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## SINGLE-NUMBER STATISTICAL PARAMETERS IN THE ASSESSMENT OF THE TECHNICAL CONDITION OF MACHINES OPERATING UNDER VARIABLE LOAD

### JEDNOLICZBOWE PARAMETRY STATYSTYCZNE W OCENIE STANU TECHNICZNEGO MASZYN PRACUJĄCYCH PRZY ZMIENNYM OBCIĄŻENIU\*

*Diagnosing machines operating under variable load requires the use of special signal analysis methods. In literature there are many methods based on synchronic analysis where vibration signal is synchronized with shaft rotation of the diagnosed machine. Varying load, however, also affects the amplitude of diagnostic parameters values. The paper deals with the problem of diagnosing machines operating under variable load. The impact of load on the diagnostics parameters values has been tested on a laboratory station. Due to varying rotational speed, the diagnostic parameters were amplitudes of orders determined by means of order analysis synchronized with the shaft rotation in the diagnosed drive unit. Examined was the impact of varying load on the diagnostics effectiveness of damage, such as incorrect motor mount (soft foot) and misalignment of shafts. Single-number statistical parameters were proposed to determine the technical condition of machines operating under variable load.*

**Keywords:** *soft foot, diagnostics, vibroacoustics, order analysis.*

*Diagnozowanie maszyn pracujących przy zmiennym obciążeniu wymaga zastosowania specjalnych metod analizy sygnału. W literaturze znaleźć można wiele metod bazujących na analizie synchronicznej, gdzie sygnał drgań zsynchronizowany jest z obrotami diagnozowanej maszyny. Jednak zmienne obciążenie ma wpływ również na zmianę wartości amplitud parametrów diagnostycznych. Praca podejmuje problem diagnozowania maszyn pracujących przy zmiennym obciążeniu. Przeprowadzono badania na stanowisku laboratoryjnym wpływu obciążenia na wartości parametrów diagnostycznych. Ze względu na zmienną prędkość obrotową jako parametry diagnostyczne przyjęto wartości amplitud rzędów, wyznaczonych za pomocą analizy rzędów zsynchronizowanej z obrotami wału diagnozowanego układu napędowego. Zbadano wpływ zmiennego obciążenia na skuteczność diagnozowania uszkodzeń takich jak nieprawidłowe posadowienie silnika (soft foot) oraz niewspółosiowość wałów. Zaproponowano jednoliczbowe parametry statystyczne do określania stanu technicznego maszyn pracujących przy zmiennym obciążeniu.*

**Słowa kluczowe:** *nieprawidłowe posadowienie maszyn, diagnostyka, wibroakustyka, analiza rzędów.*

#### 1. Introduction

In order to ensure the reliability of machines operating in a continuous mode, where each downtime generates high costs, it is necessary to use continuous monitoring systems. One of the methods of diagnosing machines is the assessment of the technical condition based on vibration measurements. Vibroacoustic diagnostics often deals with machines operating at varying conditions which hinders the diagnostics with standard methods as the values of determined estimates change together with the load. In such cases, the spectral methods require an analysis of frequency bands, which in addition to information on the parameter being diagnosed, may contain other components. Using the order analysis method allows synchronizing the vibroacoustic signal with rotational speed, which solves this problem and facilitates tracing component amplitudes responsible for a given machine component. Varying operating conditions affect also the change of characteristic component amplitudes which is the subject of this paper.

Presently, the literature describes an increasing number of methods of evaluating the technical conditions of machines; a wide review of current diagnostic methods of planetary gears can be found in [8]. For machines operating at varying rotational speed, it is recommended to apply the method of non-stationary signals analysis. The methods of analysis of non-stationary signals generated by rotor machines were known as early as in 1980s; the methods of their implementation can be found in [2, 15]. The methods involve a synchronous sampling

of measurement signals where varying sampling frequency depends on the rotational speed signal of the machine being diagnosed. Other diagnostic methods that are being developed presently include synchronization of the diagnostic signal with the rotational speed using the decimation method [3], subsampling method [5, 9, 11] or the Gabor transform [12, 16].

