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## A MODEL OF AN ADAPTIVE STRATEGY OF PREVENTIVE MAINTENANCE OF COMPLEX TECHNICAL OBJECTS

### MODEL ADAPTACYJNEJ STRATEGII PREWENCYJNEJ ODNOWY ZŁOŻONYCH OBIEKTÓW TECHNICZNYCH\*

*The paper presents results of the analysis of the developed models for complex technical objects preventive maintenance scheduling. Models based on two different sets of assumptions were developed. The general problem solved was to determine the joint time of preventive renewal for a group of parts or subassemblies. The purpose of the first model (the model of scheduled preventive maintenance strategy) is to determine the profitability of constant application of a previously developed preventive maintenance schedule for a part undergoing post-failure renewal. The second model (the model of adaptive strategy of a system's preventive maintenance) allows one to determine a new joint time of preventive renewal for a group of parts each time when one of them is undergoing a post-failure renewal. The initial preventive maintenance strategy for each part or subassembly was obtained using typical tools for maintenance planning (decision-random models based on dynamic programming and Bellman's principle of optimality). Exemplary simulation calculations with the use of both models were made and their results presented as the total maintenance costs estimated for the renewal strategies developed. The object of the analysis were the chosen geometrical features of a rail vehicle wheel changing due to its wear during operation. Based on this kind of analysis, one can choose a better preventive maintenance model for a specific application area.*

**Keywords:** preventive maintenance; simulation efficiency evaluation; reliability engineering.

*W artykule przedstawiono wyniki analizy opracowanych modeli do planowania odnowy profilaktycznej złożonych obiektów technicznych, które oparto o dwa różniące się od siebie zestawy założeń. Rozwiązywany problem dotyczy określenia wspólnego czasu odnowy profilaktycznej grupy części lub podzespołów złożonego obiektu. Pierwszy z opracowanych modeli (model planowej strategii odnowy prewencyjnej) pozwala określić zasadność przeprowadzenia ustalonego wcześniej, planowego odnowienia prewencyjnego części obiektu, która została już odnowiona poawaryjnie. Drugi model (model adaptacyjnej strategii odnowy prewencyjnej) umożliwia wyznaczenie najbliższego wspólnego czasu odnowy profilaktycznej grupy części, z których jedna aktualnie podlega odnowie poawaryjnej. Początkowe (wyjściowe) strategie odnowy profilaktycznej każdej części bądź podzespołu wyznaczone zostały za pomocą standardowych narzędzi do planowania odnawiania profilaktycznego (modeli decyzyjno-losowych wykorzystujących programowanie dynamiczne Bellmana). Posługując się opracowanymi modelami odnowy, przeprowadzono przykładowe obliczenia symulacyjne, których wyniki przedstawiono w postaci całkowitych kosztów obsługiwanego dla każdej z uzyskanych strategii. Przedmiotem analizy były wybrane cechy geometryczne koła pojazdu szynowego, których wartości zmieniają się na skutek zużycia w procesie eksploatacji. Na podstawie tego rodzaju analiz można wybrać lepszy (tj. efektywniejszy ekonomicznie) z modeli dla konkretnego zastosowania w praktyce.*

**Słowa kluczowe:** odnowa profilaktyczna; symulacyjna ocena efektywności; inżynieria niezawodności.

#### Notations

$t_i^*$  – preventive maintenance time of  $i$ -th element of the system,  
 $k_{ai}$  – failure cost of  $i$ -th element of the system,  
 $k_{oi}$  – preventive maintenance cost of  $i$ -th element of the system,  
 $t_{max}^*$  – the highest value of preventive maintenance time of one out of all the elements of the system,  
 $t_{min}^*$  – the lowest value of preventive maintenance time of one out of all the elements of the system,  
 $t_{ai}$  – time of failure of  $i$ -th element,  
 $t^*$  – joint time to preventive maintenance of elements determined prior to failure of  $i$  th element,

$F_i(t)$  – cumulative distribution function of time to failure of  $i$ -th element,  
 $k_{ai}$  – cost of failure of  $i$ -th element of the system,  
 $k_{oi}$  – cost of preventive maintenance of  $i$ -th element of the system,  
 $D(t_{ai})$  – decision indicator describing the profitability of renewal,  
 $T_h$  – finite time horizon of the object's operation,  
 $T_{hmax}$  – maximal operation time without preventive renewal and/or inspection,  
 $\Delta t$  – time interval in calculations with the use of decision-random models,

