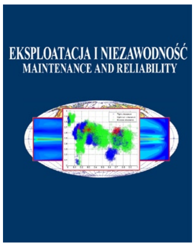

Eksploatacja i Niezawodnosc – Maintenance and Reliability

Volume 24 (2022), Issue 1

journal homepage: http://www.ein.org.pl

Article citation info:
Loska A, Paszkowski W. Geometric approach to machine exploitation efficiency: modelling and assessment. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2022; 24 (1): 114–122, http://doi.org/10.17531/ein.2022.1.13.

Indexed by: Web of Science Group

Geometric approach to machine exploitation efficiency: modelling and assessment

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Highlights

- The analysis of selected exploitation measures in discrete production processes is presented.
- A new approach to OEE modelling is proposed, based on a geometric interpretation of the time dependent components.
- The verification of this developed model is carried out with the use of analytical and simulation tools.
- The proposed method allows for the real-time mapping of the variability of the exploitation efficiency.
- There is a significant difference to the classical static approach of such an assessment

Abstract

This article presents a new approach to the exploitation assessment of machines and devices. A key aspect of this approach is the construction of the assessment model based on the geometric representation of measures associated with each other, which covers the full specifics of the exploitation process. This approach is successfully implemented by the Overall Equipment Effectiveness (OEE) model, which is fully susceptible to the geometric modelling process due to the three-way system of assessed exploitation aspects. The result of this approach is the vectored OEE model and its interpretation in terms of time series of changes in values of components. Methods of determining vector calculus measures were developed, including the second-order tensor and gradient. This is the subject of the variability of the reliability conditions of machines or production processes. It allows for the realisation of an exploitation assessment based on dynamic changes in the values of their components in the time domain. This is a significant difference to the classical static approach to such an assessment. The developed new geometric OEE model was confirmed by verification tests using the LabView software, based on two parallel data sets obtained with analytical and simulation methods using the FlexSim software.

Keywords

exploitation process, time series, exploitation efficiency, OEE, gradient, tensor, vectored model, maintenance.

1. Introduction

Exploitation decision-making problems relate to the search for ways to extend the periods of use (operate) and shorten non-use (maintenance) times with the assumed quality level of the performed work. The choices cover both technical and non-technical aspects (economic, organizational). In the exploitation assessment process, the operation of technical objects in specific environmental conditions is also taken into account. Such an approach means that the specificity of the undertaken exploitation decision problems is influenced by those factors which are related to the variability of both determinate and random features.

The need to achieve and maintain high exploitation efficiency in industrial practice applies not only to individual machines and devices, but above all to complex technical systems (e.g., industrial installations, process lines, network technical systems) [11]. For this reason, in the operational decision-making process, it is necessary to jointly consider the systems of machines and devices operating in the system. The increasing complexity of such systems may result in a potential deterioration of the values of reliability features. Different durability and diagnostic susceptibility of components of complex technical systems frequently make rational exploitation decisions and activities difficult to use [35]. However, the summary value of such systems (e.g., replacement cost) often significantly exceeds the amounts that are currently available to maintain their efficiency. These factors necessitate systematic exploitation (maintenance and repair) actions in order to achieve the desired effect over the long term.

The basis for undertaking, and consequently implementing, preventive and intervention maintenance actions is an appropriate exploitation assessment system. Such a system consists of models, methods, and tools that allow for:

- obtaining and collecting data directly describing the exploited machines,
- processing data into exploitation measures,
- interpretation of the results of measures in relation to machines and the exploitation context,
- possible feedback on the analysed operational decision-making process on the analysed exploitation decision-making process realised in the industrial environment.

One of the most important aspects of developing the exploitation assessment is the identification and verification of the measure model. Such a model, which is a set of deliberately selected indicators, must take into account, on the one hand, the specificity of the exploitation

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process, including the required direction of its analysis and interpretation, and, on the other hand, the availability of an appropriate resource of input data.

Various operational evaluation systems are most often built on the basis of a set of averaged measures clustered within the Overall Equipment Effectiveness (OEE) indicator. The popularity of this model stems primarily from the simplicity of its construction of measures (low mathematical complexity), easy accessibility of input data, and mutual comparability between different technical systems. The area and scope of the interpretation of the measurement values is also significant. It can be related not only to the analysed machines, but also to their environment and the functioning of the maintenance department.

However, the popularity of the OEE model also causes ambiguity due to its many variants. This results from both the high flexibility of the construction of individual measures and the ways of interpreting the results of the assessment. Therefore, in the latter part of this article, the use of vector calculus and simulation methods for the construction of the exploitation assessment model and the method of calculating and distributing measures within the new OEE model are discussed.

2. Analysis of the possibility and need of using OEE for the exploitation assessment of machines in discrete production processes

The effectiveness of the exploited technical systems is defined ambiguously. This is indicated by the ongoing scientific debate about the effects of working machines and devices. In the classical approach, technical and/or economic efficiency can be distinguished [4]. More complex interpretations assume the possibility of building a model of efficiency as a resultant value of features of different importance. The problem of assessing effectiveness is addressed in a wide range of research works, the majority of which are concerned with attempts to develop mathematical models of measures and the implementation of organisational procedures. Contemporary publications describing the results of research on the efficiency of exploitation cover a wide range of issues concerning Total Productive Maintenance (TPM) strategy, including methodological assumptions, among others [21, 34], and application solutions [16, 29, 30]. Attempts to build computational models and their industrial verification are also widely undertaken, based on the measures focused on the OEE model [9, 28], or operational measures concentrated in a set of KPI (Key Performance Indicators) [22, 27, 31], often associated with the issue of benchmarking [26, 37].

