

Chenxi LIU
Achim KRAMER
Stephan NEUMANN

RELIABILITY ASSESSMENT OF REPAIRABLE PHASED-MISSION SYSTEM BY MONTE CARLO SIMULATION BASED ON MODULAR SEQUENCE-ENFORCING FAULT TREE MODEL

OCENA NIEZAWODNOŚCI NAPRAWIALNEGO SYSTEMU Z MISJAMI OKRESOWYMI ZA POMOCĄ SYMULACJI MONTE CARLO W OPARCIU O MODUŁOWY MODEL DRZEWA NIEZDATNOŚCI Z BRAMKAMI SEQ

Phased-mission system (PMS) is the system subject to multiple, consecutive and non-overlapping tasks. Much more complicated problems will be confronted when the PMS is repairable since the repairable system could perform the multi-phases mission with more diversity requirements. Besides, various maintenance strategies will directly influence the reliability analysis procedure. Most researches investigate those repairable PMSs that carry out the multi-phases mission with deterministic phase durations, and the mission fails once the system switches from up to down. In this case, one common maintenance strategy is that failed components are repairable as long as the system keeps in up state. However, many practical systems (e.g., construction machinery, agricultural machinery) may be involved in such multi-phases mission, which has uncertain phase durations but limited by a maximum mission time, within which failed components can be unconditional repaired, and the system can be restored from down state. Comparing with the former type of repairable PMS, the latter will also concern phase durations dependence, and both the system and components included have the state bidirectional transition. This paper makes new contributions to the reliability assessment of repairable PMSs by proposing a novel SEFT-MC method. Two types of repairable PMS mentioned above are considered. In our method, a specific sequence-enforcing fault tree (SEFT) is proposed to correctly depict failure logical relationships between the system and components included. In order to transfer the graphical fault tree (no matter its size and complexity) into a modular reliability model used in Monte Carlo (MC) simulation, an improved linear algebra representation (I-LAR) approach is introduced. Finally, a numerical example including two cases corresponding to the two types of repairable PMS is presented to validate the proposed method.

Keywords: *repairable, phased-mission system, modular reliability modeling, improved linear algebra representation; Monte Carlo simulation.*

System z misjami okresowymi (phased-mission system, PMS) to system, który wykonuje wiele następujących po sobie i nienakładających się na siebie zadań. W przypadku naprawialnych systemów PMS, analiza niezawodności jest o wiele bardziej skomplikowana, ponieważ system naprawialny może wykonywać misje wielofazowe o bardziej różnorodnych wymaganiach. Poza tym systemy takie wymagają zastosowania różnych strategii utrzymania ruchu, co ma bezpośredni wpływ na procedurę analizy niezawodności. Większość badaczy bada naprawialne systemy PMS, które wykonują misje wielofazowe, w których czas trwania fazy jest wielkością deterministyczną, a misja kończy się niepowodzeniem, gdy system przechodzi ze stanu zdatności do stanu niezdatności. W takich przypadkach najczęściej przyjmuje się, że uszkodzone elementy można naprawić o ile system pozostaje w stanie zdatności. Jednak wiele systemów stosowanych w praktyce (t.j. maszyny budowlane czy maszyny rolnicze) może wykonywać misje wielofazowe, w których czas trwania fazy jest wielkością niepewną, ograniczoną jedynie przez maksymalny czas trwania misji, w którym to czasie uszkodzone komponenty mogą być bezwarunkowo naprawiane, dzięki czemu system może zostać przywrócony do stanu zdatności. W porównaniu z pierwszym rodzajem naprawialnego PMS, w drugim, czasy trwania faz zależne od siebie. Ponadto, w systemie tego typu, zarówno poszczególne elementy, jak i cały system mogą przechodzić ze stanu zdatności do stanu niezdatności i odwrotnie. Niniejsza praca wnosi nowy wkład w ocenę niezawodności naprawialnych systemów PMS, proponując nowatorską metodę, która polega na wykorzystaniu dynamicznego drzewa niezdatności do przeprowadzenia symulacji Monte Carlo (SEFT-MC). Rozważane są dwa wymienione powyżej typy naprawialnego PMS. W naszej metodzie zaproponowano drzewo niezdatności z bramkami SEQ (SEFT), które pozwala poprawnie zobrazować logiczne zależności między systemem a jego komponentami w zakresie uszkodzeń. Do przeniesienia graficznego drzewa niezdatności (bez względu na jego rozmiar i złożoność) do modułowego modelu niezawodności wykorzystywanego w symulacji Monte Carlo, zastosowano udoskonaloną metodę reprezentacji algebry liniowej (I-LAR). Poprawność proponowanej metody wykazano na przykładzie numerycznym obejmującym dwa przypadki odpowiadające dwóm omawianym typom naprawialnego PMS.

Słowa kluczowe: *naprawialny, system z misjami okresowymi, modułowe modelowanie niezawodności, udoskonalona reprezentacja algebry liniowej; symulacja Monte Carlo.*

1. Introduction

Phased-mission systems (PMSs) are systems that perform multiple, consecutive and non-overlapping tasks [13]. Such systems are common in many fields, like power [4], spacecraft [5-6], distributed computing system [13], and military [26]. As the name suggests, the whole mission undertaken by PMS includes multiple tasks; each specified task lasts for a duration and the system has to withstand different stress loads. Usually, the system structure, as well as component failure behaviors are various among different phases; some components participate in more than one phase, and the cumulative damage caused in phase i have to be taken into account when determining the failure rate in phase j ($i < j$). Thus, challenges in analyzing PMS comprise of two aspects: dynamic behaviors among phases, and state dependence among phases.

For non-repairable PMS, methods and applications for reliability assessment have been extensively studied [19]. Basically, existing methodologies can be categorized into the simulation and the analytical methods. The simulation methods are outstanding in their wide applicability to a variety of scenarios [23, 28]; whereas the analytical methods, including binary decision diagram (BDD)-based method [17, 24, 25], multivalued decision diagram (MDD)-based method [13, 16], Markov chains-based method [18], Markov reward model-universal generating function (UGF) technique [7], Bayesian networks approach [4], recursive algorithm [3], have advantages in obtaining accurate results with high efficiency, but may not be suitable in large-scale PMS with complex dynamic behaviors.

In contrast, the investigation on the reliability of repairable PMSs has not been studied to the same extent, though they are commonly found in many real-world engineering applications. Comparing with non-repairable PMSs, there will be more challenges have to be confronted. On the one hand, the repairable system could perform the multi-phases mission with more diversity requirements; on the other hand, various maintenance strategies will directly influence the reliability analysis procedure.

