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COMMON CAUSE AND LOAD-SHARING FAILURES-BASED RELIABILITY ANALYSIS FOR PARALLEL SYSTEMS

ANALIZA NIEZAWODNOŚCI SYSTEMÓW RÓWNOLEGŁYCH W SYTUACJI JEDNOCZEŚNIE WYSTĘPUJĄCYCH USZKODZEŃ WYWOŁANYCH WSPÓLNĄ PRZYCZYNĄ ORAZ USZKODZEŃ ELEMENTÓW DZIELĄCYCH OBCIĄŻENIE

For parallel system reliability, the mean time to failure of parallel system under common cause failure (load-sharing failure) is shorter than that of the system without common cause failure (load-sharing failure). The traditional calculation approaches of mean time to failure of parallel systems do not consider the possible effect of common cause and load-sharing failure. However, it may result in the poor accuracy of mean time to failure of parallel system and pose a threat to system reliability. This paper not only considers the effect of common cause failure with stress strength, but also investigates the joint effect of the load-sharing and common cause failures. Besides, the joint failure model of three-dependent-component parallel system are established, and the corresponding properties are analyzed. Finally, a numerical example is used to illustrate the proposed method.

Keywords: *system reliability, mean time to failure, common cause failure, load-sharing failure, parallel system.*

Gdy mowa o niezawodności systemu równoległego, średni czas do uszkodzenia, w przypadku uszkodzenia wywołanego wspólną przyczyną (lub uszkodzenia elementów dzielących obciążenie) jest krótszy niż dla systemu, w którym nie występują tego typu uszkodzenia. Tradycyjne metody obliczania średniego czasu do uszkodzenia systemów równoległych nie uwzględniają potencjalnego wpływu uszkodzeń wywołanych wspólną przyczyną oraz uszkodzeń komponentów dzielących obciążenie. Może to skutkować małą dokładnością tak obliczanego średniego czasu do uszkodzenia systemu równoległego i stanowić zagrożenie dla jego niezawodności. W prezentowanej pracy rozważono nie tylko wpływ uszkodzenia wywołanego wspólną przyczyną dla modelu typu wytrzymałość-obciążenie, ale również wpływ jednocześnie występujących uszkodzeń wywołanych wspólną przyczyną i uszkodzeń elementów dzielących obciążenie. Poza tym opracowano model, w którym omawiane dwa typy uszkodzeń występują jednocześnie w systemie równoległym składającym się z trzech zależnych elementów oraz przeanalizowano właściwości takiego systemu. W artykule przedstawiono przykład numeryczny, który ilustruje zastosowanie proponowanej metody.

Słowa kluczowe: *niezawodność systemu, średni czas do uszkodzenia, uszkodzenie wywołane wspólną przyczyną, uszkodzenie elementów dzielących obciążenie, system równoległy.*

1. Introduction

A. Background

Today's systems are becoming more complex and more sophisticated, and the problems of system reliability are drawing an increasing attention. Common cause failures are critical risk contributors in complex technological systems as they challenge multiple redundant systems simultaneously. Common cause failures can contribute significantly to the overall system unreliability [9]. Therefore, it is important to incorporate common cause failure into the system reliability analysis. Alizadeh et al. [1] introduced the impact of common cause failure on the system reliability using Markov analysis technique. Zuo et al. [23] analyzed the system failure suffering common cause failure. Fan et al. [2] developed a new model for common cause failures considering components degradation based on mathematical framework of Stochastic Hybrid Systems. Levitin [5] adapted the universal generating function method of multistate system reliability analysis to incorporate common-cause failures. Pourali [7] presented presented the importance of considering common cause failure in reliability, availability, and maintainability analysis for industrial and commercial mission-critical facilities and high-reliability organizations. Vaurio

[11] incorporated common-cause failures into system analysis by an implicit method and discussed the possible limitations and extensions. Wang et al. [13] incorporated effects of probabilistic common cause failures into system reliability analysis. Wang et al. [14] proposed an explicit method and an implicit method to analyze the reliability of systems. Xiao and Gao [15] proposed efficient simulation methods to assess the system reliability with input uncertainty. Xiao et al. [16] presented a data simulation approach to estimating the system failure probability in the presence of stochastic constraints. Yuan [17] extended the pivotal decomposition method for system availability and failure frequency from the case where components are statistically independent to that where components are also subject to common-cause failures.

Load-sharing is always an essential nature in parallel system. Huang et al. [3] presented a general closed-form expression for lifetime reliability of load-sharing redundant systems. Liu [6] developed a model to calculate the reliability of a load-sharing system which is composed of non-identical components each having an arbitrary failure time distribution. Paula et al. [8] analyzed the optimization in redundant system considering load sharing. Jiang et al. [4] formulated two load optimization models to identify the optimal loading strategy. Sutar et al. [10] modeled the load sharing phenomenon in a k-out-of-

m system through the accelerated failure time model. Wang et al. [12] presented three policies for load assignment among unequal strength components and compared three of these policies. Ye et al. [18] developed a model for a load sharing system where an operator dispatches work load to components in a manner that manages their degradation. He assumed degradation is the dominant failure type, and that the system will not be subject to sudden failure due to a shock. Yang et al. [19] proposed a novel approach for assessing a systems' reliability with dependency structures, load sharing, and damage accumulation and reversal. Zhao et al. [20] presented a reliability modeling and analysis framework for load-sharing systems with identical components subject to continuous degradation. Zhang et al. [21] proposed a new reliability analysis method for the load-sharing k-out-of-n: F system based on the load-strength model. Zhang et al. [22] presented a two-component load-sharing system. And the failure rates of the two components are time dependent and load dependent.

B. Motivation

Undoubtedly, above researches has contributed to the development of reliability of parallel system. Some of them propose excellent methods to calculate the mean time to failure of system, rest of them help to investigate the reliability of system under common cause failure or load-sharing failure respectively. However, lots of researches often ignore the joint of common cause and load-sharing failure in terms of the failure analysis of the parallel system with stress strength. Some parallel systems often happen simultaneously common cause and load-sharing failures. The results tend to be over positive than factual information. In this paper, for parallel systems, common cause failure model with stress strength and joint failure model of load-sharing and common cause failures are established respectively. Based on these models, the results are more approaching to the realistic situation considering the mean time to failure of parallel systems under common cause and load-sharing failures.