The efficiency measures most often express the level of key exploitation features in a comprehensive and aggregated manner. Mathematical models are resultant values in relation to the applied simple measures, describing selected aspects of the exploitation of technical systems. In this area, it can be distinguished [23, 28]:

- The exploitation availability indicator is described by the following relationship:

$$A = \frac{MTBF}{\left(MTBF + MFOT\right)} \tag{1}$$

where: MTBF - Mean Time Between Failures, MFOT - Mean Force Outage Time.

This expression represents the relationship between broadly understood reliability and maintainability. The expected availability assumes the maximisation of MTBF while minimising the MFOT.

The exploitation efficiency indicator is described by the following relationship:

$$E = \frac{MTBF}{\left(MTBF + MTTR\right)} \tag{2}$$

where: MTTR - Mean Time to Repair.

This expression, similar to (1), represents the relationship between reliability and maintainability. However, in this case, maintainability has a different interpretation as it more closely reflects efficiency and responsiveness in the organisational context. This method of determining efficiency shows its importance in the assessment of a company's maintenance department.

The OEE model is the most important part of the quantitative assessment of the TPM strategy. This indicator expresses the overall exploitation efficiency using three main factors: availability, efficiency, and quality (Tab. 1).

Table 1. Components of the OEE indicator [21, 28, 34]

Availability		**Efficiency**		**Quality**	
$D = \frac{t_d - t_p}{t_d}$	(3)	$E = \frac{t_c \cdot n}{t_o}$	(4)	$J = \frac{n - d}{n}$	(5)
t_d - worktime, t_p - downtime.		t_c - theoretical cycle time, n - processed quantity, t_0 - operational runtime.		n - quantity of produced products, d - quantity of incorrect products (defects).	
		$\boldsymbol{OEE = D \cdot E \cdot J}$			(6)

The OEE model is characterised by high flexibility in terms of the possible structure of data sets as the basis for determining the particular measures. This causes the goal of obtaining the values of these measures to become dominant. However, less attention is paid to the realisation of the assessment, including its representation and the selection of input data sets. The exploitation assessment of technical systems, based on the OEE model, is the result of the simultaneous observation of two related processes:

- the exploitation process describes the time course of operation of production machines and devices, expressed in the form of a set of events, which may be subjected to a technical object, with a change between possible technical states,
- in the production process, in this context, the effect of the operation of production machines and devices is in the form of products or services.

The implementation of production processes requires maintenance and repair work of machines and devices (planned and random), considered as part of exploitation processes. On the other hand, the analysis of the exploitation process as a sequence of time-period series of observations of events approximates the mapping of the work of technical systems.

The shaping and interpretation of the OEE model has for years been the subject of many concepts and solutions and, consequently, the resulting publications. These publications can be organised into the following thematic groups:

- the fundamental principles of building the OEE model in its original form, and research on the ways and scope of its application, besides user experiences and opinions, are considered. This includes proposals for interpretation at various points of reference (e.g., taking into account the specifics of various branches of industry) [3, 5, 12, 14],
- the ways to shape measures concerning the concepts of valuation and weighting of the component measures of the OEE model (D, E, J), as well as research on the impact and interpretation of input factors on the form of the OEE model, especially in terms of meaning, taking into account the specificity of the environment of the operated machines and the implemented production/exploitation process [2, 13, 32, 33],

ways of interpreting the OEE model, taking into account the context of the object and the production/exploitation process, and the effects of which may be an element of the decision-making process [7, 8, 15, 25].

Research on the construction of the exploitation assessment models, as well as the interpretation of their results, allowed for the identification of the research gap. The authors assumed that the research gap lies in the imperfection of the classical mathematical OEE model, which can be expressed with the following arguments:

- multiple complexity is manifested by the fact that the result of OEE calculation is a simple product of partial measures (components), which themselves are measures of relative values based on the input data; a product constructed in this manner is extremely sensitive to minor value changes in any of its factors,
- the internal linearity of partial measures (components) consists of the fact that the OEE model does not take into account the differences in the impact of individual partial measures on the final result,
- the mutual dependence of input factors of partial measures implies the possibility of multiple inclusion of the same features in the final computational result of the OEE model,
- the limited domain-specificity of the OEE model consists of the possibility of calculating and interpreting measures regardless of time, which is of key importance in the assessment of the exploitation events and processes due to the nature of the functioning of machines.

The identified and described research gap is the starting point for the formulation and solution of the research problem, i.e. the development of an OEE evaluation model that would be free from the limitations described above.

The research problem being solved is of a modeled-mathematical nature, but its effects are observed in the industrial environment, in the practical area of machinery and devices exploitation. In this context, the machine can be treated as a dynamic object, the features of which change over time and depend on their values in previous moments of time [6]. The dynamic and continuous nature of the machine operation influences the choice of its observation. Based on the assessment of the variability features of the machine, it is possible to infer not only the variability of its technical condition but also that of its interaction with the environment [24]. In the assessment of machines with the use of the OEE model, there are considered long-term input data sets continuous in time, which in many cases do not reflect the specifics of real production processes and, above all, real exploitation processes. These processes are characterised by interruptions directly related to the course of the technological process or breaks/downtimes resulting from the exploitation of machines. Thus, the realisation of production and exploitation processes and the specificity of their direct linkage justify the discrete assessment. This introduces a fundamental change in the approach to the exploitation assessment method in relation to the classic OEE model. This change consists of the use of the discrete form of time in the description of production and exploitation events identified at equally defined unit time intervals. The result of this approach is the consideration of data sets describing the discussed processes in the form of time series. The analysis of the time series in relation to the OEE model, apart from the assessment process itself, offers possibilities of:

- identifying changes in the discussed processes represented by the sequence of observations. That is, determining a trend (development tendency) and distinguishing cyclical and seasonal fluctuations,
- predicting and simulating future values of the component measures of the OEE model.

