Uncertainty propagation in structural reliability with implicit limit state functions under aleatory and epistemic uncertainties

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Highlights

- A novel surrogate model is given for implicit function under uncertain random variable.

- The concepts of chance reliability and chance reliability index (CRI) are proposed.

- A new level-2 uncertainty propagation model is provided for uncertain random structure.

- The principles for choosing reasonable uncertainty propagation types are presented.

Abstract

Uncertainty propagation plays a pivotal role in structural reliability assessment. This paper introduces a novel uncertainty propagation method for structural reliability under different knowledge stages based on probability theory, uncertainty theory and chance theory. Firstly, a surrogate model combining the uniform design and least-squares method is presented to simulate the implicit limit state function with random and uncertain variables. Then, a novel quantification method based on chance theory is derived herein, to calculate the structural reliability under mixed aleatory and epistemic uncertainties. The concepts of chance reliability and chance reliability index (CRI) are defined to show the reliable degree of structure. Besides, the selection principles of uncertainty propagation types and the corresponding reliability estimation methods are given according to the different knowledge stages. The proposed methods are finally applied in a practical structural reliability problem, which illustrates the effectiveness and advantages of the techniques presented in this work.

Keywords

structural reliability, uncertainty quantification, uncertainty propagation, reliability index, uncertainty theory.

1. Introduction

Structural reliability assessment has been widely recognized as vital in engineering product design and development [7]. In the context of structural reliability assessment, uncertainty propagation plays a significant role, which aims to quantify the uncertainties of input factors and calculate the overall uncertainty within the model response in reliability estimation [36].

Before propagating the structure's uncertainty, a primary issue is to choose a reasonable mathematical theory related to the types of uncertainty, to quantify the uncertainty [12,38]. In practical structural engineering problems, uncertainty can be divided into two categories: aleatory uncertainty derived from inherent randomness of physical behavior, while the epistemic uncertainty arising out of lack of knowledge [10].

Probability theory is regarded as the most effective tools to describe aleatory uncertainty in structural reliability assessment. Over the last decades, numerous reliability assessment methods based on probability theory have been developed, including first-order reliability method (FORM) [23], second-order fourth moment [29] Monte Carlo simulation (MCS) [24], FORM-sampling simulation method

[22], envelope function method [28], response surface method (RSM) [6], and Bayesian networks method [26]. Although these probabilistic methods typically make sense in uncertainty quantification and propagation when the structure is mainly affected by aleatory uncertainty, they do not work well in the scenarios involving great epistemic uncertainty [37]. For example, the distribution of input factors may not be precisely obtained due to insufficient sample data. Consequently, several alternative non-probabilistic theories have been developed to describe the epistemic uncertainty in reliability assessment.

The general non-probabilistic structural reliability assessment theories consist of fuzzy set theory [9], fuzzy random theory [13], possibility theory [1], interval theory [5, 27], and evidence theory [39]. The fuzzy and possibility measures fail to satisfy the duality property, which will make it difficult for decision-makers to understand the results [33]. Moreover, interval and evidence theories will lead to an over-conservative result due to the interval extension problems [38]. To overcome the shortcomings of the above-mentioned theories, a new mathematical framework called uncertainty theory was introduced to deal with epistemic uncertainty.

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Uncertainty theory proposed by Liu [18] in 2007 to describe the belief degree of human, has been successfully applied in various areas such as decision making [30], uncertain insurance [19, 32], uncertain risk and reliability analysis [34, 35]. Uncertainty theory is considered a reasonable and useful tool to express epistemic uncertainty, compared with the theories mentioned above [12]. Since the uncertain measure satisfies the axiom of duality, normality, and subadditivity, the results produced by the uncertainty theory are more in line with real engineering conditions [8]. Hence, in this work, uncertainty theory is chosen to express epistemic uncertainty and describe human thinking processes. In practical structural problems, there are usually two types of input factors that embody different types of uncertainties at the same time. Some input factors may suffer great epistemic uncertainty and are described by uncertainty theory, while some others may be primarily determined by aleatory uncertainty and are modeled based on probability theory. These structures comprising both aleatory and epistemic uncertainties are called uncertain random structures in this paper. It is impossible to analyze the reliability of uncertain random structures only by probability theory or uncertainty theory [38].

To solve this problem, chance theory was established by Liu [20] in 2013 to propagate aleatory and epistemic uncertainties together. Chance theory can be understood as a combination of probability theory and uncertainty theory, which also satisfies normality, duality, and subadditivity theorems. In recent years, chance theory has been successfully used in various fields such as project scheduling [11], uncertain random risk analysis [8], uncertain random programming [25], and systems reliability analysis [31, 38]. Especially, a hybrid model of structural reliability analysis based on chance theory was proposed by Zhang [37] in 2019, and a new reliability index was proposed. However, this method has the following disadvantages. Firstly, there is no corresponding reliability analysis method when the implicit limit state function (LSF) contains both random and uncertain variables. Secondly, the defined reliability and reliability index do not involve time dynamic parameters. Thirdly, this reliability analysis method does not consider the problem of uncertainty propagation.

For completeness, this paper uses a uniform design (UD) combined with the least-squares (LS) method to simulate LSF, which adapts to both random and uncertain variables. The UD is a novel kind of experimental design method founded by Fang and Wang [3], defined according to the uniform distribution in number theory [40]. Compared to the orthogonal design (OD), factorial design (FD), and Latin hypercube sampling (LHS) methods, the UD method appears to be more advanced if the number of experimental factors is large and the number of experiments is limited [4].

Besides the above research, this paper's main contribution is to provide a new uncertainty propagation method for structural reliability assessment. Uncertainty propagation aims to estimate structural output responses by propagating the input factors essential for structural reliability assessment and safety design [36]. Normally, uncertainty propagation can be classified into the form of level-1 and level-2 [14]. For level-1 propagation, the values of input factors can be characterized by epistemic or aleatory uncertainties at the same level [2]. For level-2 propagation, the values of input factors are represented by aleatory uncertainties on the first level. Epistemic uncertainties describe the parameters of probability distributions in the second level [34]. These two types of uncertainty propagation methods are commonly used in risk assessment. Comprehensive research about this was reported by Hu et al. [8], who presented a framework for propagation methods corresponding to different knowledge stages in fault tree analysis. However, there are no literature about the level-2 uncertainty propagation modeling and propagation type selection methods for structural reliability assessment. Hence, this paper aims to develop some propagation analysis methods and the principles for the selection of propagation type in structural reliability assessment.

The remainder of this work is organized as follows: Section 2 briefly discusses some important mathematical concepts of uncertainty and chance theory. A new surrogate model combining UD and LS method is proposed for implicit LSF in Section 3. Section 4 provides a novel structural reliability quantification model based on chance theory. Some principles for choosing appropriate uncertainty propagation types are discussed, and corresponding reliability calculation methods are provided in Section 5. In Section 6, a practical engineering case study is carried out to show the proposed method's rationality. Finally, some conclusions are presented in Section 7.

2. Preliminaries

In this section, some fundamental knowledge and results regarding the uncertainty theory and chance theory are introduced.

2.1. Uncertainty theory

Uncertainty theory is a fairly new branch of axiomatic mathematics, and has been widely applied in various areas. In the uncertainty theory, the human belief degree of events are quantified by defining uncertain measures.

Definition 2.1 (Uncertain measure [15]) Let Γ be a nonempty set, and $\mathcal{L}$ be a σ- algebra over Γ. Each element Λ in $\mathcal{L}$ is called an event. Then, a set function M is defined as an uncertain measure if it satisfies normality, duality, and subadditivity axioms.

Definition 2.2 (Uncertain variable [18]) An uncertain variable is a measurable function τ from an uncertainty space (Γ, $\mathcal{L}$, M) to the set of real numbers, i.e., $\{\tau \in B\}$ is an event for any Borel set B of real numbers.

Definition 2.3 (Uncertainty distribution [15]) The uncertainty distribution $\Phi(x)$ of an uncertain variable τ can be defined by $\Phi(x) = M\{\tau \leq x\}$ for any real number x.

