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APPLICATION OF THE LOGISTIC REGRESSION FOR DETERMINING TRANSITION PROBABILITY MATRIX OF OPERATING STATES IN THE TRANSPORT SYSTEMS

ZASTOSOWANIE REGRESJI LOGISTYCZNEJ DO WYZNACZANIA MACIERZY PRAWDOPODOBIEŃSTW PRZEJŚĆ STANÓW EKSPLOATACYJNYCH W SYSTEMACH TRANSPORTOWYCH*

Transport companies can be regarded as a technical, organizational, economic and legal transport system. Maintaining the quality and continuity of the implementation of transport requisitions requires a high level of readiness of vehicles and staff (especially drivers). Managing and controlling the tasks being implemented is supported by mathematical models enabling to assess and determine the strategy regarding the actions undertaken. The support for managing processes relies mainly on the analysis of sequences of the subsequent activities (states). In many cases, this sequence of activities is modelled using stochastic processes that satisfy Markov property. Their classic application is only possible if the conditional probability distributions of future states are determined solely by the current operational state. The identification of such a stochastic process relies mainly on determining the probability matrix of interstate transitions. Unfortunately, in many cases the analyzed series of activities do not satisfy Markov property. In addition, the occurrence of the next state is affected by the length of time the system remains in the specified operating state. The article presents the method of constructing the matrix of probabilities of transitions between operational states. The values of this matrix depend on the time the object remains in the given state. The aim of the article was to present an alternative method of estimating the parameters of this matrix in a situation where the studied series does not satisfy Markov property. The logistic regression was used for this purpose.

Keywords: *logistic regression, transition probability matrix, Markov chains, transport system.*

Przedsiębiorstwa transportowe mogą być traktowane jako wyodrębniony pod względem technicznym, organizacyjnym, ekonomicznym i prawnym system transportowy. Zachowanie jakości i ciągłości realizacji zleceń przewozowych wymaga wysokiego poziomu gotowości pojazdów oraz personelu (szczególnie kierowców). Kontrolowanie i sterowanie realizowanymi zadaniami wspierane jest modelami matematycznymi, umożliwiającymi ocenę i określenie strategii dotyczącej podejmowanych działań. Wsparcie procesów zarządzania polega głównie na analizie sekwencji kolejnych, realizowanych czynności (stanów). W wielu przypadkach taki ciąg czynności jest modelowany za pomocą procesów stochastycznych, spełniających własność Markowa. Ich klasyczne zastosowanie możliwe jest tylko w przypadku, gdy warunkowe rozkłady prawdopodobieństwa przyszłych stanów są określone wyłącznie przez bieżący stan eksploatacyjny. Identyfikacja takiego procesu stochastycznego polega głównie na wyznaczeniu macierzy prawdopodobieństw przejść międzystanowych. Niestety w wielu przypadkach analizowane ciągi czynności nie spełniają własności Markowa. Dodatkowo, na wystąpienie kolejnego stanu wpływa długość interwału czasowego pozostania systemu w określonym stanie eksploatacyjnym. W artykule przedstawiono metodę konstrukcji macierzy prawdopodobieństw przejść pomiędzy stanami eksploatacyjnymi. Wartości tej macierzy zależą od czasu przebywania obiektu w danym stanie. Celem artykułu było zaprezentowanie alternatywnej metody estymacji parametrów tej macierzy w sytuacji, gdy badany szereg nie spełnia własności Markowa. Wykorzystano w tym celu regresję logistyczną.

Słowa kluczowe: *regresja logistyczna, macierz prawdopodobieństw przejść, łańcuchy Markowa, system transportowy.*

1. Introduction

The concept of a transport system, as defined by Grzywacz and Burnewicz [17] as well as Andrzejczak [3] is seen in this article as a segregated system of three subsystems: technical, organizational and economic-legal ones, creating a logical, internally balanced entirety, enabling to achieve a specific goal. This makes it possible to define the analyzed enterprise as the transport system, and assume the implementation of transport tasks as its operating goal.

Transport systems can be analyzed as multi-state sequences of subsequent planned and unplanned maintenance activities carried out by the transport system operator [27]. The construction of the models that describe them and allows the prediction of the operating state of the object used, allows planning of the maintenance strategy and control of the readiness of the machine [9, 21] and vehicles fleet [13, 26] etc. Modelling the functioning of technical objects using deterministic models is not always possible because the results (implementations) are affected by external disturbances (random factors), which make it

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** impossible to accurately predict subsequent states. In such cases, we model the behaviour of technical systems using probabilistic methods, in particular stochastic processes. An important class of stochastic processes are Markov processes. Some possibilities of applications of these processes are presented in papers [7, 24]. The essential condition of their use is satisfying Markov property: the future does not depend on the past when the present is known. Many analyzes assume a priori that the time series satisfies this property and its verification is omitted [55]. Only a few authors indicate the need for checking it [53] and eliminate cases that did not achieve this assumption [47]. An alternative for the systems that do not satisfy Markov property are classic reliability methods, allowing to determine empirical characteristics such as: renewal stream, renewal function, time to the next defect or intensity of the renewal stream [6, 14, 35] and calculating the main system assessment measures based on them [15, 23, 42]. In the literature, as part of similar studies, various models are presented [30, 43], including semi-Markov [7, 24] and also those using artificial neural networks [10, 33], factorization algorithm [31], fault trees [52] or reliability models [36, 42].

In many publications, the time of remaining in a particular operating state is not taken into account. The heterogeneity of the time interval between successive states may also cause non-fulfilment of Markov property. In this article, the logistic regression was used to estimate the conditional probabilities of the test object remaining in the individual operational states [49]. The logistic regression describes the relationship between a qualitative variable and one or more predictive variables [25, 46]. In the literature, logistic regression is used in medicine [5, 44], in computed tomography [46], to identify technical systems [25], in the area of corporate finance [39, 54], banking [1, 34] broadly understood investments [12, 29] and is used to assess the level of risk [2, 8, 45], in the social and demographic research [4, 41] and others [25, 40]. Regarding transport systems, logistic regression models are proposed primarily for assessing the road dangers demonstrated by the road accidents [18, 37], making the choices of routs in the transport network [32, 49] or analyzing the impact of selected factors on the implementation of transport processes [38, 48].

