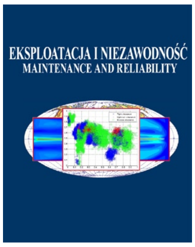

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Importance measure-based maintenance strategy considering maintenance costs

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Highlights

- A new maintenance priority is proposed to guide the preventive maintenance of components.
- A joint importance is applied into the opportunistic maintenance of components.
- Characteristics of the maintenance model in series-parallel systems are analyzed.
- Maintenance strategies of components in 2H2E architecture are discussed.
- The effectiveness of two proposed models are verified with the 2H2E architecture.

Abstract

Maintenance is an important way to ensure the best performance of repairable systems. This paper considers how to reduce system maintenance cost while ensuring consistent system performance. Due to budget constraints, preventive maintenance (PM) can be done on only some of the system components. Also, different selections of components to be maintained can have markedly different effects on system performance. On the basis of the above issues, this paper proposes an importance-based maintenance priority (IBMP) model to guide the selection of PM components. Then the model is extended to find the degree of correlation between two components to be maintained and a joint importance-based maintenance priority (JIBMP) model to guide the selection of opportunistic maintenance (OM) components is proposed. Also, optimization strategies under various conditions are proposed. Finally, a case of 2H2E architecture is used to demonstrate the proposed method. The results show that generators in the 2E layout have the highest maintenance priority, which further explains the difference in the importance of each component in PM.

Keywords

system reliability, importance measure, maintenance cost, preventive maintenance, opportunistic maintenance.

1. Introduction

Maintenance occupies a very important proportion in the whole life cycle of various systems. A good maintenance strategy can improve the reliability of the system and reduce the cost of system maintenance. To achieve the maintenance objective, it is necessary to identify some important components of the system. However, in actual systems, the system structure could be complex, and how to determine the maintenance priority of components and reduce the system maintenance cost while improving the reliability of the system become very important.

Importance measure is an important method to evaluate the influence of components on the performance of the whole system in the field of reliability, and it is widely used in repairable systems [17, 26, 27, 28]. In 1969, Birnbaum [2] firstly proposed an importance measure theory and established its theoretical framework. The Birnbaum importance measure evaluates the relationship between component reliability and system reliability. Griffith et al. [9] explained the effects of component performance improvements on system performance based on the Birnbaum importance measure. Wu and Chan [21] defined a new utility importance that overcomes some drawbacks of Griffith importance measure. In addition, Wu et al. [23] proposed a component maintenance priority importance measure to identify the order of preventive maintenance components. Based on the importance of multi-state components, Si et al. [18] proposed an integrated importance measure to identify the components which have the greatest impact on system performance. Dui et al. [7] extended the integrated importance measure and proposed a joint integrated importance measure to maximize the gain of the system performance. In order to allocate limited maintenance resources to important components in repairable systems and improve the overall reliability of the system, some scholars consider combining importance measure with preventive maintenance and opportunistic maintenance to guide the maintenance of components. Zhang et al. [29] introduced the importance measure theory into the opportunistic maintenance strategy to provide guidance for the maintenance of the heave compensation system and retard the degradation of the expected performance of the

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system. Babishin et al. [3] proposed an aperiodic strategy of joint optimization of maintenance and inspection, which provides a promising approach for system maintenance. In addition, some scholars have proposed reliability models to identify weak links of the system under specific circumstances, so as to guide component maintenance and improve system reliability. Xing et al. [24] proposed a combinatorial reliability model of correlation system probabilistic competitions and random failure propagation time to optimize the function dependence of components. Gao et al. [10] proposed a reliability model related to the failure process and the degradation impact to consider the dependency relationship between the soft and hard failure process. Sun et al. [19] proposed a dynamic linear model for fault prediction and predictive maintenance of aircraft air conditioning systems. Legát et al. [11] proposed a method to determine the optimal interval of preventive periodic maintenance and studied the relationship between preventive maintenance interval and reliability function.

In the process of maintenance, system maintenance cost restricts the number of maintenance components and the determination of maintenance degree. Considering the importance of cost-effectiveness in maintenance, Wu and Coolen [22] extended Birnbaum importance measure from the perspective of cost and proposed a cost-based importance measure. Dui et al. [8] proposed a cost-based integrated importance measure to identify components or component groups that can be used for preventive maintenance. Minwoo et al. [14] conducted a systematic analysis and assessment of the direct operating costs of wide-body airliners and identified the most cost-effective aircraft types that could be helpful to airline operators and policy makers. Tan et al. [20] proposed a maintenance strategy to effectively reduce the maintenance cost of the hemodialysis machine and ensure the high availability of the equipment. Andrzejczak et al. [1] conducted a simple fault random model for the cost of vehicle corrective maintenance, and applied the model to identify the damaged components of the vehicle. In recent years, Bayesian networks have been widely used in the reliability research of multi-level complex systems. In terms of component fault diagnosis, Cai et al. [4-6] used Bayesian network to analyze the reliability of components, which can play a guiding role in the follow-up preventive maintenance of components. In addition, it is inevitable to encounter some irresistible factors to restrict the system maintenance and reliability improvement. Considering the impact of random shocks, Zhao et al. [30] analyzed the reliability in a random shock environment and provided the optimal task termination strategy for the system. Qiu et al. [16] proposed a mission abort strategy for internal system failures and external shocks to improve the survivability of critical systems. Peng et al. [15] proposed a hybrid incomplete maintenance model with random adjustment, and studied a sequential preventive maintenance strategy with periodicity leisure interval. Moreover, Levitin et al. [12, 13] conducted a series of studies on the mission abort policy of systems.

Although the above researches made outstanding contributions, traditional importance measure-based methods seldom consider the change rate of system maintenance cost caused by the state transition of system components. In this paper, we propose an importance-based maintenance priority (IBMP) model and a joint importance-based maintenance priority (JIBMP) model to perform cost-based maintenance decision analyses. We use the model to sort the important components and determine the maintenance cost level of system in different states, which could reduce the expected maintenance cost of the system while improving the performance of the system. Thus, the models can provide theoretical guidance for the maintenance of components in the system.

The rest of this article is organized as follows. In Section 2, IBMP and JIBMP models are proposed to guide the selection of components in preventive maintenance and opportunistic maintenance, and then the features of JIBMP in series-parallel systems are discussed. In Section 3, the IBMP model and the JIBMP model are applied to the aircraft 2H2E architecture to help identify the important components of the system, and the combination of system components in each state is listed. In Section 4, the application of the model in 2H2E architecture is simulated, and the results show that the model is effective. Then we analyze the maintenance optimization strategy of the 2H2E architecture under the condition of a limited budget and determine the number of components that can be maintained under various budget constraints. Finally, the conclusions are drawn in Section 5.