A regular uncertainty distribution $\Phi(x)$ is defined as an uncertainty function that is continuous and strictly increasing with respect to x.

Example 2.1 An uncertain variable τ is defined as a normal uncertain variable if it has a normal uncertainty distribution:

$$\Phi(x) = (1 + \exp(\frac{\pi(m-x)}{\sqrt{3}\sigma}))^{-1} \quad x \in \mathrm{R} \qquad (1)$$

It is denoted by $\tau \sim \mathcal{N}(m, \sigma)$, where m is the expected value and σ is the standard deviation.

Example 2.2 An uncertain variable τ is defined as linear variable if it has a linear uncertainty distribution:

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ \dfrac{x-a}{b-a}, & \text{if } a < x \leq b \\ 1, & \text{if } b < x \end{cases} \qquad (2)$$

It is denoted by $\tau \sim \mathcal{L}(a, b)$, where a and b are real numbers with $a < b$.

Since the uncertainty theory can describe the incomplete information contained in design variables, the epistemic uncertainty (especially human) can be characterized by uncertain variables and uncertainty distribution in the uncertainty space [16, 17].

Definition 2.4 (Inverse uncertainty distribution [15]) Let τ be an uncertain variable with regular uncertain distribution $\Phi(x)$. The inverse function $\Phi^{-1}(u)$ is known as the inverse uncertainty distribution of τ.

Theorem 2.1 (Operational law [18]) Let $\tau_1, \tau_2, \cdots, \tau_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If $f(\tau_1, \tau_2, \cdots, \tau_n)$ is continuous, strictly increasing with respect to $\tau_1, \tau_2, \cdots, \tau_m$ and strictly decreasing with respect to $\tau_{m+1}, \tau_{m+2}, \cdots, \tau_n$, then $\tau = f(\tau_1, \tau_2, \cdots, \tau_n)$ is an uncertain variable with inverse uncertainty distribution:

$$\Psi^{-1}(u) = f(\Phi_1^{-1}(u), \Phi_2^{-1}(u), \cdots, \Phi_m^{-1}(u), \Phi_{m+1}^{-1}(1-u), \Phi_{m+2}^{-1}(1-u), \cdots, \Phi_n^{-1}(1-u))$$

$$(3)$$

2.2. Chance theory

As a combination of uncertainty and probability theory, the chance theory is applied as a new tool to deal with problems affected by both uncertainty and randomness. The basic concept involves the chance measure of an uncertain random event in a chance space.

Definition 2.5 (Chance measure [20]) Let $(\Gamma, \mathcal{L}, \text{M}) \times (\Omega, \mathcal{A}, \text{Pr})$ be a chance space, and let $\Theta \in \mathcal{L} \times \mathcal{A}$ be an event. Then the chance measure of Θ can be defined as:

$$\text{Ch}\{\Theta\} = \int_0^1 \text{Pr}\{\omega \in \Omega \mid \text{M}\{\gamma \in \Gamma \mid (\gamma, \omega) \in \Theta\} \geq r\} dr \quad (4)$$

Theorem 2.2 Let $(\Gamma, \mathcal{L}, \text{M}) \times (\Omega, \mathcal{A}, \text{Pr})$ be a chance space, then the chance measure $\text{Ch}\{\Lambda \times A\} = \text{M}\{\Lambda\} \times \text{Pr}\{A\}$ for any $\Lambda \in \mathcal{L}$ and any $A \in \mathcal{A}$. Especially, we have $\text{Ch}\{\varnothing\} = 0, \text{Ch}\{\Gamma \times \Omega\} = 1$ [15].

Definition 2.6 (Uncertain random variable [20]) An uncertain random variable is a measurable function ξ from a chance space $(\Gamma, \mathcal{L}, \text{M}) \times (\Omega, \mathcal{A}, \text{Pr})$ to the set of real numbers such that $\{\xi \in \text{B}\}$ is an event in $\mathcal{L} \times \mathcal{A}$ for any Borel set B of real numbers.

Definition 2.7 (Chance distribution [15]) Let ξ be an uncertain random variable. Then its chance distribution is defined as $\Phi(x) = \text{Ch}\{\xi \leq x\}$ for any real number x.

Theorem 2.3 Let $\eta_1, \eta_2, \cdots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \cdots, \Psi_m$, and let $\tau_1, \tau_2, \cdots, \tau_n$ be independent uncertain variables with uncertainty distributions $\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_n$, respectively. If f is a measurable function, then the uncertain random variable $\xi = f(\eta_1, \eta_2, \cdots, \eta_m, \tau_1, \tau_2, \cdots, \tau_n)$ has a chance distribution [20]:

$$\Phi(x) = \int_{\Re^m} F(x; y_1, y_2, \cdots, y_m) d\Psi_1(y_1) d\Psi_2(y_2) \cdots d\Psi_m(y_m) \quad (5)$$

where $F(x; y_1, y_2, \cdots, y_m)$ is the uncertainty distribution of the uncertain variable $f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n)$ for the any real numbers $y_1, y_2, \cdots, y_m$.

Besides, assume f is continuous, strictly increasing with respect to $\tau_1, \tau_2, \cdots, \tau_k$ and strictly decreasing with respect to $\tau_{k+1}, \tau_{k+2}, \cdots, \tau_n$. Then $F(x; y_1, \cdots, y_m)$ is the root u of the following equation:

$$f(y_1, y_2, \cdots, y_m, \Upsilon_1^{-1}(u), \cdots, \Upsilon_k^{-1}(u), \Upsilon_{k+1}^{-1}(1-u), \cdots, \Upsilon_n^{-1}(1-u)) = x \quad (6)$$

Theorem 2.4 (Expected value [21]) Let $\eta_1, \eta_2, \cdots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \cdots, \Psi_m$, and let $\tau_1, \tau_2, \cdots, \tau_n$ be independent uncertain variables with regular uncertainty distributions $\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_n$, respectively. If f is continuous and strictly increasing with respect to $\tau_1, \tau_2, \cdots, \tau_k$ and strictly decreasing with respect to $\tau_{k+1}, \tau_{k+2}, \cdots, \tau_n$. Then the uncertain random variable $\xi = f(\eta_1, \eta_2, \cdots, \eta_m, \tau_1, \tau_2, \cdots, \tau_n)$ has an expected value:

$$E[\xi] = \int_{\Re^m} \int_0^1 f(y_1, y_2, \cdots, y_m, \Upsilon_1^{-1}(u), \cdots, \Upsilon_k^{-1}(u),$$
$$\Upsilon_{k+1}^{-1}(1-u), \cdots, \Upsilon_n^{-1}(1-u)) du d\Psi_1(y_1) \cdots d\Psi_m(y_m) \quad (7)$$

Theorem 2.5 (Variance [15]) Let $\eta_1, \eta_2, \cdots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \cdots, \Psi_m$, and let $\tau_1, \tau_2, \cdots, \tau_n$ be independent uncertain variables with regular uncertainty distributions $\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_n$, respectively. Assuming f is continuous, strictly increasing with respect to $\tau_1, \tau_2, \cdots, \tau_k$ and strictly decreasing with respect to $\tau_{k+1}, \cdots, \tau_n$, then $\xi = f(\eta_1, \eta_2, \cdots, \eta_m, \tau_1, \tau_2, \cdots, \tau_n)$ has a variance:

$$V[\xi] = 2 \int_{\Re^m} \int_0^{+\infty} x(1 - F(e + x; y_1, \cdots, y_m) + F(e - x; y_1, \cdots, y_m))$$
$$dx d\Psi_1(y_1) \cdots d\Psi_m(y_m) \quad (8)$$

where e is the expected value $E[\xi]$ of ξ, and $F(x; y_1, \cdots, y_m)$ is the uncertainty distribution of uncertain variable $f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n)$ for any real numbers $y_1, y_2, \cdots, y_m$, which is also the root of Equation (6).