The rest of this paper is organized as follows. Section 2 analyzes the reliability model with stress strength under common cause failure. The reliability model under common cause and load-sharing failure is presented in section 3. Section 4 utilizes a numerical example to testify the validity of the proposed model. Finally, the conclusions of this paper are given in Section 5.

2. Reliability analysis with stress strength under common cause failure

Generally, x and y denote stress and strength respectively, $f_x(x)$ and $f_y(y)$ denote stress probability density function and strength probability density function respectively. We suppose parallel system is composed of n components. The probability of all components failure in the system is system conditional failure probability, so statistical average of system conditional failure probability under common cause failure is $p_s^n = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^n dx$ where $x \in (0, +\infty)$. We utilize the model to calculate conditional failure probability of two-dependent-component and three-dependent-component parallel system respectively, and compare ultimate consequence.

According to above model, conditional failure probability of two-dependent-component parallel system is $p_s^2 = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^2 dx$, and conditional failure probability of three-dependent-component parallel system is $p_s^3 = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^3 dx$. Now we need to compare them. Because $\int_0^{+\infty} f_y(y)dy = 1$, we could get $0 \leq \int_0^x f_y(y)dy \leq 1$.

$\int_0^x f_y(y)dy = 1$ when y is not more than x forever. However, y could be more than x. Thus, $0 \leq \int_0^x f_y(y)dy < 1$. Based on relative mathematical knowledge, we can get $p_s^3 < p_s^2$. Obviously, conditional failure probability of three-dependent-component parallel system under common cause failure is less than two-dependent-component parallel system, which shows that we could decrease system conditional failure probability by increasing a redundant component.

For a parallel system under common cause failure with n components, if statistical average of system conditional failure probability satisfies $p_s^n = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^n dx$, where x denotes stress, three properties could be deduced.

Property 1: conditional failure probability of three-dependent-component parallel system under common cause failure is less than two-dependent-component parallel system, where $x \in (0, +\infty)$.

Proof: $p_s^2 = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^2 dx$, $p_s^3 = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^3 dx$. Because $0 < f_x(x)\left[\int_0^x f_y(y)dy\right]^3 < f_x(x)\left[\int_0^x f_y(y)dy\right]^2$, based on relative mathematical knowledge, we could deduce $\int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^3 dx < \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^2 dx$. That is $p_s^3 < p_s^2$.

Property 2: conditional failure probability of k-dependent-component parallel system under common cause failure is less than (k-1)-dependent-component parallel system, where $x \in (0, +\infty)$.

Proof: $p_s^{k-1} = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^{k-1} dx$,

$p_s^k = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^k dx$. According to property 1, we can deduce $p_s^k < p_s^{k-1}$.

Property 3: system conditional failure probability approaches 0 when n approaches infinity, that is to say, $\lim_{n \to +\infty} p_s^n \approx 0$.

Proof: $p_s^n = \int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^n dx$, because $\int_0^x f_y(y)dy < 1$, $f_x(x)\left[\int_0^x f_y(y)dy\right]^n$ approaches infinitesimal when n approaches infinity. Thus, $\int_0^{+\infty} f_x(x)\left[\int_0^x f_y(y)dy\right]^n dx \approx 0$, that is to say, $\lim_{n \to +\infty} p_s^n \approx 0$.

Besides, $F_x(x)$ and $F_y(y)$ denote stress distribution function and strength distribution function. We suppose parallel system is composed of n components. Reliability of parallel system under common cause failure is $R_s = \int_{-\infty}^{+\infty}\left\{1 - \left[\int_{-\infty}^x f_y(y)dy\right]^n\right\} f_x(x)dx = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^n\right\} f_x(x)dx$ [13], where $x \in (0, +\infty)$.

According to above model, reliability of two-dependent-component parallel system is $R_s(2) = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^2\right\} f_x(x)dx$, and reliability of three-dependent-component parallel system is $R_s(3) = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^3\right\} f_x(x)dx$. Now we need to compare them.

$0 \le F_y(x) \le 1$, $F_y(x) = 1$ when y is not more than x forever. However, y could be more than x. Thus, $0 \le F_y(x) < 1$, according to above analysis, we can deduce $\left\{1 - \left[F_y(x)\right]^3\right\} f_x(x) > \left\{1 - \left[F_y(x)\right]^2\right\} f_x(x)$.

Based on relative mathematical knowledge, we could deduce $R_s(2) < R_s(3)$. That is to say, reliability of three-dependent-component parallel system is more than two-dependent-component parallel system. Thus, we could deduce that increasing a redundant component would enhance system reliability.

For a parallel system under common cause failure with n components, if system reliability satisfies:

$$R_s = \int_{-\infty}^{+\infty}\left\{1 - \left[\int_{-\infty}^{x} f_y(y)dy\right]^n\right\} f_x(x)dx = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^n\right\} f_x(x)dx ,$$

where x and y denote stress and strength respectively, three properties could be deduced.

Property 4: Reliability of three-dependent-component parallel system under common cause failure is more than two-dependent-component parallel system, where $x \in (0, +\infty)$.

Proof: $R_s(2) = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^2\right\} f_x(x)dx$,

$R_s(3) = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^3\right\} f_x(x)dx$. Because $F_y(x) < 1$, we could get $\left[F_y(x)\right]^3 < \left[F_y(x)\right]^2$, and we could to deduce $1 - \left[F_y(x)\right]^2 < 1 - \left[F_y(x)\right]^3$. Thus, we could take a further step to deduce $\left\{1 - \left[F_y(x)\right]^2\right\} f_x(x) < \left\{1 - \left[F_y(x)\right]^3\right\} f_x(x)$. Based on relative mathematical knowledge, we get $R_s(2) < R_s(3)$.

