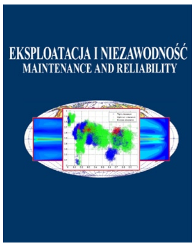

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Lifetime performance evaluation model based on quick response thinking

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Highlights

- This study explored the lifetime of products based on type II censoring.
- Adopted right censoring to find the bester estimator of the lifetime index.
- The 1-α confidence interval and UMP test model of lifetime index were found.
- A numerical example was demonstrated the application of the proposed model.
- Under above methods, the products can take market share earlier.

Abstract

In practice, lifetime performance index CL has been a method commonly applied to the evaluation of quality performance. L is the upper or lower limit of the specification. The product lifetime distribution is mostly abnormal distribution. This study explored that the lifetime of commodities comes from exponential distribution. Complete data collection is the primary goal of analysis. However, the censoring type is one of the most commonly used methods due to considerations of manpower and material cost or the timeliness of product launch. This study adopted Type-II right censoring to find out the uniformly minimum variance unbiased (UMVU) estimator of the lifetime performance index CL and its probability density function. Afterward this study obtained the 100×(1-α)% confidence interval of the lifetime performance index CL as well as created the uniformly most powerful (UMP) test and the power of the test for the product lifetime performance index. Last, this study came up with a numerical example to demonstrate the suggested method as well as the application of the model.

Keywords

lifetime performance index, Type-II right censoring, *UMVU* estimators, *UMP* test.

1. Introduction

A number of studies related to process quality have pointed out that enhancing process quality can increase product lifetime as well as reduce the rate of process scrap and rework. Not only can it raise the value of the product, but it also can contribute to the sustainable development of enterprises in the face of challenges to global warming [7, 10, 17]. Numerous studies have stated that under the thinking of circular economy and sharing economy, good product quality, high availability, and long lifetime can not only reduce operating costs but also improve operating efficiency, thereby enhancing users' satisfaction and willingness [18, 20, 24]. Obviously, improving quality and lifetime for a product can increase its value and industrial competitiveness.

Regarding product quality, quite a few quality control engineers and experts in statistics have been dedicated to their studies on process capability indicators in terms of evaluation, analysis, and improvement [3, 31, 34-35]. In addition, Six Sigma is also a method commonly adopted in the industry to improve process quality [15, 21-22, 25]. Subsequently, many scholars discussed the relation between the abovementioned process capability indicators and the Six-Sigma quality level widely used in the industry, and then they proposed the Six-Sigma quality index, hoping to apply the process capability index to the model of Six-Sigma quality management [6, 29].

Concerning the product lifetime performance, Tong et al. [28] came up with a lifetime performance index on the basis of the larger-the-better process capability index. Under the assumption of exponentially complete data, the proposed lifetime performance index is presented as follows:

$$C_L = \frac{\lambda - L}{\lambda} = 1 - \frac{L}{\lambda}, \tag{1}$$

where L refers to the minimum required time unit of the lifetime for each electronic product. According to the research of Chen et al. [5], there is a one-to-one mathematical relation between the lifetime performance index C_L and the rate of failure. When the average lifetime value of λ is larger, the rate of failure becomes relatively smaller. Then, some scholars also have invested in related research to explore

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the product lifetime performance. The above-mentioned studies all used complete sample data with a sample size of n to make statistical inferences. However, in the experiment on product lifetime performance and reliability, data collection is subject to time constraints or another factor (like money, material resources, machine or experimental difficulties), so that the experimenter is usually unable to view the data of quality for each tested product. Therefore, the censored sample is considered as one of the methods [11, 16]. There are three censoring types: Type-I censoring, Type-II censoring, and random censoring (also known as Type-III censoring) [14, 19].

In right censoring, the observations are $Y_1, Y_2, Y_3, \ldots, Y_n$, as follows:

$$Y_i = \begin{cases} T_i\ , & T_i \ge C_i \\ C_i\ , & T_i \le C_i\ , \end{cases} \quad \text{i = 1, 2, 3, …, n,} \tag{2}$$

where T_i represents time of failure, and C_i is censoring time associated with T_i.

Right censoring refers to dividing the obtained data into two parts: one is T, time of failure for the product lifetime, and the other is C, censoring time. If T of the experimented product lifetime is greater than C, then T is regarded as the censored observation, denoted by T^+. As to the censored observation T^+, the corresponding C is used as the imputation value of T for the product lifetime, and the imputation value is called censored data [19]. Type-I censoring and Type-II censoring are applied to engineering (censoring time is a fixed value), while random censoring are applied to medical science, like experimental studies using animals or experiments done in clinical research (censoring time is a random variable) [19].