Existing researches mostly investigate those repairable PMSs that carry out multi-phases missions with such requirement, i.e., phase durations are deterministic. Kim [2] supposed that failed components are repairable only when the system is up, and a Markov model is formulated to obtain the mission reliability. A series-parallel PMS is studied by the generic Monte Carlo simulator known as Raptor [15], in which only the non-critical component (i.e., generally a redundant component) can be repaired. Lu [10] proposed a decomposition approach combined with continuous-time Markov chains (CTMCs) to evaluate the reliability of PMS considering both combinatorial phase requirements and repairable components. The PMS consisting of a large number of phases and repairable components is studied in [9,11]. It is assumed that the failed component can only be repaired when the system is still operating, and it can be reused only in the next phase after its restoration. A truncation method based on the binary-decision-diagram (BDD) and Markov chains is proposed to solve the scaling issue. Considering multi-mission PMS with repairable components, and repairable PMS with common cause failures, Wu [20-21] proposed an extended object-oriented Petri net (EOOPN) model for mission reliability simulation. In Li et. al.'s research [5], redundant architecture such as cold standby (structural or functional) is applied to certain critical parts, and then, the Semi-Markov process is used to assess the reliability of the PMSs with non-exponential and partially repairable components. Zhao et.al. [27] introduced spare parts for every component to make the PMS repairable; an integrated modeling method based on the multistate multi-valued decision diagram (MMDD) and Markov chain is developed to evaluate the mission success; besides, the optimal allocation of spare parts is also studied. Overall, the PMS with deterministic phase durations refers to that each task has to be continuously executed for a specific duration. Some components in

the system are allowed to be repaired or replaced to keep the system on, until a minimum cut set is triggered, resulting in the task (mission) interruption, i.e., mission failure. On the contrary, mission success is concluded if the system completes the whole mission in continuous operation.

However, it is not necessary to require the system to perform a multi-phases mission without interruption in many fields, such as construction machinery, agricultural machinery, printing equipment, machine tools, etc., since downtime of the system is allowable ascribed to the components' maintenance. A typical example is the tractor system that performs the grass harvest mission. The mission includes 3 phases: cutting the ripe grass; raking the grass that has been cut off; loading the grass up to the trailer and transporting the grass to the pasture faraway. During each phase, a certain task has to be carried out by general tractor equipping with the related implement, i.e., mower, rake, and trailer, respectively. Throughout the whole mission, failed components are repairable regardless of whether the system is up or down. However, it is required that the entire mission has to be completed within T_{\max} days, including the system downtime (i.e. for repairs) due to certain component failures. The roles and encountered load condition of the tractor system varies in different phase; besides, the system configuration, success criteria, and component behavior change from phase to phase. Thus, the tractor system can be termed as the PMS. Moreover, even though the working time for each task is determined according to the normal operating ability, the duration of each phase is uncertain because the repair times for failure components are random variables. But all the three-phase durations have to satisfy the relationship, represented as $t_1 + t_2 + t_3 \leq T_{\max}$; otherwise, the required mission is determined as failure.

In consequence, the repairable PMS with uncertain phase durations but limited by a maximum mission time is also studied in this paper. Note that this type of PMS is different from those addressed in literature [2, 5, 9-11, 15, 20-21, 27] mentioned above, whose phase durations are assumed to be deterministic. It has to concern with phase durations dependence, except for dynamic behaviors among phases, and components state dependence among phases. Moreover, not only the repairable components have bidirectional transitions between states of up and down, but also the system has the bidirectional state transition.

In existing research, Monte Carlo (MC) simulation, as a typical simulation method, has been adopted in analyzing non-repairable PMS [23,28]. Since MC simulation is superior due to its strong adaptability, it could be taken to cope with the reliability analysis of repairable PMS regardless of the complexity of the system. MC procedure is a way of carrying out numerical trails and based on a mapping model between inputs and outputs. The accuracy of the analysis outcome could be guaranteed by the reasonable number of simulation trails. As for an individual trail, the correctness of output corresponding to certain inputs depends on the mapping model in use. Therefore, it is important to particularly explore a rational and efficient modeling method that is compatible with the problem being studied.

The fault tree is a graphical tool for system reliability analysis; it has the advantages of being straightforward, being clear logical, and having semantic specification. Thus, it is widely used in reliability analysis on system failure criteria during each phase of PMS. Additionally, some researches adopted the OR gate as the first-level logical connection to construct the whole fault tree of PMS [22], i.e., the output is the state of the system, whereas inputs are all phases subtrees. By using the OR gate, it can display the fact that the system is determined to fail once any one phase fails; however, it cannot display the sequence behavior among phases, i.e. phase j will not fail before phase i when $i < j$. Therefore, it is not appropriate for applying the OR gate as the first-level logical connection. (It must be noted that the sequence behavior and state dependence among phases do have been

taken into account in investigation [22], even though they are not be properly displayed in the fault tree.)

In fact, one type of logical gate, called sequence-enforcing gate (SEQ gate) [1] is introduced to express constraints that all inputs are forced to occur in the left-to-right order. Obviously, it just fit the sequence behavior of PMS, that system mission has to be carried out phase by phase. Thus, Sequence-enforcing Fault Tree (SEFT) is proposed in this paper, in which an SEQ gate is adopted as the first-level logical connection to construct the whole fault tree of PMS. In that case, a complete relationship between the system and components can be accurately displayed by logic gates. Furthermore, a fault tree can be regarded as a hierarchical combination of several logic modules [8]. Each logic module is centered on a gate unit, while linking an output event and more than one inputs. The existing literature [8,12] shows that once operating rules of all gates could be expressed in a standard unified form, the modular model of the whole fault tree can be established. Therefore, how to establish the unified form that is available in various static/dynamic gates including the SEQ gate will be specially studied in this paper.

This paper is organized as follows. In Section 2, the two types of repairable PMS being studied are introduced. In Section 3, the proposed SEFT-MC method (SEFT-MC is short for Monte Carlo Simulation based on Modular Sequence-enforcing Fault Tree Model) is described in detail. In Section 4, the application of the SEFT-MC method is presented, in which the influence of whether phase duration is deterministic is discussed. Finally, conclusions are drawn in Section 5, as well as the direction of future research.

2. System description

Two types of repairable PMS are considered in this paper, in which both types comply with the same system structure and failure criteria, in detail:

- N components are included in the system.
- The system is required to undertake a mission, which consists of n phases. The switching time between the two phases is negligible.
- Each component has binary states, i.e., up and down; up implies the component working normally, whereas down implies component failure or in repair.

- Component failure & repair times are mutually s-independent which can obey different distributions rather than just the exponential distribution.
- The system is either in up or down state, which is determined by related components states, as well as the structure function.

Moreover, the different characteristics of the two types of repairable PMS are listed in Table 1, including different mission requirements and maintenance strategies.

