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## OPTIMIZATION METHOD OF BEVEL GEAR RELIABILITY BASED ON GENETIC ALGORITHM AND DISCRETE ELEMENT

### METODA OPTIMALIZACJI NIEZAWODNOŚCI PRZEKŁADNI STOŻKOWEJ Z ZASTOSOWANIEM ALGORYTMU GENETYCZNEGO I ELEMENTÓW DYSKRETYCH

*Gear transmission is the most basic transmission component in mechanical transmission system. Many scholars have done a lot of research on gear reliability. When the variation coefficient is used to calculate and optimize the reliability of bevel gear, in order to calculate the reliability of bevel gear, it is often assumed that the gear works under constant torque, that is, the coefficient of variation (COV) is zero, but this is not the case in practice. In this paper, a gear reliability method based on discrete element simulation is proposed. The purpose of this method is to simulate the actual working conditions of gears, calculate more accurate coefficient of variation in the real world, and improve the accuracy of gear reliability design. Firstly, the real working conditions of the bevel gear transmission are simulated by discrete element method (DEM), and in the transmission system, the tangential force COV of the bevel gear is proved to be equal to the torque COV of the crusher central shaft. Secondly, the multi-objective function model of the gear transmission system is established based on the double tooth roll crusher (DTRC). The optimal volume and reliability of the bevel gear transmission are taken as the objective function, and the teeth number, module and face width factor of basic parameters of gear are optimized by genetic algorithm (GA). Finally, the accuracy of the optimization results is verified by Monte Carlo method. The main purpose of the manuscript is to analyse the effect of actual conditions (DEM simulation) on the optimization results. The results show that the COV of nominal tangential load of bevel gear is about 0.65 under actual working conditions, so in order to guarantee the same reliability, total volume need to be increased by 34.4%. This method is similar to the selection of gear safety factor. In practical production, the selection of safety factor is often based on experience. This paper provides a new method to optimize the reliability of bevel gear, combining with DEM simulation, which provides theoretical guidance for optimal design of bevel gear.*

**Keywords:** bevel gear, reliability, discrete element method, monte carlo simulation, double tooth roll crusher.

*Przekładnia zębata to podstawowy element mechanicznego układu napędowego. niezawodność przekładni jest przedmiotem wielu badań. Przy obliczeniach i optymalizacji niezawodności przekładni stożkowej z wykorzystaniem współczynnika zmienności, często przyjmuje się, że przekładnia pracuje w warunkach stałego momentu obrotowego, t.j. że współczynnik zmienności wynosi 1. Sytuacja taka jednak nie występuje w praktyce. W niniejszej pracy zaproponowano metodę optymalizacji niezawodności przekładni opartą na symulacji metodą elementów dyskretnych. Celem tej metody jest zasymulowanie rzeczywistych warunków pracy przekładni, dokładniejsze obliczenie rzeczywistego współczynnika zmienności oraz poprawa dokładności projektowania niezawodności przekładni. W pierwszej kolejności, na przykładzie kruszarki podwójnej, wyznaczono model działania układu przekładni stożkowej wykorzystujący wielokryterialną funkcję celu. Optymalną objętość i niezawodność przekładni stożkowej przyjęto jako funkcje celu. Następnie, za pomocą metody elementów dyskretnych, symulowano rzeczywiste warunki pracy przekładni. Wyznaczono moment obrotowy przekładni stożkowej i współczynnik zmienności siły wypadkowej, a podstawowe parametry koła zębatego: liczbę zębów, moduł zęba i współczynnik szerokości zębów, zoptymalizowano za pomocą algorytmu genetycznego. Trafność wyników optymalizacji weryfikowano metodą Monte Carlo. Wyniki pokazują, że badana metoda może skutecznie poprawiać niezawodność przekładni stożkowej.*

**Słowa kluczowe:** przekładnia stożkowa, niezawodność, metoda elementów dyskretnych, symulacja Monte Carlo, kruszarka podwójna.

#### 1. Introduction

Gear transmission is one of the most common transmission in mechanical systems, and is widely used in various precision mechanical transmission components, such as machine tools, vehicles, etc. In mechanical systems, the basic requirement of gear mechanism is accurate and smooth transmission of motion, considering the requirements of manufacturing costs and convenience, in the premise of ensuring the requirements of gear transmission. The optimal design of the volume or weight of the wheel drive system has become an important subject for many scholars [2, 3, 4, 7, 42, 43].

With the rapid development of computer technology and various optimization algorithms, there have been a lot of research results on

the optimal design of gear system. The methods and objects of optimal design have been continuously enriched [22, 38, 49]. Mendi [25] used GA to optimize the module of spur gear and then the optimal size of gear box shaft and rolling bearing is obtained. Savsani [34] used the particle swarm optimization and simulated annealing algorithm to optimize the design of multistage spur gear. Sa'id Golabi [13] established the objective function and constraint conditions of the optimum design of the volume / weight of the gearbox, compiled the optimization program using MATLAB, and verified the practicability of the design result by comparison with the known gearbox.

## Nomenclature

$Z_M$	midpoint factor	$Y_{\delta_{relT}}$	relative notch sensitivity factor
$Z_H$	node region factor	$Y_{RrelT}$	relative surface factor
$Z_E$	elasticity factor $\left(\sqrt{N/mm^2}\right)$	$Y_X$	size factor
$Z_{LS}$	stress correction factor	$\sigma_{Hlim}$	experimental surface contact fatigue strength (N/mm <sup>2</sup> )
$Z_\beta$	helix angle factor	$\sigma_{HP}$	allowable contact stress (N/mm <sup>2</sup> )
$Z_K$	bevel gear factor	$Z_{NT}$	life factor
$K$	calculating factor	$Z_L$	lubricant factor
$K_A$	application factor	$Z_V$	velocity factor
$K_V$	dynamic factor	$Z_R$	surface factor
$K_{H\beta}$	face load factor	$Z_W$	work harden factor
$K_{H\alpha}$	transverse load factor	$Z_X$	size factor
$F_t$	nominal tangential load (N)	$m_{nm}$	normal module (mm)
$d_m$	reference diameter (mm)	$K_{F\beta}$	face load factor
$l_{bm}$	length of contact line in the middle of gear (mm)	$K_{F\alpha}$	transverse load factor
$\mu$	gear ratio	$Y_{FS}$	form factor
$\sigma_{Flim}$	nominal stress number from reference test gears (N/mm <sup>2</sup> )	$Y_\epsilon$	rim thickness factor
$\sigma_{FP}$	allowable bending stress (N/mm <sup>2</sup> )	$Y_K$	bevel gear factor
$Y_{NT}$	life factor	$Y_{LS}$	stress correction factor
		$b$	face width (mm)

With the rapid development of science and technology, people have higher requirements for various machines, products and parts. For gear transmission, it is necessary not only to have enough precision and strength, but also to ensure that it can complete its specified function, namely reliability design under the specified working conditions and time [18, 45, 46]. In recent years, scholars have conducted in-depth research on reliability design, and achieved considerable results. Savage [35] established the reliability model of planetary gear system based on Weibull distribution of each unit. Thompson [41] presented a generalized optimal design formula with multi-objective, which can calculate the fatigue strength of multi-stage gear. Li. [21] established the relationship between the wind turbine system and the factors and mechanisms affecting the failure based on the global tree, the success tree and the main logic diagram. Zhang [47] established the optimization design of reliability of large ball mill gear transmission based on the Bayesian analysis algorithm of Kriging model and verified the reliability calculation results by Monte Carlo method. Huang [15] proposed a method to determine the accuracy and reliability of gear motion based on truncated random variables, and discussed the practical application value of the model in detail. Gallego-Calderon and Nejad [11, 30] studied the reliability of gear transmission system in wind turbine. Zhou [48] analysed the dynamic reliability of planetary gear drive system of shearer and verified the results by Monte Carlo simulation method.