In most cases, the change of rotational speed is caused by the load change, hence along with the change of characteristic frequencies their amplitudes change as well. Therefore, in addition to synchronizing the signals and the rotational speed, it is also necessary to determine the relationship between the amplitudes of diagnostic parameters and the load. In [13], the authors determined the relationships of orders amplitudes and the rotational speed for various load levels which significantly improved the diagnostics effectiveness, and in [6, 14] the artificial intelligence methods were used for diagnostics of machines running at varying load. There is also an application of complex Bayesian inference [17] to detect the damage type for various variants of load and rotational speed [7]. Separation of components related to variable speed and load can be found in the literature [18].

The paper presents the analysis of impact of the diagnosed drive system on the values of diagnostic parameters determined for vibration acceleration signals. A diagnostic experiment was conducted for a machine running under varying, set operation conditions. The amplitudes of characteristic orders and their changes depending on the set load for various machine states were analysed. The method for diagnosing machines working under variable load based on order

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analysis and statistical parameters, which was verified at the laboratory testing rig, has been proposed. The presented method can also be applied to other objects in a similar way.

Chapter two presents the experimental rig and describes the diagnostic experiment. The next chapter includes a description of used signal analysis methods and the method of determination of diagnostic parameters. Chapter four includes the results of analyses with the use of order spectrum and relationship between the characteristic order amplitude and the load.

## 2. Laboratory testing rig and diagnostic experiment

The laboratory testing rig (Fig. 1) comprises a planetary gear, drive motor and a braking motor. The power supply frequency was set by frequency converters controlled by the measurement card. The use of measurement card with analogue outputs supported by an application built in the LabVIEW environment allows setting the rotational speed and any function of gear load torque.

A PCB type 356B08 triaxial acceleration sensor was used to diagnose the gear. An LM35 temperature sensor was used to control the operating conditions. All measurements were made for the same gear temperature of 40°C. A DT-2234C+ tachometer with analogue output was used to measure the rotational speed.

The data were recorded by an NI PXIe-8133 controller placed in an NI PXIe-1062Q chassis, along with measurement cards: NI PXI-4472B – vibration acceleration and rotational speed measurement, NI PXIe-6361 – temperature measurement.

Placement of measuring sensors and places where damage was simulated are shown in (Fig. 2).

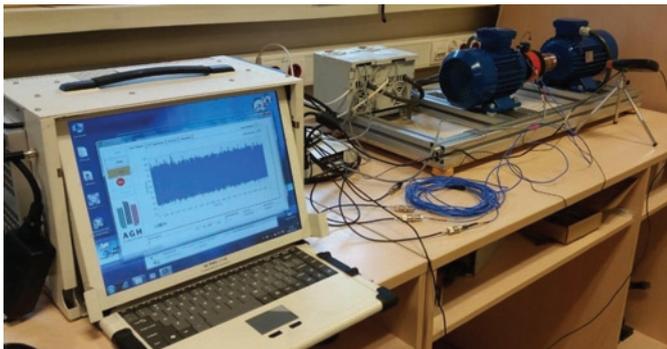


Fig. 1. Testing rig view

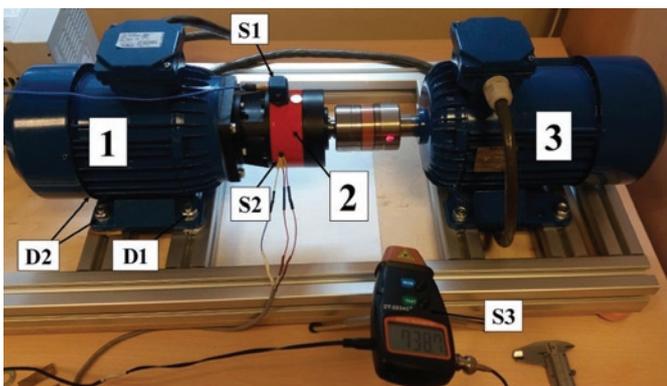


Fig. 2. Testing rig and placement of measuring sensors, 1) drive motor; 2) planetary gear; 3) braking motor; S1) acceleration sensor; S2) temperature sensor; S3) tachometer; D1) foot in which the "soft foot" damage was introduced, D2) feet that were shimmed to misalign the system

Two system damage types were simulated. The first was an incorrect motor mount, in diagnostics called the soft foot. This was done

by removing the nut (Fig. 2 - D1) on the drive motor foot. The other damage was system misalignment of varying degree, effected by shimming the drive motor rear feet (Fig. 2 - D2); the shims used were 0.5 mm, 1 mm, and 1.5 mm thick.