Based on right censored data, point and interval prediction of the censoring number of failures is discovered by means of a simulation study under indeterminate survival times and censoring status [13]. Parameters of the exponential distribution are estimated by means of complete samples of Type -I censoring, Type-II censoring, and random censoring [2, 4, 30, 36]. The Type-II progressive censoring scheme has been widely adopted to analyze lifetime data for highly reliable products. For example, construct interval estimators and hypothesis testing of the lifetime performance index based on progressive type II right-censored data. And the potentiality of the model is analyzed by a numerical example [1, 8, 16]. The fuzzy statistical estimator of the lifetime performance index is then utilized to develop a new fuzzy statistical hypothesis testing procedure [33]. Finally, by Monte Carlo power simulation, the objectives assess the behavior of the lifetime performance index [1, 32].

Therefore, this study applies the Type-II right censoring data. In Type-II right censoring, only the lifetimes of the first r components are censored, whereas the lifetimes of the remaining (n-r) components are uncensored or missing.

In Type-II right censoring, the observations are $Y_1, Y_2, Y_3, \ldots, Y_n$, displayed as follows:

$$Y_i = \begin{cases} T_i\ , & T_i \le T_{(r)} \\ T_{(r)}\ , & T_i > T_{(r)}\ , \end{cases} \quad i = 1, 2, 3, \ldots, n, \tag{3}$$

where $T_{(r)}$ is the order statistic of failure times $T_1, T_2, T_3, \ldots, T_n$, and r is the number of uncensored data, $r \le n$.

In this paper, the product lifetime T comes from the exponential distribution with the mean λ, which is the probability density function (*p.d.f.*) of T as follows:

$$f_T(t) = \frac{1}{\lambda} exp\left(-\frac{t}{\lambda}\right),\ t > 0\ . \tag{4}$$

Then, the survival function is:

$$S_T(t) = p\left(T > t\right) = 1 - F_T(t) = exp\left(-\frac{t}{\lambda}\right),\ t > 0\ , \tag{5}$$

where $F_T(t)$ is the cumulative distribution function of T .

With the exponentially complete data, the lifetime performance index which Tong et al. [28] suggested is denoted as Eq. (1). Based on the above, the graphic scheme of methods as following (Fig.1).

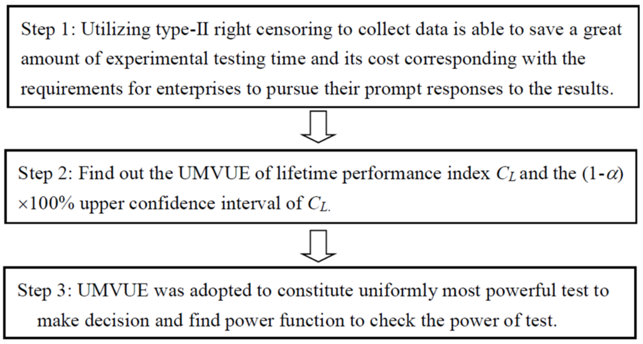

Fig. 1. The graphic scheme of methods

In fact, the statistic testing method the study proposed is equipped with more advantages shown as follows than others:

1. Find the uniformly minimum-variance unbiased estimator (UMVUE) of the lifetime performance index.
2. Derive the uniformly most powerful (UMP) test.
3. Utilizing type-II right censoring is able to save a great amount of experimental testing time and its cost corresponding with the requirements for enterprises to pursue their prompt responses to the results; moreover, the technique is more likely to assist enterprises to efficiently make sensible decision in a short time.

In this paper, the other sections are arranged as follows. In Section 2, we discover the uniformly minimum-variance unbiased estimator (UMVUE) of the lifetime performance index. Next, in Section 3, we demonstrate the (1-α) ×100% upper confidence of the performance index. Furthermore, the uniformly most powerful (UMP) test of the lifetime performance index is developed in Section 4. Also, a numerical example is employed to describe the efficacy of the suggested method in Section 5. Finally, we make conclusions in Section 6.

