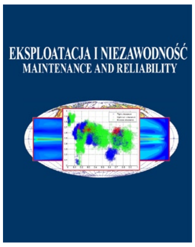

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The post-warranty random maintenance policies for the product with random working cycles

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Lijun Shang[a], Haibin Wang[b,c], Cang Wu[c], Zhiqiang Cai[b,*]

[a] Foshan University, School of Quality Management and Standardization, Foshan 528225, China
[b] Northwestern Polytechnical University, School of Mechanical Engineering, Xi'an 710072, China
[c] China United Northwest Institute for Engineering Design & Research Co., Ltd., Xi'an 710077, China
[d] Lanzhou University of Technology, School of Mechanical and Electronical Engineering, Lanzhou 730050, China

Highlights

- A novel warranty of the product with random working cycles is proposed.
- Random maintenance policies sustaining the post-warranty reliability are investigated.
- The performance of replacement last (first) with PM is more excellent.

Abstract

Advanced sensors and measuring technologies make it possible to monitor the product working cycle. This means the manufacturer's warranty to ensure reliability performance can be designed by monitoring the product working cycle and the consumer's post-warranty maintenance to sustain the post-warranty reliability can be modeled by tracking the product working cycle. However, the related works appear seldom in existing literature. In this article, we incorporate random working cycle into warranty and propose a novel warranty ensuring reliability performance of the product with random working cycles. By extending the proposed warranty to the post-warranty maintenance, besides we investigate the post-warranty random maintenance policies sustaining the post-warranty reliability, i.e., replacement last (first) with preventive maintenance (PM). The cost rate is constructed for each post-warranty random maintenance policy. Finally, sensitivity of proposed warranty and investigated polices is analyzed. We discover that replacement last (first) with PM is superior to replacement last (first).

Keywords

random working cycle, warranty, post-warranty reliability, replacement last, replacement first.

1. Introduction

In a broader sense, offering products to warranty can benefit simultaneously manufacturers and consumers. Due to this, the warranty policies (or models) have been recently researched widely from the manufacturer's perspective. The research stream on warranty policies is dependent on reliability modeling (or evaluation) technology. It is less difficult to discover that there are two types of reliability modeling technology, which are being used frequently in academia and industry. One type is that the product lifetime is modeled as a distribution function with self-announcing failure. For example, Li et al. [11] studied the reliability evaluation using limited and censored time-to-failure data, by means of the uncertainty theory; Zhang and Zhang [29] first proposed a reliability model of aviation cables by using nonlinear mixed model and Bayesian estimation, and then analyzed accuracy of reliability model by the failure time of cable. Other type is that the product failure is modeled as a type of degradation failure (which is referred to as not self-announcing failure [18]). For example, Gao et al. [5] developed methods to analyze the system reliability of two-phase degradation model with a random change point; Huang et al. [6] proposed a degradation model for soft failure by considering continuous degradation processes with recoverable shock damages for reliability assessment and lifetime prediction of products.

Along with the above frame on reliability modeling technology, similarly, the research stream on warranty policies can be distinctly divided into two research streams. The first stream concentrates on the design of the distribution-based warranty policies, namely design warranty policies by modeling the product lifetime as a distribution function with self-announcing failure. For example, Hooti et al. [7] proposed an extended two-dimensional warranty plan which includes limitation on time and the number of repairs, under the assumption that the lifetime of the system follows distribution function; Huang et al. [8] developed a model to determine the optimal sale price, warranty period and product reliability to maximize the discounted profit for a repairable product sold with a free replace-repair warranty policy, by assuming that the product failure time follows distribution function; He et al. [9] established the decision model of extended warranty price from the perspective of win-win by assuming that the product

(*) Corresponding author.
E-mail addresses: L. Shang - ljshang2020@126.com, H. Wang - 36185397@qq.com, C. Wu - 464968118@qq.com, Z. Cai - caizhiqiang@nwpu.edu.cn

failure time follows distribution function; Knopik and Migawa [10] investigated the effects of introducing preventive replacement to maintenance system implemented by age-replacement of technical objects with valid manufacturer's warranty and non-repairable, by means of Weibull distribution. Wang et al. [24] studied an optimal extended warranty policy after the expiration of base two-dimensional warranty with repair time threshold by assuming that the failure time of the equipment follows distribution function. The second stream aims to design the degradation-based warranty policies, namely design warranty policies by modeling the product failure as a type of degradation failure. For example, Cha et al. [2] and Zhang et al. [30] studied warranty policy of the product by modeling the product failure as degradation failure.

Usually, the manufacturer adopts some methods (maintenance or replacement, and so on) to ensure the product reliability performance during the warranty period. However, consumers (or users) tend to be concerned about how to sustain reliability during the post-warranty period (i.e., the post-warranty reliability). Due to increased maintenance costs, how to model a post-warranty maintenance to sustain the post-warranty reliability has recently received considerable attention. This type of problem has been also investigated extensively along with the above reliability modeling technology. For example, Liu et al. [14] investigated the optimal replacement problem for a warranty product subject to $(M+1)$ types of mutually exclusive failure modes, including M repairable failure modes and a catastrophic failure mode, by supposing that the warranty product's lifetime follows Weibull distribution; Park et al. [17] developed mathematical formulas to evaluate the long-run expected cost rates during the life cycle of the product, by considering the failure time of the product and a Weibull distribution; and Shang et al. [19] investigated an optimal maintenance-replacement policy after the warranty expiry by assuming that the product lifetime follows distribution function; Shang et al. [20] investigated the post-warranty maintenance by modeling the product failure as a degradation failure.

By modeling the product failure as a type of degradation failure, in essence, designing warranty policies and modeling post-warranty maintenance are undoubtedly driven and powered by in-situ sensor and measuring technologies, which can accurately or approximately measure/inspect health condition of the product. In addition to measuring product health condition, in-situ sensor and measuring technologies can monitor working cycle of the product effectively that performs successively projects or missions at random working cycle. In real word, lots of products work at random working cycle. For example, the intelligent air pump inflates the tire at random working cycle; and the intelligent cutter cuts the material at random working cycle, and so on. For the product with random working cycles (i.e., the product which works at random working cycle), from reliability theory, it deteriorates with respect to its working time. Considering this reality, Chang [3] and Sheu et al. [21, 22] researched preventive maintenance policies to ensure or enhance the product reliability performance of the product with random working cycles by modeling the product working cycles as an independent identically distributed random variable sequence; Nakagawa [16] and Zhao et al. [31] researched various maintenance policies of the product with random working cycles by assuming working cycle as an independent identically distributed random variable.

If the product working cycle is integrated into the warranty period and the post-warranty period, the following advantages can be brought: ① the manufacturer or the consumer can calculate the product reliability by making use of the monitored total working time in real time; ② The manufacturer can more precisely evaluate the warranty budget and more efficiently control warranty cost; ③ the post-warranty maintenance planning techniques (such as repair, replacement, imperfect preventive maintenance) of the consumer can be programmed and scheduled more reasonably, and the related maintenance cost can be reduced appropriately, and so on. However, few warranty policies and the post-warranty maintenance policies to sustain the post-warranty reliability have been developed by integrating the product working cycle.

In this article, we introduce the limited number of random working cycle to the warranty period and proposes a novel warranty from the manufacturer' perspective. The proposed warranty requires that if the failure doesn't occur until the warranty period before the completion of the limited number of random working cycle, then the proposed warranty expires at the warranty period; and if the failure doesn't occur until the completion of the limited number of random working cycle before the warranty period, then free repair (minimal repair) warranty [4, 15] will be triggered to warrant the product from the completion of the limited number of random working cycle to the warranty period. Defining that the proposed warranty is extended to the consumer's post-warranty maintenance model, we investigate two kinds of the post-warranty random maintenance policy to sustain the post-warranty reliability. The first type is replacement last with preventive maintenance (PM), where PM at the warranty period is integrated into random periodic replacement last [16]. The second type is replacement first with PM where PM at the warranty period is integrated into random periodic replacement first [16]. For each post-warranty random maintenance policy, we construct the related cost rate model by integrating the product's depreciation expense depending on the total working time. By means of the numerical experiments, we compare the performance of the post-warranty random maintenance policies.