3. Proposed SEFT-MC method for repairable PMS analysis

To evaluate the reliability of repairable PMS, a SEFT-MC method is proposed in this investigation. Utilizing this method, the whole fault tree (i.e. SEFT) is constructed to distinctly express interrelationships between the system state and components states; at this point, a modular reliability model could be developed, which is used to effectively support the further MC simulation procedure. The highlight of this method is the proposal of SEFT and how to transfer this graphical expression into a modular reliability model that is unaffected by the size and complexity of the fault tree.

3.1. Basic structure of SEFT

SEFT is proposed as the whole fault tree of PMS. Take a 3-PMS (short for PMS with 3 phases) for example, the basic structure of SEFT is shown in Fig. 1. The top event represents the state of a system that has to carry out a 3-phases mission; utilizing the SEQ-OR gate, it connects to all phase subtrees. Each subtree can be further explored by analyzing system failure criteria during the related phase.

As the core of an SEFT, SEQ-OR gate is a kind of SEQ gate, which not only restricts that the inputs must occur from left to right but also determines the output failure as long as any one input fails. By using the SEQ-OR gate as the first-level logical connection, the basic structure of SEFT is suitable for reliability analysis on PMS in various practical fields.

3.2. Improved linear algebra representation approach

An SEFT can be regarded as a hierarchical combination of several logic modules. A logic module, as shown in Fig. 2, includes a gate unit, m inputs (short for input events), and 1 output (shorts for output event). For each logic module, once inputs state transition is given,

Table 1. Differences between two types of repairable PMS

	Items	Type I	Type II
Multi-phases mission requirements	Phase durations	Determined values, i.e., T_1, T_2, \dots, T_n	Random variables, i.e., t_1, t_2, \dots, t_n
	Time of system in up state	Determined values, i.e., $T_1^{up} = T_1, T_2^{up} = T_2, \dots, T_n^{up} = T_n$	Determined values, i.e., $T_1^{up}, T_2^{up}, \dots, T_n^{up}$
	Maximum mission time	Determined values, i.e. $T_{max} = T_1 + T_2 + \dots + T_n$	Determined values, i.e. $T_{max} > T_1^{up} + T_2^{up} + \dots + T_n^{up}$
Maintenance strategies	Failed components repairable	Only repaired when the system is up	Unconditional repaired immediately
	The extent of repair	As good as a new one	As good as a new one
	System state bidirectional transition	No, down→up is not allowed	Yes, down→up is allowed unless the limited mission time is reached

according to operating rules of the gate, the output state transition can be determined. If the output is not the top event of the whole tree, it is also an input belonging to a logic module of the higher hierarchical level. Thus, as long as operating rules of various gates are established in a standard unified form, the modular reliability model of SEFT can be obtained by means of substitution layer by layer.

Liu proposed the linear algebra representation (LAR) approach in literature [8]. According to LAR, each state of a certain event is denoted by a *state unit vector*, and then the transition between any two states can be expressed as a matrix multiplication. Besides, how to express the operating rules of logic gates in a standard unified form is also introduced in literature [8], including 3 static gates (OR, AND, VOTING gates) and 3 dynamic gates (PAND, SPARE, FDEP gates). Then, we wonder if the operating rule of SEQ-OR gate can be expressed in the unified form, by directly applying the LAR approach.

Compared to other logic gates, the SEQ-OR gate has a very special feature. As shown in Fig. 1, an SEQ-OR gate connects more than one phase (as inputs), and these phases are carried out one by one. In other words, at any time during the system mission period, only 1 phase event is active, as well as its subtree. Once a phase is accomplished, it should be non-activated, and the next phase will be activated unless the whole system mission is fulfilled. However, the existing LAR approach supposes that all events included in gate operating are active. It can neither be used to distinguish whether the event is active or not; nor to support expressions of activated/non-activated action.

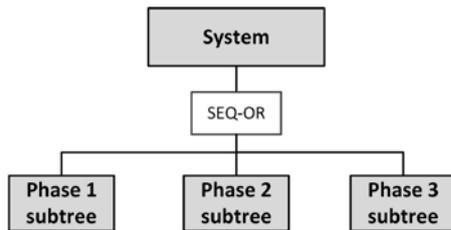


Fig. 1. Basic structure of SEFT

Thus, an improved linear algebra representation (I-LAR) approach is proposed as follows:

- For any event with 2 states, i.e., up and down, the *ordered index vector* is written as:

$$\alpha = (1, 2) \tag{1}$$

where each component of the row vector is called a *state number*[8]. Specifically, state1 and 2 denote up and down, respectively.

- The *State unit vector* S_i s used to denote state i [8]. Here, S_i is a 2-dimensional unit column vector with “1” in the i th element and “0” in the other. In detail, corresponding to state1, i.e., up $S_1 = \langle 1, 0 \rangle$ corresponding to state2, i.e., down, $S_2 = \langle 0, 1 \rangle$. Then, the *state set* of a certain event is denoted as:

$$V = \{S_1, S_2\} \tag{2}$$

- In accordance with the statement in [8], the *state transition matrix* T_{pq} is used to express the instantaneous state transition by matrix multiplication. In detail, the transition from state p to state q can be represented by:

$$T_{pq} \cdot S_p = S_q \tag{3}$$

where S_p and S_q are *state unit vectors* associated with state p and q , respectively; T_{pq} is an elementary switching matrix that transformed from the identity matrix by exchanging the p th and q th row vectors, and the dimensions of T_{pq} , S_p , S_q are the same.

In this paper, since p and q is either 1 or 2, bidirectional state transitions are specified as follows. Considering that a certain event transits from up to down at time t_i , *state transition* $state(t_i^-) \rightarrow state(t_i^+)$ can be represented by:

$$T_{12} \cdot S_1 = S_2 \tag{4}$$

where S_1 and S_2 are *state unit vectors* associated to state up and down, respectively T_{12} ; the *state transition matrix*, given by:

$$T_{12} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{5}$$

Further, regarding the state transition from down to up, it can be represented by $T_{21} \cdot S_2 = S_1$. Obviously, $T_{21} = T_{12}$, which is transformed from the 2-by-2 identity matrix by exchanging the 1st and 2nd row vectors.

- At any time during the system mission period, events included in SEFT may be either *inactive* or *active*.

① Definition 1. (Inactive event)

Inactive event is the event, whose state is impossible to transit.

② Definition 2. (Active event)

Active event is the event, whose state has the possibility to transit.

According to the definitions above, the top event is always an *active event* during the mission process. During the 1st phase, all events belonging to the phase 1 subtree are *active events*, and events belonging to other phase subtrees are *inactive events*. During other phases, things can be deduced in the same manner.

