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Huiying GAO
Xiaoqiang ZHANG

RELIABILITY-BASED DESIGN OPTIMIZATION UNDER FUZZY AND INTERVAL VARIABLES BASED ON ENTROPY THEORY

OPARTA NA TEORII ENTROPII NIEZAWODNOŚCIOWA OPTYMALIZACJA KONSTRUKCJI DLA ZMIENNYCH ROZMYTYCH I PRZEDZIAŁOWYCH

Reliability-based design optimization under fuzzy and interval variables is important in engineering practice. The interval Monte Carlo simulation (IMCS), extremum method, and saddlepoint approximation (SPA) can be used for reliability optimization issues contain only interval variables. Thus, how to deal with the fuzzy variables is critical for system reliability analysis and optimization design. The α-level cut method can be applied to deal with fuzzy variables but it is complex and computationally expensive. Therefore, an equivalent conversion method based on entropy theory is proposed in this paper, which can convert the fuzzy variables to the normal random variables to avoid the complex integral process. According to the equivalent conversion method, the entropy-based sequential optimization and reliability assessment (E-SORA) is developed in combination with the worst case analysis (WCA) for reliability-based design optimization under fuzzy and interval variables. A numerical example about the reliability design of the crank-link mechanism under fuzzy and interval variables is solved by the E-SORA, double-loops method, and α-level cut algorithm, respectively, is used to demonstrate the accuracy and efficiency, and the results show that the proposed method is feasible for reliability-based design optimization under fuzzy and interval variables.

Keywords: *fuzzy variables, interval variables, reliability-based design optimization, entropy, the worst case analysis.*

Zagadnienie optymalizacji niezawodnościowej konstrukcji w przypadkach, gdy mamy do czynienia ze zmiennymi rozmytymi i przedziałowymi odgrywa ważną rolę w praktyce inżynierskiej. Problemy optymalizacji niezawodności, w których wykorzystuje się tylko zmienne przedziałowe można z powodzeniem rozwiązywać stosując przedziałową symulację Monte Carlo, metodę ekstremum czy aproksymację metodą punktu siodłowego. Kluczowe znaczenie dla analizy niezawodności oraz projektowania optymalizacyjnego systemów ma zatem sposób postępowania ze zmiennymi rozmytymi. Wprawdzie zmienne rozmyte można przekształcać do zmiennych interwałowych za pomocą metody alfa-przekrojów, jest to jednak metoda skomplikowana i kosztowna obliczeniowo. Dlatego w niniejszym artykule zaproponowano równoważną metodę konwersji opartą na teorii entropii, która umożliwia przekształcanie zmiennych rozmytych do normalnych zmiennych losowych, pozwalając w ten sposób pominąć złożony proces całkowania. W oparciu o tę metodę, opracowano entropijną metodę optymalizacji sekwencyjnej i oceny niezawodności (E-SORA), którą, w połączeniu z analizą najgorszego przypadku, można stosować do niezawodnościowej optymalizacji konstrukcji przy zmiennych rozmytych i przedziałowych. W przykładzie numerycznym, metodę E-SORA zastosowano w połączeniu z metodą podwójnej pętli do rozwiązania problemu niezawodnościowego projektowania mechanizmu korbowego przy zmiennych rozmytych i przedziałowych. Trafność i skuteczność proponowanej metody oceniano za pomocą algorytmu alfa-przekrojów. Wyniki pokazują, że proponowana metoda stanowi odpowiednie narzędzie do przeprowadzania optymalizacji niezawodnościowej konstrukcji w przypadku gdy zmienne mają charakter rozmyty i przedziałowy.

Słowa kluczowe: *zmienne rozmyte, zmienne przedziałowe, niezawodnościowa optymalizacja konstrukcji, entropia, analiza najgorszego przypadku.*

1. Introduction

Reliability-based design optimization is a probabilistic design method which has been successfully applied into engineering fields. The main purpose of reliability-based design optimization is to assure products achieving the optimal performance with an expected reliability. Nowadays, the traditional reliability based design optimization method is hard to use in reality. For complex systems, such as spaceship, high-speed train, and nuclear power plant, often involve multi-disciplines, multi-type design variables and multi-source uncertainties which interacting and coupling with each other. Therefore, reliability-based design in engineering practice face two difficulties, one is some variables in systems are fuzzy variables or interval variables, since they cannot be obtained accurately, the other is the expensive computation.

A large amount of research works have been done on reliability assessments with interval or fuzzy variables. Huang [12] investigated the methods to determine the membership functions under three different forms of the fuzzy safety state definition. Based on the fuzzy comprehensive evaluation, Wu et al. [22] proposed a reliability analysis method by combining with the fuzzy set theory. To solve the state explosion and the parametric uncertainty problems, Li et al. [14] proposed a dynamic reliability analysis method via the continuous-time Bayesian networks under fuzzy numbers. Garg [9] proposed a fuzzy reliability analysis method based on credibility theory to solve the problem that all failure rates are usually assumed to follow the identical type of fuzzy set, and the membership and non-membership functions can be constructed by different types of intuitionistic fuzzy numbers. Tao et al. [21] developed an uncertainty model combining with multiple membership functions, to deal with the epistemic uncertainties in fuzzy reliability analysis, and the combined membership functions can be converted to equivalent probability density function through normalizing factor. Yang et al. [25] constructed a reliability analysis model via gamma process, and proposed a reliability analysis method to consider the non-competing relationship of multiple degradation processes. He and Zhang [10] developed a fuzzy reliability analysis method using cellular automata (CA) and fuzzy logic for network systems to solve the problem that the failure rates of networks may not follow the identical membership function. Based on a semi-Markov jump model, Shen et al. [20] designed a fuzzy fault-tolerant controller for Takagi-Sugeno (T-S) fuzzy delayed systems. The α-level cut method is an important approach to solve the problems with fuzzy variables, which can divide a fuzzy set into a series of intervals, and has been used for reliability analysis and design optimization [7]. Bagheri et al. [2-3] proposed a fuzzy structure dynamic reliability analysis method by the α- level cut optimization method based on genetic algorithm. Awruch et al. [1] applied fuzzy α- level cut method for optimization analysis under uncertainties. He et al. [11] introduced the fuzzy set theory, changing failure probability function and dynamic fuzzy subset into Bayesian Networks method for the reliability analysis of multi-state system reliability analysis with fuzzy and dynamic information. However, these models are very complex and computation expensive. Mi et al. [16-17] investigated the reliability analysis of complex systems under epistemic uncertainty, and proposed an extended probability-box to represent the epistemic uncertainty for reliability assessment, which could deal with test, field and design data, and the lifetime of component could be denoted as interval numbers. Based on particle swarm optimization approach, Zhang and Chen [27] formulated an interval multi-objective optimization model for reliability redundancy allocation in interval environment. With the consideration of interval uncertainties under stationary Guassian excitation, Muscolino et al. [18] presented a method to evaluate the bounds of the interval reliability function. Wang et al. [23] investigated a reliability based design optimization method with hybrid probability and interval parameters, and adopted approximate reliability analysis method to improve computational efficiency. Aim to the spatially dependent uncertainties in system inputs, Wu and Gao [24] proposed the concept of random and interval fields and extended unified interval stochastic sampling method for static reliability analysis. Chen et al. [6] proposed the interval type-2 fuzzy multi-objective optimization method for reliability redundancy allocation with different types of uncertainties. For overcome the problem of reliability analysis with fuzzy and random uncertainties, Zhang et al. [28] developed a chance theory using the multi-state performance reliability model. Peng et al. [19] demonstrated a hybrid first order reliability analysis method for structural system with interval, sparse and statistical variables, and interval variables were converted to probabilistic variables by a uniformity method. Gao et al. [8] presented a unified interval stochastic reliability sampling method to investigate the robust reliability analysis of structural with mixture of stochastic and non-stochastic uncertainty.