2. Proposed maintenance model

In this section, we first introduce the expected maintenance cost of the system. Then the definitions of IBMP and JIBMP are proposed in Sections 2.1 and 2.2. The features of JIBMP in a series-parallel system are discussed in Section 2.3. The number of PM components is discussed in Section 2.4.

Assuming that a multistate system has n components and M states, where State 0 is the system's complete failure state and State M is the system's perfect state. States from state 1 to state M-1 are the intermediate state of the system, in which some components of the system fail and the system performance deteriorates but the system can continue to operate. Then, the expected maintenance cost of the system at time t is defined as:

$$C(X(t)) = \sum_{j=0}^{M-1} c_j \Pr[\Phi(X(t)) = j] = \sum_{j=0}^{M-1} c_j \Pr[\Phi(X_1(t), X_2(t), \ldots, X_n(t)) = j], \tag{1}$$

where c_j is the failure maintenance cost when the system is in state j. Let $\{0 \le c_{M-1} \le c_{M-2} \le \cdots \le c_0\}$ be the corresponding failure maintenance cost levels. The function $\Phi(X(t))$ is the structural function of the system related to the state of each component. $\Pr[\Phi(X(t)) = j]$ is a system probability function and could also be written as $f_j(R_1(t), R_2(t), \ldots, R_n(t))$.

2.1. Definition of importance-based maintenance priority

IBMP determines how to make maintenance choices for components when a system's performance degrades; different choices of components can lead to marked differences in system expected maintenance cost. When component i changes from state m to state 0, the IBMP value of component i is defined as:

$$I_i^{IBMP}(t) = P_{i,m} \cdot \lambda_{m,0}^{i} \cdot \sum_{j=0}^{M-1} c_j \{\Pr[\Phi(0_i, X(t)) = j] - \Pr[\Phi(m_i, X(t)) = j]\}, \tag{2}$$

where $P_{i,m}$ is the probability that component i is in state m, $\lambda_{m,0}^{i}$ is the transition rate of component i from state m to state 0. State 0 is the failure state of the component. $\Pr[\Phi(0_i, X(t)) = j]$ is the probability that component i is in a failure state and the rest of components are in state j at time t. $\Pr[\Phi(m_i, X(t)) = j]$ is the probability that component i is in state m and the rest of components are in state j at time t. For each component we consider two states, the perfect state and the failure state. So the probability that the component is in state m is the probability that the component is in perfect state, and we can express that in terms of the reliability of the component. The transition rate of the component from state m to state 0 is the transition rate of the component from perfect state to failure state, which can be expressed by the failure rate of the component. Therefore, the IBMP value of component i can also be expressed as:

$$I_i^{IBMP}(t) = R_i(t) \cdot \lambda_i(t) \cdot \sum_{j=0}^{M-1} c_j \{\Pr[\Phi(0_i, X(t)) = j] - \Pr[\Phi(1_i, X(t)) = j]\}, \tag{3}$$

$I_i^{IBMP}(t)$ is the contribution of component i from perfect state to failure state to the change rate of system expected maintenance cost.

When the component $I_i^{IBMP}(t)$ value is large, it means that component i contributes the most to the rate of change in system expected maintenance cost. We know that the component failure maintenance cost is much higher than the component preventive maintenance cost. So in order to prevent component failure from increasing the change rate of system expected maintenance cost, we should give priority to the maintenance of those components with large IBMP values.

Next, we give the relation between the change rate of the expected maintenance cost of the system and the IBMP value of each component:

$$\frac{dC(X(t))}{dt}=\frac{d[\sum_{j=0}^{M-1}c_j f_j(R_1(t),R_2(t),\ldots,R_n(t))]}{dt}=\sum_{j=0}^{M-1}c_j\sum_{i=1}^{n}\frac{dR_i(t)}{dt}\frac{\partial f_j(R_1(t),R_2(t),\ldots,R_n(t))}{\partial R_i(t)}$$
$$=\sum_{j=0}^{M-1}c_j\sum_{i=1}^{n}\frac{dR_i(t)}{dt}\frac{\partial \Pr[\Phi(X(t))=j]}{\partial R_i(t)}=\sum_{i=1}^{n}\sum_{j=0}^{M-1}c_j\frac{dR_i(t)}{dt}\frac{\partial \Pr[\Phi(X(t))=j]}{\partial R_i(t)}$$

$$\begin{aligned}\Pr[\Phi(X(t))=j]&=\Pr[X_i(t)=0]\cdot\Pr[0_i,X(t)=j]+\Pr[X_i(t)=1]\cdot\Pr[1_i,X(t)=j]\\&=(1-R_i(t))\cdot\Pr[0_i,X(t)=j]+R_i(t)\cdot\Pr[1_i,X(t)=j]\end{aligned},$$

and $\lambda_i(t)=-\frac{dR_i(t)/dt}{R_i(t)}$, so we can get:

$$\frac{dC(X(t))}{dt}=\sum_{i=1}^{n}\sum_{j=0}^{M-1}c_jR_i(t)\lambda_i(t)\{\Pr[\Phi(0_i,X(t))=j]-\Pr[\Phi(1_i,X(t))=j]\}=\sum_{i=1}^{n}I_i^{IBMP}(t)\tag{4}$$

Eq. (4) shows the relation between the change rate of the expected maintenance cost of the system and the IBMP value of each component. From the formula, we can know that the change rate of expected maintenance cost of the system at time t is equal to the sum of the IBMP values of n components at time t. Therefore, the IBMP value of component i is the contribution of component i to the change rate of the expected maintenance cost of system at time t.

IBMP is a PM model, so it is performed by the size of each component's IBMP value at a given time. Next we will introduce the JIBMP model. JIBMP model is an opportunistic maintenance (OM) model. It means that when a component in the system fails and needs to be shut down for maintenance, this component needs to be repaired; at the same time, PM of several other components should be performed in order to reduce the expected maintenance cost of the system as much as possible while improving the reliability of the system. The JIBMP model is derived from the IBMP model.