- For each event, it is assigned an *event vector* \hat{H}

③ Definition 3. (Event vector)

Event vector \hat{H} is a 3-dimensional column vector, can be written as:

$$\hat{H} = \begin{pmatrix} \delta \\ H \end{pmatrix} \tag{6}$$

where H is a *state unit vector*, i.e. $H \in V$; δ is the index to distinguish whether the event is active or not, in specifically:

$$\delta = \begin{cases} 0 & \text{inactive} \\ 1 & \text{active} \end{cases} \tag{7}$$

- Thus, once an *inactive event* is activated, it can be expressed as:

$$\hat{H}|_{\delta=0} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \hat{H}|_{\delta=1} \tag{8}$$

where \mathbf{o} s a 2-dimensional column vector with all 0 elements.

- On the contrary, once an *active event* is non-activated, it can be expressed as:

$$\hat{H}|_{\delta=1} - \begin{pmatrix} 1 \\ \mathbf{o} \end{pmatrix} = \hat{H}|_{\delta=0} \quad (9)$$

- According to the LAR approach proposed in literature [8], *state matrix* and *state number vector* are two concepts corresponding to the combination of m events. Among the m events, since *inactive events* and *active events* might co-exist, it is necessary to give new definitions.

④ Definition 4. (*State matrix*)

State matrix X is a matrix corresponding to m events. Only the *state unit vectors* of those *active events* will be selected and sequentially combined into the *state matrix X*.

In detail, it can be obtained as follows:

a) *Obtaining \mathbf{x} ($j=1,2,\dots,m$)*

\mathbf{x}_j is a 2-dimensional column vector, related to event j . It can be determined by the following equation:

$$\mathbf{x}_j = \Delta \cdot \hat{H}_j = \begin{pmatrix} 0 & \delta_j & 0 \\ 0 & 0 & \delta_j \end{pmatrix} \cdot \hat{H}_j \quad (10)$$

where \hat{H}_j is the event vector of event j , and Δ s a 2-by-3 matrix, which mainly depends on the index δ_j

Obviously, Eq. (10) can be simplified as:

$$\mathbf{x}_j = \begin{cases} \mathbf{o} & \delta_j = 0 \\ \mathbf{H}_j & \delta_j = 1 \end{cases} \quad (11)$$

where \mathbf{H}_j is the *state unit vector* of event j , and \mathbf{o} s a 2-dimensional column zero vector.

b) *Obtaining X:*

As long as \mathbf{x}_j ($j=1,2,\dots,m$)s not a zero vector, it will be selected in order as a column of \mathbf{X} .

Thus, the number of columns in the *state matrix X* may be less than m .

⑤ Definition 5. (*State number vector*)

Corresponding to *state matrix X*, *state number vector XX* as a row vector is defined to denote the ordered collection of those *active events' state numbers*. It can be obtained by:

$$\mathbf{XX} = \alpha \cdot \mathbf{X} \quad (12)$$

3.3. Modular modeling of SEFT

For a logic module, as shown in Fig. 2, the state of output will not change unless one input has a state transition. Based on I-LAR approach introduced above, the operation process of a logic module can be described as follows:

- Given the following conditions:

- a) At time t_i^- , the input *state matrix* is represented as $\mathbf{X}(t_i^-)$ and the corresponding *state number vector* is expressed as $\mathbf{XX}(t_i^-)$

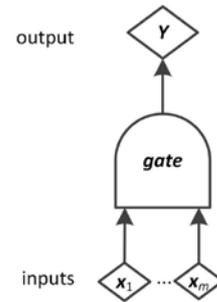


Fig. 2. Logic module of SEFT

- b) At time t_i^+ , the input *state matrix* is represented as $\mathbf{X}(t_i^+)$, and the corresponding *state number vector* is expressed as $\mathbf{XX}(t_i^+)$.
- c) At time t_i^- , the output state is represented as $\mathbf{Y}(t_i^-)$, and the corresponding *state number* is expressed as p . In other words, the output state is $\mathbf{Y}(t_i^-) = \mathcal{S}_p$
- To determine the output state at time. t_i^+ , represented as $\mathbf{Y}(t_i^+)$ it can be calculated by:

$$\mathbf{Y}(t_i^+) = (\mathcal{T}_{pq})^k \mathbf{Y}(t_i^-) \quad (13)$$

where q is the output *state number* at time t_i^+ , and $k \in \{0,1\}$; the value of k is used to reveal whether the output state transition occurs or not, in detail, $k=1$ indicates the transition is triggered, whereas $k=0$ indicates that no state transition of the output happens instantly.

Compared to the statement in literature [8], the revised operation process has no difference but only those active input events are involved, owing to new definitions of *state matrix* and *state number vector*.

Obviously, variables k and q in Eq.(13) change as the gate unit in the logic module changes. The calculation of these two variables is determined by the operation rules of each gate.

- *OR gate:* Considering m inputs, as long as one active input is in the down state, the output is determined as down. In other words, the output state is the same as the worst active input state. Thus, the variable q is represented as:

$$q = \left\| \mathbf{XX}(t_i^+) \right\|_{\infty} \quad (14)$$

where $\| \cdot \|_{\infty}$ refers to the infinity norm of a certain vector.

As for the variable k , since OR gate is a kind of static gate, the output state is only related to the combination of active inputs states at time t_i^+ , it can be determined that $k \equiv 1$

- *AND gate:* Considering m inputs, if and only if all active inputs are in the down state, the output is determined as down. In other words, the output state is the same as the best active input state. Thus, the variable q is represented as:

$$q = \left\| \mathbf{XX}(t_i^+) \right\|_{-\infty} \quad (15)$$

where $\| \cdot \|_{-\infty}$ refers to the negative infinity norm of a certain vector.

Since AND gate is a kind of static gate, it can be determined that $k \equiv 1$.

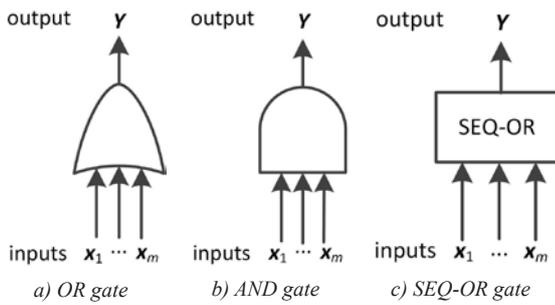


Fig. 3. Typical logical gate

• SEQ-OR gate: Considering m inputs, they have to be activated one by one from left to right; obviously, there is only 1 active input at any time; once the active input x_j is in down state, the output is determined as down. In other words, the output state is the same as the current active input state.