2. Estimation of lifetime performance index and uniformly minimum variance unbiased estimator

Let $Y_1, Y_2, Y_3, \ldots, Y_n$ be the observed data with sample size n under Type-II right censoring as follows [32]:

$$Y_i = \begin{cases} T_i\ , & T_i \le T_{(r)}\ (uncensored\ data) \\ T_{(r)}, & T_i > T_{(r)}\ (censored\ data) \end{cases} \quad \text{, i=1, 2, 3, …, n,} \tag{6}$$

where the order statistic $T_{(r)}$ is the time of censoring. Failure time T follows the exponential distribution with the mean λ. We figure out the estimator of λ under Type-II censoring. By Eq. (4) and the maximum likelihood method, we find the unbiased estimator of λ as follows [19]:

$$\hat{\lambda} = \frac{\sum_{i=1}^{n} Y_i}{r}\ , \tag{7}$$

where:

$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{r} T_{(i)} + (n - r) T_{(r)} . \quad (8)$$

When r=n (complete data), the estimator $\hat{\lambda}$ is equal to $\bar{T}$. By Eq. (1), the unbiased estimator of C_L is expressed below:

$$\hat{C}_L = 1 - \left(\frac{r-1}{r}\right)\frac{L}{\hat{\lambda}} = 1 - \frac{(r-1)L}{\sum_{i=1}^{n} Y_i} . \quad (9)$$

Furthermore:

$$\lim_{r\to\infty} E\left[\left(\hat{C}_L - C_L\right)^2\right] = \lim_{r\to\infty} Var\left(\hat{C}_L\right) = \lim_{r\to\infty} \frac{\left(1-C_L\right)^2}{r-2} = 0 . \quad (10)$$

According to the Lehmann-Scheffé theorem, $\hat{C}_L$ is the uniformly minimum-variance unbiased estimator (UMVUE) of C_L. To compare with Lee et al., their study proposed that the estimator $\hat{C}_L$ is not unbiased of C_L, an asymptotically unbiased estimator [16]. Hence, the findings from the estimator in Eq. (9) indicated that it was better than the results of Lee et al. [16].

3. Find the (1-α) ×100% upper confidence of performance index

According to Miller [19], the failure time *T* follows the exponential distribution with the mean λ. Let $X = \frac{2\sum_{i=1}^{n} Y_i}{\lambda}$, then we get X to follow the chi-square distribution with degree 2r (i.e. $X \sim \chi^2_{2r}$). Let $W = \hat{C}_L$, the cumulative distribution function of W is denoted by:

$$\begin{aligned} F_W(w) &= P(W \le w) \\ &= P\left(\frac{2(r-1)(1-C_L)}{1-W} \ge \frac{2(r-1)(1-C_L)}{1-w}\right) \\ &= P\left(X \ge \frac{2(r-1)(1-C_L)}{1-w}\right) \\ &= 1 - P\left(X \le \frac{2(r-1)(1-C_L)}{1-w}\right). \end{aligned} \quad (11)$$

By the fundamental theorem of calculus (part 1), we get the probability density function of *W* as follows:

$$f_W(w) = \frac{1}{\Gamma(r)}\left((r-1)(1-C_L)\right)^r (1-w)^{-(r+1)} \exp\left(-\frac{(r-1)(1-C_L)}{(1-w)}\right), \; w<1 \quad (12)$$

Furthermore, we can derive the level of (1-α)×100% upper confidence limit of C_L as follows:

$$1-\alpha = P\left(X \ge \chi^2_{\alpha,2r}\right) = P\left(\frac{2(r-1)}{1-W}\left(1-C_L\right) \ge \chi^2_{2r,\alpha}\right). \quad (13)$$

Therefore:

$$P\left(C_L \le 1 - \frac{\left(1-\hat{C}_L\right)}{2(r-1)}\chi^2_{\alpha,2r}\right) = 1-\alpha , \quad (14)$$

and the (1-*α*) ×100% upper confidence limit of C_L is:

$$UC_L = 1 - \frac{\left(1-\hat{C}_L\right)}{2(r-1)}\chi^2_{\alpha,2r} = 1 - \frac{(r-1)L}{2(r-1)\sum_{i=1}^{n} Y_i}\chi^2_{\alpha,2r} , \quad (15)$$

where $\chi^2_{\alpha;2r}$ is the upper *α* quintile of χ^2_{2r}.

