, consequently the reliability calculated through $\frac{200 + 150}{200 + 150 + 80} = 81\%$ for a period of 430 hours. After replicating the model, the relevant curve could as plotted as shown by Figure 4. More replication conducts to the more smooth curve.

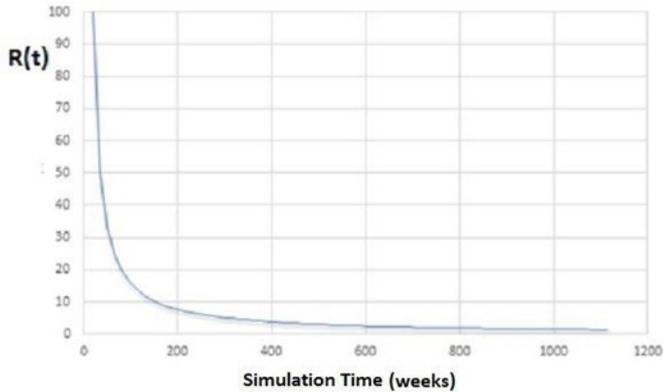


Fig. 4. A typical reliability curve

As a parallel of the basic network, two individual networks could influence each component failure by sending triggers. The first simulates the degradation process while the second models the shock occurrence. If fatal shock occurs, the relevant component immediately fails; otherwise the time to failure time drops by a constant adjustable parameter. This trick enabled us to model the accelerated breakdown process by a constant percentage. The shock network need two inputs: 1) Inter-arrival shock process TBA which could be set by any random distribution (e.g. Exponential, Gamma, Weibull, ...), and 2) The shock consequence table which embeds the updated degradation rates; $\alpha_{ji}(t)$. For a fatal shock, the practitioner may set the rates to large value to immediately reveal the failure of the component.

4. Validation of the proposed simulation model

In order to examine the computer simulation model capability, we followed a two-step process. First, the validity of the simulation model was examined and compared with an analytic method. Once the validity of the simula-

tion model was verified, it was used to estimate the system reliability in the desired complex systems. Hence, applying an analytic model is a very difficult technical task.

In order to validate the simulation model, a crystallizing shaft crack grow model was considered on a sugarloaf plant in Shahroud, Iran. A research team consisting of five skilled persons from the University studied 10 shaft samples of one type crystallizer machine for 100 consecutive days in terms of crack growth using an X-ray machine. Here, the crack initiation time was uncertain and should have been estimated. Experts believed that since the shaft under study was not a critical component and could not withstand high mechanical loads, the maximum crack length should have been less than 100 mm. Table 1 reports the values of the experimental data for the measured crack length.

In order to examine the degradation models, we overviewed the crack length of samples over time. Figure 1 displays such actual degradation. Rottenly linear, exponential and power models act as a proper candidate for the fitness function. Since the linear and exponential models did not have an adequate fitting index in terms sum of squared error, the best fit was obtained based on the power model. The general form of such fitness function is presented by the function below:

$$D_t = b(t - t_0)^a \tag{8}$$

where, D_t denotes the crack length in time. Hence t_0 begins for the crack initiation time and like a and b parameters they should be estimated using maximum likelihood estimation method from experimental data. Table 2 presents an estimation of the power model parameters. In this table, the failure time was calculated based on the $D_{Max} = 100mm$ and the growth time was calculated according to the period of time between the failure time and crack initiation time.

Fitting a different probability distribution function to the calculated crack growth time reveals that the Weibull has a good candidate to estimate crack growth time density function. Figure 5 presents the relevant probability plot justification. Low amount of the Anderson Darling statistic; 1.657 beside high p-value; greater than 0.1 reveals that we has not any evidence to reject the Weibull distribution in the goodness of fit hypothesis testing.