Therefore, at time t_i^+ , once the state of active input x_j transits, variables k and q are, respectively, represented as:

$$k = \min \left(\left\lfloor \frac{xx_1(t_i^+)}{2} \right\rfloor, 1 \right) \quad (16)$$

$$q = xx_1(t_i^+) \quad (17)$$

where $\lfloor \cdot \rfloor$ refers to the typical floor function.

In summary, by means of the proposed I-LAR method, operation rules of different logic gates can be presented in unified forms of expression. And then, constructing a modular SEFT model for PMS reliability analysis is feasible by substitution of logic module layer by layer.

3.4. MC simulation based on the modular reliability model

Once the modular reliability model related to SEFT is obtained based on the statement above, MC simulation containing M trials is adopted to evaluate the reliability of repairable PMS. The basic flow chart is shown in Fig. 4, corresponding to a 3-PMS example. During the r th simulation trial, the procedure used to distinguish whether the current phase is successful or not may be repeated up to three times. The trial will not switch to the next phase until the current phase is completed, and the mission success is determined when the final phase has been fulfilled. Furthermore, take phase1 for example, the

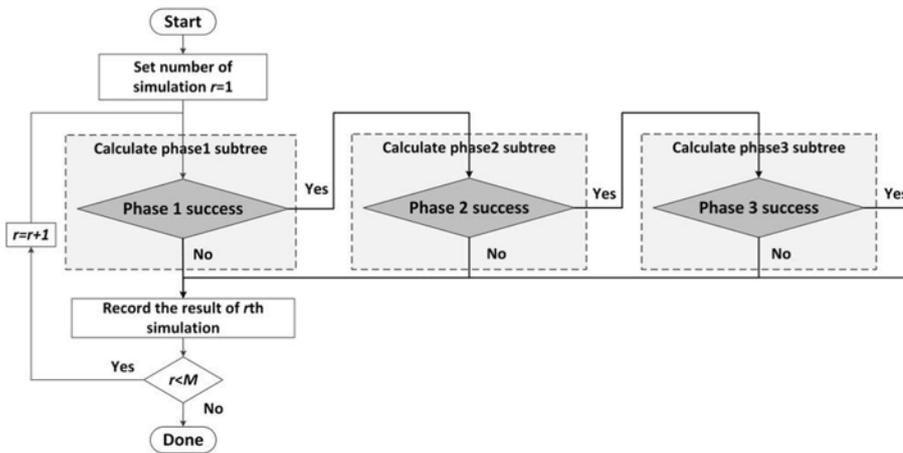
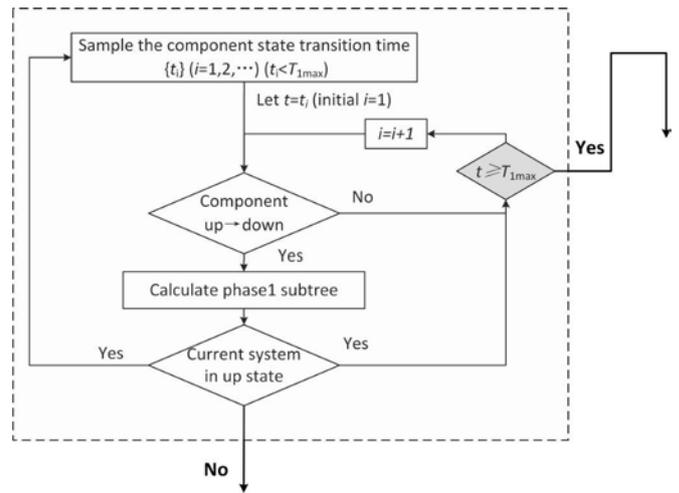
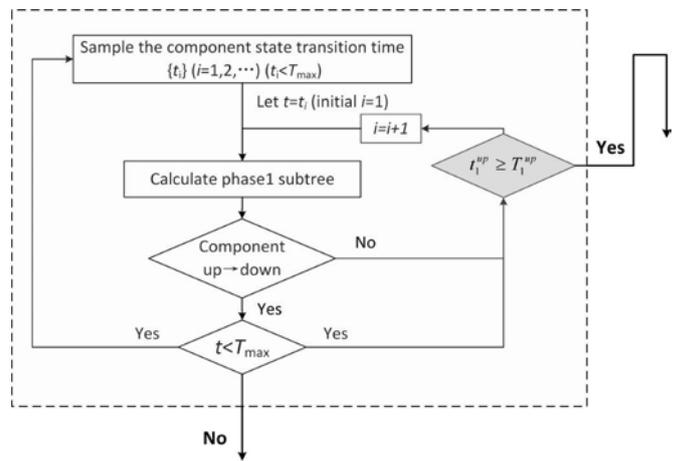


Fig. 4. Basic flow chart of MC simulation based on modular SEFT



(a) Type I



(b) Type II

Fig. 5. Detailed distinguishing process for the current phase

detailed distinguishing process for repairable PMS of Type I and Type II is different, as shown in Fig.5.

(a) Type I

In the r th simulation trial, phase1 starts at the initial time $t=0$. Phase1 success is determined iff the trail time reaches the given phase

duration T_{1max} . It is regarded that the system keeps on operating until a component state transition has reached. Since the component state transition might be bidirectional, two situations need to be discussed separately:

- Once one component state transits from up to down, the modular subtree of the current phase is calculated. As long as the system transits to the down state, the r th simulation trial ends. Otherwise, the repair time for a certain component is sampled according to its given distribution function; furthermore, the next failure time after its restoration is also sampled. Then, the trial moves to the next state transition time unless the trail time has come to the given phase duration T_{1max} .

- Once one component state transits from down to up, obviously, the system will not change the status of normal operating. Thus, as long as the trail time is still shorter than the determined phase duration T_{1max} , the trial will move to the next state transition time.

(b) Type II

In the r th simulation trial, phase1 also starts at initial time $t=0$. Phase1 success is determined iff the time of system in up state reaches the given value T_1^{up} . Different from Type I, the state of phase1 event has to be calculated once a component state transition time has reached. Then, regarding the two situations:

- Once the state transition is from down to up, as long as the system's operating time is still shorter than the given T_1^{up} , the trial in the current phase has to be continued.
- Once the state transition is from up to down, if the trail time has come to the maximum mission time T_{max} , the r th simulation trial ends. Otherwise, the repair time for a certain component is sampled according to its given distribution function; furthermore, the next failure time after its restoration is also sampled. Then, the trial moves to the next state transition time unless the system's operating time in the current phase has come to the given T_1^{up} .

4. Numerical example

In this section, the application of the proposed SEFT-MC method is illustrated under two different cases corresponding to the two types of repairable PMS mentioned above. Furthermore, comparisons of the two cases are also discussed afterward.