In summary, the main goals of the research described in the article include:

- development of a geometric model of the exploitatation assessment based on the OEE structure,
- development of the assessment method with the use of time series of changes in selected features of production and exploitation processes,
- verification of the developed models in the context of data obtained in the discrete events simulation process.

The analysis of the existing solutions described in scientific publications shows that the methodproposed by the authors is original. It has not been undertaken so far in the exploitation area.

3. Geometric interpretation of the OEE model

It is assumed that the components of the OEE model can be analysed as discrete variables represented by vectors [1, 17, 20]. In this approach, OEE is a cumulative representation of three component vectors, the values of which are subject to change over time.

This means that the OEE model in a three-dimensional system can be expressed as a vector basis whose components are: availability ($\vec{D}$), efficiency ($\vec{E}$), and quality ($\vec{J}$). They are spread over planes of a rectangular coordinate system (Fig. 1). Each of the component vectors is respectively a projection of the OEE vector on the axes x, y, and z.

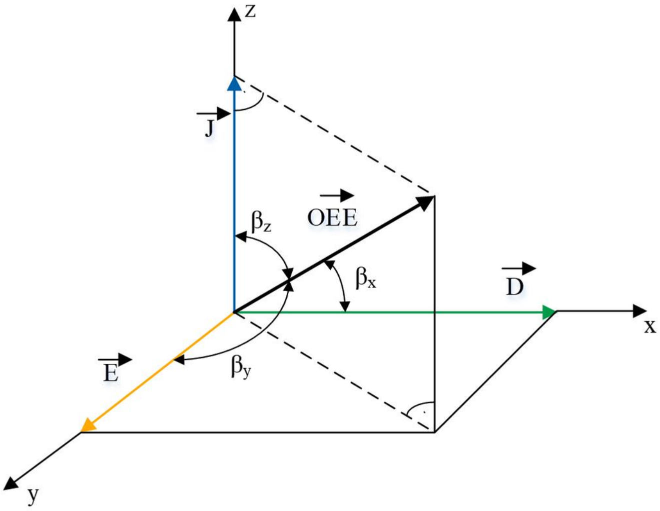

Fig. 1. Vector representation of the OEE model with projections of its components on the axes in a rectangular coordinate system

Based on Fig. 1 and (6), the OEE indicator can be represented as a vector:

$$\overrightarrow{OEE} = \left[\vec{D}, \vec{E}, \vec{J}\right] \tag{7}$$

Introducing the axis unit versors:

$$\vec{d} = [1,0,0], \vec{e} = [0,1,0], \vec{j} = [0,0,1] \tag{8}$$

then $\overrightarrow{OEE}$ takes the form:

$$\overrightarrow{OEE} = \vec{D} + \vec{E} + \vec{J} \tag{9}$$

The vector $\overrightarrow{OEE} = \left[\vec{D}, \vec{E}, \vec{J}\right]$, forms the angles $\beta_x, \beta_y, \beta_z$ β_x, β_y, β_z with the coordinate axes. For the coordinates of the vector $\overrightarrow{OEE}$ there are appropriate geometrical relationships:

$$\vec{D} = \overrightarrow{OEE} \cdot \cos\beta_x, \quad \vec{E} = \overrightarrow{OEE} \cdot \cos\beta_y, \quad \vec{J} = \overrightarrow{OEE} \cdot \cos\beta \tag{10}$$

$$\cos^2\beta_x+\cos^2\beta_y+\cos^2\beta_z=1$$

Assuming that the vectors $\vec{D},\vec{E},\vec{J}$ are non-coplanar, a parallelepiped can be stretched over them, by placing the beginnings of these vectors at a selected point *O* in the Euclidean space (Fig. 2).

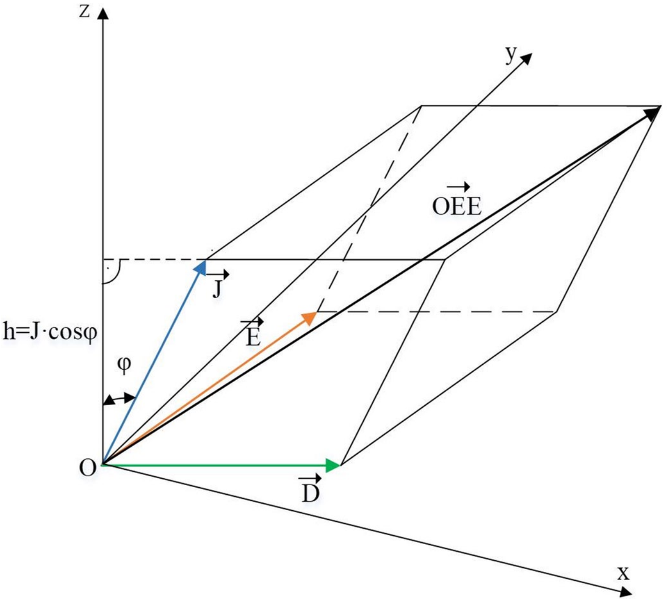

Fig. 2 Graphical representation of a parallelepiped stretched over the $\vec{D},\vec{E},\vec{J}$ vectors

