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SEMI-MARKOV SYSTEM MODEL FOR MINIMAL REPAIR MAINTENANCE

SEMI-MARKOWSKI MODEL SYSTEMU OBSŁUGI Z MINIMALNĄ NAPRAWĄ*

This paper analyzes the semi-Markov model of technical objects age-replacements. The model includes two types of repairs: perfect repairs and minimal repairs. Minimal repairs in semi-Markov models have been studied in literature only to an extent. In this paper, the asymptotic availability coefficient and profit per time unit are considered as criteria for the quality of the system operation. The paper formulates various conditions for the occurrence of the maximum of criteria functions. The two numerical examples given at the end of the paper illustrate the results obtained in the paper.

***Keywords**: age-replacement, minimal repair, perfect repair, profit per time unit, availability, semi-Markov processes, preventive maintenance, corrective maintenance.*

W pracy bada się semimarkowski model wymian według wieku obiektów technicznych. W modelu uwzględnia się dwa rodzaje napraw: naprawy dokładne i naprawy minimalne. Naprawy minimalne w modelach semimarkowskich były badane w literaturze w niewielkim stopniu. Jako kryteria jakości pracy systemu rozważa się asymptotyczny współczynnik gotowości i zysk przypadający na jednostkę czasu. W pracy sformułowano różne warunki istnienia maksimum funkcji kryterialnych. Podane na końcu pracy dwa przykłady numeryczne ilustrują wyniki uzyskane w pracy.

***Słowa kluczowe**: wymiana według wieku, naprawa minimalna, naprawa dokładna, zysk na jednostkę czasu, gotowość, procesy semi-Markowa, obsługa prewencyjna, obsługa korekcyjna.*

1. Introduction

During a long period of use, technical systems are prone to degradation processes. The resulting failure has a negative impact on the security and income of the system. Failure can, in turn, cause further failure to the system. In order to reduce the amount of failure to technical objects, various strategies of preventive actions are introduced into system management. The problem of reduction of the costs of system maintenance arises. This requires the development of effective repair and replacement strategies. Managing exchanges and repairs in industrial systems requires introducing various activities related to maintenance as well as appropriate level of reliability and availability into the system. These activities are divided into two types: preventive maintenance (PM) and repair, or corrective maintenance (CM). Corrective maintenance in practice is carried out in two variants: after repair the system is "good as new" (perfect repair) or "bad as old" (minimal repair). Minimal repair restores the system to its reliability condition just before failure. In practice, it restores the system to an intermediate state between the two possible extreme cases. The condition resulting from this activity is referred to as imperfect maintenance. Various models of imperfect maintenance are presented in detail in the review papers [12, 13].

Reduction of system maintenance costs is achieved by implementing various effective prevention strategies and repairs. These activities include the replacement of important system components and determining the frequency of inspections. The schedule of these activities is often set by the system designer or manufacturer. The maintenance department also decides about the replacement of worn components. CM always requires prior diagnosis and identification of failure, therefore it is expensive and can be done by highly trained personnel only. CM repair costs are generally higher than the costs of preventive maintenance (PM). Similarly, average repair times are higher than average times of preventive maintenance. For some industrial systems, it is also possible to repair a failed component without replacing it. This type of repair can be considered as a minimal repair (MR). Minimal repair restores the failed object to the state before the failure. From this point of view, some replacements can be considered minimal repairs. Based on this argument, many practical models of exchanges with minimal repair have been suggested in literature. As a result, developing different prevention strategies suggested by optimal decision-making models to reduce system maintenance costs and reduce the risk of undesired events is an important research topic in reliability engineering. In the last four decades, preventive maintenance models have generated growing interest in system reliability research.