The system structure and failure criteria of both cases are identical, which is based on the example presented in the literature [2]. The system consists of 4 components, as shown in Fig.6; all the 4 components participate in phase1, whereas component D and B is not involved in phase 2 and 3, respectively. Each component has two states, i.e., up and down. The bidirectional state transition time of all components is exponentially distributed; the transition rates are shown in Table 2, and there is no difference in each phase. Here, failed components will be restored as good as a new one once it is repaired. Different mission requirements of the two cases are listed in Table 3.

Obviously, the SEFTs corresponding to two cases are identical, as shown in Fig. 7. The SEQ-OR gate is used as the specified first-level connection; besides, OR gates and AND gates are adopted to construct the phase subtrees. According to the I-LAR approach, the state

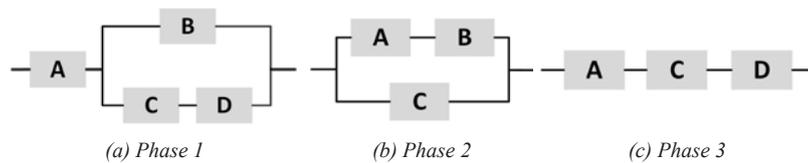


Fig. 6. The structure for each phase of the discussed PMS

Table 2. State transition rates of components

State transition	Symbol	Component			
		A	B	C	D
Up→down	λ	0.1	0.2	0.3	0.4
Down→up	μ	0.2	0.3	0.4	0.5

Table 3. Differences of mission requirements between two cases

Items	Case I	Case II
Phase durations	$T_1=1\text{days}, T_2=1\text{days}, T_3=2\text{days}$	Random variables, represented as t_1, t_2, t_3
Time of system in up state	$T_1^{up}=1\text{days}, T_2^{up}=1\text{days}, T_3^{up}=2\text{days}$	$T_1^{up}=1\text{days}, T_2^{up}=1\text{days}, T_3^{up}=2\text{days}$
Maximum mission time	$T_{max}=4\text{days}$	$T_{max}=6/10/14\text{days}$

set of each event is $V = \{S_1, S_2\}$, where the state unit vectors are $S_1 = \langle 1, 0 \rangle$ and $S_2 = \langle 0, 1 \rangle$. Then, state transitions of up→down and down→up are represented by $T_{12} \cdot S_1 = S_2$ and $T_{21} \cdot S_2 = S_1$, respectively, where $T_{12} = T_{21} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

At the beginning of the mission, the initial state of all events is set as up. Besides, all events included in phase1 subtree are activated, as well as the top event. Thus, according to Eq.(6)-Eq.(7), the event vector of above events are $\hat{H} = \langle 1 \ 1 \ 0 \rangle$, others are $\hat{H} = \langle 0 \ 1 \ 0 \rangle$.

Once phase1 is successfully completed, the mission switches to phase2 instantly. Phase1 event is non-activated by subtraction according to Eq.(8), represented as $\hat{H} = \langle 0 \ 1 \ 0 \rangle = \langle 1 \ 1 \ 0 \rangle - \langle 1 \ 0 \ 0 \rangle = \langle 0 \ 1 \ 0 \rangle$; whereas phase2 event is activated by addition according to Eq.(9), represented as $\hat{H} = \langle 0 \ 1 \ 0 \rangle + \langle 1 \ 0 \ 0 \rangle = \langle 1 \ 1 \ 0 \rangle$. As for component D, since it is not involved in phase2, during this phase, the corresponding event vector is set by subtraction, whereas the state will remain. Similar operations are also applied to phase3.

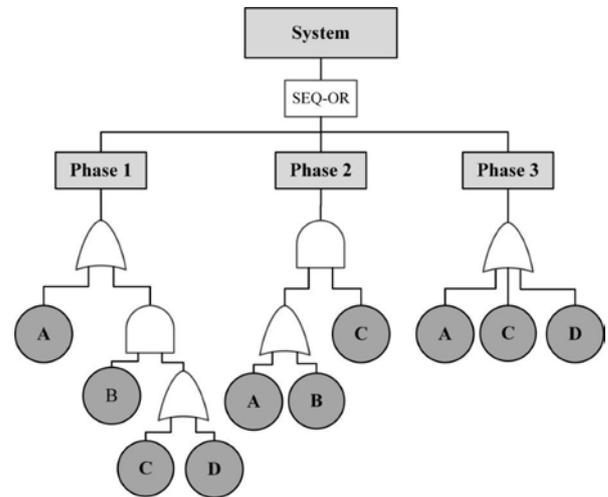


Fig. 7. SEFT of the 3-PMS example

Then, according to the modular modeling introduced in section 3.3, once a state transition of any basic event occurs, the state of top event and intermediate event can be easily obtained through a series of matrix operations.

Further, MC simulation with M trials is carried out, in which the basic flow chart is shown in Fig. 4. As for one simulation trial, it switches to phase j ($j=2, 3$) iff the phase ($j-1$) has successfully completed, and the whole mission success is determined followed by the completion of phase 3. Corresponding to the case I and case II, the process to determine whether the current phase is succeeded or not is implemented by the flow chart, as shown in Fig. 5(a) and Fig. 5 (b), respectively. For case I, the criteria for success is that the trail time has come to the given phase duration; whereas the simulation trail will be interrupted, i.e., mission failed, once the system state transits from up to down. For case II,

the criteria for success is that the time of system in up state reaches the given value; whereas the simulation trail will be interrupted once the trail time has come to the maximum mission time T_{max} .

As for MC simulation, high accuracy and short computation time are contradict each other,