In a word, different types of uncertainty variables are existing in engineering, and there are many solutions for reliability-based design optimization, the reliability analysis or optimization design methods under fuzzy and interval variables, however, are complex and computationally expensive. To make up this disadvantage, the reliability-based design optimization for system with fuzzy and interval variables is investigated in this paper. The key points contain reliability analysis based on the worst case analysis of interval variables and optimization design based on an equivalent conversion method. The rest of this paper is organized as follows. The entropy-based equivalent conversion method is proposed in Section 2, and fuzzy variables are converted to random variables in Gaussian space. Section 3 develops an entropy-based sequential optimization and reliability assessment approach based on the worst case analysis of the interval variables, and the optimization strategy under fuzzy and interval variables is also further addressed. To verify the accuracy and efficiency of the proposed method, the crank-connecting rod mechanism of an internal combustion engine under fuzzy and interval variables is analysed in Section 4. Finally, the conclusions are given in Section 5.

2. An entropy-based equivalent conversion method

2.1. The Equivalent Conversion Method

Generally, different types of variables are widely existed in engineering practice, and the probability density functions (PDFs) of some variables cannot be accurately obtained. In this case, we can use fuzzy numbers or intervals to model them. Thus the research on reliability-based design optimization method under fuzzy and interval variables is important. The interval Monte Carlo simulation (IMCS), extremum method, and saddlepoint approximation (SPA) can be used for reliability-based design optimization under interval variables. Thus, how to deal with the fuzzy variables is critical for system reliability analysis and design optimization. The fuzzy set decomposition theorem is a basic theorem in fuzzy set theory and a fuzzy set can be divided into a series of intervals by α- level cut method with high accuracy in computation [1]. Thus α - level cut method is usually applied to deal with fuzzy variables, but it is complex and computationally expensive. In this section, an equivalent conversion method based on entropy is proposed, then reliability-based design with fuzzy and interval variables can be solved based on this equivalent conversion method.

According to [5, 29], the probabilistic entropy of a random variable X can be defined as:

$$H_X = -\int_{-\infty}^{+\infty} f_X(x) \ln f_X(x) \mathrm{d}x \tag{1}$$

where H_X is the probabilistic entropy, and $f_X(x)$ is the PDF of the random variable X.

Assuming that the random variable X follows a normal distribution with the mean μ and standard deviation σ, then Eq. (1) can be transformed as:

$$H_X = -\int_{-\infty}^{+\infty} f_X(x) \ln f_X(x) \mathrm{d}x = -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \ln\left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right) \mathrm{d}x = \frac{1}{2} + \ln\left(\sqrt{2\pi}\sigma\right) \tag{2}$$

Assuming that $\tilde{X}$ is a continuous fuzzy variable which can be denoted as $\int_U \mu_{\tilde{X}}(x)/x, \left(x \in U = \left[U^{lb}, U^{ub}\right]\right)$, where $\mu_{\tilde{X}}(x)$ is the membership function, and U^{lb} and U^{ub} are the lower and upper bounds, respectively. Then the definition of fuzzy entropy can be given by [5, 30]:

$$G_{\tilde{X}} = -\int_U \mu'_{\tilde{X}}(x) \ln \mu'_{\tilde{X}}(x) \mathrm{d}x \tag{3}$$

where $G_{\tilde{X}}$ denotes the fuzzy entropy of $\tilde{X}$, $\mu'_{\tilde{X}}(x) = \mu_{\tilde{X}}(x) / \left(\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x) \mathrm{d}x\right)$, and $\mu'_{\tilde{X}}(x)$ represents the standard membership function.

As aforementioned, the reliability-based design optimization under fuzzy variables is difficult. If fuzzy variables can be converted to the variables which are easy to deal with by a reasonable approach, the difficulty of reliability-based design optimization will be reduced. Thus, the conception of equivalent random variable is proposed in this paper. According to Refs. [4-5, 29-30], the equivalent normal random variable can be defined as a variable which satisfies:

$$H_{X_{eq}} = G_{\tilde{X}} \tag{4}$$

where X_{eq} is the equivalent normal random variable, and $H_{X_{eq}}$ is the probabilistic entropy of X_{eq}.

The mean and standard deviation of $\tilde{X}$ need to be determined to obtain the PDF of X_{eq}. The standard deviation of X_{eq} can be given by combining Eq. (2) with Eq. (4):

$$\sigma_{X_{eq}} = \frac{1}{\sqrt{2\pi}} e^{G_{\tilde{X}} - 0.5} \tag{5}$$

where $\sigma_{X_{eq}}$ is the standard deviation of X_{eq}.