In order to simulate the impact of operating conditions on the tested system, a variable sine load was introduced, with maximum value of 3.9 Nm and minimum value of 1.8 Nm.

## 3. Signal analysis method

The vibration on the gear body and the rpm was measured using a tachometer with analogue output. The tachometer signal was used for the synchronous analysis, discussed further in the paper.

The signal analysis was performed using the order analysis method, one of the results of which is the order spectrum determined using the resampling of the vibration time signal in relation to the input shaft rpm. The order analysis algorithm diagram is presented in Fig. 3. In the first phase, the signal is interpolated using a cascaded integrator-comb (CIC). Then, the vibration signal resampling is performed based on the filtered signal from the tachometer in order to determine the vibration signal in relation to the angle of rotation (Even Angle Signal). Such resampled signal can be Fast Fourier Transformed (FFT). Following the transform, instead of frequency, shaft order numbers are obtained which correspond to multiples of the rotational frequency of the input shaft [10]. In this case, the measurement was performed on the output shaft, so the numbers of orders correspond to the multiples of the output shaft frequencies.

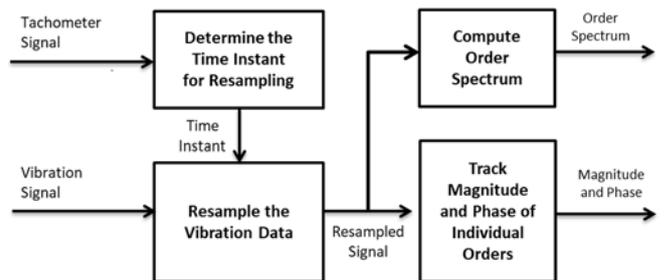


Fig. 3. Order analysis algorithm diagram [10]

Application of the order analysis simplifies significantly the machine diagnostics because instead of watching the whole spectrum it suffices to watch a single parameter which is an amplitude of a given order, which order is synchronized with machine's revolutions. Fig. 4 presents an amplitude-frequency spectrum for the acceleration signal during the gear operation at varying load. The frequency of the output shaft rotation varied from 48 Hz to 50 Hz, which can be observed in the occurrence of many peaks in this band and subsequent ones corresponding to the next harmonic. Fig. 5, on the other hand, shows the order spectrum on which single lines are clearly visible.

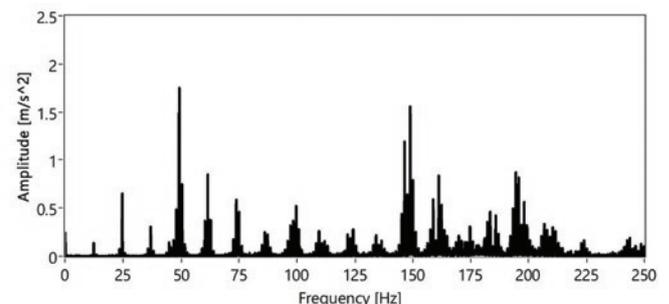


Fig. 4. Amplitude-frequency spectrum for the vibration acceleration signal

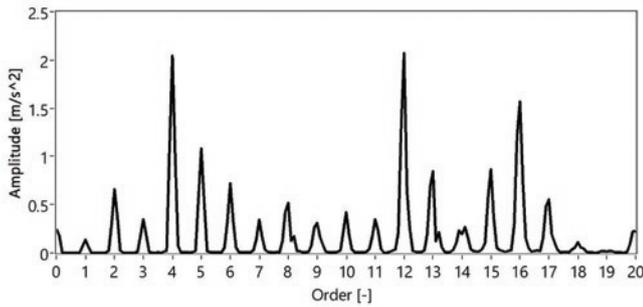


Fig. 5. Order spectrum of vibration acceleration

By monitoring the amplitudes of characteristic orders one can obtain information about the technical condition of an examined facility. However, the change of amplitudes' values can be also caused by the system load variation [13]. Therefore, the values of characteristic orders should be monitored as a function of load.