In recent decades, a large number of studies have been carried out on the failure probability of structural systems [8, 10, 23, 31, 36,

39]. Many methods have been developed to identify major failure modes, such as (A) “probabilistic” methods, including branch and bound methods [16, 19, 29, 40] and simulation-based techniques [9, 12, 24, 27, 32]; (B) “deterministic” methods, such as incremental loading method [20, 26, 28] b-unzipper method [31], mathematical programming based method [5] or heuristic technique [37, 44]. Kim [17] proposed an effective method to identify dominant failure modes in random variable space, and then analysed the system reliability and calculated the failure probability of the system, and identified the dominant failure mode in the decreasing order of its contribution to the system failure probability. Savage [33] presented a reliability model for single input pinion or equal size double input pinion reducer. Hao [14] mainly studied the influence of different failure modes of bevel gear transmission under incomplete probabilistic information on the modeling of interdependent structures.

In this paper, the probability method is used to calculate the gear reliability model based on the COV, and a new method is proposed to optimize the bevel gear parameters in combination with DEM and GA to improve the gear reliability effectively. This paper is mainly divided into three parts. In the Section 1, the bevel gear working condition is introduced, and the reliability model of single-stage transmission bevel gear is built based on reliability theory. In the Section 2, the model of bevel gear reliability is optimized by GA, and the optimization results is verified by the Monte Carlo method. The Section 3 is a summary of the full text.

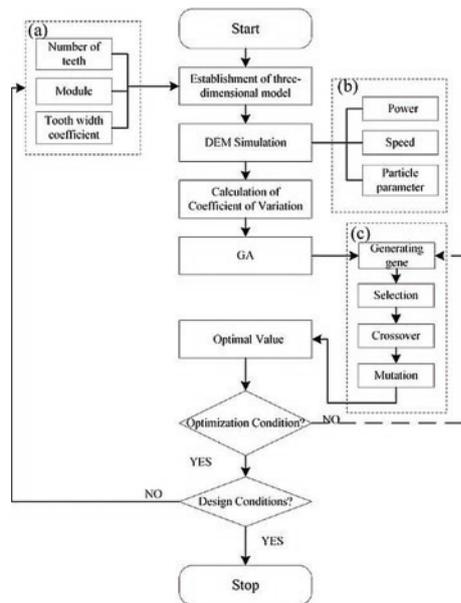


Fig. 1. Analysis flow chart

The optimal design of the bevel gear transmission system could be carried out according to the established optimization model based on DEM. The whole process of the method is shown in Figure 1. Firstly the 3D model of the DTRC is established according to the actual size, as shown in Figure 1 (a); Then the whole model is imported into EDM simulation in Figure 1 (b) according to the actual conditions of crusher (Table 2), and the torque COV of the crusher shaft torque is calculated in order to deduce the torque COV of bevel gear; Finally, based on GA, the optimization design of bevel gear is carried out in Figure 1 (c).

## 2. Modeling of bevel gear reliability system

### 2.1. Gear working condition analysis

In this paper, the double tooth roll crusher (2PGC1040×3610) is taken as an example to simulate and calculate the torque change of the central shaft of the DTRC under a certain working condition with the discrete element method (DEM). Through the mean and variance of the central shaft torque, the torque COV of the bevel gear in the transmission process is deduced.

In the reducer of the DTRC, torque change of the DTRC central shaft under a certain working condition directly affect the force situation of the bevel gear in the transmission system. According to the load of transmission system, as shown in Figure 2, a series transmission in this paper is adopted, so the load of the crusher central shaft is transferred to the bevel gear in a certain proportion. Through calculation of COV, the torque COV of the bevel gear is equal to the torque COV of the crusher central shaft and meanwhile is equal to the nominal tangential load ( $F_t$ ) COV of the bevel gear.

The basic theory of DEM is that the discontinuities of the system are discretized into elements, and each element can interact with each other and satisfy Newton's law of motion when the equations of motion of the system are satisfied. The static or dynamic relaxation iteration method is used to solve the force and motion of each unit in each time step, and then the macroscopic motion law of the whole system is obtained. Based on discrete element, the flow model of material particles in machine can be established, and the information of particle velocity, flow rate, force and wear with machine can be obtained [1,6].

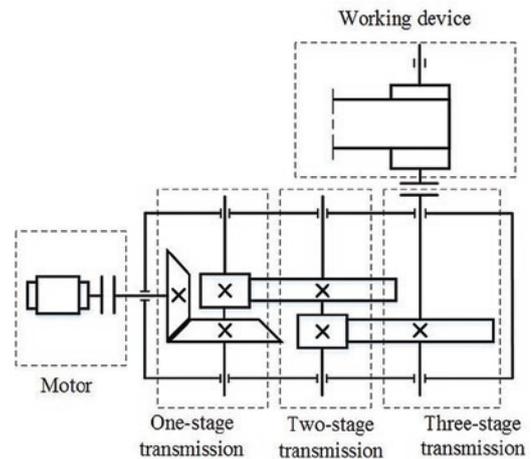


Fig. 2. Reducer of DTRC structure

Table 1. Material parameters in DEM

Material	Steel	Grit stone
Density (kg / m <sup>3</sup> )	7850	2500
Poisson ratio	0.3	0.28
Modulus of shearing (Pa)	7.9×10 <sup>10</sup>	2.7×10 <sup>8</sup>
Recovery coefficient	0.5	0.2
Static friction coefficient	0.3	0.4
Kinetic friction coefficient	0.01	0.01

The discrete element model of DTRC mainly includes: material condensation model, prototype operation model and particle factory, material parameter setting Table 1.

The equivalent form of simulated particles in DEM is usually irregular particles, as shown in Figure 3 (a). The theoretical model of particles is assumed to be spherical in order to facilitate the establishment of equivalent model. The radius of equivalent model is 300mm, which is formed by the condensation of small particles (as shown in the Figure 3 (b)) with radius 40mm. The condensation radius is set to 40.5 mm.

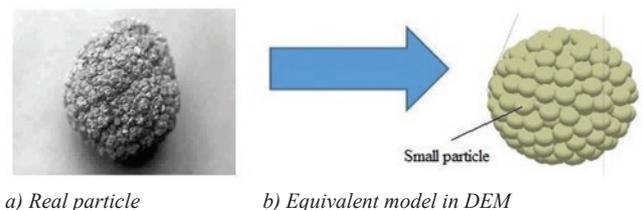


Fig. 3. Particle model transformation

Table 2. Working parameters of bevel gears

Name	Value	Name	Value
Power of motor(P)	710kW	Life expectancy(hours)	8.76×10 <sup>4</sup>
Motor speed(n)	175rad/s	Lubricant	Mine Gear Oil 100
Crossed axis angle	90°	Mode of production	Medium-duty
Accuracy of manufacture	6	Bevel gear ratio	3
Main gear	42CrMo(41~45HRC) Tempered	Pinion	20CrMnMo(56~62HRC) Tempered