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- $n_{OT}$  – number of maintenance activities in the adopted time horizon,  
 $n$  – number of simulations,  
 $dt$  – time step used during simulation,  
 $t_U$  – age of the system (counted from the beginning of simulation),  
 $t_{Ei}$  – age of the  $i$ -th element (counted from the beginning of simulation),  
 $P_{UEi}$  – probability of failure of the  $i$ -th element in simulation calculations,  
 $RND_{Ei}$  – random number from the interval [0,1], of uniform distribution, for the  $i$ -th element,  
 $t_{O\_opt}$  – system's optimal renewal time period in simulation calculations.

## 1. Introduction

To guarantee economically effective and safe use of complex technical objects it is necessary to employ an adequate strategy of their maintenance [4]. The quality of the operation process, described by various indicators [19], depends largely on the correct renewal of technical objects. This problem is of particular significance in the case of systems in which the failure of elements may threaten human health and life or result in considerable economic losses. In such context, preventive maintenance of system's elements may be justified, and the optimal time of its execution is determined by various models of preventive maintenance [15, 20]. To this end economic data on failures and maintenance as well as reliability characteristics of system's elements are used. In the literature, apart from the classical maintenance models [10], there are numerous maintenance models available on the required reliability level of an entire system [12, 21], some of them based on the application of simulation methods [5, 8]. Some models include the possibility of partial maintenance (cf. [7, 8]), some others make use of additional inspection of object's technical state when it can be performed while the system is in actual operation [2, 3]. A detailed and comprehensive classification of existing preventive renewal models is provided in [20].

A model for the determination of the preventive maintenance optimal time ( $t^*$ ) should be adapted to the specificity and operation conditions of the given object. Only then will the preventive maintenance strategy based on this model be economically justified and guarantee the reduced failure rate of the object (cf. [11, 14]).

The aim of the paper is to develop an adaptive model of renewal of complex technical objects and a simulation method to assess the efficiency of using the renewal strategy determined with this model.

The article discusses the specific problem of renewal of a complex technical object, whose parts are functioning in a reliability series structure and their failures are considered as independent of each other, however, for organizational and economic reasons, individual renewal of each of them may be less profitable than block renewal.

In complicated cases, when the maintenance of some parts of complex technical objects is performed at the same time as the servicing of other units or systems of the given object, after a failure the scheduled maintenance strategy of the failed part or even the entire object needs to be modified. The present paper is devoted to the analysis of this important practical problem.

The developed solutions are a significant complement to existing renewal models due to the needs in maintenance practice.

The article presents a new original computer simulation based method of the evaluation of the efficiency of the proposed strategies. The newly developed method was validated on a practical example of failures of wheels of an electric rail unit vehicle.

## 2. Models of preventive maintenance of complex systems

Technical objects are composed of many structural elements that have individual operational characteristics. The empirical reliability indexes of component parts are often described with various models and their plots differ considerably [7]. As a result, the decision making as to the dates and scope of the safety-orientated maintenance of entire objects or their units and parts, and, consequently, their rational and economically effective use is difficult (cf. [6, 16]).

### 2.1. Determination of joint preventive maintenance time of system's components

When preventive maintenance of systems that are composed of a number of different elements is scheduled, for organisational and economic reasons a joint time of the preventive maintenance of all the elements, that is the entire system, is most frequently searched for [17]. If the preventive maintenance model adopted includes a unit of the complex technical object considered as a whole – there is a reliability function as well as maintenance and failure cost specified for it, the indicated optimal maintenance time refers to this entire unit. Moreover, it is subject to preventive and post-failure maintenance always as a whole (all its parts are subject to maintenance simultaneously).