The paper shows the existence of a relationship between the duration of the operational state and the value of the probability of transition to the next state. In order to analyze the problem thoroughly and in detail, the introduction was firstly made, presenting the mathematical methods referred to in the article. The second chapter presents definitions regarding Markov chains and how to verify Markov property. The third chapter presents the method of estimating the transition probability matrix using logistic regression. Then, an example of the implementation of the proposed method using empirical data for a selected means of transport, carrying out transport tasks under the transport system (enterprise), has been presented. In the final stage, the results obtained are discussed, summaries of the analyzes carried out and directions for further research are indicated.

2. Markov chains

The state of an object is defined by its characteristic feature, a technical property that assigns it to a given operating system [50]. It is a vector whose components are physical values describing the object from the point of view of a given test [28]. In the literature, the state of a technical object is defined as the result of one and only one event in a series of experiments from a finite or countable set of pairs of mutually exclusive events [11, 54]. We use probability calculus tools and mathematical statistics to analyze technical systems [22, 53]. Let $(\Omega, \mathcal{F}, P)$ be a probabilistic space, N - a set of natural numbers, S - the space of the states of the analyzed phenomenon.

Definition 1 A sequence $\{X_t\}_{t\in N}$ of random variables $X_t : \Omega \to S$ for any $t \in N$ is called a stochastic process in discrete time [51, 54].

In the paper, we analyze the operating states in which the vehicles remain. The S set of operating states is a set of values of the stochastic process $\{X_t\}_{t\in N}$. At any $t \in N$ time, the object is in one of the possible states and $X_t(\omega) = x_t$, i.e. in the event of a ω random event occurring at the t moment, the system is in a state $x_t \in S$. In our research, we assume that the S set of states is a finite set and $S = \{s_1, s_2, \ldots, s_k\}$, $k \in N$, $2 \le k < \infty$. The $P(X_t = s_i) = p_i(t)$ value means the probability that the system at a moment $t \in N$ is in a state $s_i \in S$, $1 \le i \le k$, and $\sum_{i=1}^{k} p_i(t) = 1$.

Definition 2 A stochastic process $\{X_t\}_{t\in N}$ in discrete time is called a Markov chain if for each $n \in N$, of any moments $t_1, t_2, \ldots, t_n \in N$ satisfying the condition $t_1 < t_2 < \ldots < t_n$, and any $x_1, x_2, \ldots, x_n \in S$, the equality occurs [26, 47]:

$$P\left(X_{t_n} = x_n \middle| X_{t_{n-1}} = x_{n-1}, X_{t_{n-2}} = x_{n-2}, \ldots, X_{t_1} = x_1\right) = P\left(X_{t_n} = x_n \middle| X_{t_{n-1}} = x_{n-1}\right) \tag{1}$$

From the definition of the Markov chain it follows that the conditional distribution of the random variable X_n, for a given values $X_{t_0}, X_{t_1}, \ldots, X_{t_{n-1}}$ depends only on the last known value $X_{t_{n-1}}$. It is usually assumed that the t_i and t_{i+1} intervals are equal [16]. Below we assume that $t_n = n \in N$. If $\{X_t\}_{t\in N}$ is a heterogeneous Markov chain, then for any $t \in N$ and $1 \le i, j \le k$, the value:

$$P\left(X_t = s_j \middle| X_{t-1} = s_i\right) = p_{ij}(t) \tag{2}$$

we call the probability of transition from s_i state at the moment $t-1$ to the s_j state at moment t. Therefore, for the chains satisfying the Markov property (1), the conditional probability distributions of the future process states are determined only by its current state and moment t, regardless of the past (they are conditionally independent of the past states). The matrix $P(t) = \left[p_{ij}(t)\right]_{1\le i,j\le k}$ satisfying the condition $\sum_{j=1}^{k} p_{ij}(t) = 1$ for $t \in N$ and $1 \le i \le k$ is called the matrix of probabilities of transitions in one step at the moment t [7, 47, 51].

Definition 3 The Markov chain $\{X_t\}_{t\in N}$ is a homogeneous Markov chain, if the $p_{ij}(t)$ probabilities of transition do not depend on the moment $t \in N$.

Thus, for a homogeneous Markov chain $p_{ij}(t) = p_{ij}$ for $1 \le i, j \le k$ and any moment $t \in N$. The matrix $P = \left[p_{ij}\right]_{1\le i,j\le k}$ satisfying the condition $\sum_{j=1}^{k} p_{ij} = 1$, $1 \le i \le k$ we call the transition probability matrix in one step. For a homogeneous Markov chain, the probabilities of transition from a s_i state at a t moment to the s_j state at the $t+n$ moment is determined using the formula [13, 24]:

$$P\left(X_{t+n} = s_j \middle| X_t = s_i\right) = p_{ij}^{(n)} \tag{3}$$

where $\left[p_{ij}^{(n)}\right]_{1\le i,j\le k} = P^n$, $n \in N$ is the matrix of probability of transition in n steps.

Definition 4 If $\{X_t\}_{t\in N}$ is a homogeneous Markov chain and there is a distribution $\pi = (\pi_1, \pi_2, \ldots, \pi_k)$ where $\pi_i \ge 0$, $1 \le i \le k$ and $\sum_{i=1}^{k} \pi_i = 1$ satisfying the equation:

$$\pi P = \pi \tag{4}$$

then the distribution π is called the stationary distribution of the homogeneous Markov chain.

The stationary property means that if at some $n \in N$ moment the chain reaches a stationary distribution, then for each subsequent moment greater than n the distribution will remain the same. We determine the stationary distribution by solving the system of equations [16, 22]:

$$\sum_{j=1}^{k} \pi_j \cdot p_{ij} = \pi_i \tag{5}$$

$$\sum_{i=1}^{k} \pi_i = 1 \tag{6}$$

and $\pi_i \ge 0$ for $1 \le i \le k$.

An important role in the studying of processes using Markov chains is played by its boundary properties, especially the boundary probability $p_j(n)$ and $p_{ij}^{(n)}$ at $n \to \infty$, which describe the probabilistic behaviour of the process after a long time [16, 22].