4. Uniformly most powerful test

To know whether C_L is larger than or equal to *c*, it is necessary to take into account the null hypothesis: $H_0: C_L \ge c$ (showing that the product lifetime performance is good) versus the alternative hypothesis: $H_1: C_L < c$ (showing that the product lifetime performance is poor) at a desired level of significance *α*. Significance *α* represents the probability indicating that the product lifetime performance is satisfying but denied, so it can be called the producer risk. Then, the uniformly most powerful (UMP) test is defined by:

$$\phi(Z) = \begin{cases} 1, \text{ if } \hat{C}_L < c_0 \; (\textit{reject } H_0 \textit{ region}) \\ 0, \text{ otherwise} \end{cases} , \quad (16)$$

where c_0 is the critical value determined by Eq. (17):

$$\mathrm{E}\left[\phi(Z) \middle| \hat{C}_L = c\right] = p\left\{C_L < c_0 \middle| C_L = c\right\} = p\left\{X < \chi^2_{\alpha;2r} \middle| C_L = c\right\} = \alpha , \quad (17)$$

where $\chi^2_{\alpha;2r}$ is the upper *α* quintile of χ^2_{2r}. Based on Eq. (12) and Eq. (17), we derive Eq. (18) and Eq. (19) as follows:

$$\chi^2_{\alpha;2r} = \frac{1-C_L}{1-C_0}\cdot 2(r-1) \quad (18)$$

and:

$$c_0 = 1 - \frac{2(r-1)(1-c)}{\chi^2_{\alpha;2r}} . \quad (19)$$

As a matter of fact, the power of the test is critical in learning the probability that the test correctly rejects the null hypothesis H_0 as the alternative hypothesis H_1 is true. Based on Eq. (17), Eq. (18), and Eq. (19), the power of the test for C_L is given by Eq. (20) below.

$$p\left\{C_L < c_0 \middle| C_L = c_1, \; c_1 \in H_1\right\} = 1-\beta , \quad (20)$$

where *β* is the Type-II error of $C_L = c_1, \; c_1 \in H_1$. By Eq. (12) and Eq. (20), we get Eq. (21) as follows:

$$\pi(c_1) = p\left\{\frac{1-C_L}{1-\hat{C}_L}\cdot 2(r-1) < \frac{1-C_L}{1-c_0}\cdot 2(r-1) \middle| C_L = c_1, \; c_1 \in H_1\right\} = 1-\beta . \quad (21)$$

Similarly, based on Eq. (12) and Eq. (21), we have:

$$\frac{1-c_1}{1-c_0}\cdot 2(r-1) = \chi^2_{1-\beta;2r} . \quad (22)$$

Based on the null hypothesis H_0 ($C_L \ge c$) versus alternative hypothesis H_1 ($C_L < c$), Figure 1 shows the power curve for $c = 0.7$ and $r = 60, 80, 100$ (bottom-up in the plot) with $n = 100$. According to Figure 2, the greater the power function $\pi(c_1)$ with the value of c_1 ($c_1 \in H_1$), the greater the power. Also, when the power function $\pi(c_1)$

is fixed at c_1 ($c_1 \in H_1$), the greater the value of r (i.e. the number of uncensored data is greater), the greater the power. That is, the larger the number of uncensored data, the stronger the power, which meets the more the original data, the better the quality of the analysis.

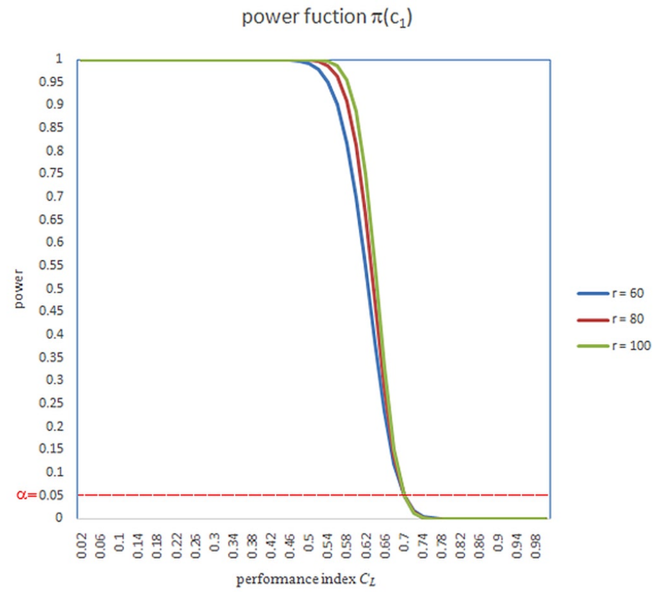

Fig. 2. Power curve for c = 0.7 and r = 60, 80, 100 (bottom-up in the plot) with n = 100

When product lifetime (T) follows the exponential distribution with the mean λ, the estimate of $\hat{C}_L$, the confidence interval for C_L, and the uniformly most powerful test are discovered as stated above. The steps of the statistical testing method are listed below:

Step 1: Collect the sample size n, Y_1, Y_2, …, Y_{100} under Type-II right censoring, as shown in Eq. (3).