It is obvious that the standard deviation of X_{eq} can be obtained via Eq. (5). However, how to determine the mean value is a challenge. Generally, for normal convex fuzzy sets, the mean of X_{eq} can be considered as the symmetrical center of the membership function. Then the PDF of X_{eq} can be expressed as:

$$f_{X_{eq}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{X_{eq}}} e^{-\frac{(x-\hat{x})^2}{2\sigma^2_{X_{eq}}}} \tag{6}$$

where $f_{X_{eq}}(x)$ is the PDF of X_{eq}, and $\hat{x}$ is the symmetrical center of the membership function.

According to Eq. (3), $\mu'_{\tilde{X}}(x)$ has the following properties:

$$\mu'_{\tilde{X}}(x) = \mu_{\tilde{X}}(x) / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x \geq 0 \tag{7}$$

$$\int_{-\infty}^{+\infty} \mu'_{\tilde{X}}(x)\mathrm{d}x = \int_{U^{lb}}^{U^{ub}} \left[\mu_{\tilde{X}}(x) / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x \right] \mathrm{d}x = \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x = 1 \tag{8}$$

From Eqs. (7) to (8), $\mu'_{\tilde{X}}(x)$ satisfies the properties of PDF, thus $\mu'_{\tilde{X}}(x)$ can be defined as the PDF of a continuous random variable X_n. Denoting X_n as the nominal random variable of $\tilde{X}$, then:

$$f_{X_n}(x) = \mu_{\tilde{X}}(x) / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x \tag{9}$$

where $f_{X_n}(x)$ represents the PDF of the continuous random variable X_n, and $x \in \left[U^{lb}, U^{ub}\right]$.

By combination with Eqs. (1) and (9), the probabilistic entropy of X_n yields:

$$H_{X_n} = -\int_{-\infty}^{+\infty} f_{X_n}(x) \ln f_{X_n}(x)\mathrm{d}x = -\int_{-\infty}^{+\infty} \mu_{\tilde{X}}(x) / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x \ln\left[\mu_{\tilde{X}}(x) / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x \right] \mathrm{d}x \tag{10}$$

According to Eqs. (3) and (10), the probabilistic entropy of continuous random variable X_n equals to the fuzzy entropy of fuzzy variable $\tilde{X}$. In this way, fuzzy variables can be converted to equivalent normal random variables.

2.2. The Equivalence Between $\tilde{X}$ and X_n

Based on stress-strength interference (SSI) model, the equivalence between the fuzzy variable $\tilde{X}$ and random variable X_n is addressed in this section. Assuming that the PDF and the membership function of the random tress S and fuzzy strength $\tilde{R}$ are $f_S(s), s \in (-\infty, +\infty)$ and $\int_U \mu_{\tilde{R}}(r)/r, r \in U = \left[U^{lb}, U^{ub}\right]$, respectively. If $\tilde{A}$ represents the fuzzy safety event, and the membership function $\mu_{\tilde{A}}(s)$ of $\tilde{A}$ is defined as [29-30]

a) For $s < U^{lb}$, the system is safe absolutely, $\mu_{\tilde{A}}(s) = 1$.

b) For $s > U^{ub}$, the system is fail absolutely, $\mu_{\tilde{A}}(s) = 0$.

c) For $U^{lb} \leq s \leq U^{ub}$, the interval $\left[U^{lb}, U^{ub}\right]$ can be divided into two parts, that is, subintervals $\left[U^{lb}, s\right]$ and $\left[s, U^{ub}\right]$. Subintervals $\left[U^{lb}, s\right]$ and $\left[s, U^{ub}\right]$ can be regarded as failure and safe domain, respectively. Herein, the membership function $\mu_{\tilde{A}}(s)$ of $\tilde{A}$ can be considered as:

$$\mu_{\tilde{A}}(s) = \int_s^{U^{ub}} \mu_{\tilde{R}}(s')\mathrm{d}s' / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(s)\mathrm{d}s, \; s \in \left[U^{lb}, U^{ub}\right] \tag{11}$$

Therefore, the membership function of the fuzzy safety event can be expressed by the piecewise function as shown in Eq. (12):

$$\mu_{\tilde{A}}(s) = \begin{cases} 1, \; -\infty < s < U^{lb} \\ \int_s^{U^{ub}} \mu_{\tilde{R}}(s')\mathrm{d}s' / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(s)\mathrm{d}s, \; U^{lb} \leq s \leq U^{ub} \\ 0, \; U^{lb} < s < +\infty \end{cases} \tag{12}$$

According to Ref. [26] and the definition of the membership function of fuzzy safety events, the probability formula of fuzzy events that can be defined as:

$$P(\tilde{A}) = \int_{-\infty}^{+\infty} \mu_{\tilde{A}}(s) f_S(s)\mathrm{d}s \tag{13}$$

Thus, the fuzzy reliability yields:

$$\begin{aligned} \tilde{R}_f &= P(\tilde{A}) = \int_{-\infty}^{+\infty} \mu_{\tilde{A}}(s) f_S(s)\mathrm{d}s \\ &= \int_{-\infty}^{U^{lb}} 1 \cdot f_S(s)\mathrm{d}s + \int_{U^{lb}}^{U^{ub}} \left[\int_s^{U^{ub}} \mu_{\tilde{R}}(s')\mathrm{d}s' / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(s)\mathrm{d}s \right] \cdot f_S(s)\mathrm{d}s + \int_{U^{ub}}^{+\infty} 0 \cdot f_S(s)\mathrm{d}s \\ &= \int_{-\infty}^{U^{lb}} f_S(s)\mathrm{d}s + \frac{\int_{U^{lb}}^{U^{ub}} \left[\int_s^{U^{ub}} \mu_{\tilde{R}}(s')\mathrm{d}s' \right] \cdot f_S(s)\mathrm{d}s}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(s)\mathrm{d}s} \end{aligned} \tag{14}$$

Assuming that S and $\tilde{R}$ are independent, according to SSI model, the fuzzy reliability is:

$$\tilde{R}_s \triangleq \Pr\left\{\tilde{R} \tilde{>} S\right\} \tag{15}$$

where $\tilde{R}_s$ is the fuzzy reliability.

