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A MOMENT-MATCHING BASED METHOD FOR THE ANALYSIS OF MANIPULATOR'S REPEATABILITY OF POSITIONING WITH ARBITRARILY DISTRIBUTED JOINT CLEARANCES

OPARTA NA DOPASOWYWANIU MOMENTÓW METODA ANALIZY POWTARZALNOŚCI POZYCJONOWANIA MANIPULATORA O DOWOLNYM ROZKŁADZIE LUZÓW NA PRZEGUBACH

The joint clearance can be the mainly concern factor in the analysis of repeatability of positioning for a manipulator. Traditionally, the joint clearance is empirically assumed to be uniform or normal variables. This hasty treatment may be not accurate enough when the precise statistic information of variables cannot be obtained. To handle the reliability evaluation problem with arbitrarily distributed joint clearances, a moment-matching based method is proposed. The highly nonlinear performance function is firstly established by the forward kinematics and then a second order Taylor expansion is performed on this function for the order reduction. Based on the maximum entropy principle, the Lagrange multipliers method is employed to derive a best-fit probability density function (PDF) with consideration of the first four moments-matching restrictions. This study shows that the prosed method can acquire a better accuracy and efficiency compared with the first order second moment method (FOSM), first order reliability method (FORM) and Monte Carlo simulation (MCS). A serial manipulator is applied as an example to demonstrate the new method.

***Keywords**: manipulator, positioning repeatability, arbitrarily distributed clearance, moment-matching, Lagrange multipliers method.*

Luzy na przegubie manipulatora mogą stanowić główny czynnik wpływający na analizę powtarzalności pozycjonowania manipulatora. Tradycyjnie przyjmuje się empirycznie podbudowane założenie, że luz na przegubie jest zmienną jednorodną lub normalną. Takie ujęcie może jednak nie być wystarczająco dokładne w przypadku, gdy nie można uzyskać precyzyjnych informacji statystycznych na temat zmiennych. Aby rozwiązać problem oceny niezawodności przy dowolnie rozłożonych luzach na przegubie, zaproponowano metodę opartą na dopasowywaniu momentów. W pierwszej kolejności, obliczono za pomocą kinematyki prostej, wysoce nieliniową funkcję stanu granicznego, a następnie wyznaczono szereg Taylora drugiego rzędu dla tej funkcji w celu obniżenia rzędu. Opierając się na zasadzie maksymalnej entropii, zastosowano metodę mnożników Lagrange'a w celu wyprowadzenia najlepiej dopasowanej funkcji gęstości prawdopodobieństwa (PDF) z uwzględnieniem pierwszych czterech ograniczeń dopasowania momentów. Badanie to pokazuje, że przedstawiona metoda pozwala uzyskać wyższą trafność i skuteczność niż metoda pierwszego rzędu drugiego momentu (FOSM), metoda analizy niezawodności pierwszego rzędu (FORM) czy symulacja Monte Carlo (MCS). Zastosowanie nowej metody zilustrowano na przykładzie manipulatora szeregowego.

***Słowa kluczowe**: manipulator, powtarzalność pozycjonowania, arbitralnie rozłożone luzy, dopasowanie momentów, metoda mnożników Lagrange'a.*

1. Introduction

Manipulators have been widely used in various types of repetitive work, because they not only ameliorate the quality of products, but also improve the efficiency significantly [1, 17, 19, 22].A typical manipulator usually consists of a series of links connected by joints, the clearance and deviation in dimension of links cannot be eliminated completely due to manufacturing and assembly errors. The instability in the manipulator's behavior can be bring about by those variations, and finally result in the actual position of the end-effector deviating from its desired position. Some other defects include vibrations, aperiodic shocks and noises [2, 9, 21]. As it well known, the analysis of the positioning repeatability is one of the two topics in the kinematic reliability and it means the probability that the manipulator falls inside a permissible region with reference to a particular point. The other issue is the interval reliability which considers the reliability over the entire path [18]. In recent decades, a lot of methods have been proposed for the interval reliability analysis [7, 16, 24], but the analysis of positioning accuracy has not drawn enough attention, especially when the manipulator has an arbitrary distribution clearance.

A probabilistic approach was presented by Rao and Bhatti[18] to study the impact that join clearances had on the kinematic and dynamic performance of a two-link manipulator. In their work, all the parameters are treated as independent random variables following Gaussian distributions. Given a specified distribution of the joint clearance, Zhu and Ting [26] put forward a probability distribution function (PDF)-based method to analyze a planar robot. With integration of the first order Taylor expansion and bivariate dimension reduction for dependent joint clearance variables, a hybrid dimension reduction method was proposed by Wang et al. [20] to evaluate the point reliability of a slider crank mechanism. The drawbacks of this method lied in that the joint clearance was also uniform variable and the additional error caused by dimension reduction was unavoidable. Kim et al. [12] firstly supposed that all the parameters of an open-loop mechanism meet the norm distribution and then the first-order reliability method (FORM) was used to calculate the reliability in a particular configuration. Luo and Du [14] constructed a probabilistic model of a planar mechanism with consideration of truncated random variables, though the positive or negative infinity of random variables was abandoned, this method was limited in the empirical assumption of normality of clearance. To maximize the tolerance as well as minimize the operation error of a manipulator, Choi and Yoo [4] proposed a single Monte Carlo simulation (MCS) to perform the reliability analysis. Time-consuming made this kind approach less attractive. By treating input errors and joint clearances as a mixture of random interval variables, Zhan et al. [23] developed an analytical method with combination the first order second moment method (FOSM) and MCS to compute the reliability of a parallel manipulator. Later on, because of the excellent learning ability of the artificial neural networks (ANN), many ANN-based methods were put forward to estimate the reliability for mechanisms[3, 6, 10]. This type of approach is suitable for a system of which the performance function is not available as an explicit function. To ensure the estimation precision, a mass of training samples are usually required.

Most of the current methods for the analysis of repeatability of positioning are developed with a foundation that all the uncertain parameters are supposed to be random variables with typical distributions. In practice, it is difficult to obtain the exact distribution due to the lack of the precise statistic information [25], especially the joint clearance. Taken together, there are two of limitations in conventional approaches.

(1) *Far-fetched assumption*. Unlike the deviations in dimension of links, which could be regarded as normally distributed, the joint clearance can take place during the rotation of the journal in the bearing and usually has a greater influence, the random nature for the motion of the journal in two dimensions makes the joint clearance rarely follow a normal or a uniform distribution [20, 25]. The simple assumption may be inappropriate.