The problem of the determination of a joint preventive maintenance time arises when for particular elements of the system optimal (following the adopted preventive maintenance model) different from each other, but close, preventive maintenance times are determined. The proposed manner of unification of preventive maintenance time assumes that individual preventive maintenance times of component parts are accounted for ( $t_i^*$ ), as well as preventive maintenance costs ( $k_{oi}$ ), failure costs ( $k_{ai}$ ) and increments of distribution functions as to the range of individual preventive maintenance times for all the elements. The distribution function increments, maintenance and failure costs will for each of the elements be the weight of its individual preventive maintenance time ( $t_i^*$ ). We propose the joint preventive maintenance time ( $t^*$ ) to be determined from the formula:

$$t^* = \frac{\sum_{i=1}^n t_i^* \cdot (k_{ai} + k_{oi}) \cdot [F_i(t_{max}^*) - F_i(t_{min}^*)]}{\sum_{i=1}^n (k_{ai} + k_{oi}) \cdot [F_i(t_{max}^*) - F_i(t_{min}^*)]} \quad (1)$$

where:

- $t_i^*$  – preventive maintenance time of  $i$ -th element of the system,
- $k_{ai}$  – failure cost of  $i$ -th element of the system,
- $k_{oi}$  – preventive maintenance cost of  $i$ -th element of the system,
- $t_{max}^*$  – the highest value of preventive maintenance time of one out of all the elements of the system,
- $t_{min}^*$  – the lowest value of preventive maintenance time of one out of all the elements of the system.

The joint preventive maintenance time of the given unit ( $t^*$ ) can also be defined including scheduled maintenance cycles and servicing of other units and parts of the same complex object [9]. Such an approach is often followed in the operational practice of large transport companies. In these companies the optimal maintenance periods based on the adopted preventive maintenance model and inspections of individual units are extended or reduced so that they constitute an integer multiple of the scheduled maintenance activities of a larger number of units and parts of the object. Such an approach results in the reduction of downtime periods of a complex technical object, which brings measurable economic effects.

**2.2. A model of scheduled preventive maintenance strategy of a system**

After the joint preventive maintenance time ( $t^*$ ) for all the elements of the system has been defined, the strategy of their joint maintenance cycle can be easily followed until the failure of one of the elements. When this happens, a decision making problem arises whether the element renewed after the failure should be included in the scheduled joint time (although by then it will not have reached time  $t^*$  to the scheduled maintenance), or it should be shifted to the next joint preventive maintenance time (Fig. 1). In the latter case, its time to preventive maintenance would be extended, which would increase the risk of failure.

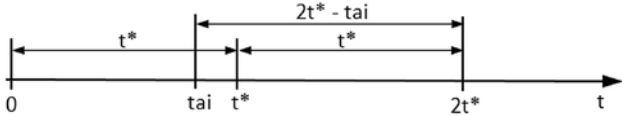


Fig. 1. Element's scheduled time to preventive maintenance and extended after a failure

To solve the problem it is necessary to consider an increased probability of a failure of the element of an extended operation time ( $2t^* - t_{ai}$ ). Moreover, the costs of its potential additional maintenance ( $k_{oi}$ ) should also be taken into account, when the element's maintenance would take place in the framework of the maintenance of the entire system although by then it will not have reached the scheduled time  $t^*$  to its preventive maintenance (it will have reached only  $t^* - t_{ai}$ ). We propose to derive the indicator on the basis of which the decision for the failed element should be taken in such case from the formula:

$$D(t_{ai}) = k_{oi} + F_i(t^* - t_{ai})k_{ai} - \frac{F_i(2 \cdot t^* - t_{ai}) - F_i(t^*)}{1 - F_i(t^*)} \cdot k_{ai}, \quad (2)$$

where:

- $t_{ai}$  – time of failure of  $i$ -th element,
- $t^*$  – joint time to preventive maintenance of elements determined prior to failure of  $i$  th element,
- $F_i(t)$  – cumulative distribution function of time to failure of  $i$ -th element,
- $k_{ai}$  – cost of failure of  $i$ -th element of the system,
- $k_{oi}$  – cost of preventive maintenance of  $i$ -th element of the system.

If  $D(t_{ai}) \leq 0$ , the maintenance of the element together with the other ones is justified, although since the post-failure maintenance at time  $t_{ai}$  it has not worked through time  $t^*$ . When  $D(t_{ai}) > 0$ , it is reasonable to extend the working time of the element and exclude it from the nearest maintenance period of the other elements. It should be included in the maintenance procedure during the subsequent maintenance of the system, after the nearest joint maintenance of its elements.