The contribution of this article can be highlighted in three key aspects: (1) we propose a novel warranty to ensure reliability performance of the product with random working cycles; (2) we investigate two types of random maintenance policy to sustain the post-warranty reliability of the product, which seldom exists in literature; (3) the performance of replacement last (first) with PM is more excellent.

The structure of this article is organized as follows. Section 2 proposes the manufacturer's warranty, derives the related warranty cost. In Section 3, replacement last with PM and replacement first with PM are defined, and the related cost rate models are derived. Section 4 presents a comparing approach, which can help manufacturer to make decision on the post-warranty random maintenance policies. In Section 5, numerical experiments are used to illustrate the proposed approach and sensitivity analysis on some key parameters is performed. Finally, conclusions are drawn in Section 6.

2. Warranty model

It is assumed that the product does projects or missions successively, and random working cycle Y_j of the j^{th} ($j = 1,2,\dots$) project is independent identically distributed to the distribution function $G(y) = \Pr\{Y_j < y\}$ with the lack-of-memory property. The product deteriorates with respect to its working time and the time-to-first-failure X of the product is subject to a general distribution function $F(x) = \Pr\{X < x\}$ with a failure rate function $r(u)$ where $u > 0$. Besides, it is assumed that the downtime resulted from each replacement or each minimal repair is completely negligible in this article.

2.1. Warranty assumptions

The particular attractiveness of renewing (or renewable) free replacement warranty [13, 19] (RFRW) is that it makes possibly consumers to obtain freely a new identical product with the same warranty. Due to this particular characteristic, RFRW is an attractive warranty which can be used as a significant advertising tool from the manufacturer's perspective. Basing on the product working cycle monitored by using in-situ sensor and measuring technologies, besides the manufacturer can design warranty policies to ensure the product reliability performance. However, the existing RFRW model neglects universally to make proper use of the product working cycle.

In view of this, we consider the particular attractiveness of RFRW and study a novel warranty of the product which performs successively projects or missions at random working cycle, as below.

Given the number m (a limited value) of random working cycle and the warranty period w, the warranty proposed in this article is described as follows:

(1) The product will be replaced by a new identical one with the warranty proposed in this section (i.e., failure replacement) if the failure occurs before the completion of the m^{th} random working cycle or before the warranty period w, whichever occurs first.

(2) The manufacturer shoulders whole failure replacement cost (including labor cost, transport cost and so on) resulted from unit failure replacement.

(3) If the failure doesn't occur until the completion of the m^{th} random working cycle before the warranty period w, then free repair warranty [24, 28] (FRW) with a time span $w-S_m$ will be triggered to warrant the product, where S_m is the total working time of the product when the m^{th} random working cycle is completed and satisfies $S_m=\sum_{j=1}^{m}Y_j$; if the failure doesn't occur until the warranty period w before the completion of the m^{th} random working cycle, then the warranty expires.

(4) FRW requires that any failure in the interval $(S_m,w]$ is removed by minimal repair and the related minimal repair cost is also shouldered by the manufacturer.

Note that ① in this warranty, m and w are obviously two types of failure replacement limit, and so the warranty region related to failure replacement can be represented as $(0,m]\times(0,w]$; ② the product goes through warranty (i.e., the proposed warranty, hereinafter similarly) at the completion of the m^{th} random working cycle before the warranty period w or at the warranty period w before the completion of the m^{th} random working cycle, whichever occurs first; ③ for some consumers with a higher product working frequency, their warranty expires very easily at the completion of the m^{th} random working cycle before the warranty period w, therefore the manufacturer offers them a FRW so that they are treated as equal as other consumers whose warranty expires at the warranty period w before the completion of the m^{th} random working cycle.

2.2. Warranty cost modeling

Let $\bar{F}(x)$ be survival function of the time-to-first-failure X of the product, where $\bar{F}(x)=1-F(x)$. And let the Stieltjes convolution $G^{(m)}(s)$ ($G^{(m)}(s)=\int_0^s G^{(m-1)}(s-u)\mathrm{d}G(u)$) and the Stieltjes convolution $\bar{G}^{(m)}(s)$ ($\bar{G}^{(m)}(s)=1-G^{(m)}(s)$) be respectively distribution function and survival function corresponding to the total working time S_m . According to the warranty proposed in Subsection 2.1, the case that the product goes through warranty can be divided into two types of case. The first case is that the product goes through warranty at the completion of the m^{th} random working cycle before the warranty period w ; and the second case is that the product goes through warranty at the warranty period w before the completion of the m^{th} random working cycle. By summing the probability of two types of case, the probability q that the product goes through warranty can be computed as:

$$q=\Pr\{S_m<w,S_m<X\}+\Pr\{w<S_m,w<X\}=\int_0^w\bar{F}(u)\mathrm{d}G^{(m)}(u)+\bar{G}^{(m)}(w)\bar{F}(w)=1-\int_0^w\bar{G}^{(m)}(u)\mathrm{d}F(u) \tag{1}$$

Since the event that the product doesn't go through warranty and the event that the product goes through warranty form jointly a complete event group, the probability p that the product doesn't go through warranty is expressed as:

$$p=1-q=\int_0^w\bar{G}^{(m)}(u)\mathrm{d}F(u) \tag{2}$$

It is less difficult to conclude that the probability that until the i^{th} ($i=1,2,\dots$) product goes through the m^{th} random working cycle or the warranty period w is a geometric distribution $p^{i-1}q$, and the number of failure replacement is precisely $i-1$. Further, the expected number of all failure replacements produced by the proposed warranty can be modeled as:

$$E[\kappa]=\sum_{i=1}^{\infty}p^{i-1}q(i-1)=\frac{p}{q}=\frac{\int_0^w\bar{G}^{(m)}(u)\mathrm{d}F(u)}{1-\int_0^w\bar{G}^{(m)}(u)\mathrm{d}F(u)} \tag{3}$$

Let X_k ($k=1,2,\dots$) be lifetime of the k^{th} product failed during the warranty region $(0,m]\times(0,w]$. According to probability theory, then every element of the sequence $\{X_k\}$ is independent identically distributed to the distribution function $H(x)$ with the below expression:

$$H(x)=\Pr\{X_k<x\mid X_k<S_m,X_k<w\}=\frac{\int_0^x\bar{G}^{(m)}(u)\mathrm{d}F(u)}{\int_0^w\bar{G}^{(m)}(u)\mathrm{d}F(u)} \tag{4}$$

where $0<x<w$.

Suppose that the depreciation expense of the product is only affected by its working time t and is increasing with respect to t , then we model the depreciation expense $D(t)$ at t as:

$$D(t)=\alpha_1 t^{\beta_1} \tag{5}$$

where α_1 ($\alpha_1>0$) is depreciation rate; $0<\beta_1\le\log_w(c_R/a_1)$ where c_R is unit failure replacement cost suffered for the manufacturer.

For the k^{th} product failed during the warranty region $(0,m]\times(0,w]$, its working time is its lifetime X_k . So, the related depreciation expense is $D(X_k)$ and the k^{th} product failed prompts the manufacturer to suffer a cost $c_R-D(X_k)$. Until the $(i-1)^{\text{th}}$ failure replacement is completed, the replacement cost WC_{i-1} of the manufacturer can be obtained as:

$$WC_{i-1}=\sum_{k=0}^{i-1}\left(c_R-D(X_k)\right) \tag{6}$$

where $X_0=0$.