(2) *Unbalance in accuracy and efficiency*. The popular methods, such as MCS, FORM and FOSM, can hardly have great efficiency as well as good accuracy when arbitrarily distributed clearance exists. MCS can acquire an accurate solution, but numerous samples make it rather time costly [23]. FOSM is efficient but its accuracy deteriorates dramatically when the system output no-longer meets a normal distribution [16, 20]. FORM exhibits a better accuracy but is still inadequate for a large system since an iterative process is required to search for the most probable point [8, 12].

Therefore, a novel method developed for the analysis of the manipulators' positioning accuracy with unknown distribution clearance is the key objective of this study, in which the available information only includes the first four moments. With combination of the second order Taylor expansion and the Lagrange multipliers method, a best-fit probability distribution is firstly derived, in which all the characteristics of the output deviation can be featured. Here, this new method can be called the second order fourth moment method (SOFM). The advantages of the proposed method include three folds: 1) a wider range of application can be achieved, since the joint clearance can be arbitrarily distributed, 2) a greater efficiency can be realized because of the use of less samples. And 3) a better accuracy of the analysis can be acquired as the more statistic information (the first four moments instead of only the mean and the variance) is utilized to search for a proper distribution of the system output.

The remainder of this paper has been organized as follows. Section 2 established the probabilistic model of a manipulator with uncertain parameters. Section 3 briefly reviewed conventional methods and presented all the details of the proposed method. The simulation is carried out in section 4 followed by the discussion in section 5. Section 6 summarized the conclusions.

2. Probabilistic modeling of the manipulator

2.1. Forward kinematics with uncertain parameters

A typical manipulator used in industry is often structured with open-loop linkages, as shown in Fig.1 (a). The way to describe the relative movement between two links is to construct the D-H coordinate, as shown in Fig.1 (b). The transformation between two coordinates can be represented by a homogenous matrix, written as [5]:

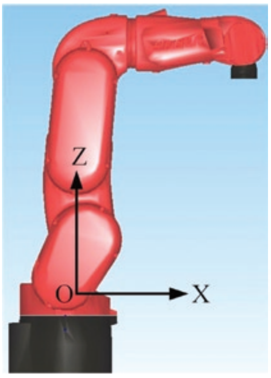
$$T_i^{i-1}=\begin{bmatrix} Cos\theta_i & -Sin\theta_i Cos\alpha_i & Sin\theta_i Sin\alpha_i & a_i Cos\theta_i \\ Sin\theta_i & Cos\theta_i Cos\alpha_i & -Cos\theta_i Sin\alpha_i & a_i Sin\theta_i \\ 0 & Sin\alpha_i & Cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where Sin and Cos are sine and cosine function respectively, $(a_i, d_i, \alpha_i, \theta_i)$ denote the D-H parameters at the ith link.

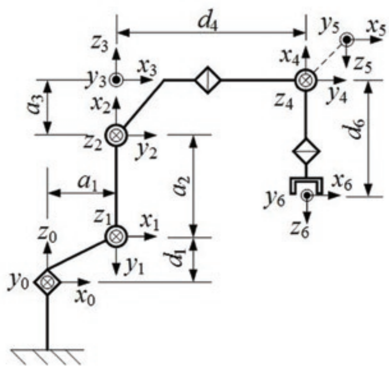
The final position and orientation of the end-effector with reference to the base frame can be determined by multiplying all the D-H matrix, formulated as:

$$T_n^0=\prod_{i=1}^{n} T_i^{i-1}=\begin{bmatrix} \boldsymbol{Rot} & \boldsymbol{Pos} \\ 0 & 1 \end{bmatrix} \quad (2)$$

where n is the number of degree of freedoms (DOF), $\boldsymbol{Rot}$ is the orientation matrix and $\boldsymbol{Pos}=\left[P_x,P_y,P_z\right]$ is the vector of position.



(a) 3-D model



(b) D-H coordinate

Fig. 1. A schematic diagram of a serial manipulator

The forward kinematics is analyzed based on the 6-DOFs manipulator, the corresponding D-H parameters are listed in Table 1.

The deviation in dimension of link and joint clearance have different influence on the performance of a manipulator. The error caused by link dimension deviations can hold constant even can be eliminated by calibration methods [15]. Whereas the latter, joint clearance, can exhibit random nature. It has been approved that clearance contributes most to the position error of the end-effector [13, 26]. In this research, geometrical variables are regarded as normally distributed, the mean and the variance are listed in Table 2. The dis-

Table 1. D-H parameters

Joint number	$a_i\,(mm)$	$d_i\,(mm)$	$\alpha_i\,(°)$	$\theta_i\,(°)$	Initial angle(°)
1	40	330	-90	θ_1	0
2	315	0	0	θ_2	-90
3	70	0	-90	θ_3	0
4	0	310	90	θ_4	0
5	0	0	-90	θ_5	90
6	0	70	0	θ_6	0

Table 2. Distributions of geometric parameters

Variables	Distribution	Mean (mm)	Standard deviation(mm)
a_1	normal	40	0.04
a_2	normal	315	0.32
a_3	normal	70	0.07
d_1	normal	330	0.33
d_4	normal	310	0.31
d_6	normal	70	0.07

tribution of joint clearance is unknown and just featured with the first four moments.

2.2. Repeatability of positioning

Due to the effect of joint clearance and link dimension deviations, the actual position of the end-effector and desired position may not overlap. This deviation is defined as the position error ε，thus [12]:

$$\varepsilon(\mathbf{X}) = \sqrt{(x_d - p_x)^2 + (y_d - p_y)^2 + (z_d - p_z)^2} \tag{3}$$

Where (x_d, y_d, z_d) denote the desired position, (p_x, p_y, p_z) represent the actual position and $\boldsymbol{X}$ is a random vector consists of dimension and join clearance variables.

The unaccepted performance of a manipulator means the end-effector falls outside a permissible region, suppose the size of such a safe area is δ, the performance function can be expressed as:

$$Z = g(\boldsymbol{X}) = \delta - \square \tag{4}$$

Therefore, the probability of failure of the manipulator can be defined as:

$$P_f = \Pr[Z \le 0] = \int_{g(\boldsymbol{X}) \le 0} f(z)dz \tag{5}$$

where $f(z)$ is the probability distribution function (PDF) of the variable Z.