Assuming, that the area of the parallelogram spanned by $\vec{D}$ and $\vec{E}$ is a vector product $\left|\vec{D}\times\vec{E}\right|$, and the height of the parallelepiped *h* is expressed through $\vec{J}\cdot\cos\phi$, then the resulting volume of the parallelepiped can be determined from the mixed product of the component vectors $\vec{D},\vec{E},\vec{J}$:

$$V=\left|\left(\vec{D}\times\vec{E}\right)\cdot\vec{J}\right| \tag{11}$$

Geometric representation of the vector $\overrightarrow{OEE}$, expressed by a mixed product of vectors $\vec{D},\vec{E},\vec{J}$, means that the same values of the parallelepipeds volumes (i.e., $V_1=V_2$) can correspond various features of these vectors in the form of their coordinates and angle φ (Fig. 2). This mixed product has all the properties of the determinant, including multi-lineage. In other words, you can get the same values of the vector $\overrightarrow{OEE}$ for different variants of combinations of the features of the component vectors $\vec{D},\vec{E},\vec{J}$. This means, that the same values of the determined volumes of the parallelepipeds will then correspond to these combinations (11).

The image of the vector $\overrightarrow{OEE}$ and the component vectors $\vec{D},\vec{E},\vec{J}$ corresponds to the geometric interpretation of the model in relation to the exploited technical systems and exploitation processes. Such an interpretation may include:

- the vector length $\left|\overrightarrow{OEE}\right|$ determined geometrically in the Euclidean space both in the entire time range and in individual periods,
- the tensor of second order,
- the directional vector of a scalar function (gradient).

Including projections of the vector $\overrightarrow{OEE}$ on the individual coordinate axes (Fig. 1), the modulus of this vector (length) can be determined by the geometrical relationship with its components (12).

$$\left|\overrightarrow{OEE}\right|=\sqrt{\left|\vec{D}\right|^2+\left|\vec{E}\right|^2+\left|\vec{J}\right|^2} \tag{12}$$

The length of the vector $\left|\overline{OEE}\right|$ may be a new measure of exploitation assessment, Its interpretation in the context of vector calculus takes into account the variability and specificity of the exploitation and production process.

For the purposes of mapping the exploitation process, in particular taking into account the change in time of the values of its features, based on (12), the vector length $\left|\overline{OEE}\right|$ can be described by a formula (13):

$$\left|\overline{OEE}\right|(\delta t)=\sqrt{\left|\vec{D}\right|^2(\delta t)+\left|\vec{E}\right|^2(\delta t)+\left|\vec{J}\right|^2(\delta t)}$$
$$\delta t=t_i-t_{i-1} \tag{13}$$

Generalizing the formula (12) to vector form, model OEE can take the special form of a tensor of second order (as a square matrix 3x3) $\vec{D},\vec{E},\vec{J}$:

$$OEE=\begin{bmatrix} D & 0 & 0\\ 0 & E & 0\\ 0 & 0 & J\end{bmatrix} \tag{14}$$

where the components are the vectors $\vec{D},\vec{E},\vec{J}$ in the Cartesian coordinate system.

In the Cartesian coordinate system the quadric equation of the OEE tensor takes the following form:

$$D_i x^2+E_i x^2+J_i x^2=1$$
$$D_i>0;E_i>0;J_i>0 \tag{15}$$

The surface resulting from the equation (15) is an ellipsoid, and the lengths of its semi-axes represented respectively by:

$$\frac{1}{\sqrt{D_i}};\frac{1}{\sqrt{E_i}};\frac{1}{\sqrt{J_i}} \tag{16}$$

The form of the second-order tensor of the OEE model, as a linear representation, corresponds to the operations of the tensor calculus. The representation of the OEE model in the form of a tensor can be used in the description of the variability of the exploitation conditions. The components are calculated for values of i in order to compare the exploitation conditions corresponding to each moment in time D_i,E_i,J_i. On this basis, the exploitation tensor OEE_i can be determined. The obtained values of the exploitation tensor OEE_i, can be compared with each other and thus assessed for various time points.

The time distribution of OEE depends on the variables that can be described by a differentiable function. Expressing OEE as a vector of n partial derivatives of this function allows one to determine the direction of its greatest increase and the magnitude of this increase at a given point. Examination of the OEE with the use of the directional vector of a scalar function (gradient) [10] allows determining its variability, taking into account the importance and impact of its individual components for the assessment of the analysed exploitation process. Assuming the gradient in the Cartesian coordinate system for the needs of the exploitation assessment, there is also included the differentiability of the functions such that they express availability (D), efficiency (E) and quality (J):

$$\nabla OEE(D,E,J)=\left(\frac{\partial OEE}{\partial D},\frac{\partial OEE}{\partial E},\frac{\partial OEE}{\partial J}\right)$$
$$\partial D\neq 0,\ \partial E\neq 0,\ \partial J\neq 0 \tag{17}$$

The gradient of OEE for the versors of the individual axes: e_D, e_E, e_J of the Cartesian coordinate system can be written as:

$$\nabla OEE = \frac{\partial OEE}{\partial D}\cdot e_D + \frac{\partial OEE}{\partial E}\cdot e_E + \frac{\partial OEE}{\partial J}\cdot e_J \qquad (18)$$

If the formula (18) is related to a given time t_i, it becomes reasonable to determine the gradient of OEE for the discrete representation of data sets:

$$\nabla OEE(t_i) = \frac{\partial OEE(t_i)}{\partial D(t_i)} + \frac{\partial OEE(t_i)}{\partial E(t_i)} + \frac{\partial OEE(t_i)}{\partial J(t_i)} \qquad (19)$$

The use of a gradient for the purpose of determining measures of the OEE model allows for the exploitation assessment on the basis of dynamic changes in the time domain of the values of its components. This is a significant difference to the classical static approach of realisation of such an assessment.