Theorem 1 (ergodic) Let $\{X_t\}_{t\in N}$ be a homogeneous Markov chain with a finite number of states $k < \infty$ $\left(k = \#S = \#\{i : s_i \in S\}\right)$, then:

a) a vector $\pi = (\pi_1, \pi_2, \ldots, \pi_k)$ exists such that $\pi_i \ge 0$ for $1 \le i \le k$ and $\sum_{i=1}^{k} \pi_i = 1$;

b) for any $1 \le i, j \le k$

$$\pi_j = \lim_{n\to\infty} p_{ij}^{(n)} \;;$$

c) π vector is the solution to the equation (6).

Below, the method of estimating the transition probability matrix for the homogeneous Markov chain and the way of verifying Markov property will be presented. Let $\{x_t\}_{0\le t\le n}$ be the realization of the Markov chain. The value $n_i = \#\{t : x_t = s_i, 0 \le t \le n\}$ means the number of moments for which the system remained in the state s_i for $1 \le i \le k$, where $\sum_{i=1}^{k} n_i = n$, while the value $n_{ij} = \#\{t : x_t = s_i, x_{t+1} = s_j, 0 \le t \le n-1\}$ means the number of transitions from the state s_i to the state s_j for $1 \le i, j \le k$ and $\sum_{j=1}^{k} n_{ij} = n_i$.

Assuming that the Markov property is satisfied, we estimate the transition probability matrix. The estimator of the transition probability from state s_i to state s_j we determine from the formula $\hat{p}_{ij} = \frac{n_{ij}}{n_i}$ for $1 \le i, j \le k$.

We use a χ^2 goodness of fit test to verify Markov property. At the significance level $\alpha \in (0,1)$ we create a working hypothesis:

$H_0 : P\left(X_t = x \middle| X_{t-1} = y, X_{t-2} = z\right) = P\left(X_t = x \middle| X_{t-1} = y\right)$ (the chain $\{X_t\}_{t\in N}$ satisfies Markov property)

and an alternative hypothesis:

$H_1 : P\left(X_t = x \middle| X_{t-1} = y, X_{t-2} = z\right) \ne P\left(X_t = x \middle| X_{t-1} = y\right)$ (the chain $\{X_t\}_{t\in N}$ does not satisfy Markov property),

where $x, y, z \in S$. As a measure of discrepancy between $P\left(X_t = x \middle| X_{t-1} = y, X_{t-2} = z\right)$ and $P\left(X_t = x \middle| X_{t-1} = y\right)$ distributions we choose the test statistics:

$$\chi_e^2 = \sum_{i=1}^{k}\sum_{j=1}^{k}\sum_{v=1}^{k} \frac{\left(n_{ijv} - n_{ij}\hat{p}_{jv}\right)^2}{n_{ij}\hat{p}_{jv}} \tag{7}$$

which has a χ^2 distribution with k^3 degrees of freedom.

The value $n_{ijv} = \#\{t : x_t = s_i, x_{t+1} = s_j, x_{t+2} = s_v, 0 \le t \le n-2\}$ means the number of transitions from state s_i to state s_j and next to state s_v for $1 \le i, j, v \le k$. From the tables for the χ^2 distribution with k^3 degrees of freedom we determine the quantile of order $1-\alpha$, which we denote as $\chi^2\left(1-\alpha, k^3\right)$. If $\chi_e^2 < \chi^2\left(1-\alpha, k^3\right)$, then at the significance level α there are no grounds for rejecting the working hypothesis H_0, so we assume that the chain $\{X_t\}_{t\in N}$ satisfies Markov property. On the other hand, if $\chi_e^2 \ge \chi^2\left(1-\alpha, k^3\right)$, then at the significance level α we reject the working hypothesis H_0 in favour of the alternative hypothesis, thus the chain $\{X_t\}_{t\in N}$ does not satisfy Markov property.

3. Logistic regression

In many cases, the stochastic process $\{X_t\}_{t\in N}$ does not satisfy Markov property. The realization of the process $\{X_t\}_{t\in N}$ depends on additional factors. In transport and logistics systems, the time of remaining in a specific state directly affects the probability of transition to other states. Below, the authors used logistic regression to define the transition probability matrix, which depends on the time the object remains in a given state. In the case under consideration a random variable $X_t, t \in N$ describing the state of the system can get k possible realizations. Because we are considering moments for which the system state changes, so if at the moment $t \in N$ the system was in a state $s_i \in S$, then in the moment $t + \tau$ the system may take the states $S \setminus \{s_i\}$ (a random variable $X_{t+\tau}$ may get $k-1$ possible realizations). Determination of the transition probability is possible thanks to polynomial logistic regression [19, 25, 44, 46]. One of the levels should be taken as a reference. For each state $s_i \in S$, $1 \le i \le k$ we determine the probabilities of transition to the other states:

$$P\left(X_{t+\tau} = s_j \middle| X_t = s_i\right) = p_{ij}(\tau), \tag{8}$$

where $t, \tau \in N$ and $s_j \in S \setminus \{s_i\}$. From the set $S \setminus \{s_i\}$, we select the reference state $s_q \in S \setminus \{s_i\}$ and determine the logarithms of chances for the remaining states:

$$ln \frac{P\left(X_{t+\tau} = s_j \middle| X_t = s_i\right)}{P\left(X_{t+\tau} = s_q \middle| X_t = s_i\right)} = \beta_{ij}^0 + \beta_{ij}^1 \tau \tag{9}$$

for $s_j \in S \setminus \{s_i, s_q\}$. The values of structural parameters in the model (9) are determined using the maximum likelihood method [20, 25, 49]. Wald's test is used to assess the significance of model parameters.