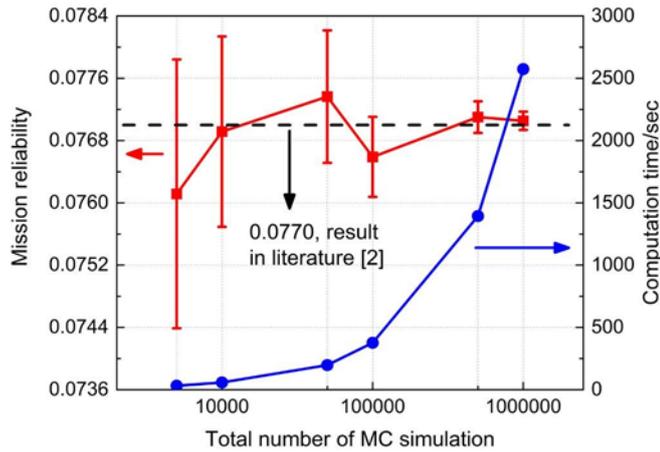


Fig. 8. Effect of total number of MC simulation on accuracy and computation time

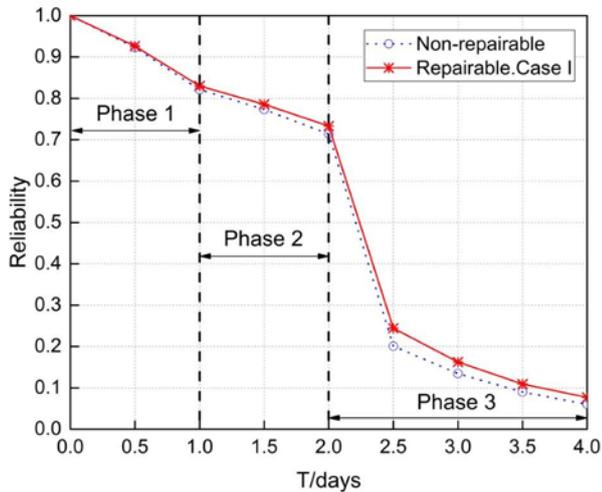


Fig. 9. Comparison of dynamic change in the reliability of the 3-PMS example

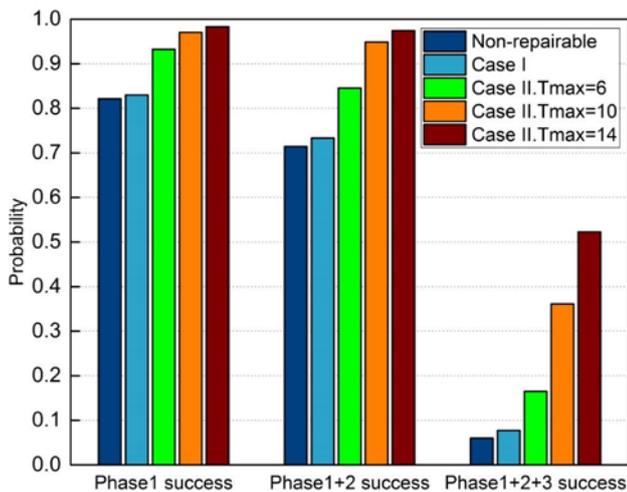


Fig. 10. Comparison of the success probability of the whole mission and each phase included

and they have different requirements for the total number M . The evaluation of mission reliability and computation time with increasing M are addressed using the proposed SEFT-MC method, as shown in Fig. 8. For a certain value of M , 10 repeated simulations are conducted and the corresponding results (including mean value and root mean squared error (RMSE) of evaluation, and average computation time)

are given. With increasing M , the resultant values of mission reliability gradually tend to 0.077, which is consistent with the results in the study [2]. Meanwhile, the reducing RMSEs indicate improving convergence of results. When the value of M reaches 5×10^5 , the reliability calculated by the SEFT-MC method is 0.077101 with the RMSE of 2.0276×10^{-4} , which is acceptable in this study.

Fig. 9 shows the dynamic changes in the reliability of repairable PMS discussed in case I. In order to discuss the effect of reliability improvement, the non-repairable PMS that has the same system structure and failure criteria is also considered according to SEFT-MC method. It is easy to find that maintenance strategy in case I can just slightly improve the system reliability, since only the component in the redundant structure may be repaired.

Due to the uncertainty of phase duration in repairable PMS discussed in case II, it is more meaningful to investigate the probability of success for each phase. As shown in Fig. 10, the system reliability in case II has significantly improved according to result comparison. Furthermore, the greater the maximum mission time is, the higher the probability to complete the whole mission and each phase included. Herein, in order to make sure the probability of mission success is higher than 50%, the maximum mission time should be set as 14 days, which is 3.5 times the required time of system in up state. That is to say, the reliability improvement is at the expense of increased mission time.

5. Conclusions and future work

Repairable PMSs abound in real-world applications. Due to the diversity of mission requirements and maintenance strategies, the analysis of repairable PMSs is much more complicated than that of non-repairable PMSs. In this paper, a novel SEFT-MC method is developed to evaluate the reliability of repairable PMS considering two types: to execute a multi-phases mission with deterministic phase durations, and within which failed components could be repaired only when the system is up; to execute a multi-phases mission with uncertain phase durations but limited by a maximum mission time, and within which failed components could be unconditional repaired immediately. The major characteristics of the proposed method are: the specific SEFT, whose core is the SEQ-OR gate, could be applied to a variety of PMS; the modular reliability modeling could make up for modeling inability of MC simulation itself; the manner to construct the modular reliability model has universal applicability due to the proposed I-LAR approach; the I-LAR approach allows the achievement that operational rules of various gates are expressed in standard form, and moreover, inputs included in the gate operating can be either active or inactive events. Furthermore, by means of a numerical example including two cases corresponding to the two types of repairable PMS, the application of the proposed method is demonstrated; in addition, the comparisons of two cases display that the significant improvement in reliability is at the expense of increasing mission time. This result could be useful for decision-makers on the optimal choice of maintenance strategies according to a comprehensive trade-off between reliability improvement and time cost. Consequently, a detailed study of such optimization problems will be conducted in our future work. Furthermore, how to improve the calculating efficiency by introducing some improved MC simulation methods will also be studied.

Considering the degradation of the system/components in PMS, the multi-state behavior will be introduced in analyzing PMS. In other words, the degradation process can be described in terms of transitions among multi-states (from perfectly working to totally failure). Therefore, the reliability assessment of non-repairable/repairable multi-state PMS is another direction of our future work.

Acknowledgment

This research is carried out with the Institute for Machine Elements and Systems Engineering (MSE) at RWTH Aachen University, which is led by Univ.-Prof. Dr.-Ing. Georg Jacobs; it is supported by the Jiangsu Government Scholarship for Overseas Studies Program and the Jiangsu Province "Double-Creation Project" Program. The authors are grateful for the reviewers' helpful comments.