3. Advanced UD-LS surrogate model for implicit limit state functions

In the practical structural reliability problems, the analytical expression of LSF is generally unknown. The traditional RSM of structural reliability analysis is iteratively obtained based on the probabilistic reliability index (PRI). Furthermore, the traditional RSM is only suitable for random variables and requires a large number of test sample data. Thus, a new surrogate model is established by combining UD with the LS method considering both of aleatory and epistemic information.

The structure's response is obtained by experiment or finite element analysis, and the sample points used to fit the surrogate model are determined by the design of experiments (DOE) methods. Compared with traditional DOE methods, the UD method is more stable and efficient [4]. UD can maintain the results with high stability and accuracy even with a small sample data. Similar to the OD approach, the UD method can be used to generate experiment points by a series of designed UD tables. The representation of a specific UD table is $U_n(m^n)$ or $U_n^*(m^n)$, where U denotes the UD table, m represents the number of levels and the number of experiments required, n is the number of input factors, and $*$ represents the UD table with a smaller deviation and better uniformity. This work presents only a brief introduction of the UD method, and interested readers can refer to relevant research literature [3, 4, 40]. The quadratic polynomial surrogate model without the cross-terms is chosen as the response surface function of the structure.

$$\widehat{f}(\boldsymbol{x}) = b_0 + \sum_{i=1}^n b_i x_i + \sum_{j=n+1}^{2n} b_j x_j^2 \quad (9)$$

where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ is the vector of input factors, x_i is a random variable or an uncertain variable. $\boldsymbol{b} = (b_0, b_1, \cdots, b_{2n})^T$ is $2n+1$ undetermined coefficients vector in the surrogate model [4]. According to the LS approach, $\boldsymbol{b}$ can be estimated based on $\boldsymbol{b} = (\boldsymbol{a}^T \boldsymbol{a})^{-1} \boldsymbol{a}^T \boldsymbol{y}$, where $\boldsymbol{a}$ is the regression coefficients vector with $m \times (2n+1)$ orders, $\boldsymbol{y} = (f(x_1), f(x_2), \cdots, f(x_m))^T$ is the real responses vector of the structure.

Some indexes are used for validation to verify the surrogate model's fitting performance and check the accuracy. Among them, the coefficient of determination R^2 is the most crucial measurement index:

$$R^2 = 1 - \frac{\sum_{i=1}^m (\widehat{f}_i(\boldsymbol{x}) - f_i(\boldsymbol{x}))^2}{\sum_{i=1}^m (f_i(\boldsymbol{x}) - \overline{f}(\boldsymbol{x}))^2} \quad (0 \leq R^2 \leq 1) \quad (10)$$

where $\overline{f}(\boldsymbol{x})$ is the expected value of all the real responses $f_i(\boldsymbol{x})$, and $\widehat{f}_i(\boldsymbol{x})$ are the simulation values of the responses. The closer the

value of R^2 to 1, the higher is the accuracy of UD-LS surrogate model fitting.

According to the stress-strength interference model and the UD-LS surrogate model, the LSF $G(\boldsymbol{x},\alpha)$ of a structural system under mixed aleatory and epistemic uncertainties can be expressed as:

$$G(\boldsymbol{x},\alpha) = S_{\text{threshold}} - \hat{f}(\boldsymbol{x},\alpha) \qquad (11)$$

where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ is the input factor that affects the structural functioning, $S_{\text{threshold}}$ is the allowable threshold of structural response, and α is a dynamic input parameter associated with time.

4. Structural reliability assessment method under mixed aleatory and epistemic uncertainties

In a complex structural system, some design variables may have enough samples for estimating their probability distribution, which can be described by random variables. Nonetheless, some other design variables may lack sufficient data, which can be estimated by domain experts and regarded as uncertain variables. The structure cannot be simply considered to be a random or uncertain structure model under mixed aleatory and epistemic uncertainties [37]. This section put forward an advanced structural reliability assessment method for this issue depending on chance measure and belief reliability theory.

4.1. Uncertainty quantification for structural reliability based on chance measure

Let $(\Gamma, \mathcal{L}, \text{M}) \times (\Omega, \mathcal{A}, \text{Pr})$ be a chance space, and the LSF of structure contains uncertain random input factors $x_1, x_2, \cdots, x_n$. In the present work, the input factors are uniformly described by uncertain random variables $\xi = (\xi_1, \xi_2, \cdots, \xi_n)$, then the chance reliability of structure based on the chance measure can be defined as follows.

Definition 4.1 Assuming that $G(\xi,\alpha)$ is the LSF of a structure, in which ξ is the vector of uncertain random variables, the chance reliability is defined as the chance measure of the reliability event $\{G(\xi,\alpha) > 0\}$ at α:

$$\text{Ch}_{\text{reliability}}(\alpha) = \text{Ch}\{G(\xi,\alpha) > 0\} \qquad (12)$$

Because of the duality of chance measure, the chance measure of a failure event $\{G(\xi,\alpha) \le 0\}$ at α can be derived as:

$$\begin{aligned} \text{Ch}_{\text{failure}}(\alpha) &= \text{Ch}\{G(\xi,\alpha) \le 0\} \\ &= 1 - \text{Ch}\{G(\xi,\alpha) > 0\} \\ &= 1 - \text{Ch}_{\text{reliability}}(\alpha) \end{aligned} \qquad (13)$$

Consequently, the uncertainty of a safety event at α in structure can be quantified by $\text{Ch}_{\text{reliability}}(\alpha)$ with a numerical value of $[0, 1]$. $\text{Ch}_{\text{failure}}(\alpha)$ describes the confidence how a failure even will be happened at α. Obviously, the higher the $\text{Ch}_{\text{failure}}(\alpha)$, more is the possibility that the failure event will occur at α. The theorem to be defined below provides computational methods for practical engineering applications.

Theorem 4.1 Let the LSF of a structure contain independent random variables $\eta_1, \eta_2, \cdots, \eta_p$ with probability distributions $\Psi_1, \Psi_2, \cdots, \Psi_p$, and independent uncertain variables $\tau_1, \tau_2, \cdots, \tau_q$ with regular uncertainty distributions $\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_q$, respectively. If the LSF $G(\eta_1, \cdots, \eta_p;\ \tau_1, \cdots, \tau_q;\ \alpha)$ is continuous and strictly increasing with respect to $\tau_1, \tau_2, \cdots, \tau_k$, and strictly decreasing with respect to $\tau_{k+1}, \tau_{k+2}, \cdots, \tau_q$, then the chance reliability of the structural system at α can be rewritten as:

$$\text{Ch}_{\text{reliability}}(\alpha) = \int_{\Re^p} F(0; y_1, y_2, \cdots, y_p;\alpha) \text{d}\Psi_1(y_1) \text{d}\Psi_2(y_2) \cdots \text{d}\Psi_p(y_p) \ (14)$$

where $F(0; y_1, y_2, \cdots, y_p;\alpha)$ is the root u of the following equation for any real numbers $y_1, y_2, \cdots, y_p$:

$$G(y_1, y_2, \cdots, y_p;\ \Upsilon_1^{-1}(1-u), \cdots, \Upsilon_k^{-1}(1-u), \Upsilon_{k+1}^{-1}(u), \cdots, \Upsilon_q^{-1}(u);\ \alpha) = 0$$
$$(15)$$

Proof. According to the Theorem 2.3 and Definition 4.1, the chance reliability can be computed as follows:

$$\begin{aligned} \text{Ch}_{\text{reliability}}(\alpha) &= \text{Ch}\{G(\eta_1, \cdots, \eta_p;\ \tau_1, \cdots, \tau_q;\ \alpha) > 0\} \\ &= \int_{\Re^p} \text{M}\{G(y_1, \cdots, y_p;\ \tau_1, \cdots, \tau_q;\ \alpha) > 0\} \text{d}\Psi_1(y_1) \cdots \text{d}\Psi_p(y_p) \end{aligned}$$
$$(16)$$

where $\text{M}\{G(y_1, y_2, \cdots, y_p;\ \tau_1, \cdots, \tau_q;\ \alpha) > 0\} = F(0; y_1, y_2, \cdots, y_p;\alpha)$ is the root u of Equation (15).