3. Moment-matching based method

3.1. Previous study

Three widely used methods, FOSM, FORM and MCS are briefly reviewed to emphasize their different accuracy and efficiency. Given a set of variables $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$ with the mean values $\boldsymbol{\mu}_{\mathbf{X}} = (\mu_{X1}, \mu_{X1}, \cdots, \mu_{Xn})$.

FOSM can provide a convenient way to evaluate the reliability for a system with the normally distributed output. This method is efficient as it directly linearizes the performance function $Z = g(\boldsymbol{X})$ with first order Taylor expansion at the means of variables $\boldsymbol{\mu}_{\mathbf{X}}$ [14].

$$Z = g(\boldsymbol{X}) \approx g(\boldsymbol{\mu}_{\mathbf{X}}) + \sum_{i=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_{\boldsymbol{X}=\boldsymbol{\mu}_{\mathbf{X}}} (x_i - \mu_{Xi}) \tag{6}$$

where n is the total number of variables.

Then the mean μ_Z and the standard deviation σ_Z of the performance function are approximated based on the above equation. The probability of failure defined in Eq. (5) can be computed by:

$$P_f = 1 - \varnothing\left(\mu_Z / \sigma_Z\right) \tag{7}$$

where $\varnothing(\cdot)$ denotes the standard normal cumulative distribution function.

FORM is commonly applied in structural engineering, in which random variables $\boldsymbol{X}$ need to be transformed into standard normal variables $\boldsymbol{U}$, the reliability can be converted into an optimal problem[12,16], formulated as:

$$\begin{cases} \text{minimize} & \beta = \sqrt{\left(\mathbf{U}^T\mathbf{U}\right)} \\ \text{satisfy} & g(\mathbf{X}) = 0 \end{cases} \tag{8}$$

where β is the reliability index.

After an iterative process for a search of the most probable point (MPP), the probability of failure in terms of the reliability index can be obtained, given as:

$$P_f = 1 - \varnothing(\beta) \tag{9}$$

MCS is used as a straightforward method to analyze various types of mechanism. Especially for systems with complicated or implicit performance functions. The key of MCS to evaluate the reliability is formulated as [4]:

$$P_f = {N_f}/{N} \tag{10}$$

where N is the size of simulation samples and N_f is the number of failed performance.

3.2. Second order fourth moment method (SOFM)

In this research, it can be found that both FOSM and FORM exhibit unsatisfied accuracy when confronted with a sophisticated manipulator system with the arbitrarily distributed joint clearance. A new method to ameliorate the accuracy with maintenance of a high efficiency is therefore put forward.

3.2.1. Moment estimation of the system output *Z*

With Taylor series expansion technique, the mean values $\boldsymbol{\mu}_{\mathbf{X}} = \left(\mu_{X1}, \mu_{X1}, \cdots, \mu_{Xn}\right)$ is utilized as the expansion point for order-reduction of the performance function. The approximated performance function can be written as:

$$Z = g(\boldsymbol{X}) \approx g(\boldsymbol{\mu_X}) + \sum_{i=1}^{n} g_i\left(x_i - \mu_{Xi}\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}\left(x_i - \mu_{Xi}\right)\left(x_j - \mu_{Xj}\right) \tag{11}$$

where g_i and g_{ij} are the first-order and second-order partial derivatives of the performance function respectively.

The advantages of using the second order Taylor expansion for SOFM includes two folds :(1) it is generally more accurate than using only the first order components, like FOSM; and (2) it requires no iterative process for a search of the most probable point, like FORM.

According to the statistic theory, the first four moments of a variable x_i can be defined as:

$$\upsilon_{x_i k} = E\left[\left(x_i - \mu_{Xi}\right)^k\right] = \int \left(x_i - \mu_{Xi}\right)^k f\left(x_i\right) dx_i \quad for\left(k = 0, 1, \ldots, 4\right) \tag{12}$$

where k denotes the order and $f\left(x_i\right)$ is the probability density function (PDF) of x_i.

According to the Eq. (12), the first four moments of x_i can be denoted by $\left[1, 0,\ \upsilon_{x_i 2},\ \upsilon_{x_i 3},\ \upsilon_{x_i 4}\right]$. In addition, the third moment of a variable represents the extent of asymmetry of the PDF about its mean which can be measured by the skewness value. Meanwhile the fourth moment describes the tailedness of the PDF which can be measured by the kurtosis value [11]. The relationship between the variance and the third and fourth moment is formulated as:

$$\begin{cases} \upsilon_{x_i 3} = C_S \sigma_{x_i}^3 \\ \upsilon_{x_i 4} = C_K \sigma_{x_i}^4 \end{cases} \tag{13}$$

where C_s is the skewness value, C_k is the kurtosis value and σ_{x_i} is the standard deviation of variable x_i.

Therefore, the first four moments of Z can be estimated with the corresponding first four moments of $\boldsymbol{X}$, the symbol υ_{Zk} is applied to denote the kth order moment of Z. Through Eq. (11), the mean of Z can be approximated as:

$$\mu_Z = E\left\{g(\boldsymbol{X})\right\} = g\left(\boldsymbol{\mu_X}\right) + \frac{1}{2}\sum_{i=1}^{n} g_{ii}\, \upsilon_{x_i 2} \tag{14}$$

Then the variance of Z is formulated as:

$$\upsilon_{Z2} = \sigma_Z^2 = E\left\{\left(Z - \mu_Z\right)^2\right\} = \sum_{i=1}^{n} g_i^2 \upsilon_{x_i 2} + \sum_{i=1}^{n} g_i g_{ii} \upsilon_{x_i 3} + \frac{1}{4}\sum_{i=1}^{n} g_{ii}^2\left(\upsilon_{x_i 4} - 3\upsilon_{x_i 2}^2\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}^2 \upsilon_{x_i 2}\upsilon_{x_j 2} \tag{15}$$

where σ_Z is the standard deviation.