Using the Minitab statistical package, the reliability plot of the shaft growth time using 95% confidence interval is illustrated in Figure 6. According to the results, the newly received shafts do not have good quality and their cracking started within the interval of 8 to 24 days reaching their maximum permissible average over a period of 110 days. Also, the reliability of these shafts has been only 51.57%

Table 1. Crack length (mm) of 10 samples of crystallizer machine shaft

Time (days)	Sample No.									
	1	2	3	4	5	6	7	8	9	10
10	13	11	13	24	8	22	16	18	12	14
20	34	46	32	71	38	60	33	48	31	35
30	45	69	40	108	54	90	42	67	40	45
40	55	88	45	139	65	120	50	84	48	50
50	63	103	52	165	75	143	55	100	53	60
60	71	118	54	195	85	165	60	112	58	63
70	77	130	60	216	92	187	61	124	60	68
80	84	144	65	243	100	210	64	135	65	70
90	89	153	67	264	106	225	70	145	70	75
100	93	165	70	285	111	245	72	158	75	80

Table 2. Degradation parameters and time to failure estimation for crack process

Parameters	Sample No.									
	1	2	c	4	5	6	7	8	9	10
t_0	13	11	13	24	8	22	16	18	12	14
a	34	46	32	71	38	60	33	48	31	35
b	45	69	40	108	54	90	42	67	40	45
Failure time	111.15	48.00	231.47	27.78	81.42	33.02	204.45	51.288	213.91	176.01
Growth time	63	103	52	165	75	143	55	100	53	60

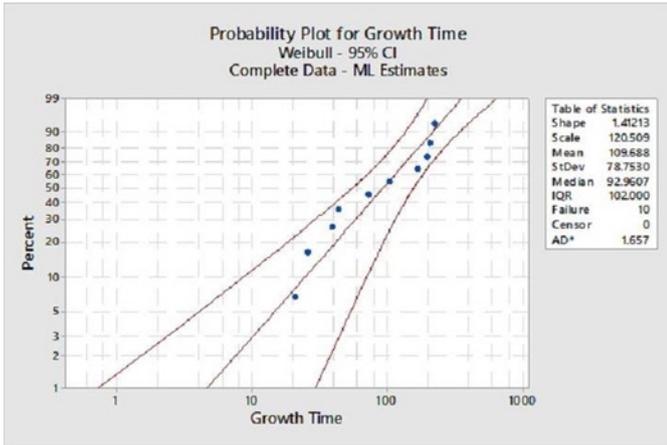


Fig. 5. The crack growth time probability plot based on the two parameter Weibull distribution

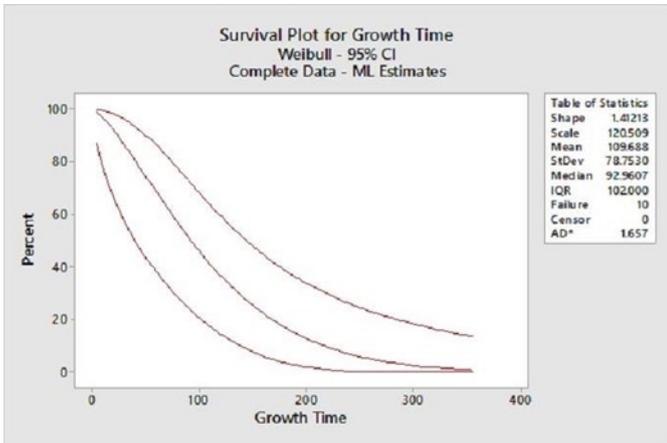


Fig. 6. Fig. 6. The 95% confidence interval for reliability of the growth time

for 90 days of operation after cracking initiation; so, this type of shaft should be replaced with higher quality types.

As mentioned earlier, applying a technical method for reliability of a case study needs many sequential statistical methods. Since the shaft breakdown tends to fail the crystallizer machine and there are a set of different machines in this department, so the reliability analysis of such departments is very time-consuming especially when the system has a complicated reliability block diagram of multiple multi-component machines engaged with different stochastic degradation processes and random soft/hard shocks. A feasible efficient solution method could be applied using a computer simulation model.