By combination with Eqs. (9) and (15), and substituting $\tilde{R}$ with R_n, yields:

$$\begin{aligned} R_s &= \Pr\{R_n > S\} \\ &= \int_{-\infty}^{+\infty} \mu_{\tilde{R}}(r) / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r \int_{-\infty}^{r} f_S(s)\mathrm{d}s\mathrm{d}r \\ &= \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r) / \int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r \int_{-\infty}^{r} f_S(s)\mathrm{d}s\mathrm{d}r \\ &= \frac{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r) \cdot \int_{-\infty}^{r} f_S(s)\mathrm{d}s\mathrm{d}r}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \end{aligned} \tag{16}$$

where R_s is the reliability after equivalent conversion, and R_n is the nominal random variable of $\tilde{R}$.

Eq. (16) can be rewritten as Eq. (17) via integration by parts:

$$\begin{aligned} R_s &= \frac{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r) \int_{-\infty}^{r} f_S(s)\mathrm{d}s\mathrm{d}r}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \\ &= \frac{\int_{U^{lb}}^{U^{ub}} \left[\int_{-\infty}^{r} f_S(s)\mathrm{d}s\right] \mathrm{d}\left[\int_{-\infty}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \end{aligned} \tag{17}$$

Noting that when $r' < U^{lb}$, $\mu_{\tilde{R}}(r') = 0$, then:

$$\mathrm{d}\left[\int_{-\infty}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right] = \mathrm{d}\left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right] \tag{18}$$

Substituting Eq. (18) into Eq. (17), yields:

$$\begin{aligned} R_s &= \frac{\int_{U^{lb}}^{U^{ub}} \left[\int_{-\infty}^{r} f_S(s)\mathrm{d}s\right] \mathrm{d}\left[\int_{-\infty}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \\ &= \frac{\int_{U^{lb}}^{U^{ub}} \left[\int_{-\infty}^{r} f_S(s)\mathrm{d}s\right] \mathrm{d}\left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \\ &= \left.\frac{\left[\int_{-\infty}^{r} f_S(s)\mathrm{d}s\right]\left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r}\right|_{U^{lb}}^{U^{ub}} - \frac{\int_{U^{lb}}^{U^{ub}} \left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right] \mathrm{d}\left[\int_{-\infty}^{r} f_S(s)\mathrm{d}s\right]}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \end{aligned} \tag{19}$$

Since $\int_{U^{lb}}^{U^{lb}} \mu_{\tilde{R}}(r')\mathrm{d}r' = 0$ and $\mathrm{d}\left[\int_{-\infty}^{r} f_S(s)\mathrm{d}s\right] = f_S(r)\mathrm{d}r$, then:

$$\begin{aligned} R_s &= \left.\frac{\left[\int_{-\infty}^{r} f_S(s)\mathrm{d}s\right]\left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r}\right|_{U^{lb}}^{U^{ub}} - \frac{\int_{U^{lb}}^{U^{ub}} \left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right] \mathrm{d}\left[\int_{-\infty}^{r} f_S(s)\mathrm{d}s\right]}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \\ &= \int_{-\infty}^{U^{ub}} f_S(s)\mathrm{d}s - \frac{\int_{U^{lb}}^{U^{ub}} f_S(r) \cdot \left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]\mathrm{d}r}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \end{aligned} \tag{20}$$

The second item of the right-hand member in Eq. (20) can be rewritten as [30]:

$$\begin{aligned} &-\frac{\int_{U^{lb}}^{U^{ub}} f_S(r)\left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]\mathrm{d}r}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \\ &= -\frac{\int_{U^{lb}}^{U^{ub}} f_S(r)\left[\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r')\mathrm{d}r' - \int_{r}^{U^{ub}} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]\mathrm{d}r}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \\ &= -\int_{U^{lb}}^{U^{ub}} f_S(r)\mathrm{d}r + \frac{\int_{U^{lb}}^{U^{ub}} f_S(r)\int_{r}^{U^{ub}} \mu_{\tilde{R}}(r')\mathrm{d}r'\mathrm{d}r}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \end{aligned} \tag{21}$$

Therefore, R_s can be further rewritten as Eq. (22):

$$\begin{aligned} R_s &= \int_{-\infty}^{U^{ub}} f_S(s)\mathrm{d}s - \frac{\int_{U^{lb}}^{U^{ub}} f_S(r)\left[\int_{U^{lb}}^{r} \mu_{\tilde{R}}(r')\mathrm{d}r'\right]\mathrm{d}r}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \\ &= \int_{-\infty}^{U^{ub}} f_S(s)\mathrm{d}s - \int_{U^{lb}}^{U^{ub}} f_S(r)\mathrm{d}r + \frac{\int_{U^{lb}}^{U^{ub}} f_S(r)\int_{r}^{U^{ub}} \mu_{\tilde{R}}(r')\mathrm{d}r'\mathrm{d}r}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(r)\mathrm{d}r} \\ &= \int_{-\infty}^{U^{lb}} f_S(s)\mathrm{d}s + \frac{\int_{U^{lb}}^{U^{ub}} \left[\int_{s}^{U^{ub}} \mu_{\tilde{R}}(s')\mathrm{d}s'\right] f_S(s)\mathrm{d}s}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{R}}(s)\mathrm{d}s} \end{aligned} \tag{22}$$

Obviously, Eqs. (14) and (22) are identical, that is, the reliability calculated by the proposed equivalent conversion method is equivalent to that by SSI model. Therefore, the equivalent conversion method based on entropy theory is reasonable.

For the fuzzy variable $\tilde{X}$, if the means of equivalent random variable X_{eq} and nominal random variable X_n are the same, that is:

$$\mu_{X_{eq}} = E(X_{eq}) \triangleq E(X_n) = \int_{-\infty}^{+\infty} x f_{X_n}(x)\mathrm{d}x = \int_{U^{lb}}^{U^{ub}} x\mu_{\tilde{X}}(x) / \left[\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x\right]\mathrm{d}x \tag{23}$$

where $\mu_{X_{eq}}$ denotes the mean of X_{eq}.