With known failure and maintenance costs and distribution function of times to failure of individual elements, the boundary value of  $t_{ai} = t_{i,gr}$  can be calculated for each of them. It is the time for which  $D(t_{ai}) = 0$ . If the failure occurs before the element reaches this working time, its additional maintenance together with the other elements in the nearest joint preventive maintenance time is reasonable. If the failure occurs after reaching this working time, it is justified to extend the element's working time with no maintenance. An example of  $D(t_{ai})$  plot is shown in figure 2. The curve was plotted following equation (2) for an element of normal distribution of time to failure  $N(m = 21,5$  [months];  $\sigma = 4,75$  [months]), with the quotient of failure

cost and repair cost  $ka/ko = 4$  and scheduled preventive maintenance time  $t^* = 14$  [months].

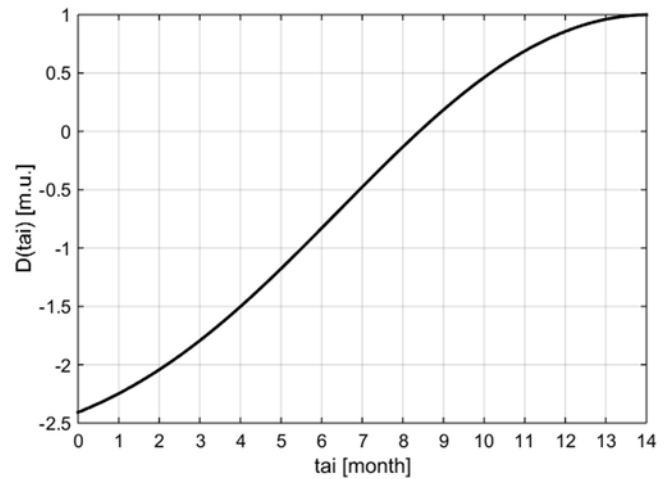


Fig. 2. An example of function  $D(t_{ai})$  plot

The value of  $t_{i,gr}$  of 8,5 [months] determined for this case is clearly visible. This means that if the element's failure occurs by the 8,5 month of operation, its additional preventive maintenance together with the other elements scheduled in the 14th month is justified. If the failure occurs after the 8,5 month, the period of its operation can be extended (by 5,5 months maximum) and it should be included in the next joint maintenance procedure rather than the nearest one.

Another method of determining  $t_{i,gr}$  discussed in the literature [13] is the use of decision-random models based on dynamic programming and Bellman's principle of optimality. With these models preventive maintenance optimal time ( $t^*$ ) in a finite time horizon ( $T_h$ ) of the operation of a technical object can be calculated. It is also possible to calculate the highest value of  $T_h$  for which the model indicates when preventive maintenance or the system's technical state inspection are not cost effective [13], which is illustrated in the flowchart in figure 3 [18]. The maximum this time ( $T_{hmax}$ : with no preventive maintenance or inspection) corresponds to time  $2t^* - t_{i,gr}$  in the model proposed in the present paper. When  $T_{hmax}$  is known from the decision-random model also  $t_{i,gr} = 2t^* - T_{hmax}$  can be calculated.

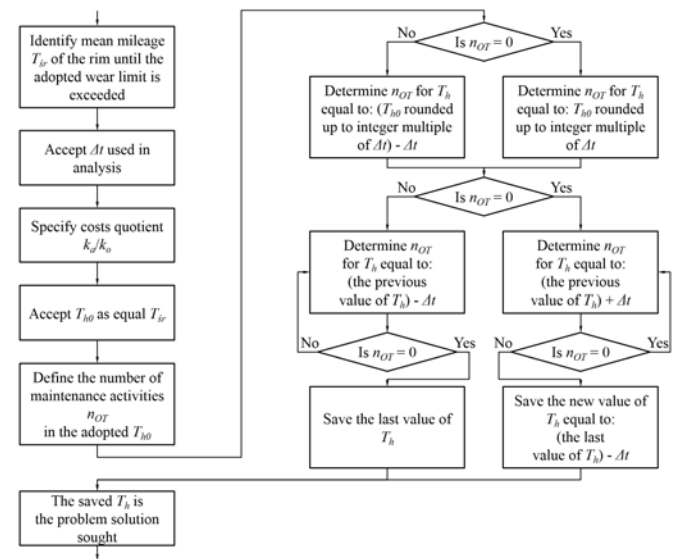


Fig. 3. A method of determining the highest mileage of wheel set rim for which the analysed servicing activities remain uneconomic [18]