Since the random variable X_k is an independent and identically distributed to $H(x)$ in (4) and the number $i-1$ of failure replacement satisfies the geometric distribution $p^{i-1}q$, the expected value $E[WC_R]$ of the replacement cost WC_{i-1} can be obtained as:

$$E[WC_R]=E\left[\sum_{i=1}^{\infty}p^{i-1}q\cdot WC_{i-1}\right]=E\left[\sum_{i=1}^{\infty}p^{i-1}q\left(\sum_{k=0}^{i-1}(c_R-D(X_k))\right)\right]=\frac{p}{q}\cdot E[c_R-D(X_k)]=\frac{\int_0^w(c_R-D(x))\bar{G}^{(m)}(x)\mathrm{d}F(x)}{1-\int_0^w\bar{G}^{(m)}(u)\mathrm{d}F(u)} \tag{7}$$

When the product goes through warranty at the completion of the m^{th} random working cycle, the total working time of the product is S_m . In this case, the distribution function $H_{S_m}(s)$ of the total working time S_m can be derived as:

$$H_{S_m}(s)=\Pr\{S_m<s\mid S_m<w,S_m<X\}=\frac{\int_0^s\bar{F}(u)\mathrm{d}G^{(m)}(u)}{\int_0^w\bar{F}(u)\mathrm{d}G^{(m)}(u)} \tag{8}$$

where $0<s<w$.

By the third term [i.e., (3)] of the proposed warranty, when the product goes through warranty at the m^{th} random working cycle before the warranty period w, its past age is equal to its total working time S_m and it is warranted by FRW with a time span $w - S_m$. Let c_m be unit minimal repair cost, then the minimal repair cost WC_m produced by FRW can be estimated as:

$$WC_m = c_m \int_0^w r(S_m + u)\mathrm{d}u \tag{9}$$

Since the past age (i.e., the total working time) S_m is subject to the distribution function $H_{S_m}(s)$ in (8), the expected value $E\left[WC_m\right]$ of the minimal repair cost WC_m can be computed as:

$$E\left[WC_m\right] = E\left[c_m \int_0^w r(S_m + u)\mathrm{d}u\right] = c_m \int_0^w \left(\int_0^w r(s+u)\mathrm{d}u\right)\mathrm{d}H_{S_m}(s) = \frac{c_m \int_0^w \left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)}{\int_0^w \bar{F}(u)\mathrm{d}G^{(m)}(u)} \tag{10}$$

It is well known that the probability that until the first product goes through the m^{th} random working cycle is $\sum_{i=1}^{\infty} p^{i-1} q_1 = q_1 / q$, where q_1 is the probability that the product goes through the m^{th} random working cycle and satisfies $q_1 = \Pr\{S_m < w, S_m < X\} = \int_0^w \bar{F}(u)\mathrm{d}G^{(m)}(u)$; q has been offered in (1). By summing, the warranty cost $E[WC]$ produced by the proposed warranty can be derived as:

$$\begin{aligned} E[WC] &= E\left[WC_R\right] + \frac{q_1}{q}\cdot E\left[WC_m\right] = \frac{\int_0^w (c_R - D(x))\bar{G}^{(m)}(x)\mathrm{d}F(x)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} + \frac{\int_0^w \bar{F}(s)\mathrm{d}G^{(m)}(s)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)}\cdot\frac{c_m \int_0^w \left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)}{\int_0^w \bar{F}(s)\mathrm{d}G^{(m)}(s)} \\ &= \frac{\int_0^w (c_R - D(x))\bar{G}^{(m)}(x)\mathrm{d}F(x) + c_m \int_0^w \left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} \end{aligned} \tag{11}$$

When $m \to \infty$, $\bar{G}^{(m)}(s) \to 1$ and $G^{(m)}(s) \to 0$. This means that the failure replacement limit m fails and the proposed warranty is reduced to RFRW. Therefore, the above model can be reduced to a warranty cost $\lim_{m\to\infty} E[WC] = \int_0^w (c_R - D(x))\mathrm{d}F(x) / \bar{F}(w)$, which is produced by RFRW.

3. The post-warranty random maintenance policies

As mentioned in above, how to model a post-warranty maintenance to sustain the post-warranty reliability has recently received considerable attention. Although the post-warranty maintenance policies to sustain the post-warranty reliability have been investigated extensively, the post-warranty random maintenance policies considering the product working cycle are investigated seldom. In this section, we incorporate the product working cycle into the post-warranty period and investigate the post-warranty random maintenance policies of the product with the warranty proposed in Section 2.

When the product goes through warranty, the total working time of the product is w. This means that reliability is lowered after the product goes through warranty. Therefore, it is necessary to improve reliability of the product through warranty so that the post-warranty period is extended and the post-warranty maintenance cost is reduced. In view of this, we integrate imperfect preventive maintenance (PM) at the warranty period w into the post-warranty maintenance model and investigate two types of the post-warranty random maintenance policies, which will be next provided in Subsection 3.1 and Subsection 3.2.

In order to model conveniently, besides we define similarly the life cycle of the product as an interval from the product installation time to the product replacement occurrence time at the consumer's expense [17, 19], which is composed of the warranty service period and the post-warranty period. By means of this definition, we can derive cost rates model, as below.

3.1. The post-warranty random maintenance policy 1

In this subsection, we introduce imperfect PM at the warranty period w to random periodic replacement last [16] and investigate a post-warranty random maintenance policy satisfying ① imperfect PM is done at the warranty period w; ② replacement is done at the replacement time T or at the completion of a random working cycle, whichever occurs last; ③ minimal repair removes every failure before replacement. In this article, we refer to this type of maintenance policy as replacement last with PM, which can sustain the post-warranty reliability of the product with random working cycles.

3.1.1. Life cycle cost modeling

In the reliability engineering practice, PM cost is increasing with both the reliability increment resulted from PM and the time where PM is done. The reliability increment resulted from PM is usually estimated by age reduction or/and failure rate reduction [26]. In this article, age reduction is used as a measure of the reliability increment resulted from PM. At the expiry of the proposed warranty, age of the product equates its total working time w. Denote the decreased function $(1-\varphi(n))w$ with respect to the decision variable n ($n = 0,1,\cdots$) by the reliability increment resulted from PM at w, then PM cost C_{PM} at w can be modeled as an increasing function with both $(1-\varphi(n))w$ and w, as follows:

$$C_{PM} = c_h\left((1-\varphi(n))w\right)^{\alpha_2}(w)^{\beta_2} = c_h(1-\varphi(n))^{\alpha_2}(w)^{\alpha_2+\beta_2} \tag{12}$$

where c_h is a cost coefficient and satisfies $c_h > 0$; α_2 is an elasticity coefficient of input on reliability improvement and satisfies $\alpha_2 > 0$; β_2 is an elasticity coefficient of input on implementation at w for PM and satisfies $\beta_2 > 0$; $\varphi(n)$ satisfies $0 < \varphi(n) < 1$ where n is the maintenance ability level. Note that when $\varphi(n) = 0$, PM is reduced to perfect PM; when $\varphi(n) = 1$, any maintenance (including PM and minimal repair) is not implemented.

In this article, we have assumed that the random working cycle Y_j ($j = 1,2,\ldots$) is independent identically distributed to the distribution function $G(y)$ with the lack-of-memory property. This assumption means that remaining completion time of a project is still subject to the distribution function $G(y)$. Besides, the probability that replacement is done at the replacement time T or at the completion of a random working cycle, whichever occurs last, can be respectively represented as $G(T)$ and $\bar{G}(T)$. Thus, the costs related to them can be respectively computed as:

$G(T)\left(c_P - D(\varphi(n)w+T) + (c_f + c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u\right)$ and

$\int_T^{\infty}\left(c_P - D(\varphi(n)w+t) + (c_f + c_m)\int_0^t r(\varphi(n)w+u)\mathrm{d}u\right)\mathrm{d}G(t)$, where

$\varphi(n)w$ is virtual age after PM at w; c_f is unit failure cost resulted from each failure; c_P ($c_P < c_R$) is unit replacement cost suffered for the consumer. By summing, further the expected value $E\left[C_l(n,T)\right]$ of the total cost during the post-warranty period is computed as:

$$\begin{aligned} E\left[C_l(n,T)\right] &= G(T)\left(c_P - D(\varphi(n)w+T) + (c_f + c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u\right) + \\ &\quad \int_T^{\infty}\left(c_P - D(\varphi(n)w+t) + (c_f + c_m)\int_0^t r(\varphi(n)w+u)\mathrm{d}u\right)\mathrm{d}G(t) + c_P \\ &= (c_f + c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u + c_P - D(\varphi(n)w+T) + \int_T^{\infty}\bar{G}(t)\left(d(\varphi(n)w+t) + (c_f + c_m)r(\varphi(n)w+t)\right)\mathrm{d}t \end{aligned} \tag{13}$$

where $d(\varphi(n)w+t)$ is first-order derivative with respect to t of the depreciation expense $D(\varphi(n)w+t)$.