We determine transition probabilities for states $s_j \in S \setminus \{s_i, s_q\}$ using the formula:

$$p_{ij}(\tau) = \frac{e^{\beta_{ij}^0 + \beta_{ij}^1 \tau}}{1 + \sum\limits_{\substack{1 \le v \le k \\ v \ne i, v \ne q}} e^{\beta_{iv}^0 + \beta_{iv}^1 \tau}}, \tag{10}$$

while for the reference state s_q the probability is:

$$p_{iq}(\tau) = \frac{1}{1 + \sum\limits_{\substack{1 \le v \le k \\ v \ne i, v \ne q}} e^{\beta_{iv}^0 + \beta_{iv}^1 \tau}}. \tag{11}$$

From the formulas (10) - (11) we obtain that the logarithm of the chances ratio for any two states $s_j, s_v \in S \setminus \{s_i, s_q\}$ is equal to:

$$ln \frac{P\left(X_{t+\tau} = s_j \middle| X_t = s_i\right)}{P\left(X_{t+\tau} = s_v \middle| X_t = s_i\right)} = ln \frac{p_{ij}(\tau)}{p_{iv}(\tau)} = \left(\beta_{ij}^0 - \beta_{iv}^0\right) + \left(\beta_{ij}^1 - \beta_{iv}^1\right)\tau \; . \tag{12}$$

4. Estimation of transition probability matrix for the selected means of transport

The subject of the study was a Belgian distribution department operating for the benefit of hypermarket chains. Transport services are carried out every day, 24 hours a day, which is why it is important to schedule transport properly, taking into account the availability of employed staff (especially drivers), as well as the readiness of vehicles.

The study used data from the company's fleet management system that integrates, processes and archives readings from the vehicle's GPS transmitter, tachograph, CAN (Controller Area Network) and on-board computer. It allows to obtain data on the driver and the vehicle in real time, allows tracking of the position and movement of cars, visualization of the location of vehicles and trailers on the map, monitoring of driving and resting times, etc. The information concerned 69 Iveco Stralis EEV 460 trucks. The collected data was segregated and 10 operational states realized by heavy goods vehicles were analyzed. These activities are detailed in tab. 1.

Table 1. Operating states highlighted in the study

No.	Name of operational state
S1	Availability
S2	Driving
S3	Manipulation
S4	Repair
S5	Maintenance
S6	Parking
S7	Layover
S8	Off-loading
S9	Refuelling
S10	Loading

The study presented in the article was carried out for one randomly selected vehicle. Markov property were checked. The χ^2 test was used for this purpose. The test statistics was 2672.74, and $p-value = 2.2 * 10^{-16}$. This means that at the significance level $\alpha = 0.001$, the working hypothesis should be rejected in favour of the alternative hypothesis, therefore the analyzed stochastic process does not satisfy Markov property. Nevertheless, the transition probability matrix of realization of the process $\{X_t\}_{1 \le t \le n}$, $n = 6822$ was estimated (for comparison purposes), which is presented graphically in Fig. 1, while the values of this matrix are presented in Table 2.

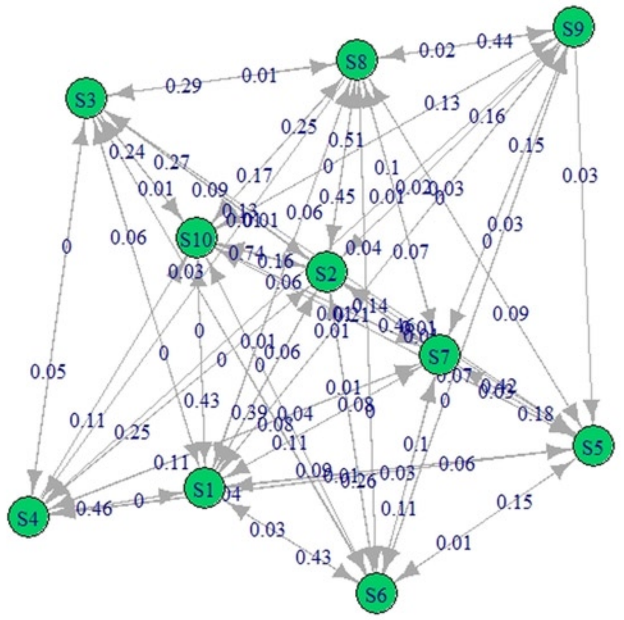

Fig. 1. Graph of the interstate transitions according to Markov chain

By solving equation (4), the boundary probabilities were estimated. The values of these probabilities are presented in Tab. 3.

Because Markov property was not satisfied for the analyzed data, the parameters of the transition probability matrix were estimated using the polynomial logistic regression model. The impact of the time

Table 2. Matrix of transition probabilities for the Markov chain

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0.392	0.002	0.002	0.012	0.032	0.113	0.009	0.007	0.431
S2	0.060	0	0.128	0.003	0.015	0.015	0.142	0.453	0.022	0.162
S3	0.065	0.274	0	0.003	0.009	0.029	0.085	0.294	0	0.241
S4	0.456	0.246	0.053	0	0.035	0	0.105	0	0	0.105
S5	0.056	0.416	0.011	0.034	0	0.146	0.180	0.090	0	0.067
S6	0.433	0.264	0.082	0	0.014	0	0.111	0.005	0.005	0.087
S7	0.084	0.456	0.003	0.044	0.027	0.103	0	0.074	0.001	0.208
S8	0.062	0.510	0.008	0.009	0.005	0.044	0.097	0	0.019	0.247
S9	0.035	0.163	0	0.012	0.035	0.035	0.151	0.442	0	0.128
S10	0.004	0.735	0.008	0	0.012	0.001	0.064	0.173	0.003	0

Table 3. The values of boundary probabilities for the Markov chain

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
π_i	0.064	0.337	0.050	0.008	0.013	0.031	0.099	0.212	0.013	0.173