The concept of minimal repair was introduced by Brown and Prochan in paper [2]. The minimum repair model assumes that once the failure occurs, perfect repair is carried out with p probability and minimal repairs are carried out with 1-p probability. Perfect repair restores the technical object to the "good as new" condition. If $p = 0$, the repair is always minimal, while if $p = 1$, the repair is always perfect. Pham and Wang in paper [13] called such a mechanism of repairing an imperfect maintenance model with the rule (p, q). In paper [2] it is assumed that the probability of perfect repair depends on the age of the technical object at the time of failure. In literature, the construction of the minimal repairs model is carried out using various mathematical methods. The review of the methods of constructing criterion functions in the models of minimal repairs with preventative maintenance by age is found in papers [12, 13]. However, only one work cited there [3] uses semi-Markov processes. A more recent review of papers on minimal repairs is featured in book [15]. Recently, in papers [4, 5, 16, 17] new results have been obtained regarding minimal repairs. The problem of minimal repairs is considered from the economic point of view in paper [6]. In addition, the article contains an up-to-date and

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** extensive literature review on minimal preventive repairs and replacements.

This paper analyzes the strategy of maintaining the system using the (p, q) rules of age replacement. The possibility of using semi-Markov processes to build a preventive replacement model in systems with minimal repair is discussed. The basis for building a criterion function is a certain border theorem for semi-Markov processes [7, 8]. This approach to the construction of the criterion function was used in the works [11, 12]. The results of paper [3] are a special case of results obtained for the 3-state model in paper [11] and this article. In this paper, unlike in most articles on maintenance, repair times are not negligible. In the article, profit per unit of time and system availability rate are tested as criterion function. The conditions for the occurrence of an exactly maximum of both criterion functions have been formulated. Chapter 2 defines a 4-state model of replacements with minimal repairs and specifies a criterion function as profit per unit of time. Chapter 3 contains sufficient conditions for the occurrence of maximum profit per unit of time and maximum of the availability rate. In Chapter 4, two numerical examples are analyzed showing the results obtained in the paper. In the first example, the availability rate is maximized, while in the second the profit per time unit is maximized. In both examples it was assumed that the time before failure has Weibull distribution.

2. Criterion function

The paper examines the system in which the technical object may belong to one of the four states: S_1 – failure free operation state, S_2 – minimal repair state, S_3 – perfect repair state, S_4 – preventive replacement state. Possible changes of states are shown in the graph in Fig. 1.

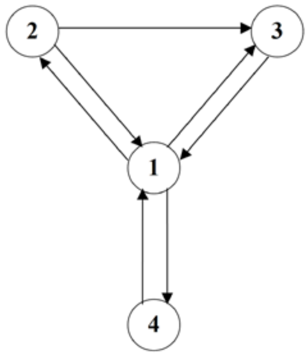

Fig. 1. Directed graph for changes of states $S = \{ S_1, S_2, S_3, S_4 \}$

In cases when we know the probabilities of transition between states, we have a given Markov chain. The matrix of Markov chain transition has the following form:

$$P = \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} \\ p_{21} & 0 & p_{23} & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

By solving the appropriate system of linear equations, limit probabilities for the Markov chain are obtained. The analyzed chain has the following limit probabilities:

$$\begin{aligned} p_1^* &= 1 / M, \\ p_2^* &= p_{12} / M, \\ p_3^* &= (p_{13} + p_{12}\, p_{23}) / M, \\ p_4^* &= p_{14} / M, \end{aligned} \qquad (1)$$

where $M = 2 + p_{12}\, p_{23}$.

The article analyzes the semi-Markov model of preventive repairs and age replacements. The 4-state semi-Markov X(t) process is discussed with the state space S = {1, 2, 3, 4}. If X(t) = i, then the analyzed technical object at moment t is at state i. The profit per time unit for state i is determined by z_i, i = 1, 2, 3, 4. In the paper it is assumed that $z_1 > 0$, $z_i \le 0$ for $2 \le i \le 4$. If the technical object is at state 1, it brings profit, whereas if the technical object is in the state i, where $2 \le i \le 4$, then the technical object generates loss.

