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A RELIABILITY MODEL FOR LOAD-SHARING *K*-OUT-OF-*N* SYSTEMS SUBJECT TO SOFT AND HARD FAILURES WITH DEPENDENT WORKLOAD AND SHOCK EFFECTS

MODEL NIEZAWODNOŚCI DLA SYSTEMÓW TYPU *K*-Z-*N* Z PODZIAŁEM OBCIĄŻENIA PODLEGAJĄCYCH USZKODZENIOM PARAMETRYCZNYM I KATASTROFICZNYM, W KTÓRYCH ZACHODZI ZALEŻNOŚĆ MIĘDZY OBCIĄŻENIEM PRACĄ A SKUTKAMI OBCIĄŻEŃ LOSOWYCH

A component in a k-out-of-n system may experience soft and hard failures resulting from exposure to natural degradation and random shocks. Due to load-sharing characteristics, once a component fails, the surviving components share an increased workload, which increases their own degradation rates. Moreover, under the larger workload, random shocks may cause larger abrupt degradation increments and larger shock sizes. Therefore, the system experiences the dependent workload and shock effects (DWSEs). Such dependence will cause the load-sharing system to fail more easily, though it is often not considered in existing methods. In this paper, to evaluate the system reliability more accurately, we develop a novel reliability model for load-sharing k-out-of-n systems with DWSEs. In the model, the joint probability density function of shock effects to soft and hard failures is developed to describe the DWSEs on a component. To derive an analytical expression of system reliability with load-sharing characteristics and DWSEs, conditional probability density function is used to model the random component failure times. A load-sharing Micro-Electro-Mechanical System (MEMS) is then utilized to illustrate the effectiveness of the reliability model.

Keywords: *reliability modeling, load-sharing k-out-of-n systems, dependent workload and shock effects, degradation, random shocks.*

Element systemu k-z-n może ulegać uszkodzeniom parametrycznym i katastroficznym wynikającym z ekspozycji na naturalne procesy degradacji i obciążenia losowe. Ze względu na równomierny podział obciążenia między wszystkie elementy systemu, gdy jeden element ulega awarii, obciążenie pracą przypadające na pozostałe komponenty zwiększa się, podnosząc tempo degradacji każdego z nich. Ponadto, przy większym obciążeniu pracą, obciążenia losowe mogą powodować większe nagłe przyrosty degradacji i zwiększać rozmiary obciążeń. Mówi się wtedy o istnieniu zależności między obciążeniem pracą a skutkami obciążeń losowych (dependent workload and schock effects (DWSE). Taka zależność powoduje, że system z podziałem obciążeń łatwiej ulega uszkodzeniom. Fakt ten jest często pomijany w obecnie stosowanych metodach oceny niezawodności. W niniejszym artykule przedstawiamy nowatorski model oceny niezawodności systemów k-z-n z podziałem obciążenia i zależnością DWSE, który pozwala dokładniej ocenić niezawodność takich systemów. W modelu, opracowano wspólną funkcję gęstości prawdopodobieństwa skutków obciążeń losowych dla uszkodzeń parametrycznych i katastroficznych, która pozwala opisać zależność DWSE dla elementu systemu. Aby wyprowadzić analityczne wyrażenie niezawodności systemu z podziałem obciążenia i DWSE, do modelowania czasów losowych uszkodzeń elementów systemu wykorzystano funkcję warunkowej gęstości prawdopodobieństwa. Skuteczność modelu niezawodności zilustrowano na przykładzie układu mikroelektromechanicznego z podziałem obciążenia (MEMS).

Słowa kluczowe: *modelowanie niezawodności, systemy k-z-n z podziałem obciążenia, zależność między obciążeniem pracą a skutkami obciążeń losowych, degradacja, obciążenia losowe.*

1. Introduction

In reliability engineering, redundancy technique is widely applied to ensure a system remain functional over a long period of time. A k-out-of-n system is a typical redundant system with n components. At a minimum, it requires k operational components for the system to work normally [25, 30, 38]. Many reliability models of k-out-of-n systems have been developed, which assume that components work independently [6, 41]. However, many systems are load-sharing, such as micro-engines in a Micro-Electro-Mechanical System (MEMS) [4, 13], common buses in a common bus performance sharing system [40], and gear pair systems in a machines transmission system [37], which makes the assumption of independent components unrealistic [10, 24]. A common feature in a load-sharing system is that the workload is shared equally or unequally by the surviving components, and when a component fails, its load is distributed to the working components [32]. The increased workload on the component strongly affect its degradation rate and failure rate [11], which has been proved by many empirical studies of mechanical systems [7, 26], and battery systems [20]. Therefore, due to load-sharing characteristics, the components are stochastically dependent on each other.

Although numerous studies have explored the reliability of load-sharing systems considering the dependence among the components, they ignore the detrimental effects of random shocks on system reliability. Taghipour et al. [25] propose a periodic inspection optimization policy of a load-sharing system, where stochastic dependence among thecomponents is considered by sharing a certain amount of load. Zhang et al. [38] develop a reliability model of a load-sharing system with dependent components that equally share the system load before and after other components have failed. Ye et al. [36] develop a reliability model of a water filtering system with multiple filters, where the workload influences the filter degradation. In their model, the dominant failure type is degradation, while hard failure due to a shock will not occur. Kong et al. [12] investigate the dependence between component lifetime and load level through a link function. Although such methods successfully consider the effects of workloads on the degradation processes or failure rates, they do not consider random shocks which can accelerate the degradation process and cause sudden hard failure.

In fact, the components in a load-sharing k-out-of-n system are subject to soft and hard failures [1]. The soft failures are mainly due to degradation processes and the hard failures are due to random shocks, while the degradation processes and random shocks may be dependent [18]. For example, MEMS may be a load-sharing system where multiple micro-engines work together to perform more reliably [4, 13] and each micro-engine experiences dependent wear degradation and random shocks [18]. Based on the reliability testing experiments in [27], the dominant failure mechanism of micro-engine is determined as wear on rubbing surfaces which usually leads to either broken pin joints or seized micro-engines [29]. In addition, Tanner et al. [28] investigate shock effects on a micro-engine through shock tests, finding that random shocks will cause wear debris, which will accelerate the wear on rubbing surfaces. Moreover, the misalignment of the springs may occur and a large enough shock can result in a spring fracture. Therefore, the micro-engines will experience soft failure (i.e., wear) and hard failure (i.e., spring fracture) due to simultaneous exposure to degradation processes and random shocks.

To develop a reliability model of systems with degradation processes, random shocks, and their dependence, many literatures assume that random shocks can (a) cause abrupt degradation increases [8, 14, 15, 18, 21, 22, 39], (b) increase the degradation rate [2, 19, 31, 39], or (c) increase the hazard rate of sudden failure [3, 33]. On the other hand, random shocks may be influenced by the current degradation level. Yang et al. [34] and Che et al. [5] suggest that the occurrence of random shocks is affected by the degradation level of the system. Yang et al. [35] develop a reliability model where the magnitude of the damage caused by a shock load is correlated to the system degradation level. As reviewed above, the reliability modeling for systems with dependent degradation processes and random shocks has been thoroughly investigated, while such systems are usually series or parallel systems without load-sharing characteristics.

In literature, only a few authors analyze the reliability of a load-sharing system with dependent degradation process and random shocks, and the studies are limited in some respects. Random shocks commonly affect the components of a load-sharing system in two respects: (i) being transmitted to a shock size to components and then inducing a hard failure suddenly if the size is huge enough; (ii) creating a shock damage and then contributing to soft failure. Liu et al. [13] develop a reliability model of a load-sharing MEMS with three micro-engines subject to continuous degradation processes under a constant load or a cumulative load. In their model, degradation is the dominant failure type, while shocks only cause degradation increases and cannot lead to hard failure. In practice, a huge shock may lead to a common cause failure of the entire system [16]. Che et al. [4] develop a reliability model of a load-sharing system with dependent degradation process and random shocks. In their model, the shock effects are independent of workload, which may not be applicable in all situations.

In fact, components are subject to both workload and shock load, and both types of loads contribute to soft and hard failures. For a load-sharing system, overload is a typical shock load, such as a surge of workload for micro-engines [13] and the over discharge for battery packs [20]. When the arrival shock is an overload, its effects (i.e. the transmitted shock sizes and transmitted shock damages) depend on the resultant load of the workload and overload. After a component fails, the workload shared by each surviving component will increase, and under the high workload, the degradation rate of components will increase. In addition, the resultant load will also increase, causing the shock effects on the components to become more serious.Therefore, shock effects to soft and hard failures are dependent on the current components' workload. Load-sharing system experiences the dependent workload and shock effects (DWSEs), and the dependence is first studied to evaluate the reliability of load-sharing systems. The reliability may be overestimated without considering the dependence scenario.

Due to load-sharing characteristics and DWSEs, the degradation rate and shock effects to soft and hard failures are all dependent on the number of failed components. In addition, the failure times of the components and the arrival times of the random shocks are both stochastic. It is more practical but also presents new challenging issues to build a reliability model. In this paper, a reliability model of load-sharing systems subject to soft and hard failures with DWSEs is developed. In the model, the joint probability density function of shock effects to soft and hard failures given the number of failed components is developed to describe the DWSEs on a surviving component. In addition, the conditional probability density function of component failure time and conditional total probability formula are utilized to model the system reliability. An analytical expression is then developed to calculate system reliability, which can save much calculation time. Finally, a load-sharing MEMS is utilized as a realistic application to illustrate the effectiveness of the reliability model.