By multiplying c_f on the expected number $E[\kappa]$ of failure replacement, the expected value of the total failure cost resulted from all failure replacements can be obtained as $E[\kappa]\cdot c_f = c_f\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u) / \left(1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)\right)$. By replacing c_m in $E[WC_m]$ as c_f, the total failure cost resulted from all failures in the interval $(S_m, w]$ can be obtained, i.e., $c_f\int_0^w\left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s) / \left(1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)\right)$. Besides, the expected value $E[C_l(n,T)]$ of the total cost during the post-warranty period have been offered in (13) and PM cost C_{PM} at w has been obtained in (12). On the basis of life cycle definition, by summing, the expected value $E[C_l(L)]$ of the life cycle cost is derived as:

$$E[C_l(L)] = \frac{c_f\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} + \frac{c_f\int_0^w\left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} + C_{PM} + E[C_l(n,T)]$$
$$= \frac{c_f\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u) + c_f\int_0^w\left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} + c_h(1-\varphi(n))^{\alpha_2}(w)^{\alpha_2+\beta_2} + c_P - D(\varphi(n)w+T) +$$
$$(c_f+c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u + \int_T^\infty \bar{G}(t)\left(d(\varphi(n)w+t)+(c_f+c_m)r(\varphi(n)w+t)\right)\mathrm{d}t \quad (14)$$

3.1.2. Life cycle length modeling

Until the i^{th} product goes through warranty, the manufacturer performs totally $i-1$ failure replacements. In this case, the total warranty service period resulted from $i-1$ failure replacements can be obtained as $\sum_{k=0}^{i-1} X_k$ where $X_0 = 0$. Since the number $i-1$ of failure replacement satisfies the geometric distribution $p^{i-1}q$, the expected value $E[W]$ of the total warranty service period resulted from all failure replacements can be expressed as:

$$E[W] = E\left[\sum_{i=1}^{\infty} p^{i-1}q\left(\sum_{k=0}^{i-1} X_k\right)\right] = \frac{p}{q}\cdot E[X_k] = \frac{\int_0^w x\bar{G}^{(m)}(x)\mathrm{d}F(x)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} \quad (15)$$

where X_k is subject to $H(x)$ in (4) and $E[X_k] = \int_0^w x\bar{G}^{(m)}(x)\mathrm{d}F(x) / \int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)$.

For the product through warranty, its warranty service period is equal to w and the probability that it is replaced at the replacement time T or at the completion of a random working cycle (whichever occurs last) can be respectively computed as $G(T)$ and $\bar{G}(T)$. The corresponding replacement times are respectively $G(T)T$ and $\int_T^\infty u\mathrm{d}G(u)$. On the basis of life cycle definition, by summing, the expected value $E[L_l]$ of the life cycle length can be expressed as:

$$E[L_l] = E[W] + w + G(T)T + \int_T^\infty u\mathrm{d}G(u) = \frac{\int_0^w x\bar{G}^{(m)}(x)\mathrm{d}F(x)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} + w + T + \int_T^\infty \bar{G}(u)\mathrm{d}u \quad (16)$$

3.1.3. Cost rate modeling

The expected value $E[C(L_l)]$ of the life cycle cost and the expected value $E[L_l]$ of the life cycle length have been presented respectively in (14) and (16). Let $A = \left(c_f\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u) + c_f\int_0^w\left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)\right) / \left(1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)\right) + c_P$ and $B = \int_0^w x\bar{G}^{(m)}(x)\mathrm{d}F(x) / \left(1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)\right) + w$, by the renewal rewarded theorem [1], the expected cost rate $CR_l(n,T)$ can be calculated as:

$$CR_l(n,T) = \frac{A + c_h(1-\varphi(n))^{\alpha_2}(w)^{\alpha_2+\beta_2} - D(\varphi(n)w+T) + (c_f+c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u + \int_T^\infty \bar{G}(t)\left(d(\varphi(n)w+t)+(c_f+c_m)r(\varphi(n)w+t)\right)\mathrm{d}t}{B + T + \int_T^\infty \bar{G}(u)\mathrm{d}u} \quad (17)$$

Since the expression of $r(u)$ is undefined and unspecific, it is difficult to obtain optimum analytical solutions. But, the existence and uniqueness of optimum solutions can be summarized by discussing the first-order derivative with respect to decision variables of cost rate models. The detail process has been presented and extensively discussed by the literature [19, 22, 31]. In view of this, the existence and uniqueness of optimum solutions are no longer summarized in this article and interested reader consults the above literature, hereinafter similarly.

3.1.4. Special cases

The expected cost rate $CR_l(n,T)$ in (17) is constructed by defining that the proposed warranty is used to ensure reliability preference during the warranty period and by defining that replacement last with PM is used to sustain the post-warranty reliability. By discussing, the expected cost rate $CR_l(n,T)$ can be reduced to some special models representing special problems, as follows:

Case 1: when $m\to\infty$, model in (17) can be reduced to:

$$CR_l(n,T) = \frac{A + c_h(1-\varphi(n))^{\alpha_2}(w)^{\alpha_2+\beta_2} - D(\varphi(n)w+T) + (c_f+c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u + \int_T^\infty \bar{G}(t)\left(d(\varphi(n)w+t)+(c_f+c_m)r(\varphi(n)w+t)\right)\mathrm{d}t}{B + T + \int_T^\infty \bar{G}(u)\mathrm{d}u} \quad (18)$$

where $A = F(w)c_f / \bar{F}(w) + c_P$ and $B = \int_0^w \bar{F}(x)\mathrm{d}x / F(w)$.

As mentioned in above, $m\to\infty$ means that the failure replacement limit m is failed and the proposed warranty is reduced to RFRW. Therefore, model in (18) represents an expected cost rate where RFRW is used to warrant the product and replacement last with PM is used to sustain the post-warranty reliability.

Case 2: when $\bar{G}(t)=0$ and $m\to\infty$, model in (17) can be reduced to:

$$CR_l(n,T) = \frac{F(w)c_f/\bar{F}(w) + c_P + c_h(1-\varphi(n))^{\alpha_2}(w)^{\alpha_2+\beta_2} - D(\varphi(n)w+T) + (c_f+c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u}{\int_0^w \bar{F}(x)\mathrm{d}x / F(w) + T} \quad (19)$$

$\bar{G}(t)=0$ means that replacement at the completion of a random working cycle is not existed. Therefore, model in (19) represents an expected cost rate where RFRW warrants the product and periodic replacement with PM sustains the post-warranty reliability.

In addition to these special models, some other models are also obtained by discussing one or more of other parameters in the model $CR_l(n,T)$, here we no longer offer them.

3.2. The post-warranty random maintenance policy 2

In this subsection, we introduce imperfect PM at the warranty period w to random periodic replacement first [16] and investigate other post-warranty random maintenance policy satisfying ① imperfect

PM is done at the warranty period w; ② replacement is done at the replacement time T or at the completion of a random working cycle, whichever occurs first; ③ minimal repair removes every failure before replacement. In this article, we refer to this type of maintenance policy as replacement first with PM to sustain the post-warranty reliability of the product.

Obviously, the unique difference between replacement last with PM and replacement first with PM is that replacement occurrence of the former is decided by 'whichever occurs last' and while replacement occurrence of the latter is decided by 'whichever occurs first'.