4. Development of verification data sets

The proposed geometrical form of the OEE model has been verified. In order to obtain interpretative results, a procedure for acquiring and adjusting the input data is proposed and will be implemented. This procedure is shown in Fig. 3.

In the first step, an inventory of the components of the technical system involved in the production process was made. First, the authors relied on the serial production of plastic components, carried out with the use of an injection moulding machine. Then, based on the identified and pre-interpreted technical system, a model of the production and exploitation processes was developed using the Flexsim software (Fig. 4) [19, 36].

The developed simulation model includes:

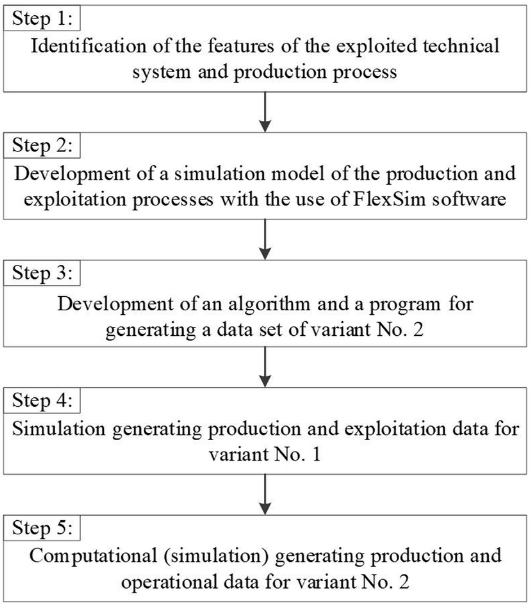

Fig. 3. The procedure for acquiring and positioning the input data

1. Source1 – token generator (logical element representing the product). The frequency of generating the next token was set at 5 minutes, i.e. an average of 12 tokens per hour according to a normal distribution with a standard deviation of 2 (*normal(12, 2, getstream(current)*).
2. Queue1 – buffer for generated tokens waiting to be processed in Processor1. The maximum queue capacity is assumed to be 1000 elements.
3. Processor1 – an object representing a production machine (in our case an injection machine). This object simulates a delay in relation to the maximum theoretical efficiency of the production process. The processing time for each product was set at 60

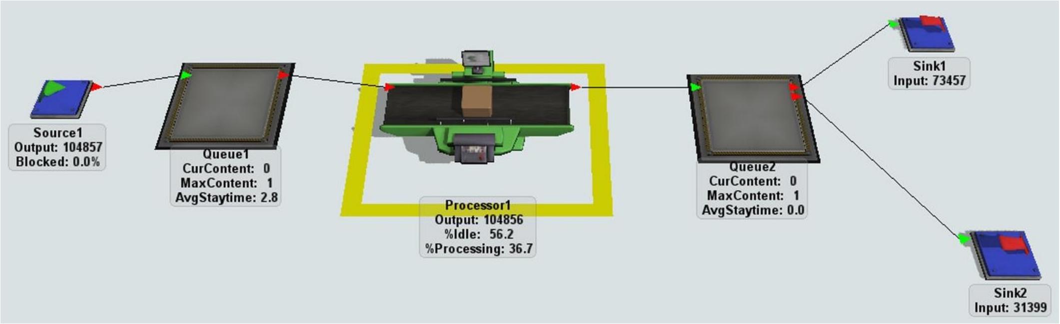

Fig. 4. The model of the production and exploitation processes developed with the use of the Flexsim software

Table 2. A part of the input data resources

Day No.	Variant No. 1				Variant No. 2			
	D_1	E_1	J_1	OEE_1	D_2	E_2	J_2	OEE_2
1	0,6963	0,7919	0,8475	**0,4673**	0,7913	0,6695	0,8822	**0,4673**
2	0,9963	0,5912	0,8623	**0,5079**	0,8119	0,9225	0,6781	**0,5079**
3	1,0000	0,8270	0,9825	**0,8126**	1,0000	1,0000	0,8126	**0,8126**
4	0,8894	0,8092	0,9231	**0,6644**	0,8608	0,8036	0,9604	**0,6644**
5	1,0000	0,4454	0,9449	**0,4209**	1,0000	0,4811	0,8748	**0,4209**
6	0,5181	0,4486	0,8443	**0,1963**	0,7138	0,3124	0,8801	**0,1963**
7	0,8406	0,5068	1,0000	**0,4260**	0,9281	0,5184	0,8853	**0,4260**
8	1,0000	0,6676	0,8893	**0,5937**	0,7300	0,8133	1,0000	**0,5937**
9	1,0000	0,8157	0,9822	**0,8012**	0,9388	0,8719	0,9789	**0,8012**
10	0,9256	0,6259	0,9320	**0,5400**	0,7781	0,8185	0,8478	**0,5400**

seconds, with a random delay described by the exponential distribution (*exponential(0.60, getstream (current)*). A simulation of the machine delay associated with downtime was described using the Weibull distribution with an average event frequency of 6 hours (*weibull(0.0, 11600, 4, getstream(current)*).