In the analysed case, the changes of rotational speed are inversely proportional to the load torque. Because the driving motor is supplied with the voltage of constant frequency, the load variations are set by reducing the voltage frequency supplied to the breaking motor.

Analysing the amplitude of order no. 4 (direction along the shaft) (Fig. 6) as a function of rotational speed, one can see that it increases for the lowest speed (727 rpm) which corresponds to the greatest load of 3.9 Nm. When the load increases, the amplitude of order no. 4 grows.

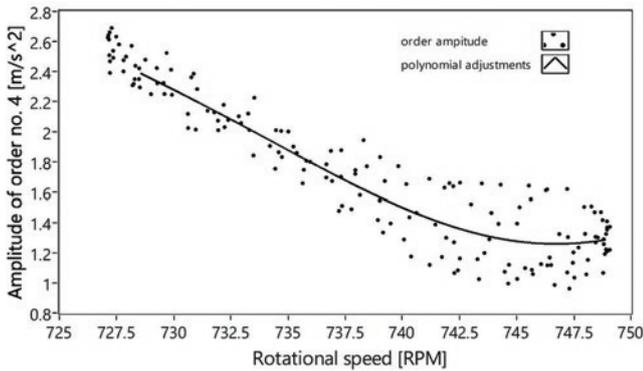


Fig. 6. Amplitude of order no. 4 (direction along the shaft) as a function of rotational speed which varies with the load – no damage in the system

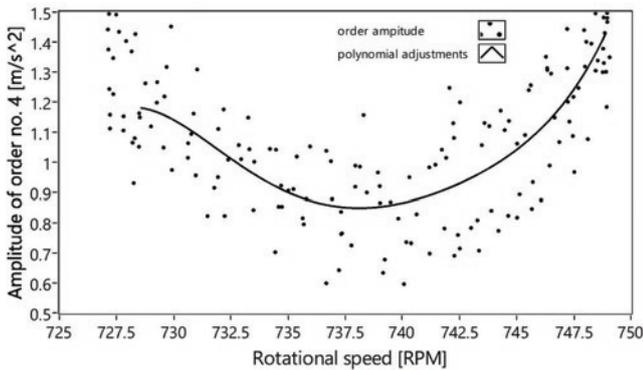


Fig. 7. Amplitude of order no. 4 (direction transversal to the shaft) as a function of rotational speed which varies with the load – no damage in the system

The situation for the transversal direction (Fig. 7) is however different, because when the load increases the amplitude initially drops and then grows. Hence, it cannot be assumed that

the vibration amplitude grows when the load increases, as one could expect.

In presented cases for the good condition (no damage) at varying load, the amplitude value changes even twofold. So, it is necessary to account for the load-induced amplitude changes, because in some cases the amplitude increase could be misinterpreted in the monitoring systems.

In order to use the presented dependencies in the diagnosis method an algorithm for designation of functional dependency between rotational speed and characteristic orders has been developed. This dependency is designated based on measurement data recorded during correct machine operation. The condition for correct operation is synchronous measurement of vibrations and rotational speed. The values of the given characteristic order and the corresponding rotational speed values are recorded.