The unit is replaced at age T or when it is failed, whichever comes first. Replacement or failure free time is defined by $T_1(x)$. The variable $T_1(x)$ can be written as:

$$T_1(x) = \begin{cases} T_1, & if \quad T_1 < x, \\ x, & if \quad T_1 \ge x. \end{cases} \qquad (2)$$

In paper [11] it was proven that profit per time unit is expressed through the formula:

$$L = \frac{\sum_{i=1}^{4} z_i p_i^* ET_i}{\sum_{i=1}^{4} p_i^* ET_i}, \qquad (3)$$

where ET_i, i = 1, 2, 3, 4 is average time of the technical object remaining at state S_i.

It is assumed that after the time x, if the object has not been failed, it goes into the prevention (replacement) state. The process of changes of states s_i, i = 1, 2, 3, 4, taking into account the preventive replacement after time x, is a new semi-Markov process with the matrix P(x) of probabilities of transition of the embedded Markov chain. In relation to the matrix P described above, only the first line of matrix P changes. In particular, based on paper [11], you can write:

$$\begin{aligned} p_{12}(x) &= p_{12}\, F_{12}(x), \\ p_{13}(x) &= p_{13}\, F_{13}(x), \\ p_{14}(x) &= p_{14}\, F_{14}(x) + R_1(x). \end{aligned}$$

where:
$F_{1i}(x)$, i = 1, 2, 3, 4 are conditional distribution functions of time spent remaining at state i, before transition to state j, defined as follows:

$$F_{ij}(t) = P\left\{ \tau_{k+1} - \tau_k < t \,\middle|\, X(\tau_{k+1}) = j,\ \ X(\tau_k) = i \right\}, \ \ for\ i, j = 1,2,3,4,$$

$R_1(x) = 1 – F_1(x)$ is a function of reliability T_1.

In order to simplify calculations, it is assumed that the following equations are true:

$$F_{12}(x) = F_{13}(x) = F_{14}(x) = F_1(x).$$

On the basis of paper [11], criterion function has the following form:

$$g(x) = \frac{z_1 ET_1(x) p_1^*(x) + z_2 ET_2 p_2^* + z_3 ET_3 p_3^* + z_4 ET_4 p_4^*}{ET_1(x) p_1^*(x) + ET_2 p_2^* + ET_3 p_3^* + ET_4 p_4^*}. \qquad (4)$$

Mean value $ET_1(x)$ is calculate from the formula:

$$ET_1(x) = \int_0^x dF_1(t) + xP\{T_1 \ge x\} \qquad .$$

By integrating through parts, we get:

$$ET_1(x) = \int_0^x R_1(t)\,dt \quad .$$

Limit probabilities $p_1^*(x)$, $p_2^*(x)$, $p_3^*(x)$ are probabilities for Markov chain:

$$P(x) = \begin{bmatrix} 0 & p_{12}(x) & p_{13}(x) & p_{14}(x) \\ p_{21} & 0 & p_{23} & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad .$$

On the basis of the formulas (1) may be written as:

$$\begin{gathered} p_1^*(x) = 1 / M(x), \\ p_2^*(x) = p_{12}(x) / M(x), \\ p_3^*(x) = (p_{13}(x) + p_{12}(x)\, p_{23}) / M(x), \\ p_4^*(x) = p_{14}(x) / M(x), \end{gathered} \tag{5}$$

where $M(x) = 2 + p_{12}(x)\, p_{23}$.

ET_2, ET_3 i ET_4 are mean values for times of the object remaining at states S_2, S_3 and S_4 of the system.