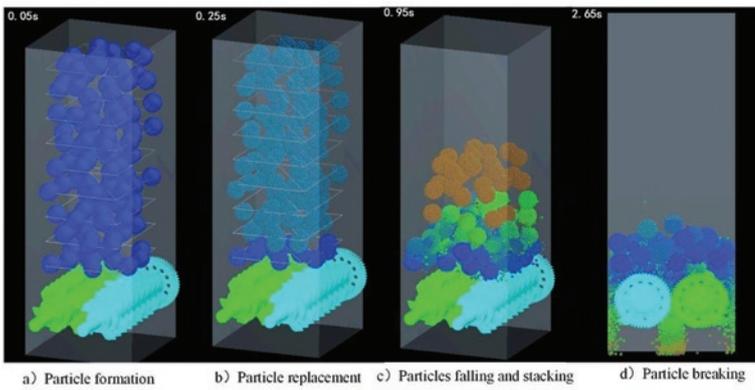


Fig. 4. Simulation process in DEM

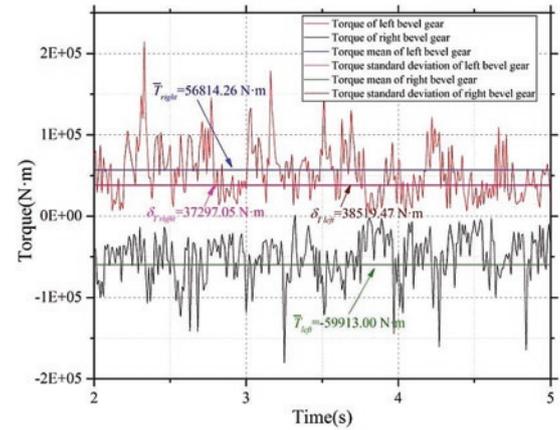


Fig. 5. Torque output curve of DTRC in DEM

Table 2 shows the working parameters of the bevel gear deceleration part of the DTRC. The rotational speed of the motor is 175 rad/s, after deceleration of the reducer (Figure 2), the rotational speed is

Table 3. Coefficient of variation of parameters in bevel gear transmission system [14]

Variables	Mean	Std	COV	Unit	Variables	Mean	Std	COV	Unit
$F_t$	$5.85 \times 10^4$	$3.8 \times 10^4$	<b>0.6495726</b>	N·m	$Z_X$	1	0.033	0.033	-
$K_A$	1	0.033	0.033	-	$Z_L$	0.9658	0.0318714	0.033	-
$K_V$	1.034147	0.034127	0.033	-	$Z_V$	0.968213	0.031951	0.033	-
$K_{H\beta}$	1.65	0.05445	0.033	-	$Z_R$	0.936408	0.030901	0.033	-
$K_{F\beta}$	1.65	0.05445	0.033	-	$Z_W$	1	0.033	0.033	-
$K_{H\alpha}$	1	0.033	0.033	-	$Y_{Fa1}$	2.243285	0.074028	0.033	-
$K_{F\alpha}$	1	0.033	0.033	-	$Y_{Fa2}$	2.249664	0.074238	0.033	-
$l_{bm}$	27.138	0.13569	0.005	mm	$Y_{sa1}$	1.880157	0.06204	0.033	-
b	28	0.14	0.005	mm	$Y_{sa2}$	1.889086	0.06234	0.033	-
$d_{V1}$	50.886	0.25433	0.005	mm	$Y_e$	0.717334	0.0035867	0.005	-
$m_{mn}$	2.54827	0.01274	0.005	mm	$Y_K$	1.000244	0.005	0.005	-
$Z_{M-B}$	0.9874	0.00494	0.005	-	$\sigma_{Flim}$	380	76	0.2	N / mm <sup>2</sup>
$Z_H$	2.49457	0.01247	0.005	-	$Y_{ST}$	2.0	0.066	0.033	-
$Z_E$	189.8117	9.4905	0.05	(N / mm <sup>2</sup> ) <sup>1/2</sup>	$Y_{NT1}$	0.912	0.030096	0.033	-
$F_t$	$5.85 \times 10^4$	$3.8 \times 10^4$	<b>0.6495726</b>	N·m	$Y_{NT2}$	0.933	0.0300789	0.033	-
$Z_{LS}$	1	0.033	0.033	-	$Y_{relT1}$	1.004782	0.033158	0.033	-
$Z_\beta$	1	0.033	0.033	-	$Y_{relT2}$	1.003269	0.033108	0.033	-
$Z_K$	0.8	0.004	0.005	-	$Y_{RelT1}$	1.024202	0.033809	0.033	-
$\sigma_{Hlim}$	1370	164.4	0.12	N / mm <sup>2</sup>	$Y_{RelT2}$	1.024202	0.033809	0.033	-
$Z_{NT}$	1.07	0.03531	0.033	-	$Y_{X1}$	1	0.033	0.033	-

reduced to 2.77 rad/s, the total gear ratio is 63.18, and the input power of motor is 710 kW. The bevel gear works at the one-stage transmission with a gear ratio of 3, a rotational speed of 175 rad/s for the input shaft, and a rotational speed of 58.3 rad/s for the output shaft. The material and processing parameters are shown in the table.

Figure 4 shows the change of particle position at different time in DEM simulation. The working speed of DTRC shaft is 2.77 rad/s. It can be seen that the particle does not contact the crusher when the simulation time reaches 1 s, so the torque of the crusher central shaft is zero. After 1 s, the crusher starts to work. The torque change curve of the central shaft under the steady working condition of 2-5s crusher is exported, as shown in Figure 5.

Figure 5 is torque output curve of DTRC in DEM, the output torque of the left and right tooth rollers can be seen to be basically equal. The load distribution information is extracted and the mean value and standard deviation are calculated as follows:

$$\begin{cases} \bar{T}_e = \frac{\bar{T}_{right} + \bar{T}_{left}}{2} = 58363.63N \cdot m \\ \delta T_e = \frac{\delta T_{right} + \delta T_{left}}{2} = 37908.26N \cdot m \end{cases} \quad (1)$$

For accuracy of calculation, the mean and standard deviation (Std) are taken as the mean values of the left and right axes.

$$COV(F_t) = COV(\bar{T}) = \frac{\delta T_e}{\bar{T}_e} \quad (2)$$

In Table 3, except that the force  $F_t$  is simulated by DEM, the other gear parameters are assumed to follow the normal distribution [14], so the COV of the parameters is calculated, and the reliability calculation process is shown in the appendix of this paper.

### 2.2. Gear modeling analysis

The theory of system reliability bound, such as Cornell bounds [4] and Ditlevsen bounds [7], is often used to estimate the reliability of the system. It is also called the narrow reliability bound. By evaluating the joint failure probability of each pair of failure modes, a narrower estimate of the system failure probability is obtained. For a series system with m failure modes, a narrow bound of system reliability estimation is given:

$$P_{f_1} + \sum_{i=2}^m \max(P_{f_i} - \sum_{j=1}^{i-1} P_{f_{ij}}, 0) \leq P_f \leq \sum_{i=1}^m P_{f_i} - \sum_{i=2}^m \max(P_{f_{ij}}) \quad (3)$$

Where,  $P_{f_1}$  is the failure probability of the first failure mode and  $P_{f_{ij}}$  is the joint failure probability of each pair of failure modes. Considering the convenience of the analysis, we only consider the primary bevel gear drive, which is a series system.