Step 2: Given the value L, we can find the estimate of $\hat{C}_L$ by Eq. (7) and Eq. (9).

Step 3: By Eq. (15), we get the level of (1-α)×100% upper confidence interval for C_L.

Step 4: We construct hypotheses as follows:

null hypothesis $H_0 : C_L \geq c$ (the product lifetime perform ance is good)

versus

alternative hypothesis $H_1 : C_L < c$ (the product lifetime performance is poor)

5. Numerical example

In this section, a numerical example is adopted to elaborate the above four steps stated in Section 4, assuming that the lifetime of an electronic product follows an exponential distribution (λ = 1) as well as assuming that sample size n = 100. By Type-II censoring, r = 80 is given. The four steps are listed as follows:

Step 1: Collect the sample data $T_1, T_2, ..., T_{100}$ from an exponential distribution with the mean (λ = 1). Given the number of uncensored data r = 80, we can find the order statistic $T_{(80)}$ = 1.6377, and then we can collect Type-II right censored data Y_1, Y_2, …, Y_{100} by Eq. (3).

Step 2: Given the ratio L/λ = 0.35 (i.e. $L<\lambda$) and then C_L = 0.65 by Eq. (1), we can find $\sum_{i=1}^{100} Y_i$ = 830392 by Eq. (3) and the estimate of $\hat{C}_L$ = 0.6644 by Eq. (9).

Step 3: Given the confidence level of 95% by Eq. (15), we can get that the upper confidence interval for C_L is (-∞, 0.7201].

Step 4: We give the level of significance (α = 0.05) and the value of requirement (c = 0.7). Then, the hypotheses are expressed as follows:

null hypothesis $H_0 : C_L \geq 0.7$ (the product lifetime performance is good)

versus

alternative hypothesis $H_1 : C_L < 0.7$ (the product lifetime performance is poor).

By Eq. (17) and Eq. (19), the reject H_0 region is { $\hat{C}_L < 0.6402$}. Since $\hat{C}_L$ = 0.6644, obviously, H_0 is not rejected, concluding that the product lifetime performance is good. Obviously, type-II right censoring method the study adopted allowed investigators to make the best of eighty samples out of one hundred valid ones to conduct the survey, which saved twenty percent of experimental testing time and its cost. Therefore, the result was totally correspondent with the needs of pursuing enterprises' requirements for prompt responses and efficiently making wise decision in no time [12, 27].

6. Conclusions

The process capability index C_L is a common method used to practically evaluate the lifetime performance index of the product. However, in the product development process, when taking time and labor costs into consideration, collecting data in the form of censoring is one of the methods to improve the defect. This study took the research and development of the electronic product manufacturing process as an example and adopted Type-II right censoring to collect data. When the data collection is incomplete, the missing data values are replaced by the experiment termination time $T_{(r)}$. This study not only derived the uniformly minimum-variance unbiased estimator (UMVVUE) $\hat{C}_L$ of the process capability index C_L and its probability density function but also derived the $(1-\alpha)$ 100% upper confidence interval of C_L . Assuming that the period of warranty was L unit time, this study created the uniformly most powerful (UMP) test for the lifetime performance index of the product and obtained the power of the test for C_L . From the graph of the power function, it is learned that the greater the number of uncensored data r, the greater the power. When other conditions remain unchanged, the complete data ($r = n$) has the greatest power. For the convenience of manufacturers, this paper then established a testing process and at the same time proposed a numerical case. According to the testing process to perform the best statistical testing model, the proposed method and the applications of the model were explained in this paper.

According to reliable life tests, apart from type-II right censoring, type-I right censoring [9, 23] and type-III right censoring [26] are also widely utilized to collect data. Thus, for the purpose of offering more relevant references to even more enterprises, the study suggests that three different right censoring types to be further investigated and analyzed in the future study.

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