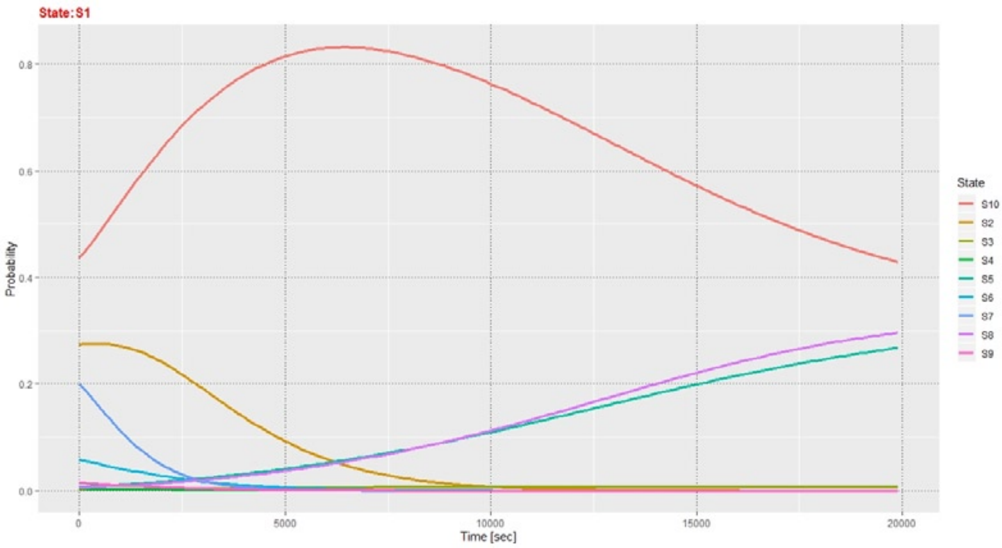

Fig. 2. Relationship between the transition probability from Availability state and its duration

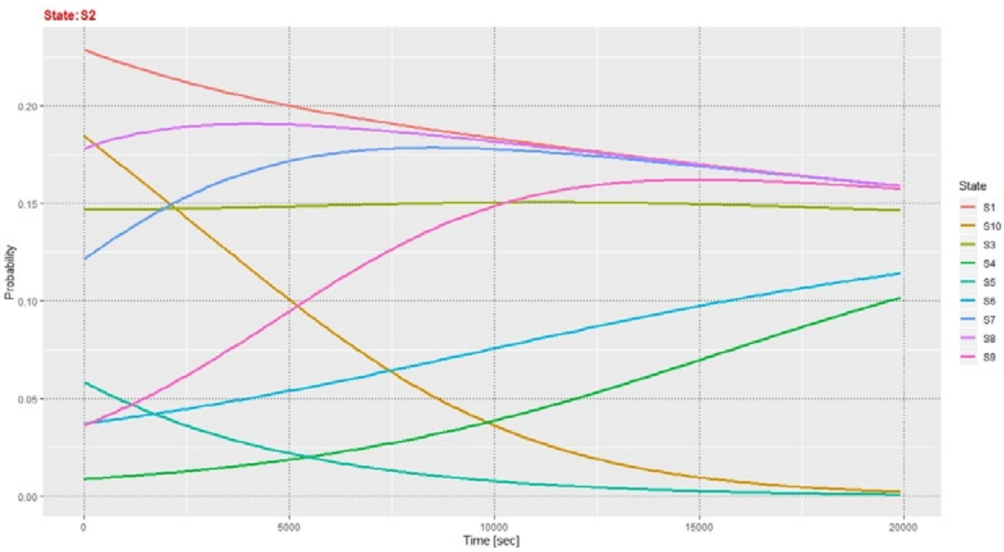

Fig. 3. Relationship between the transition probability from Driving state and its duration

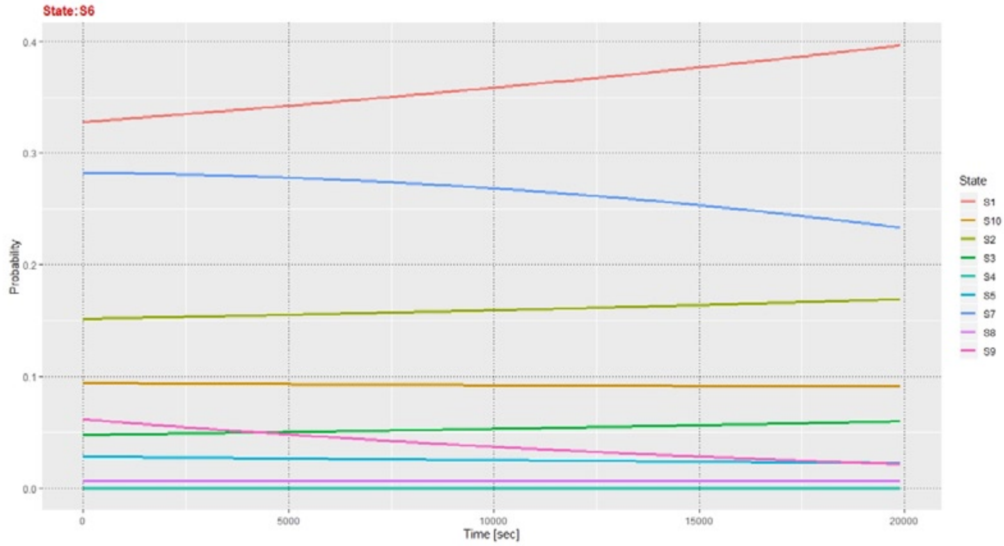

Fig. 4. Relationship between the transition probability from Parking state and its duration

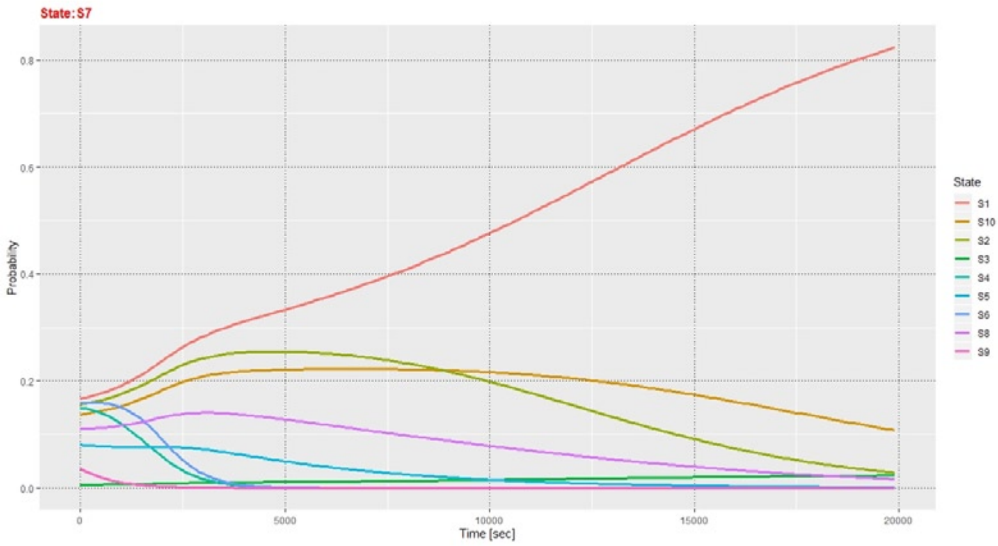

Fig. 5. Relationship between the transition probability from Layover state and its duration