In a similar way, the third moment of Z can be derived, expressed as:

$$\upsilon_{Z3} = E\left\{\left(Z - \mu_Z\right)^3\right\} = \sum_{i=1}^{n} g_i^3 \upsilon_{x_i 3} + \frac{3}{2}\sum_{i=1}^{n} g_i^2 g_{ii}\left(\upsilon_{x_i 4} - 3\upsilon_{x_i 2}^2\right) + 3\sum_{i=1}^{n}\sum_{j=1}^{n} g_i g_j g_{ij} \upsilon_{x_i 2}\upsilon_{x_j 2} \tag{16}$$

The fourth moment can be obtained, formulated as:

$$\upsilon_{Z4} = E\left\{\left(Z - \mu_Z\right)^4\right\} = \sum_{i=1}^{n} g_i^4\left(\upsilon_{x_i 4} - 3\upsilon_{x_i 2}^2\right) + 3\sum_{i=1}^{n}\sum_{j=1}^{n} g_i^2 g_j^2 \upsilon_{x_i 2}\upsilon_{x_j 2} \tag{17}$$

Recall that the key is to find a best-fit distribution $f(z)$ meeting the first four moments' matching-constraints. It has been approved that the one with a maximum entropy is most rational among all possible solutions [11].

3.2.2. Probability density function with the maximum entropy

To make the following analysis convenient, the variable Z is transformed into variable W such that its mean equals to zero and variance equals to one, the transform equation is:

$$W = \frac{Z - \mu_Z}{\sigma_Z} \tag{18}$$

The first four moments of variable W can be obtained with the statistical information of variable Z, written as:

$$\upsilon_{wK} = \frac{\upsilon_{Zk}}{\sigma_Z^k} \quad for\left(k = 0,1,\cdots 4\right) \tag{19}$$

The way to measure the uncertainty of a random variable is the information entropy proposed by Shannon from the perspective of statistic [11]. The information entropy of the variable W in terms of its probability density function $f(\mathrm{w})$ is expressed as:

$$H(W) = -\int_W f(w) ln[f(w)] dw \tag{20}$$

The first four origin moments of the variable W are defined as:

$$\upsilon_{wK} = \int_W w^k f(w) dw \ for\left(k = 0,1,\cdots 4\right) \tag{21}$$

The first four origin moments of the variable W are treated as the additional restrictions. The search of the best-fit PDF can be equivalent to a constrained optimization problem. Therefore the Lagrange multipliers method is suitable and can be applied to solve this problem, define the Lagrange function as:

$$L = -\int_W f(w) ln[f(w)] dw + \sum_{k=0}^{4} \lambda_k \left[\int_W w^k f(w) dw - \upsilon_{wk}\right] \tag{22}$$

where $\lambda = (\lambda_0, \lambda_1, \cdots, \lambda_4)$ represent the Lagrange multipliers.

The optimal solution can be solved by:

$$\frac{\partial L[\lambda, f(w)]}{\partial f(w)} = 0 \tag{23}$$

which can derive the best-fit probability density function for the variable W, formulated as:

$$f(w) = exp\left(-\sum_{k=0}^{4} \lambda_k w^k\right) \tag{24}$$

Substitute Eq. (24) into Eq. (21), a set of equations with only unknown Lagrange multipliers can be established as:

$$\upsilon_{wK} = \int_W w^k exp\left(-\sum_{k=0}^{4} \lambda_k w^k\right) dw \ for\left(k = 0,1,\cdots 4\right) \tag{25}$$

With the MATLAB functions of *quadgk* and *fsolve,* the Lagrange multipliers can be figured out. Therefore, the probability of failure of the manipulator with the cumulative density function of variable W over a range of $(-\infty, 0]$ can be reformulated as:

$$P_f = \int_{-\infty}^{-\mu_z/\sigma_z} exp\left(-\sum_{k=0}^{4} \lambda_k w^k\right) dw \tag{26}$$

4. Simulation

4.1. Arbitrarily distributed joint clearance

A typical industrial manipulator, as shown in Fig.1, is utilized as an example to illustrate the proposed method. A general non-linear path composed of 10 points is designed by the following equation, written as:

$$\begin{cases} x = 100 sin(t) + 350 \\ y = 100 cos(t) \\ z = 10t + 645 \end{cases} \tag{27}$$

where t increases from t=0 to t=4π with a step of 1.396 rad.

The resulting trajectory from the start-point A to the end-point D is drawn in Fig. 2.

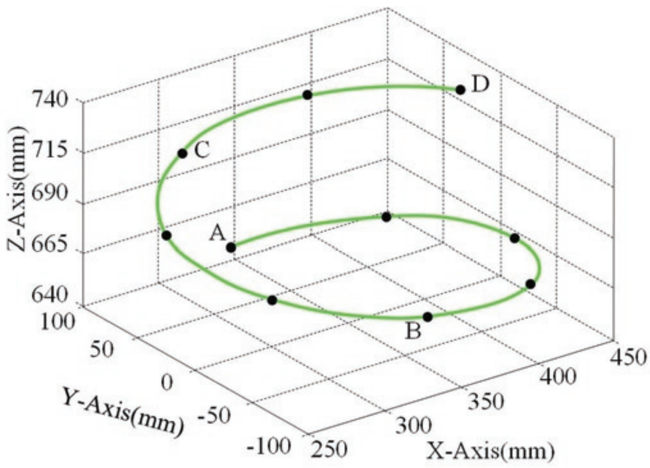


Fig. 2. Designed path of the manipulator

The first four moments of variables are used as information restrictions, which can be given in advance or computed from a set of observations, instead of relying on the inappropriate assumption of normality or uniformity. Then based on the maximum entropy principle, the Lagrange multipliers method is applied to search for a perfect distribution of the output deviations. The flow chart is drawn in Fig.3.

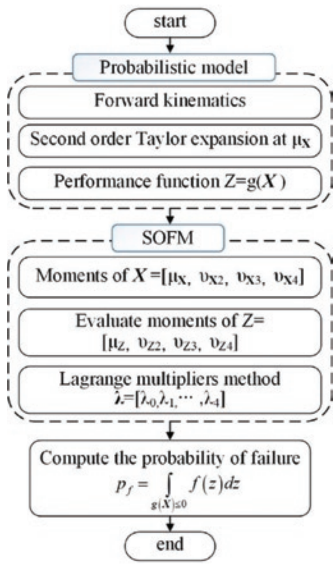


Fig. 3. Procedure of reliability analysis with SOFM

The popular approximate methods, FOSM and FORM are used for an accuracy comparison. Since MCS can produce an accurate result based on a large scale of samples. 100,000 samples are generated for MCS and its solution is regarded as the benchmark result. The simulation process is based on an Intel CORE i5 CPU.