References

1. Dugan JB, Bavuso SJ, Boyd MA. Dynamic Fault-Tree Models for Fault-Tolerant Computer Systems. *IEEE Transactions on Reliability* 1992; 41(3):363-377, <https://doi.org/10.1109/24.159800>.
2. Kim K, Park KS. Phased-Mission System Reliability under Markov Environment. *IEEE Transactions on Reliability* 1994; 43(2):301-309, <https://doi.org/10.1109/24.295013>.
3. Levitin G, Xing L, Amari, SV. Recursive algorithm for reliability evaluation of non-repairable phased mission systems with binary elements. *IEEE Transactions on Reliability* 2012; 61(2):533-542, <https://doi.org/10.1109/TR.2012.2192060>.
4. Li XT, Tao LM, Jia M. A Bayesian networks approach for event tree time-dependency analysis on phased-mission system. *Eksploatacja i Niezawodność - Maintenance and Reliability* 2015; 17(2):273-281, <https://doi.org/10.17531/ein.2015.2.15>.
5. Li XY, Huang HZ, Li YF. Reliability analysis of phased mission system with non-exponential and partially repairable components. *Reliability Engineering and System Safety* 2018; 175:119-127, <https://doi.org/10.1016/j.res.2018.03.008>.
6. Li XY, Li YF, Huang HZ, Zio E. Reliability assessment of phased-mission systems under random shocks. *Reliability Engineering and System Safety* 2018; 180:352-361, <https://doi.org/10.1016/j.res.2018.08.002>.
7. Li YF, Peng R. Availability modeling and optimization of dynamic multi-state series-parallel systems with random reconfiguration. *Reliability Engineering & System Safety* 2014; 127:47-57, <https://doi.org/10.1016/j.res.2014.03.005>.
8. Liu C, Chen N, Yang J. New method for multi-state system reliability analysis based on linear algebraic representation. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability* 2015; 229(5):469-482, <https://doi.org/10.1177/1748006X15582129>.
9. Lu JM, Lundteigen MA, Liu Y, Wu XY. Flexible truncation method for the reliability assessment of phased mission systems with repairable components. *Eksploatacja i Niezawodność - Maintenance and Reliability* 2016; 18(2):229-236, <https://doi.org/10.17531/ein.2016.2.10>.
10. Lu JM, Wu XY. Reliability evaluation of generalized phased-mission systems with repairable components. *Reliability Engineering & System Safety* 2014; 121:136-145, <https://doi.org/10.1016/j.res.2013.08.005>.
11. Lu JM, Wu XY, Liu Y, Lundteigen MA. Reliability analysis of large phased-mission systems with repairable components based on success-state sampling. *Reliability Engineering and System Safety* 2015; 142:123-133, <https://doi.org/10.1016/j.res.2015.05.010>.
12. Merle G, Roussel JM, Lesage JJ, Perchet V, Vayatis N. Quantitative Analysis of Dynamic Fault Trees Based on the Coupling of Structure Functions and Monte Carlo Simulation. *Quality and Reliability Engineering International* 2016; 32(1):7-18, <https://doi.org/10.1002/qre.1728>.
13. Mo YC, Xing LD, Amari SV. A multiple-valued decision diagram based method for efficient reliability analysis of non-repairable phased-mission systems. *IEEE Transactions on Reliability* 2014; 63(1):320-330, <https://doi.org/10.1109/TR.2014.2299497>.
14. Mo YC, Zhong F, Zhao X. New results to BDD truncation method for efficient top event probability calculation. *Nuclear Engineering and Technology* 2012; 44: 755-766, <https://doi.org/10.5516/NET.03.2011.058>.
15. Murphy KE, Carter CM, Malerich AW. Reliability analysis of phased-mission systems: A correct approach. 2007 Proceedings - Annual Reliability and Maintainability Symposium, RAMS 2007:7-12, <https://doi.org/10.1109/RAMS.2007.328042>.
16. Peng R, Zhai Q, Xing L, Yang J. Reliability of demand-based phased-mission systems subject to fault level coverage. *Reliability Engineering and System Safety* 2014; 121:18-25, <https://doi.org/10.1016/j.res.2013.07.013>.
17. Tang Z, Dugan JB. BDD-based reliability analysis of phased-mission systems with multimode failures. *IEEE Transactions on Reliability* 2006; 55(2):350-360, <https://doi.org/10.1109/TR.2006.874941>.
18. Wang C, Xing L, Peng R, Pan Z. Competing failure analysis in phased-mission systems with multiple functional dependence groups. *Reliability Engineering and System Safety* 2017; 164:24-33, <https://doi.org/10.1016/j.res.2017.02.006>.
19. Wu D, Peng R, Xing L. Recent Advances on Reliability of Phased Mission Systems. In: Li QL., Wang J., Yu HB. (eds) *Stochastic Models in Reliability, Network Security and System Safety. JHC80 2019. Communications in Computer and Information Science*, vol 1102. Springer, Singapore, https://doi.org/10.1007/978-981-15-0864-6_2.
20. Wu X, Wu X. Mission reliability modeling and evaluation of multi-mission phased mission system based on an extended object-oriented Petri net. *Eksploatacja i Niezawodność - Maintenance and Reliability* 2017; 19 (2): 244-253, <https://doi.org/10.17531/ein.2017.2.12>.
21. Wu X, Wu Y. Extended object-oriented petri net model for mission reliability simulation of repairable PMS with common cause failures. *Reliability Engineering & System Safety* 2015; 136:109-119, <https://doi.org/10.1016/j.res.2014.11.012>.
22. Xing L. Reliability evaluation of phased-mission systems with imperfect fault coverage and common-cause failures. *IEEE Transactions on Reliability* 2007; 56(1):58-68, <https://doi.org/10.1109/TR.2006.890900>.
23. Yang J, Liu F, Li F. Reliability Modeling and Simulation for Phased Mission System with Multi-mode Failures. *Fire Control and Command Control* 2011; 36(2):89-92.
24. Zhai Q, Xing L, Peng R, Yang J. Aggregated combinatorial reliability model for non-repairable parallel phased-mission systems. *Reliability Engineering and System Safety* 2018; 176:242-250, <https://doi.org/10.1016/j.res.2018.04.017>.
25. Zhang S, Sun S, Si S, Wang P. A decision diagram based reliability evaluation method for multiple phased-mission systems. *Eksploatacja i niezawodność - Maintenance and Reliability* 2017; 19 (3):485-492, <https://doi.org/10.17531/ein.2017.3.20>.
26. Zhang T, Bai G, Guo B. Success probability model of phased mission systems with limited spares. *Eksploatacja i Niezawodność - Maintenance and Reliability* 2012; 14(1):24-32.

27. Zhao J, Cai ZQ, Si W, Zhang S. Mission success evaluation of repairable phased-mission systems with spare parts. *Computers and Industrial Engineering* 2019; 132:248-259, <https://doi.org/10.1016/j.cie.2019.04.038>.
28. Zio E, Librizzi M. Direct Monte Carlo Simulation for the Reliability Assessment of a Space Propulsion System Phased Mission (PSAM-0067). *Proceedings of the Eighth International Conference on Probabilistic Safety Assessment & Management (PSAM) 2006*.

Chenxi LIU

School of Mechanical Engineering
Jiangsu University
Xuefu Road 301, 212013 Zhenjiang, China

Achim KRAMER

Stephan NEUMANN

Institute for Machine Elements and Systems Engineering
RWTH Aachen University
Schinkelstraße 10, 52062 Aachen, Germany

E-mails: liu_chenxi@ujs.edu.cn, achim.kramer@imse.rwth-aachen.de,
stephan.neumann@imse.rwth-aachen.de