### 2.3. Determination of Objective Functions

According to the definition of robust optimal design involving reliability sensitivity, the robust design optimization model of bevel gear transmission system based on reliability is established:

$$\min f(X) = \sum_{k=1}^M w_k f_k(X) \quad (4)$$

$$s.t. R[g_i(X) > 0] \leq R_i, i = 1, \dots, N \quad (5)$$

$$X^L \leq X \leq X^U, X \in X^{ndv} \quad (6)$$

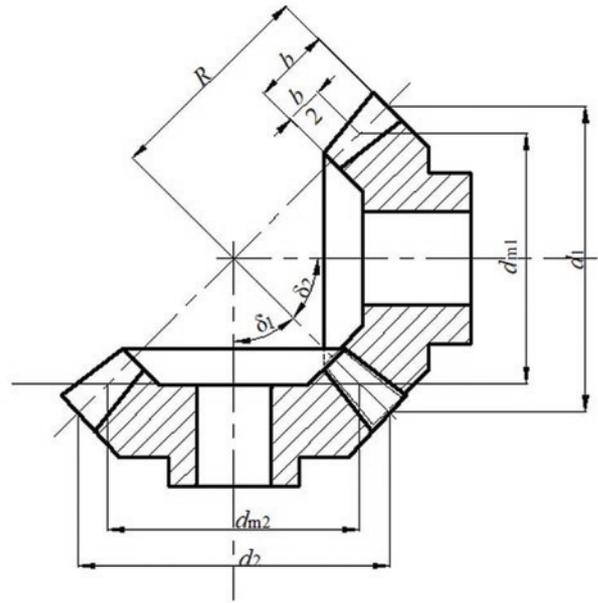


Fig. 6. Bevel gear transmission geometric parameters

Where,  $X$  is the design variables,  $X = [m_{nm}, z_1, \varphi_R]^T$ ,  $X^L$  and  $X^U$  is the lower and upper bounds of the design variables, respectively.  $M$ ,  $N$  and  $ndv$  represent the number of sub-objective functions, reliability functions and design variables, respectively.  $\omega_k$  is the weight of each sub-function which shows the importance of each sub-function in calculating reliability. The formulas are as follows:

$$\omega_k = [f_1(X^{*k} - f_1(X^{*1}))] / \{ [f_1(X^{*k}) - f_1(X^{*1})] + [f_2(X^{*(k-1)}) - f_2(X^{*2})] + \dots + [f_k(X^{*1}) - f_k(X^{*k})] \} \quad (7)$$

where, the weighting factor should satisfy the following conditions:

$$\sum_{k=1}^n \omega_k = 1 \quad (\omega_k \geq 1) \quad (8)$$

The reliability function is the objective function  $f_1$ , and the bevel gear transmission is series system.  $R_1(X)$ ,  $R_2(X)$ ,  $R_3(X)$ ,  $R_4(X)$  represent the reliability of contact strength of the gear tooth surface, bending strength of the gear tooth root, contact strength of pinion tooth surface and bending strength of pinion tooth root, respectively. In order to ensure the reliability of gear transmission, the structure of transmission will be destroyed if any gear is damaged. Therefore, the reliability objective function of bevel gear transmission can be designed as follows:

$$f_1(X) = f(X) = 1 - R_1(X)R_2(X)R_3(X)R_4(X) \quad (9)$$

where, calculation formula of  $R_1(X)$ ,  $R_2(X)$ ,  $R_4(X)$  is in the appendix.

Meanwhile, the volume of bevel gear group is also the objective function  $f_2$ , the formula is:

$$f_2(\mathbf{X}) = (f_p(\mathbf{X}) + f_g(\mathbf{X})) / V_{\max} \quad (10)$$

where,  $f_p(\mathbf{X})$ ,  $f_g(\mathbf{X})$  represent the volume of the pinion and the main gear, respectively.

$$f_p(\mathbf{X}) = \frac{\pi u d_1^2}{12 \times 10^9} \left( 1 - \left( \frac{d_1 - b/u}{d_1} \right) \right) \quad (11)$$

$$f_g(\mathbf{X}) = \frac{\pi d_2^3}{12 \times u \times 10^9} \left( 1 - \left( \frac{d_2 - bu}{d_2} \right) \right) \quad (12)$$

$$b = \varphi_R m_n \quad (13)$$

$$d_1 = z_1 m_n \quad (14)$$

$$d_2 = d_1 u \quad (15)$$

where,  $u$  is bevel gear ratio.

Therefore, the formula (1)-(13) is simultaneous, then the objective function is as follows:

$$\min f(\mathbf{X}) = \omega_1 f_1(\mathbf{X}) + \omega_2 f_2(\mathbf{X}) \quad (16)$$

$$f_1(\mathbf{X}) = 1 - R_1(\mathbf{X})R_2(\mathbf{X})R_3(\mathbf{X})R_4(\mathbf{X}) \quad (17)$$

$$f_2(\mathbf{X}) = \frac{\pi u (z_1 m_n)^2}{12 \times 10^9} \left( 1 - \left( \frac{z_1 - \varphi_R / u}{z_1} \right) + u - u \left( \frac{z_1 - \varphi_R}{z_1} \right)^2 \right) / V_{\max} \quad (18)$$

$$\text{s.t. } R[g_i(\mathbf{X}) > 0] \leq R_i \quad (19)$$

$$(R_1 \cap R_2 \cap R_3 \cap R_4) \geq 0.9 \quad (20)$$

$$g_i(\mathbf{X}) \geq 0 \quad (21)$$

$$\begin{cases} 1 \leq m_e \leq 50 \\ z_{\min} \leq z_1 \leq 44 \\ 1/4 \leq \varphi_R \leq 1/3 \end{cases} \quad (22)$$

where,  $i = 1, 2$ ,  $z_{\min}$  is the minimum number of pinion teeth, determined by the type of bevel gear in Table 4.  $z_{\min} = 13 \sim 15$ .

Table 4. Selection of gear types and minimum module

Type of Gear	Coniflex gear	Skew bevel gear	Zerol bevel gear	Gleason spiral bevel gear
Power range (kW)	<50	<500	<3700	>3700
Minimum teeth number	13	13	15	12

### 3. Case Analysis

#### 3.1. Analysis of results

Figure 7-11 shows the optimization results of DEM and GA. From those figures, it can be seen that the optimization result is good. As shown in Figure 7, the optimization result of GA tends to be stable after 70 iterations by using the objective function of weight  $\omega_1 = 0.3$  and  $\omega_2 = 0.7$ , that is, the optimal value may be reached. It can be seen that the total volume and failure rate of gears will affect the objective function, and the objective function decreases first and then rise. That indicates that the reliability of gear system has a great influence on the objective function before 25 generations, while after 25 generations, the total volume of gear system has great influence on the objective function.

Figure 8 shows number of teeth and module curve (DEM and GA) and Figure 9 shows face width factor curve (DEM and GA). Combined with Figure 7-9, GA is used to optimize bevel gear parameters (teeth number, module and face width factor), which change trend is approximate, finally after 80 iterations tend to balance. Variable approximation is  $\mathbf{X}_{op} = (m_n, z_1, \varphi_R)^T \approx (11, 32.23, 0.264)^T$ , optimal value  $f(\mathbf{X})_{\min} = 0.5661$ .

Figure 10 shows the failure rate and reliability curve (DEM and GA), after 30 iterations, the reliability gradually tends to 1, while Gear failure rate gradually approach 0. Figure 11 shows the change of contact stress on tooth surface and bending stress on tooth root during iteration. According to experience, the pinion is most easily worn out during transmission. The allowable contact stress of pinion is 783.57 MPa and the bending stress of pinion tooth root is 466.95 MPa. As the Figure 11 shows, after 30 generations, the contact strength and bending strength of gear are lower than the allowable value, and the curve tends to be stable.

Table 5 shows the differences of optimization results using DEM and without DEM. In mode 1, the result is not using DEM and the COV of nominal tangential load ( $COV_{F_t}$ ) is assumed to be 0; in mode 2, the COV of nominal tangential load ( $COV_{F_t}$ ) is obtained from DEM. The results of mode 1 and mode 2 is all optimized by GA.