What is characteristic of the presented scheduled maintenance strategy is that the initially determined joint preventive maintenance times of all the elements, performed at constant time intervals of  $t^*$  remain unchanged. The application of such strategy is particularly justified in cases when the maintenance activities of the given system are performed at constant intervals of time  $t^*$  and its multiples connected with other scheduled servicing activities of the entire object or the group of objects.

**2.3. A model of adaptive strategy of a system's preventive maintenance**

The other proposed maintenance method is an adaptive strategy in which the intervals between the times of subsequent preventive maintenance of the entire system can be modified. Such an interval is re-determined after the failure of any element of the system. In this model the starting situation is the same as in the model of scheduled strategy, that is there is a fixed time to preventive maintenance ( $t^*$ ) common for all the elements of the system. The strategy compatible with this time is followed until an element fails before reaching age  $t^*$ . When this happens, following the adaptive strategy, a post-failure maintenance of the failed component is performed. And if we know that the other elements did not fail by this time (modification of prob-

ability distribution of times to failure), another time, common for all the elements, of preventive maintenance is set.

The two maintenance strategies differ from each other and the question which will be more effective in an actual case can be solved by identifying economic indicators resulting from their application in the adopted time horizon [1].

**3. A simulation based model of scheduled and adaptive maintenance strategy**

The simulation based calculation model was developed on the assumptions formulated for the model of scheduled strategy and the model of adaptive strategy described in section 2. The calculation algorithms enable computer simulations of the operation of a system when each of the strategies is followed. Simplified algorithms showing a single simulation iteration are shown in figures 4a and 4b.

Detailed characteristics of the simulation method used for calculations were presented in [7], where the possibility of estimating the reliability of a complex technical object subjected to decomposition was shown. For the purposes of this paper, this method is a basis that was significantly supplemented with new functions related to preventive renewal of the examined objects. Thanks to this, in addition to the evaluation of the reliability of the object, it is also possible to evalu-

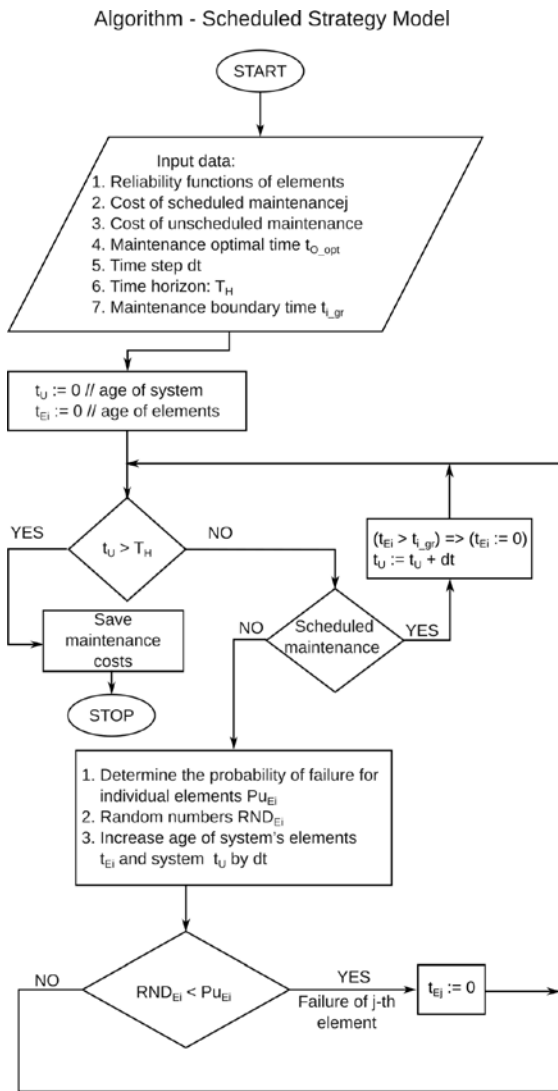


Fig. 4a. Simplified algorithm of a single simulation iteration of system's operation after scheduled strategy model