The proof is completed.

4.2. The new chance reliability index based on uncertain random variables

PRI in probability space is a vital indicator for quality of structure, and it can be used to describe the structural reliability under aleatory uncertainty. However, the PRI cannot accurately measure the reliability under mixed aleatory and epistemic uncertainties. For completeness, a novel chance reliability index (CRI) is defined using the expected value and variance of the uncertain random variable, showing the reliable degree of a structure in chance space.

Definition 4.2 Let $(\Gamma, \mathcal{L}, \text{M}) \times (\Omega, \mathcal{A}, \text{Pr})$ be an chance space, the LSF $G(\xi,\alpha) = G(\eta_1, \cdots, \eta_p; \tau_1, \cdots, \tau_q; \alpha)$ of a structure contains independent random variables $\eta_1, \eta_2, \cdots, \eta_p$ with probability distributions $\Psi_1, \Psi_2, \cdots, \Psi_p$, and independent uncertain variables $\tau_1, \tau_2, \cdots, \tau_q$ with regular uncertainty distributions $\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_q$, respectively. Then the CRI of the structural system at α can be given as follows:

$$\beta_{\text{chance}}(\alpha) = \frac{E[G(\xi,\alpha)]}{\sqrt{V[G(\xi,\alpha)]}} \qquad (17)$$

where $E[G(\xi,\alpha)]$ is the expected value, $V[G(\xi,\alpha)]$ is the variance of LSF, and $\xi = \xi_1, \xi_2 \cdots \xi_{p+q}$ is an uncertain random vector.

If the LSF is continuous and strictly increasing with respect to $\tau_1, \tau_2, \cdots, \tau_k$, and strictly decreasing with respect to $\tau_{k+1}, \tau_{k+2}, \cdots, \tau_q$, then according to the theorems 2.4 and 2.5, the expected value and variance of LSF at α can be calculated as:

$$\begin{aligned} E[G(\xi,\alpha)] = \int_{\Re^p} \int_0^1 G(y_1, \cdots, y_p; \Upsilon_1^{-1}(u), \cdots, \Upsilon_k^{-1}(u), \\ \Upsilon_{k+1}^{-1}(1-u), \cdots, \Upsilon_q^{-1}(1-u);\alpha) \text{d}u \text{d}\Psi_1(y_1) \cdots \text{d}\Psi_p(y_p) \end{aligned}$$
$$(18)$$

$$\begin{aligned} V[G(\xi,\alpha)] = 2\int_{\Re^p} \int_0^{+\infty} x(1 - F(e_{G(\xi,\alpha)} + x; y_1, \cdots, y_p;\alpha) + F(e_{G(\xi,\alpha)} - x; y_1, \cdots, y_p;\alpha)) \\ \text{d}x \text{d}\Psi_1(y_1) \cdots \text{d}\Psi_p(y_p) \end{aligned}$$
$$(19)$$

where $e_{G(\xi,\alpha)} = E[G(\xi,\alpha)]$, and $F(x; y_1, \cdots, y_p;\alpha)$ is the root u of the following equation for any real numbers $y_1, y_2, \cdots, y_p$:

$$G(y_1, y_2, \cdots, y_p; \Upsilon_1^{-1}(u), \cdots, \Upsilon_k^{-1}(u), \Upsilon_{k+1}^{-1}(1-u), \cdots, \Upsilon_q^{-1}(1-u);\alpha) = x \ (20)$$

The chance-measure-based CRI is offered as a tool to measure the confidence that a reliability event will occur in the structural system affected by both aleatory and epistemic uncertainties. A larger $\beta_{\text{chance}}(\alpha)$ indicates a better possibility that the reliability event will occur.

5 Joint uncertainty propagation method for structural reliability assessment

Uncertainty propagation plays a significant role in reliability problem, which aims to estimate structural output responses by propagating the input factors essential for reliability assessment and safety design. To make it possible for decision-makers to find an appropriate uncertainty propagation types under different knowledge stages, a new joint uncertainty propagation technique is presented in this section. Therefore, the selection principles of uncertainty propagation types are developed in section 5.1. Section 5.2 briefly introduces the level-1 uncertainty propagation method, especially the propagation of uncertain random structure. The novel level-2 joint uncertainty propagation method for structural reliability assessment is proposed in Section 5.3.

5.1. The principles of uncertainty propagation types selection

In general, uncertainty propagation can be classified into the form of level-1 and level-2. The quantification and propagation of uncertainty runs through the whole analysis process. To explain the uncertainty propagation of level-1 and level-2 types, the probability theory is utilized to express aleatory uncertainties, while the uncertainty theory is used to describe the epistemic uncertainties. $G(\boldsymbol{x})$ is assumed to be the LSF established in Section 3, where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ is the input factors vector, and G is the output. To analyze the uncertainty of output G, the uncertainty expressions of the input factors needs to be studied, in addition to their propagations through LSF.

According to the knowledge stage of reliability analyst, the reliability evaluation types can divided into different stages. For example, uncertainty propagation types can be divided into five different stages, shown in Fig.1. At stage 1, the reliability analyst has no sample data on $x_1, x_2, \cdots, x_n$. So uncertain variables are used to describe all the input factors. In this situation, the uncertainty propagations are in level-1 type. At stage 2, the reliability analyst collects more sample data and improves his knowledge. The distribution function type of x_1 is known, which is the probability distribution type. Nonetheless, the shaping parameters of this probability distribution are still lack-

ing and can be described by uncertain variables. $x_2, x_3 \cdots, x_n$ are still described by uncertain variables. In this case, the uncertainty propagations will be in level-2 type. At stage 3, the knowledge of reliability analyst improves further. The probability distribution of x_1 is determined completely, while $x_2, x_3 \cdots, x_n$ are still described by uncertain variables, and the uncertainty propagations turns into level-1 type. At stage 4, the knowledge of reliability analyst improves by obtaining the probability distributions type of x_2 and x_3, but their shaping parameters are still lacking and can be described by uncertain variables. x_1 and $x_4 \cdots, x_n$ are perfectly described by probability distributions. In this situation, the uncertainty propagations turn into level-2 type. At stage 5, the probability distributions of all input factors are determined completely due to elimination of epistemic uncertainties, and random variables are used to describe all input factors. Meanwhile, the uncertainty propagations turn into level-1 type.

In summary, the selection of uncertainty propagation types depends on the knowledge stage and sample data owned by the reliability analyst. For more general situations, the decision-makers can match any circumstances in practical engineering by increasing the number of stages.