Comparing mode 1 with mode 2, it can be seen that when mode 1 reaches the best point  $(m_n, z_1, \varphi_R)^T \approx (11, 28.57, 0.298)^T$  and mode 2 reaches the best point  $(m_n, z_1, \varphi_R)^T \approx (11, 32.23, 0.264)^T$ , the interesting phenomenon is occur that there is a small change in module relative to mode 1, which is always 11mm. The teeth number increased from 28.57 to 32.23, while the face width factor decreased from 0.298 to 0.264. The reliability changes little and the gear volume increases by 34.4%, which shows that the mode 2 design increases the cost, but the mode 2 design is more suitable for the actual situation. The method is same as the safety factor in gear design. Because of the different working conditions of gear, the selection of safety factor is often based on experience. This method provides theoretical guidance for the selection of gear safety factor.

In the mode 2, the optimization result based on the DEM increase the number of gear tooth than the mode 1, resulting in the increase of the volume, but the reliability change is relatively small.

The result shows that when the variation coefficient method is used to optimize the gear reliability, the gear working condition also has a great influence on the reliability optimization.

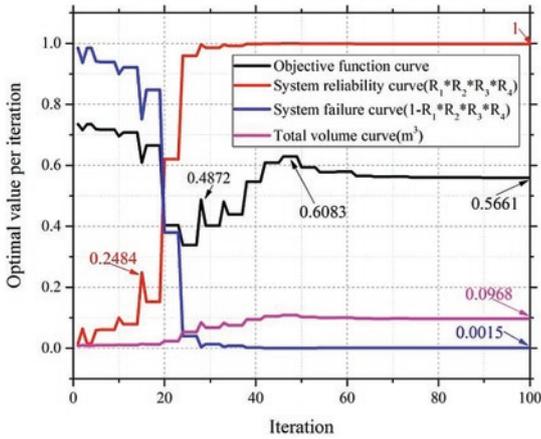


Fig. 7. Gear parameters curve

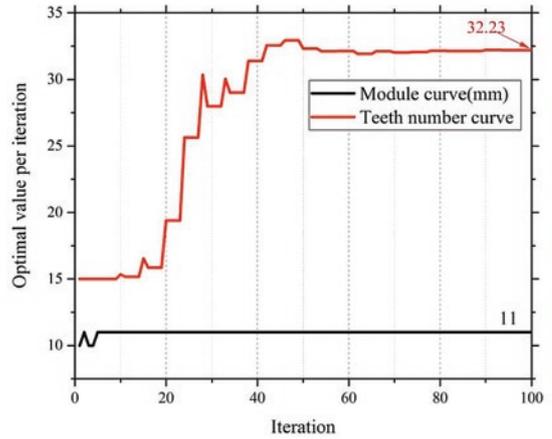


Fig. 8. Number of teeth and module curve

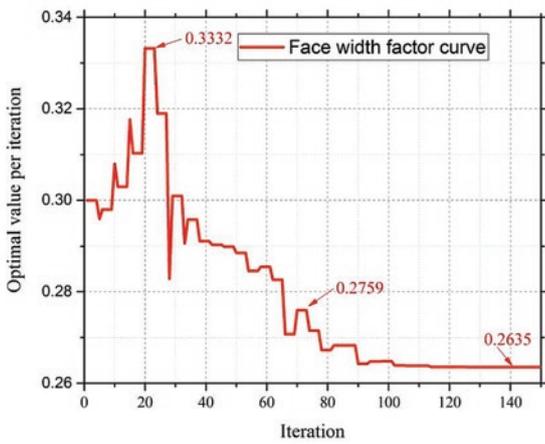


Fig. 9. Face width factor curve

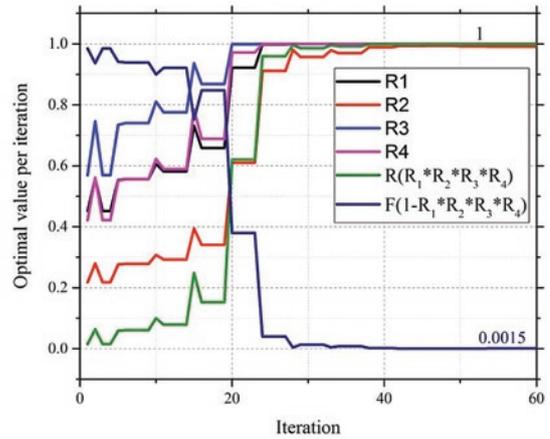


Fig. 10. Reliability and failure rate curve

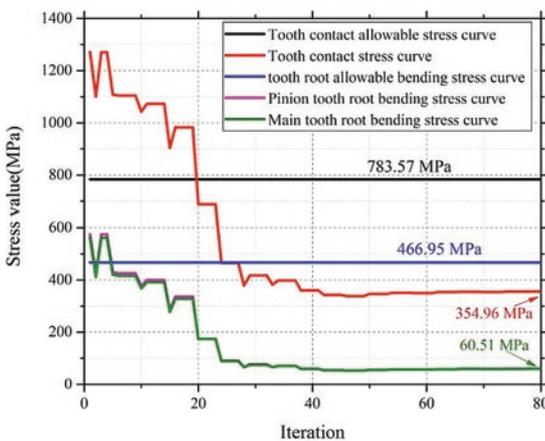


Fig. 11. Stress iterative curve

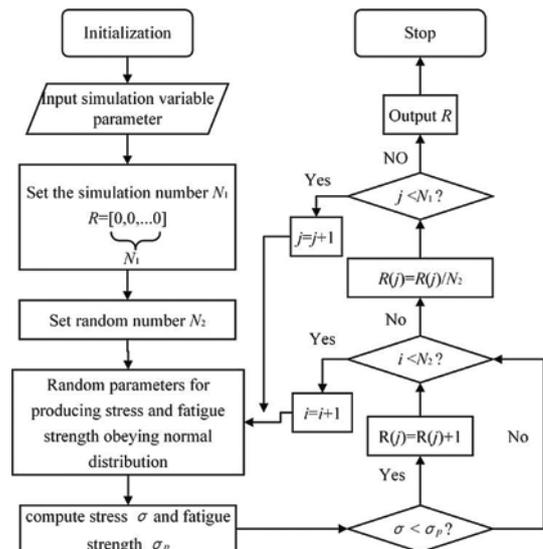


Fig. 12. Reliability calculation using Monte Carlo method

Table 5. Differences of optimization results using DEM and without DEM

Design mode	$m_e$	$z_1$	$\varphi$	$R_1$	$R_2$	$R_3$	$R_4$	System Reliability	Volume (m <sup>3</sup> )	$F_{ob}$
1 Reliability optimization design (COV <sub>F<sub>t</sub></sub> = 0)	11	28.57	0.298	0.9999	0.9903	1	1	<b>0.9903</b>	<b>0.0721</b>	0.4227
Teeth number upward rounding (COV <sub>F<sub>t</sub></sub> = 0)	11	29	0.298	0.9999	0.9925	1	1	0.9925	0.0754	0.4403
2 Reliability optimization design (DEM) (COV <sub>F<sub>t</sub></sub> = 0.6495726)	11	<b>32.23</b>	0.264	0.9998	0.9900	1	1	<b>0.9900</b>	0.0969	<b>0.5661</b>
Teeth number upward rounding (DEM) (COV <sub>F<sub>t</sub></sub> = 0.6495726)	11	33	0.264	0.9999	0.9931	1	1	0.9931	0.104	0.6050

Note:  $F_{ob} = \omega_1 \times Reliability + \omega_2 \times \frac{Volume}{V_{max}}, V_{max} = 0.052m^3, \omega_1 = 0.3, \omega_2 = 0.7$

Table 6. Results Comparison of Monte Carlo (MC) and theoretical value (TV)

Reliability	Teeth number upward rounding (COV <sub>T</sub> = 0)		Teeth number upward rounding (DEM) (COV <sub>T</sub> = 0.6495726)	
		MC	TV	MC
$R_1$	0.9999	0.998201	0.9999	0.999896
$R_2$	0.9925	0.992051	0.9931	0.994681
$R_3$	1	1	1	1
$R_4$	1	0.992494	1	0.999439

### 3.2. Verification of reliability with Monte Carlo method

Monte Carlo method is also called statistical test method, stochastic simulation method, or random sampling technique or statistical test method.