In step one of the algorithm the pairs measured at the given moment (rotational speed value and order amplitude) are structured in relation to rotational speed – in ascending order. In the next step average values for rotational speed and order amplitude are designated based on  $N$  adjacent samples, in accordance with the relationships:

$$\bar{s}_i = \frac{1}{N} \sum_{n=iN}^{iN+N} s_n \quad (1)$$

$$\bar{A}(r)_i = \frac{1}{N} \sum_{n=iN}^{iN+N} A(r)_n \quad (2)$$

where:

- $s_n$  – rotational speed temporary value [rpm];
- $\bar{s}_i$  –  $i$ -th rotational speed average value for  $N$  consecutive samples [rpm];
- $A(r)_n$  – amplitude temporary value of  $r$ -th order [ $m/s^2$ ];
- $\bar{A}(r)_i$  –  $i$ -th average amplitude value of  $r$ -th order for  $N$  consecutive samples [ $m/s^2$ ].

Then adjustment by a  $n$ -order polynomial is designated. Using this operation we obtain the rotational speed dependency on the order amplitude in the form of a function. The scheme of the described algorithm is presented in Fig. 8.

The polynomial equation determined for the correct machine operation state can serve as a pattern. For the purpose of the article, a parameter was introduced which defines the deviation of the curve determined during the machine operation from the reference curve. To determine the differences between the curves, the Root Mean Square Deviation (RMSD) parameter was used [1]:

$$RMSD(r) = \sqrt{\frac{1}{N} \sum_{s=1}^N (A(r,s) - A_G(r,s))^2} \quad (3)$$

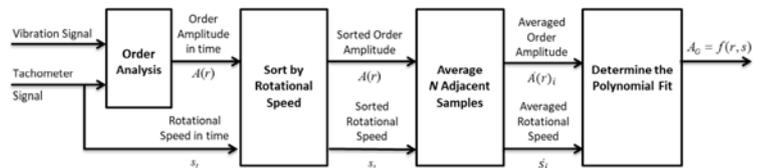


Fig. 8. The scheme of algorithm for functional dependency designation between rotational speed and characteristic orders.

- $A_G(r,s)$  – reference amplitude for the current rotational speed value ( $s$ ) and  $r$ -th order, determined from the functional dependence [m/s<sup>2</sup>];
- $A(r,s)$  – actual amplitude for the current rotational speed value ( $s$ ) and  $r$ -th order, determined from the functional dependence [m/s<sup>2</sup>].

The second proposed parameter is the maximum difference for the whole rotational speed range:

$$\Delta A(r)_{max} = |A(r,s) - A_G(r,s)|_{max} \quad (4)$$

In order to make the above parameters independent from the vibration values, normalized measures were introduced, where the values of differences between the current and the reference amplitude were divided by the value of the reference amplitude:

$$rRMSD(r) = \sqrt{\frac{1}{N} \sum_{s=1}^N \left( \frac{A(r,s) - A_G(r,s)}{A_G(r,s)} \right)^2} \quad (5)$$

$$r\Delta A(r)_{max} = \left| \frac{A(r,s) - A_G(r,s)}{A_G(r,s)} \right|_{max} \quad (6)$$

#### 4. Signal analysis results

Two types of damage were simulated in the laboratory test rig: incorrect motor mount (soft foot) and misalignment of the gear output shaft and the system loading motor.

##### 4.1. Incorrect motor mount

Incorrect motor mount, that is in this case removal of one mounting nut, can be diagnosed by monitoring the amplitude of the first and second rotational speed harmonic [4]. In the analysed case, the rotational speed was measured on the gear output, and the drive motor damage was introduced. Hence, taking into account the gear ratio equals 4, this type of damage will be represented by amplitudes of order no. 4 and no. 8.

The orders spectrum analysis for correct operation (no damage) and for incorrect mounting does not indicate significant differences (Fig. 9). If we analysed the average amplitude value for order no. 4 (which often happens in the monitoring systems), the damage would not be detected, because for the damage and no-damage state the average amplitude is 0.8 m/s<sup>2</sup>. The amplitude value for order no. 8 did not change significantly.