4. Sink1, Sink2 - process output. The model defines two outputs that separate the correct products from the defective products. This separation was made according to the discrete Bernoulli binomial distribution with an average probability of 0.9 (*bernoulli(70, 0, 1, getstream(current)*).

This enabled the generation of a complete set of input data for the OEE model. Based on the simulation model, the simulation process was launched. This allowed for the generation of a time series with a unit of one day of production and exploitation values. Finally, the values of the OEE model were calculated on the basis of the relationships (3) - (6).

In the third step, an algorithm and a programme generating the second set of input data were developed (using the Python and Pandas environments). For the needs of the latter set of input data, the same corresponding OEE result values were assumed for both data sets, at particular time points, with various values of their components. That is:

$$OEE_I(t) = OEE_{II}(t) \tag{20}$$

In practice, for each line, modifications of two component values were made in an iterative way, adjusting computationally the value of the third component in such a way that the resultant would not change.

The analyzed vectors: $\vec{D}, \vec{E}, \vec{J}, \overrightarrow{OEE}$, included the collected sets of input data represented in the discrete time points. For the purpose of simulation, equal cumulative OEE values for the tested variants were assumed. Every set contained 1810 data points for each of the vectors. A part of the input data resources for the two variants is shown in Tab. 2.

The differentiation of the components of the OEE model for individual variants and combinations of their pairs was tested using selected statistical evaluation measures, that is: skew, standard deviation, kurtosis and the coefficient of correlation (Spearman). The obtained values indicated significant differences in the input data sets of the time series of the components D, E, J, with the same forms of result sets of time series (OEE).

5. An example of the use of the developed model in the exploitation assessment

Based on the data prepared in accordance with the procedure and description presented in the previous section, verification calculations were carried out to confirm the mathematical correctness and industrial suitability of the developed method of exploitation assessment. For the purposes of this verification, calculations were carried out based on the models developed by the authors in the LabView environment. Examples of models with the results of calculations are presented in Fig. 5 - Fig. 8.

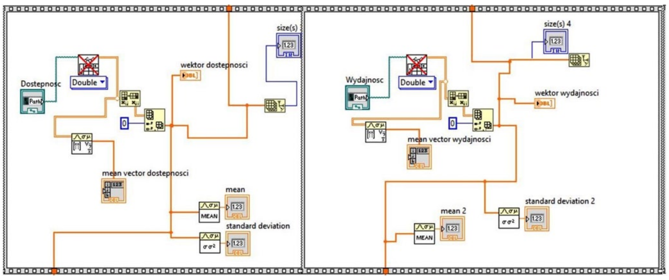

Fig. 5. A model for calculating the vector length $\left|\overrightarrow{OEE}\right|$

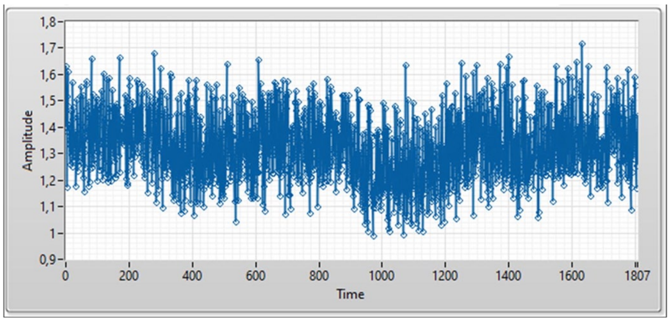

Fig. 6. A visualization of changes in length of the vector $\left|\overrightarrow{OEE}\right|$

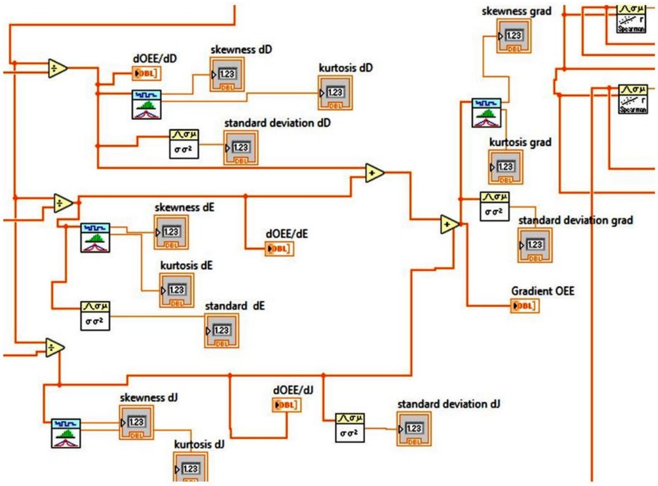

Fig. 7. A model for calculating the directional vector of the scalar function (gradient)

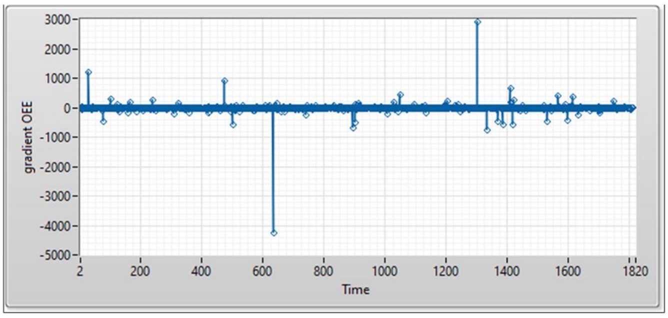

Fig. 8. A visualization of changes in the gradient ∇OEE

Based on a formula (12), the vector length values $\left|\overrightarrow{OEE}\right|$ for successive elements of the time series of the two considered variants were determined. Time changes of measures in classical and geometric versions are shown in Fig. 9. For greater readability, the set of results was limited to 50 observations.