Let us now examine the system reliability estimation through simulation model for the first case study report. This case embeds one degradation process alongside a soft shock occurrence. In that case

at any given time, the crack length develop from a Weibull distribution with 1.412 and 120.509 respectively for the shape and the scale parameters. Hence the mean of the degradation process is assumed to develop a power function based on the mean estimates of samples for a , b , t_0 parameters as function $D_t = 11.34(t - 8.33)^{0.521}$. Defining one block showing the crystallizer in the first modulus and setting a two-state system transition diagram with time to failure density function based on the Weibull (1.412,120.504) in the second modulus while recognizing the degradation function in the fourth modulus are necessary to estimate system reliability at a given time, say 90 days. Running the simulation model under 5 runs reported the values of system reliability as: 45.1 %, 64.9%, 57.1%, 54.4%, and 49.2%. Thus, the simulation model estimated the mean shaft reliability as 54.14%, only 4.7% overestimation in relation to the analytic method.

In order to further examine the simulation model validity, we run the sample model using a two-parameter gamma degradation process. According to Huang et al. [10], the reliability of such a process is given by Eq. (9):

$$R_d(t) = 1 - \int_L^\infty f_{\alpha,\beta}(x) dx = 1 - \frac{\Gamma(\alpha, L\beta)}{\Gamma(\alpha)} \quad (9)$$

Here, α denotes the shape parameter β stands for the scale parameter. Also, the threshold degradation is represented by L. By substituting the mentioned parameters by 60, 10, and 14.5 respectively, the system reliability will be 0.97, 0.53, 0.31, and 0.22 for 4, 8, 12, and 16 weeks respectively. In order to compare the simulation model under the same circumstances as with the analytic model, we set the simulation model based on the gamma random TTF while the another feature of the model was relaxed. Running the simulation model for 25 replications, the Mann-Whitney non-parametric statistical testing was applied using the SPSS statistical package. The results of this hypothesis testing are summarized below:

Point estimate for ETA1-ETA2 is 0.044
 95.0 Percent CI for ETA1-ETA2 is (0.032,1)
 W = 10285.0
 Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.9432

The report reveals that the mean differences between the simulated and analytic values of reliability is too small (0.044). The calculated p-value for the W-statistic is 0.9432 greater than the significant level. Consequently we could conclude the there is no reason to reject the null hypothesis. Hence the validation of the proposed simulation method is acceptable.

Once again it is emphasized that the simulation output illustrates reliability instead of availability when repair time sets to a very large amount to prevent the system from returning to the working state.

5. Case study

In order to fully reveal the applicability of the proposed simulation model, we focused on a more complicated system in the same crystalizing process in the Shahroud sugarloaf. Here, this production department was equipped with three crystallizer units. Units A and B worked serially while another one (unit C) acted as a cold standby unit and could be activated instantly. The desired performance requires proper operation of two out of three units carrying out their duties successfully. Progressively, the state of each crystalizing unit changes over working time through the state space of “Good” to “Faulty”, “Imperfect” and finally “Fail”. The nomenclature of this crystalizing modulus is shown in Table 3.

Table 3. Nomenclature

Nomenclature	Description
A, B, C	Crystalizing units
$0, 1, 2, 3$	States of each crystalizing unit, respectively stands as Good, Faulty, Imperfect and Fail
S, W, R, O	Condition of each crystalizing unit, respectively stands for Standby, Working, Repair, and Operable
$S_i(A_{mS}, B_{mW}, C_{nW})$	The i^{th} state of the system, in which elements A, B, and C are in state m and standby, state m and working, and state n and working, respectively. ($m, n \in (0, 1, 2, 3)$ and $i \in N$ is the size of system state space, For example state $S_7:(A_{0S}, B_{1W}, C_{2W})$ means the system is working, the unit B and C are in faulty and imperfect respectively and working while unit A is in good state and standby.
γ_{30X}	Repair rate (transition from state 3 to 0 for crystalizing unit of element X ; $X \in (A, B, C)$)
λ_{mnX}	Rate of transition from state m to n for crystalizing unit of element X ; $m, n \in (0, 1, 2, 3)$, $X \in (A, B, C)$

The state transition sequence for crystalizing unit of X is shown in Figure 7. Here $X \in (A, B, C)$.