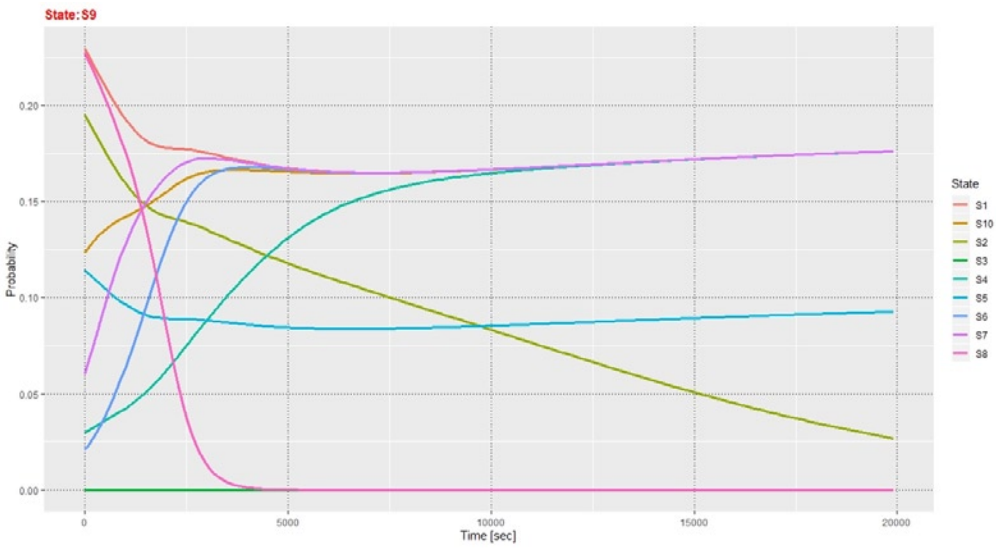

Fig. 6. Relationship between the transition probability from Refuelling state and its duration

while remaining in a specific operating state on the probability of transition to other states was investigated. It was assumed that the probability at the moment $t+\tau$ is a value conditionally dependent on the state in which the object was at the moment t and the duration length τ, as well as that after a time τ the system does not return to it. For each state, 8 logistic regression equations (9) were determined, which describe the relationships for nine possible transitions. The significance of structural parameters was examined by Wald test. As a result of using polynomial logistic regression for each system state a transition probability matrix was obtained, which depends on the duration time τ.

Then, the graphs were drawn illustrating the change in the transition probabilities depending on the duration time τ in a given operational state. For the selected states: availability, driving, parking, layover, refuelling, the relationships between the transition probabilities and the duration time are shown on Fig. 2 - Fig. 6.

The above graphs show the dependence of the transition probability value from a given operational state to the next, depending on the time the vehicle spends in it. From the graphs it is possible to see that the values of these probabilities are not constant, which shows impossibility of the use of the classical approach when estimating the transition probability matrix as for the Markov chain. The approach proposed by the authors shows how to determine the matrix of transition probabilities for the case when for a specific state the duration time significantly affects the values of the elements of this matrix. The variability of the transition probabilities is justified and reflects the specificity of the implementation of transport processes, which are partly determined by legal regulations concerning, for example, the driver's working time, as well as deadlines resulting from the operating strategy implemented in the company, regulating the periods of repairs and inspections.

The solutions presented are helpful in addition to developing a method that allows assessing the readiness of the system to carry out transport tasks. Operating states can be classified as states of suitability and unsuitability, and it is possible to determine the technical readiness factor as the sum of appropriate probabilities of reliability states.

5. Conclusion

The article estimates the matrix of transition probabilities to the identified operating states in which the tested vehicle was. The use of Markov chains is popular in such estimates, which requires the condition of the lack of memory of the analyzed process to be met. In the presented case this property was not fulfilled. In addition, it was shown that the probability of transition to a given operating state is conditionally dependent on the state in which the object was and the length of time spent in it. Therefore, an alternative method was proposed for their estimation. For this purpose, a polynomial logistic regression model was used. The transition probabilities were obtained, whose values for a given state differed depending on the length of time the vehicle stayed in the previous state.

The results obtained were compared with the values obtained according to the Markov chain - for which they are constant - showing that using so calculated transition probability matrix, when the Markov property is not met, may lead to the erroneous conclusions.

The proposed logistic regression model allows to conduct short-term forecasts regarding the implementation of the transport process. The assessment of the probability transition depending on previously carried out activities supports the process of scheduling transport tasks, as well as planning in the field of vehicle maintenance.

As part of further research, it is worth extending the proposed method by determining the values of estimators and assessing the boundary probabilities of transitions for individual states over a long period of time. This will allow a comprehensive evaluation of the system's functioning, as well as determining the level of readiness to carry out transport tasks. The division of operational states into states of suitableness and unsuitableness will allow to calculate the technical readiness coefficient as the sum of the respective boundary probabilities of the reliability states. The presented solution can also be used for modelling the driving cycles of heavy vehicles, which directly reflect the real operating conditions of the engine or components on the chassis dynamometer.