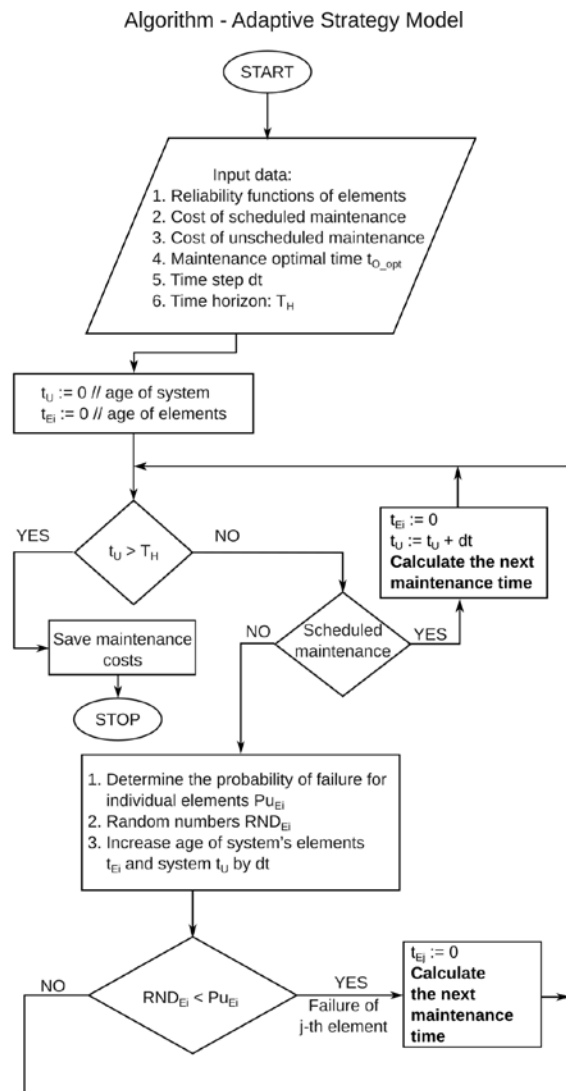


Fig. 4b. Simplified algorithms of a single simulation iteration of system's operation after adaptive strategy model

ate the economic efficiency of various strategies for the preventive renewal of its parts.

The assumptions adopted for the algorithm based on the scheduled strategy model were:

- in the case of a failure of an element, its maintenance is performed with no maintenance of undamaged elements,
- this element will undergo maintenance in the framework of preventive maintenance for time  $t^*$ , if its operation time was less than  $t_{i\_gr}$ .

The input data for the program developed on the basis of this algorithm include: elements' reliability functions, the cost of preventive maintenance of each element  $k_{oi}$ , the cost of post-failure maintenance  $k_{ai}$ , maintenance optimal time of system's elements  $t^*$ , boundary value of time  $t_{i\_gr}$ , time step  $dt$  and time horizon  $T_H$  of the simulation and the number of simulations  $n$ .

For the algorithm based on the adaptive strategy model the following assumptions were adopted:

- in the case of a failure of an element, its maintenance is performed with no maintenance of undamaged elements and the time of the nearest required preventive maintenance of all elements is calculated,
- preventive maintenance of all elements is performed at a time fixed after each failure of any element (as above).

In this case the only difference in the input data is that the determination of the boundary value of time  $t_{i\_gr}$  is not necessary.

**4. Calculation results**

The calculations were conducted on the basis of the data referring to the external research on the monobloc wheel sets of electric rail unit vehicles performed during operational practice.

It was stated that the distribution of the wheel sets' operation time is compatible with Weibull distribution of the shape parameter  $\nu = 4,1$  and scale parameter  $\beta = 170000$  [km]. The probability density function of mileage to failure for this distribution is given as equation (3) and presented in figure 5.

$$f(t) = \nu \left(\frac{1}{\beta}\right)^\nu t^{\nu-1} \exp\left(-\left(\frac{t}{\beta}\right)^\nu\right), \quad (3)$$