In Eq. (23), the integral of $\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x$ is a constant, then:

$$E(X_{eq}) = \int_{U^{lb}}^{U^{ub}} x\mu_{\tilde{X}}(x) / \left[\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x\right]\mathrm{d}x = \frac{1}{\int_{U^{lb}}^{U^{ub}} \mu_{\tilde{X}}(x)\mathrm{d}x}\int_{U^{lb}}^{U^{ub}} x\mu_{\tilde{X}}(x)\mathrm{d}x \tag{24}$$

Therefore, the PDF of X_{eq} can be expressed by Eq. (25):

$$f_{X_{eq}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{X_{eq}}} e^{-\frac{\left(x-\mu_{X_{eq}}\right)^2}{2\sigma_{X_{eq}}^2}} \tag{25}$$

Based on entropy theory, fuzzy variables can be converted to equivalent normal random variables. Reliability-based design optimization under fuzzy and interval variables can be converted to the issues with normal random variables and interval variables. Then the critical point for reliability analysis and design optimization are solving the upper and lower bounds of failure probability.

3. Reliability-Based Design Optimization Based on The Equivalent Conversion Method

3.1. The Worst Case Analysis under Fuzzy and Interval Variables

When the reliability constraints contain both random variables and interval variables, the equivalent conversion method abovementioned is applied to convert the fuzzy variables to the normal random variables firstly. And the combination of interval variables in the worst case is obtained. Finally, the double-loops method and SORA are applied to reliability-based design optimization, respectively, for the worst case.

The reliability-based design optimization model under mixed fuzzy variables and interval variables in the worst case can be expressed as:

$$\begin{aligned} &\min_{\mathbf{d}} h\left(\mathbf{d}, \tilde{\mathbf{X}}, \bar{\mathbf{Y}}\right) \\ &s.t.\quad \Pr\left\{g_i\left(\mathbf{d}, \tilde{\mathbf{x}}_{worst}, \mathbf{y}_{worst}\right) \tilde{\geq} 0\right\} \tilde{\geq} R_i,\ i = 1, 2, \cdots, m \end{aligned} \tag{26}$$

where $\tilde{\mathbf{X}}$ and $\bar{\mathbf{Y}}$ denote the vectors of fuzzy random variables and interval variables, respectively, $\tilde{\mathbf{x}}_{worst}$ is the combination of fuzzy random variables in the worst case, and $\mathbf{y}_{worst}$ is the combination of interval variables in the worst case.

The above-mentioned model contains fuzzy variables, and the equivalent conversion the fuzzy variables is used. Lei and Chen [13] calculated the fuzzy entropy of three different membership functions as follows.

a) The triangle distribution

The membership function is:

$$\mu(x) = \begin{cases} \dfrac{x - a_1}{a - a_1}, & a_1 \le x \le a \\ \dfrac{a_2 - x}{a_2 - a}, & a < x \le a_2 \end{cases} \tag{27}$$

The fuzzy entropy satisfies:

$$G'(x) = 0.5 - \ln\frac{2}{a_2 - a_1} \tag{28}$$

b) The trapezium distribution

The membership function is:

$$\mu(x) = \begin{cases} \dfrac{a_2 + x - a}{a_2 - a_1}, & a - a_2 \le x \le a - a_1 \\ 1, & a - a_1 < x \le a + a_1 \\ \dfrac{a_2 - x + a}{a_2 - a_1}, & a + a_1 < x \le a + a_2 \end{cases} \tag{29}$$

The fuzzy entropy satisfies:

$$G'(x) = \frac{a_2 - a_1}{2\left(a_2 + a_1\right)} - \ln\frac{1}{a_1 + a_2} \tag{30}$$

c) The Γ distribution

The membership function is:

$$\mu(x) = \begin{cases} e^{k(x-a)}, & a - a_1 \le x \le a \\ e^{-k(x-a)}, & a < x \le a + a_1 \end{cases} \tag{31}$$

The fuzzy entropy satisfies:

$$G'(x) = \frac{k}{2\left(e^{ka_1} + e^{-ka_1} - 2\right)}\left(\ln\frac{k}{2\left(e^{ka_1} - 1\right)} - 1\right) \tag{32}$$

When the membership function of the fuzzy variable is symmetric, the mean of the equivalent normal variable can be regarded as the symmetric center of the membership function. If the membership function of the fuzzy variable is asymmetric and included in the above-mentioned three types, the standard deviation of the equivalent normal random variable can be calculated with the corresponding fuzzy entropy and Eq. (5), then the mean of the equivalent normal random variable is solved through Eq. (24) and the corresponding membership function. Otherwise, the standard deviation of the equivalent normal random variable can be calculated with the corresponding membership function, Eqs. (3) and (5). And the mean of the equivalent normal random variable can be obtained with the membership function and Eq. (24).

Suppose that a vector $\tilde{\mathbf{X}} = \left(\tilde{X}_1, \tilde{X}_2, \cdots, \tilde{X}_n\right)$ of fuzzy variables and the equivalent vector after conversion is $\mathbf{X}^{eq} = \left(X_1^{eq}, X_2^{eq}, \cdots, X_n^{eq}\right)$, then the model described in Eq. (26) can be converted to:

$$\begin{aligned} &\min_{\mathbf{d}} h\left(\mathbf{d}, \mathbf{X}^{eq}, \mathbf{Y}\right) \\ &s.t.\quad \Pr\left\{g_i\left(\mathbf{d}, \mathbf{x}_{worst}^{eq}, \mathbf{y}_{worst}\right) \ge 0\right\} \ge R_i,\ i = 1, 2, \cdots, m \end{aligned} \tag{33}$$

where $\mathbf{X}^{eq}$ is the vector of equivalent normal random variables, $\mathbf{x}_{worst}^{eq}$ is the combination of equivalent normal random variables in the worst case.

From the equivalent conversion, the reliability based design optimization under fuzzy and interval variables is transformed into the design optimization under normal random variables and interval variables. The optimal solutions of the reliability–based design optimization can be solved quickly combined with the SORA method.