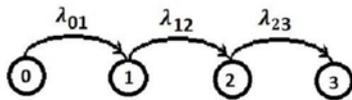


Fig. 7. State diagram for each crystalizing unit in the sugarloaf plant

Each unit is said to be working if it is in states 0, 1, or 2 and the units is said to be failed if it is in state 3. Among of all of system states, there are 54 working system states that have been listed in table 4.

The reliability becomes the probabilities that the system is in the working states and is given by:

$$R(t) = \sum_{1 \leq i \leq 54, i \in N} P_i(t) \tag{10}$$

Table 4. The states in which the system acts as working

Unit A in repair or standby		Unit B in repair or standby		Unit C in repair or standby	
$S_1:(A_{0S}, B_{0W}, C_{0W})$	$S_{10}:(A_{3R}, B_{0W}, C_{0W})$	$S_{19}:(A_{0W}, B_{0S}, C_{0W})$	$S_{28}:(A_{0W}, B_{3R}, C_{0W})$	$S_{37}:(A_{0W}, B_{0W}, C_{0S})$	$S_{46}:(A_{0W}, B_{0W}, C_{3R})$
$S_2:(A_{0S}, B_{1W}, C_{0W})$	$S_{11}:(A_{3R}, B_{1W}, C_{0W})$	$S_{20}:(A_{1W}, B_{0S}, C_{0W})$	$S_{29}:(A_{1W}, B_{3R}, C_{0W})$	$S_{38}:(A_{1W}, B_{0W}, C_{0S})$	$S_{47}:(A_{1W}, B_{0W}, C_{3R})$
$S_3:(A_{0S}, B_{2W}, C_{0W})$	$S_{12}:(A_{3R}, B_{2W}, C_{0W})$	$S_{21}:(A_{2W}, B_{0S}, C_{0W})$	$S_{30}:(A_{2W}, B_{3R}, C_{0W})$	$S_{39}:(A_{2W}, B_{0W}, C_{0S})$	$S_{48}:(A_{2W}, B_{0W}, C_{3R})$
$S_4:(A_{0S}, B_{0W}, C_{1W})$	$S_{13}:(A_{3R}, B_{0W}, C_{1W})$	$S_{22}:(A_{0W}, B_{0S}, C_{1W})$	$S_{31}:(A_{0W}, B_{3R}, C_{1W})$	$S_{40}:(A_{0W}, B_{1W}, C_{0S})$	$S_{49}:(A_{0W}, B_{1W}, C_{3R})$
$S_5:(A_{0S}, B_{0W}, C_{2W})$	$S_{14}:(A_{3R}, B_{0W}, C_{2W})$	$S_{23}:(A_{0W}, B_{0S}, C_{2W})$	$S_{32}:(A_{0W}, B_{3R}, C_{2W})$	$S_{41}:(A_{0W}, B_{2W}, C_{0S})$	$S_{50}:(A_{0W}, B_{2W}, C_{3R})$
$S_6:(A_{0S}, B_{1W}, C_{1W})$	$S_{15}:(A_{3R}, B_{1W}, C_{1W})$	$S_{24}:(A_{1W}, B_{0S}, C_{1W})$	$S_{33}:(A_{1W}, B_{3R}, C_{1W})$	$S_{42}:(A_{1W}, B_{1W}, C_{0S})$	$S_{51}:(A_{1W}, B_{1W}, C_{3R})$
$S_7:(A_{0S}, B_{1W}, C_{2W})$	$S_{16}:(A_{3R}, B_{1W}, C_{2W})$	$S_{25}:(A_{1W}, B_{0S}, C_{2W})$	$S_{34}:(A_{1W}, B_{3R}, C_{2W})$	$S_{43}:(A_{1W}, B_{2W}, C_{0S})$	$S_{52}:(A_{1W}, B_{2W}, C_{3R})$
$S_8:(A_{0S}, B_{2W}, C_{1W})$	$S_{17}:(A_{3R}, B_{2W}, C_{1W})$	$S_{26}:(A_{2W}, B_{0S}, C_{1W})$	$S_{35}:(A_{2W}, B_{3R}, C_{1W})$	$S_{44}:(A_{2W}, B_{1W}, C_{0S})$	$S_{53}:(A_{2W}, B_{1W}, C_{3R})$
$S_9:(A_{0S}, B_{2W}, C_{2W})$	$S_{18}:(A_{3R}, B_{2W}, C_{2W})$	$S_{27}:(A_{2W}, B_{0S}, C_{2W})$	$S_{36}:(A_{2W}, B_{3R}, C_{2W})$	$S_{45}:(A_{2W}, B_{2W}, C_{0S})$	$S_{54}:(A_{2W}, B_{2W}, C_{3R})$