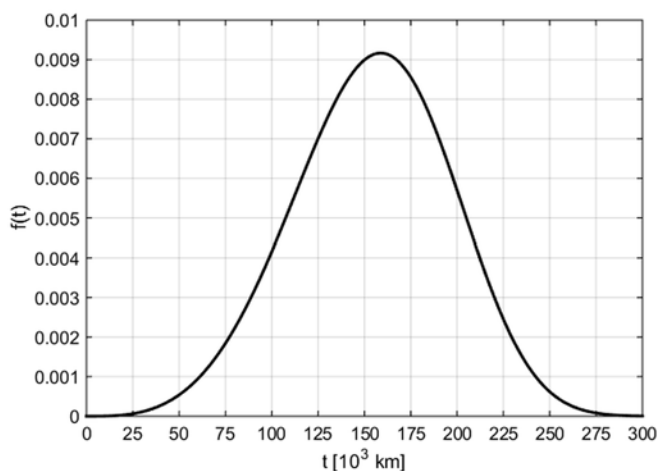


Fig. 5. The probability density function of operation time to failure of the analysed wheel sets

Only failures resulting from wear of vehicle wheels during operation, consisting in exceeding the limit values of wheel profile dimensions specified in the vehicle maintenance system documentation (DSU), are considered. The wheel set operation and maintenance

process implemented at the carrier includes inspections and renewals carried out in accordance with DSU. Inspections are carried out at predetermined times - after a specific mileage and time.

Taking into account the costs of: preventive maintenance -  $k_o = 80000$  [m.u. - monetary units] and post-failure maintenance -  $k_a = 250000$  [m.u.] an optimal preventive maintenance time period of the analysed wheel sets was determined. For this purpose the decision-random models were employed [13, 18] and the value of maintenance time was obtained (expressed in mileage kilometres) of  $t^* = 108000$  [km].

With these data the curve of function  $D(tai)$  was plotted following equation (2), shown in figure 6.

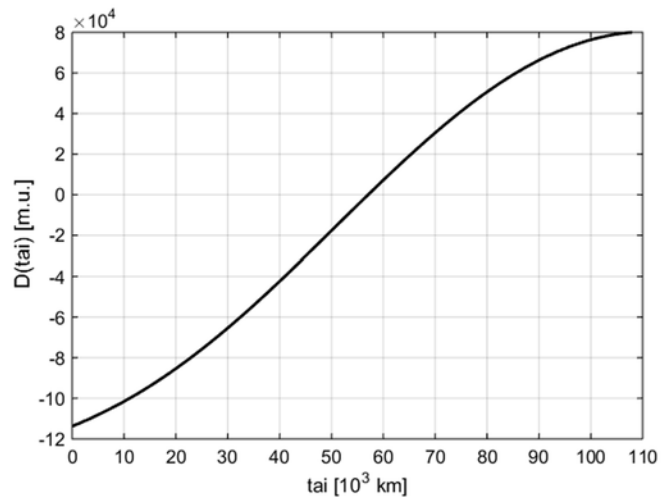


Fig. 6. Function  $D(tai)$  for the analysed wheel set

It implies that the boundary value of mileage  $t_{i\_gr}$  for a wheel set is 58000 [km]. If a failure of a wheel occurs before this mileage is reached, its post-failure maintenance will have to be performed together with the other wheels of the car of the rail unit and again together with all the wheels of the entire system - also in the nearest joint maintenance. If the failure occurs after this time, it is renewed after the failure, but not during the nearest joint maintenance of the wheels of the entire system its operation time is extended until the next maintenance after the nearest joint maintenance.

Using the newly developed simulation models calculations were performed adopting additionally the following values of the parameters for the simulation:  $T_H = 1000000$  [km],  $dt = 5$  [km], the number of simulation repetitions  $n = 10000$ .

Some results from the simulation are tabulated in Table 1. The result found the most important was the maintenance costs. This is confirmed in the operational practice in which the economic aspect of operation is frequently the basis for the assessment of the operating system and maintenance strategy adopted [5]. Consequently, the obtained results are significant indications for decision making in the operation process management.

Table 1 shows mean costs of preventive and post-failure maintenance as well as their totalled value, obtained with the application of both strategies of maintenance of the analysed wheel sets in the simulation adopted horizon.

The analysis of the results shows that in the discussed case lower total costs were obtained when the maintenance adaptive strategy was employed. This indicates that its application may be more cost effective in the real life operation of the modelled technical object.