When the Monte Carlo method is used to calculate the reliability of gear transmission, the parameters related to the geometric dimensions of the gears are regarded as constants, while the other parameters are considered to obey normal distribution.

As shown in Figure 12, this paper uses the Monte Carlo method to calculate the gear transmission reliability flow chart, the basic process is:

- Step 1. Input the parameters of simulation variables that need to verify the results;
- Step 2. Set the number of simulations  $N_1$  and make the  $N_1$  dimension line vector  $R = [0, 0, \dots, 0]$ ;
- Step 3. Generating  $N_2$  random variables of normal distribution;
- Step 4. Calculate the stress  $\sigma$  and fatigue strength  $\sigma_p$ ;
- Step 5. Comparing the magnitude of stress and fatigue strength  $\sigma_p$ , if  $\sigma < \sigma_p$ , it accumulates to the reliable database, otherwise it goes directly to the next step;
- Step 6. If the total number of random numbers reaches  $N_2$ , go to the next step, otherwise, return to Step4;
- Step 7. Calculate the reliability of  $j$ th order  $R(j) = R(j)/N_2$ ;

- Step 8. If the number of simulations reaches  $N_1$ , enter the reliability  $R$  of all the simulations and end the program. Otherwise,  $j = j + 1$ , return to Step3.

Suppose the simulation number  $N_1$  is 1 and the number of random numbers  $N_2$  is one million. In Table 6, comparison of reliability between Monte Carlo and theoretical calculations (Section 3.1) can be obtained.

In Table 6, the simulation results of reliability obtained by Monte Carlo method and theoretical value are very similar, so it can be considered that the calculation of results by normal distribution is correct.

### 4. Summary

In this paper, the multi-objective function model of bevel gear is established, and the parameters of bevel gear are optimized by GA and DEM. The feasibility of the method is proved theoretically. In the design of gear reliability, the calculation of reliability by COV often assumes that the moment of transmission is subjected to constant load, but the actual situation is not always so. The innovation of this paper is to use DEM. Combined with DEM of real working conditions, the torque COV of gear transmission is calculated, which makes the designed gear more in line with the actual situation. The results of this paper verify the feasibility of the method by Monte Carlo method and provide reference for the design and optimization of gears.

The results show that it is closer to gear design when the optimum design of gear under actual working conditions are considered, and this method is similar to the selection of gear safety factor. In practical production, the selection of safety factor is often based on experience.

This paper provides a new method to optimize the reliability of bevel gear, combining with DEM simulation, which provides theoretical guidance for optimal design of bevel gear.

There are also many defects in this paper. After all, the DEM is only theoretical simulation, and there is still a lot of gap with the actual situation. At the same time, the GA is used in the process of global optimization, because of the defects of the algorithm. It is possible to fall into a local optimal situation in the process of optimization. Of

course, with the continuous improvement of computer performance and research of algorithm, this method will have a lot of room for improvement.

#### Acknowledgement

*The research work financed with the National Natural Science Foundation of China (Grant No. 51775225) and key scientific and technological coal based research projects in Shanxi (Grant No. MJ2014-02).*

#### References

- Boemer D, Ponthot J. A generic wear prediction procedure based on the discrete element method for ball mill liners in the cement industry. *Minerals Engineering* 2017; 109: 55-79, <https://doi.org/10.1016/j.mineng.2017.02.014>.
- Chong T H, Lee J S. A Design Method of Gear Trains Using a Genetic Algorithm. *International Journal of Precision Engineering & Manufacturing* 2000; 1(1): 62-70, <http://pdfs.semanticscholar.org/3460/f3437791bde00fa9f5e21c98e5836980a17a.pdf>.
- Chong T H, Bae I, Park G J. A new and generalized methodology to design multi-stage gear drives by integrating the dimensional and the configuration design process. *Mechanism & Machine Theory* 2002; 37(3): 295-310. [https://doi.org/10.1016/S0094-114X\(01\)00078-7](https://doi.org/10.1016/S0094-114X(01)00078-7).
- Chong T H, Bae I, Kubo A. Multiobjective Optimal Design of Cylindrical Gear Pairs for the Reduction of Gear Size and Meshing Vibration 2002; 44(1): 291-298, <https://doi.org/10.1299/jsmec.44.291>
- Corotis RB, Nafday A M. Structural system reliability using linear programming and simulation. *Journal of Structural Engineering ASCE* 1989; 115(10): 2435-2447, [https://doi.org/10.1061/\(ASCE\)0733-9445\(1989\)115:10\(2435\)](https://doi.org/10.1061/(ASCE)0733-9445(1989)115:10(2435))
- Delaney G W, Morrison R D, Sinnott M D, et al. DEM modelling of non-spherical particle breakage and flow in an industrial scale cone crusher. *Minerals Engineering* 2015; 74: 112-122, <https://doi.org/10.1016/j.mineng.2015.01.013>.
- Deb K, Jain S. Multi-Speed Gearbox Design Using Multi-Objective Evolutionary Algorithms. *Journal of Mechanical Design* 2003; 125(3): 609-619, <http://link.aip.org/link/?JMDEDB/125/609/1>.
- Ditlevsen O, Madsen HO. *Structural reliability methods*. Chichester, UK: JohnWiley & Sons; 1996.
- Ditlevsen O, Bjerager P. Plastic reliability analysis by directional simulation. *Journal of Engineering Mechanics ASCE* 1989; 115(6): 1347-1362, [https://doi.org/10.1061/\(ASCE\)0733-9399\(1989\)115:6\(1347\)](https://doi.org/10.1061/(ASCE)0733-9399(1989)115:6(1347)).
- Freudenthal AM, Garrelts JM, Shinozuka M. The analysis of structural safety. *Journal of Structures Division, ASCE* 1966; 92: 267-325.
- Gallego-Calderon J, Natarajan A, Dimitrov N K. Effects of Bearing Configuration in wind Turbine Gearbox Reliability. *Energy Procedia* 2015; 80 (Pt2): 392-400, <https://doi.org/10.1016/j.egypro.2015.11.443>.
- Grimmelt M, Schueller GI. Benchmark study on methods to determine collapse failure probabilities of redundant structures. *Structural Safety* 1982;1:93-106.
- Golabi S, Fesharaki J J, Yazdipoor M. Gear train optimization based on minimum volume/weight design. *Mechanism & Machine Theory* 2014; 73(2): 197-217, <https://doi.org/10.1016/j.mechmachtheory.2013.11.002>.
- Hao L. The probabilistic analysis and optimal design of a bevel gear transmission system with failure interaction. *Eksplotacja i Niezawodność - Maintenance and Reliability* 2017; 19(2): 220-228, <http://dx.doi.org/10.17531/ein.2017.2.9>.
- Huang X, Hu S, Zhang Y, et al. A method to determine kinematic accuracy reliability of gear mechanisms with truncated random variables. *Automotive Engineering* 2015; 92: 200-212, <https://doi.org/10.1016/j.mechmachtheory.2015.04.017>.
- Karamchandani A. *Structural system reliability analysis methods*. Report no. 83. Department of Civil Engineering, Stanford University; 1987.
- Kim, D., et al., System reliability analysis using dominant failure modes identified by selective searching technique. *Reliability Engineering & System Safety* 2013; 119: 316-331, <https://doi.org/10.1016/j.res.2013.02.007>
- Li M, Xie L, Ding L. Load sharing analysis and reliability prediction for planetary gear train of helicopter[J]. *Mechanism & Machine Theory*, 2017,115:97-113.<https://doi.org/10.1016/j.mechmachtheory.2017.05.001>
- Lee YJ, Song J. Finite-element-based system reliability analysis of fatigueinduced sequential failures. *Reliability Engineering & System Safety* 2012; 108: 131-41, [https://doi.org/10.1016/0167-4730\(82\)90018-2](https://doi.org/10.1016/0167-4730(82)90018-2).
- Lee JS. Basic study on the reliability analysis of structural systems. *Journal of Ocean Engineering and Technology* 1989; 12: 145-57.
- Li Y F, Valla S, Zio E. Reliability assessment of generic geared wind turbines by GTST-MLD model and Monte Carlo simulation. *Renewable Energy* 2015; 83: 222-233, <https://doi.org/10.1016/j.renene.2015.04.035>.
- Marjanovic N, Isailovic B, Marjanovic V, et al. A practical approach to the optimization of gear trains with spur gears. *Mechanism & Machine Theory* 2012; 53(7): 1-16, <https://doi.org/10.1016/j.mechmachtheory.2012.02.004>.
- Melchers RE. *Structural reliability: analysis and prediction*. 2nd ed New York,NY: John Wiley; 1999. ISBN:0471987719
- Melchers RE. Structural system reliability assessment using directional simulation. *Structural Safety* 1994; 16: 23-37, [https://doi.org/10.1016/0167-4730\(94\)00026-M](https://doi.org/10.1016/0167-4730(94)00026-M).
- Mendi F, Boran F E. Optimization of module, shaft diameter and rolling bearing for spur gear through genetic algorithm. *Expert Systems with Applications An International Journal* 2010; 37(12): 8058-8064, <https://doi.org/10.1016/j.eswa.2010.05.082>.
- Moses F, Stahl B. Reliability analysis format for offshore structures. In: *Proceedings of the 10th annual offshore technology conference*, 1978; Paper 3046. DOI: 10.4043/3046-MS
- Moses F, Fu G. Important sampling in structural system reliability. Fifth ASCE EMD/GTD/STD specialty conference on probabilistic mechanics; 1988, <http://cedb.asce.org/CEDBsearch/record.jsp?dockey=0056556>.
- Moses F. System reliability developments in structural engineering. *Structural Safety* 1982; 1: 3-13, [https://doi.org/10.1016/0167-4730\(82\)90011-X](https://doi.org/10.1016/0167-4730(82)90011-X).
- Murotsu Y, Okada H, Taguchi K, Grimmelt M, Yonezawa M. Automatic generation of stochastically dominant failure modes of frame structures. *Structural Safety* 1984; 2: 17-25, [https://doi.org/10.1016/0167-4730\(84\)90004-3](https://doi.org/10.1016/0167-4730(84)90004-3).