The rest of this article is organized as follows. In Section 2, we presents the system description and its assumptions. In Section 3, the model of DWSEs on a component of a load-sharing system is described in detail, and then the reliability model of a load-sharing system is proposed in Section 4. In Section 5, the reliability model is illustrated by load-sharing micro-engines in MEMS developed at Sandia. Finally, Section 6 concludes the paper and makes some suggestions for further work.

2. System specifications

In this paper, we focus on a load-sharing k-out-of-n system with n identical components sharing a certain amount of load. Each component is subject to competing soft and hard failure processes due to experiencing degradation process and random shocks simultaneously. The reliability model is built based on the following assumptions, which are adapted from recent literatures [4, 13, 18, 22, 38].

1. Random shocks arrive following a Poisson process.
2. The components in the load-sharing k-out-of-n system fail due to soft failure and hard failure. Soft failure will occur when the overall degradation is beyond the threshold value of the component. Hard failure will occur suddenly when the shock load exceeds the maximum strength of the component.
3. The system consists of n identical components. It requires at least k components being operational for the system to work properly.
4. The system load is fixed and it is shared by surviving components equally after a soft failure of a component occurs. This leads to an increased component workload and a higher degradation rate.
5. The shock load is shared by surviving components equally. Once a hard failure of a surviving component occurs when the shared shock load exceeds its maximum strength, all of the other surviving components will fail due to the same shock

load at the meanwhile, which leads to the sudden failure of the load-sharing system.

The first two assumptions are taken from [18, 22], and they are widely applied to the components and systems subject to degradation processes and random shocks simultaneously. Assumptions 3 and 4 are taken from [4, 13, 38] and are basic assumptions for equal load-sharing k-out-of-n systems. Based on Assumptions 3 and 4, Liu et al. [13]conduct the reliability analysis of a load-sharing MEMS with three micro-engines. Assumption 5 is effective when the shock is an overload such as the surge of workload for micro-engines [13] and the over discharge for battery packs [20]. Overload is a typical shock load for a load-sharing system, and the load is shared by surviving components equally. Therefore, when the shock load is large enough, the load equally shared by each surviving component exceeds its maximum strength, and the hard failures of all surviving components occur suddenly based on the assumptions that the components are identical. Consequently, such shock load will result in the sudden failure of the load-sharing system.

As shown in Fig. 1, Peng et al. [18] develop a component reliability model considering two dependent competing failure processes: soft failure due to total degradation, and hard failure due to the same shocks. For each shock j , W_j is the transmitted shock size and hard failure occurs when W_j exceeds the maximum strength D , and Y_j denotes the abrupt damage in degradation process and soft failure occurs when the overall degradation $X_S(t)$ is greater than the threshold value H .

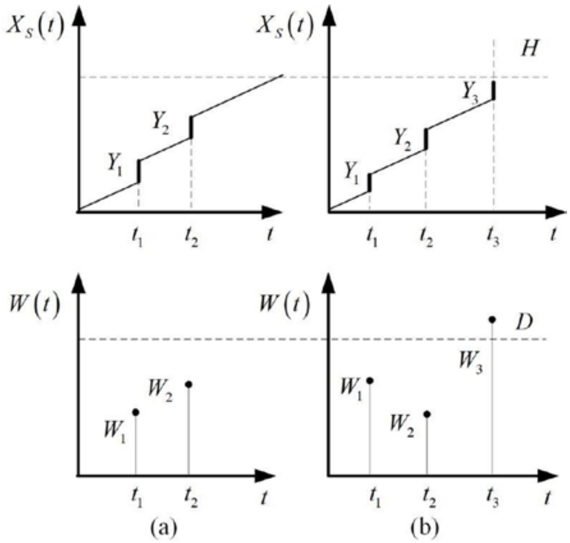

Fig. 1. Two dependent competing failure processes: (a) soft failure process, and (b) hard failure process [18]

As shown in Fig. 2, when a component in a load-sharing system fails, the system configuration changes, and the workload on each surviving component will increase, which will lead to a higher degradation rate (line a3) [17]. When a shock arrives, it can be transmitted to a shock size to the devices and induce a hard failure through line a1, and it can also create a shock damage to the devices and then contributes to soft failure through line a2. In addition, the shock load contributes to the failures together with the workload, and the workload will make shock effects more serious. Under an increased workload, the shock effects on each surviving component will be greater since the effects are caused by the resultant load of the workload and shock load. Thus the shock effects are dependent on the workload, and the load-sharing system experiences the DWSEs.

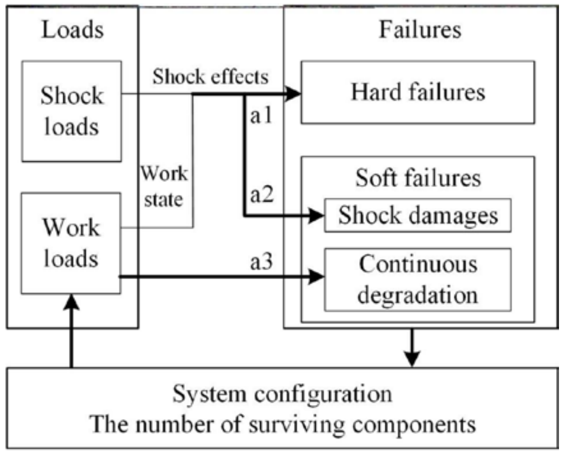

Fig. 2. The dependence analysis for the load-sharing systems

Due to the load-sharing characteristics and DWSEs, once a component fails, the workload shared on surviving components increases, resulting in that (i) the degradation process is accelerated, and (ii) the shock effects to soft and hard failures become worse. As shown in Fig. 3, for a load-sharing system with i failed components, the load of j th shock together with the shared workload will be transmitted to abrupt degradation damage Y_{ij} and shock size W_{ij} to each surviving-component in the system. When the $i+1$ th component has failed, the degradation rate increases significantly and the shock effects become more significant. As illustrated in Fig. 3, W_{i+1j} and Y_{i+1j} are greater than W_{ij} and Y_{ij} respectively.

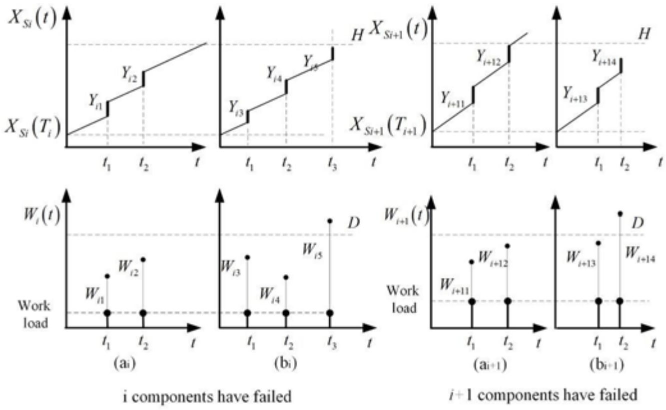

Fig. 3. Two dependent competing failure processes for a surviving component in a load-sharing system with different system configuration: (a) soft failure process, and (b) hard failure process, where $X_{Si}(t)$ is the total degradation of a component in the system with i failed components at time t .

3. Failure modeling for a component with DWSEs

In this section, we investigate the modeling for soft and hard failures of a surviving component with DWSEs. Firstly, the shock effects to soft and hard failures with DWSEs are modeled. Then, we develop the soft failure model and hard failure model of a surviving component with DWSEs.

3.1. Modeling of shocks considering DWSEs

The DWSEs on each surviving component are depicted in Fig. 4. When the j th system shock arrives with magnitude Z_j , it affects both the hard failure process and soft failure process for each C_i , where C_i is the surviving component in the load-sharing system with i failed components. Usually, the hard failure and soft failure may occur in different devices. For example, for a micro-engine, hard failure is mainly due to the spring fracture while soft failure is mainly due to wear on the rubbing surface. Therefore, Z_j can be transmitted to Z_{Hj}, which is the magnitude of the j th shock on the devices (e.g. the spring) where hard failures occur, and Z_{Sj} ,which is the magnitude of the j th shock on the devices (e.g. the rubbing surface) where soft failures occur. Z_{Hj} and Z_{Sj} are assumed to be independent, since they are applied to different devices. In addition, Z_{Hj} and Z_{Sj} apply to C_i together with the workload, and their resultant load can be transmitted to Z_{Hij} and Z_{Sij}, respectively. Then Z_{Hij} and Z_{Sij} are transmitted as shock sizes W_{ij} for the hard failure process and shock damage increments Y_{ij} for the soft failure process, respectively.