On the basis of (5), criterion function (4) is expressed through the formula:

$$g(x) = \frac{z_1 ET_1(x) + p_{12}F_1(x) z_2 ET_2 + \left[p_{13}F_1(x) + p_{12}p_{23}F_1(x)\right] z_3 ET_3 + \left[1 - p_{12}F_1(x) - p_{13}F_1(x)\right] z_4 ET_4}{ET_1(x) + p_{12}F_1(x) ET_2 + \left[p_{13}F_1(x) + p_{12}p_{23}F_1(x)\right] ET_3 + \left[1 - p_{12}F_1(x) - p_{13}F_1(x)\right] ET_4} .$$

After rearrangement, one can write:

$$g(x) = \frac{z_1 ET_1(x) + F_1(x)\left[p_{12}z_2ET_2 + p_{12}p_{23}z_3ET_3 + p_{13}z_3ET_3 - p_{12}z_4ET_4 - p_{13}z_4ET_4\right] + z_4ET_4}{ET_1(x) + F_1(x)\left[p_{12}ET_2 + p_{12}p_{23}ET_3 + p_{13}ET_3 - p_{12}ET_4 - p_{13}ET_4\right] + ET_4} .$$

Now the numerator and denominator of the criterion function can be represented as:

$$\begin{gathered} L(x) = z_1\, ET(x) + F_1(x)\, B_1 + C_1, \\ M(x) = ET(x) + F_1(x)\, B + C. \end{gathered}$$

Analogically:

$$g(x) = \frac{z_1 ET_1(x) + F_1(x) B_1 + C_1}{ET_1(x) + F_1(x) B + C} ,$$

where:

$$\begin{gathered} B_1 = p_{12}\, z_2\, ET_2 + p_{12}\, p_{23}\, z_3\, ET_3 - p_{12}\, z_4\, ET_4 + p_{13}\, ET_3\, z_3 - p_{13} \\ z_4\, ET_4, \\ C_1 = z_4\, ET_4, \\ B = p_{12}\, ET_2 + p_{12}\, p_{23}\, ET_3 - p_{12}\, ET_4 + p_{13}\, ET_3 - p_{13}\, ET_4, \\ C = ET_4. \end{gathered}$$

After simple transformations we get:

$$\begin{gathered} B_1 = p_{12}\, z_2 ET_2 + z_3 ET_3\, (p_{12}\, p_{23} + p_{13}) - z_4\, ET_4 (p_{12} + p_{13}), \\ B = p_{12}\, ET_2 + ET_3\, (p_{12}\, p_{23} + p_{13}) - ET_4\, (p_{12} + p_{13}). \end{gathered}$$

The following symbols are introduced:

$$\begin{gathered} \alpha = -B\, z_1 + B_1, \\ \beta = C\, z_1 - C_1, \\ \gamma = CB_1 - C_1B. \end{gathered}$$

The α, β and γ rates play an important role in formulating conditions sufficient for the occurrence of extremes of criterion functions.

3. Conditions for occurrence criterion function maximums

The assumptions for the parameters of the tested system are formulated below. These assumptions must reflect the actual relationship between perfect repairs, minimal repairs and preventive replacements:

Z1. $z_1 > 0$, $z_2 < 0$, $z_3 < 0$, $z_4 < 0$. The last element means that the technical object brings profit only at the S_1 state, while the remaining states require expenses.

Z2. $ET_3 > ET_4$, mean time of replacement (prevention) is shorter than mean time of perfect repair.

Z3. $z_3 < z_4$, unit cost $(-z_4)$ of replacement (prevention) is higher than unit cost $(-z_3)$ of perfect repair.

Z4. $ET_3 > ET_2$, mean time of minimal repair is shorter than mean time of perfect repair.

Z5. $z_3 < z_2$, unit cost $(-z_2)$ of minimal repair is higher than unit cost $(-z_3)$ of perfect repair.

The above assumptions do not include the relationship between the state of minimal repair and the state of preventive replacement. In practice, it is not known what is the relationship between the average values of ET_2 and ET_4 or z_2 and z_4. However, if $z_2 - z_4 \le 0$, then on the basis of the assumption Z3 is $\gamma < 0$.