Based on the historical data and expert judgments, the parameters of the model estimated and presented in Table 5.

Figure 8 illustrated the model layout in the ED software. This model verified conceptually through examining logical entity flow within the networks.

As the layout presents the sugar plant has four state, Hence a Server atom is considered to model each state. Also, time to failure and time to repair for such atom has capability to define transition rate or density function. We have considered each unit A, B, and C act as a Server atom with their relevant states. Every Server has three inputs. meanwhile, in the degradation state in the sub model, one entity enters the system. Based on a given randomly distributed failure, an entity activates one of the servers A, B, or C, and accelerate its failure rate.

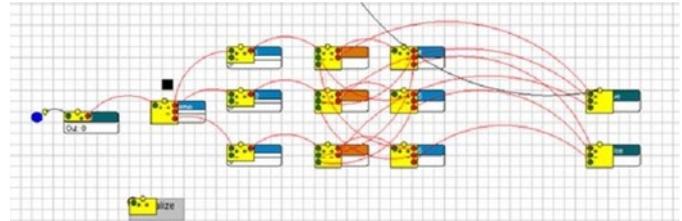


Fig. 8. The ED layout for the two four-states serially crystallization equipment with one standby

In the case of presenting a random shock, such as the Gamma distributed occurrence, for example after 100 min, a hard shock strikes the system and a unit state will change into the fail mode causing the failure of the server. Note that here the time to repair should be set to a large value to prevent system condition to return to working state after finishing the repair time. In the case of considering a small amount, the long term system availability may be calculated.

Table 5. Parameters setting of the model

Notation	Transition of States	Rate
λ_{01A}	0 → 1 for unit A	0.001
λ_{12A}	1 → 2 for unit A	0.002
λ_{23A}	2 → 3 for unit A	0.003
λ_{01B}	0 → 1 for unit B	0.002
λ_{12B}	1 → 2 for unit B	0.004
λ_{23B}	2 → 3 for unit B	0.009
λ_{01C}	0 → 1 for unit C	0.003
λ_{12C}	1 → 2 for unit C	0.006
λ_{23C}	2 → 3 for unit C	0.009
γ_{30A}	3 → 0 repair rates for A	0.01
γ_{30B}	3 → 0 repair rates for B	0.01
γ_{30C}	3 → 0 repair rates for C	0.01

Using the Experimental wizard, simulation running parameters established and the system availability chart achieved and is shown in Figure 9.