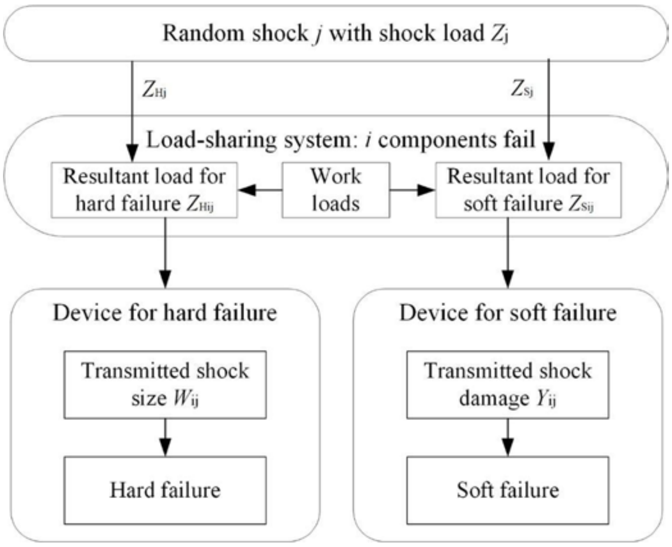

Fig. 4. The transmitted effects of system shock to the soft and hard failures

There are many ways to describe dependence characteristics in shock propagation, such as proportional correlated, additive dependent, and other more complicated models [23]. Song et al. [23] and Liu et al. [15] assume that the shock effects are linearly dependent on shock load. In this paper, a linear shock transmission model is also utilized to formulate the DWSEs. The shock size W_{ij} and the shock damage Y_{ij} to the component are transmitted linearly from Z_{Hij} and Z_{Sij} respectively, and Z_{Hij} and Z_{Sij} are also a linear function of Z_{Hj} and Z_{Sj} respectively. Then, W_{ij} and Y_{ij} can be simplified as a linear function of Z_{Hj} and Z_{Sj}, while to model the DWSEs, the transmission parameters are dependent on the workload. Based on Assumption 4, the workload shared by surviving components is only dependent on the number of failed components, i. Then, the transmission parameters are dependent on the value of i. Moreover, to consider purely random shock effects, two random terms, $\tilde{W}_{ij}$ and $\tilde{Y}_{ij}$, are present in response to a system shock, and they are not dependent on system shock loads. We assume:

$$W_{ij} = \alpha_i Z_{Hj} + \tilde{W}_{ij}, \tag{1}$$

$$Y_{ij} = \gamma_i Z_{Sj} + \tilde{Y}_{ij}, \tag{2}$$

where α_i is a transmission parameter between Z_{Hj} to the shock size for the hard failure process of C_i, and γ_i is the transmission parameter from Z_{Sj} to the shock damage for the soft failure process of C_i. The values of α_i and γ_i can be estimated from previous data, life testing, engineering judgment, and etc. As mentioned above, $\tilde{W}_{ij}$ is a random shock size contributing to C_i 's hard failure, and does not depend on Z_{Hj}. For some cases, $\tilde{W}_{ij}$ may be zero for all i or j. Similarly, $\tilde{Y}_{ij}$ is a random shock damage to soft failure process, which is not dependent on Z_{Sj}. In some special examples, $\tilde{Y}_{ij}$ may be zero, while in some other examples, the shock damage Y_{ij} is not exactly proportional to the shock magnitude Z_{Sj} and additional randomness can be introduced into Y_{ij} through $\tilde{Y}_{ij}$. Both $\tilde{W}_{ij}$ and $\tilde{Y}_{ij}$ are independent and identically distributed (i.i.d.) random variables.

The cumulative distribution function (CDF) for W_i, $F_{W_i}(w_i)$ can be derived as:

$$\begin{aligned} F_{W_i}(w_i) &= \Pr\{W_{ij} < w_i\} = \Pr\{\alpha_i Z_{Hj} + \tilde{W}_{ij} < w_i\} \\ &= \int_{z_{Hj}} \Pr\{\alpha_i z_{Hj} + \tilde{W}_{ij} < w_i\} f_{Z_{Hj}}(z_{Hj}) dz_{Hj} \end{aligned}. \tag{3}$$

Then, the probability distribution function (PDF) for W_i, $f_W(w)$ can be derived as:

$$f_{W_i}(w_i) = \int_{z_{Hj}} f_{\tilde{W}_i}(w_i - \alpha_i z_{Hj}) f_{Z_{Hj}}(z_{Hj}) dz_{Hj}. \tag{4}$$

Similarly, the CDF for Y_i, $F_{Y_i}(y_i)$ can be derived as:

$$\begin{aligned} F_{Y_i}(y_i) &= \Pr\{Y_{ij} < y_i\} = \Pr\{\gamma_i Z_{Sj} + \tilde{Y}_{ij} < y_i\} \\ &= \int_{z_{Sj}} \Pr\{\gamma_i z_{Sj} + \tilde{Y}_{ij} < y_i\} f_{Z_{Sj}}(z_{Sj}) dz_{Sj} \end{aligned}. \tag{5}$$

Then, the PDF for Y_i, $f_{Y_i}(y_i)$ can be derived as follows:

$$f_{Y_i}(y_i) = \int_{z_{Sj}} f_{\tilde{Y}_i}(y_i - \gamma_i z_{Sj}) f_{Z_{Sj}}(z_{Sj}) dz_{Sj}. \tag{6}$$

Based on Eqs. (4) and (6), the joint PDF for W and Y, $f_{W_i,Y_i}(w_i, y_i)$, is derived as:

$$f_{W_i,Y_i}(w_i, y_i) = \int_{z_{Hj}} f_{\tilde{W}_i}(w_i - \alpha_i z_{Hj}) f_{Z_{Hj}}(z_{Hj}) dz_{Hj} \times \int_{z_{Sj}} f_{\tilde{Y}_i}(y_i - \gamma_i z_{Sj}) f_{Z_{Sj}}(z_{Sj}) dz_{Sj}. \tag{7}$$

3.2. Modeling of soft and hard failures of a surviving component

Figure 1(b) shows an extreme shock model where a hard failure occurs when the shock size is beyond the maximal fracture strength D. In this paper, system shocks arrive following a Poisson process with rate λ. Based on the stress–strength model, the probability that C_i survives the applied stress from the j th system shock is:

$$P(W_{ij} < D) = F_{W_i}(D) \text{ for } j = 1,2,...,m_i. \tag{8}$$

Then, the probability that each C_i does not experience hard failure by time t, $P_i(NHF_t)$, is:

$$
\begin{aligned}
&P_i\left(NHF_t\right)=\Pr\left\{W_{01}<D,W_{02}<D,\ldots,W_{0m_0}<D,\ldots,W_{i1}<D,W_{i2}<D,\ldots,W_{im_i}<D\right\}\\
&=P\left\{W_{01}<D,W_{02}<D,\ldots,W_{0m_0}<D,\ldots,W_{i1}<D,W_{i2}<D,\ldots,W_{im_i}<D \mid m_0,m_1,\ldots,m_{i-1},m_i\right\}\\
&=P\left(\bigcap_{i=0}^{N(t)}\bigcap_{j=0}^{m_i}W_{ij}<D\right)=\prod_{i=0}^{N(t)}F_{W_i}(D)^{m_i}=\prod_{i=0}^{N(t)}\left(\int_{z_H}f_{\tilde{W}_i}\left(D-\alpha_i Z_H\right)f_{Z_H}\left(z_H\right)dz_H\right)^{m_i}
\end{aligned}
,\quad (9)
$$

where m_i is the number of shocks arrived in the time interval between T_i and T_{i+1}, as shown in Fig. 5, and T_i is the failure time of the i th component. $N(t)$ is the number of failed components by time t.

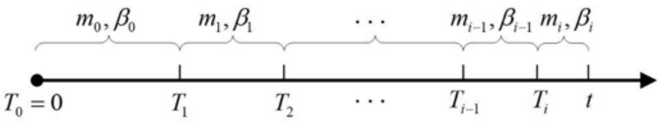

Fig. 5. Degradation process of C_i in a load-sharing system

As an example, if Z_{Hj} and $\tilde{W}_{ij}$ follow normal distributions, a more specific case for Eq. (9) can be derived as:

$$
P_i\left(NHF_t\right)=\prod_{j=0}^{N(t)}\left(\int_{z_H}\Phi\left(\frac{D-\alpha_j z_H-\mu_{\tilde{W}}}{\sigma_{\tilde{W}}}\right)\varphi\left(\frac{z_H-\mu_{Z_H}}{\sigma_{Z_H}}\right)dz_H\right)^{m_j}, \quad (10)
$$

where $\Phi(\bullet)$ and $\varphi(\bullet)$ are the CDF and PDF of a standard normally distributed variable, respectively.

As shown in Fig. 1 part (a), the soft failure of a component occurs when the overall degradation is greater than the threshold value H [18, 22, 23]. The overall degradation, $X_S(t)$, is affected by the load-sharing characteristics and DWSEs, and is accumulated by continual degradation and cumulative abrupt damage caused by shocks. According to the degradation models in many literatures [5, 18, 19], we also assume a linear degradation path to accumulate continual degradation, $X(t)=\mu+\beta t+\varepsilon$, where μ is constant and represents the initial component degradation, β is a random variable and represents degradation rate, and ε is a random error term and follows a normal distribution, $\varepsilon \sim N(0,\sigma^2)$.

Each C_i in a load-sharing system will experience the system configuration changing from no failed components to i failed components. Therefore, as shown in Fig. 5, its degradation rate will change from β_0 to β_i step by step, where β_i is the current degradation rate of each C_i. The degradation rate β is influenced by the workload on the component, and due to load-sharing characteristics, the following inequalities $\beta_i > \beta_{i-1} > \ldots > \beta_1 > \beta_0$ will exist. In addition, β_{i-1} will increase to β_i when the i th component fails. The value of $\boldsymbol{\beta}$ can be estimated through accelerated degradation test [17].

Therefore the total degradation of C_i by time t is denoted as:

$$
X(t)=\begin{cases}\mu+\sum_{l=0}^{i-1}\beta_l\left(T_{l+1}-T_l\right)+\beta_i\left(t-T_i\right)+\varepsilon, & \text{if } i=N(t)\ge 1\\ \mu+\beta_0 t+\varepsilon & \text{if } N(t)=0\end{cases}. \quad (11)
$$

Moreover, a shock will cause a damage increment to the degradation process Y_{ij}, and a cumulative shock model is used to determine accumulated shock damage increments. The cumulative degradation damage increments $S(t)$ caused by shocks until time t can be derived as:

$$
S(t)=\begin{cases}\sum_{i=0}^{N(t)}\sum_{j=1}^{m_i}Y_{ij}=\sum_{i=0}^{N(t)}\sum_{j=1}^{m_i}\left(\tilde{Y}_{ij}+\gamma_i Z_{Sj}\right), & \text{if } \sum_{i=0}^{N(t)}m_i>0\\ 0 & \text{if } \sum_{i=0}^{N(t)}m_i=0\end{cases}. \quad (12)
$$

Therefore, the total degradation accumulated by both continual degradation and cumulative abrupt damages can be expressed as $X_S(t)=X(t)+S(t)$. Then the probability that the overalldegradation at time t is less than the threshold value H can be derivedas $P(X_S(t)<H)=P(X(t)+S(t)<H)$.