Below, sufficient conditions are formulated for the following inequalities to be true $\alpha < 0$, $\beta > 0$, $\gamma < 0$. The above conditions are formulated depending on mean times ET_i, costs z_i, $i = 1, 2, 3, 4$ as well as elements of matrix $P = [p_{ij}]$, $i, j = 1, 2, 3, 4$. It is relatively easy to calculate that $\beta = ET_4\, (z_1 - z_4)$. The assumption Z1 results in $\beta > 0$. Rate α is expressed through formula:

$$\alpha = p_{12}\, ET_2\, (z_2 - z_1) + (p_{12}\, p_{23} + p_{13})\, ET_3\, (z_3 - z_1) + ET_4\, (p_{12} + p_{13})\, (z_1 - z_4). \tag{6}$$

Inequality $\alpha < 0$ is equivalent to the inequality

$$(p_{12}\, p_{23} + p_{13})\, ET_3 > p_{12}\, ET_2\, (z_2 - z_1) / (z_1 - z_3) + ET_4\, (p_{12} + p_{13})\, (z_1 - z_4) / (z_1 - z_3). \tag{7}$$

Rate γ is similarly determined

$$\gamma = ET_2 [ET_4\, p_{14}\, (z_2 - z_4) + ET_3\, (p_{12}\, p_{23} + p_{13})\, (z_3 - z_4)]. \tag{8}$$

Inequality $\gamma < 0$ is equivalent to the inequality

$$(p_{12}\, p_{23} + p_{13})\, ET_3 > ET_4\, p_{14}\, (z_2 - z_4) / (z_4 - z_3). \tag{9}$$

Let us mark the right side of the inequality (7) and (9) with δ_1 i δ_2 respectively. Let $\delta = \max\{\delta_1, \delta_2\}$. The condition $(p_{12}\, p_{23} + p_{13})\, ET_3 > \delta$ and (6), (7), (8) and (9) is implicated by inequalities $\alpha < 0$, $\gamma < 0$. Now, the following conclusion may be drawn:

Conclusion 1. If $p_{23} > (\delta / ET_3 - p_{13}) / p_{12}$, then the inequalities $\alpha < 0$, $\gamma < 0$ are true.

In the literature on minimal repairs [2, 13, 14], it is assumed that if a technical object passes from the state of operation to the state of failure, the state of minimal repair is achieved with probability equal to $1 - p$, and the state of exact repair with probability p. On the basis of elementary properties of conditional probability, the following equality is true:

$$p_{12} / p_{13} = (1 – p) / p. \quad (10)$$

Conclusion 2. If Tϵ IFR, $\lambda(t)$ is differentiable, $\alpha < 0$, $\gamma < 0$, $\beta > 0$, $\beta + \gamma f(0^+) > 0$, $\lambda(\infty)\ \alpha\ ET + \beta – \alpha < 0$, then the criterion function g(x) reaches maximum value.
Proof.

Function derivative g'(x) has the following form:

$$g'(x)= \{\alpha[f(x)\ ET(x) – R(x)\ F(x)] +\beta\ R(x) + \gamma\ f(x)\} / M^2(x),$$

where M(x) is the denominator of criterion function g(x).

It is known that if time before failure T belongs to the class of MTFR distributions, then the equality $H(x) = \lambda(x)\ ET(x) – F(x)] \geq 0$ for $x \geq 0$. The class of MTFR distributions was tested in papers [9, 10]. The MTFR class includes some lifetimes with single-mode function of failure intensity [9, 10]. From the fact that derivative H'(x) has the form $H'(x) = \lambda'(x)\ ET(x)$ it follows that if the function of intensity of failure $\lambda(t)$ increases, the function H(x) also increases. The class of distributions with the increasing failure rate function (IFR) is included in the MTFR class. The symbol of the derivative is the same as the symbol of the function:

$$h(x) = \alpha[\lambda(x)\ ET(x) – F(x)] + \beta + \gamma\ \lambda(x).$$

It is known that $H(0^+) = 0$, hence $h(0^+) = \beta + \gamma\ f(0^+) > 0$. From the fact that $\alpha < 0$, $\beta > 0$, $\gamma < 0$ and function H(x) increases it follows that the function h(x) decreases from $(0^+) = \beta + \gamma\ f(0^+) > 0$ to $h(\infty) = \lambda(\infty)\alpha\ ET + \beta – \alpha < 0$. It follows that derivative function g'(x) changes from a „+" to a „– " exactly once. Hence it is concluded the criterion function g(x) reaches precisely one maximum. □