Property 5: Reliability of k-dependent-component parallel system under common cause failure is more than (k-1)-dependent-component parallel system, where $x \in (0, +\infty)$.

Proof: $R_s(k-1) = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^{k-1}\right\} f_x(x)dx$,

$R_s(k) = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^k\right\} f_x(x)dx$. According to derivation way of property 4, we could deduce $R_s(k-1) < R_s(k)$.

Property 6: Parallel system reliability approaches 1 when n approaches infinity, that is to say, $\lim_{n \to +\infty} R_s(n) \approx 1$.

Proof: $R_s(n) = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^n\right\} f_x(x)dx$, $\left[F_y(x)\right]^n < 1$. Thus,

$1 - \left[F_y(x)\right]^n \approx 1$ and

$R_s(n) = \int_{-\infty}^{+\infty}\left\{1 - \left[F_y(x)\right]^n\right\} f_x(x)dx \approx \int_{-\infty}^{+\infty} f_x(x)dx = 1$ when n approaches infinity.

3. Reliability analysis under load-sharing and common cause failures

We assume a system is composed of three same components. All components share whole system load and failure rate of each component is λ_3, when system works normally. Failure rate will become λ_2 with one component failed. When two components fail, failure

rate will become λ_1. When there is one component working in the system, the common cause failure rate is λ_{c1}, when there are two components working in the system, the common cause failure rate is λ_{c2}, and when all of the three components are working normally, the common cause failure rate is λ_{c3}. We have merely one maintenance device which repairs randomly one failed component once, and other failed components must wait until last one has worked normally. With one component failed, μ_3 denote mean time to maintenance and maintenance rate respectively. With two components failed, μ_2 denote mean time to maintenance and maintenance rate respectively. With three components failed, μ_1 denote mean time to maintenance and maintenance rate respectively. According to the above assumption, we can describe the state transition figure of three-dependent-component parallel system under common cause and load-sharing failure as Fig. 1.
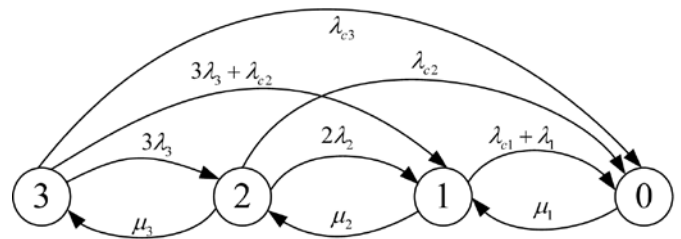

Fig. 1. state transition under common cause and load-sharing failures

As is shown in Fig. 1, based on state transition figure, we establish transition intensity matrix for calculation of system mean time to failure, and A denotes transition intensity matrix:

$$A = \begin{pmatrix} -\mu_1 & \mu_1 & 0 & 0 \\ \lambda_{c1}+\lambda_1 & -(\lambda_{c1}+\lambda_1+\mu_2) & \mu_2 & 0 \\ \lambda_{c2} & 2\lambda_2 & -(\lambda_{c2}+2\lambda_2+\mu_3) & \mu_3 \\ \lambda_{c3} & 3\lambda_3+\lambda_{c2} & 3\lambda_3 & -(6\lambda_3+\lambda_{c2}+\lambda_{c3}) \end{pmatrix}$$

The state 0 is absorbing state, therefore, we need to omit all elements in the system that is related to state 0. And B denotes a transition intensity matrix:

$$B = \begin{pmatrix} -(\lambda_{c1}+\lambda_1+\mu_2) & \mu_2 & 0 \\ 2\lambda_2 & -(\lambda_{c2}+2\lambda_2+\mu_3) & \mu_3 \\ 3\lambda_3+\lambda_{c2} & 3\lambda_3 & -(6\lambda_3+\lambda_{c2}+\lambda_{c3}) \end{pmatrix}$$

We have $C = \begin{bmatrix} q_1(0) & q_2(0) & q_3(0) \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}$, where state transition equation is $CB = D$. Therefore, we could get the following equation:

$$\begin{bmatrix} q_1(0) & q_2(0) & q_3(0) \end{bmatrix} \begin{pmatrix} -(\lambda_{c1}+\lambda_1+\mu_2) & \mu_2 & 0 \\ 2\lambda_2 & -(\lambda_{c2}+2\lambda_2+\mu_3) & \mu_3 \\ 3\lambda_3+\lambda_{c2} & 3\lambda_3 & -(6\lambda_3+\lambda_{c2}+\lambda_{c3}) \end{pmatrix} = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}$$

Considering the complexity of equation and the accuracy of calculation, we can get $q_1(0), q_2(0), q_3(0)$ by using the math software. Then the mean time to failure of three-dependent-component parallel system is $\mathrm{MTTF}_s(3) = q_1(0) + q_2(0) + q_3(0)$. But the solution is too complex, we cannot use it to get some useful message, so we should make some assumptions to simplify the solution.

Assumptions 1: No matter how many components are working in the system, λ_c denotes common cause failure rate.

Assumptions 2: The failure rate decrease linearly with the decline of the quantity of the components which are working in the system, this is, if $\lambda_1 = \lambda_e$, than we will get $\lambda_2 = 2\lambda_e$ and $\lambda_3 = 3\lambda_e$.

Assumptions 3: No matter how many components are broken, the maintenance rate is common, it is μ_1.