Conditioning on the times $T_1,T_2,\ldots,T_i$ and shock numbers $m_0,m_1,\ldots,m_{i-1}$, the probability that no soft failure will occur on C_i at time t can be derived as:

$$
\begin{aligned}
&P_i(NSF(t)\mid m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i)\\
&=\sum_{m_i=0}^{\infty}P_i(X_S(t)<H,m_i\mid m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i)\\
&=\sum_{m_i=0}^{\infty}P_i(X_S(t)<H\mid m_i,m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i)P(m_i\mid ,m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i)
\end{aligned}
, \quad (13)
$$

where $P(m_i \mid m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i)$ is the conditional probability that m_i shocks arrive in the time interval between T_i and t given the component failure times $T_1,T_2,\ldots,T_i$ and the shock numbers $m_0,m_1,\ldots,m_{i-1}$. The conditional probability is only dependent on T_i and t due to the characteristics of Poisson process and can be simplified as:

$$
P(m_i\mid m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i)=P(m_i\mid T_i)=\frac{\exp(-\lambda\left(t-T_i\right))(\lambda\left(t-T_i\right))^{m_i}}{m_i!}. \quad (14)
$$

Then Eq. (13) can be rewritten as:

$$
\begin{aligned}
&P_i(NSF(t)\mid m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i)\\
&=\sum_{m_i=0}^{\infty}P(\mu+\sum_{l=0}^{i-1}\beta_l\left(T_{l+1}-T_l\right)+\beta_i\left(t-T_i\right)+\sum_{l=0}^{i}\sum_{j=1}^{m_l}Y_{lj}+\varepsilon<H\mid m_i)\frac{\exp(-\lambda\left(t-T_i\right))(\lambda\left(t-T_i\right))^{m_i}}{m_i!}.
\end{aligned}
\quad (15)
$$

Furthermore, if $f_{Z_{Sj}}^{\langle m\rangle}$ is considered to be the PDFof the sum of m i.i.d. Z_{Sj} variables,then $P_i\left(X_S(t)<H\mid m_i,m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i\right)$ in Eq. (13) can be derived to amore specific expression based on a convolution integral:

$$
\begin{aligned}
&P_i(X_S(t)<H\mid m_i,m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_i)\\
&=\int_{u_1}\ldots\int_{u_i}P(\mu+\sum_{l=0}^{i-1}\beta_l\left(T_{l+1}-T_l\right)+\beta_i\left(t-T_i\right)+\sum_{l=0}^{i}\sum_{j=1}^{m_l}\tilde{Y}_{lj}+\sum_{l=0}^{i}\sum_{j=1}^{m_l}\gamma_l z_{Sj}+\varepsilon<H\mid \sum_{j=1}^{m_l}z_{Sj}=u_l)\\
&\times f_{Z_{Sj}}^{\langle m_1\rangle}\left(u_1\right)\ldots f_{Z_{Sj}}^{\langle m_i\rangle}\left(u_i\right)du_1\ldots du_i\\
&=\int_{u_1}\ldots\int_{u_i}P(\mu+\sum_{l=0}^{i-1}\beta_l\left(T_{l+1}-T_l\right)+\beta_i\left(t-T_i\right)+\sum_{l=0}^{i}\sum_{j=1}^{m_l}\tilde{Y}_{lj}+\sum_{l=0}^{i}\gamma_l u_l+\varepsilon<H)\\
&\times f_{Z_{Sj}}^{\langle m_1\rangle}\left(u_1\right)\ldots f_{Z_{Sj}}^{\langle m_i\rangle}\left(u_i\right)du_1\ldots du_i
\end{aligned}
. \quad (16)
$$

Conditioning on that $\sum_{l=0}^{i}\sum_{j=1}^{m_l}\tilde{Y}_{lj}=y$, if the PDF of the purely random variables $\tilde{Y}_{ij}$ is $f_{\tilde{Y}}(y)$ for all i and j, Eq. (16) can be derived as:

$$\begin{aligned}
&P_i(X_S(t)<H\mid m_i,m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_i)\\
&=\int_{u_1}\dots\int_{u_i}P(\mu+\sum_{l=0}^{i-1}\beta_l\left(T_{l+1}-T_l\right)+\beta_i\left(t-T_i\right)+\sum_{l=0}^{i}\sum_{j=1}^{m_l}\tilde{Y}_{lj}+\sum_{l=0}^{i}\gamma_l u_l+\varepsilon<H)\\
&\times f_{Z_{Sj}}^{\langle m_1\rangle}(u_1)\dots f_{Z_{Sj}}^{\langle m_i\rangle}(u_i)du_1\dots du_i\\
&=\int_{u_1}\dots\int_{u_i}\left(\int_y P\left(\begin{array}{l}\mu+\sum_{l=0}^{i-1}\beta_l\left(T_{l+1}-T_l\right)+\beta_i\left(t-T_i\right)\\ +\sum_{l=0}^{i}\sum_{j=1}^{m_l}\tilde{Y}_{lj}+\sum_{l=0}^{i}\gamma_l u_l+\varepsilon<H\mid\sum_{l=0}^{i}\sum_{j=1}^{m_l}\tilde{Y}_{lj}=y\end{array}\right)f_{\tilde{Y}}^{\left\langle\sum_{l=0}^{i}m_l\right\rangle}(y)dy\right)\\
&\times f_{Z_{Sj}}^{\langle m_1\rangle}(u_1)\dots f_{Z_{Sj}}^{\langle m_i\rangle}(u_i)du_1\dots du_i\\
&=\int_{u_1}\dots\int_{u_i}\left(\int_y P(\mu+\sum_{l=0}^{i-1}\beta_l\left(T_{l+1}-T_l\right)+\beta_i\left(t-T_i\right)+y+\sum_{l=0}^{i}\gamma_l u_l+\varepsilon<H)f_{\tilde{Y}}^{\left\langle\sum_{l=0}^{i}m_l\right\rangle}(y)dy\right)\\
&\times f_{Z_{Sj}}^{\langle m_1\rangle}(u_1)\dots f_{Z_{Sj}}^{\langle m_i\rangle}(u_i)du_1\dots du_i
\end{aligned} \tag{17}$$

As an example, if β_i, $\tilde{Y}_{ij}$, and Z_{Sj} all follow normal distributions, $\sum_{l=0}^{i}\gamma_l u_l$ is also a normal distribution $\sum_{l=0}^{i}\gamma_l u_l\sim N\left(\sum_{l=0}^{i}\gamma_l m_l\mu_{Z_S},\sum_{l=0}^{i}\gamma_l^2 m_l\sigma_{Z_S}^2\right)$. Then, a more specific case for Eq. (17) can be derived as:

$$\begin{aligned}
&P_i(X_S(t)<H\mid m_i,m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_i)\\
&=\int_u\Phi\left(\frac{H-\left(\mu+\sum_{l=0}^{i-2}\mu_{\beta_l}\left(T_{l+1}-T_l\right)+\mu_{\beta_{i-1}}\left(T_i-T_{i-1}\right)+\sum_{l=0}^{i}m_l\mu_{\tilde{Y}}+u\right)}{\sqrt{\sum_{j=0}^{i-2}\sigma_{\beta_j}^2\left(T_{j+1}-T_j\right)^2+\sigma_{\beta_{i-1}}^2\left(t-T_i\right)^2+\sum_{l=0}^{i}m_l\sigma_{\tilde{Y}}^2+\sigma^2}}\right)\varphi\left(\frac{u-\sum_{l=0}^{i}\gamma_l m_l\mu_{Z_S}}{\sqrt{\sum_{l=0}^{i}\gamma_l^2 m_l\sigma_{Z_S}}}\right)du
\end{aligned} \tag{18}$$

Then, Eq. (15) can be expressed as:

$$\begin{aligned}
&P_i(NSF(t)\mid m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_i)\\
&=\sum_{m_i=0}^{\infty}\int_u\Phi\left(\frac{H-\left(\mu+\sum_{l=0}^{i-2}\mu_{\beta_l}\left(T_{l+1}-T_l\right)+\mu_{\beta_{i-1}}\left(T_i-T_{i-1}\right)+\sum_{l=0}^{i}m_l\mu_{\tilde{Y}}+u\right)}{\sqrt{\sum_{j=0}^{i-2}\sigma_{\beta_j}^2\left(T_{j+1}-T_j\right)^2+\sigma_{\beta_{i-1}}^2\left(t-T_i\right)^2+\sum_{l=0}^{i}m_l\sigma_{\tilde{Y}}^2+\sigma^2}}\right)\\
&\varphi\left(\frac{u-\sum_{l=0}^{i}\gamma_l m_l\mu_{Z_S}}{\sqrt{\sum_{l=0}^{i}\gamma_l^2 m_l\sigma_{Z_S}}}\right)du\times\frac{\exp(-\lambda\left(t-T_i\right))(\lambda\left(t-T_i\right))^{m_i}}{m_i!}
\end{aligned} \tag{19}$$

4. Reliability modeling for a load-sharing system

As pointed out in the previous section, the reliability model of a load-sharing system should consider DWSEs, load-sharing characteristics, and dependent competing failures (i.e., soft failures and hard failures). Based on Assumption 5, once a component hard failure occurs, the hard failures of the other surviving components will occur at the meanwhile, and the load-sharing system fails immediately. On the other hand, the soft failures of the surviving components will occur one by one due to the randomness of the degradation process. When a soft failure of component occurs, the component fails to function properly, and then the workload shared by each surviving component will increase, leading to a higher degradation rate of the surviving components and the more serious shock effects to soft and hard failures. Then, the failure of the load-sharing system will be accelerated.