5.2. Level-1 uncertainty propagation in structural reliability assessment

As the example mentioned in Section 5.1, there are three different knowledge stages in level-1 uncertainty propagation type, namely, stage 1, stage 3 and stage 5. For stage 1, the uncertain variables describe all the input factors of LSF, and uncertainty propagations analysis is handled through a pure uncertainty model. The uncertain reliability and index can be obtained based on the uncertainty theory and the operational laws, and the specific calculation methods of uncertain structure can be referred to the literature [35]. In stage 3, some input factors have sufficient data, so their probability distributions can be obtained, while some input factors lack data and can only be described by uncertain variables. By analyzing the joint propagation of uncertainty and probability, the methods proposed in the Sections 4.1 and 4.2 are used to estimate the reliability and index of uncertain random structure. The structure that corresponds to stage 5 is called random structure, and the uncertainty propagations analysis for random
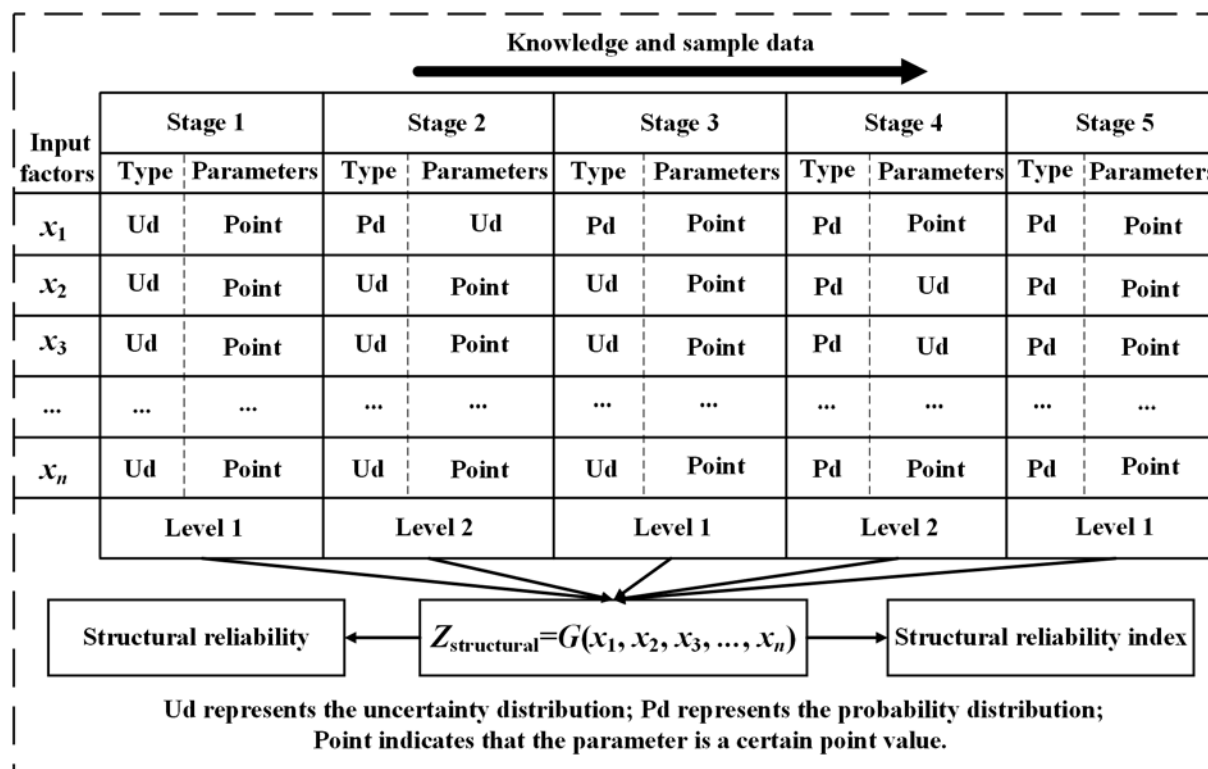
Input factors	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
	Type	Parameters	Type	Parameters	Type	Parameters	Type	Parameters	Type	Parameters
x_1	Ud	Point	Pd	Ud	Pd	Point	Pd	Point	Pd	Point
x_2	Ud	Point	Ud	Point	Ud	Point	Pd	Ud	Pd	Point
x_3	Ud	Point	Ud	Point	Ud	Point	Pd	Ud	Pd	Point
...	...	...	...	...	...	...	...	...	...	...
x_n	Ud	Point	Ud	Point	Ud	Point	Pd	Point	Pd	Point
	Level 1		Level 2		Level 1		Level 2		Level 1	

$$Z_{\text{structural}} = G(x_1, x_2, x_3, \ldots, x_n)$$

Structural reliability ← — → Structural reliability index

Ud represents the uncertainty distribution; Pd represents the probability distribution; Point indicates that the parameter is a certain point value.

Fig. 1. Uncertainty propagation types of structure corresponding to different stages

structure can be implemented by traditional pure probability model. The probabilistic reliability and index of random structure can be estimated by the classical methods such as FORM and MCS.

5.3. Level-2 joint uncertainty propagation in uncertain random structure reliability assessment

As the example mentioned in Section 5.1, there are two different knowledge stages in the level-2 uncertainty propagation type, namely, stage 2 and stage 4. For stage 2, some input factors in the LSF are expressed by probability distributions, of which the shaping parameters are described by uncertainty distributions, while other input factors can be expressed by uncertainty distributions. The next two subsections will introduce the detailed calculation methods in these two stages.

Let a LSF of structural system contain $p+q$ input factors, of which p input factors are expressed by random variables, and the shaping parameters of the probability distributions are described by uncertain variables, while q input factors are expressed by uncertain variables. The level-2 joint propagation in stage 2 can be considered as a more general situation of propagations in stage 3. Consequently, a new level-2 uncertainty analysis method and the corresponding reliability calculation model are provided for stage 2 in this work.

Assuming that p input variables are represented as $\eta_1, \eta_2, \cdots, \eta_p$, and each probability distribution of η_i is represented as $\Psi_i(\eta_i|\theta_i)$, in which θ_i represents the shaping parameters of probability distribution. The shaping parameters are described by uncertainty distributions $\Phi_i(\theta_i)$. Let $\tau_1, \tau_2, \cdots, \tau_q$ represent the q input variables, and $\Upsilon_j(\tau_j)$ represent each uncertainty distribution of τ_j. According to the method presented in Section 4, if the LSF $G(\eta_1, \cdots, \eta_p; \tau_1, \cdots, \tau_q; \alpha)$ is continuous and strictly increasing with respect to $\tau_1, \tau_2, \cdots, \tau_k$, and strictly decreasing with respect to $\tau_{k+1}, \tau_{k+2}, \cdots, \tau_q$. Therefore, the chance reliability of the structural system at α can be calculated as:

$$Ch_{reliability}(\alpha) = \int_{\Re^p} F(0; y_1, y_2, \cdots, y_p; \alpha) d\Psi_1(y_1|\theta_1) d\Psi_2(y_2|\theta_2) \cdots d\Psi_p(y_p|\theta_p)$$

(21)

where $F(0; y_1, y_2, \cdots, y_p; \alpha)$ is the root u of the Equation (15) for any real numbers $y_1, y_2, \cdots, y_p$.

Moreover, the CRI of the structural system at α can be computed from Equation (17), where the expected value and variance of LSF $G(\xi, \alpha)$ at α can be computed as follows:

$$E[G(\xi, \alpha)] = \int_{\Re^p} \int_0^1 G(y_1, \cdots, y_p; \Upsilon_1^{-1}(u), \cdots, \Upsilon_k^{-1}(u),$$
$$\Upsilon_{k+1}^{-1}(1-u), \cdots, \Upsilon_q^{-1}(1-u); \alpha) du d\Psi_1(y_1|\theta_1) d\Psi_2(y_2|\theta_2) \cdots d\Psi_p(y_p|\theta_p)$$

(22)

$$V[G(\xi, \alpha)] = 2 \int_{\Re^p} \int_0^{+\infty} x(1 - F(e_{G(\xi,\alpha)} + x; y_1, \cdots, y_p; \alpha) + F(e_{G(\xi,\alpha)} - x; y_1, \cdots, y_p; \alpha))$$
$$dx d\Psi_1(y_1|\theta_1) d\Psi_2(y_2|\theta_2) \cdots d\Psi_p(y_p|\theta_p)$$

(23)

where $e_{G(\xi,\alpha)} = E[G(\xi, \alpha)]$, and $F(x; y_1, \cdots, y_p; \alpha)$ is the root u of the Equation (20) for any real numbers $y_1, y_2, \cdots, y_p$.

Therefore, the chance reliability and CRI of the structural system is no longer a point value, but varies between the lower and upper bounds of shaping parameters with the uncertainty distribution $\Phi_i(\theta_i)$.

The level-2 joint propagation in stage 4 represents more general circumstances of random structure in stage 5. Thus, the reliability and index of random structure can also calculated by the traditional pure probability methods such as FORM and MCS. Consider a LSF

$G(\eta_1, \cdots, \eta_n; \alpha)$ of structure contains n input factors, m input factors are expressed by the probability distributions $\Psi_i(\eta_i|\theta_i)$, and the shaping parameters θ_i of $\Psi_i(\eta_i|\theta_i)$ are described by the uncertainty distributions $\Phi_i(\theta_i)$, while $n-m$ input factors are expressed by random variables with no epistemic uncertainties. The variation range of reliability and index can be calculated by replacing the original probability distributions with $\Psi_i(\eta_i|\theta_i)$ in classical FORM. Thus, the probabilistic reliability and index of the structure are also no longer a point value, and the variation range can be obtained based on the uncertainty distribution of the shaping parameters.