If $\lambda(\infty) = \infty$, then for the occurrence of criterion function g(x) the following conditions are sufficient: Tϵ IFR, differentiability $\lambda(t)$, $\alpha < 0$, $\gamma < 0$, $\beta > 0$, $\beta + \gamma\ f(0^+) > 0$. One example is Weibull distribution with increasing failure intensity function.

Conclusions 1 and 2 result in the following sufficient condition for the occurrence of maximum criterion function:

Conclusion 3. If Tϵ IFR, $\lambda(t)$ is differentiable, $\beta + \gamma\ \lambda(0^+) > 0$, $p_{23} > (\delta / ET_3 – p_{13}) / p_{12}$, $\lambda(\infty)\ \alpha\ ET + \beta – \alpha < 0$, then criterion function g(x) reaches maximum value.

Sufficient condition for occurrence of an asymptotic maximum availability rate is formulated below. In order to obtain availability rate from criterion function g(x), it is sufficient to assume the following conditions: $z_1 = 1$, $z_2 = z_3 = z_4 = 0$. After taking into account these conditions, in formula (4) we get $B_1 = 0$, $C_1 = 0$. Hence, on the basis of (2), (3) and (5) for α, β, γ the following can be calculated:

$$\alpha = – B = – p_{12}\ ET_2 – (p_{12}\ p_{23} + p_{13})\ ET_3 + (p_{12} + p_{13})\ ET_4,$$
$$\beta = ET_4,$$
$$\gamma = 0.$$

Inequality $\alpha < 0$ is equivalent to inequality

$$p_{23} > \{[ET_4(1 + p_{13} / p_{12}) – ET_2] / ET_3\} – p_{13} / p_{12}.$$

The last inequality, taking into account (7), may be written in the following form:

$$p_{23} > \{[ET_4 / (1 – p) – ET_2] / ET_3\} – p / (1 – p).$$

Taking into account the fact that $\beta > 0$ i $\gamma = 0$, we can now formulate the sufficient condition for the occurrence of maximum availability rate.

Conclusion 4. If Tϵ IFR, $\lambda(t)$ is differentiable, $\lambda(\infty)\ \alpha\ ET + \beta – \alpha < 0$, $p_{23} > \{[ET_4 / (1 – p) – ET_2] / ET_3\} – p / (1 – p)$, then the availability rate reaches precisely one maximum value.
Proof.

Function derivative g'(x) has the following form: $g'(x)= \{\alpha[f(x)\ ET(x) – R(x)\ F(x)] + \beta\ R(x)\} / M^2(x)$, where M(x) is the denominator for criterion function g(x).

If failure intensity function $\lambda(t)$ increases, then function H(x) increases. Symbol of the derivative is identical with symbol of function $h(x) = \alpha[\lambda(x)\ ET(x) – F(x)] + \beta$. It is known that $H(0^+) = 0$, hence $h(0^+) = \beta > 0$.

From the fact that $p_{23} > \{[ET_4 / (1 – p) – ET_2] / ET_3\} – p / (1 – p)$, it follows that $\alpha < 0$ and function h(x) decreases from value $h(0^+) = \beta > 0$ to values $h(\infty)$. If $h(\infty) = \lambda(\infty)\alpha\ ET + \beta – \alpha < 0$, then derivative function g'(x) changes from a „+" to a „–" exactly once. Hence it is concluded the criterion function g(x) reaches precisely one maximum. □

If $\lambda(\infty) = \infty$, then for the occurrence of maximum availability rate the following conditions are sufficient Tϵ IFR, $p_{23} > \{[ET_4 / (1 – p) – ET_2] / ET_3\} – p / (1 – p)$.