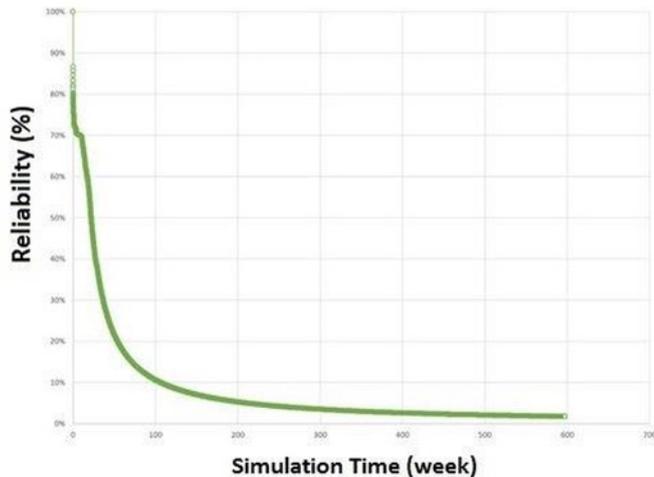


Fig. 9. Crystallizing system reliability curve

In order to look at the simulation model capability, we focused on the most critical degradation process on the crystallizations. Here their bearing displacement considered as deterioration variable and called hereinafter by D_i . Based on the historical data for a period of one cycle replacement ended to April 2018 displacement values recorded on Figure 10. As shown in the figure, 3 missing data at week 3, 19 and 31 observed. Here the maximum allowed displacement considered as 0.02 mm.

Applying curve fitting process over different alternatives patterns reveals that the crystallizing main bearing displacement sets up a parabolic curve as Eq. 11:

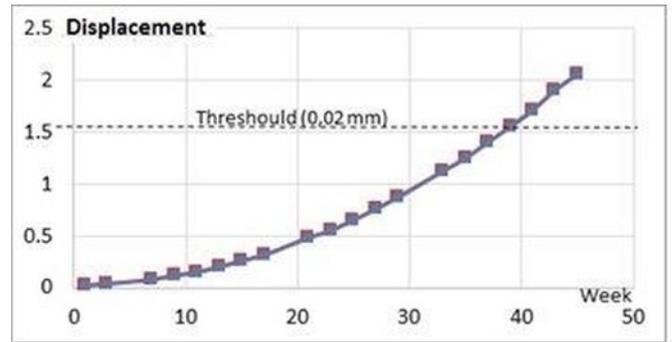


Fig. 10. Main bearing displacement in crystallizing unit (Degradation curve)

$$D_i = 0.1 t^2 + 0.2 t + 2.1 + e_i \tag{11}$$

where the error terms of e_i depicts model residuals that deploys from a Weibull distribution with 1 and 0.7 for its shape and scale parameters respectively. This fact shows that degradation process has Weibull distributed random process.

In order to simulate the system under such circumstance process, an extra network extended to the main simulation model, whereas entity flow in that sub-model acts as the main bearing displacement. Hence, any over amounts (displacement greater than or equal to nominated threshold) signals a breakdown event and a complete set of bearing parts including bushings, sleeve, two ended caps and four ball bearings should be replaced and after greasing calibration is required. This process simulated again 1000 times and reliability of crystallizing modulus illustrated by Figure 11. The figure also compares the reliability curves before and after considering the degradation process and reveals significant difference.

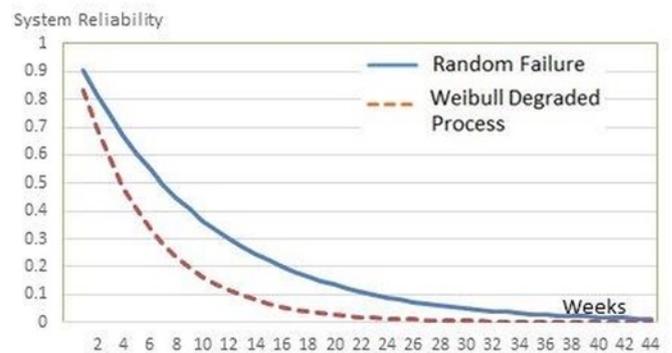


Fig. 11. Reliability comparison between before and after considering stochastic degradation