2.2. Derivation of joint importance-based maintenance priority

When component k suffers performance degradation that leads to failure, the expected maintenance cost of the system $C(X(t))$ becomes $C(0_k,X(t))$. According to Eq. (4) in the IBMP model, when component k fails, the relationship between the change rate of the expected maintenance cost of the system and the IBMP value of each component can be expressed as:

$$\frac{dC(0_k,X(t))}{dt}=\sum_{i=1,i\neq k}^{n}\sum_{j=0}^{M-1}c_jR_i(t)\lambda_i(t)\{\Pr[\Phi(0_k,0_i,X(t))=j]-\Pr[\Phi(0_k,1_i,X(t))=j]\}.\tag{5}$$

Therefore, according to Eq. (4) and Eq. (5), when component k is in a failure state, the IBMP value of component i can be expressed as:

$$I_i^{IBMP}(t)_{X_k(t)=0}=\sum_{j=0}^{M-1}c_jR_i(t)\lambda_i(t)\{\Pr[\Phi(0_k,0_i,X(t))=j]-\Pr[\Phi(0_k,1_i,X(t))=j]\}.\tag{6}$$

Here, $I_i^{IBMP}(t)_{X_k(t)=0}$ is the contribution of component i to the change rate of the expected maintenance cost of the system when component k is in a failure state. Similarly, when component k is in a perfect state, it can be seen from Eq. (5) that the relationship between the change rate of system expected maintenance cost and the IBMP value of each component can be expressed as:

$$\frac{dC(1_k,X(t))}{dt}=\sum_{i=1,i\neq k}^{n}\sum_{j=0}^{M-1}c_jR_i(t)\lambda_i(t)\{\Pr[\Phi(1_k,0_i,X(t))=j]-\Pr[\Phi(1_k,1_i,X(t))=j]\}.\tag{7}$$

So in the same way when component k is in a perfect state, the IBMP value of component i can be expressed as:

$$I_i^{IBMP}(t)_{X_k(t)=1}=\sum_{j=0}^{M-1}c_jR_i(t)\lambda_i(t)\{\Pr[\Phi(1_k,0_i,X(t))=j]-\Pr[\Phi(1_k,1_i,X(t))=j]\}.\tag{8}$$

$I_i^{IBMP}(t)_{X_k(t)=1}$ is the contribution of component i to the change rate of the expected maintenance cost of the system when component k is in a perfect state.

JIBMP is an OM model. It means that when there is a key component failure in the system, the system needs to be shut down for maintenance. In this downtime maintenance for some other potentially malfunctioning components can be performed. So when component k is in the maintenance state, the JIBMP value of component i is defined as:

$$I_i^{IBMP}(t)_{X_k(t)}=I_i^{IBMP}(t)_{X_k(t)=0}-I_i^{IBMP}(t)_{X_k(t)=1}.\tag{9}$$

$I_i^{IBMP}(t)_{X_k(t)}$ is the contribution of component i to the change rate of the expected maintenance cost of the system when component k is in the maintenance state. Therefore, if component k is under maintenance, component i with the highest JIBMP value should have the highest maintenance priority, because if component i fails, component i will contribute the most to the change rate of the expected maintenance cost of the system, so in order to avoid the failure of component i leading to an increase in the change rate of the expected maintenance cost of the system, component i should be maintained first. If component maintenance is carried out in accordance with the JIBMP model, the system performance can be improved while at the same time the growth rate of the expected maintenance cost of the system can be reduced.

Next we demonstrate the relationship between the change rate of the expected maintenance cost of the system and the JIBMP value of each component.

When component k is under maintenance, the change rate of the expected maintenance cost of the system can be expressed as:

$$\frac{dC(0_k,X(t))-dC(1_k,X(t))}{dt}=\frac{dC(0_k,X(t))}{dt}-\frac{dC(1_k,X(t))}{dt}.$$

Substituting Eq. (6) into Eq. (5), we can get:

$$\frac{dC(0_k,X(t))}{dt}=\sum_{i=1,i\neq k}^{n}I_i^{IBMP}(t)_{X_k(t)=0}.$$

In the same way, substituting Eq. (8) into (7) we have:

$\frac{dC(1_k,X(t))}{dt}=\sum_{i=1,i\neq k}^{n}I_i^{IBMP}(t)_{X_k(t)=1}$. Then,

$$\frac{dC(0_k,X(t))}{dt}-\frac{dC(1_k,X(t))}{dt}=\sum_{i=1,i\neq k}^{n}(I_i^{IBMP}(t)_{X_k(t)=0}-I_i^{IBMP}(t)_{X_k(t)=1})=\sum_{i=1,i\neq k}^{n}I_i^{IBMP}(t)_{X_k(t)}.$$

Therefore, the change rate of the expected maintenance cost of the system at time t is the sum of JIBMP values of the n-1 components at time t after removing the failure component k, so the JIBMP value of component i at time t is the contribution of component i to the change rate of the expected maintenance cost of the system. We should give priority to the maintenance of component i to prevent the failure of component i from increasing the change rate of the expected maintenance cost of the system. Thus, when using the JIBMP model to guide OM, it can improve system performance while reducing the expected maintenance cost of the system as much as possible.

2.3. Features of series-parallel system of JIBMP

In the following sections, we will discuss some characteristics of the JIBMP model in multi-state series-parallel systems. When state transition occurs after one components fails, the JIBMP illustrates the importance change of each of the rest components. The JIBMP can also be used to determine the component which induces the lowest change rate of system maintenance costs and has the highest preventive maintenance priority in remaining components.

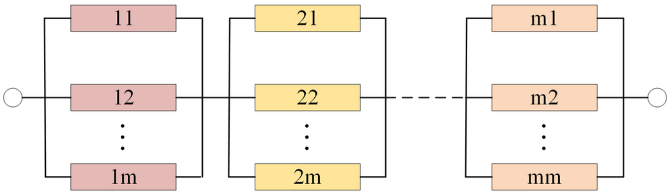

Fig. 1. Series-parallel system model

Assume that a system consists of n components. From Eq. (6), we can know that when component k is in a failure state, the JIBMP of component i from state p to state q ($p<q$) is expressed as:

$$I_i^{IBMP}(t)_{X_k(t)=0} = \sum_{j=0}^{M-1} c_j R_i(t)\lambda_i(t)\{\Pr[\Phi(0_k,p_i,X(t))=j]-\Pr[\Phi(0_k,q_i,X(t))=j]\}.$$