References

1. Abid L, Masmoudi A, Zouari-Ghorbel S. The Consumer Loan's Payment Default Predictive Model: an Application of the Logistic Regression and the Discriminant Analysis in a Tunisian Commercial Bank. Journal of the Knowledge Economy 2018; 9(3): 948-962, https://doi.org/10.1007/s13132-016-0382-8.
2. Adha M R, Nurrohmah S, Abdullah S. Multinomial Logistic Regression and Spline Regression for Credit Risk Modelling, Journal of Physics: Conference Series 2018; 1108, https://doi.org/10.1088/1742-6596/1108/1/012019.
3. Andrzejczak K. Metody prognozowania w modelowaniu eksploatacji środków transportu. Poznań: Wydawnictwo Politechniki Poznańskiej, 2013.
4. Ari E. Using Multinomial Logistic Regression to Examine the Relationship Between Children's Work Status and Demographic Characteristics, Ekonomi ve Yönetim Araştırmaları Dergisi 2016; 4(1): 77-93.
5. Bertens l C, Karel K G, Rutten F H, van Mourik Y, Hoes A W, Reitsma J B. A nomogram was developed to enhance the use of multinomial logistic regression modeling in diagnostic research. Journal of Clinical Epidemiology 2016; 71: 51-57, https://doi.org/10.1016/j.jclinepi.2015.10.016.
6. Bobrowski D. Modele i metody matematyczne teorii niezawodności w przykładach i zadaniach. Warszawa: Wydawnictwa Naukowo - Techniczne, 1985.
7. Borucka A, Niewczas A, Hasilova K. Forecasting the readiness of special vehicles using the semi-Markov model. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2019; 21 (4): 662-669, https://doi.org/10.17531/ein.2019.4.16.
8. Calster B V, Hoorde V K, Vergouwe Y, Bobdiwala S, Condous G., Kirk E., Bourne T, Steyerberg E W. Validation and updating of risk models based on multinomial logistic regression. Diagnostic and Prognostic Research 2017; 1:2, https://doi.org/10.1186/s41512-016-0002-x.
9. Cartella F, Lemeire J, Dimiccoli L, Sahli H. Hidden Semi-Markov Models for Predictive Maintenance, Mathematical Problems in Engineering 2015; 278120, https://doi.org/10.1155/2015/278120.
10. Ferdowsi A, Challita U, Saad W. Deep Learning for Reliable Mobile Edge Analytics in Intelligent Transportation Systems: An Overview, IEEE Vehicular Technology Magazine 2019; 14(1): 62-70, https://doi.org/10.1109/MVT.2018.2883777.
11. Fisz M. Rachunek prawdopodobieństwa i statystyka matematyczna. Warszawa: PWN, 1967.
12. Gautam V, Vaidya R R. Evidence on the determinants of investment-cash flow sensitivity Indian Economic Review December 2018; 53(1-2): 229-244, https://doi.org/10.1007/s41775-018-0021-3.

13. Girtler J, Ślęzak M. Application of the theory of semi-Markov processes to the development of a reliability model of an automotive vehicle, Archiwum Motoryzacji 2012; 2: 15-27, https://doi.org/10.5604/1234754X.1066721.
14. Gniedenko B.W., Bielajew J.K., Sołowiew A.D. Metody matematyczne w teorii niezawodności. Warszawa: Wydawnictwa Naukowo-Techniczne, 1965.
15. Gola A. Reliability analysis of reconfigurable manufacturing system structures using computer simulation methods, Eksploatacja i Niezawodnosc - Maintenance and Reliability 2019, 21 (1): 90-102, https://doi.org/10.17531/ein.2019.1.11.
16. Grabski F. Semi-Markov Processes: Applications in System Reliability and Maintenance, Elsevier, 2014, https://doi.org/10.1016/B978-0-12-800518-7.00004-1.
17. Grzywacz W., Burnewicz J. Ekonomika transportu. Gdańsk: Wydawnictwa Komunikacji i Łączności, 1989.
18. Halim H, Ramli M I, Adisasmita S A, Aly S H, Prasetijonternational J A. Relationship model between accident factors and the traffic accident severity using logistic regression model, Journal of Engineering and Science Application 2018; 4(2), 169-181.
19. Hastie T., Tibshirani R., Friedman J., The elements of statistical learning, Springer-Verlag New York Inc., 2009, https://doi.org/10.1007/978-0-387-84858-7.
20. James G., Witten D., Hastie T., Tibshirani R., An introduction to statistical learning, Springer-Verlag GmbH, 2013, https://doi.org/10.1007/978-1-4614-7138-7.
21. Jasiulewicz-Kaczmarek M., Gola A., Maintenance 4.0 Technologies for Sustainable Manufacturing - an Overview, IFAC PapersOnLine 2019; 52-10: 91-96, https://doi.org/10.1016/j.ifacol.2019.10.005.
22. Jaźwiński J, Grabski F. Niektóre problemy modelowania systemów transportowych. Radom: Instytut Technologii Eksploatacji, 2003.
23. Jia X, Cui L, Xing L. New insights into reliability problems for supply chains management based on conventional reliability model. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2018; 20 (3): 465-470, https://doi.org/10.17531/ein.2018.3.16.
24. Knopik L, Migawa K. Semi-Markov system model for minimal repair maintenance. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2019; 21 (2): 256-260, https://doi.org/10.17531/ein.2019.2.9.
25. Kozłowski E, Mazurkiewicz D, Żabiński T, Prucnal S, Sęp J. Assessment model of cutting tool condition for real-time supervision system. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2019; 21 (4): 679-685, https://doi.org/10.17531/ein.2019.4.18.
26. Landowski B. Example of applying markov decision process o model vehicle maintenance process. Journal of KONES Powertrain and Transport 2013;20(4): 209-218, https://doi.org/10.5604/12314005.1137618.
27. Lewitowicz J, Zieja M, Szelmanowski A, Pazur A. New Measures of Operational Readiness for Multi-states Avionics Integrated Systems with Reduced Efficiency. In: Hanus R., Mazur D., Kreischer C. (eds) Methods and Techniques of Signal Processing in Physical Measurements. Lecture Notes in Electrical Engineering, 2019; 548: 165-180, https://doi.org/10.1007/978-3-030-11187-8_13.
28. Lewitowicz J., Kustroń K. Podstawy eksploatacji statków powietrznych. Warszawa: Wydawnictwo ITWL, 2003.
29. Linhares F S, Costa F M D, Beiruth A X. Earnings management and investment efficiency. Revista Brasileira de Gestão de Negócios 2018; 20(2): 295-310, https://doi.org/10.7819/rbgn.v20i2.3180.
30. Macián V, Tormos B, Herrero J. Maintenance management balanced scorecard approach for urban transport fleets. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2019; 21 (2): 226-236, https://doi.org/10.17531/ein.2019.2.6.
31. Młynarski S, Pilch R, Smolnik M, Szybka J. Methodology of network systems reliability assessment on the example of urban transport. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2018; 20 (2): 278-283, https://doi.org/10.17531/ein.2018.2.14.
32. Murat Y S, Uludag N. Route choice modelling in urban transportation networks using fuzzy logic and logistic regression methods, Journal of Scientific & Industrial Research; 67(01): 19-27.
33. Nabian M A, Meidani H. Deep Learning for Accelerated Seismic Reliability Analysis of Transportation Networks, Computer-Aided Civil and Infrastructure Engineering 2018; 33: 443-458, https://doi.org/10.1111/mice.12359.
34. Nguyen T. Understanding Municipal Bond Ratings Using Ordered Multinomial Logistic Regression. Senior Independent Study Theses 2018; 8171.
35. Nicholson A, Schmöcker J, Bell M, Iida Y. Assessing Transport Reliability: Malevolence and User Knowledge. In The Network Reliability of Transport, Emerald Group Publishing Limited 2003; 1-22, https://doi.org/10.1108/9781786359544-001.
36. Niewczas A, Rymarz J, Debicka E. Stages of operating vehicles with respect to operational efficiency using city buses as an example. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2019; 21 (1): 21-27, https://doi.org/10.17531/ein.2019.1.3.
37. Nowakowska M. Spatial and temporal aspects of prior and likelihood data choices for Bayesian models in road traffic safety analyses. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2017; 19 (1): 68-75, https://doi.org/10.17531/ein.2017.1.10.
38. Odeck J, Bråthen S. A meta-analysis of DEA and SFA studies of the technical efficiency of seaports: A comparison of fixed and random-effects regression models. Transportation Research Part A: Policy and Practice 2012; 46(10): 1574-1585, https://doi.org/10.1016/j.tra.2012.08.006.
39. Oliva F L. A maturity model for enterprise risk management, International Journal of Production Economics 2016; 173: 66-79, https://doi.org/10.1016/j.ijpe.2015.12.007.
40. Osuji M N, Mejeha R O, Simonyan J B, Gbolagun A S. Determinants of Enterprise Preference of Cassava-based farmers by Gender in Southeast, Nigeria. Research Journal of Food and Nutrition 2018; 2(4): PP 45-52.
41. Park M, Wu A D. Investigating differential options functioning using multinomial logistic regression. International Journal of Quantitative Research in Education 2017; 4(1-2): 94 - 119, https://doi.org/10.1504/IJQRE.2017.086511.
42. Paś J, Klimczak T. Selected issues of the reliability and operational assessment of a fire alarm system. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2019; 21 (4): 553-561, https://doi.org/10.17531/ein.2019.4.3.
43. Paś J, Rosiński A. Selected issues regarding the reliability-operational assessment of electronic transport systems with regard to electromagnetic interference. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2017; 19 (3): 375-381, https://doi.org/10.17531/ein.2017.3.8.
44. Peter D, Hart A. Multinomial Logistic Regression Examination of TV Time and Two Different Measures of Obesity in U.S. Adults. American Journal of Public Health Research 2018; 6(5): 222-226, https://doi.org/10.12691/ajphr-6-5-3.
45. Raco A, Pesce A, Fraschetti F, D'Andrea G, Polli F M, Acqui M, Frati A. Risk of Postoperative Performance Status Worsening after Resection of Lesions Involving the Motor Pathway: A Multinomial Logistic Regression Model. Journal of Neurological Surgery, Part A: Central