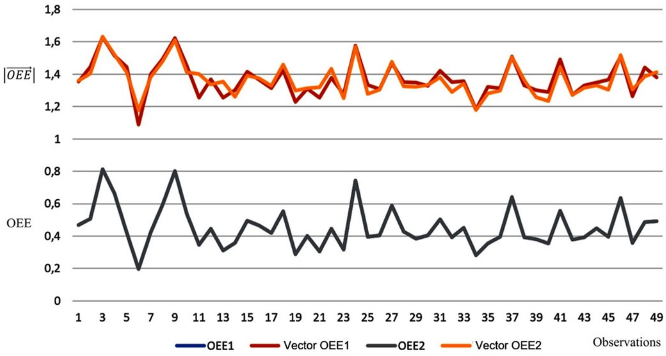

Fig. 9. Time changes of measures in classical and geometric versions for two variants of time series

Table 3. Values of selected statistical measures for two variants of time series

Statistical measure	**Variant No. 1**		**Variant No. 2**	
	OEE$_1$	$\left\lvert\overrightarrow{OEE}\right\rvert_1$	**OEE$_2$**	$\left\lvert\overrightarrow{OEE}\right\rvert_2$
Arithmetic mean	0,3931	1,3043	0,3931	1,3246
Median	0,3784	1,3026	0,3784	1,3211
St. deviation	0,1476	0,1427	0,1476	0,1241
Kurtosis	0,2059	-0,1102	0,2059	−0,2847

The results of the calculation indicate no differences in values of the statistical measures for variants of the OEE model for classic calculations (by the product of D, E, J), as well as significant differences in values and statistical measures for variants of the geometric OEE model (length of the $\left\lvert\overrightarrow{OEE}\right\rvert$ vector). This means a greater susceptibility (sensitivity) of the geometric model to the variability of the values of its components, and it has been shown, that:

- various component values (D, E, J) can lead to the same resultant values in the classic OEE model,
- various component values ($\vec{D},\vec{E},\vec{J}$) can lead to different resultant values in the geometric OEE model.