6. An illustrated example

In this section, the propagation analysis methods developed herein, are applied to a practical engineering application of turbine disk reliability assessment. The description of turbine disk and the implementation of UD-LS surrogate model are introduced in Section 6.1. Section 6.2 shows the specific application process of the uncertainty propaga-

Table 1. Input factors of three-stage turbine disk

Input factors	Physical meaning	Mean value	Standard deviation
E_1(GPa)	Elastic modulus of roulette wheel	123	5
v_1	Poisson's ratio of roulette wheel	0.33	0.015
ρ_1(g/ cm^3)	Density of roulette wheel	4.48	0.2
E_2(GPa)	Elastic modulus of hollow pin shaft	219	10
v_2	Poisson's ratio of hollow pin shaft	0.3	0.015
ρ_2(g/ cm^3)	Density of hollow pin shaft	7.76	0.3
F(KN)	Resultant force on hollow pin shaft	24.925	0.315

tion method proposed in this work. Some results and discussions on the advantages of the proposed method are given in Section 6.3.

6.1. Structure description and LSF simulation

Turbine disk is the key rotating component of modern aircraft engines, driven based on high-temperature gas in the engine combustion chamber. Because the turbine disk converts the thermal energy in the gas into mechanical energy to drive the engine, its reliability level will directly affect the performance of the entire engine.

As shown in Fig. 2(a), the three-stage turbine disk of a low-pressure compressor in a turbofan engine was selected as the research object. The pins on the roulette wheel are evenly and symmetrically distributed along the circumference. According to engineering analysis, the chief input factors affecting the reliability of roulette wheel include material characteristics, load and speed. In this work, the material of hollow pin was 3Cr13, while the material of roulette wheel was TC11. The blade load was applied perpendicularly to the hollow pin, and the average value of the load on each hollow pin was 24925N. The relevant parameters of input factors are shown in Table 1. Since the main failure mode of the turbine disk requires that the maximum stress value is greater than the allowable strength $S_{threshold}$, the maximum stress value can be obtained by finite element analysis.

Because the shape and load of the turbine disk are completely symmetrical, 1/37 part of the turbine disk is considered to describe the entire structure. The average value of each input factor was chosen as the variable value, and the turbine disk was simulated using ANSYS 18.2 at a speed of 1000 rad/s. According to the simulation results
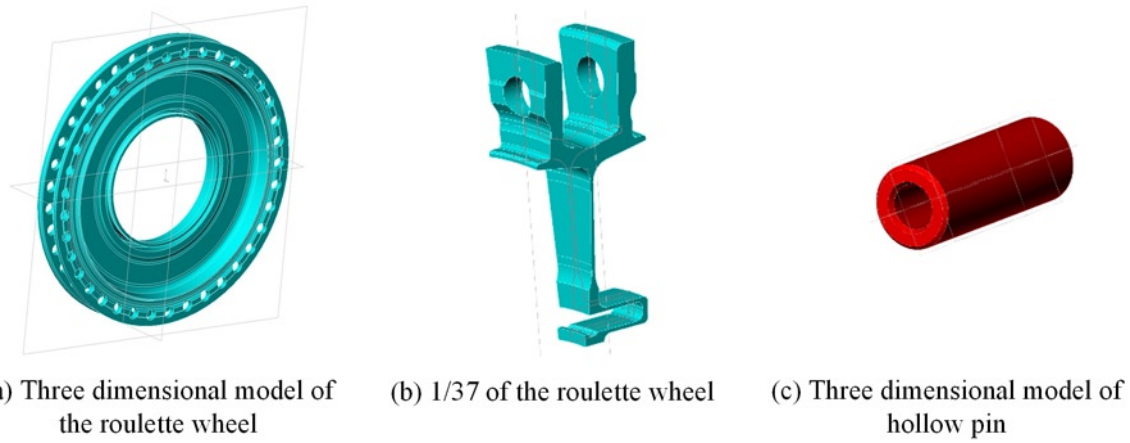
(a) Three dimensional model of the roulette wheel

(b) 1/37 of the roulette wheel

(c) Three dimensional model of hollow pin

Fig. 2. Model of three-stage turbine disk

Table 2. Finite element simulation results with uniform design

Sample number	ω(rad/s)	E_1(GPa)	v_1	ρ_1(g/cm^3)	E_2(GPa)	v_2	ρ_2(g/cm^3)	F(KN)	S_{max}(MPa)
1	0	110.50	0.3000	4.18	209.00	0.2925	7.910	25.240	631.411289
2	85	114.25	0.3187	4.53	231.50	0.3337	7.085	24.531	610.493275
3	170	118.00	0.3375	4.88	189.00	0.2775	8.210	25.870	635.223855
4	255	121.75	0.3562	3.93	211.50	0.3187	7.385	25.161	620.446302
5	340	125.50	0.3750	4.28	234.00	0.2625	8.510	24.453	653.177425
6	425	129.25	0.2962	4.63	191.50	0.3038	7.685	25.791	685.938023
7	510	133.00	0.3150	4.98	214.00	0.3450	6.860	25.082	684.244701
8	595	136.75	0.3337	4.03	236.50	0.2888	7.985	24.374	689.337371
9	680	108.00	0.3525	4.38	194.00	0.3300	7.160	25.713	739.236521
10	765	111.75	0.3712	4.73	216.50	0.2737	8.285	25.004	746.209250
11	850	115.50	0.2925	5.08	239.00	0.3150	7.460	24.295	776.036791
12	935	119.25	0.3112	4.13	196.50	0.2587	8.585	25.634	792.389327
13	1020	123.00	0.3300	4.48	219.00	0.3000	7.760	24.925	804.746581
14	1105	126.75	0.3488	4.83	241.50	0.3413	6.935	24.216	819.019637
15	1190	130.50	0.3675	3.88	199.00	0.2850	8.060	25.555	826.739102
16	1275	134.25	0.2887	4.23	221.50	0.3262	7.235	24.846	863.456674
17	1360	138.00	0.3075	4.58	244.00	0.2700	8.360	24.137	877.634253
18	1445	109.25	0.3262	4.93	201.50	0.3113	7.535	25.476	893.024436
19	1530	113.00	0.3450	3.98	224.00	0.2550	8.660	24.768	912.663743
20	1615	116.75	0.3637	4.33	246.50	0.2963	7.835	24.059	934.261452
21	1700	120.50	0.2850	4.68	204.00	0.3375	7.010	25.398	999.430433
22	1785	124.25	0.3037	5.03	226.50	0.2813	8.135	24.689	1027.33931
23	1870	128.00	0.3225	4.08	249.00	0.3225	7.310	23.980	981.648181
24	1955	131.75	0.3412	4.43	206.50	0.2662	8.435	25.319	1071.52881
25	2040	135.50	0.3600	4.78	229.00	0.3075	7.460	24.610	1114.01504

presented in Fig. 3, the stress-strain level at the junction between the roulette wheel and hollow pin is the highest, which is the dangerous failure point of the structure.

Since with the increase in rotating speed ω(rad/s), the maximum stress at the dangerous point will increase, the reliability of turbine disk will continue to degrade. According to the UD-LS surrogate model introduced in Section 3, a UD table $U_8^*(25^8)$ with 8 factors and 25 levels was designed to arrange the experiment. Let the speed range is $0\sim2040$ rad/s, and the range of other input factors is $x_i = \mu_i \pm f\sigma_i/4$, where μ_i and σ_i are the mean value and standard deviation of each input factors, respectively, and $f = 0,1,2,\cdots,12$. Then, finite element simulation can be used to calculate the maximum stress of each experiment. The simulation results corresponding to each experiment are shown in Table 2.