Than we can get the simple version of the solution as follows:

$$\text{MTTF}_s(3) - \text{MTTF}_s(2)$$

$$= \frac{-\left(\lambda_c^2 + 7\lambda_c\lambda_e + 5\frac{1}{4}_e + 12\lambda_e^2 + 9\frac{1}{4}\lambda_e + 6\frac{1}{4}\right)\left(3\lambda_c\lambda_e - 3\lambda_e^2\right)}{\left(\lambda_c^2 + 7\lambda_c\lambda_e + 2\frac{1}{4}_e + 12\lambda_e^2\right)\left(2\lambda_c^3 + 20\lambda_c^2\lambda_e + 10\lambda_c^2\frac{1}{4} + 66\lambda_c\lambda_e^2 + 39\frac{1}{4}\lambda_e + 6\lambda_c^{\frac{1}{4}} + 72\lambda_e^3 + 27\lambda_e^2\frac{1}{4}\right)}$$

However, the difference between the mean time to failure of four-dependent-component parallel system and three-dependent-component parallel system is more complex, even if it has been simplified, so it is hardly to find the same regular. Through the assumption we have made, we also can simplify the result of $q_1(0)$, $q_2(0)$, $q_3(0)$, they are:

$$q_1(0) = \frac{\lambda_c^2 + 7\lambda_c\lambda_e + 3\mu\lambda_c + 24\lambda e^2 + 9\mu\lambda_e}{2\lambda_c^3 + 20\lambda_c^2\lambda_e + 8\lambda_c^2\mu + 66\lambda_c\lambda_e^2 + 25\mu\lambda_c\lambda_e + 72\lambda_e^3 + 3\lambda_e^2\mu - 9\lambda_e\mu^2}$$

$$q_2(0) = \frac{3\lambda_c\lambda_e + 2\mu\lambda_c + 9\mu\lambda_e + 9\lambda_e^2}{2\lambda_c^3 + 20\lambda_c^2\lambda_e + 8\lambda_c^2\mu + 66\lambda_c\lambda_e^2 + 25\mu\lambda_c\lambda_e + 72\lambda_e^3 + 3\lambda_e^2\mu - 9\lambda_e\mu^2}$$

$$q_3(0) = \frac{\lambda_c^2 + 7\lambda_c\lambda_e + 4\mu\lambda_c + 12\lambda_e^2 + 5\mu\lambda_e + 3\mu^2}{2\lambda_c^3 + 20\lambda_c^2\lambda_e + 8\lambda_c^2\mu + 66\lambda_c\lambda_e^2 + 25\mu\lambda_c\lambda_e + 72\lambda_e^3 + 3\lambda_e^2\mu - 9\lambda_e\mu^2}$$

And then we can analyze the rate's influence of the component of the mean time to failure of three-dependent-component parallel system.

Firstly, we focus on the influence of λ_e to these components. As the denominator of the three components are same, when λ_e changes, there are the same changes happen in those denominators, so it is ok for us that do not care about the denominators, the only thing we should do is focusing on the numerator. We set the numerator of $q_1(0)$ is $Y_{1(\lambda_e)}$, taking the derivative of this function, we can get $Y_{1(\lambda_e)}' = 7\lambda_c + 9\mu + 48\lambda_e$, similarly we can get the derivative of the numerator of $q_2(0)$ and $q_3(0)$, it is $Y_{2(\lambda_e)}' = 3\lambda_c + 9\mu + 18\lambda_e$ and $Y_{3(\lambda_e)}' = 7\lambda_c + 5\mu + 24\lambda_e$, and we know that these rates are positive, thus it is obviously that $Y_{1(\lambda_e)}'$ is the most through the three.

Secondly, analyzing the influence of λ_c, similarly we should focus on the numerator only. We set the numerator of $q_1(0)$ is $Y_{1(\lambda_c)}$, taking the derivative of this function, we can get $Y_{1(\lambda_c)}' = 7\lambda_e + 3\mu + 2\lambda_c$, in the same way we can get the derivative of the numerator of $q_2(0)$ and $q_3(0)$, it is $Y_{2(\lambda_c)}' = 3\lambda_e + 2\mu$ and $Y_{3(\lambda_c)}' = 7\lambda_e + 4\mu + 2\lambda_c$, we can find that $Y_{3(\lambda_c)}' > Y_{1(\lambda_c)}' > Y_{2(\lambda_c)}'$.

Finally, focusing on μ, also, the numerator is the only thing that we should care about. Setting the numerator of $q_1(0)$ is $Y_{1(\mu)}$, taking the derivative of this function, we can get $Y_{1(\mu)}' = 3\lambda_c + 9\lambda_e$, similarly we can get the derivative of the numerator of $q_2(0)$ and $q_3(0)$, it is $Y_{2(\mu)}' = 2\lambda_c + 9\lambda_e$ and $Y_{3(\mu)}' = 4\lambda_c + 5\lambda_e + 6\mu$. But, as we cannot ensure the relative size of the rates, it is $Y_{1(\mu)}' > Y_{2(\mu)}'$ that we can find only.

Considering the above analysis, we can get some properties as follows.

Property 7: The failure rate λ_e has the most influence on the system when there is only a component working in.

Proof: Under the premise that all of the three rates are positive, consider the derivatives above, $Y_{1(\lambda_e)}' = 7\lambda_c + 9\mu + 48\lambda_e$, $Y_{2(\lambda_e)}' = 3\lambda_c + 9\mu + 18\lambda_e$ and $Y_{3(\lambda_e)}' = 7\lambda_c + 5\mu + 24\lambda_e$, we can find that $Y_{1(\lambda_e)}'$ is the most one, so we can say the failure rate λ_e has the most influence to the system when there is only a component working in.

Property 8: The common cause failure rate λ_c has the most influence on the system when there are three components working in, has the second most influence on it when there are two components working in, and has the least influence on the system when there is only a component working in.

Proof: Under the premise that all of the three rates are positive, consider the derivatives above, $Y_{1(\lambda_c)}' = 7\lambda_e + 3\mu + 2\lambda_c$, $Y_{2(\lambda_c)}' = 3\lambda_e + 2\mu$ and $Y_{3(\lambda_c)}' = 7\lambda_e + 4\mu + 2\lambda_c$, we can find that $Y_{3(\lambda_c)}' > Y_{1(\lambda_c)}' > Y_{2(\lambda_c)}'$. Therefore, we can get the view that the common cause failure rate λ_c has the most influence on the system when there are three components working in, has the second most influence on it when there are two components working in, and has the least influence on the system when there is only a component working in.

Property 9: The maintenance rate μ influences the system when there is only a component working in more than when there are two components working in.