To analyze the reliability of such load-sharing system, the difficulties lie in the stochastic nature of the soft failure times of components and arrival times of the random shocks and that the shock effects are dependent on the workload. In this section, the conditional probability density function and the joint probability density function are utilized to develop reliability models for a load-sharing system.

Given the times $T_1,T_2,\dots,T_{i-1}$ and the shocks numbers $m_0,m_1,\dots,m_{i-1}$, the conditional probability density function of the soft failure time of the i th component can be obtained as:

$$\begin{aligned}
&f_i\left(T_i\mid m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_{i-1}\right)=\frac{-dP(X_S(T_i)<H\mid m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_{i-1})}{dT_i}\\
&=\int_u-\varphi\left(\frac{H-\left(\mu+\sum_{l=0}^{i-2}\mu_{\beta_l}\left(T_{l+1}-T_l\right)+\mu_{\beta_{i-1}}\left(T_i-T_{i-1}\right)+\sum_{l=0}^{i-1}m_l\mu_{\tilde{Y}}+u\right)}{\sqrt{\sum_{j=0}^{i-2}\sigma_{\beta_j}^2\left(T_{j+1}-T_j\right)^2+\sigma_{\beta_{i-1}}^2\left(T_i-T_{i-1}\right)^2+\sum_{l=0}^{i-1}m_l\sigma_{\tilde{Y}}^2+\sigma^2}}\right)\\
&\times\left(\frac{\begin{array}{l}-\mu_{\beta_{i-1}}\left(\sum_{j=0}^{i-2}\sigma_{\beta_j}^2\left(T_{j+1}-T_j\right)^2+\sigma_{\beta_{i-1}}^2\left(T_i-T_{i-1}\right)^2+\sum_{l=0}^{i-1}m_l\sigma_{\tilde{Y}}^2+\sigma^2\right)\\ -\sigma_{\beta_{i-1}}^2\left(T_i-T_{i-1}\right)\left(H-\left(\mu+\sum_{l=0}^{i-2}\mu_{\beta_l}\left(T_{l+1}-T_l\right)+\mu_{\beta_{i-1}}\left(T_i-T_{i-1}\right)+\sum_{l=0}^{i-1}m_l\mu_{\tilde{Y}}+u\right)\right)\end{array}}{\left(\sum_{j=0}^{i-2}\sigma_{\beta_j}^2\left(T_{j+1}-T_j\right)^2+\sigma_{\beta_{i-1}}^2\left(T_i-T_{i-1}\right)^2+\sum_{l=0}^{i-1}m_l\sigma_{\tilde{Y}}^2+\sigma^2\right)^{\frac{3}{2}}}\right)\\
&\times\varphi\left(\frac{u-\sum_{l=0}^{i-1}\gamma_l m_l\mu_{Z_S}}{\sqrt{\sum_{l=0}^{i-1}\gamma_l^2 m_l\sigma_{Z_S}}}\right)du
\end{aligned} \tag{20}$$

The conditional probability that C_i fails in an infinitesimal interval dT_i can be expressed as $f_i\left(T_i\mid m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_{i-1}\right)dT_i$. On that condition, for a load-sharing system, there are $n-i+1$ surviving components, so, the probability that the i th component fails at time T_i is:

$$P\left(T_i\mid m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_{i-1}\right)=\left(n-i+1\right)f_i\left(T_i\mid m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_{i-1}\right)dT_i\,. \tag{21}$$

The probability that $n-i$ components in the system work reliably with no soft failure, $P_{Si}\left(NSF_t\right)$, can be derived as:

$$\begin{aligned}
&P_{Si}\left(NSF_t\right)=\frac{n!}{\left(n-i\right)!}\sum_{m_i=0}^{\infty}\dots\sum_{m_0=0}^{\infty}\int_0^t f_1(T_1\mid m_0)P(m_0)\int_{T_1}^t f_1(T_2\mid m_1,m_0,T_1)P(m_1\mid m_0,T_1)\\
&\dots\int_{T_{i-1}}^t f_i(T_i\mid m_{i-1},m_0,m_1,\dots,m_{i-2},T_1,T_2,\dots,T_{i-1})P(m_{i-1}\mid m_0,m_0,m_1,\dots,m_{i-2},T_1,T_2,\dots,T_{i-1})\\
&\times P(X_S(t)<H\mid m_i,m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_i)^{n-i}\\
&\times P(m_i\mid m_0,m_1,\dots,m_{i-1},T_1,T_2,\dots,T_i)dT_i dT_{i-1}\dots dT_1
\end{aligned} \tag{22}$$

The load-sharing system is subject to sudden failures due to random shocks. For a certain system configuration, each component is exposed to the same shock size. Then, once a hard failure occurs for a component, all of the other surviving components will fail due to the same shock at the meanwhile, which leads to a sudden failure of the load-sharing system. The probability that the load-sharing system with i failed components does not experience the sudden failure $P_{Si}\left(NHF_t\right)$ can be presented as:

$$P_{Si}\left(NHF_t\right)=\prod_{j=0}^{i}\left(\int_{z_H}\Phi\left(\frac{D-\alpha_j Z_H-\mu_W}{\sigma_W}\right)\varphi\left(\frac{z_H-\mu_{Z_H}}{\sigma_{Z_H}}\right)dz_H\right)^{m_j}. \quad (23)$$

Therefore,the probability that there are $n-i$ components working reliably R_{Si} can be denoted as:

$$\begin{aligned}
R_{Si} &= P_{Si}\left(NSF_t\right)\times P_{Si}\left(NHF_t\right)\\
&=\frac{n!}{(n-i)!}\sum_{m_i=0}^{\infty}\cdots\sum_{m_0=0}^{\infty}\int_0^t f_1(T_1\mid m_0)\int_{T_1}^t f_2(T_2\mid m_0,m_1,T_1)\ldots\int_{T_{i-1}}^t f_i(T_i\mid m_0,m_1,\ldots,m_{i-1},T_1,T_2,\ldots,T_{i-1})\\
&\times\left(\int_u\Phi\left(\frac{H-\left(\mu+\sum_{l=0}^{i-1}\mu_{\beta_l}\left(T_{l+1}-T_l\right)+\mu_{\beta_i}\left(t-T_i\right)+\sum_{l=0}^{i}m_l\mu_{\tilde{Y}}+u\right)}{\sqrt{\sum_{j=0}^{i-1}\sigma_{\beta_j}^2\left(T_{j+1}-T_j\right)^2+\sigma_{\beta_i}^2\left(t-T_i\right)^2+\sum_{l=0}^{i}m_l\sigma_{\tilde{Y}}^2+\sigma^2}}\right)\times\varphi\left(\frac{u-\sum_{l=0}^{i}\gamma_l m_l\mu_{Z_S}}{\sqrt{\sum_{l=0}^{i}\gamma_l^2 m_l\sigma_{Z_S}}}\right)du\right)^{n-i}\\
&\times\frac{\exp(-\lambda(t-T_i))(\lambda(t-T_i))^{m_i}}{m_i!}\times\prod_{l=2}^{i}\frac{\exp(-\lambda(T_l-T_{l-1}))(\lambda(T_l-T_{l-1}))^{m_{l-1}}}{m_{l-1}!}\\
&\times\frac{\exp(-\lambda T_1)(\lambda T_1)^{m_0}}{m_0!}dT_i dT_{i-1}\ldots dT_1\times\prod_{j=0}^{i}\left(\int_{z_H}\Phi\left(\frac{D-\alpha_j Z_H-\mu_W}{\sigma_W}\right)\varphi\left(\frac{z_H-\mu_{Z_H}}{\sigma_{Z_H}}\right)dz_H\right)^{m_j}.
\end{aligned} \quad (24)$$

A special case occurs when there is no soft fault by time t, and the probability of this occurrence can be obtained using

$$R_{S0}=\sum_{m_0=0}^{\infty}\left(\int_u\Phi\left(\frac{H-\left(\mu+\mu_{\beta_0}t+m_0\mu_{\tilde{Y}}+u\right)}{\sqrt{\sigma_{\beta_0}^2t^2+m_0\sigma_{\tilde{Y}}^2+\sigma^2}}\right)\times\varphi\left(\frac{u-\gamma_0m_0\mu_{Z_S}}{\sqrt{\gamma_0^2m_0\sigma_{Z_S}}}\right)du\right)^n$$
$$\times\frac{\exp(-\lambda t)(\lambda t)^{m_0}}{m_0!}\times\left(\int_{z_H}\Phi\left(\frac{D-\alpha_jZ_H-\mu_W}{\sigma_W}\right)\varphi\left(\frac{z_H-\mu_{Z_H}}{\sigma_{Z_H}}\right)dz_H\right)^{m_0}. \quad (25)$$

For a load-sharing k-out-of-n system, there must be at least k surviving components working for successful operation. Consequently, its reliability $R(t)$ can be determined according to:

$$R(t)=\sum_{i=0}^{n-k}R_{si}. \quad (26)$$

Moreover,the developed reliability model can be easily extend to a load-sharing parallel system through assigning the value k to 1.