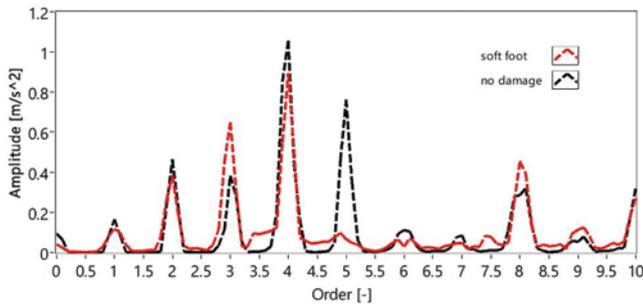


Fig. 9. Orders spectrum for no-damage and for soft foot (direction transversal to the shaft)

Situation is different when we start to analyse the amplitudes' values at shorter time intervals, and refer these values to the rota-

tional speed. The change of the rotational speed at a given constant frequency of the drive motor voltage represents the change in load. Fig. 10 shows the amplitude of order no. 4 as a function of rotational speed which varied depending on the load, for the system without damage (black) and for the system with a soft foot (red). Significant differences in the order amplitude shape can be noticed.

Table 1 lists the statistical parameters determined for the drive system with the soft foot of the drive motor. The value of the  $rRMSD$  parameter is 0.65, which means that the curve deviates significantly from the reference curve. The maximum distance between the curves is 2.01 m/s<sup>2</sup>, which is 107% of the reference value.

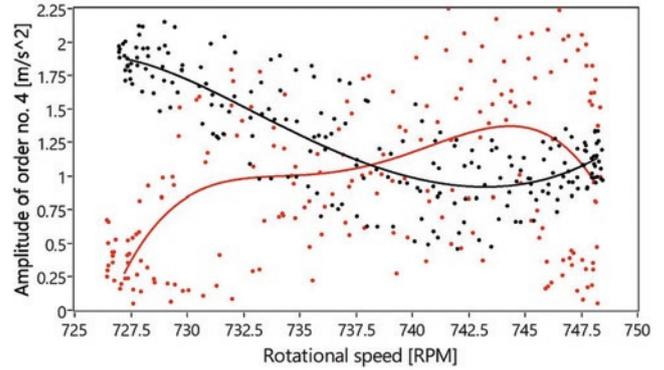


Fig. 10. Amplitude of order no. 4 as a function of rotational speed which varied depending on the load, for the system without damage (black) and for the system with a soft foot (red) - direction transversal to the shaft

A similar situation occurs for the order no. 8 presented in Fig. 11. For some rotational speeds, these values change more than doubled, which can be observed by analysing the values of statistical parameters placed in table 2.

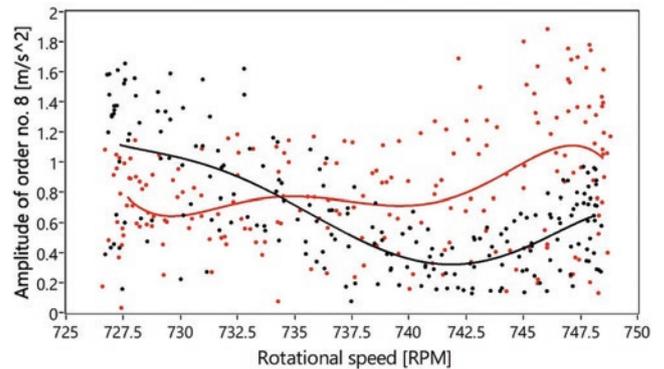


Fig. 11. Amplitude of order no. 8 as a function of rotational speed which varied depending on the load, for the system without damage (black) and for the system with a soft foot (grey) - direction transversal to the shaft

Table 1. Statistical parameters based on amplitude of order no. 4

	$RMSD$ [m/s <sup>2</sup> ]	$\Delta A_{max}$ [m/s <sup>2</sup> ]	$rRMSD$ [-]	$r\Delta A_{max}$ [-]
Soft foot	0.43	2.01	0.65	1.07

Table 2. Statistical parameters based on amplitude of order no. 8

	$RMSD$ [m/s <sup>2</sup> ]	$\Delta A_{max}$ [m/s <sup>2</sup> ]	$rRMSD$ [-]	$r\Delta A_{max}$ [-]
Soft foot	0.82	0.58	0.36	1.54

## 4.2. Shafts misalignment

The state of shafts misalignment manifests itself by a change of amplitude for the second harmonic, and if a coupling is used one can observe an increase of harmonic amplitude resulting from the number of coupling claws [4]. The coupling in this case had 4 claws. Due to the fact that misalignment occurs on the gear output shaft on which the rotational speed was measured, the order no. 4 should be analysed. The relationship between the order amplitude and the rotational speed for different misalignment is shown in Fig. 12.