Actually, unlike the geometrical parameters which remains constant after they are manufactured, the clearance occurs randomly when the journal rotates in the bearing. The center of the journal varies in two dependent dimensions which approves the unreasonable assumption of normality or uniformity [20]. In practice, the exact probability density function is generally unavailable due to the lack of sufficient information. Instead, only a few moments can be usually acquired including the mean, variance, coefficients of the skewness and kurtosis [25]. Hence, the joint clearance with arbitrary distribution can be modeled as:

$$\theta_i = \bar{\theta}_i + \chi \tag{28}$$

where θ_i is the actual value of the ith joint, $\bar{\theta}_i$ denotes the ideal value and χ is an arbitrarily distributed random variable.

The clearance has its mean value of 0°, the standard deviation of 0.5°, the skewness value of $C_s = 0.45$ and the kurtosis value of $C_k = 4.36$.With a permissible region $\delta = 2.30$ mm, the P_f is evaluated and the corresponding results are shown in Fig.4.

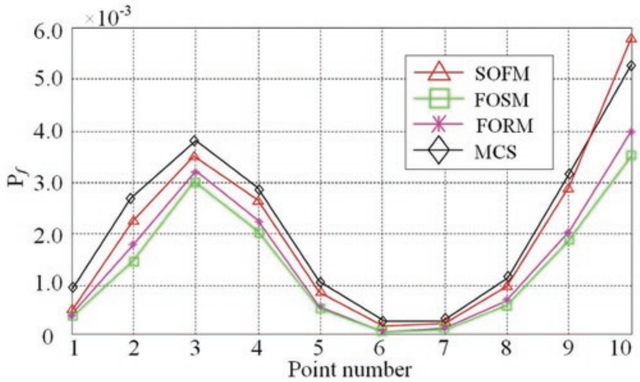


Fig. 4. P_f with arbitrarily distributed clearances

The time that the four methods cost in the calculation process is listed in Table 3.

4.2. Other two distribution types of clearance

For conducting a comprehensive comparison, other two distributions are considered in which the joint clearance is assumed to be uniformly and normally distributed respectively. Firstly, some researchers believe that the joint clearance exhibits features of uniformity during the motion and the actual angle value varies within a certain range [16, 20, 27]. With such an assumption, the joint clearance can be therefore modeled as:

$$\theta_i = \bar{\theta}_i + \xi \tag{29}$$

where ξ is a uniformly distributed random variable with a range of [-0.5°, 0.5°].

Here δ =1.30 mm. The probability of failure P_f is calculated through the four aforementioned methods, and the results are plotted in Fig. 5.

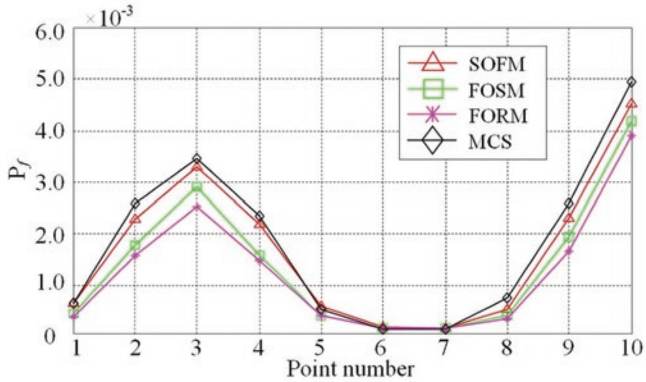


Fig. 5. P_f with uniformly distributed clearances

The CPU time each method requires during the evaluation process is listed in Table 4.

From the perspective of some other researchers, regarding the joint clearance as a normally distributed variable is reasonable and they have belief in that there is nothing different between the joint clearance and the deviation in geometrical parameters [12,14]. Thus, similar to the geometrical parameters, joint clearances can be modeled as normal variables, given as:

$$\theta_i = \bar{\theta}_i + \eta \tag{30}$$

Table 3. CPU time of four methods with arbitrary distribution clearances (unit/s)

Point number.	1	2	3	4	5	6	7	8	9	10
MCS	373.86	374.82	373.36	373.52	373.29	373.27	374.73	374.15	374.0	373.28
FOSM	12.21	13.92	12.01	13.54	13.63	13.73	12.16	12.79	12.51	13.60
FORM	27.90	27.16	27.45	28.82	27.30	28.65	28.07	28.99	27.15	27.88
SOFM	18.15	18.10	19.06	19.55	19.86	18.25	19.13	18.93	18.02	18.67

Table 4. CPU time of four methods with uniform clearances (unit/s)

Point number	1	2	3	4	5	6	7	8	9	10
MCS	374.62	374.81	373.25	374.82	374.26	373.19	373.55	374.09	374.91	374.92
FOSM	12.76	12.31	13.94	13.91	12.97	13.60	12.28	12.84	13.8	13.58
FORM	35.49	34.31	33.07	34.69	34.86	34.35	34.51	34.48	33.78	34.31
SOFM	16.06	17.41	16.55	16.09	16.19	17.64	17.38	16.63	17.90	16.06

where η is a normally distributed random variable with the mean value of 0° and the standard deviation of 0.5°.

With the permissible region δ = 2.80 mm, the probability of failure P_f with normally distributed joint clearances can be computed, results are drawn in Fig.6.

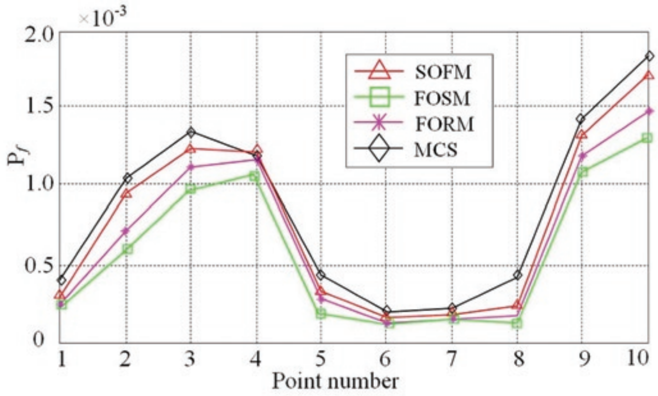


Fig. 6. P_f with normally distributed clearances

The CPU time each method takes in the calculation process is listed in Table 5.

The average CPU time A_{time} can be obtained from Table 3 to Table 5. The average estimation error A_{err} can be also computed from Fig.4 to Fig.6. The equations are formulated as:

$$\begin{cases} A_{time} = \sum_{i=1}^{N} t_i / N \\ A_{err} = \sum_{i=1}^{N} \left| P_f^i - \tilde{P}_f^i \right| / N \end{cases} \tag{31}$$

where N is the total point number, t_i is the CPU time that each method takes on the ith point, P_f^i is the benchmark result from MCS and $\tilde{P}_f^i$ is the estimated result solved by three approximate methods.