4. Numerical examples

Example 1. In this example the value of the function g(x) is determined, when g(x) is the availability rate. The following data were used in the calculations: mean values of times of the technical object remaining at states $ET_2 = 0.2$, $ET_3 = 0.5$, $ET_4 = 0.1$, assumed for Weibull distribution of time before failure T with the scale parameter b = 6. Non-zero elements were assumed for matrix P as $p_{12} = 0.2$, $p_{13} = 0.6$, $p_{14} = 0.2$, $p_{21} = 0.2$, $p_{23} = 0.7$. The values of the c form (shape) parameter for Weibull distribution c ε {5, 6, 7} were assumed. In each of the three analyzed cases, there is an optimal value for the replacement time.

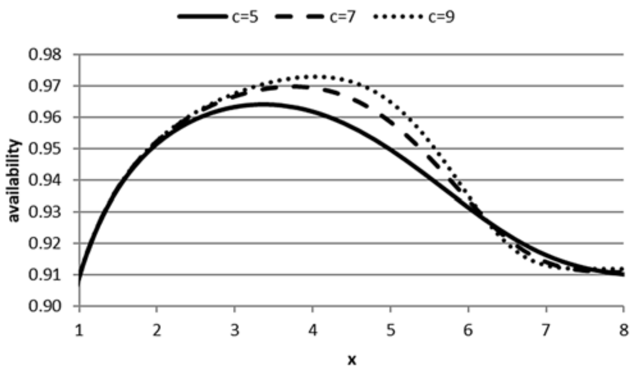

Fig. 2. Charts of changes in the value of availability rate depending on the time of preventive replacement x, for c ϵ {5, 6, 7)

Example 2. In this example the value of the function g(x) is determined, when g(x) is profit per time unit. The calculations included mean values for times of remaining at states, probability matrix P and the parameter of Weibull distribution scale the same as in Example 1. The values of parameter c form (shape) for Weibull distribution cϵ {5,6,7} were assumed. For calculations, unit profits were assumed as $z_1 = 6$, $z_2 = – 0.1$, $z_3 = – 0.8$, $z_4 = – 0.2$.

For all values of parameter c of the form of Weibull distribution, the criterion function reaches the maximum value. Analysis of the dependence of the point x_{max}, in which the criterion function g(x) reaches maximum value shows that as the value of parameter c increases,

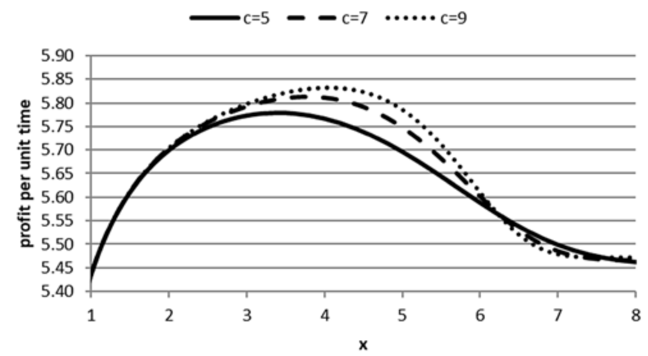

Fig. 3. Charts of changes in the value of profit per unit time rate depending on the time of preventive replacement x, for c ϵ {5, 6, 7)

the value of x_{max} and the maximum value of the criterion function increase.

6. Conclusions

Maintenance systems performing two types of repairs: minimal repairs and perfect repairs are covered by a wide range of literature. However, the use of semi-Markov processes is rare. This paper shows that the application of semi-Markov processes in determining optimal strategies for preventive actions in systems with minimal repair allows for the formulation of interesting conclusions. For the criterion functions analyzed in this paper (availability and profit per unit of time), sufficient conditions for the occurrence of maximum of these criterion functions were formulated. Criterion functions are analyzed in an infinite time horizon. Formulating stronger conditions requires establishing the relation between the mean times of technical object remaining at individual states as well as unit profits at states of minimal repair and preventive replacement.

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