According to the stress-strength interference model, the LSF of the three-stage turbine disk is established as:

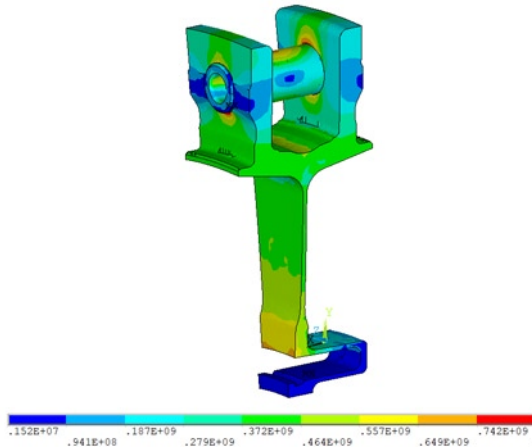$$G(\boldsymbol{x}) = S_{\text{threshold}} - (b_0 + \sum_{i=1}^{8} b_i x_i + \sum_{j=9}^{16} b_j x_j^2) \tag{24}$$

Fig. 3. *The finite element stress cloud diagram of turbine disk at* ω=1000 *rad/s.*

the LSF simulated in this experiment has a high degree of fit, which lays a good foundation for the next step of uncertainty propagation analysis.

6.2. Joint propagation of uncertainty and probability

Based on the different knowledge and sample data stages possessed by the reliability analyst on input factors, the reliability assessment can be implemented based on the uncertainty propagation model proposed in Section 5. According to the selection principles proposed in Section 5.1, the uncertainty propagation of the turbine disk can be obtained as shown in Table 3. In stage 1, the reliability analyst does not have detailed sample data on all input variables. So a domain expert is invited to estimate the values of input factors. Seven normal uncertainty distributions are used to represent the expert's beliefs corresponding to the input factors. In stage 2, the knowledge stage of reliability analyst is improved, and the distribution of roulette wheel density ρ_1 is confirmed as a normal probability distribution $N(\mu_{\rho_1},\sigma_{\rho_1})$. Nonetheless, the expected value μ_{ρ_1} of $N(\mu_{\rho_1},\sigma_{\rho_1})$ is

Table 3. *Distribution types and parameters at different knowledge stages*

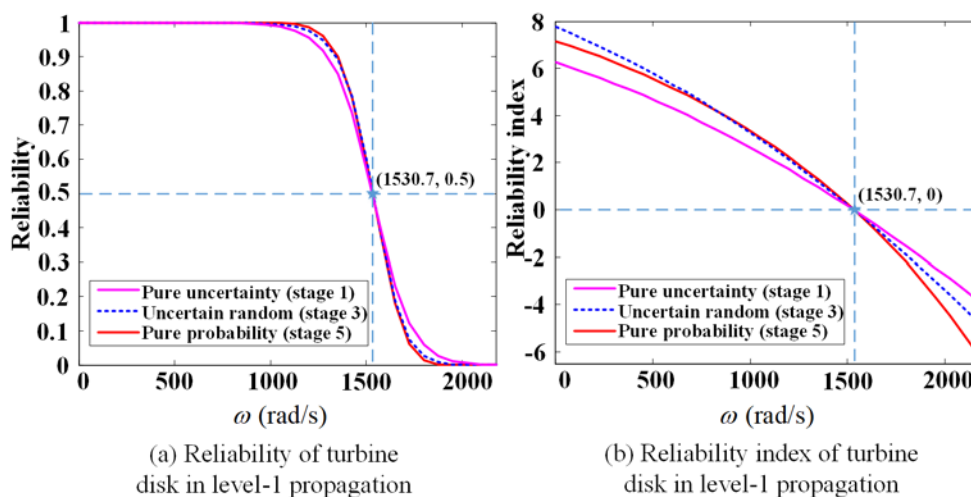| Stages	Distribution types and parameters of input variables						
	$\rho_1(g/cm^3)$	$E_1(GPa)$	v_1	$E_2(GPa)$	v_2	$\rho_2(g/cm^3)$	$F(KN)$
Stage 1	$\mathcal{N}(4.48,0.2)$	$\mathcal{N}(123,5)$	$\mathcal{N}(0.33,0.015)$	$\mathcal{N}(219,10)$	$\mathcal{N}(0.3,0.015)$	$\mathcal{N}(7.76,0.3)$	$\mathcal{N}(24.925,0.315)$
Stage 2	$N(\mu_{\rho_1},0.2)$, $\mu_{\rho_1}\sim$ $\mathcal{L}(3.78,5.18)$	$\mathcal{N}(123,5)$	$\mathcal{N}(0.33,0.015)$	$\mathcal{N}(219,10)$	$\mathcal{N}(0.3,0.015)$	$\mathcal{N}(7.76,0.3)$	$\mathcal{N}(24.925,0.315)$
Stage 3	$N(4.48,0.2)$	$\mathcal{N}(123,5)$	$\mathcal{N}(0.33,0.015)$	$\mathcal{N}(219,10)$	$\mathcal{N}(0.3,0.015)$	$\mathcal{N}(7.76,0.3)$	$\mathcal{N}(24.925,0.315)$
Stage 4	$N(4.48,0.2)$	$N(\mu_{E_1},5)$, $\mu_{E_1}\sim$ $\mathcal{L}(95,151)$	$N(\mu_{v_1},0.015)$, $\mu_{v_1}\sim$ $\mathcal{L}(0.274,0.386)$	$N(219,10)$	$N(0.3,0.015)$	$N(7.76,0.3)$	$N(24.925,0.315)$
Stage 5	$N(4.48,0.2)$	$N(123,5)$	$N(0.33,0.015)$	$N(219,10)$	$N(0.3,0.015)$	$N(7.76,0.3)$	$N(24.925,0.315)$

(a) Reliability of turbine disk in level-1 propagation

(b) Reliability index of turbine disk in level-1 propagation

Fig. 4. *Reliability assessment in level-1 propagation*

still uncertain, and a domain expert is invited to estimate the values of μ_{ρ_1}. Therefore, a linear uncertainty distribution $\mathcal{L}(3.78,5.18)$ is used to represent the expert's beliefs on the expected value μ_{ρ_1}, but the other six input factors are still expressed as normal uncertainty distributions.

In stage 3, the knowledge stage of reliability analyst is improved further by obtaining sufficient data about roulette wheel density. So the normal probability distribution of ρ_1 is determined completely. Also, the normal uncertainty distributions of the other six input factors remain unchanged. In stage 4, the knowledge stage of reliability analyst is improved, and the distributions of seven input variables is determined as a normal probability distribution. However, the expected values of $N(\mu_{E_1},\sigma_{E_1})$ and $N(\mu_{v_1},\sigma_{v_1})$ are still unknown, and domain experts believe that the expected values of $N(\mu_{E_1},\sigma_{E_1})$ and $N(\mu_{v_1},\sigma_{v_1})$ obey the linear uncertainty distributions $\mathcal{L}(95,151)$ and $\mathcal{L}(0.274,0.386)$, respectively. In stage 5, the expected values of E_1 and v_1 are determined completely thanks to the sufficient sample data.

where $S_{\text{threshold}}=935Mpa$ is the threshold of roulette wheel strength, $\boldsymbol{x}=x_1,\cdots x_8$ is a vector of eight input factors, and $\boldsymbol{b}=(b_0,b_1,\cdots,b_{16})^{T}$ is the vector of coefficients, which is estimated by the method introduced in Section 3. The coefficient of determination is calculated as $R^2=0.99784$ by Equation (10), and is very close to 1. Hence,

In other words, all input factors are perfectly described by a normal probability distribution. The above-mentioned specific reliability assessment processes under different knowledge and sample data stages are based on the joint uncertainty propagation method proposed in Section 5.