In order to examine the system reliability in the presence of random shock events due to welding operations and unusual voltage fluctuation, we applied random non-fatal shocks which would accelerate the failure process. In the simulation model, the system works as previous models up to the occurrence of a non-fatal shock; based on the expert's judgment, their mean time for arrival is on the range of [0.5 - 4] weeks with a mean of 2. Consequently, we modeled such shock arrivals by a Gamma distribution with parameters of 10 and 0.2 for the relevant shape and scale parameters, respectively. When a shock occurs, the deterioration process will rapidly be accelerated. Due to lack of reported data and by considering the experts judgment, we supposed that the failure process would accelerate by 3 times. Thus, in the simulation, we modeled the new deterioration process via Eq. 12:

$$D_i^* = 0.9t^2 + 0.6t + 2.1 + e_i \quad (12)$$

where, the residual keeps the previous value without any changes. Figure 12 compares the reliability of the system with the non-fatal shock, and degradation process with their absence.

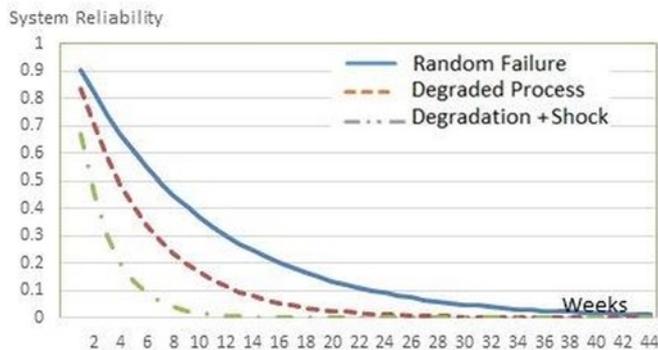


Fig. 12. Reliability comparison for crystalizing system at different failure modes

As reliability figures reveal, when the real degradation process is neglected, the system reliability will be overestimated. A similar argument can be considered when shock is also present.

6. Conclusion

The literature survey shows that system reliability estimation always accompanied by complexity in analytic methods especially when there is a great deal of uncertainties the system analysis. These uncertainties arises complicated effects and interactions in the estimations. Some usual source of uncertainty related to random time to failure, time to repair, degradation process and shock occurrences.

There is a lot of system reliability assessment methods in the literature. But almost all of them restricted to apply in the especial cases due to their relevant assumptions. For example system reliability as-

essment under the degradation process is a common task and sparse studies have reported on the presence of just only the Gamma continuous degradation process alongside non-fatal shock occurrences. Rottenly real cases may cover a wide variety of systems consisted of multi-component, multi-state, different type of time to failure, time to repair density function for each component, vast amount of continuous degradation functions, and their severities. Although many articles have discussed the necessity of simulating complex systems and some of its applications in specific cases have been delivered at yet, there is still a need to more address this issue.

The proposed object-oriented simulation model has a couple of advantages in comparison to the analytic and previous methods. The major advantages are as follows:

- 1- The method may be justified for any kind of system with complicated configurations.
- 2- It consists of a few individual moduli where each of them may be relaxed for more simplified cases.
- 3- It supports both continuous or discrete degradation deterministic or stochastic processes. Thus, it could be applied for multi-state multi-component systems.
- 4- The model has no dependence on special random variables such as exponential or gamma degradation. Any practitioner could set them to any other well-known density functions (e.g. Weibull, Logistic, Beta).
- 5- It covers the effects of all types of random fatal and non-fatal shocks with any severities.
- 6- The simulation model reports the system availability as well as reliability.

The proposed model has been established using ED simulation software capabilities which may be accounted as a disadvantage. Nevertheless, non-familiar simulation experts could follow a specific logic for implementation via other software applications. Modeling some auto-correlated degradation processes in this context will be remain as future research for interested researchers.

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