30. Nejad A R, Gao Z, Moan T. On long-term fatigue damage and reliability analysis of gears under wind loads in offshore wind turbine drivetrains. *International Journal of Fatigue* 2014; 61(2): 116-128, <https://doi.org/10.1016/j.ijfatigue.2013.11.023>.
31. Nikolaidis E, Ghiocel DM, Singhal S. *Engineering design reliability handbook*. Boca Raton, FL: CRC Press; 2005, <https://doi.org/10.1201/9780203483930>.
32. Rashedi MR. *Studies on reliability of structural systems*. Department of Civil Engineering, Case Western Reserve University; 1983
33. Savage M, Brikmanis C, Lewicki D G, Coy J J. Life and reliability modeling of bevel gear reductions. *Journal of Mechanisms, Transmissions, and Automation in Design* 1988; 110(2): 189-196, <https://doi.org/10.1115/1.3258925>.
34. Savsani V, Rao R V, Vakharia D P. Optimal weight design of a gear train using particle swarm optimization and simulated annealing algorithms. *Mechanism & Machine Theory* 2010; 45(3): 531-541, <https://doi.org/10.1016/j.mechmachtheory.2009.10.010>.
35. Savage M, Paridon C A, Coy J J. Reliability Model for Planetary Gear Trains. *Journal of Mechanical Design* 1983; 105(3): 291-297, <https://doi.org/10.1115/1.3267359>
36. Shao S, Murotsu Y. Approach to failure mode analysis of large structures. *Probabilistic Engineering Mechanics* 1999; 14: 169-177, [https://doi.org/10.1016/S0266-8920\(98\)00028-9](https://doi.org/10.1016/S0266-8920(98)00028-9).
37. Shetty NK. Selective enumeration method for identification of dominant failure paths of large structures. In: *Proceedings of OMAE conference, vol. II. ASME. Safety and Reliability 1994*: 381-391, <https://www.osti.gov/biblio/55808>.
38. Swantner A, Campbell M I. Topological and parametric optimization of gear trains. *Engineering Optimization* 2012, 44(11): 1351-1368, <https://doi.org/10.1080/0305215X.2011.646264>.
39. Thoft-Christensen P, Baker MJ. *Structural reliability theory and its applications*. Springer-Verlag; 1982, [https://doi.org/10.1007/978-3-642-68697-9\\_11](https://doi.org/10.1007/978-3-642-68697-9_11).
40. Thoft-Christensen P, Murotsu Y. *Application of structural systems reliability theory*. Berlin: Springer-Verlag; 1986, <https://link.springer.com/book/10.1007%2F978-3-642-82764-8>.
41. Thompson D F, Gupta S, Shukla A. Tradeoff analysis in minimum volume design of multi-stage spur gear reduction units. *Mechanism & Machine Theory* 2000; 35(5): 609-627, [https://doi.org/10.1016/S0094-114X\(99\)00036-1](https://doi.org/10.1016/S0094-114X(99)00036-1).
42. Wang H, Wang H P. Optimal engineering design of spur gear sets. *Mechanism & Machine Theory* 1994; 29(7): 1071-1080, [https://doi.org/10.1016/0094114X\(94\)90074-4](https://doi.org/10.1016/0094114X(94)90074-4).
43. Wang Y. Optimized tooth profile based on identified gear dynamic model. *Mechanism & Machine Theory* 2007; 42(8): 1058-1068, <https://doi.org/10.1016/j.mechmachtheory.2006.02.011>
44. Xiao Q, Mahadevan S. Fast failure mode identification for ductile structural system reliability. *Structural Safety* 1994; 13(4): 207-26, [https://doi.org/10.1016/0167-4730\(94\)90029-9](https://doi.org/10.1016/0167-4730(94)90029-9).
45. Xie L, Wu N, Qian W. Time domain series system definition and gear set reliability modeling. *Reliability Engineering & System Safety* 2016; 155: 97-104, <https://doi.org/10.1016/j.res.2016.06.009>.
46. Yang Q J. Fatigue test and reliability design of gears. *International Journal of Fatigue* 1996; 18(3): 171-177, [https://doi.org/10.1016/0142-1123\(95\)00096-8](https://doi.org/10.1016/0142-1123(95)00096-8).
47. Zhang G, Wang G, Li X, et al. Global optimization of reliability design for large ball mill gear transmission based on the Kriging model and genetic algorithm. *Mechanism & Machine Theory* 2013; 69(6): 321-336, <https://doi.org/10.1016/j.mechmachtheory.2013.06.003>.
48. Zhou D, Zhang X, Zhang Y. Dynamic reliability analysis for planetary gear system in shearer mechanisms. *Mechanism & Machine Theory* 2016; 105: 244-259, <https://doi.org/10.1016/j.mechmachtheory.2016.07.007>.
49. Zolfaghari A, Goharimanesh M, Akbari A A. Optimum design of straight bevel gears pair using evolutionary algorithms. *Journal of the Brazilian Society of Mechanical Sciences & Engineering* 2017; 39(6): 1-9, <https://doi.org/10.1007/s40430-017-0733-9>.