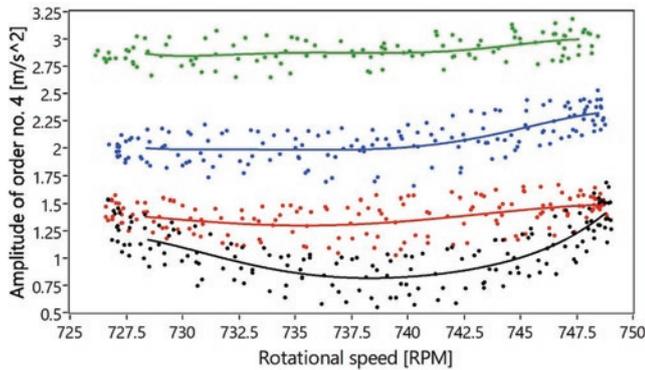


Fig. 12. Amplitude of order no. 4 as a function of rotational speed which varied depending on the load, for the system without damage (black); for the misaligned system - shim 0.5 mm (red); for the misaligned system - shim 1 mm (blue), for the misaligned system - shim 1.5 mm (green)

For the system without damage and system misaligned with a 0.5 mm shim, the differences in amplitude occur in the middle band of rotational speed. If the system operated under full load all the time (727.5 rpm), the amplitude monitoring would not detect the damage. It would not detect for the system without load (750 rpm), either. The differences are significant for other misalignment degrees, and the amplitude remains at a constant level for various loads. Table 3 presents a list of statistical parameters for different misalignment. A clear increase in all determined parameters can be observed. It should be noted that already for the smallest misalignment (0.5 mm) the relative parameters  $rRMSD$  and  $r\Delta A_{max}$  are significant, which allows the use of these parameters in continuous monitoring systems.

Table 3. Statistical parameters based on amplitude of order no. 4

	$RMSD$ [ $m/s^2$ ]	$\Delta A_{max}$ [ $m/s^2$ ]	$rRMSD$ [-]	$r\Delta A_{max}$ [-]
Misalignment - 0.5 [mm]	0.46	0.51	0.41	0.62
Misalignment - 1 [mm]	1.19	1.19	1.09	1.44
Misalignment - 1.5 [mm]	2.10	2.06	1.93	2.52

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## 5. Summary

The paper is an attempt to assess the technical condition of machines running at varying conditions. Particular attention has been paid to relationship between diagnostic parameters and the load of the examined facility. The function describing the changes of amplitudes of characteristic orders depending on the load is different for individual orders in the examined measurement point.

An active diagnostic experiment was conducted which showed that omitting the impact of load can affect the final diagnostics results. Two types of damage were analysed: incorrect engine mount and shafts misalignment. A significant impact of load on the analysed diagnostic parameters was proved in both types of damage. In the analysed case, the impact on orders amplitudes was observed. Accounting for changes of parameters depending on the system load can be crucial in initial stages of damage or in case of damage generating vibration of relatively small amplitudes.

The paper is based on vibration amplitudes of orders determined using order analysis, which allows a precise monitoring of harmonic components of a vibroacoustic signals of machines running at varying rotational speeds.

The author proposes the use of statistical parameters  $RMSD$ ,  $\Delta A_{max}$  and relative parameters  $rRMSD$ ,  $r\Delta A_{max}$ , which are a measure of the distance of the curve determined for the diagnosed machine from the reference curve (determined during the correct operation of the machine). The tests carried out at the laboratory testing rig revealed that changes in these parameters carry information about the technical condition of the machine operating under variable load.

The method can be adapted to another examined object by determining the functional relations between the rotational speed and the amplitude of the vibration acceleration characteristic orders of the state without damage. Then, treat the designated dependencies as a reference in diagnosing a given object.

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