3.2. Entropy-Based Sequential Optimization and Reliability Assessment(E-SORA)

The basic idea of SORA is decomposing the original reliability-based design optimization into a series of independent deterministic optimizations and reliability analyses. During the overall process, the number of design optimization equals to reliability analyses. Therefore, the number of function evaluations is greatly reduced comparing with the double-loops method [15]. Therefore, SORA is further developed based on entropy theory, namely, E-SORA, which is based on the equivalent conversion method proposed in section 2. The E-SORA method is an extension of SORA, and it can be used for system with the mixture of fuzzy and interval variables. It can be divided into two parts: one is the equivalent conversion based on the entropy theory, the other is the reliability-based design optimization with SORA under the worst case. The flow chart of the proposed E-SORA is shown in Fig. 1.

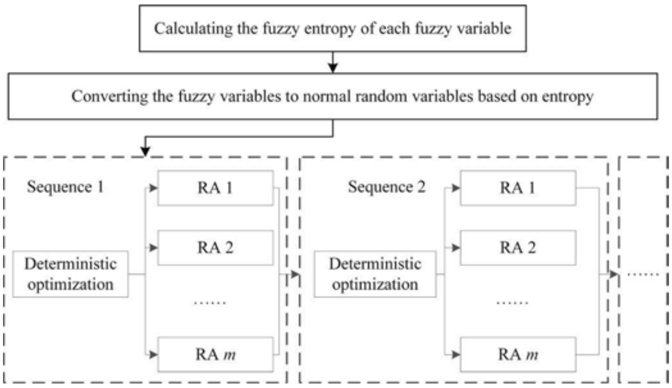

Fig. 1. The flow chart of E-SORA

When the deterministic optimization in Sequence 1 is performed, the combination of interval variables in the worst case and the most probable point (MPP) are unknown. For convenience, they can be set as the means of interval and random variables, respectively. Firstly, the optimal design $\mathbf{d}^1$ can be obtained through the deterministic optimization in Sequence 1, and then reliability analysis is performed to find the MPP $\mathbf{u}_{worst,i}^{MPP,R_i,1}$ in the worst case and the combination $\mathbf{y}_{worst,i}^{1}$ of interval variables. At this point, some reliability constrains may not be satisfied, such as $z_{worst,i}^{R_i} = g_i\left(\mathbf{d},\mathbf{u}_{worst,i}^{MPP,R_i,1},\mathbf{y}_{worst,i}^{1}\right) \geq 0$. Therefore, $\mathbf{u}_{worst,i}^{MPP,R_i,1}$ and $\mathbf{y}_{worst,i}^{1}$ are set as the new initial points, which are applied to build the deterministic design optimization in Sequence 2. The new model can be denoted as:

$$\begin{aligned} &\min_{\mathbf{d}} h\left(\mathbf{d},\mu_x,\bar{\mathbf{Y}}\right) \\ &s.t.\quad z_{worst,i}^{R_i} = g_i\left(\mathbf{d},\mathbf{u}_{worst,i}^{MPP,R_i,1},\mathbf{y}_{worst,i}^{1}\right) \geq 0,\ i=1,2,\cdots,m \end{aligned}, \quad (34)$$

Assuming that the reliability requirements in Sequence 1 are not satisfied, then the design variables are modified to improve the reliability during the deterministic optimization process of Sequence 2. If the optimal solutions $\mathbf{d}^2$ obtained through the deterministic optimization of Sequence 2 do not satisfy some reliability constraints, then a new optimization model in Sequence 3 is built by combining with the MPP $\mathbf{u}_{worst,i}^{MPP,R_i,2}$ and the combination $\mathbf{y}_{worst,i}^{2}$ of interval variables under the worst case. Repeating above processes until all reliability constraints and stopping criterions are satisfied.

4. Numerical Example

The reliability based design optimization of the crank link mechanism of an internal combustion engine is used to illustrate the proposed method. The structure of the crank link mechanism is shown in Fig. 2.

Simplifying the crank-link mechanism in Fig. 2 and a kinematic scheme of mechanism as shown in Fig. 3 is obtained.

Due to the uncertainty in manufacturing and assembling, the lengths of O_1O_2 and O_2O_3 are treated as random variables l_1 and l_2, respectively. The working process of an internal combustion engine contains different operating conditions such as startup, acceleration, turn and brake. The thrust of the piston varies with different operating conditions and the thrust is also uncertain. Therefore, the thrust $\tilde{F}$ is regarded as a fuzzy variable. The elastic modulus is defined as the ratio of the stress to the strain in an ideal material with small deformation. In practical engineering, the materials are often not ideal with the uncertainty in the manufacturing and the uncertainty

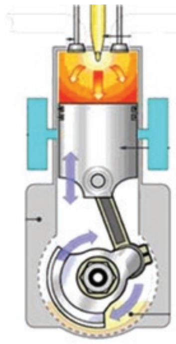

Fig. 2. The crank-link mechanism of an internal combustion engine

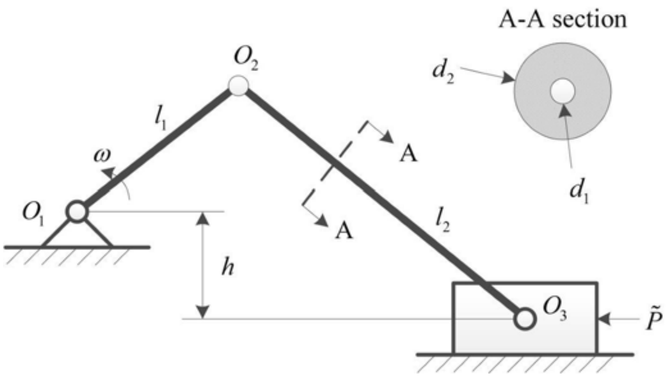

Fig. 3. The kinematic scheme of mechanism of an internal combustion engine

of the material properties, the elastic modulus $\tilde{E}$ is a fuzzy variable. The yield strength fluctuates near a fixed value due to the uncertainty of testing error, measurement error and personnel factor during the measurement process, thus it is denoted as a fuzzy variable $\tilde{\sigma}_s$. The distribution types and parameters of the random variables l_1 and l_2 are listed in Tab. 1.