European Neurosurgery 2018; 79(06): 453-463, https://doi.org/10.1055/s-0037-1617756.
46. Rymarczyk T.; Kozłowski E.; Kłosowski G.; Niderla K. Logistic Regression for Machine Learning in Process Tomography. Sensors 2019, 19, 3400, https://doi.org/10.3390/s19153400.
47. Sanusi W, Jemain A A, Zin W Z, Zahari M. The Drought Characteristics Using the First-Order Homogeneous Markov Chain of Monthly Rainfall Data in Peninsular Malaysia, Water Resources Management 2015; 29(5): 1523-1539, https://doi.org/10.1007/s11269-014-0892-8.
48. Schwartz J, Guasch J L, Wilmsmeier G, Stokenberga A. Logistics, Transport and Food Prices in LAC: Policy Guidance for Improving Efficiency and Reducing Costs. Latin America and Caribbean Region Environment and Water Resources occasional paper series 2009; 2.
49. Shao C, Xia J C, Lin T G, Goulias K G, Chen C. Logistic regression models for the nearest train station choice: A comparison of captive and non-captive stations, Case Studies on Transport Policy 2015; 3(4): 382-391, https://doi.org/10.1016/j.cstp.2015.06.002.
50. Sowa A. Stan obiektu jako wieloznaczne pojęcie we współczesnej eksploatacji technicznej. Czasopismo Techniczne. Mechanika 2012; 109(7-M): 269-278.
51. Stewart W.J. Probability, Markov Chains, Queues, and Simulation, 2009, https://doi.org/10.2307/j.ctvcm4gtc.
52. Szkoda M, Kaczor G. Reliability and availability assessment of diesel locomotive using fault tree analysis. Archives of Transport 2016; 40(4): 65-75, https://doi.org/10.5604/08669546.1225470.
53. Wang Z, Huang X, Liang Y J. Oil-gas reservoir lithofacies stochastic modeling based on one- to three-dimensional Markov chains. Journal of Central South University 2018; 25(6): 1399-1408, https://doi.org/10.1007/s11771-018-3835-3.
54. Zhu Y, Xie C, Sun B, Wang G-J, Yan X-G. Predicting China's SME Credit Risk in Supply Chain Financing by Logistic Regression, Artificial Neural Network and Hybrid Models. Sustainability 2016; 8(5): 1-17, https://doi.org/10.3390/su8050433.
55. Ziółkowski J., Borucka A. Markov model in logistic management of enterprise. Journal of Konbin 2016; 38(1): 271-290, https://doi.org/10.1515/jok-2016-0027.

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