This equation can also be written as:

$$I_i^{IBMP}(t)_{X_k(t)=0} = \sum_{j=0}^{M-1} (c_j - c_{j+1}) R_i(t)\lambda_i(t)\{\Pr[\Phi(0_k,p_i,X(t))\le j]-\Pr[\Phi(0_k,q_i,X(t))\le j]\}. \quad (10)$$

where $\Pr(\Phi(0_k,p_i,X(t))\le j)$ means the probability that the state of other components is lower than system state j when the component k is in the fault state and the component i is in state p. Similarly, $\Pr(\Phi(0_k,q_i,X(t))\le j)$ means the probability that the state of other components is lower than system state j when the component k is in the fault state and the component i is in state q. Simplifying Eq. (10):

$$\begin{aligned}I_i^{IBMP}(t)_{X_k(t)=0} &= \sum_{j=0}^{M-1} (c_j - c_{j+1}) R_i(t)\lambda_i(t)\{\Pr[\Phi(0_k,p_i,X(t))\le j]-\Pr[\Phi(0_k,q_i,X(t))\le j]\}\\ &= \sum_{j=0}^{M-1} (c_j - c_{j+1}) R_i(t)\lambda_i(t)[(1-\Pr(\Phi(0_k,p_i,X(t))>j))-(1-\Pr(\Phi(0_k,q_i,X(t))>j))]\\ &= \sum_{j=0}^{M-1} (c_j - c_{j+1}) R_i(t)\lambda_i(t)[\Pr(\Phi(0_k,q_i,X(t))>j)-\Pr(\Phi(0_k,p_i,X(t))>j)]\end{aligned}$$

and:

$$\begin{aligned}&\sum_{j=0}^{M-1}(c_j-c_{j+1})\Pr(\Phi(0_k,q_i,X(t))>j)\\ &= \sum_{j=0}^{q-1}(c_j-c_{j+1})\Pr(\Phi(0_k,q_i,X(t))>j)+\sum_{j=q}^{M-1}(c_j-c_{j+1})\Pr(\Phi(0_k,q_i,X(t))>j)\end{aligned}$$

we know that during the operation of the system without intervention, components will degrade from a perfect state to a failed state, so the system state will gradually decrease; therefore:

$$\begin{aligned}&\Pr(\Phi(0_k,q_i,X(t))>j)=0,\ j=q,q+1,\ldots,M-1,\\ &\Pr(\Phi(0_k,p_i,X(t))>j)=0,\ j=p,p+1,\ldots,M-1.\end{aligned}$$

So we have:

$$\sum_{j=0}^{M-1}(c_j-c_{j+1})\Pr(\Phi(0_k,q_i,X(t))>j)=\sum_{j=0}^{q-1}(c_j-c_{j+1})\Pr(\Phi(0_k,q_i,X(t))>j).$$

In the same way, we have:

$$\begin{aligned}&\sum_{j=0}^{M-1}(c_j-c_{j+1})\Pr(\Phi(0_k,p_i,X(t))>j)\\ &=\sum_{j=0}^{p-1}(c_j-c_{j+1})\Pr(\Phi(0_k,p_i,X(t))>j)+\sum_{j=p}^{M-1}(c_j-c_{j+1})\Pr(\Phi(0_k,p_i,X(t))>j)\\ &=\sum_{j=0}^{p-1}(c_j-c_{j+1})\Pr(\Phi(0_k,p_i,X(t))>j).\end{aligned}$$

Therefore, the $I_i^{IBMP}(t)_{X_k(t)=0}$ of component i in a multistate series-parallel system can be expressed as:

$$I_i^{IBMP}(t)_{X_k(t)=0}=R_i\cdot\lambda_i\cdot[\sum_{j=0}^{q-1}(c_j-c_{j+1})\Pr(\Phi(0_k,q_i,X(t))>j)-\sum_{j=0}^{p-1}(c_j-c_{j+1})\Pr(\Phi(0_k,p_i,X(t))>j)].$$

Similarly, when component k is in a perfect state, the $I_i^{IBMP}(t)_{X_k(t)=1}$ of component i from state p to state q can be expressed as:

$$I_i^{IBMP}(t)_{X_k(t)=1}=R_i\cdot\lambda_i\cdot[\sum_{j=0}^{q-1}(c_j-c_{j+1})\Pr(\Phi(1_k,q_i,X(t))>j)-\sum_{j=0}^{p-1}(c_j-c_{j+1})\Pr(\Phi(1_k,p_i,X(t))>j)].$$

From Eq. (9) and the analysis above, we know that $I_i^{IBMP}(t)_{X_k(t)}$ in a multistate series-parallel system can be expressed as:

$$\begin{aligned}I_i^{IBMP}(t)_{X_k(t)}&=I_i^{IBMP}(t)_{X_k(t)=0}-I_i^{IBMP}(t)_{X_k(t)=1}\\ &=R_i\cdot\lambda_i\cdot[\sum_{j=0}^{q-1}(c_j-c_{j+1})\Pr(\Phi(0_k,q_i,X(t))>j)+\sum_{j=0}^{p-1}(c_j-c_{j+1})\Pr(\Phi(1_k,p_i,X(t))>j)\\ &\quad-\sum_{j=0}^{p-1}(c_j-c_{j+1})\Pr(\Phi(0_k,p_i,X(t))>j)-\sum_{j=0}^{q-1}(c_j-c_{j+1})\Pr(\Phi(1_k,q_i,X(t))>j)].\end{aligned} \quad (11)$$

2.4. Discussion on the number of preventive maintenance components

When we do PM, after determining the maintenance budget C, we should determine which maintenance components have priority and the total number of components to be maintained. From the above analysis, we know that the maintenance strategy can be expressed as:

$$\max_{d_i}\sum I_i^{IBMP}(t)\cdot d_i\,, \quad (12)$$

and the limitation function of maintenance cost is $\sum c_i d_i \le C$, where c_i is the cost of component i in PM, d_i is a variable that determines whether component i needs to be maintained, $d_i\in\{0,1\}$, and C is the total cost of the budget. When decisions are made on maintenance optimization, we know that there are 2^n combinations. The number of PM components can be expressed as $\sum d_i$. When a component has a maximum IBMP value, it should be maintained first. When the component with the highest change rate of the expected maintenance cost of the system is maintained first, the maximum increase in system cost per unit time due to failure of that component is reduced. However, the maintenance cost of each component is different. When the PM budget is fixed, components with the maximum IBMP value may not be maintained first because the maintenance cost exceeds the budget. Therefore, the number of component maintenance in various time periods should be taken into account in combination with the above analysis.

3. Case study for 2H2E architecture

Hydraulic energy systems are crucial in ensuring flight security. State-of-the-art Airbus A380 airplane uses a dual-architecture hydraulic energy system. This is a hybrid flight control actuator power distribution system that combines a distributed electric actuator used as a backup system with a conventional telex hydraulic servo control for active control, forming four independent main flight control systems. Two of the systems are hydraulically powered and the other two are electrically powered. Therefore, this architecture is also known as the 2H2E architectural layout. 2H is the pump source of the traditional hydraulic power actuating system, consisting of eight engine-driven pumps (EDPs) and four AC electric motor pumps (EMPs). They provide hydraulic power for the aircraft's main flight control, landing gear, front-wheel turning, and other related systems. 2E is an electrically powered, distributed, electro-hydraulic actuator system, which consists of electro-hydraulic actuators and backup electro-hydraulic actuators. Each of the four systems can be individually controlled, bringing the independence, redundancy and reliability of the A380 hydraulic energy system to a new level.