3.2.1. Life cycle cost modeling

The probability that replacement is performed at the replacement time T or at the completion of a random working cycle, whichever occurs first, can be respectively represented as $\bar{G}(T)$ and $G(T)$. Besides, the costs related to them can be respectively computed as $\bar{G}(T)\left(c_P - D(\varphi(n)w+T) + (c_f+c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u\right)$ and $\int_0^T \left(c_P - D(\varphi(n)w+t) + (c_f+c_m)\int_0^t r(\varphi(n)w+u)\mathrm{d}u\right)\mathrm{d}G(t)$. By summing, the expected value $E[C_f(n,T)]$ of the total cost during the post-warranty period is computed as:

$$
\begin{aligned}
E[C_f(n,T)] &= \bar{G}(T)\left(c_P - D(\varphi(n)w+T) + (c_f+c_m)\int_0^T r(\varphi(n)w+u)\mathrm{d}u\right) + \\
&\quad \int_0^T \left(c_P - D(\varphi(n)w+t) + (c_f+c_m)\int_0^t r(\varphi(n)w+u)\mathrm{d}u\right)\mathrm{d}G(t) \\
&= \int_0^T \bar{G}(t)\mathrm{d}\left(-D(\varphi(n)w+t) + (c_f+c_m)\int_0^t r(\varphi(n)w+u)\mathrm{d}u\right) + c_P - D(\varphi(n)w)
\end{aligned}
\tag{20}
$$

PM cost C_{PM} at w has been obtained in (12) and the total failure cost resulted from the proposed warranty is $\left(c_f\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u) + c_f\int_0^w\left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)\right) / \left(1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)\right)$, which is similar to (14). On the basis of life cycle definition, by summing, the expected value of the life cycle cost is derived as:

$$
\begin{aligned}
E[C(L_f)] &= \frac{c_f\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u) + c_f\int_0^w\left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} + C_{PM} + E[C_f(n,T)] \\
&= \frac{c_f\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u) + c_f\int_0^w\left(\int_0^w r(s+u)\mathrm{d}u\right)\bar{F}(s)\mathrm{d}G^{(m)}(s)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} + c_h\left(1-\varphi(n)\right)^{\alpha_2}(w)^{\alpha_2+\beta_2} + \\
&\quad \int_0^T \bar{G}(t)\mathrm{d}\left(-D(\varphi(n)w+t) + (c_f+c_m)\int_0^t r(\varphi(n)w+u)\mathrm{d}u\right) + c_P - D(\varphi(n)w)
\end{aligned}
\tag{21}
$$

3.2.2. Life cycle length modeling

For the product through warranty, the probability that it is replaced at the replacement time T or at the completion of a random working cycle, whichever occurs first, can be respectively computed as $\bar{G}(T)$ and $G(T)$, and the corresponding replacement times are respectively $\bar{G}(T)T$ and $\int_0^T u\mathrm{d}G(u)$. On the basis of life cycle definition, by summing, the expected value $E[L_f]$ of the life cycle length can be expressed as:

$$
E[L_f] = E[W] + w + \bar{G}(T)T + \int_0^T u\mathrm{d}G(u) = \frac{\int_0^w x\bar{G}^{(m)}(x)\mathrm{d}F(x)}{1-\int_0^w \bar{G}^{(m)}(u)\mathrm{d}F(u)} + w + \int_0^T \bar{G}(u)\mathrm{d}u
\tag{22}
$$

where $E[W]$ has been offered in (15).

3.2.3. Cost rate modeling

The expected value $E[C(L_f)]$ of the life cycle cost and the expected value $E[L_f]$ of the life cycle length have been offered respectively in (21) and (22). Then, the expected cost rate $CR_f(n,T)$ can be calculated as:

$$
CR_f(n,T) = \frac{A + c_h\left(1-\varphi(n)\right)^{\alpha_2}(w)^{\alpha_2+\beta_2} + \int_0^T \bar{G}(t)\mathrm{d}\left(-D(\varphi(n)w+t) + (c_f+c_m)\int_0^t r(\varphi(n)w+u)\mathrm{d}u\right) - D(\varphi(n)w)}{B + \int_0^T \bar{G}(u)\mathrm{d}u}
\tag{23}
$$

where A and B have been offered in (17).

3.2.4. Special cases

The expected cost rate $CR_f(n,T)$ in (23) is constructed by defining that the proposed warranty warrants the product and by defining that replacement first with PM sustains the post-warranty reliability of the product. The expected cost rate $CR_f(n,T)$ can be reduced to some special models representing special problems, as below.

Case I: when $m \to \infty$, model in (23) can be reduced to:

$$
CR_f(n,T) = \frac{F(w)c_f / \bar{F}(w) + \int_0^T \bar{G}(t)\mathrm{d}\left(-D(\varphi(n)w+t) + (c_f+c_m)\int_0^t r(\varphi(n)w+u)\mathrm{d}u\right) + c_P - D(\varphi(n)w)}{\int_0^w \bar{F}(x)\mathrm{d}x / \bar{F}(w) + \int_0^T \bar{G}(u)\mathrm{d}u}
\tag{24}
$$

When $m \to \infty$, $\bar{G}^{(m)}(s) \to 1$ and $G^{(m)}(s) \to 0$. Similar to the above discussions, this means that the failure replacement limit m fails and the proposed warranty is reduced to RFRW. Therefore, model in (24) represents an expected cost rate where RFRW is used to warrant the product and replacement first with PM is used to sustain the post-warranty reliability.

Case II: when $\bar{G}(t) = 1$, $n = 0$ and $m \to \infty$, model in (23) can be reduced to:

$$
CR_f(n,T) = \frac{F(w)c_f / \bar{F}(w) + c_P - D(T) + (c_f+c_m)\int_0^T r(w+u)\mathrm{d}u}{\int_0^w \bar{F}(x)\mathrm{d}x / \bar{F}(w) + T}
\tag{25}
$$

$\bar{G}(t) = 1$ means that replacement at the completion of a random working cycle is removed. $n = 0$ means that PM is not performed and replacement first with PM is reduced to classic periodic replacement policy. Therefore, model in (25) represents an expected cost rate where RFRW warrants the product and classic periodic replacement policy sustains the post-warranty reliability.

Besides, some other models are also offered by discussing one or more of other parameters in the model $CR_f(n,T)$, here we no longer present them.

4. Comparison

Both replacement last with PM and replacement first with PM can sustain the post-warranty reliability of the product. However, making decision on which to sustain the post-warranty reliability is a concerned problem for consumers. In view of this, we present a comparing approach, which can help consumers to make decision on the post-warranty random maintenance policies.

Firstly, let $E[L_l^*]$ and $E[L_f^*]$ be respectively optimum expected values of the life cycle length, which are corresponding to two types of the post-warranty random maintenance policy; secondly, let $E[C(L_l^*)]$ and $E[C(L_f^*)]$ be respectively optimum expected values of the life cycle costs related to two types of the post-warranty random maintenance policy; thirdly, let L_l^{**} and L_f^{**} be respectively cycle lengths related to two types of the post-warranty random maintenance policy, under the case that total costs related to two types of the post-warranty random maintenance policy are equal. Finally, the comparing approach presented in this article can be summarized as below:

Step 1: Let $L_l^{**} = E[C(L_f^*)] \cdot E[L_l^*]$ and $L_f^{**} = E[C(L_l^*)] \cdot E[L_f^*]$.

Step 2: If $L_l^{**} > L_f^{**}$, then replacement last with PM should be selected to sustain the post-warranty reliability of the product; if $L_f^{**} > L_l^{**}$, then replacement first with PM should be selected to sustain the post-warranty reliability of the product; if $L_l^{**} = L_f^{**}$, then any one of them can sustain the post-warranty reliability of the product because both are equivalent from the performance's perspective.

Note that decision-making result between post-warranty random maintenance policies can also be obtained by comparing total costs related to two types of the post-warranty random maintenance policy, under the case that cycle length of each post-warranty random maintenance policy is equal to a common value. Besides, the comparing approach presented in above can be extended to make decision on three or more the post-warranty random maintenance policies (or maintenance policies).

5. Numerical experiments

The intelligent mobile equipment is frequently used to inspect the remote hidden trouble of the high-voltage electric power equipment. Management can detect operating information of the intelligent mobile equipment by means of the advanced network technology, such as turn on, turn off, failure and working time. The intelligent mobile equipment is powered on when used and is powered off when use is completed. The time interval between power on and power off is a random working cycle.

From the perspective of reliability engineering practice, it is an impossible reality that the product after maintenance is „as good as new". This means the maintenance ability is limited, namely value n of the maintenance ability level is not infinite. This article uses $\varphi(n) = (n+1)e^{-n}$ to model the reliability alteration resulted from PM, where n ($n = 0,1,2,3,4,5$) represents maintenance ability level. The maximum value of maintenance ability level is reached when $n = 5$ and PM is not needed to be performed when $n = 0$.