Table 1. The distribution types and parameters of the random variables

Symbols	Random variables	Distribution types	Means	Standard deviations
x_1	l_1	Normal	100mm	1mm
x_2	l_2	Normal	300mm	3mm

The membership function of the thrust $\tilde{F}$ of the piston follows a triangular distribution, and the expression can be described by Eq. (35), the function graph is shown in Fig. 4.

$$\mu_{\tilde{F}}(x) = \begin{cases} \dfrac{x-200}{50}, & 200 \leq x \leq 250 \\ \dfrac{300-x}{50}, & 250 < x \leq 300 \end{cases}, \quad (35)$$

The membership function of the elastic modulus $\tilde{E}$ follow triangular distribution, and the expression can be described by Eq. (36), the function graph is shown in Fig. 5.

$$\mu_{\tilde{E}}(x) = \begin{cases} \dfrac{x-190}{10}, & 190 \leq x \leq 200 \\ \dfrac{210-x}{10}, & 200 < x \leq 210 \end{cases}, \quad (36)$$

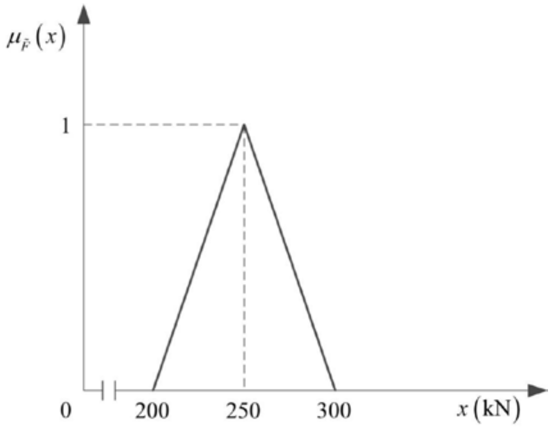

Fig. 4. The membership function of the thrust $\tilde F$

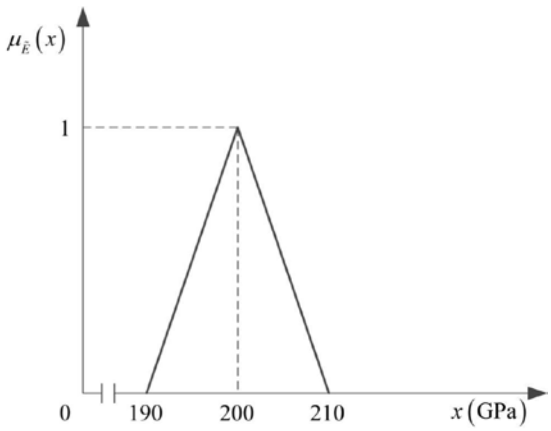

Fig. 5. The membership function of the elastic modulus $\tilde E$

The membership function of the yield strength $\tilde\sigma_s$ is also a triangular distribution, and the expression can be described by Eq. (37), the function graph is shown in Fig. 6.

$$\mu_{\tilde\sigma_s}(x)=\begin{cases}\frac{x-275}{5}, & 275\le x\le 280\\ 1, & 280<x<300\\ \frac{305-x}{5}, & 300<x\le 305\end{cases}, \quad (37)$$

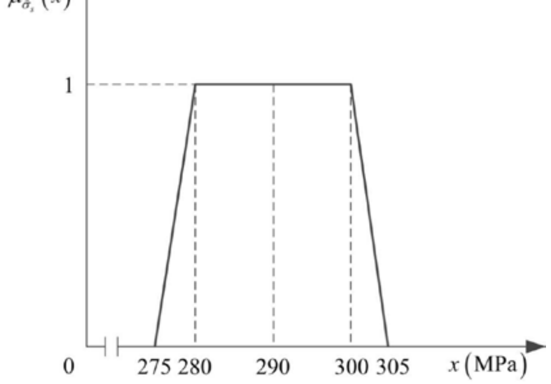

Fig. 6. The membership function of the yield strength $\tilde\sigma_s$

Table 2. The value ranges of interval variables

Symbols	Interval variables	The lower bounds	The upper bounds
y_1	h	100mm	150mm
y_2	μ_k	0.15	0.25

Owing to the technological uncertainty, manufacturing errors and other uncertainty factors, the probability density function of the dynamic friction coefficient μ_k is unknown, except for the approximate value range. To satisfy diverse vehicle models or meet different working requirements, the offset h is changeable, and its range is known. Therefore, h and μ_k can be set as interval variables and the value ranges are listed in Tab. 2.

The optimization is to determine the inner and outer diameters $d_1\left(1\le d_1\le 80\right)$ and $d_2\left(10\le d_2\le 100\right)$ of the connecting rods, which make the total weight of the crank and the connecting rods lightest under the constraints that the contact and the bending reliabilities are 0.9999. This problem is equivalent to find d_1 and d_2 which minimize the cross sectional area under the required reliability. The cross sectional area of the connecting rods can be expressed as:

$$S(\mathbf{d})=\frac{\pi}{4}\left(d_2^2-d_1^2\right). \quad (38)$$

The strength constraint and bending stress constraint of the connecting rod can respectively be expressed as:

$$\tilde g_1(\mathbf{d},\mathbf{X},\mathbf{Y})=\tilde\sigma_s-\frac{4\tilde F\left(l_2-l_1\right)}{\pi\left(\sqrt{\left(l_2-l_1\right)^2-h^2}-\mu_k h\right)\left(d_2^2-d_1^2\right)}\tilde\geq 0, \quad (39)$$

and:

$$\tilde g_2(\mathbf{d},\mathbf{X},\mathbf{Y})=\frac{\pi^3\tilde E\left(d_2^4-d_1^4\right)}{64l_2^2}-\frac{\tilde F\left(l_2-l_1\right)}{\sqrt{\left(l_2-l_1\right)^2-h^2}-\mu_k h}\tilde\geq 0, \quad (40)$$

where $\mathbf{X}=\left(x_1,x_2\right)=\left(l_1,l_2\right)$ and $\mathbf{Y}=\left(y_1,y_2\right)=\left(h,\mu_k\right)$.