A configuration diagram of the 2H2E architecture used in the A380 is shown in Fig. 2 the main components include four engines, eight EDPs, four EMPs, four fuel shut-off valves (FSOVs), four generators, two auxiliary power unit (APU) generators, two hydraulic reservoirs, and one ram air turbine (RAT). Based on statistics, some components do not fail often, including engines, FSOVs, and RAT. However, EDPs, generators, EMPs, and hydraulic reservoirs run most of the time that an aircraft is in flight, and hence may become vulnerable components. Failure of any of these components may result in system performance degradation or failure [31]. Therefore, to ensure flight safety, we must do PM on important components. But due to maintenance cost, PM cannot be done on all components, so maintenance must be prioritized based on the requirements of each component.

Table 1 lists 29 important components that play an important role in the safety of an A380 airplane. There is redundancy in some important components, and when one of the components fails, the backup components still function but the system performance will inevitably degrade. If the backup component fails, the system fails.

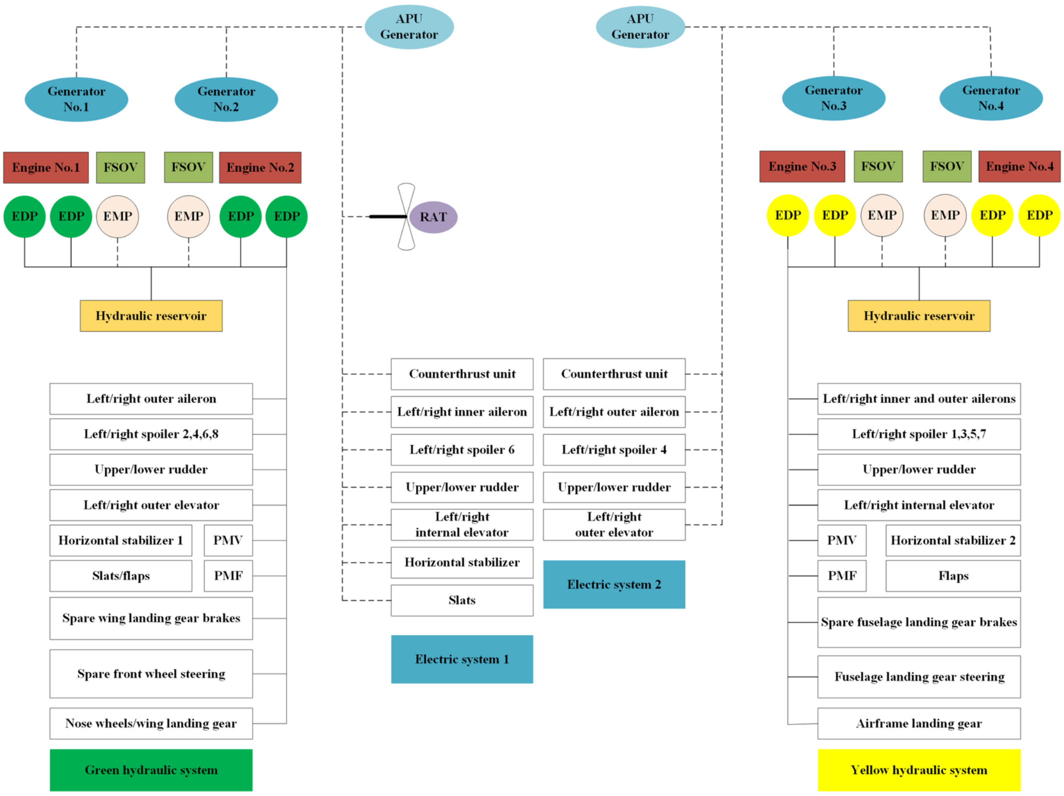

Fig. 2. Configuration diagram of a two-hydraulically-powered and two-electrically-powered architecture used by Airbus A380 airplane

Table 1. Major components in the 2H2E architecture of an A380 airplane

Code	Component	Code	Component
X1	Engine No.1	X16	APU generator No.2
X2	Electric motor pump No.1	X17	Electric motor pump No.2
X3	Engine-driven pump No.1	X18	Engine No.3
X4	Engine-driven pump No.2	X19	Engine-driven pump No.5
X5	Generator No.1	X20	Engine-driven pump No.6
X6	Hydraulic reservoir No.1	X21	Generator No.3
X7	APU generator No.1	X22	Electric motor pump No.3
X8	Fuel shut-off valve No.1	X23	Fuel shut-off valve No.3
X9	Ram air turbine	X24	Engine No.4
X10	Engine No.2	X25	Generator No.4
X11	Engine-driven pump No.3	X26	Engine-driven pump No.7
X12	Engine-driven pump No.4	X27	Engine-driven pump No.8
X13	Generator No.2	X28	Electric motor pump No.4
X14	Hydraulic reservoir No.2	X29	Fuel shut-off valve No.4
X15	Fuel shut-off valve No.2		

The Weibull distribution is a widely used statistical distribution, especially in the life analysis of mechanical components [25]. On the basis of engineering practice, we assumed that all of the above 29 important components follow the Weibull distribution $W(\theta,\gamma,t)$ with the parameters shown in Table 2.

By the properties of the Weibull distribution, the reliability function of component is $R(t)=\exp[-(\frac{t}{\theta})^{\gamma}]$ and the failure rate function is $\lambda(t)$, which is given by $\lambda(t)=\frac{\gamma}{\theta}\cdot(\frac{t}{\theta})^{\gamma-1}$. Of the above 29 compo-

Table 3. System states and corresponding state maintenance cost in descending order of maintenance cost. j is the system state and cj is the failure maintenance cost.