5. Case study

In most cases, MEMS is a load-sharing system where multiple micro-engines work together to perform more reliably [4, 13]. The system has been widely applied to many intelligent mechatronic systems, and its reliability has been studied extensively [9, 27]. A 2-out-of-4 load-sharing MEMS with four identical micro-engines is utilized as a realistic application to illustrate the effectiveness and modeling capabilities of the proposed model in this paper. The micro-engine consists of multiple orthogonal linear comb drive actuators mechanically connected to a rotating gear. By applying voltages, the linear displacement of the comb drives is transformed into the circular motion of the gear via a pin joint.

Based on the results of the reliability tests from [28], each micro-engine is subject to two dependent competing failure processes: hard failure due to the spring fracture caused by the huge shock; and

Table 1. The corresponding parameters for the reliability analysis of micro-engines

Parameters	Value	Sources
H	0.00125 μm³	[26]
D	1.5 GPa	[26]
μ	0	[26]
β_i	$\beta_0\sim N\left(\mu_{\beta_0},\sigma_{\beta_0}^2\right)$, $\mu_{\beta_0}=8.4823\times10^{-9}\mu m^3$, $\sigma_{\beta_0}=6.0016\times10^{-10}\mu m^3$ $\beta_1\sim N\left(\mu_{\beta_1},\sigma_{\beta_1}^2\right)$, $\mu_{\beta_1}=1.2\times10^{-8}\mu m^3$, $\sigma_{\beta_1}=6.0\times10^{-9}\mu m^3$ $\beta_2\sim N\left(\mu_{\beta_2},\sigma_{\beta_2}^2\right)$, $\mu_{\beta_2}=2.0\times10^{-8}\mu m^3$, $\sigma_{\beta_2}=9.0\times10^{-9}\mu m^3$	β_0: [26]; β_1 and β_2: Asumption
ε	$\varepsilon\sim N\left(0,\sigma^2\right)$, $\sigma=10^{-10}\mu m^3$	[19]
λ	2.5×10^{-5}	[19]
Z_H	$Z_H\sim N\left(\mu_{Z_H},\sigma_{Z_H}^2\right)$, $\mu_{Z_H}=1.2$GPa, $\sigma_{Z_H}=0.2$GPa	[19]
Z_S	$Z_S\sim N\left(\mu_{Z_S},\sigma_{Z_S}^2\right)$, $\mu_{Z_S}=1.2$GPa, $\sigma_{Z_S}=0.2$GPa	[19]
$\tilde{Y}$	$\tilde{Y}\sim N\left(\mu_{\tilde{Y}},\sigma_{\tilde{Y}}^2\right)$, $\mu_{\tilde{Y}}=1.0\times10^{-9}\mu m^3$, $\sigma_{\tilde{Y}}=1.0\times10^{-10}\mu m^3$	Asumption
$\tilde{W}$	$\tilde{W}\sim N\left(\mu_{\tilde{W}},\sigma_{\tilde{W}}^2\right)$, $\mu_{\tilde{W}}=0$, $\sigma_{\tilde{W}}=1.0\times10^{-3}$	Asumption
γ_i	$\gamma_0=2\times10^{-5}$, $\gamma_1=1\times10^{-4}$, $\gamma_2=5\times10^{-4}$	Asumption
α_i	$\alpha_0=0.9$, $\alpha_1=1.0$, $\alpha_2=1.2$	Asumption

soft failure due to the continual wear process on rubbing surfaces and substantial wear debris caused by shocks, which usually results in a seized micro-engine or a broken pin joint and then the micro-engine is deemed to have failed. Due to load-sharing characteristics, the degradation rate of a surviving micro-engine will increase after a micro-engine failure because of its increased shared workload. In addition, the shock effects to the soft failure and hard failure are dependent on the workload, and the load-sharing MEMS experiences DWSEs. In the application, both transmission parameters α_i and γ_i will increase after a component fails. Based on the data in [4, 18, 19, 26], along with some reasonable assumptions, the corresponding parameters for the reliability model are shown in Table 1.

5.1. Results analysis

Using Eq. (3), the probability that MEMS with zero failed micro-engine, one failed micro-engine, and two failed micro-engines will survive a shock with no sudden failure is calculated as 99.02%, 93.32%, and 59.87%, respectively. Based on Eqs. (24) and (25), for the MEMS, the probability that four micro-engines work reliably (i.e., R_{S0}), three micro-engines work reliably (i.e., R_{S1}) , and two micro-engines work reliably (i.e., R_{S2}) , is shown in Fig. 6, respectively. It can be seen that R_{S0} is very close to 1 and changes slowly at first, and then its declination increases sharply at approximately $t=8\times10^4$. In addition, R_{S1} and R_{S2} are very close to 0 before the time $t=8\times10^4$, and then increase to the peak point, and finally decrease to 0. This is mainly due to the following reasons:

(1) For each micro-engine, the arrival shocks will cause abrupt wear debris from the contact surface of the pin joint and the gear. At the beginning, the wear extent is far from the failure threshold *H*, and soft failure rarely occurs and hard failure is the main failure mode. Moreover, the arrival shocks are relatively few, and then the probability that hard failure occurs is also small. Therefore, before the time $t=8\times10^4$, R_{S0} is close to 1 and decreases slowly, and R_{S1} and R_{S2} are very close to 0.

(2) As time goes on, the wear extent increases and may approach the threshold. Therefore, for micro-engines, soft failure is more likely to occur when is large, which leads to the sharp decrease of R_{S0}. The micro-engines will fail in order due to wear. When the first micro-engine fails, R_{S1} will increase, and when the second micro-engine fails, R_{S2} will increase. Finally, they will decrease to 0 since all micro-engines will fail *t* when is large enough.

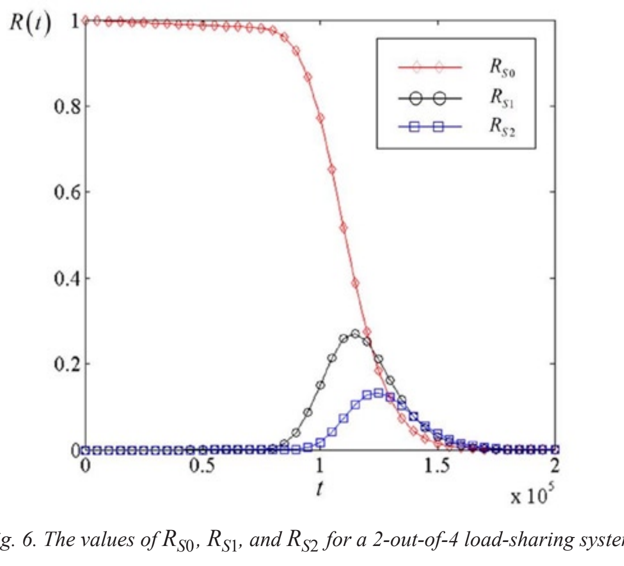

Fig. 6. The values of R_{S0}, R_{S1}, and R_{S2} for a 2-out-of-4 load-sharing system

By setting different values of the parameters in Eq. (24), we can calculate the system reliability without load-sharing characteristics and DWSEs (i.e., $\beta_0=\beta_1=\beta_2\sim N\left(\mu_{\beta_0},\sigma^2_{\beta_0}\right)$, $\gamma_0=\gamma_1=\gamma_2=2\times10^{-5}$ and $\alpha_0=\alpha_1=\alpha_2=0.9$), and only with load-sharing characteristics (i.e., $\gamma_0=\gamma_1=\gamma_2=2\times10^{-5}$ and $\alpha_0=\alpha_1=\alpha_2=0.9$). For a MEMS without load-sharing characteristics and DWSEs, the probabilities that four micro-engines work reliably (denoted by R'_{S0}), three micro-engines work reliably (denoted by R'_{S1}), and two micro-engines work reliably (denoted by R'_{S2}), are plotted in Fig. 7(a). For a MEMS with only load-sharing characteristics, the probabilities that four micro-engines work reliably (denoted by R''_{S0}), three micro-engines work reliably (denoted by R''_{S1}), and two micro-engines work reliably (denoted by R''_{S2}), are plotted in Fig. 7(b). It can be seen that R'_{S0}, and R''_{S0} are all the same, R_{S1} and R_{S2} are lower than R''_{S1} and R''_{S2} respectively, and R''_{S1} and R''_{S2} are lower than R'_{S1} and R'_{S2} respectively. In addition, compared with the reduction from R'_{S1} to R''_{S1}, R''_{S2} decreases more significantly. This can be explained by the following facts:

(1) When considering the load-sharing characteristics, once a micro-engine fails, the degradation rate of the other surviving micro-engines will increase along with their shared workload. Thus, R''_{S1} and R''_{S2} are lower than R'_{S1} and R'_{S2} respectively. Moreover, when two micro-engines fail, the shared workload and degradation rate of the surviving micro-engines will increase further, and the third micro-engine will fail more easily. Therefore, R''_{S2} decreases more significantly.

(2) When considering the DWSEs, the transmitted shock damage and size are dependent on the workload. The shock damage and size transmitted to the surviving micro-engines will rise as an increasing number of micro-engines fail, and then both soft failure and hard failure will occur more easily. Therefore, R_{S1} and R_{S2} are lower than R''_{S1} and R''_{S2}, respectively.

(3) When no micro-engine has failed, the workload and shock effects on each micro-engine are the same when considering load-sharing characteristics and DWSEs or not, and thus R_{S0}, R'_{S0}, and R''_{S0} are all the same.