Proof: Under the premise that all of the three rates are positive, consider the derivatives above, $Y_{1(\mu)}' = 3\lambda_c + 9\lambda_e$, $Y_{2(\mu)}' = 2\lambda_c + 9\lambda_e$ and $Y_{3(\mu)}' = 4\lambda_c + 5\lambda_e + 6\mu$, we can easily find that $Y_{1(\mu)}' > Y_{2(\mu)}'$, so we can say the maintenance rate μ influences the system when there is only a component working in more than when there are two components working in.

4. Numerical example

In this section, we will have an analysis about a parallel system of three components under common cause and load-sharing failure. This section mainly studies the effect of single variance on the reliability of parallel system. We assume the reliability parameters are $\lambda_a = 1.12 \times 10^{-3} h^{-1}$, $\lambda_m = 2 \times 10^{-3} h^{-1}$, $\lambda_f = 5 \times 10^{-3} h^{-1}$, $\lambda_c = 3 \times 10^{-4} h^{-1}$, $MDT_a = 16h$, $\mu_a = 5.2 \times 10^{-3} h^{-1}$, $MDT_k = 12h$, $\mu_k = 8.3 \times 10^{-2} h^{-1}$, $MDT_f = 8h$, $\mu_f = 1.25 \times 10^{-1} h^{-1}$.

4.1. The effect of each failure rate on mean time to failure of parallel system

(1) Effect of λ_a variation on MTTF_s

λ_a is defined as independent variable, and its range of values is $\left[0, 2 \times 10^{-3}\right]$. Dependent variable is MTTF_s. We can calculate mean time to failure of two-dependent-component and three-dependent-component parallel system under common cause and load-sharing failure.

$$\text{MTTF}_s(2) = p_1(0) + p_2(0) = \frac{\lambda_c + \lambda_f + 2\lambda_m + \mu_k}{\left(\lambda_c + 2\lambda_m\right)\left(\lambda_c + \lambda_f + \mu_k\right) - 2\lambda_m\mu_k} = 1935.4(h)$$

$$\text{MTTF}_s(3) = q_1(0) + q_2(0) + q_3(0) =$$

$$\frac{6\lambda_a\lambda_c + 3\lambda_a\lambda_f + 12\lambda_a\lambda_m + 2\lambda_c^2 + \lambda_c\lambda_f + 4\lambda_c\lambda_m + 2\lambda_f\lambda_m + 3\lambda_a\mu_a + 6\lambda_a\mu_k + 2\lambda_c\mu_a + 2\lambda_c\mu_k + \lambda_f\mu_a + \mu_a\mu_k}{\left(\lambda_c + \lambda_f + \mu_k\right)\left(\lambda_c + 2\lambda_m + \mu_a\right)\left(6\lambda_a + 2\lambda_c\right) - 3\lambda_a\mu_a\left(\lambda_c + \lambda_f + 2\mu_k\right) - \mu_k\left(12\lambda_a\mu_a + 4\lambda_c\lambda_m + \lambda_c\mu_a\right)} =$$

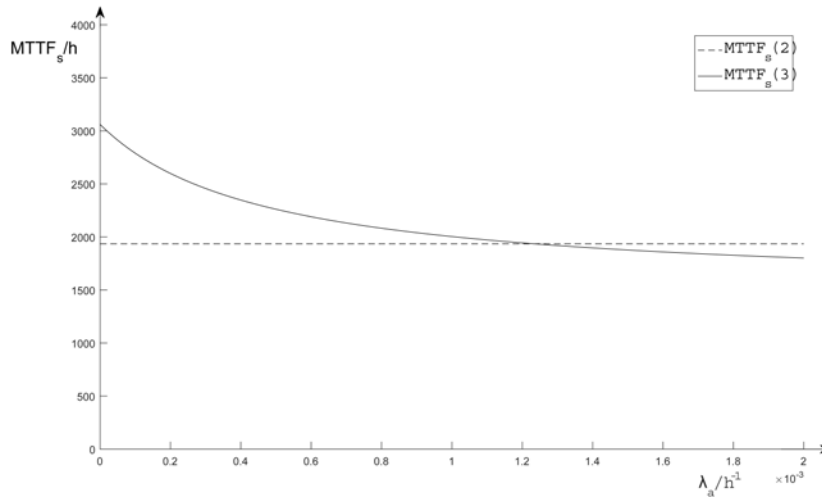$$\frac{0.5544\lambda_a + 5.346 \times 10^{-4}}{3.6882 \times 10^{-4}\lambda_a + 1.7463 \times 10^{-7}}$$

Fig. 2. Effect of λ_a variation on $\mathrm{MTTF_s}$

Fig. 2 describes the effect λ_a variation on $\mathrm{MTTF_s}(2)$ and $\mathrm{MTTF_s}(3)$. Firstly, $\mathrm{MTTF_s}(2)$=1935.4(h), λ_a have no effect on mean time to failure of two-dependent-component parallel system. Secondly, mean time to failure of three-dependent-component parallel system is negatively correlated with λ_a. Thirdly,

$$\begin{cases} \mathrm{MTTF_s}(3)>\mathrm{MTTF_s}(2), 0 \le \lambda_a < 1.233 \times 10^{-3} \\ \mathrm{MTTF_s}(3)=\mathrm{MTTF_s}(2), \lambda_a = 1.233 \times 10^{-3} \\ \mathrm{MTTF_s}(3)<\mathrm{MTTF_s}(2), 1.233 \times 10^{-3} < \lambda_a \le 2 \times 10^{-3} \end{cases}$$, so the three-

dependent-component parallel system is prior to two-dependent-component parallel system when $0 \le \lambda_a < 1.233 \times 10^{-3}$.