The significant advantages of the use of computer simulation for the analysis of a system's operation following the presented models is the possibility of obtaining the maintenance costs distributions, which is illustrated in figure 7.

Table 1. Results of simulations

Type of value of maintenance costs	Model of scheduled strategy	Model of adaptive strategy
Mean cost of preventive maintenance [m.u.]	614480	618962
Mean cost of post-failure maintenance [m.u.]	1093025	1069500
Mean total cost of maintenance [m.u.]	1707505	1688462

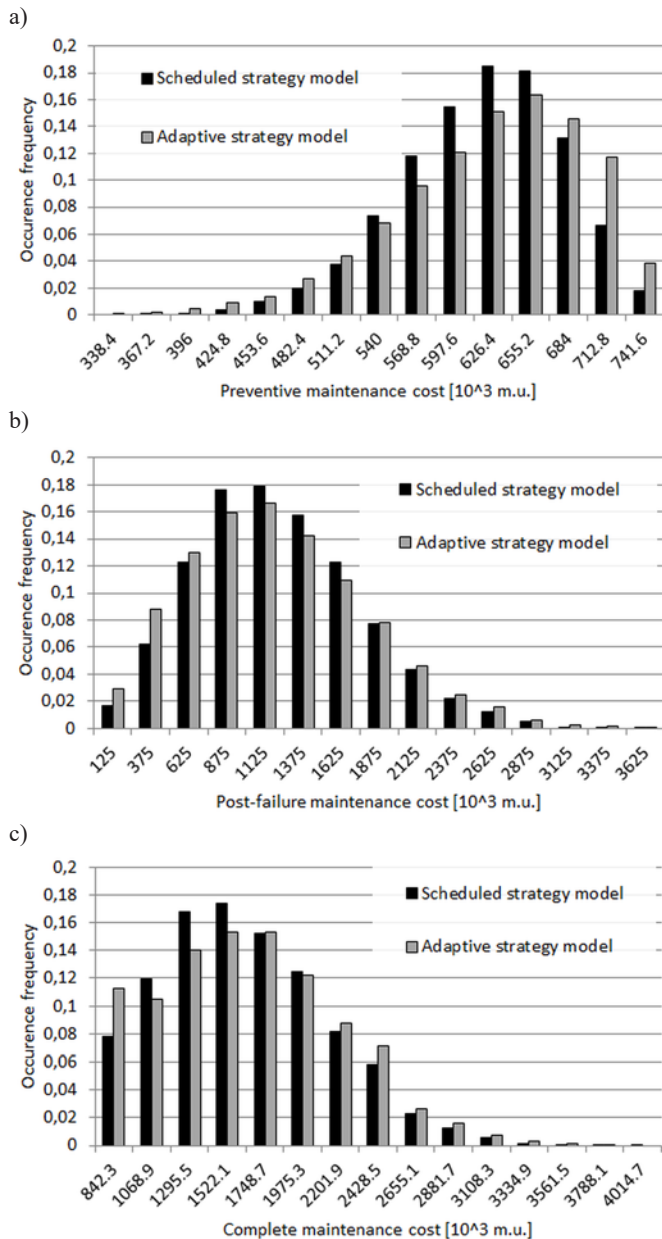


Fig. 7. Distributions of preventive, post-failure and overall maintenance costs for the analysed strategies

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Consequently, simulation enables the assessment of the probability of a maintenance cost not exceeding the intended level.

## 5. Conclusions

The economic indicators of the operation process are one of the most important indicators of the use of vehicles in transport systems. This is because it is the operation management that largely determines the proper functioning of the entire business company. The methods proposed in the paper may significantly affect the rational organisation of inspections and preventive maintenance. Consequently, the choice of a maintenance strategy should include the criterion of cost efficiency. Although the presented strategies do not apparently differ significantly, in actual practice they result in serious economic implications due to considerable costs of maintenance activities.

The presented methods of preventive maintenance scheduling in systems of vehicle operation and maintenance are a useful tool aiding decision making processes and rational use of technical objects. Each model is applicable in various conditions of operation and for complex and structurally differentiated technical objects.

The presented adaptive model of preventive maintenance strategies determination contributes to the development of methods of performing maintenance of complex technical objects.

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