In order to illustrate the proposed warranty and the policies investigated in this article, assume that lifetime of the intelligent mobile equipment is subject to a two-parameter Weibull function $F(x)$ with a failure rate $r(u) = a(u)^b$, where $a > 0$ and $b > 0$; and assume that working cycle is subject to an exponential distribution function $G(y)$ with a constant failure rate λ (i.e., $G(y) = 1 - \exp(-\lambda y)$) and some constant parameters are offered in Table 1. Other parameters (except decision variables) not to be assigned value in Table 1 are provided when needed.

Table 1. Parameter value

c_m	c_f	α_1	β_1	α_2	β_2	c_h	c_R	c_p	a
0.1	0.1	0.1	1	1	1	0.1	10	12	0.1

5.1. Sensitivity analysis of the proposed warranty

In order to illustrate characteristic of the proposed warranty, we plot Figure 1 where $w = 2$ and $b = 1$. As shown in Figure 1, when the failure replacement limit m increases for a given λ, the warranty cost produced by the proposed warranty increases first and then tends to the warranty cost (i.e., constant warranty cost) produced by RFRW. As mentioned in above, $m \to \infty$ means that the proposed warranty is transformed into RFRW. Therefore, the above change law with respect to m is existed. This indicates that when the limited number of random working cycle is used as warranty term of the proposed warranty, then the warranty cost produced by the proposed warranty can be reduced compared with traditional RFRW and the manufacturer can control the warranty cost produced by the proposed warranty by adjusting m. From Figure 1, besides we can find that the warranty cost produced by the proposed warranty is decreasing with respect to λ when the failure replacement limit m is same and is a smaller number.

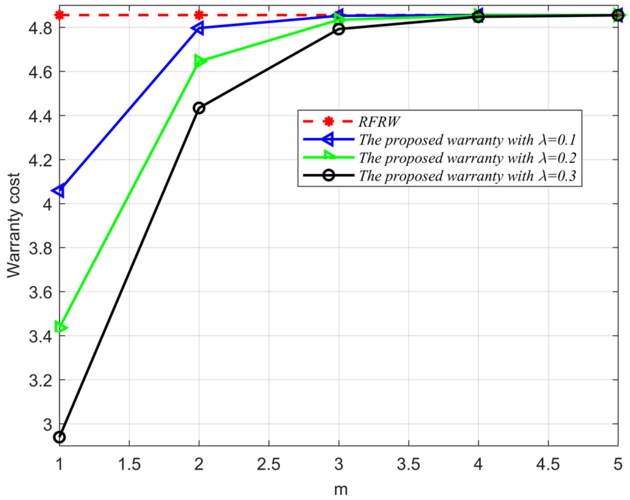

Fig. 1. Warranty cost versus m and λ

In order to further illustrate characteristic of the proposed warranty, we make Figure 2, where $\lambda = 0.1$ and $b = 1$.

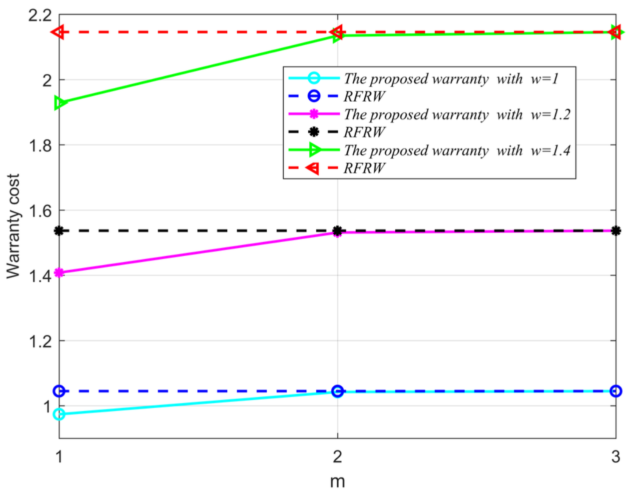

Fig. 2. Warranty cost versus m and w

As shown in Figure 2, the warranty cost produced by the proposed warranty increases first and then tends to a constant warranty cost produced by RFRW as m increases when w is given. This change law indicates that the warranty cost produced by the proposed warranty can be reduced compared with traditional RFRW and the manufacturer can control the warranty cost produced by the proposed warranty by adjusting m, which is similar to conclusions in Figure 1.

5.2. Sensitivity analysis of the post-warranty random maintenance policy 1

For description convenience, in this subsection we represent random periodic replacement first as replacement first, and represent random periodic replacement last as replacement last.

In order to display the existence and uniqueness of the optimum solutions (i.e., n^* and T^*) and the optimum value $CR_l(n^*, T^*)$, we make Figure 3 where $m = 2$, $w = 2$ and $b = 1$.

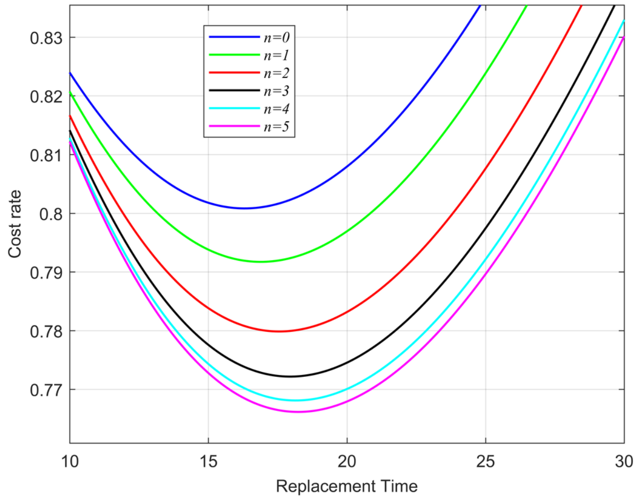

Fig. 3. Optimum solution and optimum value

As shown in Figure 3, the optimum replacement time T^* and the optimum cost rate $CR_l(n^*,T^*)$ are existed uniquely. From Figure 3, we can find that the optimum replacement time T^* is increasing with respect to n, whereas the optimum cost rate $CR_l(n,T^*)$ is decreasing with respect to n. From Figure 3, besides, we can conclude that $n^*=5$ and replacement last with PM (i.e., $n \neq 0$) is superior to replacement last (i.e., $n=0$) because replacement last with PM can produce a longer T^* and a lower $CR_l(n^*,T^*)$.

In order to indicate the effect of the failure replacement limit m on replacement last with PM, we make Table 2, where $\lambda = 0.1$, $b=1$ and $w=2$.

As shown in Table 2, the optimum replacement time T^* and the optimum cost rate $CR_l(n,T^*)$ decreases gradually to a constant with respect to the failure replacement limit m for a given n. As mentioned in Subsection 3.1.4, the proposed warranty is reduced to RFRW when the failure replacement limit m increases. As m increases, therefore, the optimum replacement time T^* and the optimum cost rate $CR_l(n,T^*)$ decreases gradually to a constant, which is obtained by optimizing the model in (18). From Table 2, besides, we can conclude that $n^*=5$ and replacement last with PM (i.e., $n \neq 0$) is superior to replacement last (i.e., $n=0$) for a given m because replacement last with PM can produce a longer T^* and a lower $CR_l(n^*,T^*)$.

5.3. Sensitivity analysis of the post-warranty random maintenance policy 2

In this subsection, we display the existence and uniqueness of the optimum solutions (i.e., n^* and T^*) and the optimum cost rate $CR_f(n^*,T^*)$, and the effect of the failure replacement limit m on replacement first with PM.