(2) The effect of λ_m variation on $\mathrm{MTTF_s}$

λ_m is defined as independent variable, and its range of values is $\left[0, 5 \times 10^{-3}\right]$. Dependent variable is $\mathrm{MTTF_s}$. We can calculate mean time to failure of two-dependent-component and three-dependent-component parallel system under common cause and load-sharing failure.

$$\mathrm{MTTF_s}(2)= p_1(0)+ p_2(0)= \frac{\lambda_c + \lambda_f + 2\lambda_m + \mu_k}{(\lambda_c + 2\lambda_m)(\lambda_c + \lambda_f + \mu_k) - 2\lambda_m \mu_k} = \frac{2\lambda_m + 8.830 \times 10^{-2}}{1.060 \times 10^{-2}\lambda_m + 2.649 \times 10^{-5}}$$

$$\mathrm{MTTF_s}(3) = q_1(0)+ q_2(0)+ q_3(0)=$$

$$\frac{6\lambda_a\lambda_c + 3\lambda_a\lambda_f + 12\lambda_a\lambda_m + 2\lambda_c^2 + \lambda_c\lambda_f + 4\lambda_c\lambda_m + 2\lambda_f\lambda_m + 3\lambda_a\mu_a + 6\lambda_a\mu_k + 2\lambda_c\mu_a + 2\lambda_c\mu_k + \lambda_f\mu_a + \mu_a\mu_k}{(\lambda_c + \lambda_f + \mu_k)(\lambda_c + 2\lambda_m + \mu_a)(6\lambda_a + 2\lambda_c) - 3\lambda_a\mu_a(\lambda_c + \lambda_f + 2\mu_k) - \mu_k(12\lambda_a\lambda_m + 4\lambda_c\lambda_m + \lambda_c\mu_a)} =$$

$$\frac{2.464\lambda_m + 1.106 \times 10^{-1}}{7.759 \times 10^{-3}\lambda_m + 4.325 \times 10^{-5}}$$

Now we describe the effect λ_m variation on $\mathrm{MTTF_s}(2)$ and $\mathrm{MTTF_s}(3)$ in Fig. 3.

Firstly, mean time to failure of three-dependent-component and two-dependent-component parallel system is negatively correlated

with λ_m. Secondly, $\begin{cases} \mathrm{MTTF_s}(3)<\mathrm{MTTF_s}(2), 0 \le \lambda_m < 1.832 \times 10^{-3} \\ \mathrm{MTTF_s}(3)=\mathrm{MTTF_s}(2), \lambda_m = 1.832 \times 10^{-3} \\ \mathrm{MTTF_s}(3)>\mathrm{MTTF_s}(2), 1.832 \times 10^{-3} < \lambda_m \le 5 \times 10^{-3} \end{cases}$,

so the three-dependent-component parallel system is prior to two-dependent-component parallel system when $\lambda_m \in \left[1.832 \times 10^{-3}, 5 \times 10^{-3}\right]$.

(3)The effect of λ_f variation on $\mathrm{MTTF_s}$

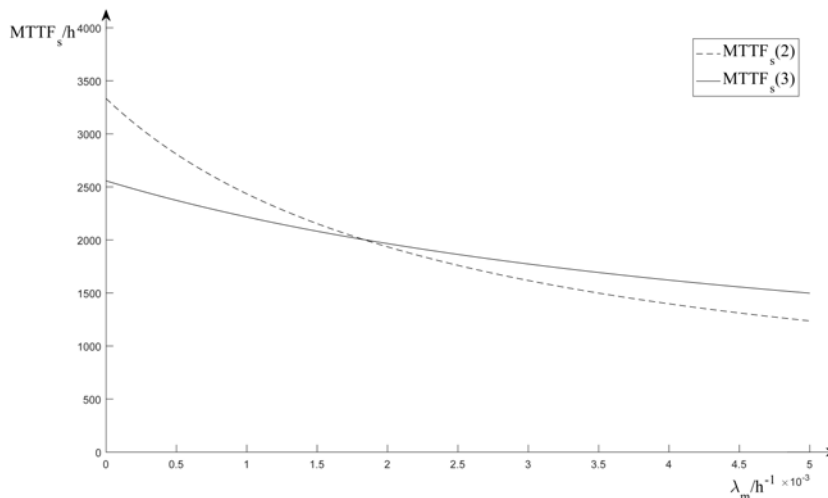λ_f is defined as independent variable, and its range of values is $\left[2 \times 10^{-3}, 1 \times 10^{-2}\right]$.
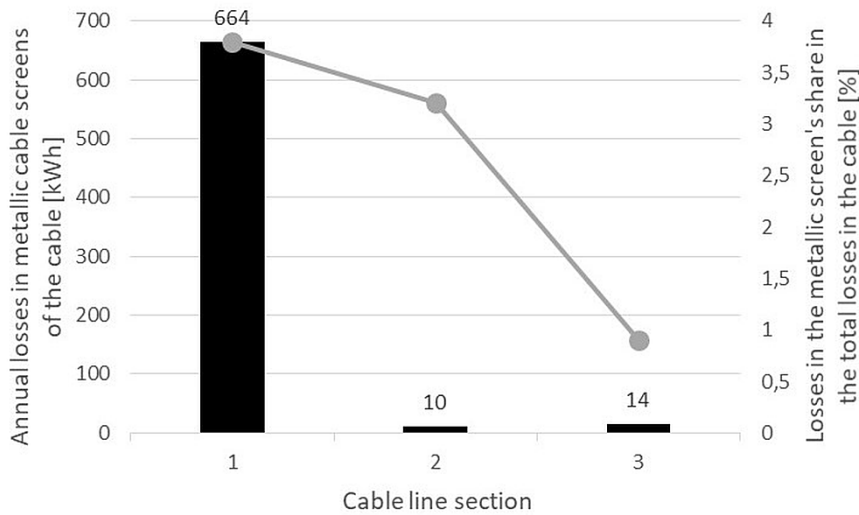