j	System state						c_j
0	Complete failure state						1
1	X5/X7/X13	X2/X17	X3/X4/X11/X12	X8/X15	X9	X6	0.95
2	X5/X7/X13	X2/X17	X3/X4/X11/X12	X8/X15	X9	–	0.90
3	X5/X7/X13	X2/X17	X3/X4/X11/X12	X8/X15	–	X6	0.87
4-5	X5/X7/X13	X2/X17	X3/X4/X11/X12	–	X9	X6	0.85
6-9	X5/X7/X13	X2/X17	–	X8/X15	X9	X6	0.83
10-11	X5/X7/X13	–	X3/X4/X11/X12	X8/X15	X9	X6	0.80
12	X5/X7/X13	X2/X17	X3/X4/X11/X12	X8/X15	–	–	0.77
13-15	–	X2/X17	X3/X4/X11/X12	X8/X15	X9	X6	0.75
16-17	X5/X7/X13	X2/X17	X3/X4/X11/X12	–	X9	–	0.75
18-21	X5/X7/X13	X2/X17	–	X8/X15	X9	–	0.73
22-23	X5/X7/X13	X2/X17	X3/X4/X11/X12	–	–	X6	0.72
24-25	X5/X7/X13	–	X3/X4/X11/X12	X8/X15	X9	–	0.70
26-29	X5/X7/X13	X2/X17	–	X8/X15	–	X6	0.70
30-37	X5/X7/X13	X2/X17	–	–	X9	X6	0.68
38-39	X5/X7/X13	–	X3/X4/X11/X12	X8/X15	–	X6	0.67
40-42	–	X2/X17	X3/X4/X11/X12	X8/X15	X9	–	0.65
43-46	X5/X7/X13	–	X3/X4/X11/X12	–	X9	X6	0.65
47-54	X5/X7/X13	–	–	X8/X15	X9	X6	0.63
55-57	–	X2/X17	X3/X4/X11/X12	X8/X15	–	X6	0.62
58-59	X5/X7/X13	X2/X17	X3/X4/X11/X12	–	–	–	0.62
60-65	–	X2/X17	X3/X4/X11/X12	–	X9	X6	0.60
66-69	X5/X7/X13	X2/X17	–	X8/X15	–	–	0.60
70-81	–	X2/X17	–	X8/X15	X9	X6	0.58
82-89	X5/X7/X13	X2/X17	–	–	X9	–	0.58
90-95	–	–	X3/X4/X11/X12	X8/X15	–	X6	0.58
96-97	X5/X7/X13	–	X3/X4/X11/X12	X8/X15	–	–	0.57
98-103	–	–	X3/X4/X11/X12	X8/X15	X9	X6	0.55
104-107	X5/X7/X13	–	X3/X4/X11/X12	–	X9	–	0.55
108-115	X5/X7/X13	X2/X17	–	–	–	X6	0.55
116-123	X5/X7/X13	–	–	X8/X15	X9	–	0.53
124-127	–	X2/X17	X3/X4/X11/X12	X8/X15	–	–	0.52
128-131	X5/X7/X13	–	X3/X4/X11/X12	–	–	X6	0.52
132-137	–	X2/X17	X3/X4/X11/X12	–	X9	–	0.50
138-145	X5/X7/X13	–	–	X8/X15	–	X6	0.50
146-157	–	X2/X17	–	X8/X15	X9	–	0.48
158-173	X5/X7/X13	–	–	–	X9	X6	0.48
174-179	–	X2/X17	X3/X4/X11/X12	–	–	X6	0.47
180-185	–	–	X3/X4/X11/X12	X8/X15	X9	–	0.45
186-197	–	X2/X17	–	X8/X15	–	X6	0.45
198-205	X5/X7/X13	X2/X17	–	–	–	–	0.45
206-229	–	X2/X17	–	–	X9	X6	0.43
230-233	X5/X7/X13	–	X3/X4/X11/X12	–	–	–	0.42
234-245	–	–	X3/X4/X11/X12	–	X9	X6	0.40
246-253	X5/X7/X13	–	–	X8/X15	–	–	0.40
254-277	–	–	–	X8/X15	X9	X6	0.38
278-293	X5/X7/X13	–	–	–	X9	–	0.38
294-299	–	X2/X17	X3/X4/X11/X12	–	–	–	0.37
300-311	–	X2/X17	–	X8/X15	–	–	0.35
312-327	X5/X7/X13	–	–	–	–	X6	0.35
328-351	–	X2/X17	–	–	X9	–	0.33
352-357	–	–	X3/X4/X11/X12	X8/X15	–	–	0.32
358-369	–	–	X3/X4/X11/X12	–	X9	–	0.30
370-393	–	X2/X17	–	–	–	X6	0.30
394-417	–	–	–	X8/X15	X9	–	0.28
418-429	–	–	X3/X4/X11/X12	–	–	X6	0.27
430-453	–	–	–	X8/X15	–	X6	0.25
454-469	X5/X7/X13	–	–	–	–	–	0.25
470-517	–	–	–	–	X9	X6	0.23
518-541	–	X2/X17	–	–	–	–	0.20
542-553	–	–	X3/X4/X11/X12	–	–	–	0.17
554-577	–	–	–	X8/X15	–	–	0.15
578-625	–	–	–	–	X9	–	0.13
626-649	–	–	–	–	–	X6	0.10
650	Perfect state						0

Table 2. Parameters of component failure times. θ is a scale parameter and γ is a shape parameter

No.	Component	Codes	θ	γ
1	Engine	X1, X10, X18, X24	20000	1.95
2	Electric motor pump	X2, X17, X22, X28	14000	2.13
3	Engine-driven pump	X3, X4, X11, X12, X19, X20, X26, X27	16000	2.43
4	Fuel shut-off valve	X8, X15, X23, X29	32000	2.24
5	Generator	X5, X13, X21, X25	14000	1.68
6	Hydraulic reservoir	X6, X14	30000	1.21
7	APU generator	X7, X16	18000	1.79
8	Ram air turbine	X9	10000	1.46

nents, there are a total of 8 types of components, which include engines X1, X10, X18, X24; generators X5, X13, X21, X25; EDPs X3, X4, X11, X12, X19, X20, X26, X27; EMPs X2, X17, X22, X28; FSOVs X8, X15, X23, X29; APU generators X7, X16; RAT X9; and hydraulic reservoirs X6, X14. Considering the common cause failure of the redundant components of the aircraft, we only analyze and discuss the energy components of one hydraulic system and one electrical system in the 2H2E structure layout.

Based on the above analysis, we listed a combination of all the failed-component situations. Components that may fail comprise various states of the hydraulic energy system. These states are shown in Table 3. Each column indicates that there is a type of component failure in the system, and the "/" in each column means "or". States 1 to 649 are the intermediate states; they represent system performance degradation but no failure. State 0 is the complete failure state, which represents that the system has failed. State 650 is the perfect state. The components in each state represent that failure has occurred. Therefore, c_j represents the combination of failure maintenance cost for components in each state. For the failure maintenance cost c_j of each state, we did normalization processing.

4. Results analysis

In this section, we simulated the above model. The reliability and failure rate of the model follow the Weibull distribution of two parameters, i.e. the scale parameter θ and the shape parameter γ. Fig. 3 shows the plot of the IBMP values of each component over time. Fig. 4 shows the JIBMP values for each component at 3,000 h. Fig. 5 shows the JIBMP values at 6,000 h. Then we analyzed the simulation results. On this basis, we analyzed the maintenance optimization strategy of a hydraulic energy system under the condition of a limited budget and determined the number of PM components under various budget constraints.