The traditional deterministic optimization model can be denoted by:

$$\begin{aligned}&\min_{\mathbf{d}} S(\mathbf{d})=\frac{\pi}{4}\left(d_2^2-d_1^2\right)\\ &s.t.\ \tilde g_1(\mathbf{d},\mathbf{X},\mathbf{Y})=\tilde\sigma_s-\frac{4\tilde F\left(l_2-l_1\right)}{\pi\left(\sqrt{\left(l_2-l_1\right)^2-h^2}-\mu_k h\right)\left(d_2^2-d_1^2\right)}\tilde\geq 0;\\ &\tilde g_2(\mathbf{d},\mathbf{X},\mathbf{Y})=\frac{\pi^3\tilde E\left(d_2^4-d_1^4\right)}{64l_2^2}-\frac{\tilde F\left(l_2-l_1\right)}{\sqrt{\left(l_2-l_1\right)^2-h^2}-\mu_k h}\tilde\geq 0;\\ &1\le d_1\le 80, 10\le d_2\le 100, d_1<d_2\end{aligned}. \quad (41)$$

Since the reliabilities of tension and bending are 0.9999, then the reliability based design optimization model is given by:

Table 3. The solutions for different optimal methods

	Deterministic optimization	E-SORA	Double-loops	α- level cut
The optimal variables $\mathbf{d}=(d_1,d_2)$	(20.30,40.61)	(27.58,55.99)	(27.58,55.99)	(27.52,55.85)
The objective function $S(\mathbf{d})$	971.60	1864.71	1864.71	1855.01
g_1 in the worst case	-	0	0	0.12
g_2 in the worst case	-	4.15	4.15	3.24
(y_1,y_2) in the worst case	-	(150,0.25)	(150,0.25)	(150,0.25)
The number of iterations	19	408	1032	1053

$$\begin{aligned}&\min_{\mathbf{d}} S(\mathbf{d})=\frac{\pi}{4}\left(d_2^2-d_1^2\right)\\&s.t.\ \Pr\left\{\tilde{g}_1(\mathbf{d},\mathbf{X},\mathbf{Y})=\tilde{\sigma}_s-\frac{4\tilde{F}(l_2-l_1)}{\pi\left(\sqrt{(l_2-l_1)^2-h^2}-\mu_k h\right)\left(d_2^2-d_1^2\right)}\gtrsim 0\right\}\gtrsim 0.9999;\\&\Pr\left\{\tilde{g}_2(\mathbf{d},\mathbf{X},\mathbf{Y})=\frac{\pi^3\tilde{E}\left(d_2^4-d_1^4\right)}{64l_2^2}-\frac{\tilde{F}(l_2-l_1)}{\sqrt{(l_2-l_1)^2-h^2}-\mu_k h}\gtrsim 0\right\}\gtrsim 0.9999;\\&1\le d_1\le 80, 10\le d_2\le 100, d_1<d_2\end{aligned} \tag{42}$$

In Eq. (42), the thrust $\tilde{F}$, the elastic modulus $\tilde{E}$, and the yield strength $\tilde{\sigma}_s$ are fuzzy variables. Therefore, this is a reliability-based design optimization under the mixture of fuzzy variables and interval variables. According to the equivalent conversion method described in Section 2, the fuzzy variables can be converted to normal random variables, and the original optimization model is changed to an optimization model under the normal random variables and interval variables.

Firstly, the thrust $\tilde{F}$, the elastic modulus $\tilde{E}$, and the yield strength $\tilde{\sigma}_s$ can be equivalently converted to normal random variables F^{eq}, E^{eq}, and σ_s^{eq}, with $N\left(250,19.95^2\right)$, $N\left(200,3.99^2\right)$, $N\left(290,6.69^2\right)$, respectively, where $N(\cdot)$ represents a normal distribution. After the equivalent conversion, the reliability-based design optimization model can be described by Eq. (43).

With the equivalent conversion, the optimal solutions can be obtained through the E-SORA and the double-loops method. The α-level cut method is an important approach to solve the problems under fuzzy variables with high accuracy [7]. Thus it is used for comparison in this paper, and the resluts with different methods can be seen in Tab. 3.

From Tab. 3, when the objective function reaches the minimum form the deterministic design optimization, the design no satisfies the reliability constrains. The E-SORA and the double-loops method have considered the reliability constrains. To verify the accuracy of the equivalent conversion method, the optimal solutions of the α- level cut method are used to compare with those of the E-SORA method. The reliability-based design optimization with both the double-loops method and the α- level cut method are repeated double loops process. The proposed E-SORA method decouples the original double loops to improve computational efficiency.

$$\begin{aligned}&\min_{\mathbf{d}} S(\mathbf{d})=\frac{\pi}{4}\left(d_2^2-d_1^2\right)\\&s.t.\ \Pr\left\{g_1(\mathbf{d},\mathbf{X},\mathbf{Y})=\sigma_s^{eq}-\frac{4F^{eq}(l_2-l_1)}{\pi\left(\sqrt{(l_2-l_1)^2-h^2}-\mu_k h\right)\left(d_2^2-d_1^2\right)}\ge 0\right\}\ge 0.9999;\\&\Pr\left\{g_2(\mathbf{d},\mathbf{X},\mathbf{Y})=\frac{\pi^3E^{eq}\left(d_2^4-d_1^4\right)}{64l_2^2}-\frac{F^{eq}(l_2-l_1)}{\sqrt{(l_2-l_1)^2-h^2}-\mu_k h}\ge 0\right\}\ge 0.9999;\\&1\le d_1\le 80, 10\le d_2\le 100, d_1<d_2\end{aligned} \tag{43}$$

5. Conclusions

Based on entropy theory, the fuzzy entropy of a fuzzy variable is defined, and the fuzzy variables can be equivalently converted to normal random variables. Then the reliability-based design optimization under the mixture of fuzzy variables and interval variables can be changed to a model with normal random variables and interval variables. Finally, by combining with the worst case analysis method, the SORA method are applied after conversion. The proposed entropy-based equivalent conversion method provides a new idea for the reliability-based design optimization under different types of uncertain variables. The results has shown that the optimal solutions based on the worst case analysis is conservative, and the proposed method can ensure the safety for systems with limited information.

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Huiying GAO
School of Automobile and Transportation
Xihua University
No. 999, Jinzhou Road, Jinniu Zone, Chengdu 610039, China

Xiaoqiang ZHANG
Institute of Electronic Engineering
China Academy of Engineering Physics
No. 64, Mianshan Road, Youxian Zone, Mianyang 621900, China

E-mails: gaohuiying@std.uestc.edu.cn
zhangxiaoqiang@std.uestc.edu.cn