Fig. 3. Effect of λ_m variation on $\mathrm{MTTF_s}$

Fig. 4. Effect of λ_f variation on $\mathrm{MTTF_s}$

$$\mathrm{MTTF_s}(2)= p_1(0)+ p_2(0)= \frac{\lambda_c + \lambda_f + 2\lambda_m + \mu_k}{(\lambda_c + 2\lambda_m)(\lambda_c + \lambda_f + \mu_k)- 2\lambda_m\mu_k}= \frac{\lambda_f + 8.730\times10^{-2}}{4.300\times10^{-3}\lambda_f + 2.619\times10^{-5}}$$

$$\mathrm{MTTF_s}(3)= q_1(0)+ q_2(0)+ q_3(0)=$$

$$\frac{6\lambda_a\lambda_c + 3\lambda_a\lambda_f + 12\lambda_a\lambda_m + 2\lambda_c^2 + \lambda_c\lambda_f + 4\lambda_c\lambda_m + 2\lambda_f\lambda_m + 3\lambda_a\mu_a + 6\lambda_a\mu_k + 2\lambda_c\mu_a + 2\lambda_c\mu_k + \lambda_f\mu_a + \mu_a\mu_k}{(\lambda_c + \lambda_f + \mu_k)(\lambda_c + 2\lambda_m + \mu_a)(6\lambda_a + 2\lambda_c)- 3\lambda_a\mu_a(\lambda_c + \lambda_f + 2\mu_k)- \mu_k(12\lambda_a\lambda_m + 4\lambda_c\lambda_m + \lambda_c\mu_a)}=$$

$$\frac{1.286\lambda_f + 1.091\times10^{-1}}{5.207\times10^{-3}\lambda_f + 3.273\times10^{-5}}$$

As is shown in Fig. 4, the three-dependent-component parallel system is prior to two-dependent-component parallel system when $\lambda_f \in \left[2\times10^{-3}, 1\times10^{-2}\right]$.

(4)The effect of λ_c variation on $\mathrm{MTTF_s}$

λ_c is defined as independent variable, and its range of values is $\left[0, 1\times10^{-3}\right]$. Dependent variable is $\mathrm{MTTF_s}$. We can calculate mean time to failure of two-dependent-component and three-dependent-component parallel system under common cause and load-sharing failure:

$$\mathrm{MTTF_s}(2)= p_1(0)+ p_2(0)= \frac{\lambda_c + \lambda_f + 2\lambda_m + \mu_k}{(\lambda_c + 2\lambda_m)(\lambda_c + \lambda_f + \mu_k)- 2\lambda_m\mu_k}= \frac{(\lambda_c + 9.200\times10^{-2})}{(\lambda_c + 8.800\times10^{-2})(\lambda_c + 4\times10^{-3})- 3.32\times10^{-4}}$$
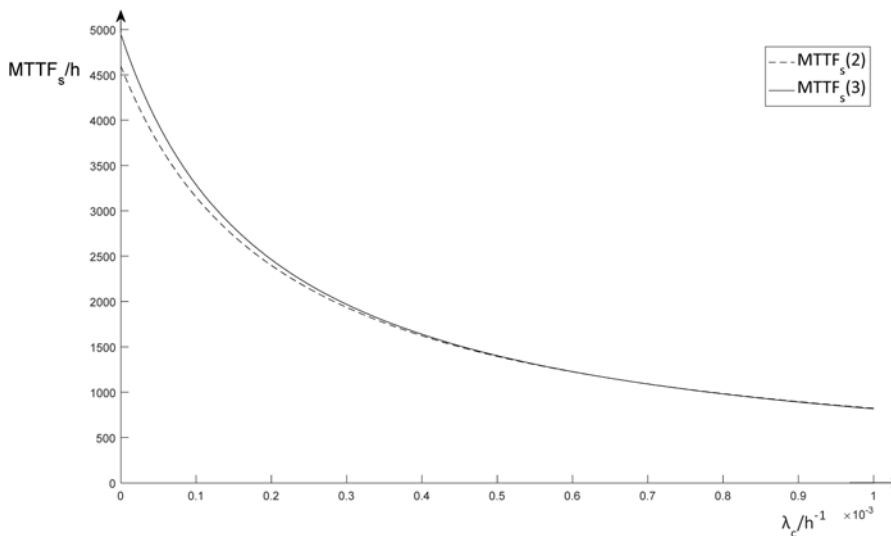
$$\mathrm{MTTF_s}(3)= q_1(0)+ q_2(0)+ q_3(0)=$$

$$\frac{6\lambda_a\lambda_c + 3\lambda_a\lambda_f + 12\lambda_a\lambda_m + 2\lambda_c^2 + \lambda_c\lambda_f + 4\lambda_c\lambda_m + 2\lambda_f\lambda_m + 3\lambda_a\mu_a + 6\lambda_a\mu_k + 2\lambda_c\mu_a + 2\lambda_c\mu_k + \lambda_f\mu_a + \mu_a\mu_k}{(\lambda_c + \lambda_f + \mu_k)(\lambda_c + 2\lambda_m + \mu_a)(6\lambda_a + 2\lambda_c)- 3\lambda_a\mu_a(\lambda_c + \lambda_f + 2\mu_k)- \mu_k(12\lambda_a\lambda_m + 4\lambda_c\lambda_m + \lambda_c\mu_a)}=$$

$$\frac{2\lambda_c^2 + 0.196\lambda_c + 1.097\times10^{-3}}{(2\lambda_c + 6.72\times10^{-3})(\lambda_c + 8.8\times10^{-2})(\lambda_c + 9.2\times10^{-3})- 1.113\times10^{-3}\lambda_c - 5.219\times10^{-6}}$$

Now we describe the figure of the effect λ_c variation on $\mathrm{MTTF_s}(2)$ and $\mathrm{MTTF_s}(3)$ in Fig. 5.

Firstly, mean time to failure of three-dependent-component and two-dependent-component parallel system is negatively correlated with λ_m. Secondly, $\begin{cases} \mathrm{MTTF_s}(3) > \mathrm{MTTF_s}(2), 0 \le \lambda_c < 5\times10^{-4} \\ \mathrm{MTTF_s}(3) = \mathrm{MTTF_s}(2), 5\times10^{-4} \le \lambda_c \le 1\times10^{-3} \end{cases}$, so the three-dependent-component parallel system is prior to two-dependent-component parallel system when $\lambda_c \in \left[0, 5\times10^{-4}\right]$.