## Appendix:

### 1. Calculation of Reliability of Bevel Gear Transmission system

According to the IOS standard, reliability design is to treat the design variables such as load, strength and their influencing factors as random variables, and apply the reliability theory and method to make the designed products meet the expected reliability requirements. In this paper, the reliability design of bevel gear transmission system mainly includes: the reliability of tooth surface contact strength of the pinion and the gear, the reliability of tooth surface contact strength of the pinion and the gear are expressed as  $R_1, R_2, R_3, R_4$ , respectively.

#### 1.1. Reliability calculation of tooth surface contact strength ( $R_1, R_2$ )

In the calculation of tooth surface contact fatigue strength, destructive pitting is the limit state of tooth surface work. The strength condition of the tooth surface without fatigue pitting is that the contact stress of the tooth surface joint is not greater than the contact allowable stress. Its reliability calculation formula is as follows:

$$R = P\{\sigma_H < \sigma_{HP}\} \quad (23)$$

#### 1.1.1. Bevel gear tooth surface contact stress

Computational formula:

$$\sigma_H = Z_M Z_H Z_E Z_{LS} Z_\beta Z_K \sqrt{\frac{K K_A K_V K_{H\beta} K_{H\alpha} F_t}{d_m l_{bm}}} \times \frac{\sqrt{u^2 + 1}}{u} \quad (24)$$

Formula of COV:

$$\sigma_H = Z_M Z_H Z_E Z_{LS} Z_\beta Z_K \sqrt{\frac{K K_A K_V K_{H\beta} K_{H\alpha} F_t}{d_m l_{bm}}} \times \frac{\sqrt{u^2 + 1}}{u} \quad (25)$$

where,  $\bar{Z}_E, \bar{F}_t, \bar{K}, \bar{K}_{H\beta}, \bar{K}_{H\alpha}$  are the mean values of the corresponding parameters.

$$V_{\sigma_H} = [V_{Z_E}^2 + \frac{1}{4}(V_K^2 + V_{F_t}^2 + V_{K_V}^2 + V_{K_{H\beta}}^2 + V_{K_{H\alpha}}^2 + V_{K_V} V_{K_{H\alpha}} + V_{K_V} V_{K_{H\beta}} + V_{K_{H\alpha}} V_{K_{H\beta}})]^{0.5} \quad (26)$$

$$S_{\sigma_H} = \overline{\sigma_H} V_{\sigma_H} \quad (27)$$

$$R = P\{\sigma_F < \sigma_{FP}\} \quad (36)$$

The average tangential force on the bevel gear face is:

$$\bar{F}_t = \frac{2000T}{d_{m1}} \quad (28)$$

### 1.1.2. Contact fatigue strength of bevel gears

Formula of tooth surface contact fatigue strength:

$$\sigma_{HP} = \sigma_{Hlim} Z_{NT} Z_L Z_V Z_R Z_W Z_X \quad (29)$$

Formula of COV:

$$\bar{\sigma}_{HP} = \bar{\sigma}_{Hlim} \bar{Z}_{NT} \bar{Z}_L \bar{Z}_V \bar{Z}_R \bar{Z}_W Z_X \quad (30)$$

$$V_{\sigma_{HP}} = \left( V_{\sigma_{Hlim}}^2 + V_{Z_{NT}}^2 + V_{Z_L}^2 + V_{Z_V}^2 + V_{Z_R}^2 + V_{Z_W}^2 \right)^{0.5} \quad (31)$$

$$S_{\sigma_{HP}} = \bar{\sigma}_{HP} V_{\sigma_{HP}} \quad (32)$$

Assuming that the contact stress limit function is:

$$g_1(\mathbf{X}_1) = \sigma_H - \sigma_{HP} \quad (33)$$

The formula of the contact fatigue strength of tooth surfaces is as follows:

$$Z_R = \frac{\bar{\sigma}_{HP} - \bar{\sigma}_H}{\left( S_{\sigma_{HP}}^2 + S_{\sigma_H}^2 \right)^{0.5}} \quad (34)$$

$$R = \Phi(Z_R) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_R} e^{-\frac{x^2}{2}} dx \quad (35)$$

## 1.2. Reliability calculation of tooth root bending strength ( $R_3, R_4$ )

In the calculation of tooth root bending strength, the fracture of tooth root is regarded as the limit state. The condition that the tooth root does not break is that the bending stress of the tooth root joint is not greater than the bending allowable stress. The reliability calculation method is as follows:

### 1.2.1. Root bending stress

Formula of bending stress of tooth root:

$$\sigma_F = K K_A K_V K_{F\beta} K_{F\alpha} F_t \frac{Y_{FS} Y_\epsilon Y_K Y_{LS}}{b m_{nm}} \quad (37)$$

Formula of COV:

$$\bar{\sigma}_F = \bar{K} K_A \bar{K}_V \bar{K}_{F\beta} \bar{K}_{F\alpha} \bar{F}_t \frac{Y_{FS} Y_\epsilon Y_K Y_{LS}}{b m_{nm}} \quad (38)$$

$$V_{\sigma_F} = \left( V_{\bar{K}}^2 + V_{\bar{F}_t}^2 + V_{\bar{K}_V}^2 + V_{\bar{K}_{F\beta}}^2 + V_{\bar{K}_{F\alpha}}^2 + V_{K_{F\alpha}} V_{K_{F\beta}} + V_{K_{F\alpha}} V_{K_V} + V_{K_{F\beta}} V_{K_V} \right)^{0.5} \quad (39)$$

$$S_{\sigma_F} = \bar{\sigma}_F V_{\sigma_F} \quad (40)$$

### 1.2.2. Tooth root bending fatigue strength

Formula of tooth root bending fatigue strength:

$$\sigma_{FP} = \sigma_{Flim} Y_{NT} Y_{\delta rel} Y_{Rrel} Y_X \quad (41)$$

Formula of COV:

$$\bar{\sigma}_{FP} = \bar{\sigma}_{Flim} \bar{Y}_{NT} \bar{Y}_{\delta rel} \bar{Y}_{Rrel} Y_X \quad (42)$$

$$V_{\sigma_{FP}} = \left( V_{\bar{\sigma}_{Flim}}^2 + V_{\bar{Y}_{NT}}^2 \right)^{0.5} \quad (43)$$

Assuming that the bending stress limit function is:

$$g_2(\mathbf{X}_2) = \sigma_F - \sigma_{FP} \quad (44)$$

The formula for the reliability of tooth root bending fatigue strength is:

$$Z_R = \frac{\bar{\sigma}_{FP} - \bar{\sigma}_F}{\left( S_{\sigma_{HF}}^2 + S_{\sigma_F}^2 \right)^{0.5}} \quad (45)$$

$$R = \Phi(Z_R) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_R} e^{-\frac{x^2}{2}} dx \quad (46)$$

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