Table 2. Sensitivity analysis

n	$m=2$		$m=3$		$m=4$		$m=5$	
	T^*	$CR_l(n,T^*)$	T^*	$CR_l(n,T^*)$	T^*	$CR_l(n,T^*)$	T^*	$CR_l(n,T^*)$
0	16.3068	0.8008	16.2934	0.8004	16.2926	0.8004	16.2926	0.8004
1	16.8873	0.7917	16.8747	0.7913	16.8740	0.7913	16.8740	0.7913
2	17.5560	0.7799	17.5443	0.7795	17.5437	0.7795	17.5437	0.7795
3	17.9494	0.7722	17.9382	0.7718	17.9375	0.7718	17.9375	0.7718
4	18.1470	0.7681	18.1361	0.7677	18.1354	0.7677	18.1354	0.7677
5	18.2395	0.7661	18.2286	0.7658	18.2280	0.7658	18.2280	0.7658

In order to display the existence and uniqueness of the optimum solutions (i.e., n^* and T^*) and the optimum value $CR_f(n^*,T^*)$, we make Figure 4 where $m=2$, $w=2$ and $b=2$. As shown in Figure 4, the optimum replacement time T^* and the optimum cost rate $CR_f(n^*,T^*)$ are existed uniquely. From Figure 4, we can find that the optimum replacement time T^* is increasing with respect to n, whereas the optimum cost rate $CR_f(n^*,T^*)$ is decreasing with respect to n. As shown in Figure 4, besides, $n^*=5$ and replacement first with PM (i.e., $n \neq 0$) is superior to replacement first (i.e., $n=0$) because replacement first with PM can produce a longer T^* and a lower $CR_f(n^*,T^*)$.

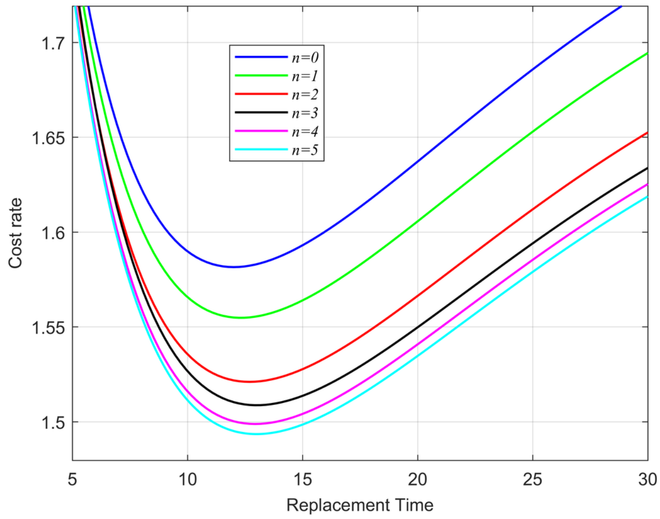

Fig. 4. Optimum solution and optimum value

We make Table 3 where $\lambda = 0.1$, $b=2$ and $w=2$. As shown in Table 3, the optimum replacement time T^* and the optimum cost rate $CR_f(n,T^*)$ decreases gradually to a constant with respect to the failure replacement limit m for a given n. The cause of this result is similar to the above analysis. From Table 3, additionally, the optimum replacement time T^* resulted from replacement first with PM (i.e., $n \neq 0$) is greater than the optimum replacement time T^* resulted from replacement first (i.e., $n=0$), and the optimum cost rate $CR_f(n,T^*)$ resulted from replacement first with PM (i.e., $n \neq 0$) is lower than the optimum cost rate $CR_f(n,T^*)$ resulted from replacement first (i.e., $n=0$) for a given m. This again means that replacement first with PM is superior to replacement first. From Table 3, thirdly, $n^*=5$.

5.4. Comparison

Consumers' concern is that which post-warranty random maintenance policy should be used to sustain the post-warranty reliability. This concern is a decision problem. From the consumer's perspective, the post-warranty random maintenance policy with most superior performance is an ideal selection. This indicates that consumers need to make decision on the post-warranty random maintenance policy by comparing performance. In Subsection 5.2 and 5.3, we illustrate performance by comparing optimum replacement time and optimum cost rate. Similarly, here we illustrate performance by comparing optimum replacement time and optimum cost rate.

We make Figure 5 where $\lambda = 0.1$, $m = 2$, $w = 1$, $b = 2$ and $n^* = 5$.

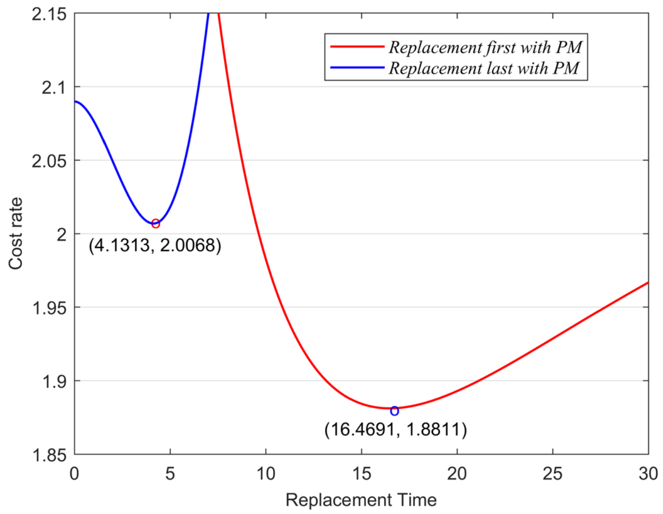

Fig. 5. Comparison

As indicated in Figure 5, optimum cost rate produced by replacement last with PM is greater than optimum cost rate produced by replacement first with PM, whereas optimum replacement time produced by replacement last with PM is not greater than optimum replacement time produced by replacement first with PM. These changes can't rank the post-warranty random maintenance policies because the information provided by them can't manifest any priority order of the post-warranty random maintenance policies.

Next, we use the comparing approach presented in Section 4 to rank the post-warranty random maintenance policies. We make Table 4 where $\lambda = 0.1$, $m = 2$, $b = 2$ and $w = 2$.

Table 4 shows that the cycle length L_l^{**} related to replacement last with PM is less than the cycle length L_f^{**} related to replacement first with PM, i.e., $L_l^{**} < L_f^{**}$, when total costs related to two types of the post-warranty random maintenance policy are equal and n is same. This means that the performance of replacement first with PM is superior to the performance of replacement last with PM.

Table 4. Comparison

n	Replacement last with PM		Replacement first with PM		Cycle lengths	
	$E[L_l^*]$	$E[C(L_l^*)]$	$E[L_f^*]$	$E[C(L_f^*)]$	L_l^{**}	L_f^{**}
0	13.7122	25.3621	10.6994	16.9864	232.9209	271.3593
1	13.8448	25.0106	10.7892	16.8387	233.1284	269.8444
2	14.0455	24.5192	10.8965	16.6368	233.6722	267.1735
3	14.1880	24.2005	10.9613	16.5000	234.1020	265.2689
4	14.2664	24.0346	10.9942	16.4253	234.3299	264.2412
5	14.3047	23.9561	11.0097	16.3890	234.4397	263.7495

In order to indicate the robustness of the above conclusion, we further make Table 5 where $\lambda = 0.1$, $n^* = 5$, $b = 2$ and $w = 2$. As shown in Table 5, the cycle length L_l^{**} related to replacement last with PM is lower than the cycle length L_f^{**} related to replacement first with PM, i.e., $L_l^{**} < L_f^{**}$, under the case that total costs related to two types of the post-warranty random maintenance policy are equal and m is same. This indicates again that replacement first with PM is superior to replacement last with PM.

Note that we only analyzed sensitivities of n and m, then we obtained the above conclusion that replacement first with PM is superior to replacement last with PM. If analyzing sensitivities of other parameters, then the conclusion obtained in above may not be established. In either case, the comparing approach presented in Section 4 is a forceful priority method for selection problem of the post-warranty random maintenance policies (or maintenance policies).