Fig. 5. Effect of λ_c variation on $\mathrm{MTTF_s}$

4.2. The effect of each maintenance rate on mean time to failure of parallel system

(1)The effect of μ_a variation on $\mathrm{MTTF_s}$

μ_a is defined as independent variable, and its range of values is $\left[0, 8.3 \times 10^{-2}\right]$. Dependent variable is $\mathrm{MTTF_s}$. We can calculate mean time to failure of two-dependent-component and three-dependent-component parallel system under common cause and load-sharing failure.

$$\mathrm{MTTF_s}(2) = p_1(0) + p_2(0) = \frac{\lambda_c + \lambda_f + 2\lambda_m + \mu_k}{(\lambda_c + 2\lambda_m)(\lambda_c + \lambda_f + \mu_k) - 2\lambda_m\mu_k} \approx 1935.4(h),$$

$$\mathrm{MTTF_s}(3) = q_1(0) + q_2(0) + q_3(0) = \frac{9.196\mu_a + 6.773 \times 10^{-2}}{4.589 \times 10^{-3}\mu_a + 3.491 \times 10^{-5}}.$$

In Fig. 6, the mean time to failure of three-dependent-component parallel system is weakly positive correlation with μ_a. When $\mu_a = 0$,
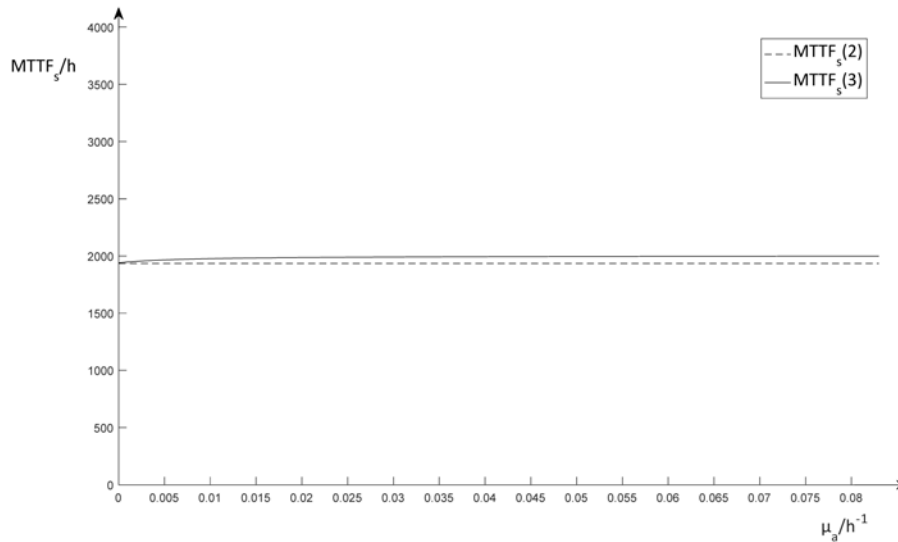
$\mathrm{MTTF_s}(3) = \mathrm{MTTF_s}(2)$. The three-dependent-component parallel system is prior to two-dependent-component parallel system when $\mu_a \in \left[0, 8.3 \times 10^{-2}\right]$.

(2)The effect of μ_k variation on $\mathrm{MTTF_s}$

μ_k is defined as independent variable, and its range of values is $\left[5.2 \times 10^{-3}, 0.125\right]$. Dependent variable is $\mathrm{MTTF_s}$.

$$\mathrm{MTTF_s}(2) = p_1(0) + p_2(0) = \frac{\lambda_c + \lambda_f + 2\lambda_m + \mu_k}{(\lambda_c + 2\lambda_m)(\lambda_c + \lambda_f + \mu_k) - 2\lambda_m\mu_k} = \frac{\mu_k + 9.3 \times 10^{-3}}{3 \times 10^{-4}\mu_k + 2.279 \times 10^{-5}},$$

$$\mathrm{MTTF_s}(3) = q_1(0) + q_2(0) + q_3(0) =$$

$$\frac{6\lambda_a\lambda_c + 3\lambda_a\lambda_f + 12\lambda_a\lambda_m + 2\lambda_c^2 + \lambda_c\lambda_f + 4\lambda_c\lambda_m + 2\lambda_f\lambda_m + 3\lambda_a\mu_a + 6\lambda_a\mu_k + 2\lambda_c\mu_a + 2\lambda_c\mu_k + \lambda_f\mu_a + \mu_a\mu_k}{(\lambda_c + \lambda_f + \mu_k)(\lambda_c + 2\lambda_m + \mu_a)(6\lambda_a + 2\lambda_c) - 3\lambda_a\mu_a(\lambda_c + \lambda_f + 2\mu_k) - \mu_k(12\lambda_a\lambda_m + 4\lambda_c\lambda_m + \lambda_c\mu_a)} =$$

$$\frac{1.252\mu_k + 1.164 \times 10^{-2}}{3.756 \times 10^{-4}\mu_k + 2.760 \times 10^{-5}}$$

From Fig. 7, the mean time to failure of three-dependent-component and two-dependent-component parallel system is positive correlation with μ_k. The three-dependent-component parallel system is

Fig. 6. Effect of μ_a variation on $\mathrm{MTTF_s}$

Fig. 7. Effect of variation on

prior to two-dependent-component parallel system when $\mu_k \in \left[5.2 \times 10^{-3}, 0.125 \right]$.

5. Conclusions

This paper presents parallel system model under common cause and load-sharing failures. According to this model, mean time to failure of three-dependent-component and two-dependent-component parallel systems is calculated. Besides, we calculate and discuss the conditional failure probability and reliability of three-dependent-component and two-dependent-component parallel system model under common cause failure. The reliability of three-dependent-component parallel system model under common cause failure is more than two-dependent-component. We could observe that mean time to failure of three-dependent-component parallel systems is not always longer than two-dependent-component. Hence, some measures could be taken to control the range of variables to ensure mean time to failure of three-dependent-component parallel systems is more than two-dependent-component.

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