Based on Eq. (26), knowing R_{S0}, R_{S1}, and R_{S2}, we can get the reliability $R(t)$ considering load-sharing characteristics and DWSEs. Similarly, we can get the reliability $R'(t)$ without load-sharing characteristics and DWSEs, as well as the reliability $R''(t)$ only considering load-sharing characteristics. A comparison plot of $R(t)$, $R'(t)$, and $R''(t)$ is shown in Fig. 8. It is easy to find that $R(t)$ is lower than $R''(t)$ and $R''(t)$ is lower than $R'(t)$, which indicates that both the load-sharing characteristics and DWSEs decrease the system reliability.

5.2 Sensitive analysis

A sensitive analysis is conducted to study the effects of important parameters on system reliability. The transmission parameter from the system's shock magnitude to the transmitted shock size on C_i, α_i, and the transmission parameter from the system's shock magnitude to the transmitted shock damage on C_i, γ_i, are the parameters of interest. The results of the sensitive analysis of α_i and γ_i are shown in Fig. 9. The red line with rhombus shows the system reliability with α_i increasing from $[\alpha_0=0.9,\alpha_1=1.0,\alpha_2=1.2]$ to $[1.0,1.2,1.4]$. The system reliability decreases more sharply before $t=10^5$, and the system reliability decreases by 15.3% (from to 0.940 to 0.815) at time

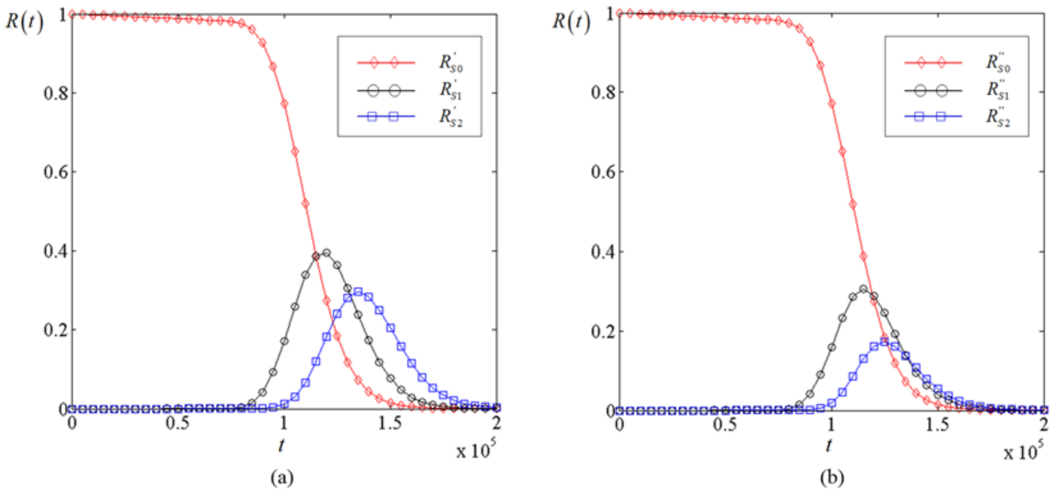

Fig. 7. The values of reliabilities for different system configurations: (a) without the load-sharing characteristic and the DWSEs and (b) only considering the load-sharing characteristic

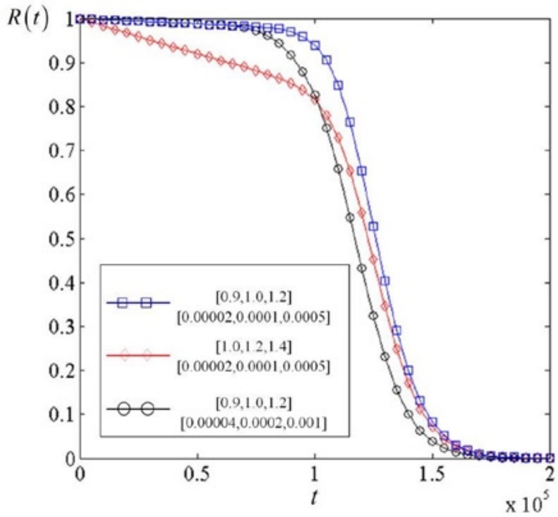

Fig. 8. The comparison plot of the reliabilities with different type of dependence

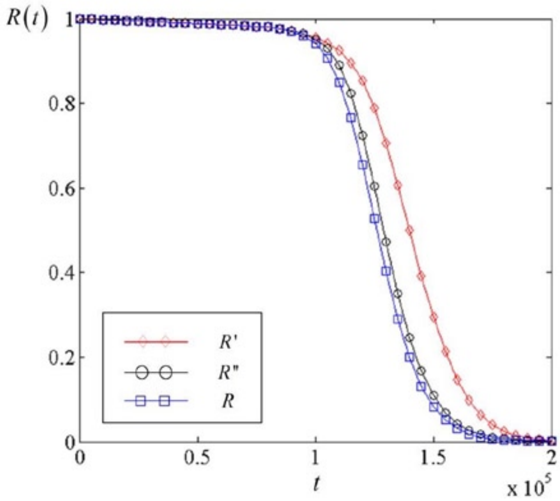

Fig. 9. The sensitivity analysis of reliability on α_i and γ_i

$t = 10^5$.This is mainly due to transmission parameters α_i having a significant affect on hard failures. A high α_i will therefore lead to a large shock size and more hard failures will occur.

The black line with circles shows the system reliability with γ_i increasing from $[\gamma_0 = 0.00002, \gamma_1 = 0.0001, \gamma_2 = 0.0005]$ to $[0.00004, 0.0002, 0.001]$. It can be seen that the two types of reliability curves are almost the same before $t = 7.5 \times 10^4$, and then the reliability with $[\gamma_0 = 0.00004, \gamma_1 = 0.0002, \gamma_2 = 0.001]$ decreases earlier. The system reliability curve moves to the left, which indicates that a larger value of γ_i results in a lower reliability performance.The main reasons for this phenomenon are the following:

(1) A higher γ_i will lead to a larger magnitude of wear debris caused by shocks, while the total wear debris is relatively small and is far from the threshold H at the beginning. Therefore, soft failure rarely occurs and the two types of reliability curves are almost the same.

(2) As t increases, for the system with higher γ_i, the wear debris increases more and approaches the threshold earlier. Thus, the reliability decreases earlier.

6. Conclusion

In this paper, a reliability model is developed for load-sharing k-out-of-n systems subject to the dependent competing soft and hard failures. A new dependence between workload and shock effects is investigated and the dependence is addressed in the model as a major extension from previous reliability models for load-sharing systems. The proposed reliability model is more realistic but difficult to develop due to the load-sharing characteristics and DWSEs. To derive an analytical reliability model, the joint probability density function of shock effects to the soft and hard failures and the conditional probability density function of random component failure times are proposed. A MEMS with four identical micro-engines is then utilized as a realistic application to demonstrate the proposed model. The results show that both the DWSEs and load-sharing characteristics lead to a lower reliability performance. Thus, the reliability evaluation of a load-sharing k-out-of-n system may be more accurate when considering the dependence between workload and shock effects.In future work, maintenance can be studied based on the proposed model and the optimal maintenance policies can be obtained to enhance the system reliability.

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Appendix

The notations used in formulating the reliability models are now listed.

DWSEs	Dependent workload and shock effects
MEMS	Micro-Electro-Mechanical System
CDF	Cumulative distribution function
PDF	Probability density function
n	Number of components in the load-sharing system
$N(t)$	Number of failed components by time t
C_i	Surviving components in the load-sharing system with i failed components
Z_j	Magnitude of the j th system shock
Z_{Hj}	Magnitude of the j th shock on the devices for hard failures
Z_{Sj}	Magnitude of the j th shock on the devices for soft failures
W_{ij}	Total transmitted shock size to C_i for the hard failure process from Z_{Hj} and workload
Y_{ij}	Total transmitted shock damage to C_i for the soft failure process from Z_{Sj} and workload
$\tilde{W}_{ij}$	Purely random shock effect for the j th system shock to C_i of the hard failure process
$\tilde{Y}_{ij}$	Purely random shock effect for the j th system shock to C_i of the soft failure process
α_i	Transmission parameter between Z_{Hj} to the shock size for the hard failure process of C_i
γ_i	Transmission parameter from Z_{Sj} to the shock damage for the soft failure process of C_i
β_i	Current degradation rate of C_i
T_i	Failure time of the i th component
m_i	Number of shocks have arrived in the time interval between T_i and T_{i+1}
$X(t)$	Degradation extent at t due to continuous degradation
$S(t)$	Cumulative degradation damage increments
$X_S(t)$	Overall degradation at t due to continuous degradation and shock damages
H	Critical degradation threshold
D	Maximum fracture strength for hard failures
$F_{W_i}(w_i)$	CDF of W_{ij} for C_i
$f_{W_i}(w_i)$	PDF of W_{ij} for C_i
$F_{Y_i}(y_i)$	CDF of Y_{ij} for C_i
$f_{Y_i}(y_i)$	PDF of Y_{ij} for C_i
$f_{Y_i}^{<m>}(y_i)$	PDF of the sum of m i.i.d. Y_{ij} variables
$f_{Z_{Sj}}(z_{Sj})$	PDF of Z_{Sj}

$f_{Z_S}^{<m>}(z_S)$	PDF of the sum of m i.i.d. Z_{Sj} variables
$f_{Z_{Hj}}(z_{Hj})$	PDF of Z_{Hj}