The efficiency and accuracy of each method in three joint distributions are listed in Table 6.

4.3. Reliability analysis with reference to a particular point

To better understand the reason why four methods behave so differently with the different type of clearance distribution. A deeper insight is provided into what will happen when the joint clearance exists. Without loss of generality, the reliability analysis at the start point A on the path is used as an example. The target position of the end-effector is ***Pos*** = [350, 100, 645].Through the invers kinematics, the corresponding desired angle values can be computed and listed as $\bar{\theta}$ =[15.94°, -87.43°, -2.62°, 0°, 90.06°, 61.68°].

When the joint clearance is treated as an arbitrarily distributed variable χ. Its mean value, standard deviation, coefficients of skewness and kurtosis are 0°, 0.5°, 0.45 and 4.36 respectively. All the geometric parameters have normal distributions, in which C_s = 0 and C_K =3. As described in previous. During the motion of the manipulator, the actual position of the end-effector will deviate from its target position due to the joint clearance. Based on 100,000 simulation samples, the stochastic positions are plotted on the following Fig.7 (a), Fig.7 (b) and Fig.7(c), the corresponding position errors are drawn in Fig.7 (d).

As we can see from Fig. 7(a) to Fig.7(c), the probabilistic position of the end-effector is randomly scattered around the desired position. The shape is irregular, since the distribution of the joint clearance is neither normal nor uniform. It can be noted from Fig.7 (d) that the distribution of position error is non-normal. Actually, it right skewed. With the proposed method, the reliability can be approximated by the following steps.

Step1) Obtain the first four moments of variables which can be listed as the following Table 7.

Step2) Compute the first four moments of Z. By utilizing the data obtained from step 1. Through Eq. (14) to Eq. (17), the corresponding moments for Z are calculated and they are $\mu_z = 2.4286$, $\upsilon_{Z0} = 1$, $\upsilon_{Z1} = 0$, $\upsilon_{Z2} = 0.2099$, $\upsilon_{Z3} = -0.0828$ and $\upsilon_{Z4} = 0.1794$. Make Z transform into the standard variable W, the corresponding moments can be obtained as: $\mu_w = 0$, $\upsilon_{w0} = 1$, $\upsilon_{w1} = 0$, $\upsilon_{w2} = 1$, $\upsilon_{w3} = -0.8607$ and $\upsilon_{w4} = 4.0731$.

Step3) Derive the best-fit distribution. Substitute the results obtained from step2 into Eq. (25) to establish a set of equations. With MATLAB functions of *quadgk* and *fsolve*, the Lagrange multipliers can be solved. They are listed as: λ_0=0.0263, λ_1 = 0.1964, λ_2 = 0.5391, λ_3 = -0.4986 and λ_4 = 0.8816.

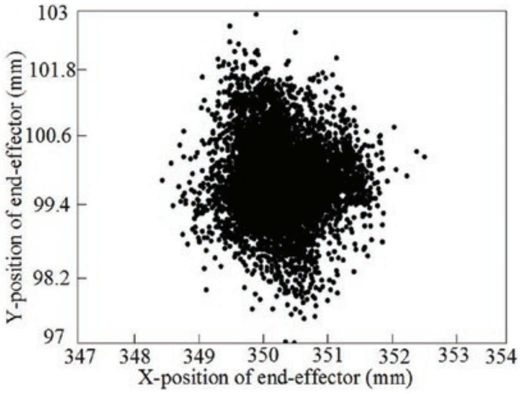
Step4) Calculate the probability of failure. Since the Lagrange multipliers are obtained. The P_f can be calculated with a permitted

Table 5. CPU time of four methods with normal clearances (unit/s)

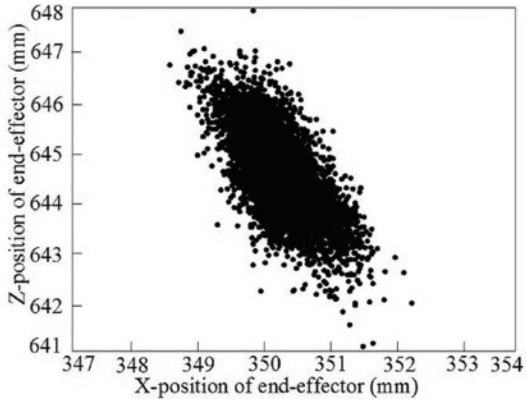
Point number.	1	2	3	4	5	6	7	8	9	10
MCS	373.87	373.76	374.53	374.59	373.37	373.97	373.89	374.29	374.41	374.50
FOSM	12.55	13.35	13.31	12.32	12.23	12.99	13.91	12.68	13.17	12.44
FORM	27.25	25.76	26.51	27.09	27.67	27.87	26.64	25.41	25.44	25.77
SOFM	17.68	16.50	17.62	16.48	17.85	16.70	16.39	16.50	17.23	16.94

Table 6. Comparison in accuracy and efficiency of four methods

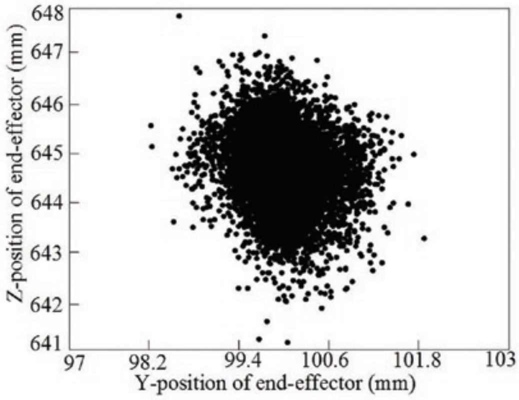
Method	Arbitrary distribution		Uniform distribution		Normal distribution	
	A_{time} (/s)	A_{err} (%)	A_{time} (/s)	A_{err} (%)	A_{time} (/s)	A_{err} (%)
MCS	373.83	—	374.24	—	374.12	—
FOSM	13.01	0.082	13.20	0.045	12.90	0.026
FORM	27.94	0.065	34.39	0.060	26.54	0.018
SOFM	18.77	0.028	16.79	0.018	16.99	0.009