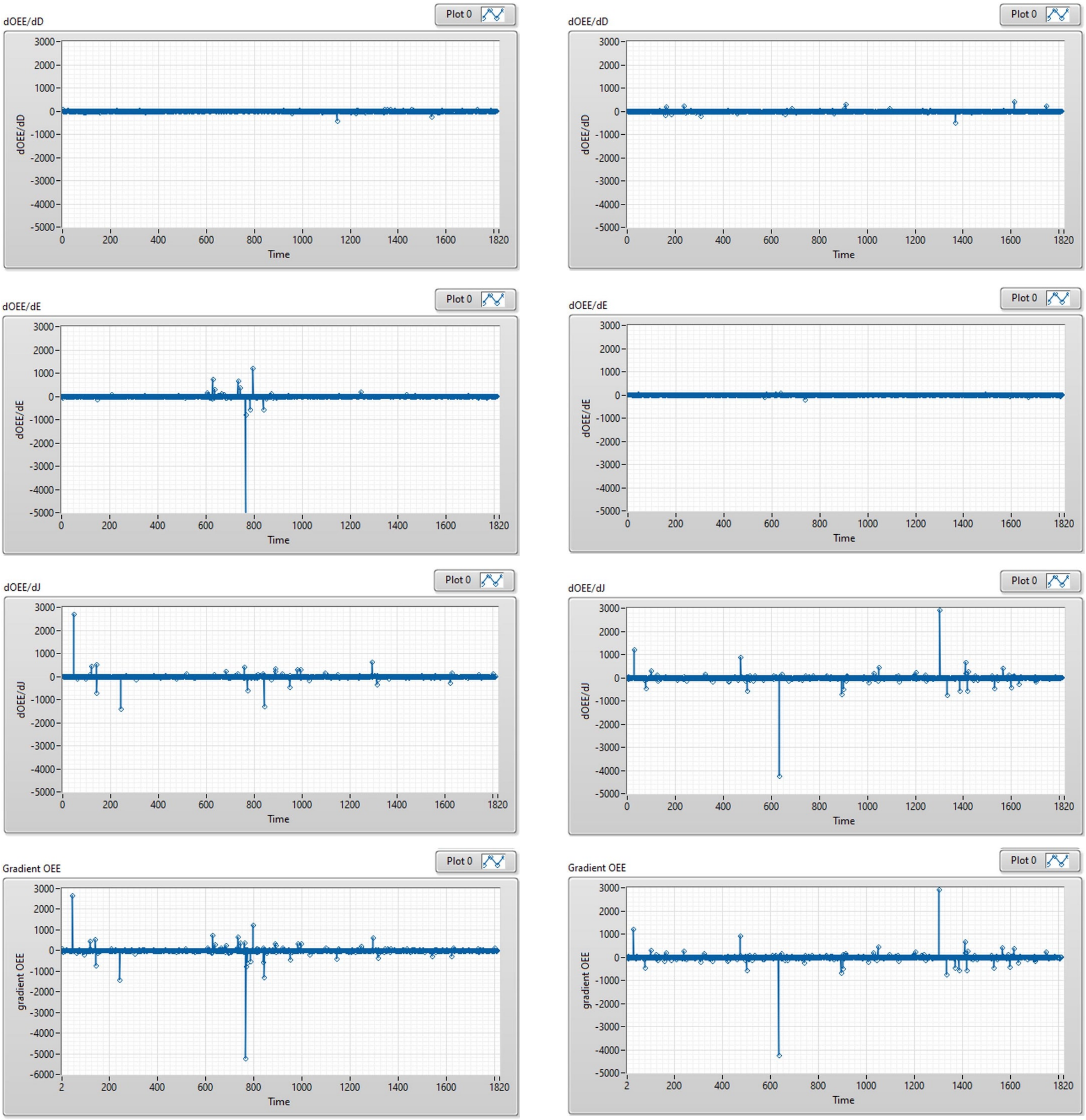

Fig. 10. Graphical interpretation of the gradient: a. the variant no. 1, b. the variant no. 2

Then, selected statistical measures were determined for all elements of the time series.

The above conclusions confirm the significance and practical usefulness of the geometric model for the evaluation of the implementation of the exploitation and production processes, because it takes into account individual components in the vector approach. This makes it possible to apply vector calculus operations to represent the components of the OEE model.

By analysing and interpreting the variability of vectors $\vec{D},\vec{E},\vec{J}$ in relation to the change in the vector $\overrightarrow{OEE}$, the gradient values ∇OEE, for the considered variants of time series, based on the formulas (17) - (19) were determined. The results are presented graphically in Fig. 10.

Based on the results obtained (Fig. 10), it can be concluded that the variability of the characteristics of vectors $\vec{D},\vec{E},\vec{J}$ in relation to the change in the vector $\overrightarrow{OEE}$ shows differentiation in a given time period. Thus, Thus, for the analysed variants, different gradient values, ∇OEE were obtained. Selected statistical measures presenting these differences are shown in Tab. 4.

Table 4. Statistical evaluation of the variability of the examined vectors

Statistical measure	**Variant No. 1**				**Variant No. 2**			
	$\frac{\partial OEE}{\partial D}$	$\frac{\partial OEE}{\partial E}$	$\frac{\partial OEE}{\partial J}$	$\nabla \boldsymbol{OEE}$	$\frac{\partial OEE}{\partial D}$	$\frac{\partial OEE}{\partial E}$	$\frac{\partial OEE}{\partial J}$	$\nabla \boldsymbol{OEE}$
Skewness	13,20	−33,62	11,77	**−17,19**	0,15	−14,92	−10,73	**−10,26**
St. deviation	14,89	131,95	88,62	**159,23**	22,08	7,50	136,26	**137,95**
Kurtosis	363,26	1354,05	548,40	**688,51**	224,57	436,98	639,85	**606,16**

Simulation studies allow for detailed variability analyzes of the vectors $\vec{D},\vec{E},\vec{J}$, in relation to the change in the vector $\overrightarrow{OEE}$ and the assessment of their impact on the obtained values of the gradient ∇OEE.

6. Conclusions

In the course of exploitation processes, there are complex relationships of technical features, the change of which may cause an increase or decrease in the effectiveness of the use of machines.

According to the authors, in the method of calculating OEE (as a product of the values of partial indicators), the absolute value of this indicator is not so important as the information resulting from its time variability. Therefore, the mathematical interpretation of its variability should have the character of a dynamic assessment. Taking into account the components of the OEE indicator, machine efficiency can be analysed in three-dimensional space based on the geometric (vector) representation of the component features. In this context, by presenting the OEE model in a vector form in three-dimensional space, it is possible to formulate conclusions not only about the instantaneous value of OEE but also about the speed and level of change in a given direction. Furthermore, this may determine the dynamics of changes in selected features of exploitation and production processes.

The results obtained for the differences in the determination of OEE measures in a classical and geometric manner for the tested variants may be more significant for the analysis of a larger amount of data. However, this requires further simulation tests.

The advantage of the presented solution is the possibility of identifying anomalies in the exploitation conditions of machines based on the assessment of the variability of the particular vectors $\vec{D},\vec{E},\vec{J}$ and the $\overrightarrow{OEE}$ vector itself, in a given time period.

The inclusion of time series in the analysis of exploitation and production processes opens new possibilities for assessing the exploitation efficiency of machines. In particular, the analysis, evaluation, and interpretation of the variability of the vectors $\vec{D},\vec{E},\vec{J}$, permits the determination of the operational efficiency for any time point.

In the authors' opinion, the proposed method allows for the real mapping of the variability of the tested exploitation efficiency. The implementation of such a solution may consist in the collecting and processing data in real time, with the simultaneous evaluation of the realization of the process. For this purpose, there can be used real-time wireless data transmission devices between the exploited production system and the data analysis system. The proposed method can be used to monitor the effectiveness of production and exploitation processes with the use of industry 4.0 solutions. This is of particular importance for the technical and economic exploitation assessment of machines and devices, which undoubtedly contributes to the reduction of the costs of enterprises.

The approach to the method proposed in the article consists of not only taking into account the influence of its components and its time variability, but also giving the opportunity to generate development scenarios and forecast the future exploitation policy of the enterprise [18].

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