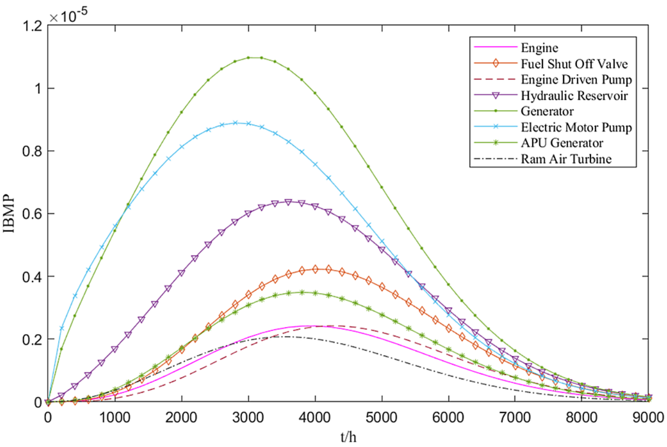

Fig. 3. Change in importance-based maintenance priority over time for various components

Fig. 3 shows the change in IBMP values over time. Their changing trend is affected by their reliability, failure rate and state transfer of each component at time *t*. Since the changing trend and degradation rate of the reliability and failure rate of each component are not the same, the probability of state transition of each component of the system is constantly changing due to their joint action. As can be seen from Fig. 3 the IBMP value of each component is zero at the beginning. This is because each component is in a perfect state at the beginning, so the contribution of each component to the change rate of the expected maintenance cost of the system is zero. With the operation of components, the performance of each component of the system degrades faster, so the expected maintenance cost of each component increases faster. Hence the contribution of each component to the change rate of the expected maintenance cost of the system increases, and the IBMP value increases. In the later period, components run for a long time, which makes all components unreliable. Therefore, the expected maintenance cost tends to be the largest, so the contribution of each component to the change rate of the expected maintenance cost of the system tends to zero.

From Fig. 3 we can see that the IBMP value of the generator is the highest. On one hand, generators are relatively important and responsible for the entire electrical system of the aircraft. On the other hand, generators have a higher failure rate compared with other components. We can also see from Fig. 3 that the hydraulic reservoir also has a high maintenance priority. That is because on one hand the redundancy of the hydraulic reservoir is low in the aircraft hydraulic energy system, and on the other hand, when the hydraulic reservoir fails, the entire hydraulic system starts to malfunction, leading to a hydraulic actuator failure, which affects the safety of the aircraft. Fig. 3 shows that the engine has a relatively low IBMP value because the failure rate of the engine is extremely low, and it has high redundancy. Therefore, although it plays an extremely important role in the operation of the aircraft, it has a very low maintenance priority. By sorting the IBMP values at a certain moment, the maintenance priority of each component can be determined, and the maintenance strategy of each component can be carried out based on this order.

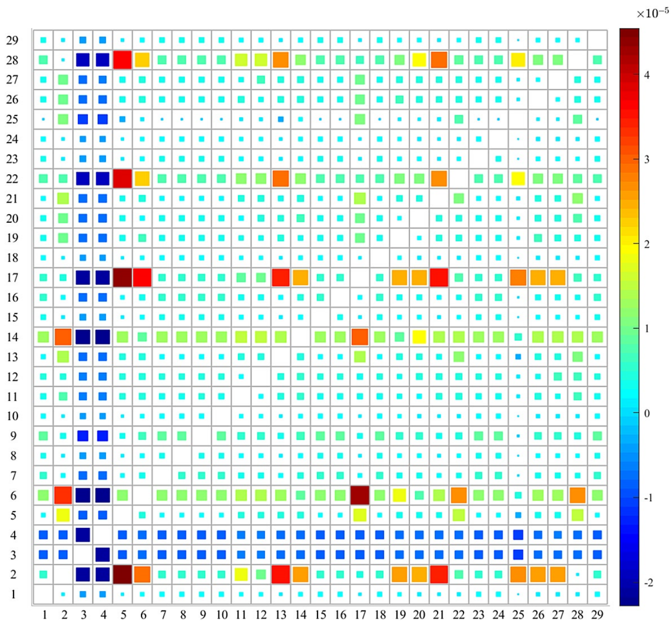

Fig. 4. Components of joint importance-based maintenance priority values at 3,000 h. Sizes and colors of squares represent levels of JIBMP values.

Fig. 4 shows the JIBMP interrelationship of each component at 3,000 h, and Fig. 5 at 6,000 h. The size and color of each grid cell

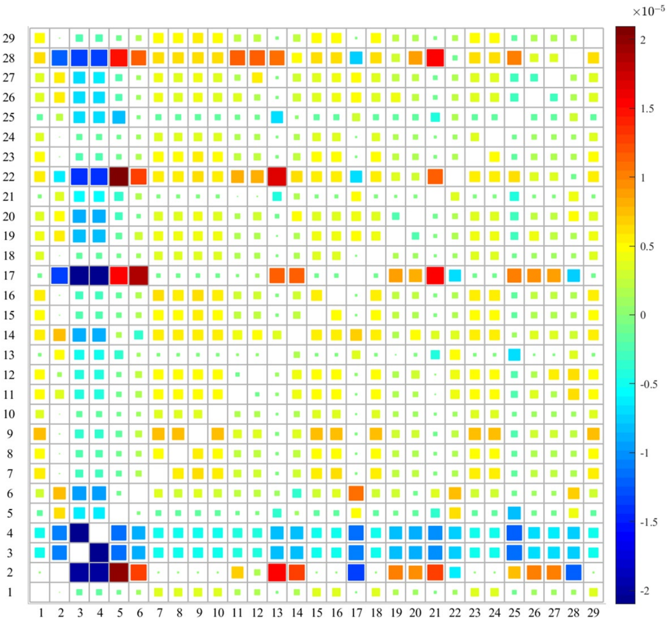

Fig. 5. Components of joint importance-based maintenance priority values at 6,000 h. Sizes and colors of squares represent levels of JIBMP values.

represent the level of JIBMP values between the components. From Fig. 4, we can see that when the hydraulic energy system has run for 3,000 h, the JIBMP values of components 5 and 2, and 5 and 17, are the highest. Combined with Table 1, it shows that when the generator X5 is under maintenance, the maintenance priority should be given to the EMPs X2 and X17. Components 6 and 17 have relatively large JIBMP values, indicating that when EMP X17 is under maintenance, hydraulic reservoir X6 is the best choice for PM, and vice versa. Fig. 5 shows that the JIBMP interrelationships of components 5 and 2, 6 and 17, and 5 and 22 are similar to those in Fig. 4. Also, JIBMP values between some components are negative, indicating that the maintenance sequence of these components has a negative effect on reducing the rate of change of system expected maintenance cost. Next we use the IBMP model to discuss components maintenance strategy.