References

1. Babishin V, Hajipour Y, Taghipour S. Optimisation of non-periodic inspection and maintenance for multicomponent systems. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2018; 20(2): 327-342, https://doi.org/10.17531/ein.2018.2.20.
2. Bocchetti D, Giorgio M, Guida M, Pulcini G. A competing risk model for the reliability of cylinder liners in marine Diesel engines. Reliability Engineering & System Safety 2009; 94(8):1299-1307, https://doi.org/10.1016/j.ress.2009.01.010.
3. Cha JH, Finkelstein M, Levitin G. Bivariate preventive maintenance of systems with lifetimes dependent on a random shock process. European Journal of Operational Research 2018; 266(1):122-134, https://doi.org/10.1016/j.ejor.2017.09.021.
4. Che H, Zeng S, Guo J. Reliability Analysis of Load-Sharing Systems Subject to Dependent Degradation Processes and Random Shocks. IEEE Access 2017; 5: 23395-23404, https://doi.org/10.1109/ACCESS.2017.2762727.
5. Che H, Zeng S, Guo J, Wang Y. Reliability modeling for dependent competing failure processes with mutually dependent degradation process and shock process. Reliability Engineering & System Safety 2018;180:168-178, https://doi.org/10.1016/j.ress.2018.07.018.
6. Griffith WS. Optimal Reliability Modeling: Principles and Applications. Technometrics 2004;46(1):112-112, https://doi.org/10.1198/tech.2004.s742.
7. Guida M, Pulcini G. A continuous-state Markov model for age- and state-dependent degradation processes. Structural Safety 2011; 33(6): 354-366, https://doi.org/10.1016/j.strusafe.2011.06.002.
8. Guo C, Wang W, Guo B, Peng R. Maintenance optimization for systems with dependent competing risks using a copula function. Eksploatacja i Niezawodnosc -Maintenance and Reliability 2013;15(1):9-17.
9. He G, Geng Z. Dynamics and Robust Control of an Underactuated Torsional Vibratory Gyroscope Actuated by Electrostatic Actuator. IEEE/ASME Transactions on Mechatronics 2015;20(4):1725-1733, https://doi.org/10.1109/TMECH.2014.2350535.
10. Hu Z, Du X. System reliability prediction with shared load and unknown component design details. Artificial Intelligence for Engineering Design Analysis & Manufacturing 2017;31(3):223-234, https://doi.org/10.1017/S0890060417000130.
11. Jiakai C, Yan H, Wei W. Reliability analysis and optimization of equal load-sharing k-out-of-n phased-mission systems. Eksploatacja i Niezawodnosc -Maintenance and Reliability 2015;17(2):250-259, https://doi.org/10.17531/ein.2015.2.12.
12. Kong Y, Ye Z. Interval estimation for k-out-of-n load-sharing systems. IISE Transactions 2017;49(3):344-353, https://doi.org/10.1080/0740817X.2016.1217102.
13. Liu B, Xie M, Kuo W. Reliability modeling and preventive maintenance of load-sharing systems with degrading components. IIE Transactions 2016(8):699-709, https://doi.org/10.1080/0740817X.2015.1125041.
14. Liu B, Xie M, Xu Z, Kuo W. An imperfect maintenance policy for mission-oriented systems subject to degradation and external shocks. Computers & Industrial Engineering 2016;102:21-32, https://doi.org/10.1016/j.cie.2016.10.008.
15. Liu H, Yeh R, Cai B. Reliability modeling for dependent competing failure processes of damage self-healing systems. Computers & Industrial Engineering 2017;105:55-62, https://doi.org/10.1016/j.cie.2016.12.035.
16. Młynarski S, Pilch R, Smolnik M, Szybka J, Wiązania G. A Method for rapid evaluation of k-out-of-n systems reliability. Eksploatacja i Niezawodnosc -Maintenance and Reliability 2018;21(1):170-176, https://doi.org/10.17531/ein.2019.1.20.
17. Nelson W. Accelerated Testing: Statistical Models, Test Plans, and Data Analysis; 2008.
18. Peng H, Feng Q, Coit DW. Reliability and maintenance modeling for systems subject to multiple dependent competing failure processes. IIE Transactions 2010;43(1):12-22, https://doi.org/10.1080/0740817X.2010.491502.
19. Rafiee K, Feng Q, Coit DW. Reliability modeling for dependent competing failure processes with changing degradation rate. IIE Transactions 2014;46(5):483-496, https://doi.org/10.1080/0740817X.2013.812270.
20. Rothgang S, Baumhöfer T, Hoek HV, Lange T, Doncker RWD, Sauer DU. Modular battery design for reliable, flexible and multi-technology energy storage systems. Applied Energy 2015;137:931-937, https://doi.org/10.1016/j.apenergy.2014.06.069.
21. Ruiz-Castro JE. Markov counting and reward processes for analysing the performance of a complex system subject to random inspections. Reliability Engineering & System Safety 2016;145:155-168, https://doi.org/10.1016/j.ress.2015.09.004.
22. Song S, Coit DW, Feng Q, Peng H. Reliability Analysis for Multi-Component Systems Subject to Multiple Dependent Competing Failure Processes. IEEE Transactions on Reliability 2014;63(1):331-345, https://doi.org/10.1109/TR.2014.2299693.
23. Song S, Coit DW, Feng Q. Reliability analysis of multiple-component series systems subject to hard and soft failures with dependent shock effects. IIE Transactions 2016;48(8):720, https://doi.org/10.1080/0740817X.2016.1140922.
24. Suprasad AV, Krishna MB, Hoang P. Tampered Failure Rate Load-Sharing Systems: Status and Perspectives; 2008.
25. Taghipour S, Kassaei ML. Periodic Inspection Optimization of a k-out-of-n Load-Sharing System. IEEE Transactions on Reliability 2015;64(3):1116-1127, https://doi.org/10.1109/TR.2015.2421819.
26. Tanner DM, Dugger MT. Wear mechanisms in a reliability methodology. In: Ramesham R. Proceedings of the society of photo-optical instrumentation engineers(SPIE) 2003: 22-40, https://doi.org/10.1117/12.476345.
27. Tanner DM, Miller WM, Peterson KA, et al. Frequency dependence of the lifetime of a surface micromachined microengine driving a load. Microelectronics Reliability 1999;39(3):401-414, https://doi.org/10.1016/S0026-2714(98)00248-0.
28. Tanner DM, Walraven JA, Helgesen K, et al. MEMS reliability in shock environments. IEEE 2000: 129-138.
29. Walraven JA, Headley TJ, Campbell AB, Tanner DM. Failure analysis of worn surface-micromachined microengines. Proc SPIE 1999;3880:30-39, https://doi.org/10.1117/12.359369.
30. Wang Y, Guo L, Wen M, Yang Y. Availability analysis for a multi-component system with different k-out-of-n:G warm standby subsystems subject to suspended animation. Ekspolatacja i Niezawodnosc - Maintenance and Reliability 2019;21(2):289-300, https://doi.org/10.17531/

ein.2019.2.14.
31. Wang Z, Huang H, Li Y, Xiao N. An Approach to Reliability Assessment Under Degradation and Shock Process. IEEE Transactions on Reliability 2011;60(4):852-863, https://doi.org/10.1109/TR.2011.2170254.
32. Yang C, Zeng S, Guo J. Reliability Analysis of Load-Sharing K-out-of-N System Considering Component Degradation. Mathematical Problems in Engineering 2015;2015:1-10, https://doi.org/10.1155/2015/726853.
33. Yang L, Zhao Y, Peng R, Ma X. Hybrid preventive maintenance of competing failures under random environment. Reliability Engineering & System Safety 2018;174:130-140, https://doi.org/10.1016/j.ress.2018.02.017.
34. Yang L, Ma X, Zhao Y. A condition-based maintenance model for a three-state system subject to degradation and environmental shocks. Computers & Industrial Engineering 2017;105:210-222, https://doi.org/10.1016/j.cie.2017.01.012.
35. Yang L, Zhao Y, Ma X. Multi-level maintenance strategy of deteriorating systems subject to two-stage inspection. Computers & Industrial Engineering 2018;118:160-169, https://doi.org/10.1016/j.cie.2018.02.038.
36. Ye ZS, Tang LC, Xu HY. A Distribution-Based Systems Reliability Model Under Extreme Shocks and Natural Degradation. IEEE Transactions on Reliability 2011;60(1):246-256, https://doi.org/10.1109/TR.2010.2103710.
37. Yu H, Eberhard P, Zhao Y, Wang H. Sharing behavior of load transmission on gear pair systems actuated by parallel arrangements of multiple pinions. Mechanism & Machine Theory 2013;65(65):58-70, https://doi.org/10.1016/j.mechmachtheory.2013.02.010.
38. Zhang T, Zhang Y, Du X. Reliability analysis for k-out-of-n systems with shared load and dependent components. Structural and Multidisciplinary Optimization 2018;57(3):913-923, https://doi.org/10.1007/s00158-017-1893-z.
39. Zhang Y, Ma Y, Ouyang L, Liu L. A novel reliability model for multi-component systems subject to multiple dependent competing risks with degradation rate acceleration. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2018;20(4): 579-589, https://doi.org/10.17531/ein.2018.4.9.
40. Zhao X, Wu C, Wang S, Wang X. Reliability analysis of multi-state k-out-of-n: G system with common bus performance sharing. Computers & Industrial Engineering 2018;124:359-369, https://doi.org/10.1016/j.cie.2018.07.034.
41. Tian Z, Zuo M, Yam RM. Multi-state k-out-of-n systems and their performance evaluation. IIE Transactions 2008;41(1):32-44, https://doi.org/10.1080/07408170802322655.

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