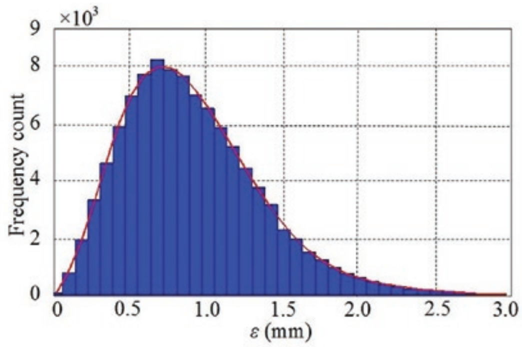
(a) X-Y plane



(b) X-Z plane

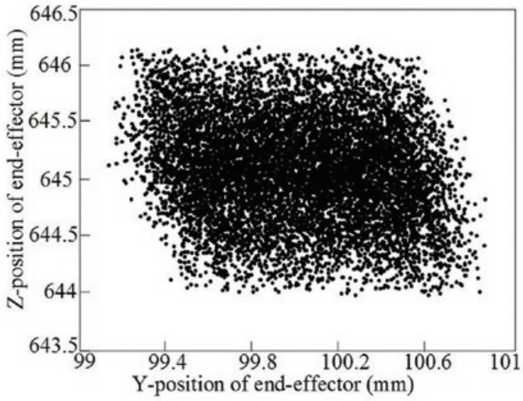


(c) Y-Z plane

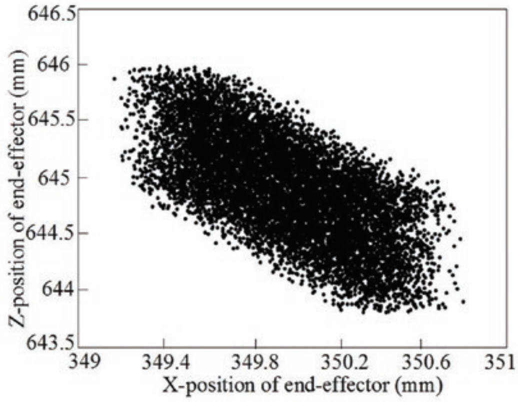


(d) Frequency histogram of the position error

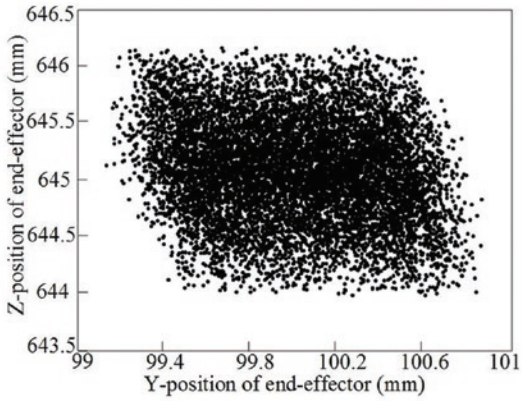
Fig. 7. Stochastic position and position error with the arbitrary distribution clearance



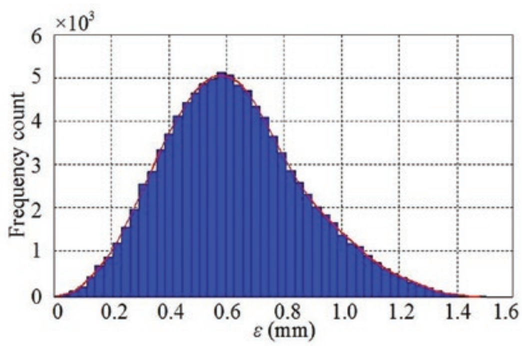
(a) X-Y plane



(b) X-Z plane



(c) Y-Z plane

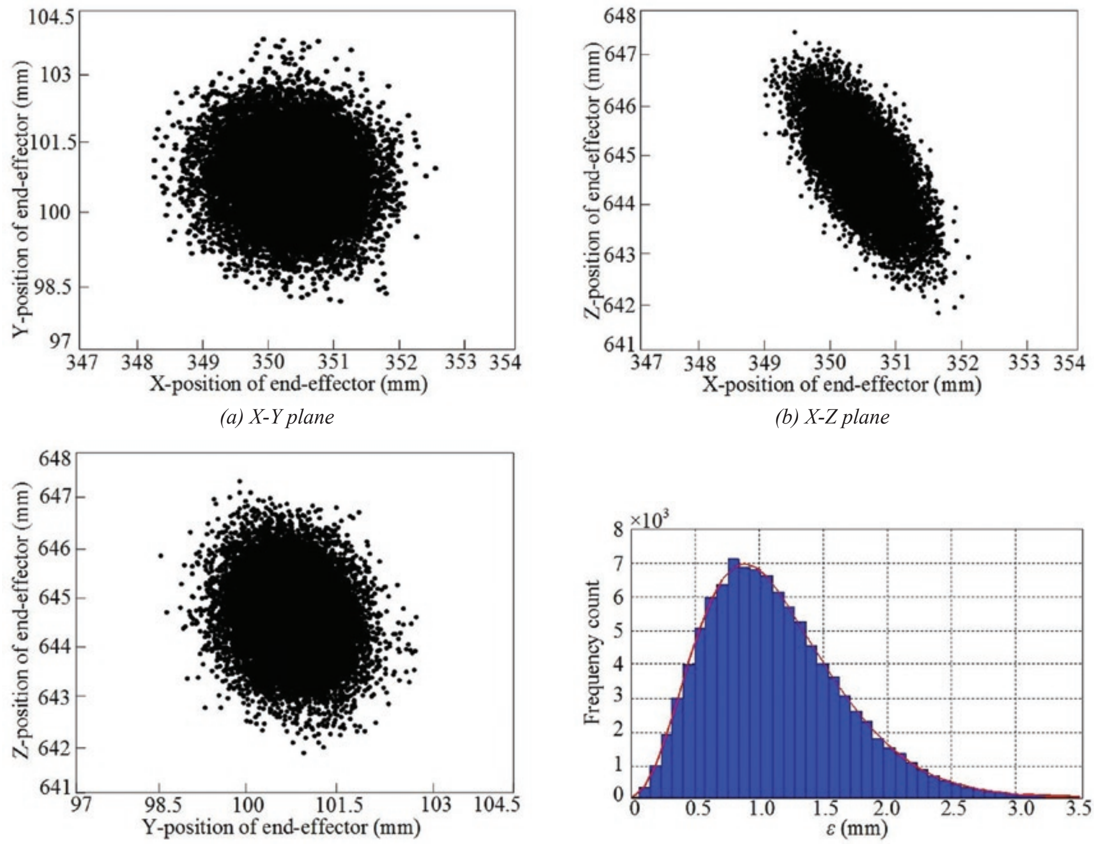


d) Frequency histogram of the position error

Fig. 8. Stochastic position and position error with the uniform distribution clearance