Table 4 lists the sequence of the IBMP value of each component at 3,000 and 6,000 h. We can see that the sequences of IBMP values are different for the two durations. At 3,000 h, the top three values in the PM sequence are generator, EMP, and hydraulic reservoir. However, at 6,000 h, the top three are generator, hydraulic reservoir, and EMP. This is because the reliability and failure rate of components change over time, which leads to changes in the change rate of system maintenance costs. Therefore, according to the IBMP value, we can determine the best PM sequence, which can effectively guide the selection of PM components on a limited budget.

Table 4. Values of the importance-based maintenance priority at 3,000 h and 6,000 h

Component	Value at 3,000 h		Value at 6,000 h	
	IBMP ($\times 10^{-5}$)	Order	IBMP ($\times 10^{-5}$)	Order
Engine	0.206	6	0.125	7
Electric motor pump	0.886	2	0.278	3
Ram air turbine	0.196	7	0.085	8
Fuel shut-off valve	0.341	4	0.234	4
Generator	1.097	1	0.374	1
Hydraulic reservoir	0.602	3	0.293	2
APU generator	0.309	5	0.167	5
Engine-driven pump	0.178	8	0.150	6

From the analysis in Section 2.4, we can know that the maintenance strategy can be expressed as $\max_{d_i} \sum I_i^{IBMP}(t) \cdot d_i$, and the constraint function of maintenance cost is $\sum c_i d_i \le C$. There are 29 important components in the aircraft hydraulic energy system, so d_i has 2^{29} cases. The IBMP value of each component changes with time, so the maintenance strategy also changes. The maintenance cost for each component is listed in Table 5.

The maintenance strategy according to the rank of IBMP value and the constraint function of the maintenance budgets are shown in Table 6. We set two maintenance periods of 3,000 h and 6,000 h. When the total maintenance budget is within $30,000, priority should be given to the maintenance of generators and EMPs. When the total maintenance budget is within $70,000, the best choice is to add hydraulic reservoirs and FSOVs for maintenance. When the total maintenance budget is within $100,000, we need to add APU generators to the PM.

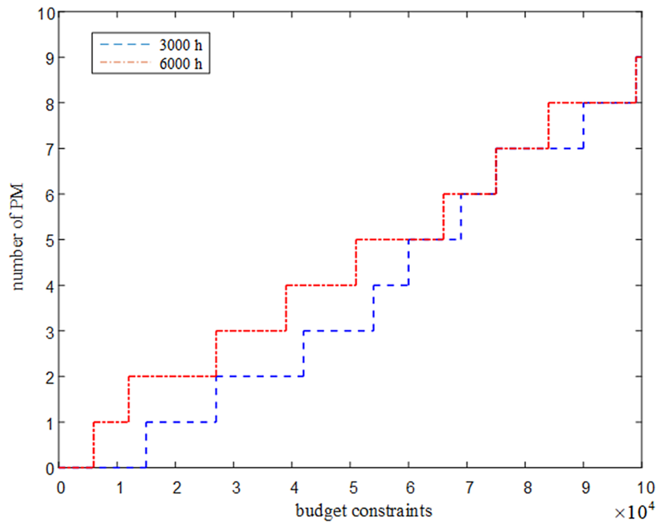

Fig. 6. Number of PM instances for various budget constraints

Fig. 6 shows the relation between budget constraints and the number of PM instances. The number of components available for PM gradually increases with increases in the budget. However, because the IBMP values for each component change over time, the two curves for the number of PM components do not overlap. As can be seen from Fig. 6, when the budget is less than $30,000, two or three components should be considered for PM. When the budget is within $70,000, six components should be considered for PM. When the budget is under $100,000, nine components should be considered for PM. The sequence of PM for components under various budget constraints can be seen in Table 6.

5. Conclusions and future work

In this paper, two maintenance measures are proposed to guide cost-based maintenance for the priority issue of component maintenance. The proposed methods are applied to aircraft 2H2E architecture, and the following conclusions are drawn: a) Preventive maintenance (PM) priority of different components changes over time, and importance-based maintenance priority (IBMP) value increases first and then decreases over time, indicating that the expected change rate of maintenance cost of components increases with the decrease of component reliability until the maintenance cost tends to the maximum and the maintenance cost change rate tends to zero; b) When a key component of the system fails, the expected change rate of maintenance cost of the system is different for the opportunistic maintenance (OM) of different components

Table 5. Maintenance cost in US dollars for each component

Component	Maintenance cost	Component	Maintenance cost
Engine	30,000	Generator	15,000
Electric motor pump	12,000	Hydraulic reservoir	6,000
Engine-driven pump	10,000	APU generator	15,000
Fuel shut-off valve	9,000	Ram air turbine	8,000

Table 6. Maintenance strategy for different budgets at different operation durations

	30,000 dollars	70,000 dollars	100,000 dollars
3,000 h	Generator No. 1	Generator No. 1	Generator No. 1
	Electric motor pump No. 1	Generator No. 2	Generator No. 2
		Electric motor pump No. 1	Electric motor pump No. 1
		Electric motor pump No. 2	Electric motor pump No. 2
		Hydraulic reservoir No. 1	Hydraulic reservoir No. 1
		Fuel shut-off valve No. 1	Hydraulic reservoir No. 2
			Fuel shut-off valve No. 1
			Fuel shut-off valve No. 2
			APU generator No. 1
6,000 h	Generator No. 2	Generator No. 3	Generator No. 3
	Hydraulic reservoir No. 1	Generator No. 4	Generator No. 4
	Hydraulic reservoir No. 2	Hydraulic reservoir No. 2	Hydraulic reservoir No. 1
		Electric motor pump No. 3	Hydraulic reservoir No. 2
		Electric motor pump No. 4	Electric motor pump No. 3
		Fuel shut-off valve No. 2	Electric motor pump No. 4
			Fuel shut-off valve No. 3
			Fuel shut-off valve No. 4
			APU generator No. 2

at different time periods, and the joint importance-based maintenance priority (JIBMP) values of different components are significantly different; c) After determining the planned expenditure cost of the airline for regular maintenance of the aircraft, i.e. the budgeted cost, with the increase of the budgeted cost, the number of components which need to be maintained is gradually increasing. With the change of maintenance time, the components which need to be maintained are also changing.

Future work will combine IBMP and JIBMP models proposed with component resilience measures to conduct joint maintenance decision analysis for key components at different stages of the system.

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