6.3. Results and discussion

Let the range of turbine disk speed ω be $[0, 2175]$. Then the reliability and index depending on ω in different stages can be calculated based on the methods developed in this paper. It is clear that with the increase in speed, the reliability and indexes of the turbine disk will degenerate because of the increase in stress.

As shown in Fig. 4, the reliability and indexes under three different stages in level-1, namely pure uncertainty in stage 1, uncertain random in stage 3 and pure probability in stage 5 are compared. The results estimated from level-2 in stage 2 are shown in Fig. 5, where the reliability and index of turbine disk fluctuate with the unknown parameter μ_{ρ_1}. In particular, when the rotation speed $\omega = 1200\text{rad}/\text{s}$, the reliability and index takes the values $(0.9648, 0.9895)$ and

$(1.8533, 2.3081)$, respectively. Fig. 6 shows the variation of reliability and index with unknown parameters μ_{E_1} and μ_{v_1} at speed $\omega = 1200\text{rad}/\text{s}$, where the reliability and index take values in $(0.9445, 0.9975)$ and $(1.5935, 2.8018)$, respectively. The practical engineering example illustrates the specific implementation process of the presented method in detail, and the reliability of turbine disk are obtained in different knowledge and sample data stages.

Besides, the simulation results of the reliability and index for different knowledge stages at a specific speed $\omega = 1200\text{rad}/\text{s}$ are presented in Table 4. It is worth noting that the specific speed is selected arbitrarily and the same comparisons can be implemented at any speed value. As presented in Table 4, the reliability is transformed from the interval $(0.9648, 0.9895)$ in stage 2 to the determined value 0.9748 in stage 3 due to the increase in sample data. Moreover, the reliability is transformed from the interval $(0.9445, 0.9975)$ in stage 4 to the determined value 0.9871 in stage 5, which is a good explanation for the process of eliminating epistemic uncertainties. Similar conclusions can be obtained from the reliability indexes in different stages. Besides, as shown in Fig. 4 and Fig.5, when the reliability values are close to 1 at some speed values, the reliability index can be employed to distinguish the reliability differences at these speed values.

In the context of structural reliability assessment, the description of epistemic uncertainty is an inevitably common problem. Classical probability theory cannot be employed to express epistemic uncertainty since the real frequency cannot be obtained due to lack of data. Fuzzy measure and possibility measure do not satisfy duality property, and hence the description of epistemic uncertainty is not reasonable enough. Evidence and interval theory leads to the problem of interval expansion in practical applications. Uncertainty theory is a newly proposed mathematical framework that firmly conforms to the normality, duality and subadditivity theorems. This paper uses the uncertainty theory to describe epistemic uncertainty because it is more suitable for describing the human thinking processes. Also, the probability theory is chosen to represent aleatory uncertainty, and the chance theory is selected to deal with the situation when aleatory and epistemic uncertainties exist simultaneously. The results of case study shows that the level-1 and level-2 joint propagation can be explained very well by combining the above three theories. Consequently, the practical engineering application shows that the various knowledge stages outcome the different reliability levels, and the results highlight that the presented methods are effective and could deliver clear messages to decision-makers.
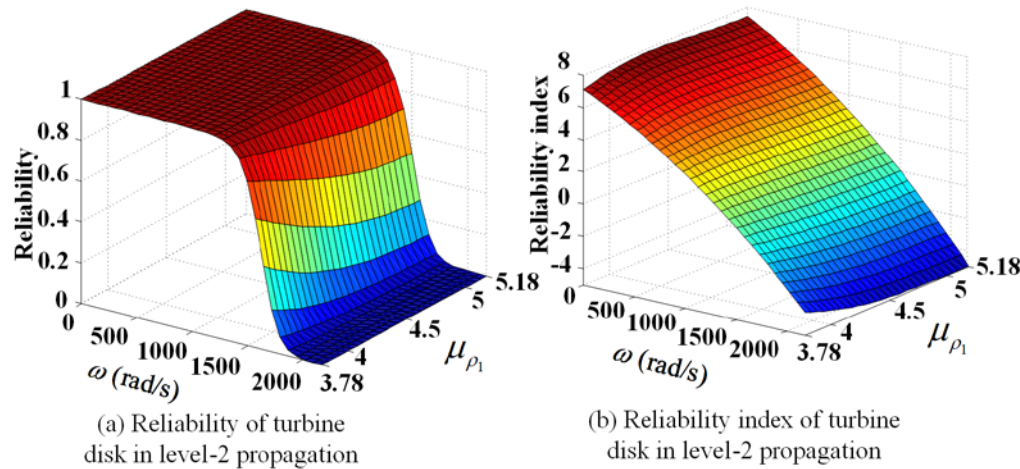

(a) Reliability of turbine disk in level-2 propagation

(b) Reliability index of turbine disk in level-2 propagation

Fig. 5. Reliability assessment in level-2 propagation (stage 2)

Table 4. *Results of reliability and index corresponding to different stages*

At ω=1200 rad/s	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Reliability	0.9565	(0.9648,0.9895)	0.9748	(0.9445,0.9975)	0.9871
Reliability index	1.7038	(1.8533,2.3081)	2.1092	(1.5935,2.8018)	2.2277

(a) Reliability of turbine disk in level-2 propagation

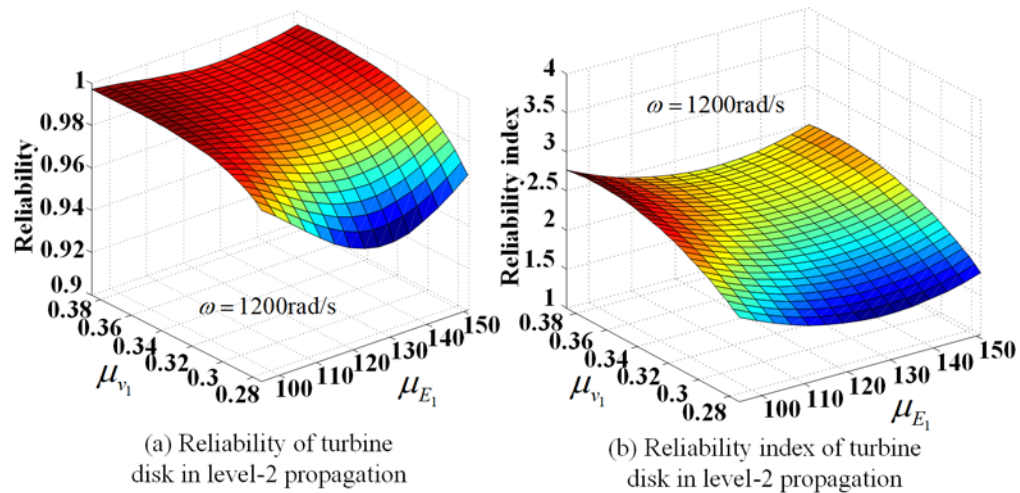(b) Reliability index of turbine disk in level-2 propagation

Fig. 6. Reliability assessment of level-2 propagation at ω =1200 rad/s (stage 4)

7. Conclusions

In this paper, a novel uncertainty propagation method is proposed for the structural reliability assessment under mixed aleatory and epistemic uncertainties. To enable the analyst to calculate the structural reliability according to the different knowledge stages, the principles of selecting the uncertainty propagation types and the corresponding reliability estimation methods are presented. In summary, the main contributions of this paper are as follows:

(1) A new UD-LS surrogate model is proposed to solve the implicit LSF problem involving random and uncertain variables.

(2) The concepts of chance reliability and CRI are defined to describe structural reliability under mixed aleatory and epistemic uncertainties.

(3) A novel level-2 uncertainty analysis method and the corresponding reliability calculation model are provided for uncertain random structures.

(4) The principles for choosing reasonable uncertainty propagation types are presented for structural reliability assessment.

Decision-makers can evaluate the structural reliability corresponding to the different knowledge and sample data stages based on the uncertainty propagation method is proposed in this paper. As the model presented in this work is based on monotonic conditions, further research is required to focus on non-monotonic situations. Another interesting and important issue is to determine the distribution type of input factors based on small sample data.

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