Table 5. Comparison

m	Replacement last with PM		Replacement first with PM		Cycle lengths	
	$E[L_l^*]$	$E[C(L_l^*)]$	$E[L_f^*]$	$E[C(L_f^*)]$	L_l^{**}	L_f^{**}
1	13.9115	24.0210	10.7308	16.6252	231.2815	257.7645
2	14.3047	23.9561	11.0097	16.3890	234.4397	263.7495
3	14.3401	23.9480	11.0343	16.3650	234.6757	264.2494
4	14.3421	23.9470	11.0356	16.3636	234.6884	264.2695

6. Conclusions

Taking advanced technologies as the technical background and by designing number of random working cycle as a warranty term, in this article, we proposed a manufacturer's warranty, which can ensure the product reliability performance by monitoring working cycle during the warranty period. The warranty cost produced by the proposed warranty was derived and special model was offered by discussing warranty term. From the consumer's perspective, we extended the proposed warranty to the post-warranty maintenance and proposed replacement last (and first) with PM, which can sustain the post-warranty reliability by tracking the post-warranty working cycles. Some classic cost rate models representing some special cases were provided by discussing parameters in each cost rate model. We presented a comparing approach to make decision on the post-warranty random maintenance policies. Sensitivities on some key parameters about both the proposed warranty and the proposed post-warranty random maintenance policies were analyzed in numerical experiments. It was discovered that the manufacturer can control the warranty cost when the limited number of random working cycle is used as a warranty term, and it was further discovered that replacement last (first) with PM is more superior compared with replacement last (first).

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References

1. Barlow RE, Proschan F. Mathematical theory of reliability. John Wiley & Sons, Hoboken 1965, http://10.1214/aoms/1177699826.
2. Cha JH, Finkelstein M, Levitin G. Optimal warranty policy with inspection for heterogeneous, stochastically degrading items. European Journal of Operational Research 2021; 289(3): 1142–1152, https://doi.org/10.1016/j.ejor.2020.07.045.
3. Chang CC. Optimum preventive maintenance policies for systems subject to random working times, replacement, and minimal repair. Computers & Industrial Engineering 2014; 67: 185–194, https://doi.org/10.1016/j.cie.2013.11.011.
4. Chen CK, Lo CC, Weng TC. Optimal production run length and warranty period for an imperfect production system under selling price dependent on warranty period. European Journal of Operational Research 2017; 259(2): 401–412, https://doi.org/10.1016/j.ejor.2016.10.038.
5. Gao H, Cui L, Dong Q. Reliability modeling for a two-phase degradation system with a change point based on a Wiener process. Reliability Engineering & System Safety 2020; 193: 106601, https://doi.org/10.1016/j.ress.2019.106601.
6. Huang T, Zhao Y, Coit DW, Tang L. Reliability assessment and lifetime prediction of degradation processes considering recoverable shock damages. IISE Transactions 2021; 53(5): 614–628, https://doi.org/10.1080/24725854.2020.1793036.
7. Hooti F, Ahmadi J, Longobardi M. Optimal extended warranty length with limited number of repairs in the warranty period. Reliability Engineering & System Safety 2020; 203: 107111, https://doi.org/10.1016/j.ress.2020.107111.
8. Huang HZ, Liu ZJ, Murthy DNP. Optimal reliability, warranty and price for new products. IIE Transactions 2007; 39(8): 819–827, https://doi.org/10.1080/07408170601091907.
9. He Z, Wang D, He S, Zhang Y, Dai A. Two-dimensional extended warranty strategy including maintenance level and purchase time: A win-win perspective. Computers & Industrial Engineering 2020; 141: 106294, https://doi.org/10.1016/j.cie.2020.106294.
10. KnopiK L, MigAwA K. Optimal age-replacement policy for non-repairable technical objects with warranty. Eksploatacja i Niezawodnosc– Maintenance and Reliability 2017; 19(2): 172–178, http://dx.doi.org/10.17531/ein.2017.2.4.
11. Li XY, Chen WB, Li FR, Kang R. Reliability evaluation with limited and censored time-to-failure data based on uncertainty distributions. Applied Mathematical Modelling 2021; 94: 403–420, https://doi.org/10.1016/j.apm.2021.01.029.
12. Luo M, Wu S. A comprehensive analysis of warranty claims and optimal policies. European Journal of Operational Research 2019; 276(1): 144–159, https://doi.org/10.1016/j.ejor.2018.12.034.
13. Liu B, Wu J, Xie M. Cost analysis for multi-component system with failure interaction under renewing free-replacement warranty. European Journal of Operational Research 2015; 243(3): 874–882, https://doi.org/10.1016/j.ejor.2015.01.030.
14. Liu P, Wang G, Su P. Optimal replacement strategies for warranty products with multiple failure modes after warranty expiry. Computers & Industrial Engineering 2021; 153: 107040, https://doi.org/10.1016/j.cie.2020.107040.
15. Marshall S, Arnold R, Chukova S, Hayakawa Y. Warranty cost analysis: Increasing warranty repair times. Applied Stochastic Models in Business and Industry 2018; 34(4): 544–561, https://doi.org/10.1002/asmb.2323.
16. Nakagawa T. Random maintenance policies. Springer, London 2014, https://10.1007/978-1-4471-6575-0.
17. Park M, Jung KM, Park DH. A generalized age replacement policy for systems under renewing repair-replacement warranty. IEEE Transactions on Reliability 2016; 65(2): 604–612, http://dx.doi.org/ 10.1109/TR.2015.2500358.
18. Sánchez-Silva M, Klutke G. Reliability and Life-Cycle Analysis of Deteriorating Systems. Springer, London 2015, https://doi.org/10.1007%2F978-3-319-20946-3.
19. Shang L, Si S, Cai Z. Optimal maintenance–replacement policy of products with competing failures after expiry of the warranty. Computers & Industrial Engineering 2016; 98: 68–77, https://doi.org/10.1016/j.cie.2016.05.012.
20. Shang L, Si S, Sun S, Jin T. Optimal warranty design and post-warranty maintenance for products subject to stochastic degradation. IISE Transactions 2018; 50(10): 913–927, https://doi.org/10.1080/24725854.2018.1448490.
21. Sheu SH, Liu TH, Zhang ZG, Zhao X, Chien YH. A generalized age-dependent minimal repair with random working times. Computers & Industrial Engineering 2021; 156: 107248, https://doi.org/10.1016/j.cie.2021.107248.
22. Sheu SH, Liu TH, Zhang ZG. Extended optimal preventive replacement policies with random working cycle. Reliability Engineering & System Safety 2019; 188: 398–415, https://doi.org/10.1016/j.ress.2019.03.036.
23. Taleizadeh AA, Mokhtarzadeh M. Pricing and two-dimensional warranty policy of multi-products with online and offline channels using a value-at-risk approach. Computers & Industrial Engineering 2020; 148: 106674, https://doi.org/10.1016/j.cie.2020.106674.
24. Wang L, Pei Z, Zhu H, Liu B. Optimising extended warranty policies following the two-dimensional warranty with repair time threshold. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2018; 20(4): 523–530, http://dx.doi.org/10.17531/ein.2018.4.3.
25. Wang X, He K, He Z, Li L, Xie M. Cost analysis of a piece-wise renewing free replacement warranty policy. Computers & Industrial Engineering 2019; 135: 1047–1062, https://doi.org/10.1016/j.cie.2019.07.015.
26. Wu S, Zuo MJ. Linear and Nonlinear Preventive Maintenance Models. IEEE Transactions on Reliability 2010; 59(1): 242–249, http://dx.doi.org/10.1109/TR.2010.2041972.
27. Ye ZS, Murthy DNP. Warranty menu design for a two-dimensional warranty. Reliability Engineering & System Safety 2016; 155: 21–29, https://doi.org/10.1016/j.ress.2016.05.013.
28. Yeh RH, Ho WT, Tseng ST. Optimal production run length for products sold with warranty. European Journal of Operational Research 2000; 120(3): 575–582, https://doi.org/10.1016/S0377-2217(99)00004-1.
29. Zhang S, Zhang Y. Nonlinear mixed reliability model with non-constant shape parameter of aviation cables. Applied Mathematical Modelling 2021; 96: 445–455, https://doi.org/10.1016/j.apm.2021.03.011.
30. Zhang N, Fouladirad M, Barros A. Evaluation of the warranty cost of a product with type III stochastic dependence between components. Applied Mathematical Modelling 2018; 59: 39–53, https://doi.org/10.1016/j.apm.2018.01.013.
31. Zhao X, Nakagawa T. Optimization problems of replacement first or last in reliability theory. European Journal of Operational Research 2012; 223(1): 141–149, https://doi.org/10.1016/j.ejor.2012.05.035.