Table 7. Statistic information for the joint clearance and geometric parameters

moment	a_1	a_2	a_3	d_1	d_4	d_6	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
μ	40	315	70	330	310	70	15.94	−87.43	−2.62	0	90.06	61.68
υ_0	1	1	1	1	1	1	1	1	1	1	1	1
υ_1	0		0	0	0	0	0	0	0	0	0	0
$\upsilon_2(\sigma^2)$	0.0016	0.1024	0.0049	0.1089	0.0961	0.0049	0.25	0.25	0.25	0.25	0.25	0.25
υ_3	0	0	0	0	0	0	0.0563	0.0563	0.0563	0.0563	0.0563	0.0563
υ_4	0.0002	0.0983	0.0010	0.1078	0.0894	0.0010	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281



(a) X-Y plane (b) X-Z plane

(c) Y-Z plane (d) Frequency histogram of the position error

Fig. 9. Stochastic position and position error with the normal distribution clearance

threshold value δ = 2.30 mm. Herein, with the MATLAB function of *integral*, P_f is 4.8×10^{-4}.

In a similar way, when the joint clearance meets a uniform and norm distribution. Each are based on the another 100,000 samples, the possible locations of the end-effector and position errors can be also drawn in the following Fig.8 and Fig.9 respectively.

As shown from Fig.8 (a) to Fig.8(c). In three planes (X-Y, X-Z, and Y-Z planes), the stochastic positions of the end-effector with the uniformly distributed joint clearance are uniformly scattered around the target position with a shape like a parallelogram, which is quite different from that for the arbitrarily distributed joint clearance. There is also a positive skewness value for the distribution of position error, which can be observed from Fig.8 (d).

Meanwhile. As shown from Fig.9 (a) to Fig.9(c). Under the influence of normally distributed joint clearance, it is interesting to observe that the shape of the stochastic positions is an ellipse in three planes. The random locations are scattered around the desired position with a more intensive trend. The position error has a similar distribution to that for the uniformly and arbitrarily distributed clearance, as shown in Fig.9 (d).

5. Discussion

1) *Influence of the distribution type of clearance*

As we can see from Fig.4 to Fig.6, at the same point on the path, the probability of failure of the manipulator varies with the distribution type of the clearance. The reason is that the impact caused by joint clearance on the deviation between the desired position and actual position is quite different. The possible location of the end-effector is related to the joint clearance distribution, as approved by the scatter pictures shown in from Fig.7 to Fig.9. With a given clearance distribution, the scatter shape formed by 10^6 samples is regular. For example, the shape of a parallelogram with a uniform distribution clearance or an ellipse with a norm clearance. Whereas, the shape can be more irregular compared with that for other two distributions, if the exact distribution is unknown. Therefore, the distribution type of joint clearance has a significant influence on the reliability of a manipulator and an appropriate assumption is essential.

2) *Accuracy analysis*

As shown from Fig.4 to Fig.6. There is a much agreement between the results of MCS and SOFM. The average estimation errors of the two traditional methods, FOSM (about 0.051%) and FORM(about 0.048%), are larger than that of the prosed method SOFM(0.018%),which can be observed from Table 6. Recall the basis on which each method performs. FOSM requires that the output deviations of the system meet the norm distribution. Unfortunately, the scattered data of the position error is never normally distributed no matter whether the clearance is a normal variable or not, as shown from Fig.7 (d) to Fig.9 (d). As for FORM, this method can be converted into an optimization problem after a transformation of abnormal variables to standard normal variables. This procedure may result in large errors, especially when the clearance is arbitrarily distributed. With a second order Taylor expansion, SOFM derives a best-fit distribution by utilizing the first four moments and thus can overcome the drawbacks existing in FOSM and FORM. Compared with FOSM and FORM, the accuracy of SOFM can be improved by at least twice in all cases

3) *Efficiency analysis*

In terms of efficiency, as we can observe from Table 6. FOSM is the most efficient method since it directly linearizes the performance function at the mean of variables, it only takes about 13s. Other things equal, the efficiency of SOFM (about 17s) is close to FOSM and it comes to the second place, because the higher order Taylor series is used and four moments instead of just the mean and the variance are considered. The efficiency of FORM (about 30s) is only better than MCS (about 370s) but much lower than both FOSM and FOSM, the reason is that an iterative process is required to search for the most probable point. MCS is based on a large size of samples to evaluate the reliability and naturally is very time-consuming.

6. Conclusion

In this paper, the repeatability of positioning is analyzed with consideration of the influence of arbitrarily distributed clearance. Unlike conventional methods, the joint clearance is empirically assumed to be normally or uniformly distributed. A new approach has been put forward with integration of Taylor series expansion method and Lagrange multipliers method. Achievements and conclusions can be summarized as follows.

1) **Distribution type of the clearance has significant influence on the reliability.** The quality of the performance of a manipulator is significantly affected by the distribution type of the joint clearance. Therefore, it is better to use the first four moments instead of an inappropriate assumption of normality or uniformity when there is not sufficient information.

2) **Better accuracy and efficiency make SOFM more attractive.** SOFM exhibits the better accuracy as well as greater efficiency compared to that of FOSM, FORM and MCS. The result of SOFM can be improved by at least two times with respect to accuracy. Furthermore, the prosed method is nearly twenty times much more efficient than MCS. Overall consideration, SOFM can outperform all the other three methods when they are used to handle the problem of positioning accuracy analysis with arbitrarily distributed joint clearances.

3) **Moment-matching method can have a wider range of application.** Based on the moment-matching principle, only the first four moments are needed to derive an optimal distribution of the system output. The proposed method exhibits impressive quality for the arbitrarily distributed clearance, which can be easily extended to the normal and uniform clearance. From this point of view, SOFM is robust and has a wider range of application.

However, SOFM still belongs to an approximate method. Some errors can always exist and originate from the following sources: 1) order-reduction for the highly non-linear performance function; 2) moment estimation process. If a more accurate result is expected, may be more than four moments are needed to be taken in to account.

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